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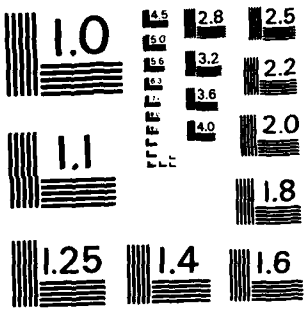

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Following an introductory status review, the paper presents a new astrogeodetic-gradiometric-inertial method for vertical deflection determination under consideration of horizontal channel interactions and suitable for the utilization of single or multiple track data in conjunction with supporting astrogeodetic deflections. It then discusses the problem of optimal deflection area adjustments under consideration of other work. Subsequently, accuracy aspects of astrogeodetic-gradiometric gravity vector determination based on the Bell gravity gradiometer, the simultaneous utilization of gradiometric and inertial data in terrestrial surveys, and potential applications are addressed.

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ON RECENT PROGRESS IN AND UTILIZATION OF ASTROGEODETIC-INERTIAL AND ASTROGEODETIC-GRADIOMETRIC GRAVITY VECTOR DETERMINATION

ZU JÜNGSTEM FORTSCHRITT IN DER UND ANWENDUNG VON ASTROGEODÄTISCH-INERTIALER UND ASTROGEODÄTISCH-GRADIOMETRISCHER SCHWEREVEKTORBESTIMMUNG

PROGRES RECENTS ET UTILISATION DU CALCUL DU VECTEUR PESANTEUR PAR ASTROGEODESIE INERTIELLE ET ASTROGEODESIE GRADIOMETRIQUE

H. Baussus von Luetzow

SUMMARY

Following an introductory status review, the paper presents a new astrogeodetic-inertial method for vertical deflection determination under consideration of horizontal channel interactions and suitable for the utilization of single or multiple track data in conjunction with supporting astrogeodetic deflections. It then discusses the problem of optimal deflection area adjustments under consideration of other work. Subsequently, accuracy aspects of astrogeodetic-gradiometric gravity vector determination based on the Bell gravity gradiometer, the simultaneous utilization of gradiometric and inertial data in terrestrial surveys, and potential applications are addressed.

ZUSAMMENFASSUNG

Im Anschluss an eine einführende Statusübersicht beschreibt dieser Artikel eine neue astrogeodätisch-inertiale Methode für die Bestimmung von Lotabweichungen unter Berücksichtigung gegenseitiger horizontaler Kanaleinflüsse und geeignet für die Verwendung von Einzel-oder Mehrfachtrassendaten. Alsdann erörtert er das Problem optimaler Flächenausgleichung unter Betrachtung anderer Arbeiten. Anschliessend wird auf Genaugkeitsaspekte astrogeodätisch-gradiometrischer Schwerevektorbestimmung, die gleichzeitige Verwendung gradiometrischer und inertialer Daten bei Messungen auf der Erde und Anwendungsmöglichkeiten eingegangen.

RESUME

Après avoir rappelé l'historique de la question, cet article présente une nouvelle méthode d'astrogéodésie inertielle pour le calcul de la déflexion verticale en tenant compte des interactions des canaux horizontaux et permettant l'utilisation des données d'une ou plusieurs transversales conjointement avec les déflexions astrogéodésiques. Il examine ensuite le problème des meilleurs réglages en matière de déflexion pour d'autres emplois. Enfin sont présentées l'importance de la precision pour la mesure astrogéodésique et gradiométrique du vecteur pesanteur à l'aide de l'appareil Bell de mesure du gradiant de pesanteur, l'emploi simultané des données gradiometriques et inertielles pour les relevés topographiques terrestres ainsi que d'autres emplois possibles.

