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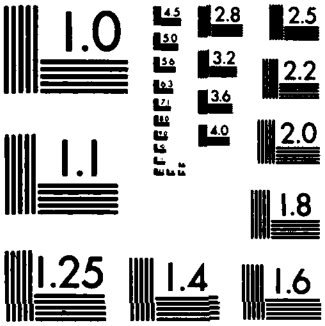

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Power-Series Solutions of the Gasdynamic Equations for Mach Reflection of a Planar Shock by a Wedge

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The self-similar solutions to the problem of a planar shock with Mach number M_s reflecting obliquely from a wedge with vertex angle θ_w are obtained to arbitrary accuracy by expanding the fluid quantities in power series in ~~the~~ scaled variables $\xi = x/t$, $\eta = y/t$. For single Mach reflection, there are four distinct regions: (a) the ambient gas ahead of the incident shock; (b) the gas behind the incident shock and outside the reflected (bow) shock; (c) the region bounded by the Mach stem, the wedge, and the contact surface (slip line) extending from the triple point; and (d) the doubly

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20. ABSTRACT (Continued)

mach number and wedge angle

cont

shocked medium bounded by the contact surface, wedge, and reflected shock. In region (b) the solution is known immediately in terms of M_s, θ_w, and the conditions in (a). The shapes of the Mach stem, contact surface and bow shock are expressed parametrically as $\xi = F(s)$, $\eta = G(s)$. Then ρ_c, u_c, v_c, p_c, and ρ_d, u_d, v_d, p_d are obtained by expanding variables in double power series, e.g.,

$$\rho_c(\xi, \eta) = \sum_{i,j} \rho^c_{ij} \xi^i \eta^j,$$

substituting in the ideal fluid equations, and equating coefficients of like powers through some order $N = \max(i+j)$. The resulting algebraic equations are solved subject to the additional relations obtained by applying the reflection conditions on the wedge, together with the jump conditions on the boundaries ac and bd, approximated by power series expansions of the F and G functions. Since all these equations are nonlinear, solutions are obtained by iteration with N increasing until convergence is obtained. The Ben-Dor equation for the fluid quantities in regions c, d at the triple point is used to give initial values. Because variation within each region is smooth, effectively exact descriptions of most features of interest can be obtained using series with $\lesssim 20$ terms. There are thus $\lesssim 200$ quantities in the discretization ~~of the~~ problem, compared with $\gtrsim 10^4$ in a conventional finite-difference treatment. The method generalizes readily to complex and double Mach reflections.

CONTENTS

POWER-SERIES SOLUTIONS OF THE GASDYNAMIC EQUATIONS FOR MACH REFLECTION OF A PLANAR SHOCK BY A WEDGE

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INTRODUCTION

The problem of a planar shock reflecting from an oblique surface goes back over a hundred years to Ernst Mach. Although this problem is important in its own right, much of the interest in it arises because of the need for better understanding of Mach reflection in more complicated situations. The field has been the object of particular interest during the last thirty years; the experimental and theoretical research carried out during this period have been reviewed by Ben-Dor.[1]

A constant planar shock propagating into a uniform ambient gas gives rise in the absence of reflection to a second medium with uniform thermodynamic properties in the region behind the shock. If it propagates in a shock

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tube whose walls are not parallel to the direction of propagation (because a wedge has been inserted along the side), a reflected shock wave propagates back into the interior of the shock tube. When the wedge angle is large, so that the primary shock is incident nearly normally on it, the reflected shock is also planar and no other gasdynamic discontinuities appear near the reflection point. As the wedge angle decreases, so that the incident shock becomes more and more nearly glancing, it becomes impossible for a fluid particle to traverse both incident and reflected shocks and still "turn the corner" enough to end up moving parallel to the wedge surface. At least one additional shock (the Mach stem) must appear (Fig. 1), intersecting the others at a so-called triple point. Because some of the material in the zone between the Mach stem and the reflected wave has been shocked once and some twice, another gasdynamic discontinuity (a contact surface) must also extend from the triple point into this region, terminating somewhere on the wedge surface. The reflected shock may terminate at the corner of the wedge (attached shock), or upstream from this point (detached shock), or may run into a second triple point between the first one and the corner (double Mach reflection). The latter case occurs in general for smaller wedge angles than does single Mach reflection; an intermediate case (complex Mach reflection) is also observed.

