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THE GENERATION OF THREE-DIMENSIONAL BODY-FITTED  
COORDINATE SYSTEMS FOR VI. (U) MISSISSIPPI STATE UNIV  
MISSISSIPPI STATE DEPT OF AEROPHYSICS A. . . Z U WARSI  
JUL 83 AA5E-83-256 AFOSR-TR-83-1274

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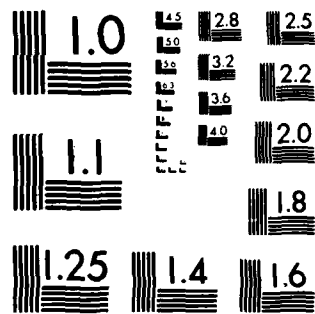
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COORDINATE SYSTEMS FOR VISCOUS FLOW PROBLEMS

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The Generation of Three-Dimensional Body-Fitted  
Coordinate Systems For Viscous Flow Problems

by

Z. U. A. Warsi\*

Abstract

An analytical model for the generation and redistribution of surface coordinates, which is to be used along with the full 3D code, has been developed. This essentially completes the development of spatial coordinate generation around multibodies and particularly around a wing-body combination in the 3D space. Numerical results for some multibody problems have been obtained.

Introduction

The problem of numerical grid generation for three-dimensional configurations through elliptic PDE's has been pursued under this grant for the last several years. Various reports and journal publications have explained the mathematical model in detail. This model, which is based on the numerical solution of three nonlinear PDE's has been programmed on Cray 1, and has been tested for single and two-body configurations as enclosed in a single outer boundary.

In the period of this report it was found necessary to have the capability of generating those surface coordinate systems which simplify the procedure of 3D coordinates generation through those equations which have already been developed in the previous reporting years. The main thrust for this development has come from the problem of wing-body

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combination. In the wing-body configuration we must, first of all, generate a suitable surface coordinate system which provides the Dirichlet boundary conditions for the full 3D code. Therefore a coordinate system is needed which reflects the trace of the body and wing intersection as one of the coordinates, and the coordinate along the wing as the outgoing coordinate from the fuselage. The developed analytical model which has been described in the next section generates those surface coordinates on the fuselage in which the trace of wing and body intersection is one of the surface curves. Based on the data so generated, the 3D equations as described in References 4-8 have been solved. Some results of the 3D coordinates for various multibodies are shown in Figures 1-5.

### Equations For The Transformed Coordinates In A Surface

Let  $(u,v)$  be a coordinate system in a surface (say a fuselage) on which the coordinate  $\eta = \text{const.}$ , and which has been generated by some means, e.g., from Craidon's routine (Ref. 9). We now wish to introduce other coordinates  $\xi = \xi(u,v)$  and  $\zeta = \zeta(u,v)$  in the same surface. From Warsi (Ref. 4), the fundamental equations for the generation of the cartesian coordinates  $\underline{r} = (x, y, z)$  as functions of  $(\zeta, \xi)$ , are

$$L\underline{r} + P\underline{r}_{\zeta} + Q\underline{r}_{\xi} = nR, \quad \eta = \text{const.}, \quad (1)$$

where

$$L = g_{11}^{\partial \zeta \zeta} - 2g_{13}^{\partial \xi \zeta} + g_{33}^{\partial \xi \xi}$$

$$\underline{n} = (X, Y, Z), \text{ surface normal.}$$

$$g_{11} = \underline{r}_{\xi} \cdot \underline{r}_{\xi}, \quad g_{13} = \underline{r}_{\xi} \cdot \underline{r}_{\zeta}, \quad g_{33} = \underline{r}_{\zeta} \cdot \underline{r}_{\zeta},$$

$$G_2 = g_{11}g_{33} - (g_{13})^2.$$

Now

$$\underline{r}_{\zeta} = \underline{r}_{u\zeta} + \underline{r}_{v\zeta},$$

$$\underline{r}_{\xi} = \underline{r}_{u\xi} + \underline{r}_{v\xi},$$

so that on substituting the terms  $\underline{r}_{\zeta\zeta}$ ,  $\underline{r}_{\xi\zeta}$ ,  $\underline{r}_{\xi\xi}$  in (1) and collecting terms, we get after some manipulations



