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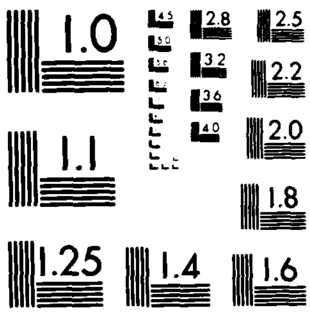

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ON RETRANSMISSION CONTROL POLICIES IN MULTIACCESS CHANNELS

Submitted to:

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COMMUNICATIONS SYSTEMS LABORATORY
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ITEM #11, TITLE: ON RETRANSMISSION CONTROL POLICIES IN MULTIACCESS CHANNELS.

On Retransmission Control Policies in Multiaccess Channels

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Aloha-type retransmission control policies have been proposed recently by Hajek and Van Loon [1] which can be implemented on a random access channel with ternary or binary feedback. They have shown that such schemes achieve a stable throughput of $e^{-1} = .3678$ for an infinite-population Poisson arrival model, by using simple first-order recursive retransmission policies. We derive measures of the speed of convergence and steady state accuracy for the "local" model of these policies. Using these measures, we compare two first-transmission policies, namely IFT and DFT. We extend the policies to cover the Success/Failure binary feedback case, which is not covered in [1]. Finally, we study the effects of channel errors on the performance of the random access system.

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1. The Retransmission policies of Hajek and Van Loon

The channel model is of the slotted packet-switched satellite broadcast type which is described in [2, sect. 5.11]. There is an infinite population of packet transmitting, bursty users. The cumulative input traffic is modeled as a homogeneous Poisson point process with intensity λ messages per slot. If each of two or more users transmits a packet during a slot, the packets "collide" and are not successfully broadcasted. Such packets join the backlog of packets which must be rebroadcasted at a later time. During each slot, each user possessing a backlogged packet must decide whether or not to transmit the packet in that slot. It is assumed that at the end of the slot $(t, t+1]$ (t is integer valued) each user learns the value of a common feedback random variable, Z_t.

The feedback informs the users whether slot $(t, t+1]$ was empty ($Z_t = 0$), or contained one packet ($Z_t = 1$), or contained ≥ 2 packets ($Z_t = e$). Let N_t denote the channel backlog at time t. Each user with a backlogged packet independently transmits the packet with the same probability f_t in the slot $(t, t+1]$. The sequence $\{f_t\}_{t \epsilon Z}$ must satisfy $0 \leq f_t \leq 1$ and f_t must be a function of the channel output history $(Z_s : s < t)$ for each t. The retransmission control policy examined in [1] is of the form

$$f_{t+1} = \min(a(Z_t)^{\gamma} f_t, \beta) \tag{1}$$

for some positive constants γ, β, $a(0)$, $a(1)$, $a(e)$. Next, it is specified how the new packets access the channel. Let Y_t be the number of packets which first arrive during the slot ending at time t. The random variables $\{Y_t\}_{t \varepsilon Z}$ are independent with Poisson distribution (λ). Two first-time transmission policies are considered, namely Immediate-First-Transmission (IFT) and Delayed-First-Transmission (DFT). Under the IFT policy, each of the Y_t packets which arrive during slot (t-1, t] are first transmitted in slot (t, t+1] with probability one. Under the DFT policy, new packets join the backlog before their first transmission is attempted, i.e. each of the Y_t packets is independently transmitted in slot (t, t+1] with probability f_t.

Let G_t be the total traffic intensity at time t and let S_t be the expected throughput during slot (t, t+1], given N_t and f_t. Thus, $S_t = P[Z_t = 1 | N_t, f_t]$. Using a Poisson approximation to the binomial distribution, a "local" Poisson approximation is introduced in [1]. The "local" Poisson approximation consists in approximating the conditional distribution of the number of retransmitted packets in slot (t, t+1] by a Poisson distribution with mean $\mu_t = N_t f_t$. Thus,

$$S_t \doteq G_t e^{-G_t}$$

where $G_t = \lambda + \mu_t$ for the IFT policy and $G_t = \mu_t$ for the DFT policy.

It is not difficult to see that the pair process (N_t, f_t) forms a Markov chain with state space $Z_+ \times [0, 1]$. The importance of the "local" Poisson approximation hinges on the fact that the transition probabilities of the chain (N_t, f_t) depend on N_t and f_t only through the product $\mu_t = N_t f_t$.

Another important point is that under the retransmission policy considered in [1] the dynamics of the total channel traffic suffer disturbances only of order $O(1/n)$ due to the fluctuation of the amount of backlogged traffic, when the backlogged size N_t is near n. Hence the total traffic level is nearly decoupled from the size of the channel backlog, as long as n is large enough. This fact justifies the use of the following "local model" for the retransmitted traffic.

Fix an integer $n > 0$. The Markov chain (f_t^n) obtained by localizing the Markov chain (f_t, N_t) to $(N_t) = n$ has the transition probabilities:

$$f_{t+1}^n = \min\left(\beta,\ f_t^n \times \begin{cases} a(0)^\gamma & \text{with probability } e^{-G_t} \\ a(1)^\gamma & \text{with probability } G_t e^{-G_t} \\ a(e)^\gamma & \text{with probability } 1-(1+G_t)e^{-G_t} \end{cases}\right) \quad (2)$$

where $G_t = \lambda + nf_t^n$ for the IFT policy and $G_t = nf_t^n$ for the DFT policy.

