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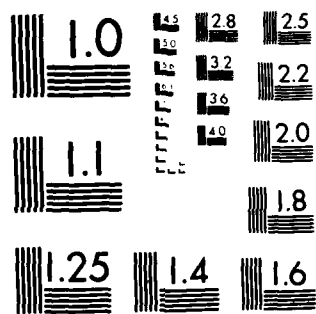
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TWO MOTION STRATEGIES
by
R. N. FORREST
December 1982

(Revised December 1983)

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Prepared for: Strategic Systems Project Office, Arlington, VA 20376

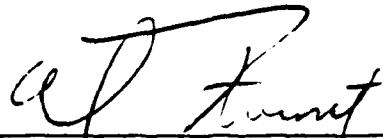
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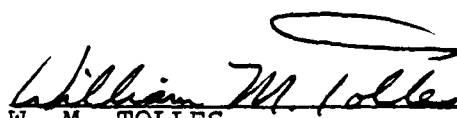
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20. ABSTRACT (Continue on reverse side if necessary and identify by block number) The report discusses two motion strategies that reduce the localization information provided by a surveillance system. In the revision, an expression in Appendix 6 has been changed and a new reference has been cited. Also, several typographical errors have been corrected.		

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I. Introduction

The relative effectiveness of two motion strategies to reduce a target's vulnerability to localization is analyzed in this report. A target is a patrolling submarine and the vulnerability arises because of the ability of a surveillance system to determine the submarine's position from time to time.

The analysis is based on the following model: A target is exposed to a surveillance system periodically. The distance the target moves during an exposure is essentially zero, but the exposure is long enough so that detection of the target by the surveillance system can occur. The target is aware of its exposures, but the exposures are unavoidable.

When a target is detected by the surveillance system, the target's position is determined, but no other information is obtained. The position, which will be called a datum, determines a search region in a plane that is bounded by circle of radius $u_M t$ that is centered on the datum where u_M is the target's maximum patrol speed and t is the time since detection. A searcher is cued by the surveillance system; but, due to various causes, a delay (time late) occurs before the start of a search.

Although its exposure times are known to a target, information about the location and the activity of searchers is not. However, the number of searchers and the searchers' sweep rates are small enough so that, without a surveillance system cue, the searchers do not represent a threat to the target.

The first motion strategy is a member of a class of motion strategies. For this class, a target course and speed are determined by independent stochastic processes. At each exposure time, the target's course θ is determined by the uniform distribution defined by the density function $f_{\theta}(\theta) = 1/2\pi$ where $0 \leq \theta < 2\pi$ and its speed u is determined by the triangular distribution defined by the density function $f_U(u) = 2u/u_m^2$ where $0 \leq u \leq u_m$.

The second motion strategy is also a member of a class of motion strategies. For this class, a target course is determined by a stochastic process but its speed is deterministic and remains constant throughout the motion. At each time determined by a Poisson process with parameter α , the target's course θ is determined by the same uniform distribution that determines the course in the first class of strategies.

The members of the first class are identified by the parameter u_m and the exposure period. The members of the second class are identified by the parameter u , the target's speed, and the parameter α . The two classes are defined in more detail in Appendices 1 and 2.

II. The First and Second Motion Strategies

Danskin has shown in Reference 1 that the strategy of the first class with the parameter u_m equated to the maximum patrol speed u_M is an optimum strategy under certain conditions. The conditions are those that exist in the model for $t < t_e$ where t is the time from a detected exposure and t_e is the time between exposures. During this interval, the strategy is optimum in the following sense: Given a target's motion strategy is known to a searcher, then, for $t < t_e$ and prior to the start of a search, the searcher's information about the target's position is a minimum; since, for the searcher, the target's position is uniformly distributed over the region bounded by the circle of radius $u_M t$ that is centered on the datum. All the remaining members of the first class and all the members of the second class are inferior to it in this sense. This strategy will be called the first strategy.

Based on Washburn's results in Reference 2, for a searcher that knows a target's strategy, the target's position distribution for a strategy of the second class is uniform over the region bounded by the circle of radius ut centered at the datum at t given the target has made exactly two course changes by that time. This suggests that the motion strategy of the second class should be considered with speed $u = u_M$ and $\alpha t_e = 2$. Here t_e could represent an average time between exposures if the exposures were not periodic. This strategy will be called the second strategy. With $\alpha t_e = 2$, a most probable number of

course changes by t_e is 2; and, after a detected exposure, if there has not been a subsequent detection for a searcher that knows the target's strategy, a most probable position distribution at t_e for the target is the same as the position distribution at t_e for a target using the first strategy.

