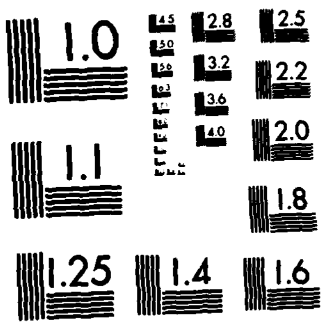
AD-A136 506 ASPECTS OF PATTERN THEORY(U) BROWN UNIV PROVIDENCE RI 1/1
DIV OF APPLIED MATHEMATICS U GRENANDER ET AL.
30 NOV 81 AFOSR-TR-83-1249 AFOSR-78-3514

UNCLASSIFIED F/G 12/1 NL

END
DATE
FILMED
2-84
DTIC



MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS-1963-A

(12)

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE

REPORT DOCUMENTATION PAGE

1a. REPORT SECURITY CLASSIFICATION UNCLASSIFIED	1b. RESTRICTIVE MARKINGS
2a. SECURITY CLASSIFICATION AUTHORITY	3. DISTRIBUTION/AVAILABILITY OF REPORT Approved for public release; distribution unlimited.
2b. DECLASSIFICATION/DOWNGRADING SCHEDULE	
4. PERFORMING ORGANIZATION REPORT NUMBER(S)	5. MONITORING ORGANIZATION REPORT NUMBER(S) AFOSR-TR- 83-1249
6a. NAME OF PERFORMING ORGANIZATION Brown University 6b. OFFICE SYMBOL (If applicable)	7a. NAME OF MONITORING ORGANIZATION Air Force Office of Scientific Research
6c. ADDRESS (City, State and ZIP Code) Division of Applied Mathematics Providence RI 02912	7b. ADDRESS (City, State and ZIP Code) Directorate of Mathematical & Information Sciences, Bolling AFB DC 20332
8a. NAME OF FUNDING/SPONSORING ORGANIZATION AFOSR 8b. OFFICE SYMBOL (If applicable) NM	9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER AFOSR-78-3514
8c. ADDRESS (City, State and ZIP Code) Bolling AFB DC 20332	10. SOURCE OF FUNDING NOS. PROGRAM ELEMENT NO.: 61102F PROJECT NO.: 2304 TASK NO.: A5 WORK UNIT NO.:
11. TITLE (Include Security Classification) "ASPECTS OF PATTERN THEORY"	

12. PERSONAL AUTHOR(S)
U. Grenander and D.E. McClure

13a. TYPE OF REPORT	13b. TIME COVERED	14. DATE OF REPORT (Yr., Mo., Day)	15. PAGE COUNT
FINAL	FROM 1/10/77 TO 30/11/81	NOV 30, 81	24

16. SUPPLEMENTARY NOTATION

17. COSATI CODES			18. SUBJECT TERMS (Continue on reverse if necessary and identify by block number)
FIELD	GROUP	SUB. GR.	

19. ABSTRACT (Continue on reverse if necessary and identify by block number)

The research in this project is motivated by pattern analysis, the study of regular structures in natural and man-made phenomena. Problems of inferring structural representations of observed patterns raise new problems of nonparametric statistical inference. The method of sieves has been developed as a general approach for adapting classical techniques of inference, such as maximum likelihood for estimation, to nonparametric settings. To develop the basic probabilistic models that form the foundation for statistical inference of patterns, characterization results have been obtained that prescribe the kinds of probability models generated by the regularity constraints of pattern theory. The mathematical questions have been studied both by analytical and computational methods. The computer experiments have led to the development of a substantial library of APL programs for mathematical experimentation. Numerous applications are described in the publications from the project.

20. DISTRIBUTION/AVAILABILITY OF ABSTRACT UNCLASSIFIED/UNLIMITED ☒ SAME AS RPT. ☒ DTIC USERS ☐	21. ABSTRACT SECURITY CLASSIFICATION UNCLASSIFIED
22a. NAME OF RESPONSIBLE INDIVIDUAL Dr. Robert T. Smythe	22b. TELEPHONE NUMBER (Include Area Code) (202) 767-5028 22c. OFFICE SYMBOL AFOSR/NM

DD FORM 1473, 83 APR EDITION OF 1 JAN 73 IS OBSOLETE.

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE

AFOSR-TR- 83-1249

FINAL SCIENTIFIC REPORT

ASPECTS OF PATTERN THEORY

Air Force Office of Scientific Research
Directorate of Mathematical and Information Sciences
Grant No. ~~78-3514~~
AFOSR-78-3514

1 October 1977 - 30 November 1981

Principal Investigator: Ulf Grenander (333-28-2844)
"L. Herbert Ballou" University Professor

Donald E. McClure (543-46-4785)
Professor of Applied Mathematics

Division of Applied Mathematics
Brown University
Providence, RI 02912

I. RESEARCH OBJECTIVES.

Our research in this project is motivated by pattern theory, the study of regular structures in natural and man-made phenomena. The emphasis of the investigations supported by the Air Force Office of Scientific Research has been on statistical inference problems for abstract parameter spaces. Many specific instances of such problems are described in the principal investigator's published volumes on pattern analysis and in the technical reports and papers written by contributors to the research effort (118 reports in our Reports in Pattern Analysis Series).

Typical inference problems in pattern analysis are concerned with "parameters" which are functions, sets, operators, transformations, etc., that is, they are members of abstract spaces. Traditional approaches to inference, specifically, estimation based on the principle of maximum likelihood, break down in this general setting if they are applied without modification from the way they are used in classical statistical practice. To understand how to circumvent this general problem, we examined a number of concrete examples (e.g., from nonparametric regression, density estimation, surface reconstruction, estimation of closed planar sets). These examples pointed the direction for the development of a general method -- the method of sieves -- for regularizing maximum likelihood estimators and least-squares estimators of parameters in abstract spaces. As the project progressed we were able to direct our studies toward the development of a general theoretical foundation for the method of sieves.

A second broad area which we planned to study was concerned with the characterization of stochastic processes and of probability measures induced by the structural and compatibility constraints on the regular constructs of pattern theory. The point of view of pattern theory has led to fundamentally new ways of conceiving of the generation of certain familiar models for random processes. In many cases, the pattern theoretic approach leads to a characterization of a measure in terms of Markovian dependence. The motivation for studying these problems derives from inference problems for regular structures; the mathematical models guide the selection of appropriate analytical strategies for estimating structural "parameters" on the basis of observed patterns.