If we define $\phi_t^n \triangleq \ln(nf_t^n)$ for $n > 0$, then $\mu_t = \exp(\phi_t^n)$ represents the total retransmitted traffic intensity for the "local model". Further, (ϕ_t^n) is itself a Markov chain with transition probabilities

$$\phi^n_{t+1} = \min\left(\ln(n\beta),\ \phi^n_t + \gamma \begin{cases} C(0) \text{ with probability } e^{-G_t} \\ C(1) \text{ with probability } G_t e^{-G_t} \\ C(e) \text{ with probability } 1-(1+G_t)e^{-G_t} \end{cases}\right) \qquad (3)$$

where $C(i) = \ln[a(i)]$ for $i = 0$, 1 or e. The important point here is that the transition probabilities of ϕ^n_t do not depend on n, except through the term $\ln(n\beta)$. The minimum in (3) simply reflects the fact that $\phi^n_t = \ln(nf^n_t) \leq \ln(n\beta)$ for all t. As n increases, the constraint $\phi^n_t \leq \ln(n\beta)$ becomes less crucial. Hence, we study only the process Φ_t with transition probabilities

$$\Phi_{t+1} = \Phi_t + \gamma \begin{cases} C(0) \text{ with probability } e^{-G_t} \\ C(1) \text{ with probability } G_t e^{-G_t} \\ C(e) \text{ with probability } 1-(1+G_t)e^{-G_t} \end{cases} \qquad (4)$$

where $G_t = G_t(\Phi_t)$

2. Drift and Variance Analysis of the Local Model

Equation (4) defines the following stochastic recursion for the log-intensity ϕ_t of the "local model"

$$\phi_{t+1} = \phi_t + \gamma C_t(\phi_t), \qquad \phi_0 \varepsilon R \tag{5}$$

Given $\phi_t = \phi$ the sequence $\{C_t(\phi)\}_{t\varepsilon z^+}$ is a random variable sequence with stationary distribution

$$P\{C_t(\phi_t) = C(0) \,|\, \phi_t = \phi\} = e^{-G}$$

$$P\{C_t(\phi_t) = C(1) \,|\, \phi_t = \phi\} = Ge^{-G}$$

$$P\{C_t(\phi_t) = C(e) \,|\, \phi_t = \phi\} = 1-(1+G)e^{-G}$$

where $G = G_I(\phi) = \lambda + \exp(\phi)$ for the IFT policy and $G = G_D(\phi) = \exp(\phi)$ for the DFT policy.

Recursions of the same form to the one given above have been studied in the context of recursive stochastic algorithms in the estimation and control literature. To analyze the recursion described by [5], we shall employ some known theoretical results for evaluating: (1) the motion of the "mean value" $\bar{\phi}_t$ of ϕ_t which will be associated with the solution of an ordinary differential equation, (2) the evolution of the difference $\tilde{\phi}_t = \phi_t - \bar{\phi}_t$, which will be described by a linear diffusion model.

2.1. The associated ordinary differential equation

It is known that $\{\phi_t\}_{t\in Z}$ can be seen as a perturbed discrete-time approximation of the solution of an associated ordinary differential equation with discretization step γ. The corresponding result can be given in the form of a theorem (see [3] - [6], for details and explicit required conditions).

Theorem 1: For every $\varepsilon > 0$ and $T < \infty$

$$\lim_{\gamma \to 0} P\{ \max_{1 \le \gamma t \le T} |\phi_t - \phi(\gamma t)| > \varepsilon\} = 0$$

where $\phi(t_c)$ denotes the solution of the ordinary differential equation associated to (5):

$$\frac{d\phi}{dt_c} = w(\phi), \quad \phi(0) = \phi_0 \tag{6}$$

where $w(\phi) = m(\phi)/\gamma$

$$m(\phi) = E\{\Delta\phi_t | \phi_t = \phi\} = E\{\gamma C_t(\phi_t) | \phi_t = \phi\}$$

and

$$\Delta\phi_t = \phi_{t+1} - \phi_t$$

In the above theorem t_c indicates the continuous time of the ordinary differential equation (6) and t the discrete time of the recursion. Under the correspondence $t_c \sim \gamma t$, $\phi(\gamma t)$ represents a discretization of the solution $\phi(t_c)$ of (6) with step γ.

2.2. The diffusion approximation

We now introduce a diffusion approximation model describing the evolution of the difference $\phi_t - \phi(t_c)$ (for $t_c = \gamma t$) as $\gamma \to 0$. For $T<\infty$, let $(X^{\gamma}_{t_c})_{0 \le t_c \le T}$ be the continuous-time stochastic process such that

$$X^{\gamma}_{t_c} = \frac{\phi_t - \phi_{t_c}}{\sqrt{\gamma}}, \quad \text{for } t_c = \gamma t$$

Under realistic assumptions (see [3] - [6]), we have the following theorem.

Theorem 2: The process $(X^{\gamma}_{t_c})_{0 \le t \le T}$ converges weakly as $\gamma \to 0$ to the Gaussian process $(X_{t_c})_{0 \le t \le T}$ which is the solution of the following linear stochastic differential equation:

$$dX_{t_c} = A(\phi(t_c))X_{t_c}\, dt_c + Q^{1/2}(\phi(t_c))\, dW_{t_c}, \quad X_0 = 0 \tag{7}$$

where (W_{t_c}) is the standard Wiener process, and

$$A(\phi) = \frac{d}{d\phi} w(\phi)$$

$$Q(\phi) = \sum_{t \in Z^+} \mathrm{Cov}(C_t(\phi), C_0(\phi))$$

the series being assumed convergent.

Next we are interested in studying the evolution of the difference $\phi_t - \phi(t_c)$ when ϕ_t lies near some stable equilibrium of the ordinary differential equation (6), denoted by ϕ_*.

