

TECHNICAL REPORT ARLCB-TR-83041

**PREDICTION OF RESIDUAL STRESSES IN AN  
AUTOFRETTAGED THICK-WALLED CYLINDER**

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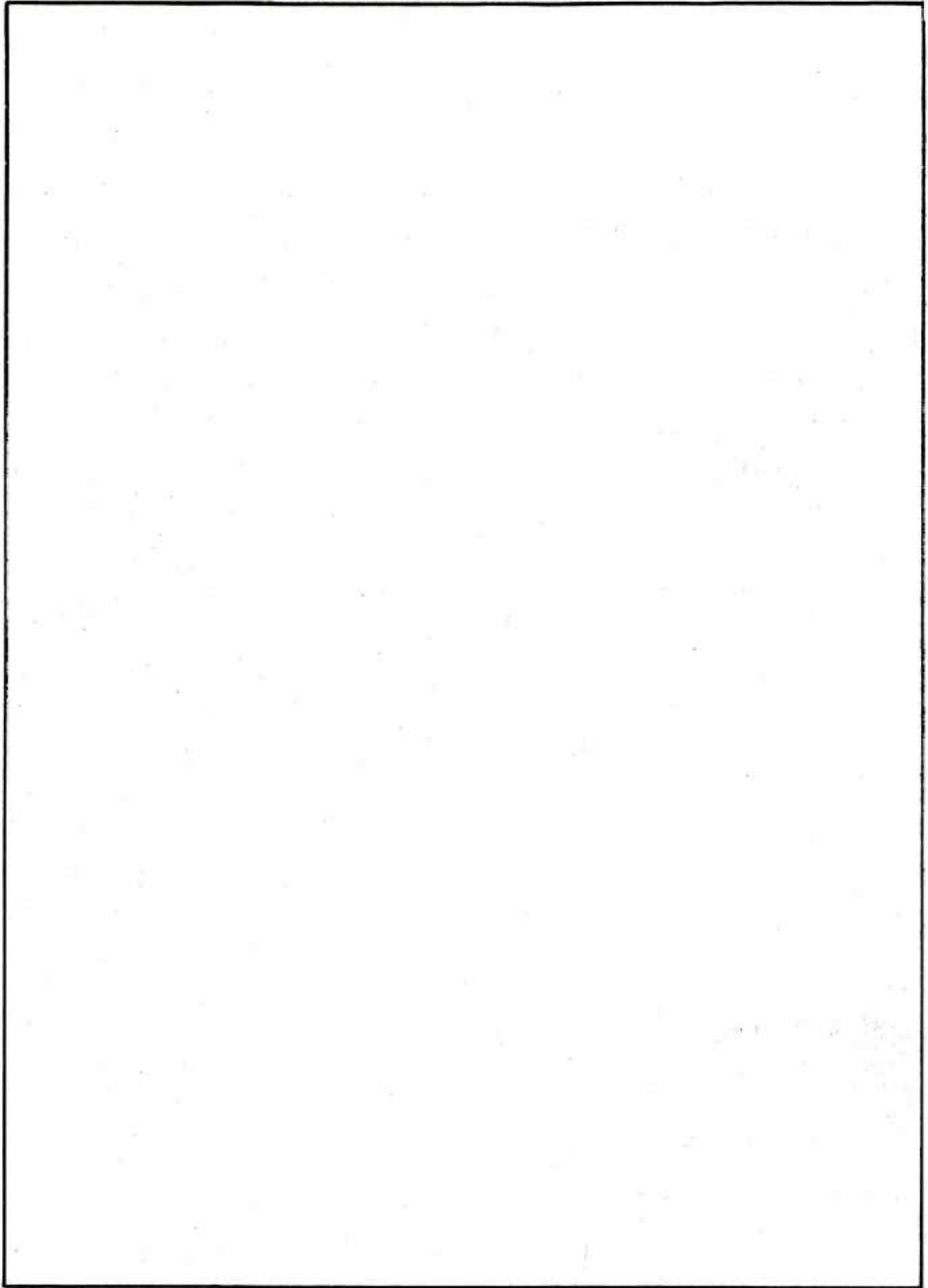
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20. ABSTRACT (Continue on reverse side if necessary and identify by block number) Most of the earlier results for residual stresses are based on the assumption of elastic unloading. In this report, the prediction of residual stresses for the case of reverse yielding including the combined Bauschinger and hardening effect will be reported for an autofrettaged thick-walled cylinder. The Bauschinger effect factor is varying as a function of overstrain. The strain- hardening effect is considered with different parameters used for loading and unloading process. The new results indicate that the influence of the combined Bauschinger and hardening effect on residual stress distribution is significant.		