One would like to derive a theoretical description of Mach reflection which would complement the experimental results and address some of the questions the latter leave unanswered, such as the structure of the contact surface near the wedge surface, and whether a "triple Mach" regime exists. An analytical solution is out of the question, though pieces of the problem (e.g., the flow in the neighborhood of the triple point[1]) can be solved. Recourse must therefore be had to numerical methods.

Figure 1. Interferogram of single Mach reflection in N_2 (M_s = 4.72, Θ_w = 10°, P_a = 15 torr) after Ben-Dor[1], with X and Y axes and gasdynamic discontinuities drawn in, and regions labeled with a, b, c, d.

Straightforward differencing of the fluid equations (using either an ideal or realistic equation of state) has been carried out with considerable success[1,2]. A rectilinear Eulerian mesh with ~100 - 200 zones along each axis, possibly varying in size as a function of position to improve resolution near the Mach stem, is used. The conditions ahead of and behind the incident shock are used as boundary conditions, along with a reflection condition on the wedge. State-of-the art shock capturing techniques such as Flux-Corrected Transport (FCT)[3] resolve the envelope (reflected) waveform accurately and permit almost all the other gasdynamic discontinuties in the problem to be distinguished (Fig. 2). To date, however, such numerical solutions have not surpassed experimental interferometric data in accuracy, nor have they succeeded in answering any of the outstanding questions associated with Mach reflection.

Moreover, such calculations leave a distinct impression of brute force. Advancing the four fluid equation 10^3 - 10^4 timesteps on 10^4 or more zones seems profligate, particularly in view of the smoothness of most of the gasdynamic discontinuities and the gentle variation of the fluid quantities in the regions they bound. In much of the domain of the calculation the solution is known a priori and does not change in time.

Furthermore, the desired solution is actually self-similar. In the scaled variables $\xi = x/t$, $\eta = y/t$, where t is time and the origin of the coordinate system is fixed at the wedge corner, the observed wave forms are stationary. It is thus natural to seek a solution to the ideal fluid equations in terms of these similarity variables.

