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STABILITY OF COMPRESSIBLE WAKE AND JET FLOWS

G.R. Verma S.J. Scherr W.L. Hankey Aerodynamics and Airframe Branch Aeromechanics Division

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FOREWORD

This report is the result of work carried on in Computational Aerodynamics Group, Flight Dynamics Laboratory, Wright-Patterson Air Force Base by Dr G.R. Verma, Dr W.L. Hankey and Mr S.J. Scherr, from June 1, 1982 to August 17, 1982. During this period, Dr Verma's work was supported by a grant from Air Force Office of Scientific Research (Grant #AFOSR 82-0130). Additional support was provided under project 2307N603. The authors would like to thank the Air Force Systems Command, Air Force Office of Scientific Research and Wright-Patterson Air Force Base for providing resources for the senior author to spend the summer of 1982 at WPAFB.



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SECTION I

INTRODUCTION

In References 1 and 2, the stability of the lower branch solution of the Falkner-Skan similar boundary layer equations was investigated. These velocity profiles possess <u>one inflection point</u> and give rise to the "Rayleigh Instability." The analysis of this instability proved extremely useful in interpreting self excited oscillation occurring in cavities, over spike tipped bodies, and in inlets (References 3 through 9).

Other classes of self-excited oscillations have been observed in jets (e.g., edge tones) and in the wakes of bluff bodies (e.g., periodic shedding of vortices behind cylinders). The velocity profile for this class of flows possesses two inflection points which give rise to two different modes of instability (References 10, 11). To assist in the interpretation of these observed instabilites it was felt useful to further investigate the stability features of compressible wake and jet profiles. For this reason, eigenvalue solutions for a series of typical profiles were computed for the following types,

(e)	Asymmetric wake	U = -	-U (of case d)	
			0	1.6 < y<∞
			1.7857(y-1.6) ²	1.25 <y<1.6< td=""></y<1.6<>
		U =	15 y ²	8 <y<1.25< td=""></y<1.25<>
			.23529(y+2.5) ²	-2.5 <y<8< td=""></y<8<>
(d)	Asymmetric jet		0	-∞ <y<-2.5< td=""></y<-2.5<>
(c) (Anti-symmetric Combined wake and jet)	U = 2	$\frac{3}{2}\sqrt{3}$ sech ² y tanh y	
(b)	Symmetric wake	U = -	-sech ² y	
(a)	Symmetric jet	U = 9	sech ² y	

The results of the stability analysis are compiled and cataloged to enable us to draw conclusions regarding the behavior of these flows.

Objective

The objective was to determine the amplification factor, disturbance propagation speed and wave number for typical velocity profiles with two or three inflection points at various Mach numbers. It was anticipated that some overall characteristics for wake/jet flows could be deduced from these series of calculations.

SECTION II

GOVERNING EQUATIONS

In this report, we study the stability of compressible wakes and jets in two dimensional flows. Let u represent the velocity component in the x direction and v the velocity component in the y direction. p, ρ and T are pressure, density and temperature respectively.

The basic equations are

$$\frac{1}{\rho} \left[\frac{\partial \rho}{\partial t} + u \frac{\partial \rho}{\partial x} + v \frac{\partial \rho}{\partial y} \right] + \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) = 0$$
(1)

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial x}$$
(2)

$$\frac{\partial \mathbf{v}}{\partial t} + \mathbf{u} \frac{\partial \mathbf{v}}{\partial x} + \mathbf{v} \frac{\partial \mathbf{v}}{\partial y} = -\frac{1}{\rho} \frac{\partial \rho}{\partial y}$$
(3)

$$\frac{\partial p}{\partial t} + u \frac{\partial p}{\partial x} + v \frac{\partial p}{\partial y} = \frac{\gamma p}{\rho} \left[\frac{\partial \rho}{\partial t} + u \frac{\partial \rho}{\partial x} + v \frac{\partial \rho}{\partial y} \right]$$
(4)

