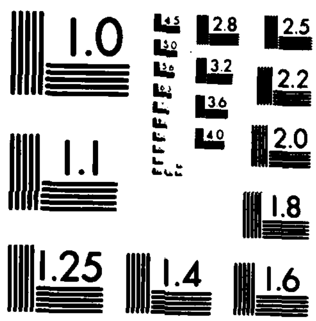
AD-A136 571 ESTIMATION AND RECONSTRUCTION FOR STOCHASTIC PROCESSES AND DETERMINISTIC..(U) JOHNS HOPKINS UNIV BALTIMORE MD DEPT OF MATHEMATICAL SCIENCES.. A F KARR 14 FEB 83 1/1

UNCLASSIFIED AFOSR-TR-83-1239 AFOSR-82-0029 F/G 12/1 NL

END
DATE
FILMED
1 84
DTIC



MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS-1963-A

AD A136571

(2)

UNCLASSIFIED
SECURITY CLASSIFICATION OF THIS PAGE

REPORT DOCUMENTATION PAGE

1a. REPORT SECURITY CLASSIFICATION: UNCLASSIFIED	1b. RESTRICTIVE MARKINGS
2a. SECURITY CLASSIFICATION AUTHORITY	3. DISTRIBUTION/AVAILABILITY OF REPORT: Approved for public release; distribution unlimited.
2b. DECLASSIFICATION/DOWNGRADING SCHEDULE	
4. PERFORMING ORGANIZATION REPORT NUMBER(S)	5. MONITORING ORGANIZATION REPORT NUMBER(S): AFOSR-TR- 83-1239
6a. NAME OF PERFORMING ORGANIZATION: Johns Hopkins University; 6b. OFFICE SYMBOL (If applicable)	7a. NAME OF MONITORING ORGANIZATION: Air Force Office of Scientific Research
6c. ADDRESS (City, State and ZIP Code): Department of Mathematical Sciences, Baltimore MD 21218	7b. ADDRESS (City, State and ZIP Code): Directorate of Mathematical and Information Sciences, Bolling AFB DC 20332
8a. NAME OF FUNDING/SPONSORING ORGANIZATION: AFOSR; 8b. OFFICE SYMBOL (If applicable): NM	9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER: AFOSR-82-0029
8c. ADDRESS (City, State and ZIP Code): Bolling AFB DC 20332	10. SOURCE OF FUNDING NOS.
11. TITLE (Include Security Classification): SEE REMARKS	

PROGRAM ELEMENT NO.	PROJECT NO.	TASK NO.	WORK UNIT NO.
61102F	2304	A5	

12. PERSONAL AUTHOR(S): Alan F. Karr

13a. TYPE OF REPORT	13b. TIME COVERED	14. DATE OF REPORT (Yr., Mo., Day)	15. PAGE COUNT
Interim	FROM 1/1/82 TO 31/12/82	14 FEB 83	11

16. SUPPLEMENTARY NOTATION

17. COSATI CODES			18. SUBJECT TERMS (Continue on reverse if necessary and identify by block number)
FIELD	GROUP	SUB. GR.	

19. ABSTRACT (Continue on reverse if necessary and identify by block number)

Methods for statistical estimation of parameters of partially observed stochastic processes and for minimum mean squared error reconstruction of unobserved portions of sample paths (state estimation) were developed. Some of these methods apply to models of random distributions of particles in space or events in time; others apply to Markov processes. Statistical estimators are asymptotically exact even though certain of the unknown "parameters" are infinite-dimensional. For several classes of processes the problem of simultaneously performing parameter estimation and state estimation was solved. Refined techniques for reconstructing a deterministic signal from hard-limited data were devised.

DTIC FILE COPY

20. DISTRIBUTION/AVAILABILITY OF ABSTRACT: UNCLASSIFIED/UNLIMITED ☒ SAME AS RPT ☐ DTIC USERS ☐	21. ABSTRACT SECURITY CLASSIFICATION: UNCLASSIFIED

22a. NAME OF RESPONSIBLE INDIVIDUAL	22b. TELEPHONE NUMBER (Include Area Code)	22c. OFFICE SYMBOL
Dr. Robert T. Smythe	(202) 767-5028	NM

DD FORM 1473, 83 APR — EDITION OF 1 JAN 73 IS OBSOLETE.