1. INTRODUCTION

The latest consolidated presentations concerning gravity vector determination by means of astrogeodetic-inertial and astrogeodetic-gradiometric data can be found in the proceedings of the Second International Symposium on Inertial Technology for Surveying & Geodesy, held in Banff, Canada, June 1-5, 1981. At that time, Adams and Hadfield (1981) addressed GEO-SPIN/IPS-2 improvements for precision gravity mesurement[1], Bose and Huddle (1981) described a regional adjustment method for gravity vector determination, Harris (1981) presented IPS-2 test results, Todd (1981) reviewed modified IPS-1 test data, Heller (1981) elaborated on prospects for gradiometric aiding of inertial survey systems, Metzger and Jincitano (1981) summarized the application of Bell's gravity gradiometer and gravity meters to airborne and land vehicle gravity surveys, Trageser (1981) gave a floated gravity gradiometer status report[2], Paik (1981) reported on the superconducting gravity gradiometer, and Brown (1981) dealt with methods of processing gradiometer data for geophysical applications. With respect to improved astrogeodetic-inertial gravity vecto determination, Honeywell (1981) studied modified hardware and software requirements to achieve high deflection change accuracies of the order of 0. arcsec average rms for a survey length of about 60 km with present hardware in the context of multiple traverses (area adjustment) and of the order of 0.1 arcsec rms following installation of improved accelerometers and velocity quantizers. Litton (1982) claimed equivalent accuracies. As to an average rms requirement of 0.1 arcsec, Litton proposed utilization of screened G 300 G2 gyroscopes with correlated random noise parameters of 2.10^{-4} deg hr^{-1} and min (correlation length) and A 1000 accelerometers with corresponding random parameters of 1 mgal and 2 min.[3] Baussus von Luetzow (1982) developed a coupled horizontal channel optimal method for vertical deflection determination in semi-flat terrain in the context of Litton's local-level system. In the field of gravity gradiometry, White (1980) researched error models and related aspects, and Chan (1982) presented progress in the development and testing of the superconducting gravity gradiometer. The emphasis in this paper is on the analysis of a framework for optimal astrogeodetic-inertial determination of vertical deflections under consideration of the area adjustment problem including the smoothing method designed by Bose and Huddle (1981), and on a discussion of significant aspects of gravity vector determination under application of the Bell system, simultaneous utilization of gradiometric and inertial data, and potential applications under inclusion of the superconducting gravity gradiometer. Apart from technical information of interest, this presentation has been designed to arrive at some useful conclusions.

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[1] IPS-2 relates to Honeywell's inertial positioning system incorporating a space-stable platform, IPS-1 stands for Litton's corresponding local-level system.

[2] Draper Laboratories floated gravity gradiometer development and Hughes rotating gravity gradiometer development were suspended in 1980 and 1979, respectively.

[3] The parameters are approximate.

2. COUPLED HORIZONTAL CHANNEL OPTIMAL DETERMINATION OF VERTICAL DEFLECTIONS IN SEMI-FLAT TERRAIN

Higher accuracy requirements necessitating installation of gyros and accelerometers with small error variances and short correlation times and of high accuracy velocity quantizers, identified in section 1, require the integration of the system of pertinent differential equations for best possible estimation of deflection components. In this context, consideration of constant gyro biases require ultimately a Wiener-Kolmogorov optimization since optimal Kalman smoothing is only possible in the case of observable linear aggregates of random errors. The pertinent local-level equations are:

$$\ddot{x} = S_N\Phi_z - g\Phi_N + g\eta + a_E - a_{Eo} \tag{1}$$

$$\ddot{y} = -S_E\Phi_z + g\Phi_E - g\xi + a_N - a_{No} \tag{2}$$

$$\dot{\Phi}_z = R^{-1}\tan\phi\,.\dot{x} + R^{-1}(\Omega_N + \rho_N \sec^2\phi)y \quad -\omega_E\Phi_N \quad +\omega_N\Phi_E + \alpha \tag{3}$$

$$\dot{\Phi}_N = R^{-1}\dot{x} \quad +\omega_E\Phi_z \quad -\omega_z\Phi_E + \beta \tag{4}$$

$$\dot{\Phi}_E = -R^{-1}\dot{y} \quad -\omega_N\Phi_z \quad +\omega_z\Phi_N \quad +\gamma \tag{5}$$

Symbols used above including total time derivatives of first and second order are:

ξ	meridian vertical deflection
η	prime vertical deflection
g	vertical gravity vector component
R	earth's mean radius
ϕ	geographic latitude
Ω	earth's inertial angular velocity
V_x	system's east velocity
V_y	system's north velocity
Φ_z	azimuth platform attitude error
Φ_N	platform tilt error about north axis
Φ_E	platform tilt error about east axis
S_E	east acceleration of survey vehicle
S_N	north acceleration of survey vehicle
a_E-a_{Eo}	correlated east accelerometer error under consideration of initial calibration
a_N-a_{No}	correlated north accelerometer error uner consideration of initial calibration
ω_z	$= \Omega_z + R^{-1} V_x \tan\phi$ vertical spatial rate, $\Omega_z = \Omega \sin\phi$
ω_N	$= \Omega_N + V_x R^{-1}$ north spatial rate, $\Omega_N = \Omega\cos\phi$
ω_E	$= -R^{-1}V_y$ east spatial rate
α	azimuth axis angular drift rate error
β	north axis angular drift rate error
γ	east axis angular drift rate error