A third major effort in our research project has been the systematic development of a library of APL programs for mathematical experiments on the computer. Most of our work on the development of theory and methods for abstract statistical inference problems has been strongly influenced by interactive experiments done in APL. The software library, which now includes hundreds of substantial programs for numerical mathematics, simulations, graphics, etc., gradually evolved during our project as a powerful general-purpose utility for studying new problems and guiding the formulation of hypotheses.

II. RESULTS.

The main achievements during this research project fall within the three broad areas identified in our research plan: methods of statistical inference for parameters in infinite dimensional spaces, characterization of probability measures on ensembles of patterns, and development of general purpose software for mathematical experimentation.

The list of publications in section III below is annotated to indicate how the separate reports and papers relate to these three main areas of the research effort. Here we shall mention the highlights of the accomplishments.

The method of sieves for regularization of estimators obtained from the principle of maximum likelihood or via least-squares is introduced in Grenander's book Abstract Inference. Numerous examples are introduced in the book in order to illustrate and to motivate the general approach. The significant general consistency results for maximum-likelihood and the method of sieves are derived in the paper "Nonparametric maximum likelihood estimation by the method of sieves" by S. Geman and C-R. Hwang, Ann. Statist., 10, pp. 401-414, 1982. In addition, several of the reports in our series of Reports in Pattern Analysis give careful developments of specific applications of the method of sieves and contribute to the underlying theory for the method (Reports in Pattern Analysis Nos. 62, 76, 79, 81, 85, 87, 88, 92, 99, 101, 102, 104, 105, 107, 109, 110, 112, 113, 114, 115 and 118).

The second major area of analysis in the project was concerned with the characterization of random processes and probability measures generated by the models of pattern theory and their regularity constraints. The first results in this direction are reported in Pattern Analysis: Lectures in Pattern Theory, Volume II, the second volume of Grenander's three volume development of general pattern theory. The Ph.D. theses of C-R. Hwang, P. Thrift, and C. Plumeri all contribute to this area. The thrust of many of the results is that the local regularity constraints of pattern theory induce Markovian processes on quite general structures. A unified presentation of the main results is given in Volume III of Lectures in Pattern Theory.

In the area of software development for mathematical experimentation, we have assembled and documented the APL library. A summary of the contents of the library is listed in section III below. The development of the library is continuing, and we have been providing copies of it to mathematicians and statisticians elsewhere who are interested in using the computer as an experimental research tool.

III. PUBLICATIONS AND TECHNICAL REPORTS.

A. Books.

1. U. Grenander, Regular Structures: Lectures in Pattern Theory, Volume III, Volume 33 in the Applied Mathematical Sciences Series, Springer-Verlag, Ney York, 1981.

This is the final volume in the author's series of lecture notes on the development of pattern theory. It crystallizes the elements of pattern theory introduced in Volumes I and II on Pattern Synthesis and Pattern Analysis, respectively. Volume III includes many new results in metric pattern theory, the characterization of probability measures on ensembles of regular structures, and in the algebraic study of regularity. The book also presents examples from taxonomy and mathematical semantics.

2. U. Grenander, Abstract Inference, John Wiley & Sons, New York, 1981.

This volume gives a thorough and systematic treatment of statistical inference problems for infinite dimensional sample and parameter spaces. It covers the thirty-year development of the theory underlying methods for stochastic processes and the more recent development of theories and methods for dealing with abstract parameter spaces. The author's method of sieves is introduced and illustrated with numerous examples.

3. U. Grenander, Mathematical Experiments on the Computer, Pure and Applied Mathematics Series, Academic Press, New York, 1982.

This book describes work that started in 1966 with the author's project on computational probability and statistics and that is continuing still with our work on the development of APL software for mathematical experimentation. The first section of the book presents eleven case studies on the use of the computer as an aid in the process of mathematical discovery. The second section is a mathematician's guide to the APL language. The third section outlines strategies for designing mathematical experiments on the computer. The fourth section documents the contents of most of our APL software library. The final version of the book was completed in early 1981.

B. Software Library.

U. Grenander and D. E. McClure have directed the development since September 1978 of an extensive library of APL programs for use in mathematical experiments on the computer. The library is maintained by the Brown University Computer Center and copies have been provided to mathematicians and statisticians elsewhere. The currently documented version contains about 325 programs for problems in areas such as combinatorics, algebra, nonparametric statistics, descriptive statistics, spectral analysis of time series, function approximation, graphics, matrix algebra, number theory, complex analysis, geometry, and differential and integral equations.

A list of the contents of the documented library is given below. Work on the incorporation of additional programs is continuing.

I. Utility Programs
- A) Function Editing & Testing (40 UTIL3)
  - 1) SPLICE - Splices together two APL functions
  - 2) RENAME - Renames several APL functions at once
  - 3) TITLEC - Changes the name of a function, or adds the lines of fn1 onto the end of fn2
  - 4) TIMER - Computes amount of CPU time used in executing a program
- B) Formatting Tables (64 UTIL2)
  - 1) PRINTAB - Formats a set of ordered pairs into a neat, tabular form
- C) Setting up Describe Variables and editing functions (66 UTIL1)
  - 1) COMPOSE - Creates describe variables for apl documentation
  - 2) EDITDV1 - Edits describe variables
  - 3) EDIT - Edits APL functions
- D) Displaying functions and describe variables (68 UTIL4)
  - 1) CMS - Allows user to execute certain CMS commands while in APL
  - 2) LISTALL - Outputs the code of all APL functions in the active workspace
  - 3) LISTFNS - Outputs the code of a chosen list of functions
  - 4) LISTDV - Displays a chosen group of describe variables
  - 5) SORT1 - Sorts a character matrix alpha-numerically

II. Plotting on TSP/HP Plotters (86 PLOT)
- 1) PLOTR - Group of functions for producing graphs
- 2) OPS - Group of fiunctions dealing with the linear algebra of piecewise linear arcs