3. Speed of Convergence Versus Steady State Accuracy for the Local Model

For the recursion described by (5) we have:

$$m(x) = \gamma \underline{C} \cdot \underline{b}^T(x)$$

$$w(x) = m(x)/\gamma = \underline{C} \cdot \underline{b}^T(x)$$

where

$$\underline{C} = (C(0), C(1), C(e))$$

$$\underline{b}(x) = [\exp(-G(x)), G(x)\cdot\exp(-G(x)), 1-(1+G(x))\cdot\exp(-G(x))]$$

If we define $V(\phi) \triangleq Var(\Delta\phi_t | \phi_t = \phi)$ then

$$V(\hat{\phi}) = \gamma^2 (C(0)^2, C(1)^2, C(e)^2) \cdot \underline{b}^T(x) - m^2(x)$$

By definition, $Q(\phi) = \sum_{t \varepsilon z^+} cov(C_t(\phi), C_0(\phi))$. But $\{C_t(\phi)\}_{t \varepsilon z+}$ is, given ϕ, a sequence of i.i.d. random variables, which are also assumed independent of $C_0(\phi)$. Thus

$$Q(x) = cov(C_t(x), C_t(x)) = G^2(x)/\gamma^2 =$$

$$= (C^2(0), C(1)^2, C(e)^2) \cdot \underline{b}(x) - w^2(x)$$

By the "local" Poisson approximation, the expected throughput in slot (t, t+1], given ϕ_t, is $S_t = G_t e^{-G_t}$ where $G_t = G(\phi_t)$, which is maximized when $G_t = 1$. If the goal of the transmission policy is to maximize the throughput then we should choose $C(0)$, $C(1)$ and $C(e)$ such that the equilibrium point ϕ_* of the ordinary differential equation (6) is unique, stable and optimal. A stable equilibrium point ϕ_* is optimal if it maximizes the throughput $S_t = S(\phi_t)$, i.e. if $G(\phi_*) = 1$, since then $S(\phi_*) = \max_{G_t}(G_t e^{-G_t}) = e^{-1}$. Hence, for ϕ_t lying near ϕ_* it is reasonable to replace the model (7) by the following simplified time-invariant linear stochastic differential equation:

$$dX_{t_c} = AX_{t_c}\, d_{t_c} + Q^{1/2}\, dW_{t_c} \tag{8}$$

where

$$A = A(\phi_*); \qquad A(x) = \frac{d}{dx} w(x)$$

$$Q = Q(\phi_*)$$

and $w(x)$ is as defined in equation (6).

It is noteworthy that equation (8) is simply the Langevin random differential equation [7, Section 7.2]. The density function of X_{t_c} is the familiar one associated with the Ornstein-Uhlenbeck process. The stationary solution of (8) is Gaussian with mean zero and variance $-Q/2A$, i.e.

$$(X_{t_c})_s = \frac{\phi_t - \phi_*}{\sqrt{\gamma}} \sim N(0, -Q/2A) \tag{9}$$

If we use the "unnormalized" difference $\phi_t - \phi_*$, then (9) is equivalent to:

$$\phi_t - \phi_* \sim N(0, -\gamma Q/2A) \tag{10}$$

In other words, for $t \to \infty$ and for sufficiently small γ (theoretically, $\gamma \to 0$) the difference $\phi_t - \phi_*$ is a Gaussian random variable with mean zero and stationary variance $V_s = -\gamma Q/2A$.

Now using the above results we are able to derive measures of the speed of convergence and steady state accuracy for the "local model" of the recursive retransmission policies under consideration.

The following conditions ensure that the point ϕ_* is an optimal stable equilibrium point of the ordinary differential equation (6).

$$w(\phi_*) = 0, \quad A(\phi_*) = \frac{d}{dx} w(x)\Big|_{x=\phi_*} < 0 \quad \text{and } G(\phi_*) = 1 \tag{11}$$

Conditions (11) are equivalent to the following conditions on $C(0)$, $C(1)$, $C(e)$ that were derived in [1]

$$\underline{C} \neq 0, \quad C(e) \leq 0 \leq C(0) \text{ and}$$
$$C(e) = -(C(0) + C(1))\frac{e^{-1}}{1-2e^{-1}} \tag{12}$$

As we can see from (4) the step sizes of the Markov chain (ϕ_t) are proportional to γ. If γ is fairly large, ϕ will quickly approach the vicinity of its optimal value. A glance at the associated differential equation leads us to the same conclusion since γ represents the discretization step size.

A first order approximation of (6) around $\phi = \phi_*$ gives:

$$\frac{d\phi}{dt_c} = A(\phi_*)(\phi - \phi_*) + o((\phi - \phi_*)^2) \tag{13}$$

From (13) we conclude that the speed of convergence of ϕ to the vicinity of ϕ_* is related to

$$A = A(\phi_*) = \frac{dw(x)}{dx}\Big|_{x=\phi_*}$$

Hence, it is desirable to have A as negative as possible. Notice that A represents the slope of the normalized expected drift $w(\cdot)$ of (4) at the equilibrium point. On the other hand, the stationary variance of the linear diffusion model $V_s = -\gamma Q/2A$ is a linear function of γ (for γ sufficiently small). Hence a larger γ will result in larger fluctuations of ϕ around its optimal value ϕ_*. This in turn will result in a larger stationary deviation of the throughput $S(\phi_t)$ from the optimal $S(\phi_*) = e^{-1}$.

If γ is very small, then ϕ approaches ϕ_* more slowly, but on the average remains closer to ϕ_* after the longer transient period. Therefore, we have here another manifestation of the usual trade-off between speed of convergence and steady state accuracy.

