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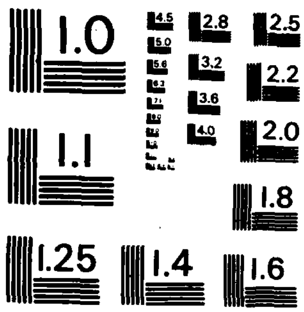

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A SINGULAR FREE BOUNDARY PROBLEM

Klaus Höllig and John A. Nohel

Mathematics Research Center
University of Wisconsin—Madison
610 Walnut Street
Madison, Wisconsin 53705

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A SINGULAR FREE BOUNDARY PROBLEM

Klaus Höllig[1][2] and John A. Nohel[1]

Dedicated to Karl Nickel on his 60th birthday

Technical Summary Report #2582

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ABSTRACT

We study the singular Cauchy problem

$$v_t = \phi(v)_{xx} \quad , \quad (x,t) \in \mathbb{R} \times [0,T], \ T > 0$$

$$v(x,0) = g(x) \quad , \quad x \in \mathbb{R} \ .$$

The constitutive function $\phi(\xi) = \max(0,\xi)$; the initial datum g is smooth (bounded) on $\mathbb{R}\setminus\{0\}$ and satisfies

$$g(0) = 0; \ xg(x) > 0, \ x \in \mathbb{R}; \ g'(0^+) \cdot g'(0^-) \neq 0 \quad ,$$

where the superscript "+" ("-") denotes the limit from the right (left). We show that the free boundary s given by $v(s(t)^+,t) = 0$ satisfies

$$s(t) = -\kappa\sqrt{t} + o(\sqrt{t}) \qquad (t \to 0^+) \quad ,$$

where $\kappa > 0$ is a monotone function of $p = g'(0^+)/g'(0^-)$, implicitly defined by the equation

$$(*) \qquad p = \frac{\kappa^2}{2} + \frac{\kappa^3}{4} e^{\kappa^2/4} \int_{-\kappa}^{\infty} e^{-t^2/4} dt, \ \kappa > 0 \quad .$$

This generalizes an earlier analysis [7] for the special case of smooth data g in which $g'(0^+) = g'(0^-) \neq 0$, and $p = 1$. In this case the numerical value $\kappa = .903446...$ as computed from (*) is consistent with our previous result.

AMS (MOS) Subject Classifications: 35K055, 35K65, 45G05

Key Words: Cauchy problem, parabolic, nonlinear, regularity of free boundary, self similar solutions, nonlinear singular integral equation.

Work Unit Number 1 - Applied Analysis

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SIGNIFICANCE AND EXPLANATION

The Cauchy problem stated in the Abstract is similar to the well-known one phase Stefan problem (in one space dimension). In the latter one would assume $g(x) \equiv -1$ for $x < 0$, as well as $g(x) > 0$ for $x > 0$, so that g would have a jump discontinuity at $x = 0$. Our assumptions on the initial data g yield a different behavior of the solution v and of the resulting free boundary. Indeed, the free boundary is not (infinitely) differentiable at $t = 0$, contrary to the situation for the classical Stefan problem.

This problem also serves as a prototype of nonlinear parabolic problems which arise as monotone "convexifications" of nonlinear diffusion equations with nonmonotone constitutive functions ϕ (see [5], [6]). That analysis shows the existence of infinitely many solutions v of the nonmonotone problem each having v bounded, but oscillating more and more rapidly as $t \to 0(+)$. Thus each solution v exhibits phase changes. Numerical experiments further suggest the conjecture that the "physically correct" solution of the nonmonotone problem is the one which for $t > 0$ sufficiently large approaches the unique solution of the appropriately related convexified monotone problem. This paper is another step towards the understanding of this intriguing phenomenon. Our earlier analysis of the Cauchy problem stated in the abstract was for smooth data g with $g'(0^+) = g'(0^-) > 0$, [7]. The present analysis shows how the free boundary $s(t)$ depends near $t = 0$ on more general data for which $p = g'(0^+)/g'(0^-)$ need no longer be 1. We present two approaches: (i) a generalization of the integral equation for the free boundary studied in [7], (ii) a preliminary analysis via self similar solutions which will be exploited in [8].