UNCLASSIFIED
SECURITY CLASSIFICATION OF THIS PAGE

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE

ITEM #11, TITLE: ESTIMATION AND RECONSTRUCTION FOR STOCHASTIC PROCESSES AND DETERMINISTIC FUNCTIONS.

Accession For		
NTIS GRA&I	☒	
DTIC TAB	☐	
Unannounced	☐	
Justification		
By		
Distribution/		
Availability Codes		
Dist	Avail and/or Special	
A-1		

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE

AFOSR-TR- 83-1239

ESTIMATION AND RECONSTRUCTION

FOR STOCHASTIC PROCESSES AND DETERMINISTIC FUNCTIONS

(Grant Number AFOSR 82-0029)

Scientific Report for the Period

1 January, 1982 through 31 December, 1982

Submitted to

AIR FORCE OFFICE OF SCIENTIFIC RESEARCH

DEPARTMENT OF THE AIR FORCE

by

Alan F. Karr

DEPARTMENT OF MATHEMATICAL SCIENCES

THE JOHNS HOPKINS UNIVERSITY

BALTIMORE, MD 21218

14 February, 1983

I. Statement of Research Objectives

The principal thrust of this research project is in the general areas of statistical inference for stochastic processes and reconstruction of stochastic processes and deterministic functions from only partial information or limited observations. In the context of stochastic processes, emphasis is on point processes, in particular Cox processes, and Markov (also semi-Markov) processes. For reconstruction of deterministic functions, the main objective is to provide explicit bounds on appropriate measures of error. In more detail, research objectives for the period 1 January, 1982-31 December, 1982, were as follows.

A) Inference and State Estimation for Derived Random Measures. As introduced by Karr (1978), a derived random measure $X*M$ is of the form

$$X*M(A) = \int_A X_t \; M(dt),$$

where M is a random measure on the underlying space E (a general LCCB space, possibly but not necessarily the real line $\mathbb{R}$) and X is a nonnegative random process. The state estimation problem for X and M is to estimate with minimum mean-squared error (MMSE) the components X and M given only observations of the derived random measure $X*M$. Of course, the estimators are then suitable conditional expectations; the key issues are to obtain explicit and recursive expressions for them as functions of the observations. Carrying out this program is particularly important for the following cases, which have application in tracking and signal processing.

1) *Poisson M; 0-1 semi-Markov X.* Initial study in Karr (1982) treated the case of 0-1 Markov X, but relied heavily on both the Markov nature of X and the property (assuming that X and M are independent) that $X*M$ is a

Cox process (doubly stochastic Poisson process). The latter continues to hold for 0-1 semi-Markov X, and solution of this case will significantly extend applicability of the model.

2) *Poisson M; general semi-Markov X.* Here even the Cox nature of $X*M$ fails to hold and entirely new techniques are necessary.

B) Semi-Markov-modulated Cox Processes on $\mathbb{R}_+$. Let N be a Cox process directed by the random measure

$$M(A) = \int_A X_t dt,$$

where X is a nonnegative semi-Markov process on $\mathbb{R}_+$. Objectives are to develop techniques for estimation of the probability law of X from single realizations of N (which is possible only if X has ergodic long-run behavior) and to devise methods for realization-by-realization reconstruction of X from observations of N. The Markov case has been studied and applied extensively (see, e.g., Snyder (1975)) but little is known about the semi-Markov case.

C) Point Process Sampling of Stochastic Processes. Given a stochastic process (X_t) and a point process N with points $T_1, T_2, \ldots$, suppose that X is observable only at the random times T_n, so that the data are the pairs $(T_n, X(T_n))$ or possibly only the observed values $X(T_n)$. Furthermore, while the structural relationship of N to X is assumed specified, N itself may involve unknown parameters. The two principal problems are to provide methods for inference (estimation and hypothesis testing) concerning the law of X or unknown parameters of X, together with unknown parameters of N, and to develop explicit and, especially, recursive techniques for MMSE reconstruction of unobserved portions of the sample paths of X. Finally there is the concatenated problem of carrying out state estimation procedures when parameters of X must themselves be estimated. Even the case of Markov X has been studied only very incompletely.