For land vehicles, eqs. (3) - (5) may be simplified by omission of ρ_N, ω_E, and $\rho_z = R\, V_x \,\mathrm{tn}\phi$, and by use of constant accelerations S_N and S_E which should be approximately achieved. Then, $\omega_z = \Omega \sin\phi$ and $\omega_N = \Omega\cos\phi$. The initial conditions at $t_o = 0$ are, under consideration of plumbline levelling, $\omega_z(0) = 0$, $\phi_N(0) = \eta_o$, $\phi_E(0) = \xi_o$, $\dot{x}(o) = \dot{y}(o) = x_o = y(o) = 0$. The accelerations S_N and S_E can probably be neglected. If the system is treated as one with constant coefficients as a good approximation, a closed solution as a function of time is possible. Because of intermittent Kalman filter corrections and the need for numerical weight factors, it appears to be advantageous to attempt a numerical solution for x and y under utilization of terminal deflection and azimuth data. For economic reasons it is assumed that the survey vehicle travels approximately at a constant speed when in motion and stops every 3 minutes for 1 minute. The speed should not exceed 10 msec^{-1} in order to restrict the length of travel intervals.

Solutions for ϕ_E, ϕ_N, ϕ_z, $\dot{x}$, $\dot{y}$, x, y are obtained in accordance with the integration schemes

$$F_1 = F_o + \dot{F}_o\, \Delta t \tag{6}$$

$$F_\nu = 2F_{\nu-1} - F_{\nu-2} + (\Delta t)^2\, \ddot{F}_{\nu-1} \tag{7}$$

with Δt = 30 sec and possibly 60 sec. The solution structure at the end of the first stop interval, indicated by the subscript s = 1, is

$$F_{\nu 1} = \sum_1^\nu a_\nu\, v_\nu + \sum_1^\nu b_\nu\, w_\nu + \sum_1^\nu c_\nu \alpha_\nu + \sum_1^\nu d_\nu \beta_\nu + \sum_1^\nu e_\nu \gamma_\nu \tag{8}$$

In eq. (8), $v = g\,(\eta - \eta_o) + a_E - a_{Eo}$, $w = -g(\xi - \xi_o) + a_N - a_{No}$.

Under consideration of Kalman filter tilt corrections, assumed for simplicity to eliminate the integrated first two terms in eq. (3) and the first term in both eqs. (4) and (5),

$$F_{\nu 1}^{(2)} = \sum_1^\nu A_\nu\, v_\nu + \sum_1^\nu B_\nu\, w_\nu + \sum_1^\nu C_\nu\, \alpha_\nu + \sum_1^\nu D_\nu\, \beta_\nu + \sum_1^\nu E_\nu\, \gamma_\nu + T_1 \tag{9}$$

where T_1 and subsequent T_s represents an aggregate of tilt-induced random errors. During the stop interval, eqs. (3) - (5) are integrated with $x = y = y = 0$ with a resultant effect on eqs. (1) and (2). The integration is then continued and yields under utilization of average biases the solution structure

$$F_{\nu s}^{(2)} = \sum_{\nu=1}^{\nu s} A_\nu\, v_\nu + \sum_{\nu=1}^{\nu s} B_\nu\, w_\nu + \hat{C}_\nu\, \bar{\alpha} + \hat{D}_\nu\, \bar{\beta} + \hat{E}_\nu \bar{\gamma} + R_{\nu s} + T_s \tag{10}$$

where $R_{\nu s}$ designates residual, random-type terms including $\bar{\alpha} - \alpha_\nu$, $\bar{\beta} - \beta_\nu$, and $\bar{\gamma} - \gamma_\nu$.