From the equilibrium analysis of the diffusion model corresponding to the local model (4) we have:

$$\lim_{\gamma\to 0} \bar{S}_t = e^{-1} = S(\phi_*)$$

where $\bar{S}_t = E_s\{S(\phi_t)\}$ denotes the expectation of $S(\phi_t)$ relative to the stationary distribution of ϕ_t given by (10). Let us expand $S(\phi_t)$ around ϕ_*; we have

$$S(\phi_t) = S(\phi_*) + (\phi_t - \phi_*)S'(\phi_*) + \frac{(\phi_t - \phi_*)^2}{2} S''(\phi_*) + O((\phi_t - \phi_*)^2$$

Taking expectations with respect to the stationary distribution yields

$$\bar{S}_t = e^{-1} - \gamma R + o(\gamma)$$

where

$$R = \frac{Q}{2A} (1/2\ S''(\phi_*))$$

Hence, R reflects the effect of the fluctuations of ϕ_t around its optimal value ϕ_* on the average throughput induced by different transmission policies.

For a fixed value of γ the values of $|A|$ and R^{-1} represent measures of the speed of convergence and the steady state accuracy respectively for different transmission policies.

4. Comparison of IFT and DFT in Terms of Speed and Accuracy

In [1] the DFT policy is proposed as a robust alternative first-transmission policy to the IFT. The DFT policy can ensure stability (in the sense of [8, Theorem 5.1]) for any i.i.d. arrival process (Y_t) with $\lambda = E(Y_t) < e^{-1}$ and $E[e^{\varepsilon Y_t}] < +\infty$ where $\varepsilon>0$.

We are interested in comparing the two policies in terms of speed and accuracy using the measures $|A|$ and R^{-1}.

Evaluation of A

We have

$$A \triangleq \left.\frac{dw(x)}{dx}\right|_{x=\hat{\phi}_*} = \underline{C}\,\frac{d}{dx}(\underline{b}^T(x))\Big|_{x=\hat{\phi}_*} = e^{-1}(C(e) - C(0))G'(\hat{\phi}_*) \qquad (14)$$

Remark: A is independent of C(1)

Evaluation of R

We have

$$Q \triangleq Q(\hat{\phi}_*) = e^{-1}(C^2(0) + C^2(1)) + (1-2e^{-1})C^2(e)$$

and

$$S''(\hat{\phi}_*) = -e^{-1}(G'(\hat{\phi}_*))^2$$

Hence

$$R = \frac{Q}{4(C(0) - C(e))}\,G'(\hat{\phi}_*) \qquad (15)$$

<u>Case A</u>: Immediate-First-Transmission (IFT)

In this ,ase the "local" traffic intensity is

$$G_I(\phi_t) = \lambda + \exp(\phi_t) \text{ and } \phi_* = \ln(1-\lambda) \tag{16}$$

Substitution of (16) into (14) and (15) yields

$$A_I = e^{-1}(C(e) - C(0))(1 - \lambda) \tag{17.a}$$

$$R_I = \frac{Q}{4(C(0) - C(e))}(1 - \lambda) \tag{17.b}$$

<u>Case B</u>: Delayed-First-Transmission (DFT)

In this case the "local" traffic intensity is

$$G_D(\phi_t) = \exp(\phi_t) \text{ and } \phi_* = 0 \tag{18}$$

Substitution of (18) into (14) and (15) yields

$$A_D = e^{-1}(C(e) - C(0)) \tag{19.a}$$

$$R_D = \frac{Q}{4(C(0) - C(e))} \tag{19.b}$$

Comparing (17) to (19) we conclude that, under Poisson statistics, the two policies are equivalent in terms of speed and steady state accuracy for the local model, if

$$\gamma_D = \gamma_I(1 - \lambda)$$

<u>A note on the selection of C</u>

The only constraint imposed so far on the three parameters C(0), C(1), C(e) is that they have to satisfy the conditions given by (12).

We propose the following criterion. Choose C(0), C(1), C(e) so that the steady state accuracy is maximized under fixed speed. In other words, the objective is to minimize R for fixed A. We fix A by setting C(0) - C(e) = 1. There is no loss of generality in doing so since according to conditions given by (12), the parameters C(0), C(1), C(e) are determined up to a multiplicative constant which can be incorporated in the value of the gain γ. After this, the problem is the following:

$$\text{minimize } R$$

$$\text{subject to} \quad \underline{C} \neq 0, \quad C(e) \leq 0 \leq C(0), \quad C(0) - C(e) = 1$$

and

$$C(e) = -(C(0) + C(1)) \frac{e^{-1}}{1-2e^{-1}}$$

The solution to the above problem is the following:

$$C(0) = \frac{1-2e^{-1}}{1-e^{-1}} \approx .418, \quad C(1) = 0, \; C(e) = -\frac{e^{-1}}{1-e^{-1}} \approx -.582 \qquad \text{(C.1)}$$

for both IFT and DFT policies. It is noteworthy that the above choice of $\underline{C}$ is identical to the one given in [1], derived after the "economical" but otherwise arbitrary choice of C(1) = 0.

Substitution of the above choice of $\underline{C}$ into (14) and (15) gives:

$$A = -e^{-1} G'(\hat{\phi}_*)$$

$$R = \frac{e^{-1}(1-2e^{-1})}{4(1-e^{-1})} G'(\hat{\phi}_*)$$

where

$$G'(\hat{\phi}_*) = \begin{cases} 1 & \text{for the IFT policy} \\ 1-\lambda & \text{for the DFT policy} \end{cases}$$

5. Retransmission Control Policies under Binary Feedback

The retransmission control policies examined so far assume that at the end of the slot (t, t+1], each user learns the value of a common ternary feedback random variable $Z(t)$. We have $Z(t) = 0$, or 1, or e depending on whether slot (t, t+1] was empty, or contained one packet, or contained ≥ 2 packets respectively.

In a random multiple-access channel like the one considered here, there are situations where ternary feedback might not be available. A very interesting feature of the retransmission policies of Hajek and Van Loon is that they can still achieve a stable throughput of $e^{-1} = .3678$ under certain types of "binary" feedback.