The responsibility for the wording and views expressed in this descriptive summary lies with MRC, and not with the authors of this report.

A SINGULAR FREE BOUNDARY PROBLEM

Klaus Höllig[(1)(2)] and John A. Nohel[(1)]

1. Introduction.

We study the Cauchy problem

$$(1) \qquad \begin{cases} v_t = \phi(v)_{xx}, \ (x,t) \in \mathbb{R} \times [0,T] \ , \\ v(\cdot,0) = g \end{cases}$$

with the piecewise linear constitutive function $\phi(\xi) = \max(0,\xi)$; the initial data g are assumed bounded, smooth on $\mathbb{R}\setminus\{0\}$ and satisfy

$$(2) \qquad \begin{cases} g(0) = 0; \ xg(x) > 0, \ x \in \mathbb{R} \ , \\ g'(0^+) \cdot g'(0^-) \neq 0 \ . \end{cases}$$

Here, the superscript "+" ("-") denotes the limit from the right (left).

One motivation for the study of the Cauchy problem (1) - (2) is that it serves as a prototype of nonlinear parabolic problems which arise as monotone "convexifications" of nonlinear diffusion equations with nonmonotone constitutive functions ϕ (see [5] and [6]); in [6, section 4] the reader will find the formulation and preliminary analysis of such a convexified problem. In this note we are primarily interested in the behavior of the free boundary s, given by $v(s(t)^+,t) = 0$, for small t and for this purpose it is sufficient to consider the simplified model (1) - (2).

Problem (1) - (2) is similar to the one phase Stefan problem [3, 4, 9, 10] where $g(x) = -1$ for $x < 0$. However, the assumption (2) yields a different behavior of the free boundary and of the solution v. In fact, for smooth initial data $(g'(0^+) = g'(0^-))$, v_x is not continuous at $(x,t) = (0,0)$, whereas solutions to the Stefan problem are smooth on the set $\{(x,t) : x > s(t), \ t \in [0,T]\}$.

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For a sufficiently regular solution, problem (1) - (2) is equivalent to the free boundary problem

$$(3)\qquad \begin{cases} v_t = v_{xx} \quad , \quad s(t) < x < \infty \quad , \\ v(s(t),t) = 0 \\ v(x,0) = g(x) \end{cases}$$

$$(4)\qquad \begin{cases} g(s(t))s'(t) = v_x(s(t),t), \ t > 0 \quad , \\ s(0) = 0 \quad . \end{cases}$$

One easily verifies that a solution of (2) - (4) which is extended by

$$(5)\qquad v(x,t) = g(x) \quad , \quad -\infty < x < s(t) \quad ,$$

is a weak solution of problem (1), (2).

In [7] we studied the equivalent integrated version of problem (1) in the form

$$(1')\qquad \begin{cases} u_t = \phi(u_x)_x \\ u(x,0) = f(x) := \int_0^x g(y)dy \quad . \end{cases}$$

Assuming sufficient regularity, the solutions of problems (1) and (1') are related by

$$u(x,t) = \int_0^x v(y,t)dy + \int_0^t \phi(v)_x(0,\tau)d\tau \quad .$$

The boundary condition (4) can be also easily derived from (1'): continuity of u across the free boundary s implies

$$f(s(t)) = u(s(t),t) \quad .$$

Differentiating with respect to t and using $u_x(s(t)^+,t) = v(s(t)^+,t) = 0$ and $f'(x) = g(x)$ we obtain (4).

We assumed in [7] that the initial data f for the Cauchy problem (1') are smooth: $f \in C^3(\mathbb{R})$ with bounded derivatives, $xf'(x) > 0 (x \in \mathbb{R})$ and $f''(0) > 0$ (i.e. for (1) $g'(0^-) = g'(0^+) > 0$). We showed that the free boundary $s(t)$ for (1') (and also for (1) with $g'(0^-) = g'(0^+) > 0$) satisfies

$$s(t) = -\kappa\sqrt{t} + o(\sqrt{t}) \quad (t \to 0^+) \quad , \tag{6}$$

where the constant $\kappa = 0.903446\ldots$ does not depend on g. This is not consistent with (4) which formally implies

$$g'(0)s(t)s'(t) = g'(0) + o(1) \quad ,$$

i.e. (6) with $\kappa = \sqrt{2}$. Therefore, the solution v of (1) cannot have v_x continuous at $(x,t) = (0,0)$ (equivalently the solution u of (1') cannot have u_{xx} continuous at $(x,t) = (0,0)$).