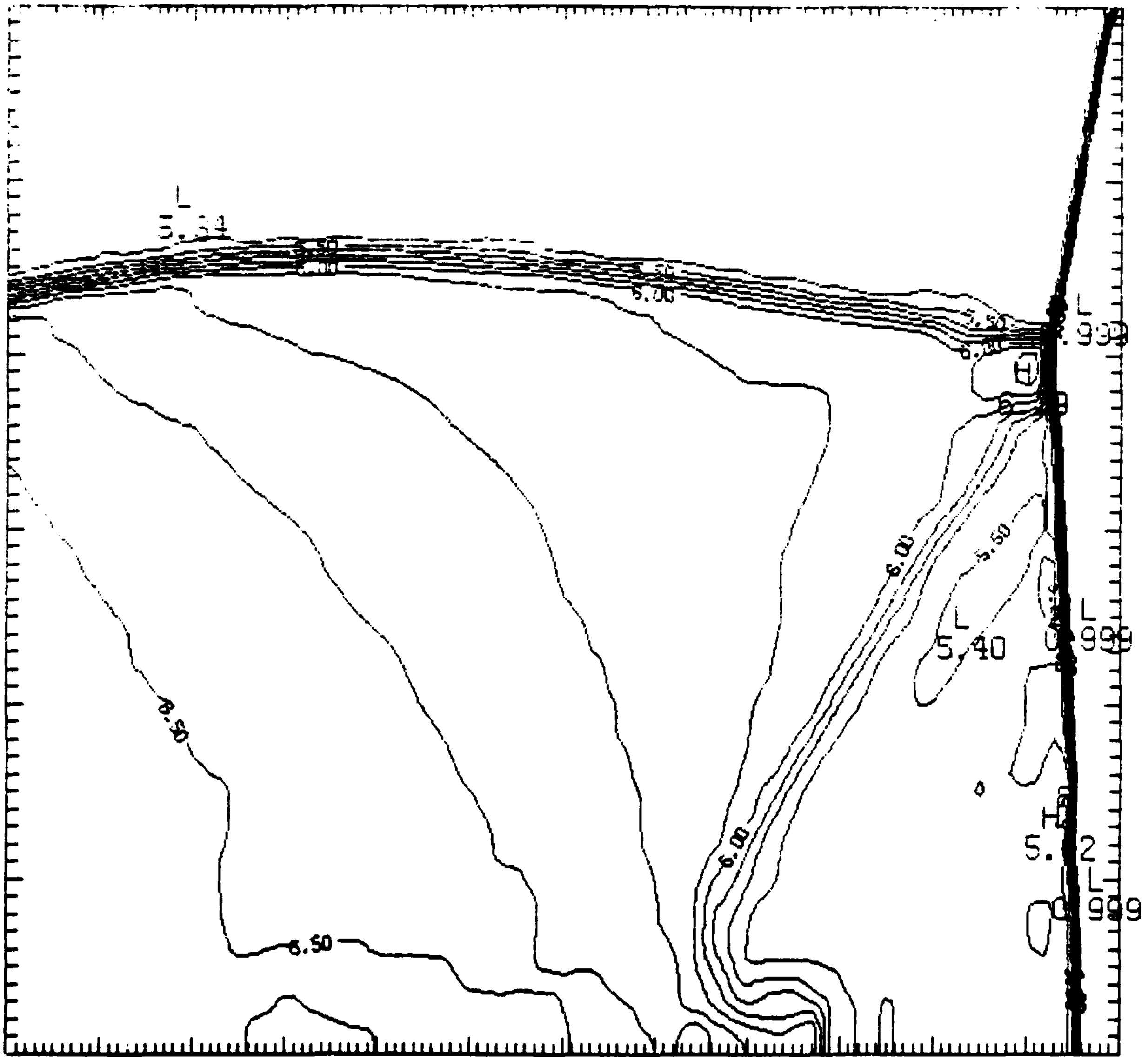

Figure 2. Triple-point region in ideal gas ($\gamma = 1.35$) corresponding to Fig. 1, calculated using FCT code FAST2D.

In the present paper we treat the shock-on-wedge problem by expanding the fluid variables and the functional forms of the boundaries in power series in ξ and η. The expansion coefficients are found by imposing the boundary conditions (Rankine-Hugoniot conditions on the shock, perfect reflection on the wedge surface).

Numerous fluid dynamics problems have been solved by power series techniques, as described by Van Dyke.[4] Here we follow the general approach he advocates: first do the lowest-order terms "by hand," then afterward automate the procedure, finally using sophisticated mathematical techniques to find the limiting values of critical numbers (e.g., boundaries between two reflection regimes). Unlike most of the problems described by Van Dyke, the algebraic equations determining the power series coefficients here are highly nonlinear and need to be solved by iteration; the proper iteration scheme is not obvious, but must be found by experimentation.

In the following section we derive the equations to be solved. The next section describes the iteration procedure, the program implementing it, and the display. The final section summarizes our conclusions and discusses extension of the calculation to higher order and to the more complicated Mach reflection cases.

DERIVATION OF EQUATIONS

In what follows we use the label "a" for quantities in the ambient gas (ahead of the incident shock); "b" behind the incident shock; "c" between the contact surface and the Mach stem; and "d" for the doubly shocked region between the reflected shock and the contact surface. Surfaces of gasdynamic discontinuities are labelled by the two regions they separate, e.g. "cd" for the contact surface. The triple point is labeled by the subscript "T," and the origin is at the point of attachment (the wedge corner).

In terms of the similarity variables ξ, η, the fluid equations become

$$(u-\xi)\,\frac{\partial\rho}{\partial\xi} + (v-\eta)\,\frac{\partial\rho}{\partial\eta} + \rho\,\left(\frac{\partial u}{\partial\xi} + \frac{\partial v}{\partial\eta}\right) = 0; \tag{1}$$

$$\rho\,\left[(u-\xi)\,\frac{\partial u}{\partial\xi} + (v-\eta)\,\frac{\partial u}{\partial\eta}\right] + \frac{\partial p}{\partial\xi} = 0; \tag{2}$$

$$\rho\,\left[(u-\xi)\,\frac{\partial v}{\partial\xi} + (v-\eta)\,\frac{\partial v}{\partial\eta}\right] + \frac{\partial p}{\partial\eta} = 0; \tag{3}$$

$$(u-\xi)\,\frac{\partial p}{\partial\xi} + (v-\eta)\,\frac{\partial p}{\partial\eta} + \gamma p\,\left(\frac{\partial u}{\partial\xi} + \frac{\partial v}{\partial\eta}\right) = ' \tag{4}$$

where γ is the adiabatic index.