Binary feedback uses less feedback information compared to ternary feedback and can be available in three different types (classification suggested by Mehravari and Berger [9]):

$$Z_{CNC}(t) = \begin{cases} C & \text{if } \geq 2 \text{ packets in slot } t \text{ (conflict)} \\ NC & \text{if } \leq 1 \text{ packet in slot } t \text{ (no conflict)} \end{cases}$$

$$Z_{SN}(t) = \begin{cases} S & \text{if } \geq 1 \text{ packet in slot } t \text{ (something)} \\ N & \text{if no packet in slot } t \text{ (nothing)} \end{cases}$$

$$Z_{SF}(t) = \begin{cases} S & \text{if one packet in slot } t \text{ (success)} \\ F & \text{if either no or } \geq 2 \text{ packets in slot } t \text{ (failure)} \end{cases}$$

The conflict/no conflict (CNC) feedback informs the users only about whether or not there was a conflict in the previous slot. The retransmission policy given by (1) can be implemented under CNC feedback by merely choosing a different vector $\underline{C}$, to accommodate an additional condition introduced by the reduction in the feedback information. The additional condition is: $C(0) = C(1)$. This results in the following choice of $\underline{C}$ [1].

$$\underline{C}_{CNC} = (0.209,\ 0.209,\ -0.582)$$

CNC binary feedback has recently been studied by Mehravari and Berger [9]. Their scheme uses a collision resolution algorithm to achieve a stable throughput of .4422. Also in [10] a very easy to implement limited sensing-type algorithm with CNC binary feedback is proposed which achieves a stable throughput of at least .363.

Something/nothing (SN) feedback informs the users about whether or not the previous slot was empty. This type of feedback is characteristic of public-key secure computer communications. The retransmission policy given by (1) can be implemented under SN feedback by choosing $\underline{C}_{SN} = (.462, -0.269, -0.269)$ [1]. In [9] a stable throughput of .279 is achieved by using a collision resolution algorithm.

5.1. Retransmission control policy under success/failure (SF) feedback

The success/failure (SF) feedback informs the users about whether or not the previous slot contained exactly one message. This situation arises if the receiver cannot distinguish between channel noise and collision noise. As it is pointed out in [9], spread-spectrum random access systems result in SF-like feedback in the event that the users attempt to disguise the fact that they are communicating by keeping the transmitted power very low. In this case, the noiselike waveform that results from the collision of two ore more transmitted signals is difficult to distinguish reliably from noise alone.

The policies introduced by Hajek and Van Loon [1] do not cover success/failure feedback. The reason for this is simple. SF feedback introduces the following condition on C(0), C(1), C(e) in addition to the conditions given by (12):

$$C(0) = C(e) \triangleq C(0e) = \text{common constant} \tag{20}$$

It is easy to see that there is no choice of $\underline{C}$ that satisfies both (12) and (20).

Let us now use the associated ordinary differential equation model introduced in section 2.1 to get some insight. The stability of the differential equation given by (6) depends on the sign of $A(\phi_*)$ of the linearized equation given by (13). Consequently, the existence of a stable equilibrium point ϕ_* depends on whether $A(\phi^*) < 0$, or equivalently on whether

$$(C(1) - C(0e))(1 - G_*)G'_*\exp(-G_*) < 0 \tag{21}$$

where

$$G_* \triangleq G(\phi_*)$$

$$G'_* \triangleq \frac{d}{dx} G(x) \bigg|_{x=\phi_*}$$

(In deriving (21) we have used condition (20).)

Now the difficulty in the SF feedback case is apparent. If we insist in establishing a stable equilibrium point ϕ_* which is optimal in a throughput maximizing sense, i.e. $S(\phi_*) = e^{-1}$ or equivalently $G_* = 1$, then we cannot guarantee stability since from (21) we have $A(\phi_*) = 0$.

The only way to get around this difficulty is to relax the optimality requirement $G_* = 1$. If we fix $G_s \neq 1$, then any ϕ_s such that $G(\phi_s) = G_s$ can be made the only stable equilibrium point of the associated differential equation by an appropriate choice of the parameters $C(1)$ and $C(oe)$. The required conditions are:

$$w(\phi_s) = 0 \quad \text{and} \quad A(\phi_s) < 0$$

or equivalently

$$C(0e)(1 - G_s e^{-G_s}) + C(1) G_s e^{-G_s} = 0 \tag{22}$$

and

$$(C(1) - C(oe))(1 - G_s) G_s' \, e^{-G_s} < 0 \tag{23}$$

For fixed $G_s > 1$ conditions (22), (23) are equivalent to the following:

$$C(1) > 0, \quad C(oe) < 0, \text{ and}$$

$$\frac{C(1)}{C(oe)} = 1 - \frac{e^{G_s}}{G_s} \tag{24}$$

If we further impose the normalizing condition $|C(1)| + 2|C(oe)| = 1$, then we have:

$$\underline{C}(G_s) = (C(0), C(1), C(e)) = (e^{G_s} + G_s)^{-1} \cdot (-G_s, e^{G_s} - G_s, -G_s) \tag{25}$$

For this choice of $\underline{C}$ we have:

$$\lim_{\gamma \to 0} \overline{S}_t = G_s e^{-G_s} \quad \text{for any fixed } G_s > 1 \tag{25}$$

Hence, the retransmission control policy given by (1) can cover the case of success/failure (SF) binary feedback with the choice of $\underline{C}$ given by (25) and is stable for $\lambda < G_s e^{-G_s}$ for any fixed $G_s > 1$, if γ is sufficiently small. For example, if $G_s = 1.2$ then for $\underline{C}(1.2) = (-.265, .469, -.265)$ and sufficiently small γ the scheme is stable for $\lambda < .3614$.