III. Combinatorics (130 COMB)
- 1) CONNECT - Finds all connected components of an undirected graph
- 2) COMREACH - Computes the commutative reachability matrix of a digraph
- 3) CONVERT - Outputs the vector representation of all lines

of a digraph
4) MATMAK - Creates the adjacency matrix of a digraph
5) REACH - Computes the reachability matrix of digraph

IV. Descriptive Statistics (131 STATA)
1) ANALYZE - Calculates mean, variance, mode, median, etc.
2) FREQTAB1 - Calculates a one-way frequency table
3) FREQTAB2 - Calculates a two-way frequency table
4) MODE - Calculates the mode (special formula)
5) QUANT - Computes sample quantiles
6) STEMLEAF - Stem and leaf representation of a batch of numbers
7) TRENDS - Creates two-way frequency table from paired consecutive values of the data
8) GNNDENS - Group of functions for nearest neighbor density estimation

V. Plotting Histograms and Histosplines (131 HISTO)
1) HISTOGRAM1 - prints simple, horizontal histogram at terminal
2) HISTOGRAM2 - prints more complicated, vertical histogram at terminal
3) HISTOSPLINE - sets up coordinates for the plotting on tsp/hp plotter of an unbounded histospline
4) BOUNDSPLINE - sets up coordinates for the plotting on tsp/hp plotter of a bounded histospline
5) HISTER - Converts a regular histogram to one which can be used by histospline or boundspline
6) DRAW1 - Draws on the plotter the spline which resulted from histospline or boundspline (also plots corresponding histogram and x-axis)
7) Several other functions used for plotting

VI. Nonparametric Statistical Testing (133 STATB)
1) RUNS1 - One-sample runs test for randomness
2) RUNS2 - Two-sample runs test for randomness
3) SIGN - Two-sample sign test for comparing prob distributions
4) KOLMOG - Computes Kolmogorov confidence bands

VII. Fourier Analysis (134 FOURIER)
1) MRFFT - Mixed-Radix Fast-Fourier-Transform
2) NINVFFT - Inversion of discrete fourier transform
3) PDGRAM - Periodogram of discrete parameter time series
4) SPECTRI - Spectral est. for discrete parameter time series
5) TRISMOOTH - Vector smoothing with triangular weights

VIII. Simulation of Discrete Probability Distributions (135 PROBA)
1) BERN - Bernoulli distribution
2) BIN - Binomial distribution
3) DISC - Discrete distribution
4) DISCRETE - Discrete distribution (alias method)
5) GEOM - Geometric distribution
6) MARKOV - Markov chain
7) MULTINOM _ Multinomial distribution
8) NEGBIN - Negative binomial distribution
9) POIS - Poisson distribution

IX. Simulation of Continuous Probability Distributions (135 PROBB)

1) BETA - Beta distribution
2) CHI2 - Chi-square distribution
3) EXPON - Exponential distribution
4) FISHER - Fisher (F) distribution
5) GAMMA - Gamma distribution
6) GAUSS - Gaussian (normal) distribution
7) PARETO - Pareto distribution
8) SAMPLE - User defined distribution
9) SGAUSS - Standard normal distribution
10) STUDENT - Student's t distribution
11) UNIF - Uniform distribution

X. Calculation of Probabilities and Quantiles from Specific Distributions (135 PROBC)

1) BETAR - Incomplete Beta ratio
2) BINFR - binomial probabilities
3) BINDF - Cumulative binomial probabilities
4) CHISQ - (1-Cumulative Chi Square probabilities)
5) CONVOL - Convolution of two probability vectors
6) FDIST - Cumulative Fisher's F probabilities
7) GAUSSDF - Cumulative gaussian probabilities
8) NORMDEV - Quantiles of the standard normal distribution
9) POISSONDF - Cumulative poisson probabilities - for small integers
10) POISSONDF2 - Cumulative poisson probabilities - unlimited range
11) POISSONFR - Poisson probabilities - for small integers
12) POISSONFR2 - Poisson probabilities - unlimited range
13) TQUANT - Positive quantiles of the Student's T distribution

XI. Demonstration Programs (301 DISPLAY)

1) HEAT - Plots solutions to the heat equation

XII. Prime Numbers (400 PRIME)

1) FACTOR - Prime factorization of an integer
2) NPRIMES - Computes the number of primes less than or equal to a given integer
3) PRIMGEN - Generates all primes between two given integers

XIII. Complex Arithmetic (402 ARITH)

1) ADD, MINUS, MULT, DIVBY POWER, MAGNITUDE - Arithmetic operations on complex numbers

XIV. Convex Geometry (415 CONVEX)

1) ADDC - Adds polygons K1 and K2
2) ADDC1 - Adds any number of polygons in stacked form
3) AREA - Finds area of a polygon
4) ATAN - Finds angle between two line segments
5) AXF - Computes a rectangle (with sides parallel to axes) to circumscribe any given polygon
6) CIRCUM - Computes a polygon with given face angles which circumscribes a given polygon
7) DCNP - Computes a stacked form array of triangles whose sum is

some translation of a given polygon (with no parallel sides)

8) DECOMP - Computes a stacked form array of triangles (and/or line segments) whose sum is a given polygon
9) DEQUAD - Computes a stacked form array of triangles (and/or line segments) whose sum is a given quadrilateral
10) DIST - Computes Hausdorff Distance between K1 and K2
11) DISTANCE - Distance from K1 to K2
12) DISTPT - Maximum distance from a point to a vertex of a polygon
13) DRAWA - Used to plot one polygon on tsp/hp plotter
14) DRAWB - Used to plot more than one polygon at a time on tsp/hp plotter
15) EVALPWL - Evaluates piecewize linear functions
16) GRAPH - Used to produce plots at the terminal
17) HULL1 - Computes the convex hull of a given set of points
18) LINSERIES - For a matrix L, a polygon K and a maximum power n, computes

$$K + LK + L^2 K + \ldots + L^n K$$

19) MATRANS - Computes matrix transformation of a polygon
20) PERIM - Computes perimeter of a polygon
21) POLY - Converts from support form to standard form
22) RANDANG - Produces a random angle on the closed interval (0,2pi)
23) REMFVT - Removes false vertices
24) ROTATE - Rotates a polygon by a given angle about the origin
25) SCALE - Scales a polygon by a factor of N
26) SFACE - Converts from raw or standard form to support form
27) STFORM - Converts from raw to standard form
28) SUPPORT - Calculates support function at any given angles in the closed interval (0,2pi)
29) TRANS - Translates a polygon horizontally and/or vertically
30) VADD - Adds elements of a vector
31) WITH - Used in conjunction with the program 'DRAWB' to graph more than one polygon at a time on tsp/hp plotter