III. Some Comparisons of the Two Motion Strategies

To establish the basis for the first comparison of the two motion strategies, consider the following localization search: At a time t after a detected exposure, a sensor is placed at the datum. The detection range of the sensor is r_c , and the time the target can be observed is short enough so that during that time the target's movement is negligible. This search suggests as a measure of motion strategy effectiveness the probability $p(t)$ at t that a target is not on or within the circle of radius $r_c < u_M t$ that is centered on the sensor; and it will be used for the first comparison.

In terms of $F_{R(t)}(r;t)$ the cumulative distribution function for a target's range from a datum, $p(t) = 1 - F_{R(t)}(r_c;t)$. Figures 1 and 2 in Appendix 3 give graphical comparisons of $F_{R(t)}(r;t)$ for the first and second strategies at times equal to t_e , $2t_e$, $3t_e$ and $4t_e$ given no subsequent detections. As can be seen from Figures 1 and 2, for the first comparison and these times, the second strategy is superior to the first. It can be conjectured that this is the case for all $t \geq 0$. However, since the measure of effectiveness rewards strategies that concentrate probability at $u_M t$, the maximum possible target range, the following strategy is superior to both strategies for $r_c < u_M t$: At the beginning of the motion, choose any course but choose the speed equal to u_M ; then hold the course and this speed throughout the motion. Clearly, the measure of effectiveness is not appropriate for every kind of localization search.

To establish a basis for a second comparison, consider a localization search that is identical to the search that was considered for the first comparison except for the placement of the sensor. For this search, the sensor's placement is not restricted to the datum. This search also suggests as a measure of motion strategy effectiveness the probability $p(t)$ at t that the target is not on or within a circle of radius $r_c < u_M t$ that is centered on the sensor. Note, however, that for this comparison its value will depend on the sensor placement strategy of the searcher. To account for this, $p_m(t)$ will be used for the second comparison where $p_m(t)$ is the minimum value of $p(t)$ as a function of sensor placement strategy.

Consider a bounding circle of radius $u_M t - r_c$, $r_c < u_M t$, centered on a datum. For the first strategy, the probability $p(t)$ that a target is not on or within the circle of radius r_c is independent of the location of its center if the center is on or within the bounding circle and $p(t) = 1 - r_c^2 / (u_M t)^2$ for $t < t_e$. If the center is outside the bounding circle, $p(t) > 1 - r_c^2 / (u_M t)^2$. Therefore, for the first strategy, the measure of effectiveness $p_m(t) = 1 - r_c^2 / (u_M t)^2$ for $t < t_e$.

For the second strategy, the evaluation of $p_m(t)$ in terms of r_c and $u_M t$ is not as tractable. However, it is shown in Appendix 5 that $p(t) < 1 - r_c^2 / (u_M t)^2$ for certain values of t and r_c . Therefore, for the second comparison and these values, the first strategy is superior to the second.

Suppose as a placement strategy a sensor is placed randomly on or within a circle of radius $u_M t + r_C$ centered on a datum. Then the probability that the sensor will be at or within a distance r_C from a point on or within a circle of radius $u_M t$ centered on the datum is $r_C^2 / (u_M t + r_C)^2$. In this case, $p(t) = 1 - r_C^2 / (u_M t + r_C)^2$ regardless of the motion strategy. Consequently, for any motion strategy, for $r_C < u_M t$, $p_m(t) \leq 1 - r_C^2 / (u_M t + r_C)^2$. For $r_C \geq u_M t$, $p_m(t) = 0$.

Both of the comparisons above are related to a search of a circular region for a time that is short enough so that during the search a target's movement is negligible. A comparison related to a search of several circular regions either sequentially or simultaneously would be more general. In particular, a sequential search would allow the consideration of a placement strategy in which sensor placement is determined by using the information gained as a search progresses. In Reference 1, Danskin has analyzed a sequential placement strategy of this kind in a game theory context for a target using the first strategy in a situation where a datum is established but there are no subsequent exposures and consequently no subsequent course or speed changes. For $t < t_e$, the analysis provides a means of obtaining an upper bound on the probability that a target will be contained in one of the circles of radius r_C at the times and places that are determined by a sequential placement strategy. However, the analysis does not apply to the second strategy; because with that strategy a target's course can change between exposures. It appears, as suggested by Danskin in Reference 1, that a corresponding analysis for the second strategy would be difficult.

The model that has been used to analyze the motion strategies does not account for factors that could be significant in an operational situation; for example, navigation constraints, transit requirements (drift) and evasive maneuvers. In particular, the model does not allow a searcher to determine a target's course.