Eliminating ρ between Equations 1 and 4 we obtain

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$$\frac{\partial p}{\partial t} + u \frac{\partial p}{\partial x} + v \frac{\partial p}{\partial y} + \gamma p \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) = 0$$
(5)

Equations 2, 3 and 5 have a steady state solution

$$u = \overline{u}(y), v = 0, p = \overline{p} = constant$$
 (6)

we assume the time dependent perturbed flow as (References 12, 13)

$$u = \overline{u}(y) + u'(x,y,t)$$
(7)

$$v = v'(x,y,t)$$
 (8)

$$p = \overline{p} + p'(x,y,t)$$
(9)

Substituting these values of u, v and p in Equations 2, 3 and 5; and retaining only linear terms in u', v' and p' we obtain

$$\frac{\partial u}{\partial t} + \overline{u} \frac{\partial u}{\partial x} + v' \frac{\partial \overline{u}}{\partial y} + \frac{1}{\rho} \frac{\partial p}{\partial x} = 0$$
(10)

$$\frac{\partial v}{\partial t}' + \overline{u} \frac{\partial v}{\partial x}' + \frac{1}{\rho} \frac{\partial p}{\partial y}' = 0$$
(11)

$$\frac{\partial \mathbf{p}}{\partial t}' + \overline{\mathbf{u}} \frac{\partial \mathbf{p}}{\partial \mathbf{x}}' + \gamma \overline{\mathbf{p}} \left(\frac{\partial \mathbf{u}}{\partial \mathbf{x}}' + \frac{\partial \mathbf{v}}{\partial \mathbf{y}}' \right) = 0$$
(12)

We seek the periodic solutions of the form

$$u' = \hat{u}(y) e^{i\alpha(x-ct)}$$
(13)

$$\mathbf{v}' = \hat{\mathbf{v}}(\mathbf{v}) \, \mathbf{e}^{\mathbf{i}\alpha(\mathbf{x} - \mathbf{c}\mathbf{t})} \tag{14}$$

$$p' = \hat{p}(y) e^{i\alpha(x-ct)}$$
(15)

where \hat{u} , \hat{v} , \hat{p} are complex, c is a complex constant and α is a real constant.

Substituting Equations 13, 14 and 15 into Equations 10, 11 and 12 we obtain

$$i_{\alpha}(\overline{u} - c) \hat{u} + u_{y} \hat{v} = -i_{\alpha} \frac{\hat{p}}{\rho}$$
 (16)

$$i\alpha(\overline{u} - c) \hat{v} = -\frac{1}{\rho} \hat{p}_{y}$$
 (17)

$$i\alpha(\overline{u} - c)\hat{\rho} = -\gamma\overline{p}(i_{\alpha}\hat{u} + \hat{v}_{y})$$
 (18)

We eliminate \hat{p} and \hat{u} from the above equations, use the relation $\bar{p} = \bar{\rho} R \bar{T}$, and obtain

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$$\left[\frac{(\overline{u} - c) \hat{v}_{y} - \overline{u}_{y} \hat{v}}{\gamma R \overline{T} - (\overline{u} - c)^{2}}\right]_{y} = \frac{\alpha^{2}(\overline{u} - c) u}{\gamma R \overline{T}}$$
(19)

Now using

$$\gamma R \overline{T} = \frac{1 + .2 M_{\omega}^{2} (1 - \overline{u}^{2})}{M_{\omega}^{2}}$$
(20)

and doing some calculations we obtain

$$\frac{(\bar{u} - c) \bar{v}_{y} - \bar{u}_{y} \bar{v}}{(1 + .2M^{2}) - M_{0}^{2} (\bar{u} - c)^{2} y} = \alpha^{2} (1 + .2M^{2}) (\bar{u} - c) \hat{v}$$
(21)

where

$$M_0^2 = \frac{M_{\omega}^2}{1+.2 \ M_{\omega}^2}$$
(22)

and

$$M^{2} = \frac{M_{\infty}^{2} \quad \overline{u}^{2}}{1 + .2 \quad M_{\infty}^{2} (1 - \overline{u}^{2})}$$
(23)