At the termination of the survey, when η_n, ξ_n, and A_n are available, it is possible to solve for the gyro biases in the form

$$\begin{bmatrix} \bar{\alpha} \\ \bar{\beta} \\ \bar{\gamma} \end{bmatrix} = \begin{bmatrix} F_1 \\ F_2 \\ F_3 \end{bmatrix} + \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \end{bmatrix} + \begin{bmatrix} \delta_1 \\ \delta_2 \\ \delta_3 \end{bmatrix} \tag{11}$$

where the F's are computable, the ε's are an aggregate of v and w-errors, and the δ's are aggregates of gyro random and tilt errors. Substitution of $\bar{\alpha}$, $\bar{\beta}$, $\bar{\gamma}$ from eq. (11) in eq. (10) and separation of v and w into deflection and accelerometer errors leads, under restriction to solutions for $\ddot{x}$ and $\ddot{y}$, to the final results

$$\eta_s + \sum_{\nu=0}^{n} k_\nu \eta_\nu + \sum_{\nu=0}^{n} l_\nu \xi_\nu = \eta_0 + \bar{g}^{-1} \ddot{x}_s^{(2)} + \bar{\Phi}_{N_s}^{(2)} + r_{gs} + r_{as} + r_{ts} + r_{ds} \tag{12}$$

$$\xi_s + \sum_{\nu=0}^{n} m_\nu \eta_\nu + \sum_{\nu=0}^{n} n_\nu \xi_\nu = \xi_0 - g^{-1} \ddot{y}_s^{(2)} + \bar{\Phi}_{E_s}^{(2)} + \rho_{gs} + \rho_{as} + \rho_{ts} + \rho_{ds} \tag{13}$$

The last 4 terms on the right side of both eqs. (12) and (13) are aggregates of random errors associated with gyros, accelerometers, tilt corrections, and initial and terminal deflection errors. Equations (12) and (13) are reformulated as

$$\eta_s + \sum_{\nu=0}^{n} k_\nu \eta_\nu + \sum_{\nu=0}^{n} l_\nu \xi_\nu = S_{\eta_s} = M_{\eta_s} - N_{\eta_s} \tag{14}$$

$$\xi_s + \sum_{\nu=0}^{n} m_\nu \eta_\nu + \sum_{\nu=0}^{n} n_\nu \xi_\nu = S_{\xi_s} = M_{\xi_s} - N_{\xi_s} \tag{15}$$

where S, M, N denote signal, measurable message, and non-measurable noise, respectively.

Under utilization of vertical deflection covariance functions, the prime deflection can, for example, be optimally estimated in the form

$$\eta_e = A_{ie} \left[S_{\eta_i} + N_{\eta_i} \right] + B_{ie} \left[S_{\xi_i} + N_{\xi_i} \right] \tag{16}$$

where $i = s$, A_{ie} and B_{ie} are matrices of regression coefficients, and the terms in brackets are message matrices. With $k = 0, 1, \ldots, n$, A_{ie} and B_{ie} can be determined from the equations

$$\overline{\eta_e S_{\eta_k}} = A_{ie} \left[\overline{S_{\eta_i} S_{\eta_k}} + \overline{N_{\eta_i} N_{\eta_k}} \right] + B_{ie} \left[\overline{S_{\xi_i} S_{\eta_k}} + \overline{N_{\xi_i} N_{\eta_k}} \right] \tag{17}$$

$$\overline{\eta_e S_{\xi_k}} = A_{ie} \left[\overline{S_{\eta_i} S_{\xi_k}} + \overline{N_{\eta_i} N_{\xi_k}} \right] + B_{ie} \left[\overline{S_{\xi_i} S_{\xi_k}} + \overline{N_{\xi_i} N_{\xi_k}} \right] \tag{18}$$

In eqs. (17) and (18), where the bar symbol stands for covariance, the noise covariances need only be computed once.

Simplified solutions, particularly in the case of approximately straight traverses, are possible. If averaged message-type data from repeated surveys are employed, the instrument-generated noise covariances in eqs. (17) and (18) are to be reduced.

3. OPTIMAL REGIONAL VERTICAL DEFLECTION ADJUSTMENT

The single channel Wiener-Kolmogorov solution of the astrogeodetic-inertial vertical deflection determination problem was first outlined by Baussus von Leutzow (1981) and also considered by Litton (1982). It simultaneously represents the optimal framework for a regional adjustment under consideration of multiple track data. In this respect, it is advantageous to conduct surveys along approximately parallel traverses and cross traverses, preferably with new system's calibration at the start of a new traverse and without significant and rapid course changes in order to reduce the effects of platform heading sensitivity. In practice, the computation of regression coefficients for a particular ξ, η-solution by means of the system of equations (17) and (18) requires only consideration of up to 50 measurements. Spatial signal covariance functions may only be employed in moderately mountainous terrain.