Suppose the model were extended so that a searcher could determine a target's course when the target was at point if it detected a target wake at the point. In this case, the decay of wake detectability and t_2 , the searcher's minimum time late, would become critical factors. In particular, suppose the decay of at least one kind of wake and t_2 were such that there was a non-zero probability that a searcher could determine a target's course at a datum. Also, suppose the time to do this was negligible. Now consider a searcher that had determined a target's course at a datum and knew the target's motion strategy, but by $t < t_e$ had no other information. With the first strategy, the target's position along the target's course line would be determined by the triangular distribution defined by the density function

$$f_{X(t)}(x;t) = 2x/(u_M t)^2$$

where $X(t)$ is the target's distance from the datum and $0 \leq x \leq u_M t$. With the second strategy, according to Washburn in Reference 2, the target's position would be determined by the distribution defined by the density function

$$f_{X(t),Y(t)}(x,y;t) = [1/2\pi(u_M t)^2][at/(1-x/u_M t)]\exp\{-at[1-(1-\rho^2)^{1/2}]\}$$

for $\rho < 1$ where $\rho = (x^2 + y^2)^{1/2} / u_M t$ and the probability $\exp(-\rho t)$ for the point $(ut, 0)$. The datum is the origin of the coordinate system and the direction of the positive x-axis equals the target's course at the datum. With respect to the first and second strategies, for the case being considered, a target's choice is between a distribution confined to a line and one confined within a circle and at a point. In Reference 3, Belkin has argued that of the strategies of the second class, the second strategy is also a good choice in this case.

IV. Some Conclusions

The comparisons that are made in Section III emphasize the dependence of target motion strategy effectiveness on search capability. The game nature of the problem of choosing an optimum strategy is also emphasized by the second comparison.

To some extent, the choice of the second strategy from the second class of motion strategies was arbitrary. Figures 3, 4, 5 and 6 in Appendix 4 give graphical comparisons of $F_{R(t)}(r;t)$ for the first strategy and two other members of the second class of motion strategies. In Figures 3 and 4, the comparison is between the first strategy and the strategy of the second class with $\alpha = 1/t_e$. In Figures 5 and 6, the comparison is between the first strategy and the strategy of the second class with $\alpha = 4/t_e$. As can be seen from Figure 5, for the first comparison, the second strategy is superior to the strategy with $\alpha = 4/t_e$ for $t = t_e$. For the second comparison, the second strategy is superior to the strategy with $\alpha = 1/t_e$ based on the arguments in Appendix 5, since for this strategy the probability that a target is on the circle of radius $u_M t$ that is centered on a datum is $\exp(-t/t_e)$ rather than $\exp(-2t/t_e)$ which is the case for the second strategy. The difference can be seen by comparing the plots for $t = t_e$ in Figure 1 and Figure 3.

It is not appropriate here to discuss the relevance of the model on which the comparisons are based; however, to the extent it is relevant, the comparisons provide some basis for the choice of a motion strategy.

Appendix 1

"THE FIRST CLASS OF MOTION STRATEGIES"

The first class of motion strategies is defined as follows: At the beginning of the motion, choose a course θ_1 from a uniform distribution with density function $f_\theta(\theta) = 1/2\pi$ where $0 \leq \theta < 2\pi$ and choose a speed u_1 from a triangular distribution with density function $f_U(u) = 2u/u_m^2$ where $0 \leq u \leq u_m$. Maintain the course θ_1 and speed u_1 until the first exposure. After the first exposure, choose a new course θ_2 and a new speed u_2 by repeating the above procedure. Continue in this manner until the end of the motion.

Suppose a surveillance system detects an exposure of a target using a motion strategy from this class and suppose the strategy is known to a searcher. In this case, the information about the target's position that is available to the searcher between the time of detection and the next exposure is that the target's position is uniformly distributed over a plane region bounded by a circle with radius $u_m t$ that is centered on the datum.