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## INTRODUCTION

To increase the maximum pressure a cylinder can contain, it is common practice to produce a more advantageous stress distribution involving residual compressive hoop stresses near the bore by autofrettage treatment of the cylinder prior to use (ref 1). The determination of residual stresses is important in the stress intensity factor calculation (ref 2) and the fatigue life estimation (ref 3). There is, however, considerable disagreement among solutions obtained by different investigators for the residual stress distribution in the cylinder after the autofrettage process (refs 4-6). This discrepancy in residual stress is a result of different mathematical methods, end conditions, and material models. Different assumptions for the material properties such as compressibility, yield criterion, flow rule, hardening rule, Bauschinger effect, etc. can lead to many material models. Most of the earlier solutions for residual stresses were based on the assumption of elastic unloading and only a few considered reverse yielding (refs 5-7). For unloading with reversed yielding, there is no general agreement in the literature over which material model should be used. Many plasticity theories have been proposed and reviewed (ref 8), yet no theory is completely adequate. In particular, it seems that no theoretical model has been given to represent accurately the actual material behavior in a high strength steel (ref 9) as reported by Milligan, Koo, and Davidson.

In this report a new theoretical model is used with one attempt to give a close representation of the actual loading/unloading behavior in a high

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References are listed at the end of this report.

strength steel. The Bauschinger effect factor is treated as a function of overstrain. The strain-hardening effect is taken into account with different parameters used for loading and unloading process. The application of this model to the prediction of residual stresses in an autofrettaged thick-walled cylinder is reported.

#### THEORETICAL MODEL

Figure 1 shows the stress-strain curve during loading and unloading after overstrain in tension. The stress-strain curve during loading can be replaced with sufficient accuracy by a bilinear elastic-plastic model as shown also in Figure 1. For the plastic portion, the yield stress ( $\sigma$ ) is related to the plastic strain ( $\epsilon^p$ ) by

$$\sigma/\sigma_0 = 1 + m\zeta/(1-m) \text{ and } \zeta = (E/\sigma_0)\epsilon^p \quad (1)$$

where the Young's modulus ( $E$ ), tangent modulus ( $mE$ ), initial yield stress ( $\sigma_0$ ), and the Poisson's ratio ( $\nu$ ) are the material constants.

Choosing a new coordinate system ( $\sigma', \epsilon'$ ) with the origin at the point before unloading, we have for the plastic portion of the reverse yielding curve

$$\sigma'/\sigma_0 = \sigma_0'/\sigma_0 + m'\zeta'/(1-m') \text{ and } \zeta' = (E/\sigma_0)\epsilon'^p \quad (2)$$

where  $m'E$  is the slope of the reverse yielding curve,  $\epsilon'^p$  is the additional plastic strain during unloading and  $\sigma_0'$  is the linear drop in stress until reverse yielding begins. According to Eq. (1) and experimental data [9],  $\sigma_0$  can be expressed as a function of plastic strain just prior to unloading by

$$\sigma_0'/\sigma_0 = [1 + m\zeta/(1-m)] [1 + f(\zeta)] = g(\zeta) \quad (3)$$

where  $f(\zeta)$  is a function of pre-strain as shown in Figure 2.

## ANALYTICAL SOLUTION

Consider a thick-walled cylinder, internal radius  $a$  and external radius  $b$ , which is subjected to internal pressure  $p$ . The material is assumed to be elastic-plastic, obeying the Tresca's yield criterion, the associated flow theory, and a bilinear hardening rule for loading and unloading. The elastic-plastic interfaces before and after unloading are represented by  $\rho$  and  $\rho'$ , respectively. The elastic-plastic solution during loading and unloading can be obtained. Due to space limitations, only some of the final results are given below. During elastic-plastic loading, the equivalent plastic strain and the tangential stress in  $a < r < \rho$  can be calculated by

$$\zeta = \beta_1(\rho^2/r^2 - 1) \quad (4)$$

$$\sigma_\theta/\sigma_0 = \frac{1}{2} (1 + \rho^2/b^2) + \frac{1}{2} \beta_2(\rho^2/r^2 - 1) - (1 - \beta_2)\log \rho/r \quad (5)$$

where

$$\beta_1 = (1 - m) / \left[ m + \frac{\sqrt{3}}{2} \frac{(1 - m)}{(1 - \nu^2)} \right], \quad \beta_2 = m\beta_1 / (1 - m) \quad (6)$$