XV. Operations on Polynomials (420 POL)

1) ADD, MINUS, MULT, DIVBY
   POWER, MAGNITUDE - Arithmetic operations on complex numbers
2) MATMULT - Multiplication of two complex matrices
3) GRAMS - Computes coefficients of polynomials orthonormal about a closed curve
4) INTEGRAL - Computes integrals about a closed curve
5) CHOLESKYC - Complex cholesky decomposition of a hermitian positive definite matrix
6) INVERSE - Computes inverse of a complex, lower triangular matrix
7) CONFORMAL - Conformal mapping from a plane region to a disc, by Rayleigh-Ritz method
8) POLYC - Multiplies two complex polynomials
9) POLY - Multiplies two real polynomials
10) EVAL - Evaluates several complex polynomials at same point
11) EVAL2 - Evaluates a complex polynomial at several points

12) POLFACT - Factors mod 2 polynomials
13) PDIVBY - Division of one polynomial by another

XVI. Evaluation of Polynomials (421 EVALU)
1) EVAL - Evaluates several complex polynomials at same point
2) EVAL2 - Evaluates a complex polynomial at several points

XVII. Roots of Polynomials (422 ROOTS)
1) SYNDIV1 - Synthetic division
2) SYNDIV - Synthetic division of a complex polynomial by a complex root
3) MULLERM - Finds both real and complex roots of a polynomial equation by Muller's method
4) NEWTON - Uses Newton's method to find a root of an arbitrary function F(X)
5) ROOT - Uses a modification of Newton's method to solve for a root of an arbitrary function F(X)

XVIII. Computing Orthonormal Functions (423 ORTHO)
1) CHEB1 - Calculates nth Chebyshev polynomial of the first kind at given points
2) CHEB2 - Calculates nth Chebyshev polynomial of the second kind at given points
3) HAAR - Evaluates values of a Haar function at one or several X-values
4) HERMITE - Computes values of orthonormal Hermite polynomial of given order at specified points
5) LAGUERRE - Computes values of orthonormal Laguerre polynomial of given order at given points
6) LEGENDRE - Computes values of orthonormal Legendre polynomial of given order at given set of values

XIX. Calculating Special Functions (423 FUNCTION)
1) LOGAM - Evaluates log(Gamma(x))

XX. Systems of Non-linear Equations (424 SYSTEMS)
1) DNEWT - Solves a system of n equations in n unknowns

XXI. Numerical Integration (431 INTEGR)
1) SIMPSON1 - Approximates a definite integral by Simpson's rule
2) TRAPEZOIDAL - Approximates a definite integral by Trapezoidal rule
3) INTEGRAL - Computes integrals about a closed curve
4) COSFILON - Approximates the integral F(x)Cos(Tx) by Filon's method
5) SINFILON - Approximates the integral F(x)Sin(Tx) by Filon's method
6) DOUBLE - Approximates double integrals
7) GAUSSINT - Approximates integrals by Gaussian quadrature
8) GAUSSQ - Calculates the nodes and weights for Gaussian quadrature
9) HERMITEQ - Calculates the nodes and weights when the orthogonal functions are the Hermite polynomials
10) JACOBIQ - Calculates the nodes and weights when the

orthogonal functions are the Jacobi polynomials
11) LAGUERREQ - Calculates the nodes and weights when the orthogonal functions are the Laguerre polynomials

XXII. Ordinary Differential Equations (432 DIFF)

1) KUTTA - Solves a system of r first order initial value problems numerically using the runge-kutta method

XXIII. Interpolation (442 INTERPOL)

1) BSPL - Evaluates B-Splines at a given point
2) SPLDER - Differentiates the cubic spline of SPLINE
3) SPLINE - Computes cubic spline coefficients for a given set of points and provides interpolation values where desired
4) SPLINT - Integrates the cubic spline of SPLINE

XXIV. Linear Algebra (451 LINEAR)

1) CONVERT - Converts a symmetric matrix to vector form
2) REVERSE1, REVERSE2 - Two programs to reverse above conversion
3) CHOLESKYR - Cholesky decomposition of a symmetric positive matrix (in vector form)
4) CHOLESKYC - Complex cholesky decomposition of a hermitian positive definite matrix
5) EQDIST - Computes the equilibrium distribution of an aperiodic, irreducible markov chain
6) GENINV - Finds the Moore-Penrose generalized inverse of a real matrix
7) INVERSE - Computes the inverse of a complex lower triangular matrix
8) MATMULT - Multiplication of two complex matrices

XXV. Eigenvalues (452 EIGEN)

1) BALANCE - Balances row and column norms of a real matrix
2) ELMHES - Reduces a real matrix (preferably balanced) to upper-hessenberg form
3) HSHLDR - Produces the householder reduction of a real symmetric matrix to a symmetric tridiagonal matrix
4) INTEQ - Computes eigenvalues and eigenvectors of a real symmetric kernal
5) TQL - Computes eigenvalues and eigenvectors of a symmetric tridiagonal matrix by the QL algorithm
6) JACOBI - Computes eigenvalues (and eigenvectors) of a real symmetric matrix by jacobi method
7) HQR - Computes the eigenvalues of a matrix that is in upper-hessenberg form

XXVII. Calculating Minima of Functions (990 MINIMIZE)

1) OPTIM - Calculates minima using a gradient method
2) POWELL - Caluculates minima without using derivatives
3) QUAD - Calculates the minimum of a function of n variables along a specified search direction

XXVII. Asymptotic Patterns in Data (999 ASYMPTOT)

1) ASYMPTOTICS - Performs a heuristic search for asymptotic

patterns in data from a computable function
whose asymptotic behavior is not known

C. Technical Reports, Theses and Articles.

The items listed below are briefly annotated to explain their content, their relationship to the research project, current publication status, and supervisors of the work. Most of the technical reports have appeared in a series named Reports in Pattern Analysis, distributed through the Division of Applied Mathematics at Brown University. For brevity in the citations below, these are referred to by "RPA No. xx".