The purpose of this note is to extend these results and study the effect of a discontinuity in the initial data g' for (1) at $x = 0$ on the local behavior of the free boundary s at $t = 0$. Our result shows that (6) still holds, but with κ now determined by the ratio $g'(0^+)/g'(0^-)$.

Theorem. *Let g satisfy the assumptions (2). The problem (3), (4) has a unique solution (v,s) having v bounded and the free boundary s satisfies (6). The constant κ is a monotone function of $p := g'(0^+)/g'(0^-)$ which is implicitly given by*

$$p = \frac{\kappa^2}{2} + \frac{\kappa^3}{4} e^{\kappa^2/4} \int_{-\kappa}^{\infty} e^{-y^2/4} dy \quad . \tag{7}$$

Figure 1, at the end of the paper, shows κ as a function of p as computed from (7). We observe from (7) that

$$p(\kappa) = \frac{\kappa^2}{2} + o(1) \quad \text{as} \quad \kappa \to 0 \quad ,$$

$$p(\kappa) = \sqrt{\pi}\, \frac{\kappa^3}{2} e^{\kappa^2/4}[1+o(1)] \quad \text{as} \quad \kappa \to \infty \quad .$$

For smooth initial data (i.e. $g'(0^-) = g'(0^+)$) we have $p = 1$ and $\kappa = 0.9034\ldots$ which is consistent with our result in [7].

A complete proof of the Theorem will be included in a forthcoming joint paper with J. Vazquez [8]. It is based on comparison arguments using self similar solutions of problem (1). More generally, we shall consider initial data of the form

$$g(x) = \begin{cases} x^{m_+} g_+(x) , & x > 0 , \\ -|x|^{m_-} g_-(x) , & x < 0 , \end{cases}$$

with $g_\pm(0^\pm) > 0$ and determine the local behavior of s in all cases.

In this note we prove the Theorem by extending the method used in [7]. This approach, compared to that using similar solutions, yields additional regularity for the free boundary, namely for any $\alpha \in (0,\frac{1}{2})$ we have

$$\frac{d}{dt}\,[g(s(t))s'(t)] = O(t^{\alpha-1}), \quad \text{as} \quad t \to 0^+ \quad . \tag{8}$$

However, for technical reasons, we have to assume that $p < 1$.

We begin by constructing in Section 2 self similar solutions to problem (2) - (4) and derive the relation (7). The behavior of these solutions is typical for the general case treated in Section 3. We prove (6) and (8) by solving a nonlinear integral equation for the function $r(t) := g(s(t))s'(t)$.

2. Self similar solutions.

For the special initial data

$$g(x) = \begin{cases} p_+ x , & x > 0 , \\ p_- x , & x < 0 , \end{cases} \tag{9}$$

it will be shown below that problem (2) - (4) has self similar solutions of the form

(10) $$v(x,t) = t^{1/2}\psi(xt^{-1/2}) \quad .$$

Substituting (10) into equations (3) and (4) and assuming that $s(t) = -\kappa\sqrt{t}$ we see that ψ must satisfy the ordinary differential equation