During elastic-plastic unloading, the additional equivalent plastic strain and the tangential stress in  $a < r < \rho'$  can be calculated by

$$\zeta' = \beta_1' [(\rho'/r)^2 g(\zeta_{\rho'}) - g(\zeta_r)] \quad (7)$$

$$\sigma_\theta'/\sigma_0 = p/\sigma_0 - \sigma'/\sigma_0 + \frac{1}{2} (\beta_2'/\beta_1)(\rho'/\rho)^2 (\zeta_r - \zeta_a)g(\zeta_{\rho'})$$

$$- \frac{1}{2} (1 - \beta_2') \int_{\zeta_r}^{\zeta_a} g(\zeta)(\zeta + \beta_1)^{-1} d\zeta \quad (8)$$

where

$$\beta_1' = (1 - m') / \left[ m' + \frac{\sqrt{3}}{2} \frac{(1 - m')}{(1 - \nu^2)} \right], \quad \beta_2' = m'\beta_1' / (1 - m') \quad (9)$$



## RESULTS AND DISCUSSION

The residual stress system, which will be denoted by two primes, is the sum of the system produced by loading and that produced by unloading, i.e.,

$$\sigma_{\theta}'' = \sigma_{\theta} + \sigma_{\theta}' \quad (10)$$

The numerical results for an open-end thick-walled cylinder with  $b/a = 2$ ,  $\nu = 0.3$ ,  $m = 0.01$  are shown in Figures 3 and 4, for  $\rho/a = 1.6$  and  $2.0$ , respectively. The other material parameters used in three cases are (a)  $m' = 0.01$ ,  $f=1$ ; (b)  $m' = 0.01$ ,  $f = 0.55-0.06\zeta$ ; (c)  $m' = 0.3$ ,  $f = 0.55-0.06\zeta$ . The first case represents isotropic hardening model with no Bauschinger effect (ref 5). According to this model, there is no reverse yielding. The second case shows the Bauschinger effect but almost no hardening during loading and unloading. The result is very close to that in Reference 7. The third case is a close representation of the actual loading/unloading behavior in a high strength steel (ref 9). The result is shown in solid curve in Figures 3 and 4. The new results indicate that the influence of the combined Bauschinger and hardening effect on the residual stress distribution is significant.

## REFERENCES

1. T. E. Davidson and D. P. Kendall in: *Mechanical Behavior of Materials Under Pressure*, H.L.D. Pugh, Ed., (Elsevier, Amsterdam, 1970), Chapter 2.
2. S. L. Pu and P. C. T. Chen, *ASME J. Pressure Vessel Technology* 105, 117-123 (1983).
3. J. F. Throop and H. S. Reemsnyder, *Residual Stress Effects in Fatigue*, ASTM STP 776 (1982).
4. P. C. T. Chen, *Proceedings of Army Symposium on Solid Mechanics*, 243-253 (1972).
5. D. R. Bland, *J. Mech. Phys. Solids* 4, 209-229 (1956).
6. G. J. Franklin and J. L. M. Morrison, *Proceedings Inst. Mech. Eng.* 174, 947-974 (1961).
7. A. P. Parker, K. A. Sleeper, and C. P. Andrasic, *Proceedings Army Numerical Analysis and Computers Conference*, 351-362 (1981).
8. H. Armen in: *Workshop on Inelastic Constitutive Equations For Metals*, E. Krempl, et al, Eds., (RPI, Troy, NY, 1975) 56-78.
9. R. V. Milligan, W. H. Koo, and T. E. Davidson, *ASME J. Basic Eng.* 88, 480-488 (1966).

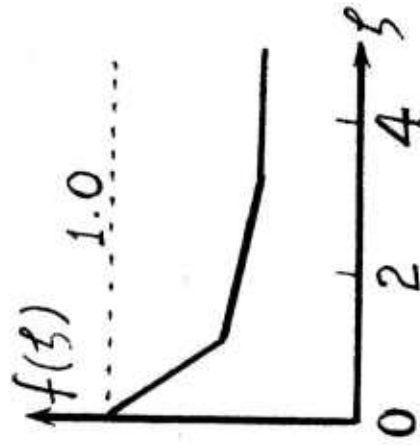


Figure 2. Bauschinger Effect.