1. I. Frolow, Abstract growth patterns, RPA No. 58, June 1978.

Ph. D. dissertation supervised by U. Grenander. The thesis studies forms that evolve from growth mechanisms for two and three-dimensional patterns. The growth mechanisms studied included ordinary and partial differential equations, contact transformations, and continuous Lie groups of transformations. Limiting shapes for objects are determined and asymptotic stability conditions are investigated.

2. U. Grenander, A representation theorem for image algebras in terms of <EQUAL,LINEAR>-regularity, RPA No.60, December 1977.

Necessary and sufficient conditions for isomophism between image algebras are derived. This report is incorporated in Volume III of the author's <u>Lectures in Pattern Theory</u>.

3. U. Grenander, On the concept of homomorphisms in pattern theory, RPA No. 61, December 1977.

This report develops a generally applicable definition of homomorphisms between image algebras in pattern theory. This report is incorporated with the material on algebraic properties of patterns in Volume III of the author's <u>Lectures in Pattern Theory</u>.

4. U. Grenander, Restoration of convex sets with D3-deformations, RPA No. 62, December 1977.

This paper studies problems of estimating planar sets that can only be partially observed. Consistent estimators are devised. The results in this report are published in Volume III of the author's <u>Lectures in Pattern Theory</u>.

5. U. Grenander, A representation theorem for image algebras in terms of <INCLUSION,LINEAR>-regularity, RPA No. 63, December 1977.

This paper develops a characterization theorem for isomorphisms between image algebras with linear structure. The results are published in Volume III of the author's <u>Lectures in Pattern Theory</u>.

6. U. Grenander, A representation theorem for image algebras in terms of <EQUAL,TREE>-regularity, RPA No. 64, December 1977.

Necessary and sufficient conditions are developed for the existence of isomorphisms between image algebras with tree structure. The results are published in Volume III of the author's Lectures in Pattern Theory.

7. U. Grenander, Another law of large numbers for symmetric <EQUAL,LINEAR>-regular patterns, RPA No. 65, January 1978.

The main result of this paper describes the convergence in probability of waveforms generated by the pattern theoretic <EQUAL,LINEAR> structure. The results are reported in Volume III of Lectures in Pattern Theory.

8. D. Town, Restoration analysis of noise disrupted density functions, RPA No. 67, June 1978.

Ph.D. dissertation supervised by U. Grenander. The thesis reports on methods of identifying discrete components in a blurred spectrum. The work is closely related to other recent investigations of methods for identifying the components in a density function formed from a finite mixture of Gaussian densities.

9. C-R. Hwang, Frozen patterns and minimal energy states, RPA No. 68, June 1978.

Ph.D. dissertation supervised by U. Grenander. The main results characterize the limiting form of a Gibbs measure (the statistical mechanical model for the probability measure of a system) as the temperature of the system goes to zero. Notions of pattern theory are related to the fundamental concepts of statistical mechanics.

10. U. Grenander, An extremal property of the Gaussian distribution, RPA No. 69, May 1978.

It is proved that among all distributions of a pair of iid random variables X and Y for which the ratio X/Y is Cauchy, the Gaussian distribution concentrates mass closest to the origin.

11. L. W. Cohen, Geodesics in cluster patterns: the simulation of geodesic curves and disks on a TSP plotting system, RPA No. 70, May 1978.

This report documents APL programs and provides examples of their use for implementation of a scheme for defining the affinity or similarity between points in terms of geodesic disks.

12. W. Freiberger and U. Grenander, Restoration of discrete star images, RPA No. 72, April 1979.

The paper develops methods of restoring images in the plane that are subjected to random elastic deformations. The results are published in Quarterly of Applied Mathematics, vol. 39, pp. 383-404, 1981.

13. Y-S. Chow, Bayesian estimation for periodic and stationary Gaussian

processes, RPA No. 73, November 1978.

This report develops sufficient conditions for the uniform consistency of Bayesian estimates of the mean value function for a Gaussian process. Estimates of rate of convergence, depending on the spectrum of the mean-value process, are obtained.

14. U. Grenander, Synthesis of social patterns of domination, RPA No. 74, January 1979.

This report builds models of social interaction on the foundation of pattern theory. Probabilistic models of "typical configurations" of social dominance are based on regularity controlled probabilities. Computer simulations illustrate the models.

15. C-R. Hwang, Gaussian measures of large balls in a Hilbert space, RPA No. 75, January 1979.

The problems are motivated by problems of characterizing properties of Gaussian measures on infinite-dimensional spaces. The results have applications to the characterization of the behavior of Gaussian random processes. The report is published in Proceedings of the Amer. Math. Soc., vol.78, pp.107-110, 1980.

16. U. Grenander, Gaussian measure of small balls in a Hilbert space, RPA No. 76, February 1979.

This report derives an asymptotic expression for the probability in a small ball in a Hilbert space when the probability measure is Gaussian. The result is useful for determining when estimators, based on the method of sieves, of sets in a Hilbert space are consistent.

17. P. Thrift, Autoregression in homogeneous Gaussian configurations, RPA No. 77, February 1979.

Ph.D. dissertation supervised by U. Grenander. The main result gives the form of an autoregressive representation for a Gaussian Markovian field induced by the pattern theoretic process of regularity controlled probabilities. The results may be useful for inference problems concerned with estimating parameters of regular configurations.

18. C-R. Hwang and L. Andersson, On conditioning along the diagonal, RPA No. 78, February 1979.

This report develops results that are later used for the characterization of probability measures induced by regularity constraints on configurations in pattern theory.

19. S. Geman, On a common sense estimator for the drift of a diffusion, RPA No. 79, May 1979.

This report develops a method for consistent estimation of the value at an arbitrary point of the drift function of a diffusion process. The results are useful for system identification and contribute to the concrete examples of use of the method of sieves.

20. S. Geman and C-R. Hwang, Nonparametric maximum likelihood estimation by the method of sieves, RPA No. 80, May 1979.

The report is recognized as a seminal paper in the general theory of the method of sieves. It develops sufficient conditions for consistency of estimators obtained by the method of sieves and the principle of maximum likelihood. Several examples are described in detail. The paper was published in Ann. Statist., vol. 10, pp.401-414, 1982.