Note that we can satisfy conditions (22) and (23) by fixing $0 < G_s < 1$. The above results are valid for this case too except that now we have to choose $C(1) < 0$ and $C(oe) > 0$. For $0 < G_s < 1$ the choice of $\underline{C}$ is the following:

$$\underline{C}(G_s) = (e^{G_s} + G_s)^{-1}(G_s, G_s - e^{G_s}, G_s) \tag{27}$$

5.2 Measures of the speed of convergence and steady state accuracy for the S/F retransmission control policy

After simple calculations, we have:

$$A(G_s) = (C(1) - C(oe))(1 - G_s)G_s' e^{-G_s} \tag{28}$$

$$R(G_s) = Q(G_s) \frac{[(G_s'' - (G_s')^2)(1 - G_s) - (G_s')^2]}{4(C(1) - C(oe))(1 - G_s)G_s'} \tag{29}$$

where

$$Q(G_s) = C^2(1)\, G_s e^{-G_s} + C^2(oe)(1 - G_s e^{-G_s})$$

$$G_s' = \frac{d}{dx} G(x) \Big|_{x=\phi_s}$$

$$G_s'' = \frac{d^2}{dx^2} G(x) \Big|_{x=\phi_s}$$

and C(1), C(oe) are given by (25) or (27).

For the IFT policy $G_s = G(\phi_s) = \lambda + \exp(\phi_s)$, hence

$$G_s' = G_s'' = G_s - \lambda$$

For the DFT policy $G_s = G(\phi_s) = \exp(\phi_s)$, hence

$$G_s' = G_s'' = G_s$$

From (28) and (29), it is not difficult to see that in order to maintain "sufficiently good" speed and steady state accuracy we have to keep G_s "sufficiently" away from 1. This, of course, will result in reduced stable throughput, since according to (26) $\lim_{\gamma \to 0} \bar{S}_t = G_s e^{-G_s}$. It is noteworthy that if we accept the ratio $q = \frac{|A|}{R}$ as a measure of the efficiency of the policies considered here under different feedback information, then:

$$q_{Tern.} > q_{CNC} > q_{SN} > q_{SF}$$

The order of the above inequalities is in agreement with the classification given in [9], where the SF binary feedback is classified as the least informative.

6. Effects of Channel Errors

Thus far it has been assumed that all users receive perfect information regarding the channel output. In other words, after each transmission slot, all users correctly detect whether the slot was empty ("0"), or contained one packet ("1"), or contained a collision ("e").

We now consider the more realistic situation where channel noise can affect the detections. In this case the updation of the retransmission probabilities will be based on information that inaccurately represents the outcome of the previous transmission.

In modeling detection errors we assume only system-wide errors, i.e. when an error is made, all users make the same error. This is realistic if the users are relatively close together, or if the feedback information is received in the form of an acknowledgement from a central facility. Although in some applications different users may receive different information about the channel status, the system-wide error analysis could still be used as a measure of the degradation of the system's performance.

The effect of channel feedback errors will be analyzed using the local model. We consider the following six possible types of detection errors:

$$P_{01} = P_r\ (1 \text{ detected} | 0 \text{ sent})$$
$$P_{oe} = P_r\ (\geq 2 \text{ detected} | 0 \text{ sent})$$
$$P_{10} = P_r\ (0 \text{ detected} | 1 \text{ sent})$$
$$P_{1e} = P_r\ (\geq 2 \text{ detected} | 1 \text{ sent})$$
$$P_{e0} = P_r\ (0 \text{ detected} | \geq 2 \text{ sent})$$
$$P_{e1} = P_r\ (1 \text{ detected} | \geq 2 \text{ sent})$$

The notation P_{ij} is used to indicate both the type of error and the corresponding probability.

At time k let the total traffic intensity be $G(\phi_k) = G$. Also let $b_o = P_r$ (o sent), $b_1 = P_r$ (1 sent) and $b_e = P_r$ (≥ 2 sent).

If p_o, p_1, p_e denote the probability of detecting an empty, successful, collided transmission respectively, given G, then

$$\underline{p} = \underline{b}P$$

where $\underline{p} = (p_0, p_1, p_e)$, $\underline{b} = (b_0, b_1, b_2) = (\exp(-G), G\exp(-G), 1-(1+G)\exp(-G))$ and

$$P = \begin{bmatrix} 1-P_{01}-P_{oe} & P_{01} & P_{oe} \\ P_{10} & 1-P_{10}-P_{1e} & P_{1e} \\ P_{eo} & P_{e1} & 1-P_{eo}-P_{e1} \end{bmatrix}$$

The local model recursion is now the following

$$\phi_{t+1} = \phi_t + \begin{array}{l} \gamma C(0) \text{ with prob. } p_0(G_t) \\ \gamma C(1) \text{ with prob. } p_1(G_t) \\ \gamma C(e) \text{ with prob. } p_e(G_t) \end{array} \qquad (30)$$

where $G_t = G(\phi_t)$

Given a specific choice for $\underline{C}$, an equilibrium point $\hat{\phi}_e$ of (30) satisfies the following equation.

$$\underline{C}\ P^T\ \underline{b}^T = 0 \tag{31}$$

The equilibrium point $\hat{\phi}_e$ will be stable if and only if

$$A = \frac{\partial}{\partial \phi} (\underline{C}\ P^T\ \underline{b}^T) \Big|_{\phi = \phi_e} < 0 \tag{32}$$

Notice that in the error-free case P is the unit matrix and (31), (32) reduce to the conditions given by (12).

In what follows, we study (31), (32) under the assumption that $P_{01} = P_{10} = P_{e1} = P_{e0} = 0$.