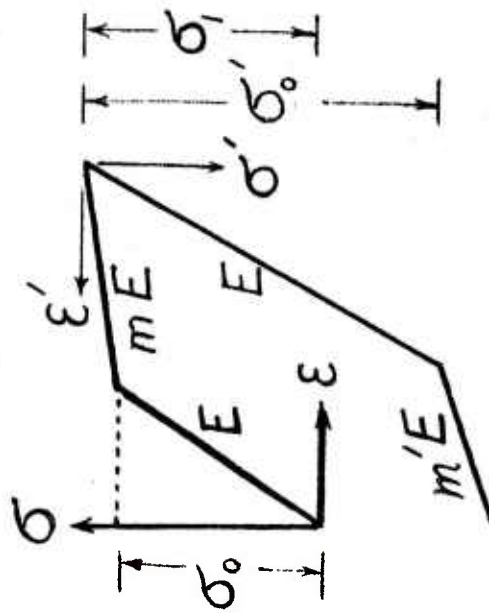


Figure 1. Stress-Strain Curve.

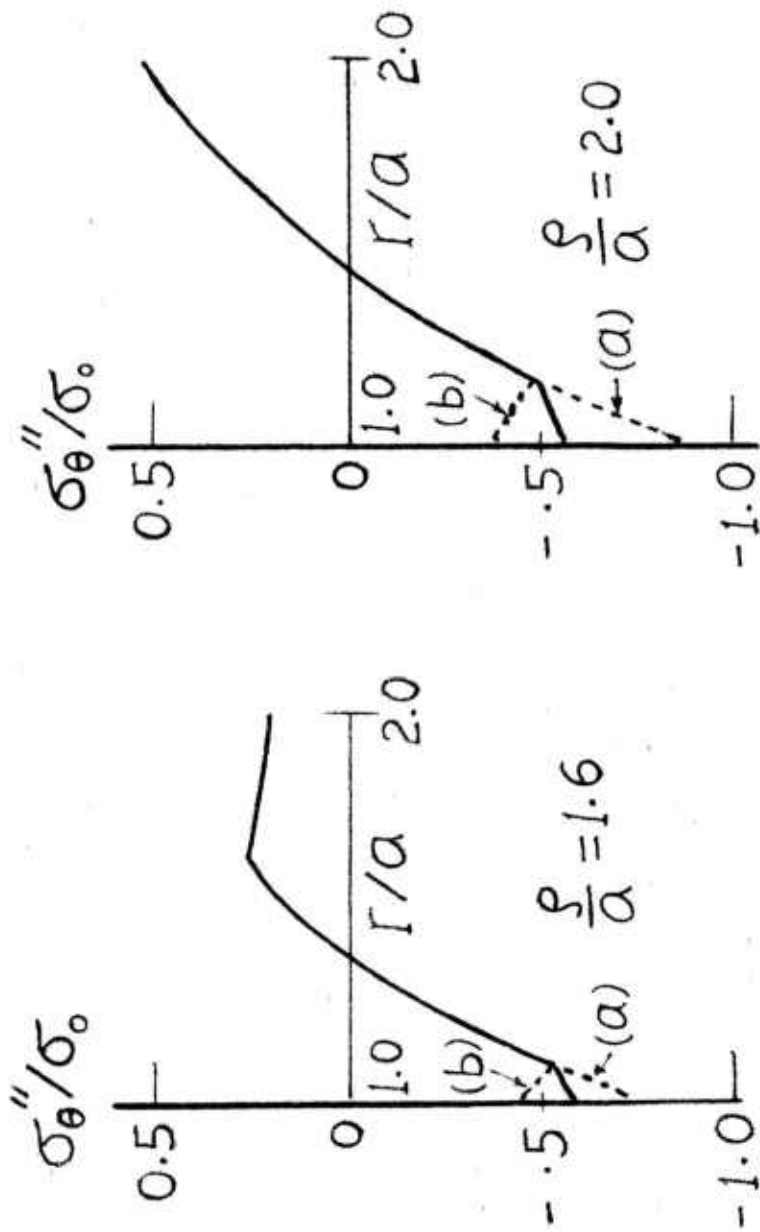


Figure 3. Residual Stress in a Thick-Walled Cylinder (60 percent overstrain).

Figure 4. Residual Stress in a Thick-Walled Cylinder (100 percent overstrain).

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