21. S. E. Howe, Estimating regions and clustering spatial data: analysis and implementation of methods using the Voronoi diagram, RPA No. 81, October 1978.

Ph.D. dissertation supervised by D. E. McClure. The paper develops consistent methods of estimating a partition of a planar region on the basis of multivariate observations at isolated sites. The methods are applied to geological data and the algorithms for partition estimation and clustering are implemented by FORTRAN programs.

22. U. Grenander, Structures in statistics, August 1979.

Invited opening address at the European Statistics Congress, Sofia, Bulgaria, September 1979.

23. S. Geman and C-R. Hwang, A chaos hypothesis for some large systems of random equations, RPA No. 82, August 1979.

Laws of large numbers and central limit theorems are deduced for systems with many interacting components which are approximately independent. The paper was published in Z. Wahrschein. verw. Gebiete, vol.60, pp.291-314, 1982.

24. U. Grenander, Solve the second limit problem in metric pattern theory!, RPA No. 83, September 1979.

This report formulates problems concerned with characterizing probability measures and random processes constrained by regularity conditions in pattern theory. The characterizations are important for doing statistical analyses of such pattern models. The problems posed in this report were later analyzed by C. Plumeri and C-R. Hwang.

25. U. Grenander, Heuristic software for experimental asymptotics, RPA No. 84, December 1979.

T. Ferguson, Programming changes in the software for experimental asymptotics, RPA No. 84A, January 1981.

These reports documents a package of APL programs designed to determine the asymptotic behavior of computable functions whose limiting behavior is unknown. The programs implement heuristic recognition algorithms. Numerous examples are included. The results of these reports are published in Grenander's book on Mathematical Experiments on the Computer.

26. U. Grenander, Thirty-one open problems in pattern theory, RPA No. 85, January 1980.

Open problems in pattern theory are formulated and conjectures and speculation about their solutions are developed. The problems are chosen on the basis of their potential for nontrivial generalization and for significant contributions to pattern theory and to our understanding of regular structures.

27. C-R. Hwang, Laplace's method revisited: weak convergence of probability measures, Ann. Probability, vol. 8, pp. 1177-1182, 1980.

Results based on the author's Ph.D. dissertation, RPA No. 68 described above.

28. C. Plumeri, Conditioning by EQUALITY when connection type is LINEAR, RPA No. 86, January 1980.

The report based on thesis research supervised by D. E. McClure applies arguments from functional analysis to derive the Markovian characterization of a random process induced by the pattern-theoretic mechanism of regularity controlled probabilities. The characterization is useful for pattern analysis and statistical inference.

29. D. E. McClure, Image models in pattern theory, RPA No. 87, January 1980.

The basic elements of pattern theory are described and exemplified with specific pattern-theoretic models for pictorial data. Three examples relate substantive results for problems of (i) optimal spatial quantization for transforming continuous pictures into discrete ones, (ii) characterization of random fields indexed by points in a square lattice, and (iii) restoration of the image of a planar convex set which is incompletely observed. This paper was presented as an invited lecture at the August 1979 NSF/ONR Workshop on Image Modeling in Chicago. It was published in Computer Graphics and Image Processing, vol. 12, pp. 309-325, 1980 and was republished in Image Modeling, A. Rosenfeld (ed.), Academic Press, New York, pp. 259-275, 1981.

30. Y-S. Chow, Estimation of conformal mappings, RPA No. 88, April 1980.

Ph.D. thesis supervised by U. Grenander. The analysis of morphogenesis in biology and in theoretical geography requires inference methods for estimating a set or a transformation determining a set. This thesis develops methods for estimation of highly structured transformations of two dimensional domains and of arcs in the plane. The method of sieves is used to construct consistent estimation schemes.

31. D. E. McClure, Cells and tissues, Group report with P. H. Bartels, K. S. Fu, G. H. Granlund, N. J. Pressman, K. Preston, Jr., J. M. S. Prewitt, B. Stenkvist, and G. Zajicek, in Biomedical Pattern Recognition and Image Processing, K. S. Fu and T. Pavlidis (eds.), Dahlem Konferenzen, Verlag Chemie, Weinheim, West Germany, pp. 331-361, 1979.

Reviews the state of the art and suggests future work on the modeling of problems in pattern recognition and image processing for cells and tissues.

32. R. M. Bates, The Gaussian measure of balls in a Hilbert space via stochastic integrals, RPA No. 89, May 1980.

Ph.D. Thesis supervised by U. Grenander. The problems analyzed in this thesis are motivated by applications of the method of sieves to the estimation of sets in high dimensional (or infinite dimensional) spaces. The density function of the norm of a Gaussian random vector is characterized by an integral representation and its behavior near zero and infinity is deduced. Numerical examples are included.

33. S. Geman, Notes on a self organizing machine, RPA No. 90, May 1980.

The paper proposes a design for a system to discover temporal and spatial regularities in a high dimensional environment. The development begins with a statistical model of the system's environment and then proposes mechanisms for learning, memory, and recall compatible with this model. The paper is published in Parallel Models for Associative Memory, J. Anderson and G. Hinton (eds.), Lawrence Erlbaum Associates, Hillsdale, NJ, 1981.

34. U. Grenander, Preliminaries to a thought experiment in mathematical morphogenesis, RPA No. 91, June 1980.

This report outlines a research program for 1980-81 to investigate with computational experiments and mathematical analysis certain models that are potentially useful in biological morphogenesis. Conjectures about the models are formulated and some preliminary results are sketched.

35. D. E. McClure, Optimized Grouping Methods, I and II, Statistisk tidskrift, vol. 18, pp. 101-110 and 189-198, 1980.

This two part paper investigates information loss from the grouping of continuous data, characterizes optimal grouping methods, and develops examples of the optimal schemes for Gaussian univariate and bivariate samples.

36. S. Geman, An application of the method of sieves: functional estimator for the drift of a diffusion, RPA No. 92, June 1980.

Consistent estimation schemes, based on the method of sieves and the principle of maximum likelihood, are developed when the target parameter is the drift function of an observed diffusion process. The paper is published in Proceedings of the Janos Bolyai Mathematical Society, 1980 Colloquium on Nonparametric Statistical Inference, North-Holland, 1980.