The above statement can be established as follows: Let the datum be the origin of a rectangular coordinate system with east-west coordinate x and north-south coordinate y . Then a point's bearing ϕ and range r from the datum are related to the point's rectangular coordinates by $x = r \sin \phi$ and $y = r \cos \phi$. If the point represents the target's position, then relative to the searcher all of these quantities are random variables. And, for $0 \leq t \leq t_e$, $X(t) = R(t) \sin \phi$ and $Y(t) = R(t) \cos \phi$ where $R(t) = ut$ and $\phi = \theta$. Since U and θ are independent for $0 \leq t \leq t_e$ where t_e is the time of the next exposure, their joint density

function is given by $f_{U,\theta}(u,\theta) = u/\pi(u_m)^2$ where $0 \leq u \leq u_m$ and $0 \leq \theta < 2\pi$. Therefore, for the $0 < t \leq t_e$, the joint density function of $R(t)$ and ϕ is given by $f_{R(t),\phi}(r,\phi;t) = r/\pi(u_m t)^2$ where $0 \leq r \leq u_m t$ and $0 \leq \phi < 2\pi$. And, for $0 < t \leq t_e$, the joint density function of $X(t)$ and $Y(t)$ is given by $f_{X(t),Y(t)}(x,y;t) = 1/\pi(u_m t)^2$ where $x^2 + y^2 \leq (u_m t)^2$.

For $0 \leq t \leq t_e$, the marginal density function for the target's range from the datum is $f_{R(t)}(r;t) = 2r/(u_m t)^2$ where $r \leq u_m t$. The cumulative distribution function for $R(t)$ is given by

$$F_{R(t)}(r;t) = r^2/(u_m t)^2$$

where $r \leq u_m t$.

After a detected exposure, if a target is not subsequently detected, the target's position distribution is no longer uniform after its next exposure. Histograms which correspond to cumulative distribution functions for the target's range from the origin at subsequent exposure times are shown in Appendix 3. The distribution of the target's bearing from the origin remains uniform. In the limit, the target's position distribution approaches a circular normal distribution centered on the origin.

Appendix 2

"THE SECOND CLASS OF MOTION STRATEGIES"

The second class of motion strategies is defined as follows: At the beginning of the motion, choose a patrol speed u . This speed will be maintained throughout the motion. Next, choose a course θ_1 from a uniform distribution with density function $f_{\theta}(\theta) = 1/2\pi$ where $0 \leq \theta < 2\pi$ and choose a time t_1 from an exponential distribution with density function $f_T(t) = \alpha \exp(-\alpha t)$ for $0 \leq t$. Maintain the course θ_1 for a time t_1 . At the end of that time, determine a new course θ_2 and time t_2 by repeating the above procedure. Continue in this manner until the end of the motion.

Suppose a surveillance system detects an exposure of a target using a motion strategy from this class and suppose the strategy is known to a searcher. In this case, the information about the target's position that is available to the searcher at a time $t > 0$ is that the target's position is described by a joint density function determined by Washburn in Reference 2.

The marginal density function for the target's range from the datum is $f_{R(t)}(r;t) = (1/ut)[\alpha t \rho / (1-\rho^2)^{3/2}] \exp\{-\alpha t [1-(1-\rho^2)^{1/2}]\}$; where $r < ut$ and $\rho = r/ut$. For $r < ut$, the cumulative distribution function for $R(t)$ is given by

$$F_{R(t)}(r;t) = 1 - \exp\{-\alpha t [1-(1-\rho^2)^{1/2}]\}$$

The probability the target's range equals ut is $\exp(-\alpha t)$. For all r , the target's bearing is uniformly distributed between 0 and 2π .

Plots of $F_{R(t)}(r;t)$ are shown in Appendix 3. In the limit, the target's position distribution approaches a circular normal distribution centered on the origin.

Appendix 3

"A GRAPHICAL COMPARISON OF THE FIRST & SECOND MOTION STRATEGIES"

Range cumulative distribution functions for a target's range from a datum given no subsequent detections for the first and second strategies are shown in Figures 1 and 2. The range units are $u_M t_e$ and the distributions are for times equal to t_e , $2t_e$, $3t_e$ and $4t_e$.

The cumulative distribution functions for the first strategy are represented by histograms. The histograms were generated by using a computer simulation. The cumulative distribution functions for the second strategy are represented by plots. The plots were generated using a computer plot routine and the cumulative distribution function given in Appendix 2 with the course change rate $\alpha = 2/t_e$.

In Appendix 6, an analytical comparison of the two range distributions is given in terms of the first and second moments about the origin.