It can be argued that under usual operation of the system, P_{oe} and P_{1e} are the more likely errors to occur. It is unlikely that noise would be interpreted as a "single packet" (P_{01}) because, in practice, each packet transmitted would be encoded with a sufficiently powerful error detecting code. For the same reason it is unlikely that a "collision" would be interpreted as a "single packet" (P_{e1}), unless the system is operating as a public-key secure computer communication system where any user other than the intended recipient cannot distinguish between a "single packet" and a "collision." A signal to noise ratio argument could be used to justify that is also very unlikely that one or more messages would be undetected (P_{10} or P_{e0}). This is not true only in special purpose random access systems like the one employing spread spectrum modulation techniques where the users cannot distinguish between channel noise and collision noise because they attempt to disguise the fact that they are communicating by keeping the transmitted power low.

On the other hand, an empty slot could, because of noise on the transmission channel, reach the users as a "garble" that they are forced to interpret as a "collision" (Poe). Also a "single packet" slot might result in a "collision" interpretation because of detected bit errors in the encoded packet on the noisy channel. The probability P_{1e} of this type of error could be significantly decreased if some error-correction were employed in addition to error detection.

In a realistic random-access situation one would anticipate the inequality

$$P_{oe} < P_{1e} << 1$$

Under the previous assumption equation (31) yields

$$C(0)\,\alpha\,\exp(-G) + C(1)\beta\,G\,\exp(-G) + C(e)(1 - (\alpha+\beta G)\exp(-G)) = 0 \tag{33}$$

where

$$\alpha = 1 - P_{0e},\ \beta = 1 - P_{1e}$$

and

$$G = G(\phi_e) = \begin{cases} \lambda + \exp(\phi_e) & \text{for the IFT policy} \\ \exp(\phi_e) & \text{for the DFT policy} \end{cases}$$

From the stability condition (32) we have

$$A = (\alpha(C(e) - C(0) + \beta(C(1) - C(e))(1-G))G'\exp(-G) < 0 \tag{34}$$

where

$$G' = \left.\frac{dG(x)}{dx}\right|_{x=\phi_e} = \exp(\phi_e)$$

If we know or have an estimate of P_{0e} and P_{1e} using (33) we can find a vector $\underline{C}$ such that the equilibrium point ϕ_e is unique, stable and optimal in the throughput maximizing sense, i.e. $\phi_e = \phi_*$, where $G(\phi_*) = 1$. This is true if and only if

$$C(e) = -(\alpha\, C(0) + \beta C(1)) \frac{e^{-1}}{1-(\alpha+\beta)e^{-1}} \tag{35}$$

and

$$\underline{C} \neq 0, \quad C(e) \leq 0 \leq C(0) \tag{36}$$

For the measure $|A|$ of the algorithm's speed of convergence to the neightborhood of ϕ_* we have

$$A = -\alpha(C(0) - C(e))e^{-1}\, G'(\phi_*) \tag{37}$$

For the measure R^{-1} of the stationary accuracy we have

$$R = \frac{Q}{4A} S''(\phi_*) = \frac{Q}{4\alpha(C(0) - C(e))} G'(\phi_*) \tag{38}$$

Next, as it was done in the error-free case, we choose $C(0)$, $C(1)$, and $C(e)$ to be the solution of the following optimization problem:

Maximize $q = \frac{|A|}{R}$, subject to conditions (35) and (36)

The solution to this problem is

$C(1) = 0$

$C(0)$, $C(e) \varepsilon R$ such that $C(e) < 0 < C(0)$ and

$$C(e) = -\frac{\alpha e^{-1}}{1-(\alpha+\beta)e^{-1}} \; C(0)$$

The parameters $C(0)$, $C(e)$ are determined up to a multiplicative constant. Without loss of generality we fix $|A|$ by setting $C(0) - C(e) = 1$.

Under this normalizing condition we have the following choice for $\underline{C}$

$$\underline{C} = \underline{C}(P_{0e}, P_{1e}) = \frac{1}{1-(1-P_{1e})e^{-1}} \, (1 - (2-P_{0e}-P_{1e})e^{-1}, \;\; 0, \;\; -(1-P_{0e})e^{-1}) \tag{39}$$

Substitution of (39) into (37) and (38) gives

$$A = A(P_{0e}) = -(1-P_{0e})e^{-1} \; G'(\phi_*) \tag{40}$$

$$R = \frac{e^{-1}(1 - (2-P_{0e}-P_{1e})e^{-1})}{4(1-(1-P_{1e})e^{-1})} \; G'(\phi_*) \tag{41}$$

where

$$G'(\phi_*) = \begin{cases} 1 & \text{for the DFT policy} \\ 1-\lambda & \text{for the IFT policy} \end{cases}$$

Some comments are in order now. From the form of the solution to the maximization problem of q, it is apparent that the choice of $\underline{C}$ as a function of P_{0e}, P_{1e} given by (39) is the choice that maximizes the steady state accuracy R^{-1} of the algorithm with $|A|$

fixed as in (40). The speed of convergence of the algorithm is proportional to $\gamma|A|$ and can be set to any desired level by an appropriate choice of the scalar γ. Hence the choice of $\underline{C}$ given by (39) also achieves the greatest maximum stable throughput $\bar{S}_t \approx e^{-1} - \gamma R$, for any fixed value of γ, i.e. for any desired speed of convergence.

As it can be seen from (40) the speed of convergence depends only on P_{0e} which is intuitively pleasing, since with $C(1) = 0$ (inaction) the updating process of the algorithm is based solely on the discrimination between the rewarding event "empty" ($C(0) > 0$) and the penalizing event "collision" ($C(e) < 0$).