37. U. Grenander, Configuration dynamics under local chaos, RPA No. 93, June 1980.

A system of differential equations characterizing the macroscopic dynamics of a random field is derived from a model for localized transformations and chaotic motion of the configurations defining the field. The mathematical problems are motivated by models proposed by Grenander for morphogenesis.

38. C. Plumeri, Conditioning by INCLUSION when connection type is LINEAR, RPA No. 94, July 1980.

This report is based on thesis research supervised by D. E. McClure. It extends Plumeri's earlier results on how probability measures are affected by the regularity constraints of pattern theory. The characterization of the probability measures on regular configurations by their Markov dependence gives a basis for methods of rigorous statistical analysis of patterns generated by these models.

39. C. Plumeri, On convergence of sums of Markov random variables, RPA No. 96, September 1980.

This paper develops upper bounds for large deviation probabilities for sums of Markov dependent rvs. The results are used to assess the performance of tests based on linear statistics of hypotheses about the structure of Markov chains. These results are incorporated in Plumeri's Ph.D. thesis supervised by D. E. McClure.

40. U. Grenander and L-D. Wu, Estimating homomorphisms in mathematical semantics, RPA No. 98, October 1980.

The paper determines necessary and sufficient conditions for the identifiability of a function that maps strings over one finite alphabet into strings over another finite alphabet. Estimation algorithms are formulated.

41. S. Geman, Sieves for nonparametric estimation of densities and regressions, RPA No. 99, January 1981.

This report develops detailed analysis of specific sieves for density estimation and nonparametric regression. Convolution sieves for density estimation are related to the familiar Parzen-Rosenblatt kernel estimators. Sufficient conditions are determined for rates of convergence of the meshsize of a sieve so that the resulting estimators are strongly consistent.

42. C. Plumeri, Probability measures on regular structures, RPA No. 100, January 1981.

Ph.D. thesis supervised by D. E. McClure. This thesis develops a common functional-analytic method of establishing the Markovian dependence of random processes on linear graphs and on tree structures generated by the regularity constraints of pattern theory. Large-deviation results for Markov chains are also proved. The results address problems posed in U. Grenander's article on the second limit problem in pattern theory, RPA No. 83 (item 24 above).

43. Y-S. Chow, Estimation of conformal mapping, I, RPA No. 101, January 1981.

This article is based in part on the author's Ph.D. dissertation supervised by U. Grenander. It analyzes sieves that are designed to yield consistent estimators of conformal mappings of the complex plane. The mathematical analysis is motivated by models for biological morphogenesis.

44. Y-S. Chow, Estimation of conformal mapping, II, RPA No. 102, February 1981.

This paper is also based on part of the author's Ph.D. dissertation supervised by U. Grenander. It develops consistency results for estimators of conformal mappings obtained by the method of sieves.

45. Y-S. Chow and L-D. Wu, On algebraic closures of random samples, RPA No. 103, March 1981.

This paper develops sharp results on the rate of convergence of the algebraic closure of a random sample to the algebraic structure that contains the sample. The general results are illustrated with several specific examples.

46. C-R. Hwang, Conditioning by <EQUAL,LINEAR>, to appear in Trans. Amer. Math. Soc., 1981.

Hwang's paper obtains results related to those in RPA No. 100 (item 42 above) concerning the Markovian dependence of processes generated by regularity constraints from pattern theory on linear graphs. The results address problems posed by U. Grenander in RPA No. 83 (item 24 above).

47. W. Freiberger and U. Grenander, A triangulation sieve for restoring surface patterns, RPA No. 104, March 1981.

A sieve is designed and analyzed for nonparametric regression when the regression functions depend on two independent variables. The mathematical problems are motivated by applications to remote sensing, where a surface height is observed with additive noise.

48. J. Anderson, Experiments with the method of sieves for density estimation,RPA No. 105, March 1981.

This work, supervised by S. Geman and D. E. McClure, focuses on the small sample properties of density function estimators based on the principle of maximum likelihood and the method of sieves. Simulation studies compare alternative sieves and the dependence of the estimators on the selection of smoothing parameters. The experiments reported here were important for guiding the successful analysis of the consistency properties of cross-validated density estimators, RPA No. 110 (item 56 below).

49. W. Freiberger and U. Grenander, A method in pattern theory, Proc. Second Scandinavian Conference on Image Analysis, June 1981.

This paper presented at the Second Scandinavian Conference on Image Analysis reports on the work by Freiberger and Grenander on the design of sieves for surface estimation. The methods are motivated by reconstruction problems from remote sensing.

50. C-R. Hwang, A generalization of Laplace's method, Proc. Amer. Math. Soc., vol. 82, pp.446-451, 1981.

This paper is based on part of the author's Ph.D. dissertation. It presents results on the limiting behavior of Gibbs measures as the temperature of the system tends to zero. The results are interpreted in terms of Laplace's classical method of function optimization.

51. C-R. Hwang, Limit problem for some linear global constraints, Technical report, Instutute of Mathematics, Academia Sinica (Taiwan), 1981.

This report builds on Hwang's earlier analyses of regularity controlled probabilities. It characterizes processes that are globally constrained, in contrast to the local structural constraints from pattern theory. The work was carried out in part during Hwang's 1981 visit to Brown, supported by AFOSR.

52. C-R. Hwang, A note on an invariant property of linear global constraints, Technical report, Institute of Mathematics, Academia Sinica (Taiwan), 1981.

This report is concerned with regularity controlled probabilities, continuing the analysis of problems addressed in items 42, 46 and 51 above. Part of the research was done during Hwang's 1981 visit to Brown.

53. U. Grenander, Restoration of images deformed by "subjective time": sketch of an approach, RPA No. 107, August 1981.

This article maps out a research strategy for analyzing deformations of patterns defined on a one-dimensional background space. The problems are motivated by the need for methods of inference for time series observed in the presence of a random clock. The background deformations are of general interest in pattern analysis.

54. Y-S. Chow and L-D. Wu, On some limit theorems of regularity controlled probabilities, RPA No. 108, October 1981.

This paper extends classical limit theorems of probability (specifically, the law of large numbers) to dependent random variables induced by regularity constraints of pattern theory. The constraints are applied by conditioning the original probablity measure of the process by a particular event determined by the regularity constraints and having probability zero.