Bose and Huddle (1981) developed a different regional adjustment system free of the use of empirical signal covariance functions. In their approach, they impose an orthogonality restriction on the covariance of Fourier coefficients appearing in $\overline{T_o T_e}$ where T_o and T_e are disturbing potentials at points P_o and P_e. This permits the representation of $T_o T_e$ in terms of eigenfunctions if

$$\overline{\Delta^2 T_o \Delta^2 T_e} = \overline{\left(\frac{\partial^2 T}{\partial z^2}\right)_o \left(\frac{\partial^2 T}{\partial z^2}\right)_e} = \overline{\left(\frac{\partial \xi}{\partial x} + \frac{\partial \eta}{\partial y}\right)_o \left(\frac{\partial \xi}{\partial x} + \frac{\partial \eta}{\partial y}\right)_e} = \sigma^2 = \text{const.} \qquad (19$$

Within a limited rectangular area containing gridded message data ξ_i, η_i, Δgi it is then possible, under general assumption that T vanishes on the boundary and that the error variances relating to the foregoing variables are constant throughout the whole domain, to establish weight factor solutions in the form

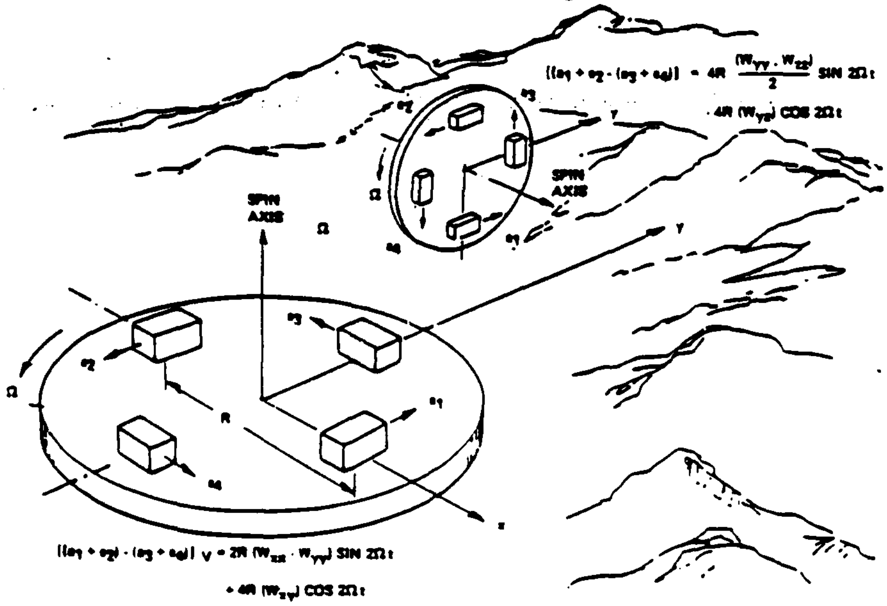
$$\hat{\xi}_e = \sum a_{ei}\xi_i + \sum b_{ei}\eta_i + \sum c_{ei}\Delta g_i \qquad (20)$$

The above method suffers from the serious restriction (19) which is less accurate than empirical covariance functions derived under the assumption of homogeneity and isotropy concerning $T_o T_e$. As to the structure thereof, reference is made to Jordan (1981). In conjunction herewith, the assumption $T = 0$ on the boundary is incompatible with a finite domain. The requirements of homogeneous error variances and of regularly spaced gridded data are also not realistic or not easily to be achieved, respectively.

4. ACCURACY ASPECTS OF THE BELL GRAVITY GRADIOMETER INSTRUMENT (GGI)

Bell Aerospace TEXTRON, under the technical leadership of E. Metzger, completed development of the ball-bearing rotating accelerometer gravity gradiometer instrument (GGI) in 1981. Its operational principle is evident from the following, Bell-furnished figure.