If we define dimensionless variables by

$$X = \xi/c, \quad Y = \eta/c\ ; \tag{5}$$

$$U = (u-\xi)c^{-1}\ , \quad V = (v-\eta)c^{-1}\ , \tag{6}$$

$$R = \rho/\rho_a\ , \quad P = p/p_a\ , \tag{7}$$

where

$$c^2 = p_a/\rho_a, \tag{8}$$

then the fluid equations become

$$U\,\frac{\partial R}{\partial X} + V\,\frac{\partial R}{\partial Y} + R\,\left(\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} + 2\right) = 0; \tag{9}$$

$$R\ (U\frac{\partial U}{\partial X} + V\frac{\partial U}{\partial Y} + U) + \frac{\partial P}{\partial X} = 0; \tag{10}$$

$$R\ (U\frac{\partial V}{\partial X} + V\frac{\partial V}{\partial Y} + V) + \frac{\partial P}{\partial Y} = 0; \tag{11}$$

$$U\frac{\partial P}{\partial X} + V\frac{\partial P}{\partial Y} + \gamma P(\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} + 2) = 0. \tag{12}$$

In each of regions c and d we expand R, U, V, P in double power series,

$$R(X,Y) = R_{00} + R_{10}X + R_{01}Y + R_{20}X^2 + R_{11}XY + R_{02}Y^2 + \tag{13}$$

$$+ R_{30}X^3 + R_{21}X^2Y + R_{12}XY^2 + R_{03}Y^3 + \ldots .$$

$$U(X,Y) = U_{00} + U_{10}X + U_{01}Y + U_{20}X^2 + U_{11}XY + U_{02}Y^2 + \tag{14}$$

$$+ U_{30}X^3 + U_{21}X^2Y + U_{12}XY^2 + U_{03}Y^3 + \ldots .$$

$$V(X,Y) = V_{00} + V_{10}X + V_{01}Y + V_{20}X^2 + V_{11}XY + V_{02}Y^2 + \tag{15}$$

$$+ V_{30}X^3 + V_{21}X^2Y + V_{12}XY^2 + V_{03}Y^3 + \ldots .$$

$$P(X,Y) = P_{00} + P_{10}X + P_{01}Y + P_{20}X^2 + P_{11}XY + P_{02}Y^2 + \tag{16}$$

$$+ P_{30}X^3 + P_{21}X^2Y + P_{12}XY^2 + P_{03}X^3 + \ldots .$$

To apply the reflection boundary condition, we set

$$\frac{\partial \rho}{\partial \eta} = \frac{\partial u}{\partial \eta} = \frac{\partial p}{\partial \eta} = v = 0 \tag{17}$$

at $\eta = 0$ for all ξ. It follows that we must have

$$R_{01} = R_{11} = R_{21} = R_{31} = \ldots = 0; \tag{18}$$

$$U_{01} = U_{11} = U_{21} = U_{31} = \ldots = 0; \tag{19}$$

$$P_{01} = P_{11} = P_{21} = P_{31} = \ldots = 0; \tag{20}$$

and

$$V_{00} = V_{10} = V_{20} = V_{30} = \ldots = 0. \tag{21}$$

To proceed further, we substitute Eqs. (13)-(16) in Eqs. (9)-(12) and collect terms of like powers $X^i Y^j$, equating their coefficients to zero order by order. The task of doing this by conventional methods rapidly becomes hopeless, but it can readily be automated. To see what is going on in the lowest orders of our "hand" calculation, we expand using MACSYMA, the symbolic manipulation program developed at the Mathlab of MIT. This works up to about order $N = 6$, where we define the order of the expansion by $N = \max(i+j)$. Beyond that point storage limitations make it necessary to use "tricks," and eventually terminate the process entirely.

