AD-A136 418 COMPUTATIONAL SOLUTION OF RANDOM EQUATIONS(U) GEORGIA 1/1
INST OF TECH ATLANTA SCHOOL OF MATHEMATICS
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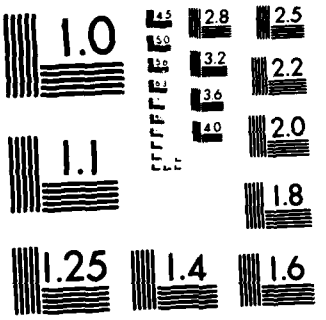

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# COMPUTATIONAL SOLUTION OF RANDOM EQUATIONS



FINAL REPORT



27 NOVEMBER 1983

U.S. ARMY RESEARCH OFFICE

CONTRACT NO. DAAG-29-81-K-0174

GEORGIA INSTITUTE OF TECHNOLOGY



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|---|---|---|
| 1. REPORT NUMBER<br>1 | 2. GOVT ACCESSION NO.<br>AD A136 418 | 3. RECIPIENT'S CATALOG NUMBER |
| 4. TITLE (and Subtitle)<br>COMPUTATIONAL SOLUTION OF RANDOM EQUATIONS | | 5. TYPE OF REPORT & PERIOD COVERED<br>Final Report<br>9/1/81 - 11/30/83<br>6. PERFORMING ORG. REPORT NUMBER |
| 7. AUTHOR(s)<br>A. T. Bharucha-Reid | | 8. CONTRACT OR GRANT NUMBER(s)<br>DAAG29-81-K-0174 |
| 9. PERFORMING ORGANIZATION NAME AND ADDRESS<br>School of Mathematics<br>Georgia Institute of Technology<br>Atlanta, GA 30332 | | 10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS |
| 11. CONTROLLING OFFICE NAME AND ADDRESS<br>U. S. Army Research Office<br>Post Office Box 12211<br>Research Triangle Park, NC 27709 | | 12. REPORT DATE<br>30 November 1983<br>13. NUMBER OF PAGES |
| 14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)<br>O.N.R., Resident Representative<br>GIT, 206 O'Keefe Bldg.<br>Atlanta, GA 30332 | | 15. SECURITY CLASS. (of this report)<br>Unclassified<br>15a. DECLASSIFICATION/DOWNGRADING SCHEDULE |
| 16. DISTRIBUTION STATEMENT (of this Report)<br>Approved for public release; distribution unlimited. | | |
| 17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report) | | |
| 18. SUPPLEMENTARY NOTES<br>THE VIEW, OPINIONS, AND/OR FINDINGS CONTAINED IN THIS REPORT ARE THOSE OF THE AUTHOR(S) AND SHOULD NOT BE CONSTRUED AS AN OFFICIAL DEPARTMENT OF THE ARMY POSITION, POLICY, OR DECISION, UNLESS SO DESIGNATED BY OTHER DOCUMENTATION. | | |
| 19. KEY WORDS (Continue on reverse side if necessary and identify by block number)<br>Random equations, numerical methods, simulation, fixed point theorems, limit theorems. | | |
| 20. ABSTRACT (Continue on reverse side if necessary and identify by block number)<br>This report presents a review of results obtained in connection with ARO Contract No. DAAG29-81-K-0174 for the period 1 October 1981 - 30 November 1983. Results presented concern numerical results for random polynomials, random integral equations, random integrodifferential equations and fixed point theorems. | | |

1. Statement of the Problems Studied

The research project was concerned with the systematic development of computational methods for random equations. An earlier ARO research project (DAAG29-77-G-0164) was concerned primarily with the development of computational methods for the solution of random integral equations. This project was concerned with the computational solution of random integral equations as well as other classes of random equations, with special reference to computer implementation of general methods for obtaining approximate solution of other classes of random equations. In particular, we were concerned with computer implementation of (1) approximate methods for solving random linear algebraic systems of equations, (2) projection methods for solving random operator equations, and (3) iterative methods for solving random operator equations.

2. Summary of the Most Important Results Obtained

All of the results obtained can, in a given context, be regarded as important.

(a) Paper A-1 was presented at the Montreal meeting by Bharucha-Reid as a keynote address. It gives an up-to-date account of general methods for solving random integral equations, and surveys studies on the approximate solution of random equations.

(b) In Paper A-2 the authors consider a system of random ODEs and the corresponding deterministic system obtained

by averaging first. Methods due to Christensen and Bharucha-Reid on the roots of random algebraic polynomials are used. The estimates obtained enable workers to estimate how much the determinstic solution differs from the random solution.

(c) In Paper A-3 we give estimates of the variance of the number of real roots in the general case of random algebraic polynomials with dependent coefficients. Using simulation methods we also determine the coefficient of variation of the number of real roots.

(d) In Paper B-1 the author discusses the importance of the study of the limiting distribution of systems of linear algebraic equations with random coefficients in the numerical analysis of random integral equations.

(e) In Paper B-2 the authors use methods developed by von Scheidt and Purkert (see their forthcoming book on random eigenvalue problems) to obtain the best estimate to-date of the difference between the roots of a random algebraic polynomial and the roots of the mean (or deterministic) polynomial. The estimates are better than those obtained earlier by Christensen and Bharucha-Reid.

(f) In Paper B-3 the authors prove a theorem on the limiting distribution of random algebraic polynomials. (We are now considering the application of this result to random integral equations.)

(g) In Paper B-4 the authors obtain estimates of the number of real roots of random trigonometric polynomials with dependent coefficients.