If we write

 $(1 + .2M^2)^{-1} - M_0^2 (\overline{u} - c) = g$, and replace \hat{v} by ϕ and \overline{u} by U in Equation 21 we obtain

$$\frac{(U-c)\phi_y - U_y\phi}{g} = \alpha^2(1 + .2M^2) (U-c)\phi$$
(24)

For boundary conditions, we assume that for unbounded flows the initial disturbances die down far from the disturbances. Therefore we get

$$\phi(-\infty) = 0, \ \phi(\infty) = 0 \tag{25}$$

For fixed wave numbers (α = constant) Equations 24 and 25 are an eigenvalue problem. ϕ is eigenfunction and c is eigenvalue.

We solve this eigenvalue problem for the following velocity profiles

 $U(y) = \frac{3}{2} \sqrt{3} \operatorname{sech}^{2} y \operatorname{tanh} y, \operatorname{anti-symmetric} (\operatorname{combined wake and jet})$ (28)

$$U(y) = \begin{cases} 0 & -\infty < y < -2.5 \\ .23529(y+2.5)^2 & -2.5 < y < -.8 \\ 1 - .5y^2 & -.8 < y < 1.25 \\ 1.7857(y-1.6)^2 & 1.25 < y < 1.6 \\ 0 & 1.6 < y < \infty \\ asymmetric jet \end{cases}$$
 (29)

U(y) = -U(y) of Equation 29 asymmetric wake (30)

SECTION III

NUMERICAL PROCEDURE

Eigenvalues of ϕ were determined by a shooting method (Reference 1): starting with boundary conditions at y_{\min} , integrating over the range of y, and comparing the result with the outer boundary condition, namely $\phi = 0$ at y_{\max} . The process involved minimization of the error caused by the deviation. This was chosen to be the square of the norm of ϕ , $|\phi|^2 = \phi_r^2 + \phi_i^2$. The integration was done using a fourth-order Runge-Kutta method.

Boundary conditions at y_{min} were determined by observing the behavior of Equation 24 as $y \rightarrow -\infty$. The equation reduces to

$$\phi_{yy} = \alpha^2 \phi \tag{31}$$

Since we desire $\phi(-\infty) = 0$, we choose

$$\phi(y_{\min}) = e^{|\alpha|y_{\min}}, \quad \phi'(y_{\min}) = |\alpha|e^{|\alpha|y_{\min}} \quad (32)$$

as our boundary conditions.

The method of finding eigenvalues utilized the same minimization routine as in previous investigations (References 1, 2). The user provides a starting guess, for c in this case, and the routine begins by searching along a constant line of c_i with increasing steps until the error begins to increase. It then uses the last three calculated values to determine a parabola, with the c_r value at the vertex used as the new approximation. Then this value of c_r is held constant and a search along a line of changing c_i is carried out. After a new relative minimum is found, the quadratic approximation is used to determine a new value for c_i . The third step involves searching the line connecting the original guess and the new point in the same manner. If the error is not less than a preset limit, here 10^{-6} , the routine starts again with the latest value used in place of the original guess.

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SECTION IV

RESULTS

The eigenvalue problem represented by Equations 24 and 25 was solved numerically for the velocity profiles given by Equations 26 through 30. The results are tabulated for a wide range of wave numbers (α) and Mach numbers (M_{ω}). The instability characteristics for a symmetric jet, asymmetric jet, and anti-symmetric jet are given in Tables 1a through 3h. For $M_{\omega} = 0$ these values agree with those given in References 10 and 11.

The velocity profiles are plotted in Figures 1 through 3. The values of α , versus c_i , α versus c_r , and c_i versus c_r are plotted in Figures 4 through 19. ϕ , \hat{u} and \hat{p} are plotted for some special values of M_{∞} , α and c in Figures 20a through 21c, and magnitudes and phases of ϕ , \hat{u} and \hat{p} are plotted in Figures 22a through 22c, and 28a through 28c.