$$au_{\zeta\zeta} - 2bu_{\xi\zeta} + cu_{\xi\xi} + Pu_{\zeta} + Qu_{\xi} = J_2^2 \Delta_2 u , \quad (2)$$

$$av_{\zeta\zeta} - 2bv_{\xi\zeta} + cv_{\xi\xi} + Pv_{\zeta} + Qv_{\xi} = J_2^2 \Delta_2 v , \quad (3)$$

where

$$J_2 = u_{\zeta}v_{\xi} - u_{\xi}v_{\zeta} ,$$

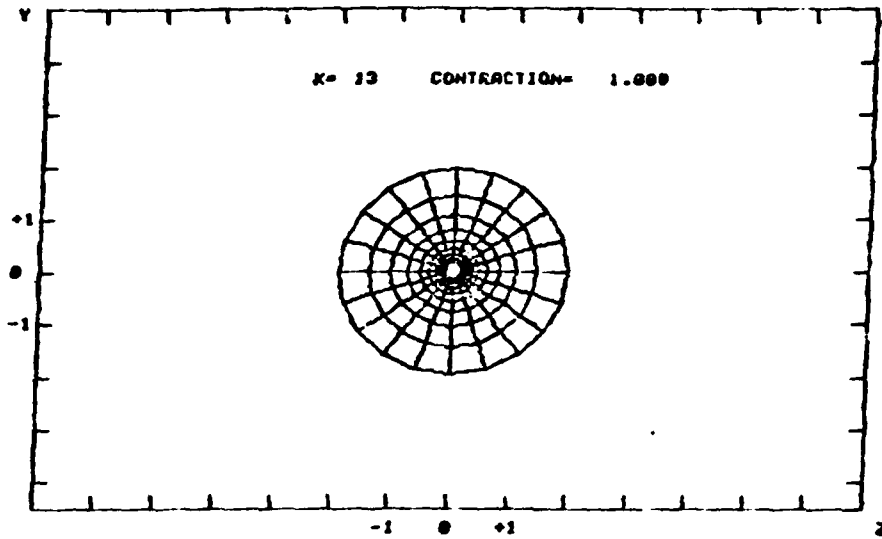
and a,b,c are the coefficients which have the derivatives  $u_{\xi}$ ,  $u_{\zeta}$ ,  $v_{\xi}$ ,  $v_{\zeta}$ , also  $x_u$ ,  $x_v$ , etc. and  $\Delta_2 u$ ,  $\Delta_2 v$  are the Beltramians. By imposing the appropriate boundary conditions on u and v the equations (2) and (3) can be solved as a 2D grid generation problem for the new coordinates  $\xi$  and  $\zeta$ . This routine can therefore be used to redistribute the surface coordinates in any manner from any given surface coordinates u and v.

The preceding equations have been programed as a subroutine with the general 3D program and is used when the coordinates for a wing-body combination are to be generated. The trace of the wing with the fuselage is considered to form an inner closed curve and the outer curve is any arbitrary closed curve near the mid section of the fuselage (Fig. 6). Equations (2) and (3) then provide the curves which are consistent with the 3D routine for the generation of space coordinates.

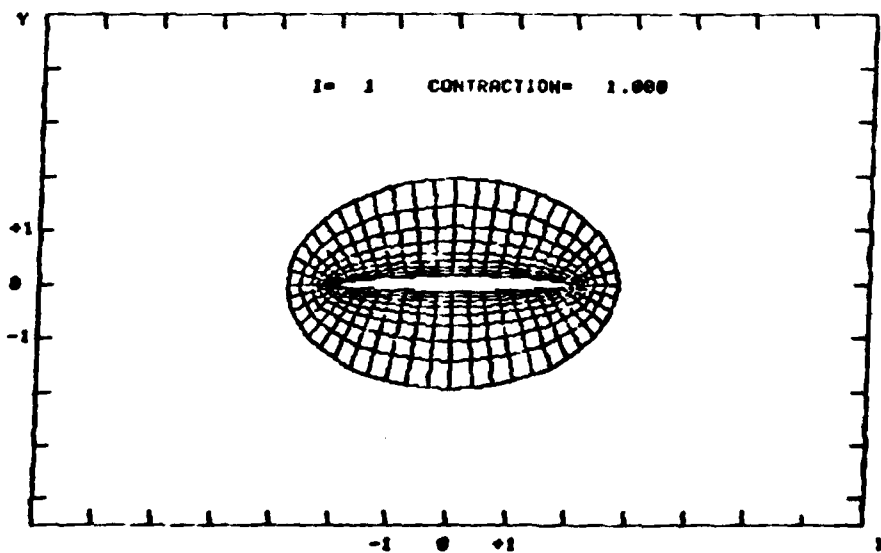
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(Note: Papers and reports written under the contract have been marked by an asterisk)