(h) In Paper B-5 the authors obtain the best lower bound to-date of the expected number of real roots of random algebraic polynomials.

(i) In Paper B-6 the authors prove a beautiful result on the limiting distribution of the roots of random trigonometric polynomials. They show that the roots are uniformly distributed in a symmetric region around the imaginary axis.

(k) The results of Paper B-7 are clearly indicated by the title.

(l) In Paper C-1 the authors use random Chebyshev polynomials to solve random Fredholm equations. A concrete example is presented.

(m) In Paper C-2 and 3 the authors study the random integro-differential equation.

$$x'(t,\omega) = A(t) \times (t, \omega) + \int_0^t B(t,r) \times (\tau,\omega)d\tau + f(t,\omega).$$

The titles of these papers indicate the problems considered.

(n) In Paper C-4 the authors introduce for the first time various measures of noncompactness in order to obtain fixed point theorems for random operators.

(o) In Papers C-5, 6, and 7 the authors consider in a very

rigorous way the notions of random projection methods for random equations. Those papers are now in the second draft stage, and will be finished by March 1984.

## 3. List of Publications

### A. Published Papers


1. Bharucha-Reid, A. T. and Christensen, M. J., "Approximate solution of random integral equations: General methods," Proc. 10th IMACS World Congress on System Simulation and Scientific Computation, 4 (1982), 299-304. (A revised version of this paper will appear in a special issue of the journal Mathematics and Computers in Simulation, which will be edited by Bharucha-Reid and Tsokos.

2. Ladde, G. S. and Sambandham, M., "Stochastic and deterministic," Math. Comp. Simulation 24 (1982), 507-514.

3. Sambandham, M., Thangaraj, V., and Bharucha-Reid, A.T., "On the variance of the number of real roots of random algebraic polynomials," Stochastic Anal. Appl. 1 (1983), 215-238.


### B. Papers in Press


1. Bharucha-Reid, A. T., "Systems of linear algebraic equations with random coefficients and the numerical solution of random integral equations," Wissenschaftliche Beiträge Ingenieurhochschule, 1983. (This is an invited paper.)

2. vom Schedit, T. and Bharucha-Reid, A. T., "On the averaging problem for the roots of random algebraic polynomials," Beiträge zur numerische Mathematik.

3. vom Scheidt, J. and Bharucha-Reid, A. T., "On the distribution of the roots of random algebraic polynomials," Zeitschrift für angewandte Mathematik.

4. Sambandham, M. and Thangaraj, V., "On the real zeros of random trigonometric polynomials," J. Indian Math. Soc.

5. Ladde, G. S., Lakshmikantham, V. and Sambandham, M., "Error estimates of solutions and mean solutions of random differential equations through comparison principles," Stochastic Anal. Appl.

C. Papers in Preparation

1. Bharucha-Reid, A. T. and Sambandham, M., "On the solution of random Fredholm integral equations using random Chebyshev polynomials."

2. Kanna, D., Bharucha-Reid, A. T. and Martin, B., "On a class of random integrodifferential equations: I. Existence, uniqueness, and uniqueness of solution, and properties of the solution process.

3. Bharucha-Reid, A. T. and Martin, B., "On a class of random integrodifferential equations: II. Numerical methods of solution."

4. Chandrasekharan, P. S. and Bharucha-Reid, A. T., "On measure of noncompactness and fixed points of random operators."

5. Römisch, W. and Bharucha-Reid, A. T., "Projection methods for the solution of random equations: I. Convergence of projection method."

6. Römisch, W. and Bharucha-Reid, A. T., "Projection methods for the solution of random equations: II. Convergence of solution measures."

7. Römisch, W. and Bharucha-Reid, A. T., "Projection methods for the solution of random equations: III. Applications."

8. Christensen, M. J. and Sambandham, M., "A limit theorem for random trigonometric polynomials."

D. Books and Monographs in Preparation

1. Bharucha-Reid, A. T. and Sambandham, M., "Random Polynomials," Academic Press (to appear in 1984).

2. Bharucha-Reid, A. T. and Christensen, M. J., "Approximate Solution of Random Integral Equations."

4. List of all Participating Scientific Personnel

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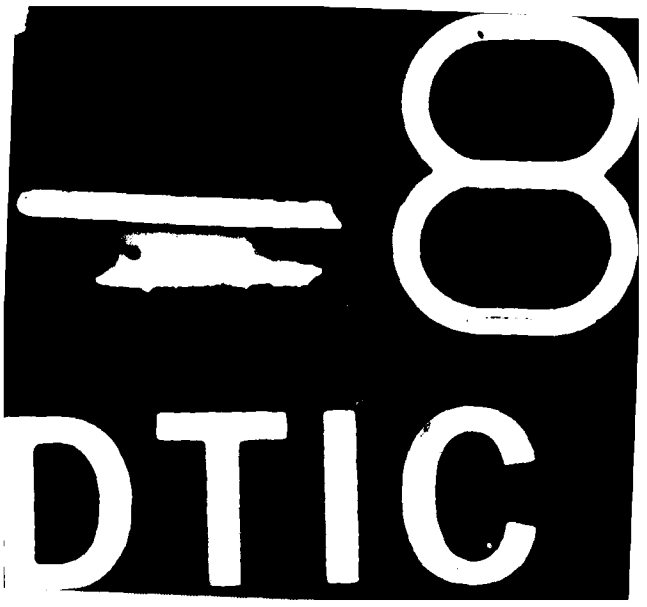
*Supported by ARO funds.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