D) Reconstruction of Deterministic Functions. Let $f: [0,1] \to \mathbb{R}$ be an unknown, deterministic function satisfying $|f| \le B$ for a known constant B. Suppose that f is to be reconstructed nonparametrically from hard-limited data of the form $\{\mathrm{sgn}\{f(t_{jk}) + x_{jk}\}: 1 \le j \le J, 1 \le k \le K\}$, where $t_{jk} \in [0,1]$ are times at which f is sampled and $x_{jk} \in [-B,B]$ are additive "corruptions" deliberately imposed before hardlimiting the signal. Karr/Serfling (1983) place the problem in the context of numerical integration and obtain explicit error bounds, for example, on $\|\hat{f}-f\|_\infty$ and $\|\hat{f}-f\|_p$, $1 \le p < \infty$, for the step function estimator

$$\hat{f}(t) = \frac{B}{K} \sum_{j=1}^{K} \mathrm{sgn}\{f(t_{jk}) + x_{jk}\}, \qquad t \in I_j. \tag{1}$$

These bounds, for suitably chosen t_{jk} and x_{jk}, in turn yield rates of convergence as $n = JK \to \infty$ (n is the number of evaluations of f) that significantly improve those known for alternative estimators (and the latter are in addition much more laborious to calculate). Objectives are to extend and refine the techniques, and hence broaden their applicability, in the following directions.

1) *Inclusion of other transformations of signal plus "noise"* (other than the signum function); in particular, multi-level quantizers should be analyzed.

2) *Treatment of averaged observations.* In practice one would expect that rather than exact point observations $\mathrm{sgn}\{f(t) + x\}$, only averaged observations of the form

$$\frac{1}{2\varepsilon}\int_{-\varepsilon}^{\varepsilon} \mathrm{sgn}\{f(t_{jk}+t) + x_{jk}\}dt$$

can be obtained. Extension to this case is of apparent physical importance.

3) *Use of the Hausdorff metric.* This metric (cf. Sendov (1969)) lies in some sense between the uniform metric and the L^p metrics for $1 \leq p < \infty$. It is particularly applicable to elucidation of pointwise behavior of the estimation error for reconstruction of discontinuous functions.

II. Status of the Research

This discussion is parallel to that in the statement of research objectives.

A); B). Inference and State Estimation for Point Processes. Neither of the problem areas described in I.A and I.B above has yielded to direct attack, and it has been necessary instead to examine more general questions of inference and state estimation, mainly for Cox processes but also, in some cases, in more general contexts. This line of research stems from Karr (1983) and, after additional general development as discussed below, is expected to lead to solution of the problems posed in I.A and I.B. Specific accomplishments are as follows.

1) *Combined Inference and State Estimation for Mixed Poisson Processes.* A mixed Poisson process N is a Cox process with directing measure $M = \alpha m^*$, where α is a nonnegative random variable with distribution σ and m^* is a fixed measure on the underlying space E. With motivation in part from Bartoszyński/Brown/McBride/Thompson (1981), where the spread of cancer is modeled as a nonstationary Poisson process, techniques were developed for:

a) Nonparametric estimation of σ and m^* from independent, identically distributed (hereafter, i.i.d.) copies N_i of the Cox process.

The estimators - as stochastic processes - are uniformly strongly consistent and the process of estimation error is asymptotically Gaussian.

b) State estimation for the (n+1)st process N_{n+1} when it is only partially observed and σ, m^* are unknown, using a "pseudo-state" estimator formed by using data from the previous processes $N_1,\ldots,N_n$ to estimate relevant functionals of σ and m^*. The main results show that the difference between the true state estimator and the pseudo-state estimator converges to zero at rate $n^{-\frac{1}{2}}$ as n (the number of observed processes) converges to infinity, in the sense that $n^{-\frac{1}{2}}$ times the difference converges in distribution.