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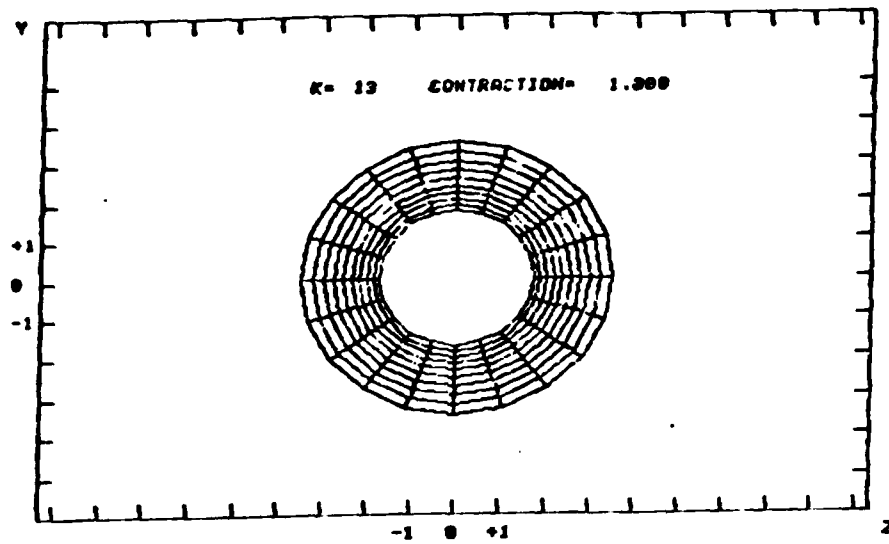


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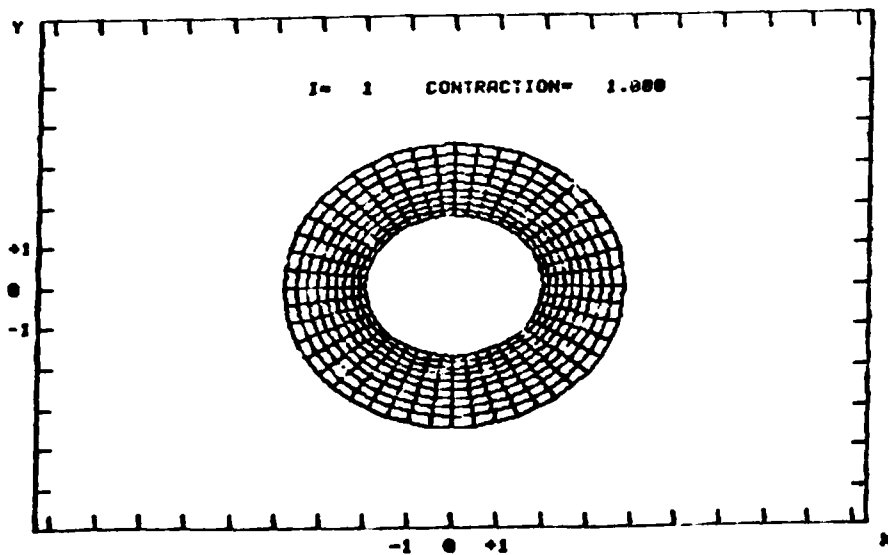


(b)

Fig. 1 Three-dimensional coordinates between a thin ellipsoid surrounded by an ellipsoid. (a) Front section, (b) Meridinal section.



(a)



(b)

Fig. 2 Three-dimensional coordinates between a thick ellipsoid surrounded by an ellipsoid. (a) Front section, (b) Meridional section.