Solutions were obtained with convergence error criteria of at least 10^{-6} for all cases.

SECTION V

SUMMARY

The stability of compressible inviscid jets and wakes has been investigated by utilizing the linearized equations resulting from a small perturbation analyis. The resulting eigenvalue problems were solved numerically for various wave numbers (Reference 2) and Mach numbers (M_{∞}) for different velocity profiles. In the cases of symmetric jets and wakes and asymmetric jets and wakes we found two propagation modes corresponding to two inflection points: the sinuous mode for even eigenfunctions and varicose mode for odd eigenfunctions.

In varicose modes, the magnitude of amplification decreased as Mach number (M_{∞}) increased and the flow became completely stable at $M_{\infty} = 2$. In sinuous modes the amplification did decrease a little with the increase of Mach number but we did not find any upper limit in Mach number above which the flow was completely stable.

In the case of anti-symmetric profile there are three modes corresponding to the three inflection points: two propagating modes, one propagating to the right and the other propagating to the left; and one standing mode. The magnitude of amplification for propagating modes decreased as the Mach number increased, and completely died down at Mach number of 1.5. On the other hand, we could not find an upper limit of Mach number for the standing mode above which the flow was completely stable. The authors believe that these results will be useful for analyzing aerodynamic instabilities encountered in wakes and jets.

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INSTABILITY CHARACTERISTICS FOR THE SYMMETRIC JET $\mathbf{u} = \mathbf{sech}^T \mathbf{y}$

TABLE 1.

1a) $M_{\infty} = 0.0$, SINUOUS MODE

α	°r	° _i
.1	.061256	.119380
.2	.137549	.205118
.3	.207237	.241188
.4	.266554	.249623
.5	.316088	.244302
. 6	.357248	.231763
.7	.392290	.215421
. 8	.422860	.197142
. 9	.450120	.177992
1.0	.474924	.158612
1.1	.497882	.139405
1.2	.519408	.120635
1.3	.539851	.102479
1.4	.559444	.085063
1.5	. 578370	.065476
1.6	.596770	.052789
1.7	.614738	.038056
1.8	.632352	.024322
1.9	.649655	.011026
2.0	.666667	.282007(10) ⁻¹¹

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16)	M	1.0.	SINUOUS	MODE
107	1.44	1.0,	***********	1.1.1.1.1.1

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a	°r	° _i
.1	.067994	.128268
.2	.156481	.216384
.3	.233560	.245 433
.4	.296861	. 246748
.5	.346972	.236112
. 6	.387995	.219914
.7	.422773	.201173
.8	.453158	.181434
.9	. 480444	.161569
1.0	.505465	.142098
1.1	.528781	.123328
1.2	.550770	.105484
1.3	.571686	.088689
1.4	.591693	.073020
1.5	.610908	.058512
1.6	.629391	.045172
1.7	.647 185	.032987
1.8	.664327	.021922
1.9	.680823	.011935
2.0	.696684	.022972

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1c) $M_{\infty} \approx 2.0$, SINUOUS MODE			
α	° r	° i	
.1	.086251	.149243	
.2	. 20367 5	.235800	
.3	.293797	.247300	
.4	.359668	.234900	
.5	.410110	.214300	
.6	.451142	.191700	
.7	.486228	.169100	
.8	.517361	.147500	
.9	.545685	.127500	
1.0	.571836	.109323	
1.1	.596155	.093041	
1.2	.618823	.078622	
1.3	.639958	.065944	
1.4	.659655	.054849	
1.5	.678011	.045186	
1.6	.695111	.036856	
1.7	.710884	.029902	
1.8	.724660	.023967	
1.9	.737094	.017381	
2.0	.749815	.011191	
2.1	.761940	.006051	

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1 .