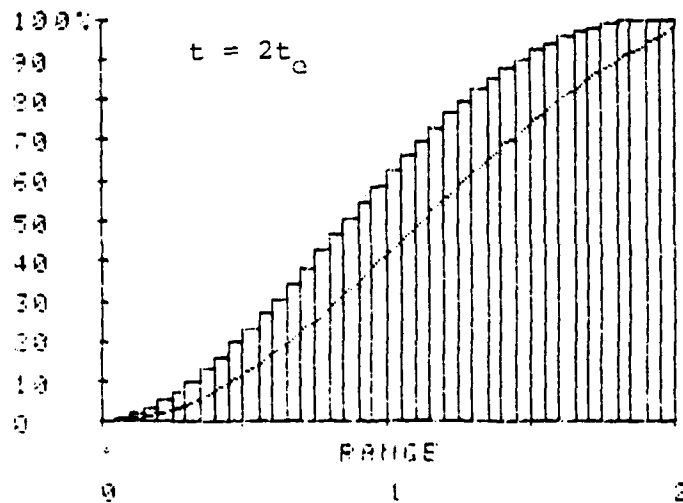
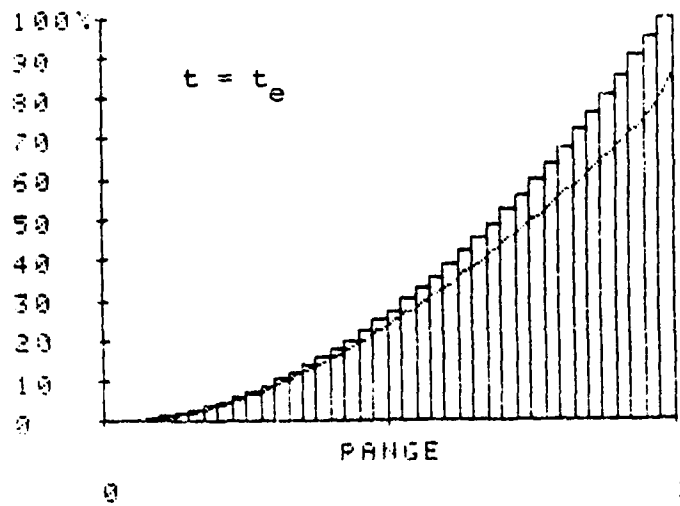


Figure 1. Range cumulative distribution functions at the indicated times. The histograms are for the first strategy and the plots are for the strategy of the second class with $\alpha = 2/t_e$ (the second strategy).

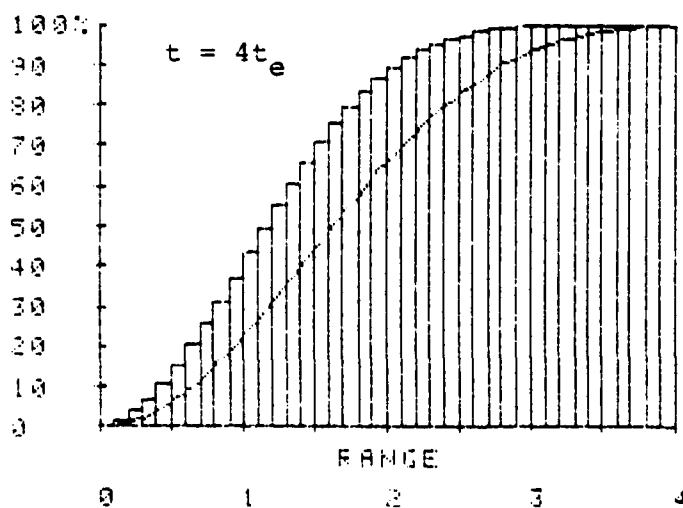
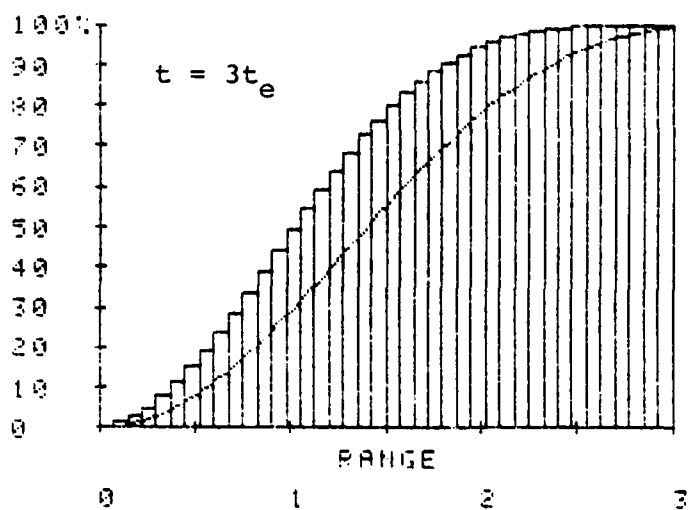


Figure 2. Range cumulative distribution functions at the indicated times. The histograms are for the first strategy and the plots are for the strategy of the second class with $\alpha = 2/t_e$ (the second strategy).

Appendix 4

"A GRAPHICAL COMPARISON OF THE FIRST & OTHER MOTION STRATEGIES"

Range cumulative distribution functions for a target's range from a datum given no subsequent detections for the first strategy and two members of the second class of motion strategies are shown in Figures 3, 4, 5 and 6.