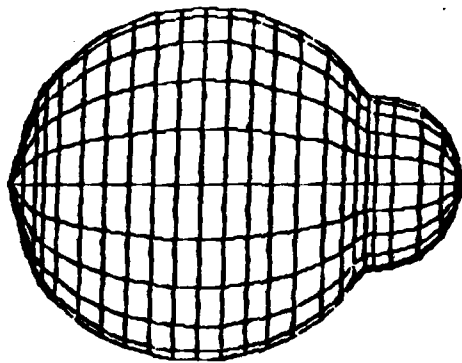
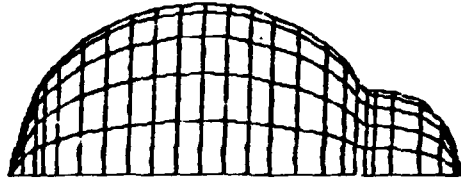
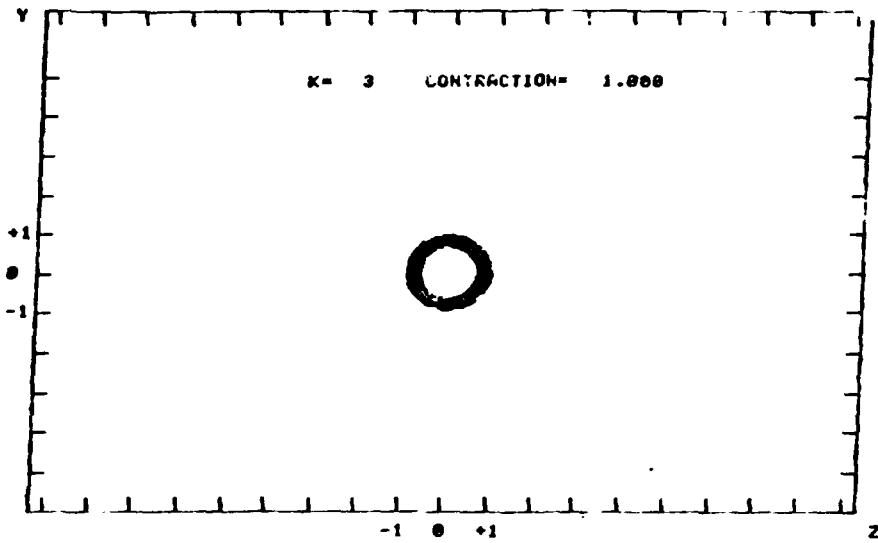
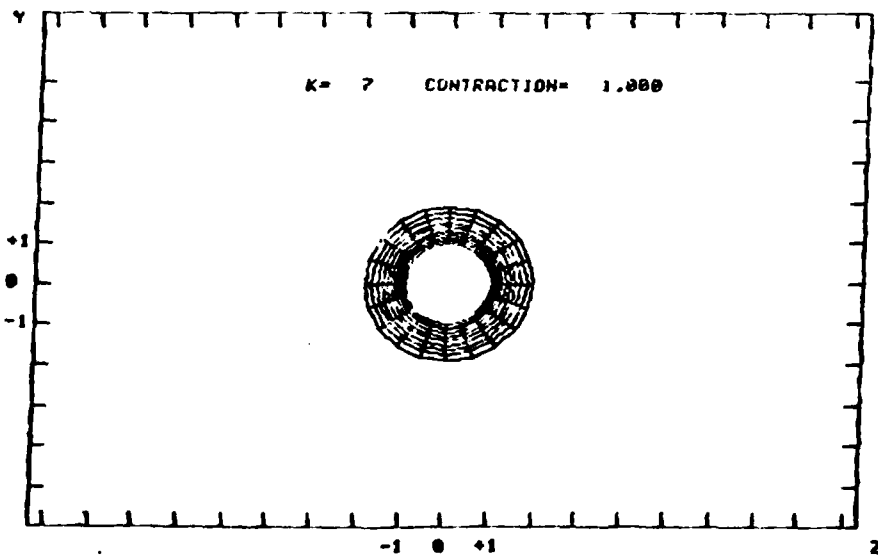


Fig. 3 Two-dimensional surface coordinates on the surface formed of two intersecting spheres.