These results appear in

Karr, A.F., Combined nonparametric inference and state estimation for a class of Cox processes. Technical Report 354, Department of Mathematical Sciences, The Johns Hopkins University, 1982.

and have application in a variety of fields. The manuscript has been accepted tentatively for publication in *Z. Wahrscheinlichkeitstheorie und verw. Geb.*

2) *Inference for Thinned Point Processes.* Given a point process $N = \sum \varepsilon_{X_i}$ on E and a function $p: E \to [0,1]$, the p-thinning of N is the point process $N' = \sum U_i \varepsilon_{X_i}$, where the U_i are 0-1 random variables that are conditionally independent given N with $P\{U_i = 1|N\} = p(X_i)$. Many derived random measures and some semi-Markov-modulated Cox processes are thinned versions of an underlying process of interest. In the context of inference for thinned point processes the following problems have been solved.

a) Development of nonparametric estimators of the law of N and of the thinning function p given data comprising i.i.d. copies N_i' of the thinned process N'. Minimal knowledge of the law of N, namely the mean measure but nothing more, is necessary; the estimators are strongly consistent and asymptotically Gaussian (as stochastic processes with a rather complicated parameter set).

b) Development of consistent tests, given observation of i.i.d. pairs (N_i, N_i') of point processes satisfying $N_i' \leq N_i$ for each i, of the hypothesis that the N_i' are p-thinnings of the N_i for some unspecified p.

c) Explicit calculation of state estimators for the underlying process N from observation of the thinned process N', when N is a Cox process with known law and p is known. MMSE reconstruction of the directing measure M of N is also effected.

d) Characterization of asymptotic properties of pseudo-state estimators when the N_i are Cox processes with unknown law and p also is unknown.

These results are described in

> Karr, A.F., Inference for thinned point processes, with application to Cox processes. Technical Report 362, Department of Mathematical Sciences, The Johns Hopkins University, 1982,

which has been submitted for publication in the *Journal of Multivariate Analysis* and is currently under review.

3) *Estimation of Palm Distributions.* As demonstrated in Karr (1983) the Palm distributions of a Cox process play a fundamental role in state estimation. In order to provide more widely applicable methodologies for problems of combined inference and state estimation, it is necessary to derive techniques for estimation of the Palm distributions of a point

process or a random measure. (As discussed, e.g., in Kallenberg (1982), the Palm distributions of a point process N can be interpreted as conditional distributions of N given that atoms of N are located at prescribed points in E.) These techniques, based on i.i.d. samples M_i of a random measure M, yield strongly consistent and asymptotically Gaussian estimators, and are presented in

> Karr, A.F., Estimation of the Palm distributions of a random measure. Technical Report 372, Department of Mathematical Sciences, The Johns Hopkins University, 1982,

which has been submitted for publication in the *Annals of Probability*. The results in it are very general and have extensive potential for application.

Current research is concentrating on amalgamating and extending the concepts and techniques in Karr (1983) and Technical Reports 354 and 372 to develop techniques applicable to virtually all Cox processes. Significant progress has been made, and completion of a technical report by 31 March, 1983, is anticipated. Thereafter, the research will turn back to the more specialized problems pinpointed in the Statement of Research Objectives, as well as to related point process inference problems.

C) Point Process Sampling of Stochastic Processes. For binary (i.e., 0-1 valued) Markov processes, important progress has been made concerning several modes of sampling, resulting in the paper

> Karr, A.F., Estimation and reconstruction for zero-one Markov processes. Technical Report 355, Department of Mathematical Sciences, The Johns Hopkins University, 1982,

which has been accepted for publication in *Stochastic Processes and their Applications*. Sampling schemes include regular samples, regular samples with time jitter, Poisson samples, Poisson samples with state 0 unobservable, observability defined by an alternating renewal process, averaged samples, observations only of times of transitions into state 1, and point process observation of a random time change of the underlying Markov process. Estimators $\hat{a}(t)$, $\hat{b}(t)$ of the two transition intensities a and b, based on observation of a single realization of the process over the time interval $[0,t]$, are devised and demonstrated to be strongly consistent and jointly asymptotically normal. In many cases unknown parameters of the observation process must and can be estimated as well. Recursive representations of state estimators are obtained and the problem of state estimation using estimated parameters is solved in rather great generality.