(11) $$\psi''(\xi) + \frac{1}{2}\xi\,\psi'(\xi) - \frac{1}{2}\psi(\xi) = 0, \ \xi > -\kappa \quad ,$$

with the boundary conditions

(12) $$\psi(-\kappa) = 0, \ \psi'(-\kappa) = p_- \frac{1}{2}\kappa^2$$

(13) $$\lim_{\xi\to\infty} \xi^{-1}\psi(\xi) = p_+ \quad .$$

Note, that (13) is equivalent to the initial condition $v(x,0) = p_+x$. Observing that $c_1\xi$ is a particular solution of (11) for any constant c_1, one easily finds that the general solution of (11) is

$$\psi(\xi) = c_1\xi + c_2(2e^{-\xi^2/4} + \xi\int_0^\xi e^{-y^2/4}dy) \quad .$$

From (12) we obtain

$$c_1 = p_-(\frac{\kappa^2}{2} + \frac{\kappa^3}{4}e^{\kappa^2/4}\int_0^\kappa e^{-y^2/4}dy)$$

$$c_2 = p_-\frac{\kappa^3}{4}e^{\kappa^2/4} \quad .$$

Therefore, by (13), p_+, p_- and κ must satisfy

$$c_1 + c_2\sqrt{\pi} = p_+ \quad ,$$

which is the relation (7). This establishes the existence of self similar solutions for the model initial data (9). Since $s(t) = -\kappa\sqrt{t}$ the assertions (6) and (8) of the Theorem are trivially valid in this case.

3. Proof of the Theorem.

Existence and uniqueness of weak solution for problem (1) follow from nonlinear semigroup theory [2]. It is therefore sufficient to show existence of a solution (v,s) for problem (2) - (4) and prove the assertions (6) - (8) for the free boundary s. Making the restrictive assumption $p \leq 1$ this can be done by extending the method in [7].

Let $\Gamma(x,t) := \frac{1}{\sqrt{2\pi t}} \exp(-\frac{x^2}{4t})$ denote the fundamental solution of the heat equation. Assume that (v,s) is a solution of problem (2) - (4) for which the function

$$r(\tau) := v_x(s(\tau),\tau) \tag{14}$$

is bounded and continuous on $(0,t]$. Integrating Green's identity

$$(\Gamma(x-\xi,t-\tau)w_\xi(\xi,\tau) - \Gamma_x(x-\xi,t-\tau)w(\xi,\tau))_\xi - (\Gamma(x-\xi,t-\tau)w(\xi,\tau))_\tau = 0$$

over the domain $\{(\xi,\tau) : s(\tau) < \xi < \infty,\ \tau \in (0,t)\}$, following [7] one can derive an integral equation for r:

$$r(t) = 2\int_0^\infty \Gamma(s(t)-\xi,t)g'(\xi)d\xi - 2\int_0^t \Gamma_x(s(t)-s(\tau),t-\tau)r(\tau)d\tau \quad . \tag{15}$$

With the abbreviation

$$A(s,t,\tau) := \frac{s(t)-s(t\tau)}{2(t-t\tau)^{1/2}}$$

this equation can be rewritten as

$$\begin{aligned} r(t) &= \frac{1}{\sqrt{\pi}}\int_0^\infty \exp(-\frac{1}{4}(\frac{s(t)}{\sqrt{t}} - \xi)^2)g'(\xi\sqrt{t})d\xi \\ &+ \frac{1}{\sqrt{\pi}}\int_0^1 \frac{A(s,t,\tau)}{1-\tau}\exp(-A(s,t,\tau)^2)r(t\tau)d\tau \\ &:= (Fr)(t) + (Kr)(t) \quad . \end{aligned} \tag{16}$$

The Theorem is a consequence of the following existence result for (16).

Proposition. Assume that

$$p = g'(0^+)/g'(0^-) =: p_+/p_- \leq 1 \quad .$$

Let s and r in (16) be related by

(17) $$r(t) = g(s(t))s'(t) \quad .$$

Then, for any $\alpha \in (0,1/2)$, there exists $T > 0$ such that the integral equation (16) has a solution Γ for which

(18) $$\sup_{t\in(0,T)} t^{1-\alpha}|f'(t)| < \infty \quad .$$

We postpone the proof of this proposition and complete the proof of the Theorem.