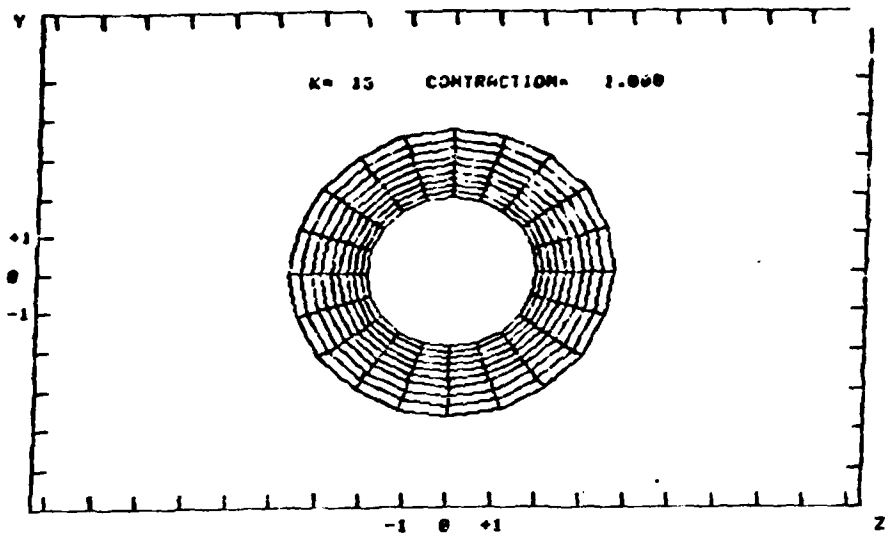


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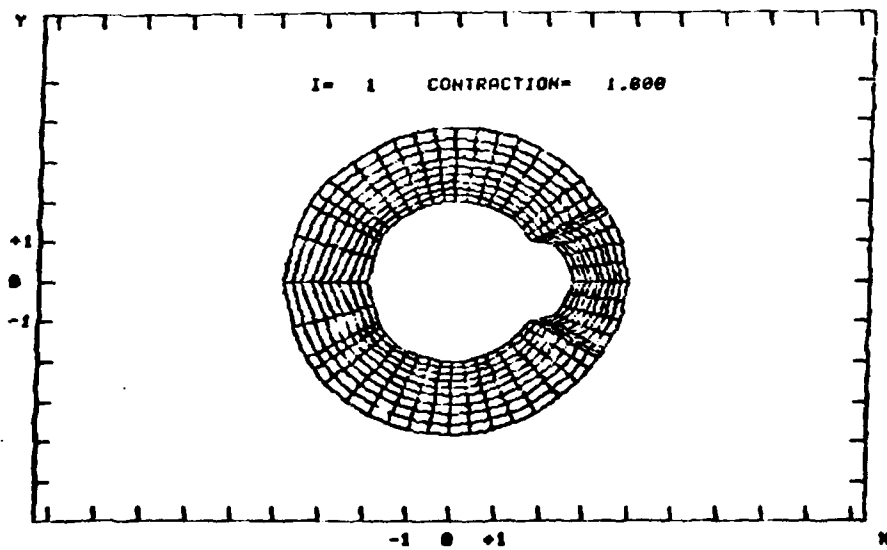


(b)

Fig. 4 caption on next page.



(c)



(d)

Fig. 4 Three-dimensional coordinates between the surface of Fig. 3 and an outer sphere. (a) Front section of the small sphere, (b) Front section at the junction, (c) Front section of the large sphere, (d) Meridinal section.

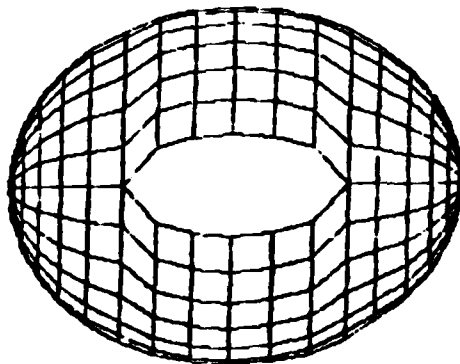
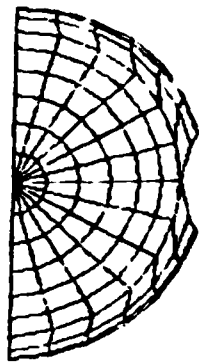


Fig. 5 Surface coordinates on an ellipsoid with an elliptical slit.