When this is done, certain patterns emerge. We find that we must have

$$V_{02} = V_{12} = V_{22} = \ldots = 0, \tag{22}$$

$$P_{03} = P_{13} = P_{23} = \ldots = 0. \tag{23}$$

Some of the equations then vanish identically. In each of regions c and d there are more variables than nontrivial equations, as shown by the following table:

	Equations					Variables				
Order	R	U	V	P	Total	R	U	V	P	Total
0	1	1	0	1	3	2	2	1	2	7
1	1	1	1	1	4	2	2	1	2	7
2	2	2	1	2	7	3	3	2	2	10
3	3	3	2	3	11	4	4	3	3	14
4	4	4	3	4	15	5	5	4	4	18
5	5	5	4	5	19	6	6	5	5	22
•										
•										
•										

Table 1

Table 1 shows that in each region there are three more unknowns than equations in each order, except for N = 0, where the number is four. The additional equations are supplied by the jump conditions, imposed on surfaces ac and bd for regions c and d, respectively.

If we represent a shock boundary parametrically by X = F(s), Y = G(s), then the Rankine-Hugoniot conditions become

$$X' (U - \bar{U}) + Y' (V - \bar{V}) = 0; \qquad (24)$$

$$Y' \; (R\,U - \bar{R}\,\bar{U}) = X' \; (R\,V - \bar{R}\,\bar{V}); \tag{25}$$

$$P + R\,(U^2 + V^2) = \bar{P} + \bar{R}\,(\bar{U}^2 + \bar{V}^2); \tag{26}$$

$$U^2 + V^2 + \frac{2\gamma P}{\gamma-1} = \bar{U}^2 + \bar{V}^2 + \frac{2\gamma \bar{P}}{\gamma-1}, \tag{27}$$

while on a contact surface

$$P = \bar{P}; \tag{28}$$

$$Y'\,U = X'\,V; \tag{29}$$

$$Y'\,\bar{U} = X'\,\bar{V}. \tag{30}$$

Here the variables in front of and behind the discontinuity are denoted by unbarred and barred symbols, respectively.

We know that ac is nearly a straight line normal to the wall, so the representation

$$X = X_0 + X_1 Y + X_2 Y^2 + X_3 Y^3 + \ldots . \tag{31}$$

is valid. Substituting Eq. (31) and the ambient conditions $u^a = v^a = 0$ in Eq. (24), the condition for continuity of parallel velocities at ac, which takes the form

$$X' \, (U^c - U^a) + V^c - V^a = 0, \qquad (32)$$

we find

$$X_1 = 0; \qquad (33)$$

$$X_2 = \frac{1 + V^c_{01} + V^c_{11}X_0 + V^c_{21}X_0^2 + \, . \, . \, . \, .}{2 \, (X_0 + U^c_{00} + U^c_{10}X_0 + U^c_{20}X_0^2 + \, . \, . \, . \, .} \, , \qquad (34)$$

etc. When the coefficients of Eq. (31) are known, substitution of Eq. (31) in Eqs. (25)-(27) (with $Y' = 1$) yields three conditions in each order on R^c_{ij}, U^c_{ij}, V^c_{ij}, P^c_{ij}.

Analogously, we write the equation for the reflected shock bd as

$$Y = Y_1X + Y_2X^2 + Y_3X^3 + \, . \, . \, . \, . \qquad (35)$$

Substitution in the condition for continuity of the parallel velocities at bd,

$$U^d - U^b + Y' (V^d - V^b) = 0, \qquad (36)$$

yields

$$Y_1 = -\frac{u^b - U^d_{00}}{v^b} ; \tag{37}$$

$$Y_2 = \frac{(V^d_{01} + 1)\ Y_1^2 + U^d_{10} + 1}{2v^b} ; \tag{38}$$

$$Y_3 = \frac{3(V^d_{01} + 1)Y_1Y_2 + (V^d_{11} + U^d_{02})\ Y_1^2 + U^d_{20}}{3v^b} , \tag{39}$$

etc. (Note that u^b and v^b, like u^a and v^a, are constants, whereas U^b, V^b, U^a, V^a are not.) Substitution of Eq. (35) is Eqs. (25)-(27) with $X' = 1$ then yields three conditions per order on the "d" variables.

At this point we need three more conditions: two for the remaining variables in order zero, and another to determine X_0. For this purpose we use Eqs. (28)-(30). Note that to apply these conditions everywhere on cd would overdetermine the system. This is equivalent to asserting the impossibility of continuing the solution across ab from a to b, across ac from a to c, across bd from b to d, and then across cd from c back to d, where it is already known. The only freedom we have is to impose Eqs. (28)-(30) at the triple point.