Figures 3 and 4 show the cumulative distribution functions for the first strategy and the strategy of the second class with $\alpha = 1/t_e$. Figures 5 and 6 show the cumulative distribution functions for the first strategy and the strategy of the second class with $\alpha = 4/t_e$.

The range units are $u_M t_e$ and the distributions are for times equal to t_e , $2t_e$, $3t_e$ and $4t_e$. The histograms and plots were generated in the same manner as those in Appendix 3.

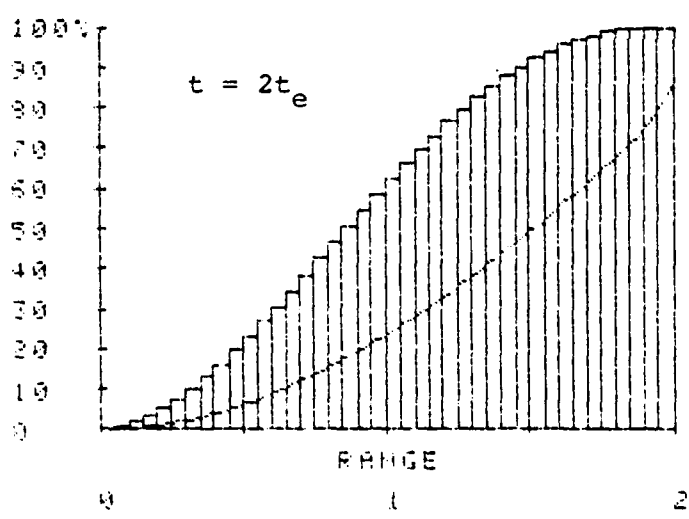
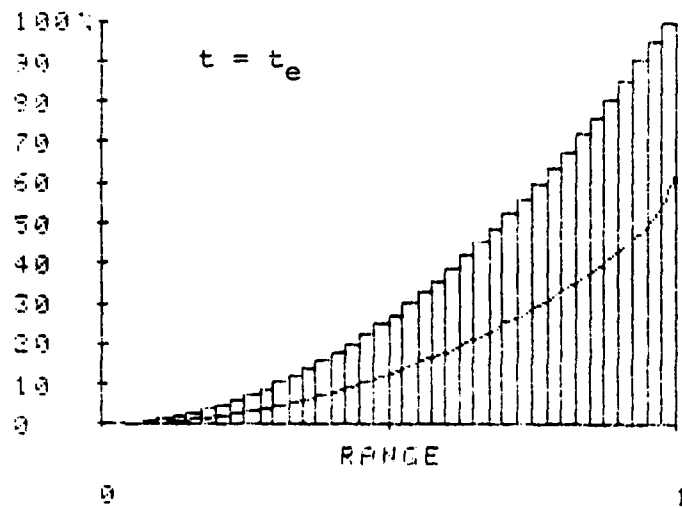


Figure 3. Range cumulative distribution functions at the indicated times. The histograms are for the first strategy and the plots are for the strategy of the second class with $\alpha = 1/t_e$.

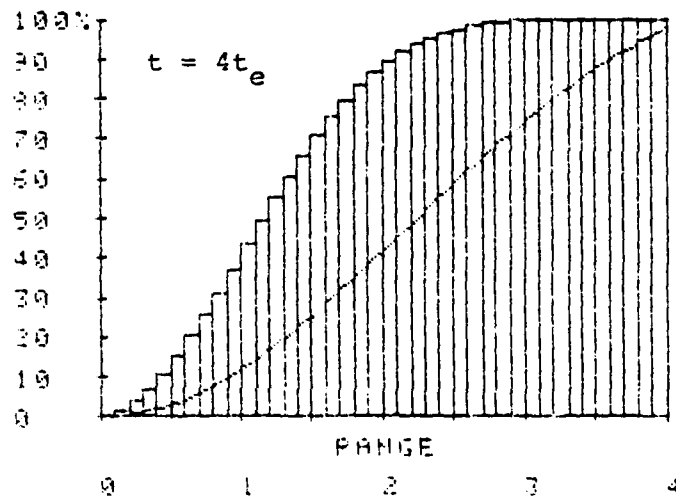
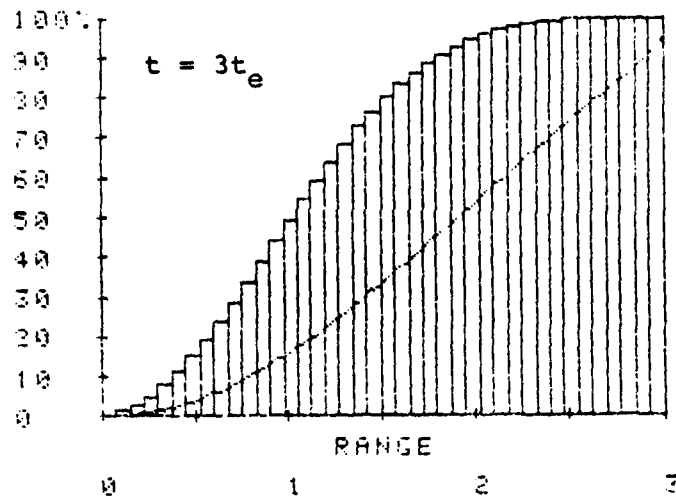