	1d) M _m =3.0, SINUOUS MODE	
	°,	° i
α	•	
.1	. 112176	.172854
.2	.259379	.247000
.3	.355744	.237003
.4	.422588	.214158
.5	.474158	.136233
.6	.517294	.159762
.7	.555257	.136500
.8	.589379	.117193
.9	.619734	.102317
1.0	.645257	.091204
1.1	.665674	.031286
1.2	.683340	.070539
1.3	.700638	.059729
1.4	.717718	.050427
1.5	.733757	.042849
1.6	.748628	.036578
1.7	.762489	.031398
1.8	.775722	.027867
1.9	.785647	.027098
2.0	.791696	.022900

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1e) M_{ω} ~4.0, SINUOUS MODE			
α	^c r	° _i	
.1	.143092	.193015	
.2	.311005	.249100	
.3	. 409095	.225500	
.4	.477423	.191509	
.5	.532312	.159900	
.6	.579928	.135133	
.7	.620580	.119050	
.8	.650447	.109428	
.9	. 671343	.098700	
1.0	.689858	.035042	
1.1	.709335	.071677	
1.2	.728751	.061123	
1.3	.746833	.053174	
1.4	.763496	.047336	
1.5	.778596	.044463	
1.6	.788371	.043617	
1.7	.795081	.038781	
1.8	. 804006	.032273	
1.9	.813824	.027343	
2.0	.823122	.023604	

-

1f) M _∞ =0.0, VARICOSE MODE			
α	° _r	°,	
.05	.862061	.030296	
.îO	.867554	.108700	
.20	.796327	.121800	
.30	.759672	.114815	
. 40	.733435	.102812	
.50	.713113	.088200	
.60	.697180	.071556	
.70	.684964	.053968	
.80	.676071	.035862	
.90	. 670036	.017700	
1.00	.666667	.000000	

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1g) M _{cc} ⁻¹ .0, VARICOSE MODE			
()L	°r	° ₁	
.050	.814895	011590	
.100	.834826	151395	
.200	.765617	.075559	
.300	.737390	056629	
. 400	.721227	030567	
.500	.710643	.022751	
.525	.708624	019570	
.550	.706811	.016559	
. 575	.705210	.010064	
.600	.703712	005774	
.625	.702614	•000774 001594	
.650	.701492	.001584	

-

1h) $M_{\infty}=2.0$, VARICOSE MODE

α	°r	°i
.54	•988650	.000784
.55	.980628	.001722
.56	.973019	.002772
.57	.965803	.003849
.58	.958940	004919
.59	.952416	.005956
.60	.946200	006928
.61	.940272	007830
.62	.934607	.008647
.63	.929187	009374
.64	. 923988	010011
.65	.918995	.010557
.66	. 914191	.011011
. 67	.909559	.011378
.68	. 905084	.011660

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INSTABILITY CHARACTERISTICS FOR

THE ANTISYMMETRIC JET $U = \frac{3}{2}\sqrt{3} \operatorname{sech}^2 y$ tanh y

TABLE 2.

2a) M_{ω} 0.0, PROPAGATING MODE

(X	° _r	° _i
. 05	.920689	.090656
.10	.879349	.116531
. 20	.820211	.136200
. 30	.771224	.138417
.40	.730500	.130034
. 50	.699088	.115459
.60	.675954	.098057
.70	.659475	.079866
. 80	.648131	.061943
. 90	.640730	.044786
1.00	.636391	.028606
1.10	.634469	.013483

2b) M_w=1.0, PROPAGATING MODE

α	° r	^c i
. 10	.824508	.095961
.15	.790995	.094567
. 20	.766441	.090119
. 25	.746227	.083887
. 30	.728650	.075980
. 35	.713325	.066800
. 40	. 700178	.056500
.45	.689115	.045423
. 50	.680052	.034040
. 60	.667009	.011367
.65	.662642	.000350

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2c) $M_{\infty} = 1.2$, PROPAGATING MODE