FIGURE
Schematic Illustration of Rotating Bell GGI Fixture



During 1981 Bell also concluded investigations concerning gimbal and survey vehicle self gradients. Preliminary GGI tests on board a ship have been encouraging. Aircraft applications simultaneously require continued positioning data as a function of time. In the context of land vehicle applications, accurate positioning is facilitated by Kalman filter error control under utilization of velocity errors observed during periodic vehicle stops, possibly augmented by terminal position information. The total GGI, including racks and power supply, is considerably more complex, voluminous and heavier than a pure inertial system and accordingly more expensive as to acquisition and operation. Intrinsic GGI errors consist of instrument self-generated noise in the platform environment and environmental sensitivities (acceleration, pressure, temperature, magnetic, etc.) Self-noise power spectral densities for the GGI mounted at the umbrella angle (spin axis = 35° from horizontal) have been approximated in the open literature by

$$\phi_s(f) = \frac{A}{f^2} + B \qquad (21)$$

where f denotes frequency in cycles per second, A is a constant relating to low frequency or red noise, B is a constant indicating high frequency or white noise, E is Eötvös unit = 10^{-9} sec^{-2}, and Hz is Hertz = 1 sec^{-1}. White (1980) listed A = 2.10^{-6} E^2.Hz and 16.10^{-6} E^2. Hz for configurations with vertical and horizontal spin axis, respectively and corresponding B = 81 $E^2.Hz^{-1}$ and 86 $E^2.Hz^{-1}$. As to environmental sensitivities, linear acceleration influences are the most severe, particularly at multiples of the nominal rotation frequency Ω = 0.25 Hz. Environmental errors represent approximately white noise and thus add to B in equation (21). The low frequency error may be essentially interpreted as random drift. In the form (21), ϕ_s does not permit a Fourier cosine transform to determine the associated covariance function. However, in a certain low frequency range, the first term in eq. (21) may be replaced by $A\,(a+f^2)^{-1}$ where a is an appropriately computed constant. This results in an exponential covariance function. The total error variance, both self-generated and environmental, is in excess of $3E^2$. Essentially different moving GGI information becomes essentially available every 2 seconds. Gradient filters are nominally fourth order Butterworth filters to attenuate 1 Ω, 3 Ω and other noise manifestations. Signal information is obtained as a ten second moving window average. For a land vehicle with a speed of 10 $msec^{-1}$, usable messages are thus generated every 20 meters. Because of nonstationary information in strongly mountainous areas it is, therefore, necessary to travel at low speeds, i.e., at 5 $msec^{-1}$. For the same reason, aircraft over strongly mountainous regions should travel at lowest possible speed. Elimination of systematic drifts and biases has to be accomplished at the start of a mission. In this respect, initial gravity tensor information is of considerable value. Otherwise, observable gravity vector component errors at mission termination would consist of mixtures of systematic and random error aggregates. These would degrade the estimation of post-mission corrections. They may be obtained in the following way: let $\hat{\xi}_o + \delta\xi_o$ and $\hat{\xi}_e + \delta\xi_e$ be observable initial and terminal vertical deflection components, $D_{\hat{\xi}} + \Delta\xi_e$ the GGI-determined difference between these deflections, and $\delta\xi_o$, $\delta\xi_e$, and $\Delta\xi_e$ uncorrelated random errors. The problem is to estimate the variable correction in the form

$$\begin{aligned}\delta\xi(t) = {} & a_{11}\left[\hat{\xi}_e + \delta\xi_e - (\hat{\xi}_o + \delta\xi_o) - (D_{\hat{\xi}} + \Delta\xi_e)\right] \\ & + a_{12}\left[\hat{\eta}_e + \delta\eta_e - (\hat{\eta}_o + \delta\eta_o) - (D_{\hat{\eta}} + \Delta\eta_e)\right] \\ & + a_{13}\left[\hat{\Delta g}_e + \delta g_e - (\hat{\Delta g}_o + \delta g_o) - (D_{\hat{\Delta g}} + \Delta\Delta g_e)\right] \qquad (2\end{aligned}$$

and two corresponding additional equations with $\delta\eta(t)$ and $\delta\Delta g(t)$ to be estimated. The first of three covariance equations reads then, with bars indicating covariances,

$$\overline{\delta\xi\Delta\xi_e} = a_{11}\left[2\overline{(\delta\xi)^2} + \overline{(\Delta\xi_e)^2}\right] + a_{12}\left[2\overline{(\delta\eta)^2} + \overline{\Delta\xi_e\Delta\eta_e}\right] + a_{13}\overline{\Delta\xi_e\Delta\Delta g_e} \qquad (23$$

where 2 var δg has been omitted because of the relatively high vertical gravity component measurement accuracy. As an alternative post-mission adjustment method, empirical corrections applying to fixed traverse intervals may be calculated under determination of a few polynomial constants under consideration of independently provided gravity vector information. Metzger and Jircitano (1982) have published preliminary, non-optional deflections rms error estimates summarized below under neglect of initial and terminal astrogeodetic deflection errors.