Let r be a solution of (16) for which (18) holds. Then, it is easy to see that s, given by (17), satisfies

(19) $$s(t) + \kappa\sqrt{t} = O(t^{1/2+\alpha}) \quad ,$$

with $\kappa = (2r(0)/p_-)^{1/2}$. To verify (7) we pass to the limit $t \searrow 0$ in (16). Since

$$A(s,0,\tau) = -\frac{\kappa}{2}\frac{1-\sqrt{\tau}}{\sqrt{1-\tau}} \quad ,$$

we obtain

(20) $$\begin{aligned} p_-\frac{\kappa^2}{2} &= \frac{1}{\sqrt{\pi}}\int_0^\infty \exp(-\tfrac{1}{4}(\kappa+\xi)^2)p_+ \, d\xi \\ &- \frac{1}{\sqrt{\pi}}\int_0^1 \frac{\kappa}{2}\frac{1}{\sqrt{1-\tau}(1+\sqrt{\tau})}\exp(-\frac{\kappa^2}{4}\frac{1-\sqrt{\tau}}{1+\sqrt{\tau}})p_-\frac{\kappa^2}{2}\, d\tau \quad . \end{aligned}$$

Substituting $y^2 = \dfrac{1-\sqrt{\tau}}{1+\sqrt{\tau}}$ in the second integral and solving for $p = p_+/p_-$ we get

(20') $$p = \left(\frac{\kappa^2}{2} + \frac{\kappa^3}{4}\frac{2}{\sqrt{\pi}}\int_0^1 \frac{1-y^2}{1+y^2}e^{-\kappa^2y^2/4}dy\right) \Big/ \frac{1}{\sqrt{\pi}}\int_\kappa^\infty e^{-y^2/4}dy \quad .$$

A simple calculation shows that this is equivalent to (7): Subtracting the right hand sides of equations (7) and (20') and multiplying by $\sqrt{\pi}\,\kappa^{-3}[\cdots]$ gives

$$\left(\frac{1}{2\kappa} + \frac{1}{4} e^{\kappa^2/4} \int_{-\kappa}^{\infty} e^{-y^2/4} dy\right) \cdot \int_{\kappa}^{\infty} e^{-y^2/4} dy - \left(\frac{\sqrt{\pi}}{2\kappa} + \frac{1}{2} \int_0^1 \frac{1-y^2}{1+y^2} e^{-\kappa^2 y^2/4} dy\right) .$$

Using $\int_{\pm\kappa}^{\infty} e^{-y^2/4} dy = \sqrt{\pi} \mp \int_0^{\kappa} e^{-y^2/4} dy$, the last expression simplifies to

$$\frac{1}{4} e^{\kappa^2/4} \left[- \int_0^1 \frac{4}{1+y^2} e^{-\kappa^2 (y^2+1)/4} dy + \left(\pi - \left(\int_0^{\kappa} e^{-y^2/4} dy\right)^2\right)\right] .$$

The term in square brackets vanishes for $\kappa = 0$. Since also

$$\frac{d}{d\kappa}[\cdots] =$$

$$2\kappa \int_0^1 e^{-\kappa^2 (y^2+1)/4} dy - e^{-\kappa^2/4}\, 2 \int_0^{\kappa} e^{-y^2/4} dy$$

$$= 0 ,$$

this finishes the argument.

It remains to prove the Proposition. To this end we define for $\alpha \in (0,1/2)$ the seminorm

$$|r|_T := \sup_{t \in (0,T)} t^{1-\alpha} |r'(t)| .$$

Further, we denote by c a generic constant which may depend on g, α, κ and monotonely on T and $|r|_T$.

If $r(0) = p\frac{\kappa^2}{2}$ with κ given by (7) and s is related to r by (17), we claim that the following estimates hold:

(21) $$|Fr|_T \le c(T) + (c_1(\kappa,\alpha) + c(T))\, |r|_T$$

(22) $$|Kr|_T \le (c_2(\kappa,\alpha) + c(T))\, |r|_T$$

with

$$c_1(\kappa,\alpha) = \frac{1}{\sqrt{\pi}} \frac{1}{1+\alpha} \frac{p(\kappa)}{\kappa} \exp(-\kappa^2/4)$$

$$c_2(\kappa,\alpha) =$$

$$\frac{1}{2\sqrt{\pi}} \kappa \int_0^1 \frac{\tau^\alpha}{\sqrt{1-\tau}\ (1+\sqrt{\tau})} \exp(- \frac{\kappa^2}{4} \frac{1-\sqrt{\tau}}{1+\sqrt{\tau}}) d\tau +$$