Figure 4. Range cumulative distribution functions at the indicated times. The histograms are for the first strategy and the plots are for the strategy of the second class with $\alpha = 1/t_e$.

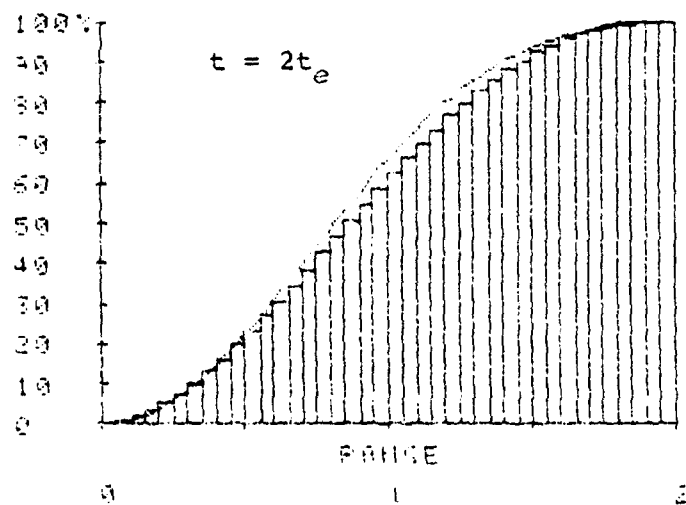
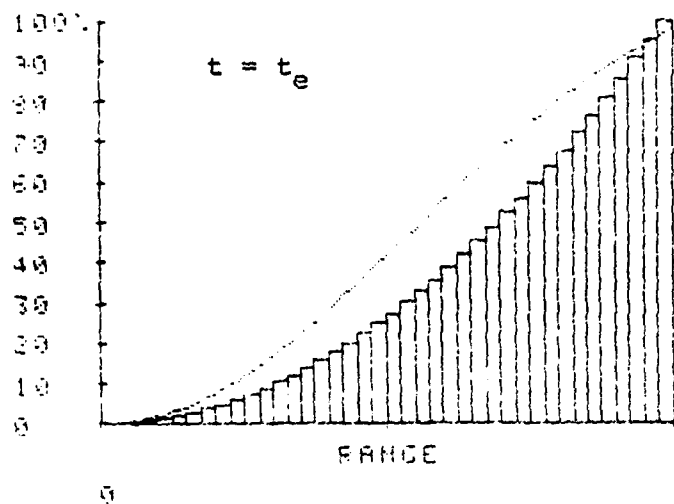


Figure 5. Range cumulative distribution functions at the indicated times. The histograms are for the first strategy and the plots are for the strategy of the second class with $\alpha = 4/t_e$.