D) Reconstruction of Deterministic Functions. Because of concentration of research effort on other problems described above, relatively less effort has been allocated to this area. The problem of signal reconstruction from averaged observations has been solved, with convergence rates established for an estimator analogous to that in (1), and partial results have been obtained for reconstruction of discontinuous functions using the Hausdorff metric to measure approximation error. Neither set of results has yet been written in the form of a technical report.

III. List of Written Publications

The following technical reports have been prepared under the auspices of this grant during the period 1 January, 1982-31 December, 1982.

[1] Karr, A.F., Combined nonparametric inference and state estimation for a class of Cox processes. Technical Report 354, Department of Mathematical Sciences, The Johns Hopkins University (tentatively, to appear in Z. *Wahrscheinlichkeitstheorie und verw. Geb.*)

[2] Karr, A.F., Estimation and reconstruction for zero-one Markov processes. Technical Report 355, Department of Mathematical Sciences, The Johns Hopkins University (to appear in *Stochastic Process. Appl.*).

[3] Karr, A.F., Inference for thinned point processes, with application to Cox processes. Technical Report 362, Department of Mathematical Sciences, The Johns Hopkins University (submitted to *J. Multivariate Anal.*).

[4] Karr, A.F., Estimation of the Palm distributions of a random measure. Technical Report 372, Department of Mathematical Sciences, The Johns Hopkins University (submitted to *Ann. Probability*).

The following manuscript is expected to be complete by 31 March, 1983.

[5] Karr, A.F., Simultaneous inference and state estimation for general Cox processes.

It will probably be submitted to *Stochastic Process. Appl.*

IV. Additional Information

A) Mr. David M. Zucker, Ph.D. candidate in the Department of Mathematical Sciences at Johns Hopkins, has participated as Research Assistant in various phases of the project. He is now engaged in dissertation research on a point process regression model that generalizes the model of Andersen/Gill (1982) to allow time-varying regression parameters. This work will be an important contribution to the "inference for point processes" theme of the project.

B) The following conference papers and seminars were presented that discussed various aspects of the project:

1) "Estimation and Reconstruction for Binary Markov Processes," Eleventh Conference on Stochastic Processes and their Applications, Clermont-

Ferrand, France, June, 1982.

2) "Nonparametric Inference and State Estimation for Cox Processes," Institute of Mathematical Statistics Annual Meeting, Cincinnati, OH, August, 1982 (invited).

3) "Estimation and Reconstruction for Binary Markov Processes," Operations Research Colloquium, Cornell University, Ithaca, NY, September, 1982.

4) "Inference for Thinned Point Processes," Mathematical Sciences Department Seminar, The Johns Hopkins University, Baltimore, MD, December, 1982.

References

[1] Andersen, P.K., and Gill, R.D. (1982). Cox's regression model for counting processes: a large sample study. *Ann. Statist.* 10 1100-1120.

[2] Bartoszyński, R., Brown, B.W., McBride, C., and Thompson, J.R. (1981). Some nonparametric techniques for estimating the intensity function of a cancer related nonstationary Poisson process. *Ann. Statist.* 9 1050-1060.

[3] Kallenberg, O. (1982). Conditioning in point processes. Technical report, Department of Mathematics, University of Göteborg.

[4] Karr, A.F. (1978). Derived random measures. *Stochastic Process. Appl.* 8 159-169.

[5] Karr, A.F. (1982). A partially observed Poisson process. *Stochastic Process. Appl.* 12 249-269.

[6] Karr, A.F. (1983). State estimation for Cox processes on general spaces. *Stochastic Process. Appl.* 14 (to appear).

[7] Karr, A.F., and Serfling, R.J. (1983). Error bounds for reconstruction of a function f from a finite sequence $\{\mathrm{sgn}(f(t_i) + x_i)\}$. *SIAM J. Appl. Math.* 43 (to appear).

[8] Sendov, B. (1969). Some questions of the theory of approximations of functions and sets in the Hausdorff metric. *Russian Math. Surveys* 24 143-183.

[9] Snyder, D.L. (1975). *Random Point Processes.* Wiley-Interscience, New York.

LMED

8