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a	°r	° _i
.10	.808530	.007919
.15	.768575	.067700
.20	.746.123	.060019
.25	.728869	.051536
. 30	.713971	.041967
. 35	.701157	.031200
.40	.690315	.019372
.45	.681388	.006778
.47	.678352	.001596

2d) M_∞≈0, STANDING MODE

· a	° r	^c i
.1	0.0	.231871
.2	0.0	.351572
.3	0.0	.421749
.4	0.0	.467320
.5	0.0	.495630
.6	0.0	.509976
.7	0.0	.512729
.8	0.0	.506042
.9	0.0	.491820
1.0	0.0	.471650
1.1	0.0	. 446790
1.2	0.0	.418208
1.3	0.0	. 386635
1.4	0.0	. 352607
1.5	0.0	.316510
1.6	0.0	.278608
1.7	0.0	.239067
1.8	0.0	. 197972
1.9	0.0	.155345
2.0	0.0	.111149
2.1	0.0	.065246

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	$2e$) $M_{\odot} = 1.0$, STANDING SUDE	
α	°r	c _i
.15	0.0	. 325303
.20	0.0	.376416
. 30	0.0	.434305
.40	0.0	.454416
. 50	0.0	.450267
.60	0.0	.429563
.70	0.0	. 396980
.80	0.0	.355153
.90	0.0	. 305120
1.00	0.0	.246296
1.10	0.0	.175520
1.20	0.0	.081841
1.23	0.0	.043534
	2f) $M_{\infty} = 1.4$, STANDING MODE	
0	°,	°,
(L	L L	1
. 1	0.0	.267995
.2	0.0	. 395734
.3	0.0	.441200
.4	0.0	.441578
.5	0.0	.414618
.6	0.0	.368 -37
.7	0.0	. 306499
.8	0.0	.224023
.9	0.0	.093603
	2g) M_{∞} =2.0, STANDING MODE	

2e) M =1.0, STANDING MODE

α	^c r	c _i
.15	0.0	. 381035
.20	0.0	.425659
. 30	0.0	.444403
. 40	0.0	.409147
.50	0.0	.339340
.60	0.0	.229747
.65	0.0	.137767

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TABLE 3.			
3a) $M_{\infty}=0.0$, SINUOUS MODE			
α	с _г	^c i	
.1	.022716	.034685	
.2	.057622	.075168	
.3	.096191	.106869	
. 4	. 1 34 36 7	.126540	
.5	. 169211	.135130	
.6	.199485	.134785	
.7	.223771	.127902	
.8	. 241919	.116944	
.9	.254235	.104212	
1.0	. 261494	.091611	
1.1	.265033	.080302	
1.2	.266183	.070682	
1.3	. 265879	.062672	
1.4	. 264760	.056016	
1.5	. 263194	.050437	
1.6	. 261 399	.045706	
1.7	.259499	.041642	
1.8	.257556	.038108	
1.9	.255643	.035004	
2.0	.253755	.032250	
2.5	.245207	.022082	
3.0	.238341	.015553	
3.5	. 233025	.011077	
4.0	.229011	.007910	

INSTABILITY CHARACTERISTICS FOR THE ASYMMETRIC JET

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α	° r	^c i
1	025250	038126
2	066401	083135
.3	.114810	.116629
. 4	.163526	.132723
.5	. 206953	.132538
.6	. 240919	.119500
7	263146	098537
.8	271909	076103
.9	271495	059204
• 2	.2/1499	:039204
1.0	. 268 354	.048282
1.1	.264800	.041100
1.2	.261773	.036078
1.3	.259106	.032331
1.4	.256705	.029392
1.5	.254633	.026992
1.6	252606	02/0//
1.0	250012	.024900
1.8	2/9280	.023213
	.249200	.021664
1.9	.247718	.020275
2.0	.246241	.019014
2.5	.239841	.014013
3.0	234696	010404
3.5	230608	.010404
4.0	.230000	005654
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3b) $M_{\infty} = 1.0$, SINUOUS MODE