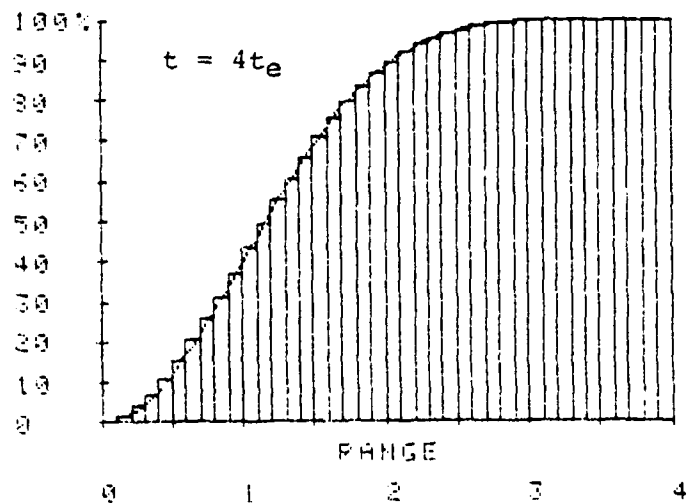
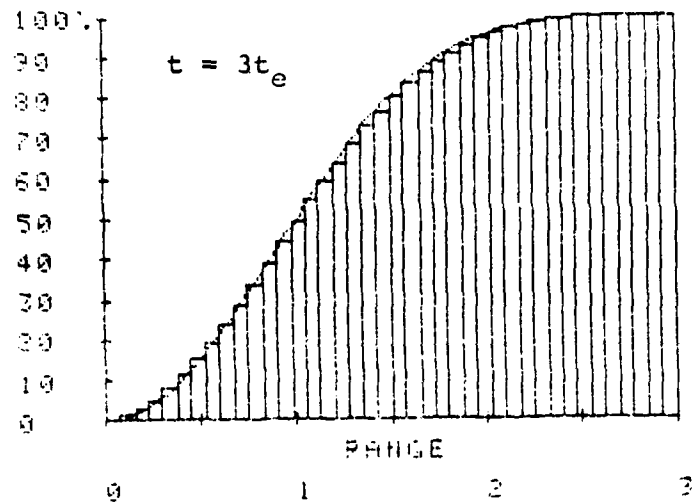


Figure 6. Range cumulative distribution functions at the indicated times. The histograms are for the first strategy and the plots are for the strategy of the second class with $\alpha = 4/t_e$.

Appendix 5

"A BOUND ON THE SECOND COMPARISON MEASURE OF EFFECTIVENESS"

A bound on the second comparison measure of effectiveness for the second motion strategy can be computed as follows: The probability that a target is on an arc length s of a circle of radius $u_M t$ that is centered on a datum is $(s/2\pi u_M t) \exp(-2t/t_e)$. The maximum arc length that can be intercepted by a circle of radius $r_C < u_M t$ is equal to $s_M = 2u_M t \sin^{-1}(r_C/u_M t)$. Therefore, the probability that a target will be on or within a circle of radius r_C that is positioned so that it intercepts an arc length s_M is at least equal to

$$(1/\pi) \sin^{-1}(r_C/u_M t) \exp(-2t/t_e).$$

For the first strategy, the probability is equal to $(r_C/u_M t)^2$. Setting $x = r_C/u_M t$ and equating these two expressions, one has the following equation in x :

$$x^2 = (1/\pi) (\sin^{-1} x) \exp(-2t/t_e).$$

Since for the second strategy only the probability that the target is on an arc is being considered, the solutions to this equation are such that at least for $r_C < x u_M t$, the first motion strategy is superior to the second. For $t = t_e$, $x = .0431$; for $t = t_e/2$, $x = .1174$ and for $t = t_e/4$, $x = .1943$.

Appendix 6

"AN ANALYTICAL COMPARISON OF THE FIRST & SECOND MOTION STRATEGIES"

An analytical comparison of the range cumulative distribution functions for a target's range from a datum given no subsequent detections for the first and second strategies is given in this appendix. The comparison is in terms of the first moment $m_1(t)$ and second moment $m_2(t)$. Because of the nature of the distribution for the second strategy, it is convenient to use the following formulas:

$$m_1(t) = \int_0^{u_M t} [1 - F_{R(t)}(r;t)] dr$$

$$m_2(t) = \int_0^{u_M t} 2r[1 - F_{R(t)}(r;t)] dr$$

For the strategies of the first class, for $t \leq t_e$,
 $m_1(t) = 2(u_m t)/3$ and $m_2(t) = (u_m t)^2/2$.

For the strategies of the second class,

$$m_1(t) = (ut) \exp(-\alpha t) \int_0^{\pi/2} \exp(\alpha t \cos \theta) \cos \theta d\theta$$

This can be shown by first substituting $\rho = r/ut$, then $\sin \theta = \rho$. The integral could not be evaluated in closed form; however, it was evaluated numerically for specified values of α and t . In addition,

$$m_2(t) = 2(u^2/\alpha^2) [\exp(-\alpha t) - 1 + \alpha t] .$$

This can be shown by first substituting $\rho = r/ut$ and then $x^2 = 1 - \rho^2$.

Evaluating $m_1(t)$ and $m_2(t)$ for the first and second strategies at $t = t_e$ gives the following results. For the first strategy, $m_1(t_e) = .67(u_M t_e)$ and $m_2(t_e) = .5(u_M t_e)^2$. For the second strategy, $m_1(t_e) = .71(u_M t_e)$ and $m_2(t_e) = .57(u_M t_e)^2$. Clearly, for $t = t_e$, the first and second moments of the two distributions do not differ significantly.

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