First we define X_T, Y_T by setting

$$Y_T = Y_1 X_T + Y_2 X_T^2 + Y_3 X_T^3 + \ldots . \tag{40}$$

$$X_T = X_0 + X_1 Y_T + X_2 Y_T^2 + X_3 Y_T^3 + \ldots . \tag{41}$$

Then we require

$$P^c(X_T, Y_T) = P^d(X_T, Y_T); \tag{42}$$

$$U^c(X_T, Y_T)\ \Delta Y_T = V^c(X_T, Y_T)\ \Delta X_T; \tag{43}$$

$$U^d(X_T, Y_T)\ \Delta Y_T = V^d(X_T, Y_T)\ \Delta X_T, \tag{44}$$

where

$$\begin{aligned} 0 = P^c(X,Y) - P^d(X,Y) &= P^c(X_T, Y_T) - P^d(X_T, Y_T) \\ &+ \Delta X_T \left(\frac{\partial P^c}{\partial X} - \frac{\partial P^d}{\partial X}\right)_T + \Delta Y_T \left(\frac{\partial P^c}{\partial Y} - \frac{\partial P^d}{\partial Y}\right)_T \end{aligned} \tag{45}$$

determines the ratio $\Delta X_T / \Delta Y_T$.

For our trial calculation we work to order $N = 5$. Including the quantities which vanish identically, we then have 183 equations in 183 unknowns, which are to be solved simultaneously.

SOLUTION OF EQUATIONS

The solution is obtained by iteration. Several points are important in the design of a satisfactory iteration scheme:

(i) Good values of the quantities X_0, X_T, Y_T, etc., can be obtained from the experimental data (Fig. 1) and used as initial guesses.

(ii) The system can be made quasilinear if we solve order by order, starting with $N = 0$, and using in any given order the previously obtained values of all variables not being solved for in that order.

(iii) To reduce the effect of possible instability in parts of the scheme, all variables are updated using some form of (under)relaxation, e.g., new value = old value + relaxation factor × corrections.

The current version of the code works as follows:

(i) For $N = 0$, use the X- and Y- independent terms of Eqs. (9)-(12), together with the zeroth-order form of Eqs. (25)-(27) and Eq. (43) to update R^c_{00}, R^c_{10}, U^c_{00}, U^c_{10}, V^c_{01}, P^c_{00}, P^c_{10};

(ii) Do the same thing [using Eq. (44) instead of (43)] for R^d_{00}, ..., P^d_{10};

(iii) Solve Eqs. (9)-(12) and Eqs. (25)-(27) on ac order by order for $N = 1$ to 5 to obtain the region-c variables;

(iv) Solve Eqs. (9)-(12) and Eqs. (24)-(27) on bd order by order for $N = 1$ to 5 to obtain the region-d variables plus Y_{N+1};

(v) Iterate Eqs. (40)-(42) together with the successive orders of Eq. (32) [e.g., Eq. (34) for X_2] to solve for X_T, Y_T, and X_i, $i = 0, 1, \ldots, 6$. (To a good approximation, $X_i = 0$ for all $i \neq 0, 2$.)

This scheme is repeated until no further change (to some preset tolerance) in the variables occurs. The program, written in Fortran, runs on a VAX 11/780 at about one second per iteration.

Results are most conveniently displayed as contour plots in pressure (or density). Although it is possible to drive the plotting pen directly using the exact formula P(X,Y), it is easier to declare a very large array (e.g., 500×500), fill it with pressures calculated at every X, Y, and use a standard contour plotting package on this.

CONCLUSION

The program described above is still under development, and no results have yet been generated. For this sample problem, it seems straightforward to carry the method to a successful solution. We have encountered, and apparently overcome, two types of difficulties: determining the formulation for the problem, and solving the resulting set of equations. Only if both stages are handled correctly will useful answers results. There remains the (programming) task of automating the solution, so as to work to arbitrarily high order N. This is of interest chiefly in connection with studying the behavior of the roll-up in the contact surface.

Finally, extension to other forms of Mach reflection is of interest. Attached double (or triple) Mach reflection presents no problem in principle, and can be handled using the techniques described here. Complex Mach reflection and detached shocks are less clear. At present we do not know how to formulate the problem in either situation so as to make it well-posed. This will be addressed in future work.

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