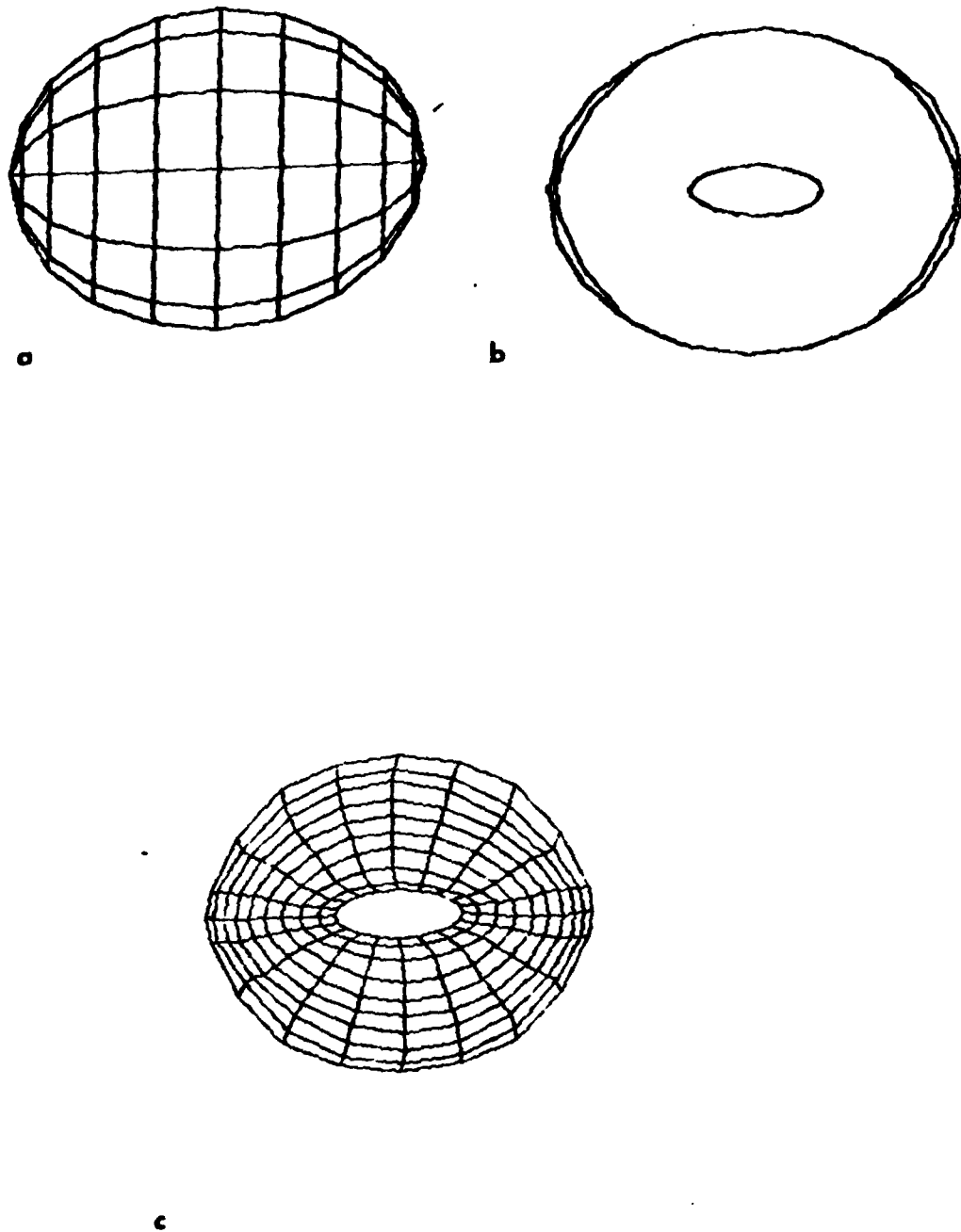


Fig. 6 Coordinate redistribution in a surface obtained by solving Eqs. (2) and (3). (a) Coordinates  $u, v$  on an ellipsoid through Ref. 9, (b) Choice of the inner and outer curves  $\xi = \xi_1$  and  $\xi = \xi_2$ . (c) Generated coordinates  $\xi = \text{const.}$ , and  $\zeta = \text{const.}$

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