55. L-D. Wu, Consistent piecewise linear approximation, RPA No. 109, October 1981.

This report develops consistent methods of estimating the structure of a piecewise linear waveform from noisy observations of the waveform at equally spaced points. The results provide a theoretical basis for algorithms of approximating noisy functions by linear splines.

56. Y-S. Chow, S. Geman and L-D. Wu, Consistent cross-validated density estimation, RPA No. 110, October 1981.

This paper develops the first consistency results for cross-validation as a general technique of data-driven selection of smoothing parameters. The results apply to histogram and kernel estimators of univariate density functions. The paper will appear in Ann. Statist., 1983.

57. Y-S. Chow and L-D. Wu, On the stability of absorbing times of Markov chains, RPA No. 111, October 1981.

The paper investigates the dependence of the coefficient of variation of the absorption time of a Markov chain with one absorbing state as a function of the initial distribution of the chain and of the transition probabilities. Connections are drawn to motivating problems from biological morphogenesis.

58. L-D. Wu, Consistent piecewise linear approximation II: two-dimensional case, RPA No. 112, November 1981.

Consistent estimation schemes are designed for determining the structure of a piecewise linear function of two variables on the basis of noisy observations of the function at regularly spaced sites. The analysis provides a foundation for statistical methods of surface fitting.

59. Y-S. Chow, On density estimates, RPA No. 113, January 1982.

This paper develops sharper results than were previously known on the rate of convergence to zero of the width of a kernel function so that the kernel-estimates of a density function will converge in L1-norm with probability one. The main result also holds for multivariate density function estimation.

60. S. Geman and D. E. McClure, Characterization of a maximum-likelihood nonparametric density estimator of kernel type, RPA No. 114, March 1982.

A convolution sieve is defined for estimation of density functions by the method of maximum likelihood. The resulting estimators are shown to have the same general form as the familiar kernel estimators. At the same time it is shown that the kernel estimators do not maximize the likelihood function. The paper appears in the proceedings of the NASA Workshop on Density Estimation and Function Smoothing, Texas A&M University, March 1982 (L. F. Guseman, ed.).

61. D. E. McClure, Estimation of planar sets from Poisson projections, RPA No. 115, April 1982.

Consistent schemes are developed for set-estimation problems motivated by applications to single photon emission computed tomography. The results were presented in an invited talk at the NASA Workshop on Density Estimation and Function Smoothing at Texas A&M University, March 1982.

62. Y-S. Chow and U. Grenander, Restoration of images deformed by "subjective time" mechanisms, RPA No. 118, May 1982.

Sufficient conditions are described for the existence of consistent schemes of reconstructing patterns on a one-dimensional background when the background is deformed by a random "clock". The problems were posed by Grenander in RPA No. 107 (item 53 above).

IV. PERSONNEL.

Ulf Grenander, L. Herbert Ballou University Professor, Division of Applied Mathematics, Brown University.

Donald E. McClure, Professor of Applied Mathematics, Division of Applied Mathematics, Brown University.

Walter F. Freiberger, Professor of Applied Mathematics, Division of Applied Mathematics, Brown University.

Stuart Geman, Associate Professor of Applied Mathematics, Division of Applied Mathematics, Brown University.

Chii-Ruey Hwang, Visiting Assistant Professor of Applied Mathematics (1981), Research Associate (1978-79), Ph.D. Candidate (1977-78), Division of Applied Mathematics, Brown University. Ph.D. awarded June 1978. Dissertation topic: Frozen patterns and minimal energy states.

Igor Frolow, Ph.D. Candidate (1977-78). Ph.D. awarded June 1978. Dissertation topic: Abstract growth patterns.

Philip Thrift, Ph.D. Candidate (1977-78). Ph.D. awarded June 1978. Dissertation topic: Autoregression in homogeneous Gaussian configurations.

Donald E. Town, Ph.D. Candidate (1977-78). Ph.D. awarded June 1978. Dissertation topic: Restoration analysis of noise disrupted density functions.

Sally E. Howe, Ph.D. Candidate (1977-79). Ph.D. awarded June 1979. Dissertation topic: Estimating regions and clustering spatial data: analysis and implementation of methods using the Voronoi diagram.

Y-S. Chow, Research Associate (1980-81). Ph.D. Candidate (1977-80). Dissertation topic: Estimation of conformal mappings.

R. Bates, Ph.D. Candidate (1977-80). Ph.D. awarded June 1980. Dissertation topic: The Gaussian measure of balls in a Hilbert space via stochastic integrals.

C. Plumeri, Ph.D. Candidate (1978-81). Dissertation topic: Asymptotic probability measures on regular structures.

J. Anderson, Brown University Honors Student. Thesis topic: Experiments with the method of sieves for density estimation.

V. INTERACTIONS.

September 1977-July 1978, Donald E. McClure, Visiting Scholar, Department of Statistics, Stanford University.

October 1977, Donald E. McClure, Annual Meeting, SIAM, Albuquerque, NM.

April 1978, Donald E. McClure, Invited address, Special Session on Approximation Theory, Meeting of the American Mathematical Society, Houston TX.

May 1979, Donald E. McClure, Dahlem Conference on Biomedical Pattern Recognition and Image Processing, Commentator, Berlin.

August 1979, Donald E. McClure, Invited address, NSF/ONR Workshop on Image Modeling, Chicago IL.

September 1979. Ulf Grenander, Invited opening address, European Statistics Congress, Sofia, Bulgaria.

January-August 1980, Ulf Grenander, Visiting Professor, Mittag-Leffler Institute.

August 1980, Donald E. McClure, Invited address, Naval Postgraduate School, Monterey, CA and visit to Department of Statistics, Stanford University.

March 1981, Donald E. McClure, Thirteenth Symposium on the Interface between Computer Science and Statistics, Pittsburgh, PA.

June 1981, Walter Freiberger and Ulf Grenander, Invited address, Second Scandinavian Conference on Image Analysis.

June 1981, Donald E. McClure, Panelist, NSF Workshop on Structural and Syntactic Pattern Recognition, Saratoga Springs, NY.

August 1981, Donald E. McClure, Presentation of paper, IMS Annual Meeting, Vail, CO.

September-November 1981, C-R. Hwang, Visiting Assistant Professor, Division of Applied Mathematics, Brown University.