$$\frac{1}{\sqrt{\pi}} \frac{3+2\alpha}{(2+2\alpha)(2+4\alpha)} \kappa \int_0^1 \frac{1-\tau^{1/2+\alpha}}{(1-\tau)^{3/2}} \left|1 - \frac{\kappa^2}{2} \frac{1-\sqrt{\tau}}{1+\sqrt{\tau}}\right| \exp(- \frac{\kappa^2}{4} \frac{1-\sqrt{\tau}}{1+\sqrt{\tau}}) d\tau \quad .$$

Here, $T \to c(T)$ denotes a continuous function with $c(0) = 0$. Note, that the constants c_1 and c_2 do not depend on T and hence are not small for small T. This reflects the fact that the operators F and K are not compact. Our existence proof relies on the fact that

$$w := c_1(\kappa,\tfrac{1}{2}) + c_2(\kappa,\tfrac{1}{2}) < 1 \tag{23}$$

which, however, is valid only for $\kappa < 1.05...$, or equivalently $p < 1.59...$. *This explains the artificial restriction on p in the Proposition.*

Assuming (23), we can proceed in a standard manner. We iterate the integral equation (16) in the form

$$\begin{aligned} r_1(t) &\equiv p_- \frac{\kappa^2}{2} \\ r_{n+1} &= Fr_n + \kappa r_n \quad , \quad n \in \mathbb{N} \quad , \end{aligned} \tag{24}$$

with p related to κ by (7). By (20) $r_n(0) = r_1$ for all u. Moreover, if we choose α close to $1/2$ and $T > 0$ so small that

$$(c_1(\kappa,\alpha) + c(T)) + (c_2(\kappa,\alpha) + c(T)) =: w' < 1 \quad ,$$

the estimates (21) and (22) imply

$$|r_{n+1}|_T \leq c(T) + w'|r_n|_T \quad .$$

It follows that

$$|r_n|_T \leq c(T) / (1-w'), \ n \in \mathbb{N} \quad ,$$

and we can select a subsequence which converges to a solution r_∞ of (16) in $C[0,T]$ which satisfies (18).

The proof of the estimates (21), (22) is fairly technical and we refer to [7], since only minor modifications are necessary to incorporate the case $g'(0^+) \neq g'(0^-)$. It is not quite satisfactory that one has to compute the constants c_1 and c_2 explicitly. It may perhaps be possible to avoid this by estimating the nonlinear operators F and K in a suitable weighted norm. In fact we conjecture that a stronger result can be proved namely that for smooth initial data g, r and s are smooth function of $\sqrt{t}$ at $t = 0$.