The steady state accuracy depends on both P_{0e} and P_{1e}, as it can be seen from (41). In Fig. 1 the stationary throughput $\bar{S}_t$ is plotted as a function of the probability of error $P_e = P_{0e} = P_{1e}$ for the optimal choice of $\underline{C}$ given by (39) and for different values of γ. The values of $C(0)$, $C(e)$ used were normalized so that all the curves in Fig. 1 correspond to $|A| = e^{-1}$. Notice that for γ small the decrease of $\bar{S}_t$ is very moderate even for unrealistic values of the probability of error.

Next we consider the case where the error-free optimal choice of $\underline{C}$ given by (C.1) is used in the presence of errors. In this case it can be proved by studying (33) and (34) that an equilibrium point ϕ_e exists for any $0 \leq P_{1e} < 1$ if and only if $P_{0e} < C(0) \approx .418$. Furthermore, it can be proved that if an equilibrium point exists then it is unique and stable. The unique stable equilibrium point is no longer optimal and the distance from its optimal value ϕ_* increases as the probability of error increases. For example, if

$P_e = .05$ then ϕ_E is such that $G(\phi_E) \approx .94$, while if $P_e = .2$ then ϕ_E is such that $G(\phi_E) \approx .63$. This results in degradation of the maximum stable throughput $\bar{S}_t$ that becomes severe as the probability of error increases. This is apparent comparing the curves of $\bar{S}_t$ for the error-free choice of $\underline{C}$ to the corresponding curves for the optimal choice in Fig. 1.

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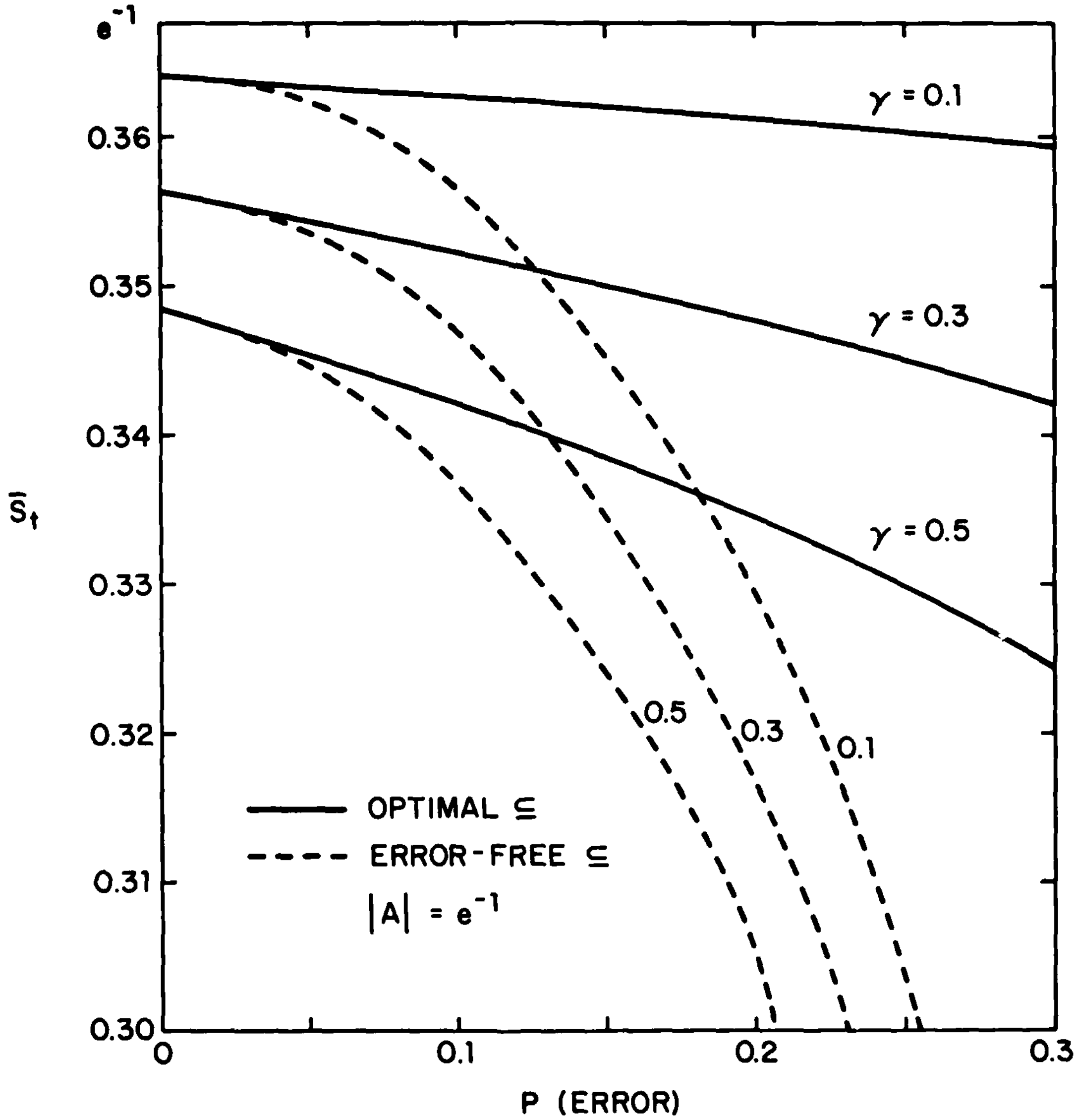

e^{-1}
0.36
0.35
0.34
0.33
0.32
0.31
0.30
$\bar{S}_t$
$\gamma = 0.1$
$\gamma = 0.3$
$\gamma = 0.5$
0.5
0.3
0.1
OPTIMAL $\subseteq$
ERROR-FREE $\subseteq$
$|A| = e^{-1}$
0
0.1
0.2
0.3
P (ERROR)

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