TABLE

Bell-Estimated Pure Gradiometric Deflection RMS Errors
for a Tie Point Separation of 80 Km and Vehicle Speed of 30to 150 $Kmhr^{-1}$

Track Length (Km)	Along Track Error (arcsec)	Cross Track Error (arcsec)
0	0.000	0.000
20	0.053	0.091
40	0.063	0.111
60	0.053	0.091
80	0.000	0.000

A reduction of GGI-generated systematic and random errors can be achieved by intentional gradiometer platform motion, primarily about the vertical spin axis. This results in modulated signal outputs. White (1980) established that the rotation or carousel frequency $\omega_c > \pi/_5V$, where V designates vehicle velocity. The minimum carousel frequency for a land vehicle with speed 10 $msec^{-2}$ would thus be 3.6 cycles hr^{-1}. For plane velocities, the minimum requirement becomes prohibitive. For aircraft applications under coverage of areas of the order $(300\ km)^2$, parallel and cross traverses separated by about 5 km and multiple tie points are a prerequisite for high accuracy gravity vector determination. To minimize the effect of hight attenuation, the aircraft altitude should not exceed a height of 500 m above terrain level. The downward analytical continuation error of gravity vector components amounts to 3% and is expected to be somewhat greater over strongly mountainous terrain.

5. COMBINED USE OF GRADIOMETER AND INERTIAL SURVEY DATA

The availability of inertial data at vehicle stops during terrestrial surveys permits simultaneous astrogeodetic-gradiometric and astrogeodetic-inertial gravity vector determination. Despite an inherent greater accuracy of the gradiometric method, the inertial method can profitably serve control purposes. In the case of acceptable agreement, weighted averages may be computed under use of covariance analysis. Because of the inclusion of linear combinations and correlations of astrogeodetic deflections, a weighted mean of the form $var_2\ (var_1 + var_2)^{-1}\ x_1 + var_1\ (var_1 + var_2)^{-1}x_2$ cannot be employed, i.e., the second, "inertial" weight factor becomes relatively smaller through covariance analysis. Further, the weights differ from point to point. On the other hand, astrogeodetic deflections with rms errors of 0.25 arcsec make the inertial multiple track method attractive since the variable astrogeodetic-gradiometer and astrogeodetic-inertial error variances would not differ appreciably.

6. APPLICATIONS

The astrogeodetic-inertial and astrogeodetic-gradiometric methods allow the determination of the detailed structure of the gravity field in continental and adjacent areas with the former method restricted to semi-flat terrestrial surveys. Satellite-based gravity gradiometers with spatial deflection control data over land areas could provide the means for the computation of gravity potential coefficients of about degree and order 90 in rather inaccessible areas. Gravity tensor information in selected regions would facilitate fine structure analyses. Large scale or small scale utilization of the available technology would result in improved space vehicle trajectories and orbits, global positioning and flight navigation, gravity programmed inertial positioning systems, subterraneous mass detection, and scientific investigations including tests of Newton's mass attraction law by means of the superconducting gravity gradiometer.

7. CONCLUSIONS

Litton and Honeywell test data produced by essentially first generation inertial sytems, existing gyros, accelerometers and velocity quantizers having considerably higher performance characteristics than presently installed instruments, and advanced optimization methods indicate potential for gravity vector component determination with rms accuracies of about 0.3 arcsec and 0.3 mgal or somewhat better in the context of area adjustments under use of multiple astrogeodetic deflections with statistically independent errors. The astrogeodetic-inertial method is in general sufficiently accurate in semi-flat terrain. The Wiener-Kolmogorov horizontal channel deflection determination method is optimal for single and multiple track data and may be slightly modified or perfected for most profitable application. The Bell gravity gradiometer is efficient and indispensable for large area aircraft surveys and over strongly mountainous terrain. Optimal post-mission adjustment methods have to be perfected in connection therewith. Combined astrogeodetic-gradiometric and astrogeodetic-inertial terrestrial surveying is feasible and advantageous for control purposes. Appropriate weighting factor computations require covariance analyses. The identified advanced systems and mathematical methods increase significantly the state-of-the-art in physical geodesy and surveying and permit multiple applications.

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Author's address:

Dr. H. Baussus von Luetzow, US Army Engineer Topographic Laboratories, Fort Belvoir, VA 22060

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