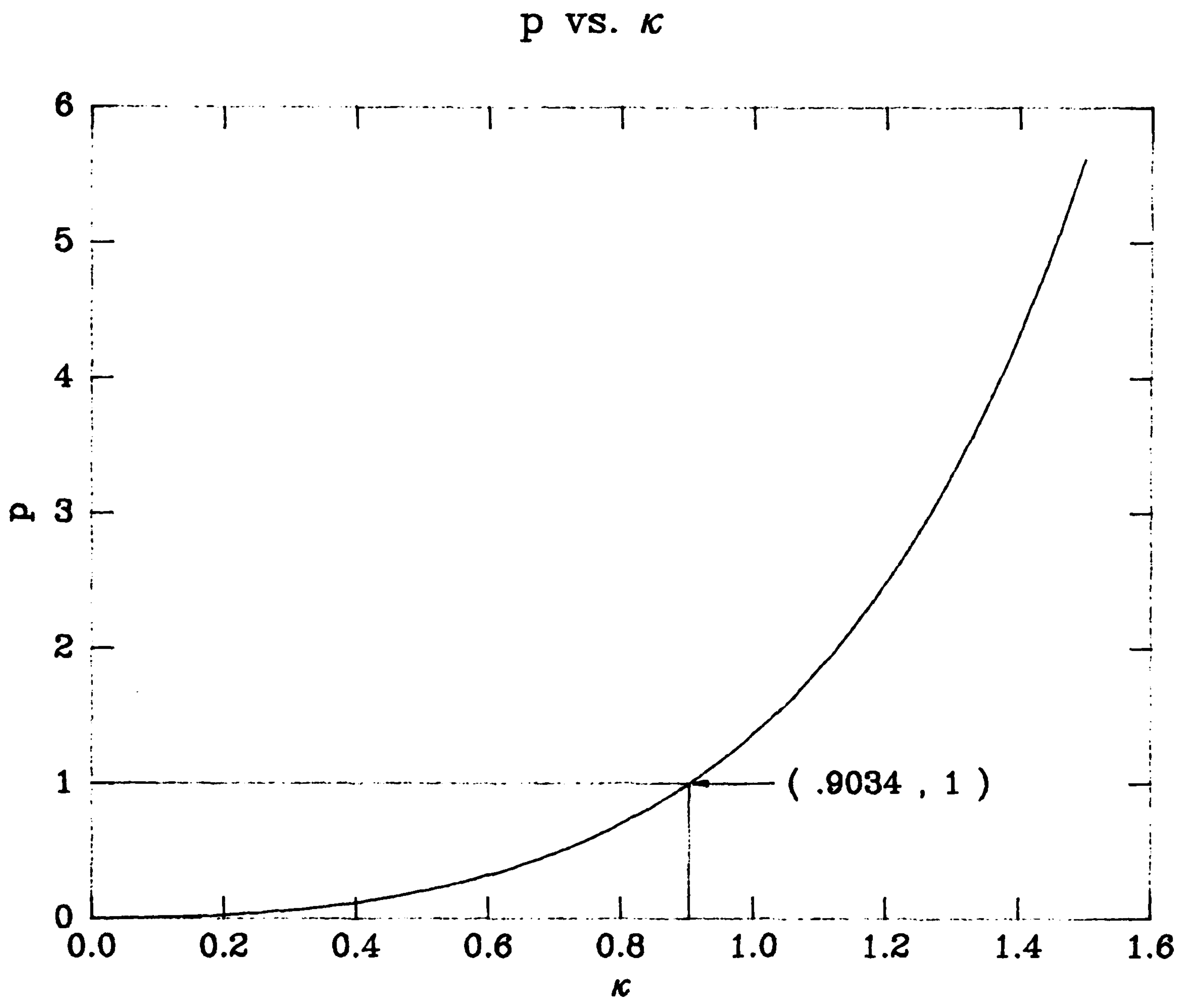

Figure 1

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19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Cauchy problem, parabolic, nonlinear, regularity of free boundary, self similar solutions, nonlinear singular integral equation.

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

We study the singular Cauchy problem

$$v_t = \phi(v)_{xx} \quad , \quad (x,t) \in \mathbb{R} \times [0,T], \ T > 0$$

$$v(x,0) = g(x) \quad , \quad x \in \mathbb{R} \ .$$

The constitutive function $\phi(\xi) = \max(0,\xi)$; the initial datum g is smooth (bounded) on $\mathbb{R}\backslash\{0\}$ and satisfies

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ABSTRACT (continued)

where the superscipt "+" ("-") denotes the limit from the right (left). We show that the free boundary s given by $v(s(t)^+,t) = 0$ satisfies

$$s(t) = -\kappa\sqrt{t} + o(\sqrt{t}) \qquad (t \to 0^+) ,$$

where $\kappa > 0$ is a monotone function of $p = g'(0^+)/g'(0^-)$, implicitly defined by the equation

$$(*) \qquad p = \frac{\kappa^2}{2} + \frac{\kappa^3}{4} e^{\kappa^2/4} \int_{-\kappa}^{\infty} e^{-t^2/4} dt, \; \kappa > 0 .$$

This generalizes an earlier analysis [ ] for the special case of smooth data g in which $g'(0^+) = g'(0^-) \neq 0$, and $p = 1$. In this case the numerical value $\kappa = .903446...$ as computed from (*) is consistent with our previous result.

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