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a	°r	°i
. 2	.092437	. 102199
.3	.169565	.130870
.4	.242842	123709
.5	.315747	081390
.6	• 405591	.064530
.7	.470175	050658
.8	.525611	.050058
.9	. 571746	.036654
1.0		
1.0	.607611	.035926
1.1	.633812	.035146
1.2	.652273	.035042
1.3	.665219	031003
1.4	.674078	027/58
1.5	.679530	.022257
1.6	682270	017241
1.7	683250	.01/361
1.8	683357	.013417
	166001	.010509
1.9	.683125	.008409
2.0	.682793	.006866
2.5	.681431	.003002

3c) $M_{\infty}=2.0$, SINUOUS MODE

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a	°r	° i
.1	.041923	. 058484
• 2	.134351	.120942
.3	.250400	.125258
.4	. 364 384	.112313
• 5	.441116	.093284
.6	.508862	.070646
.7	. 568281	.057581
.8	.613491	.053023
.9	.645471	.049900
1.0	.668287	.045140
1.1	.684931	.037703
1.2	.696740	.026809
1.3	. 700800	010626
1.4	.692436	.002767
1.5	.688292	.001691
1.6	.686144	.001325
1.7	.684820	.001130
1.8	.683920	.001000
1.9	.683265	.000899
2.0	.682766	. DOGN 14
	3_{0} M = 4_{0} SINDOUS MODE	
α	c r	° _i
.1	.054213	.071056
• 2	. 185692	.129058
. 3	. 34 30 2 4	. 122561
.4	. 440501	.105400
.5	. 523520	.078015
• 6	.590604	.067213
7	636500	061840
. /	668800	.001040
.9	.692983	.046829
.,		101002)
1.0	.713555	.034536
1.1	.736208	.020525
1.2	.760259	.011891
1.3	. 780733	007100
1.4	.798010	.003777
1.5	.812881	.001170

3d) M_m=3.0, SINUOUS MODE

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3f) $M_{\infty}=0$, VARICOSE MODE		
α	^c r	° _i
.1	.791496	176117
• 2	. 694068	187159
.3	.634863	.173131
. 4	.596700	1526/2
•5	.572033	130739
• 6	.556473	.110405
.7	. 548758	092825
•8	. 548309	078848
.9	. 553977	.068563
1.0	.564245	061259
1.1	.577329	055839
1.2	. 591419	.051351
1.3	.605205	0/ 7001
1.4	.617916	.047201
1.5	.629175	.038880
1.6	.638852	034763
1.7	.646961	030638
1.0	.653605	.026674
1.9	.658945	000075
2.0	.663174	.022975
2.5	.673978	.008493
3.0	.677488	002760
3.5	.678856	·UU3/68
4.0	.679452	.000846

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3g) $M_{\infty} = 1.0$, VARICOSE MODE

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α	^c r	c _i
.000	.999458	.000934
.050	.823680	.124270
.100	.749102	.137267
. 200	.666039	.122735
. 300	.621290	.093539
. 400	.596116	.060028
. 500	.583795	.024059
. 525	.582989	.014683
. 550	.582512	.005117
. 575	. 583089	.602216(10) ⁻⁵

	$m_{\omega}=1.3$, $RICOSE MODE$	
a	^c r	° _i
.01 .05 .10	.888249 .793824 .727671	.045163 .096293 .103920
. 15 . 20 . 30	.685409 .656887 .628048	.095711 .080776 .041152
. 35 . 40	.628602 .634545	.016927

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α = .30 M_∞ = 1.0





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 ${}^{c}r = .737390$ ${}^{c}i = .056628$

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Solid lines M_w = 1.0 a = .30

Dotted lines M_∞ = 1.0

Figure 22a



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Figure 23b





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α = .60 = 1.0 Σ⁸ Figure 26a.

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Figure 26b

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c = .621290+.093539i .30 П ರ = 1.0 Σŝ Figure 27a.

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Figure 28a

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