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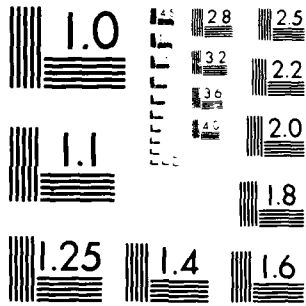
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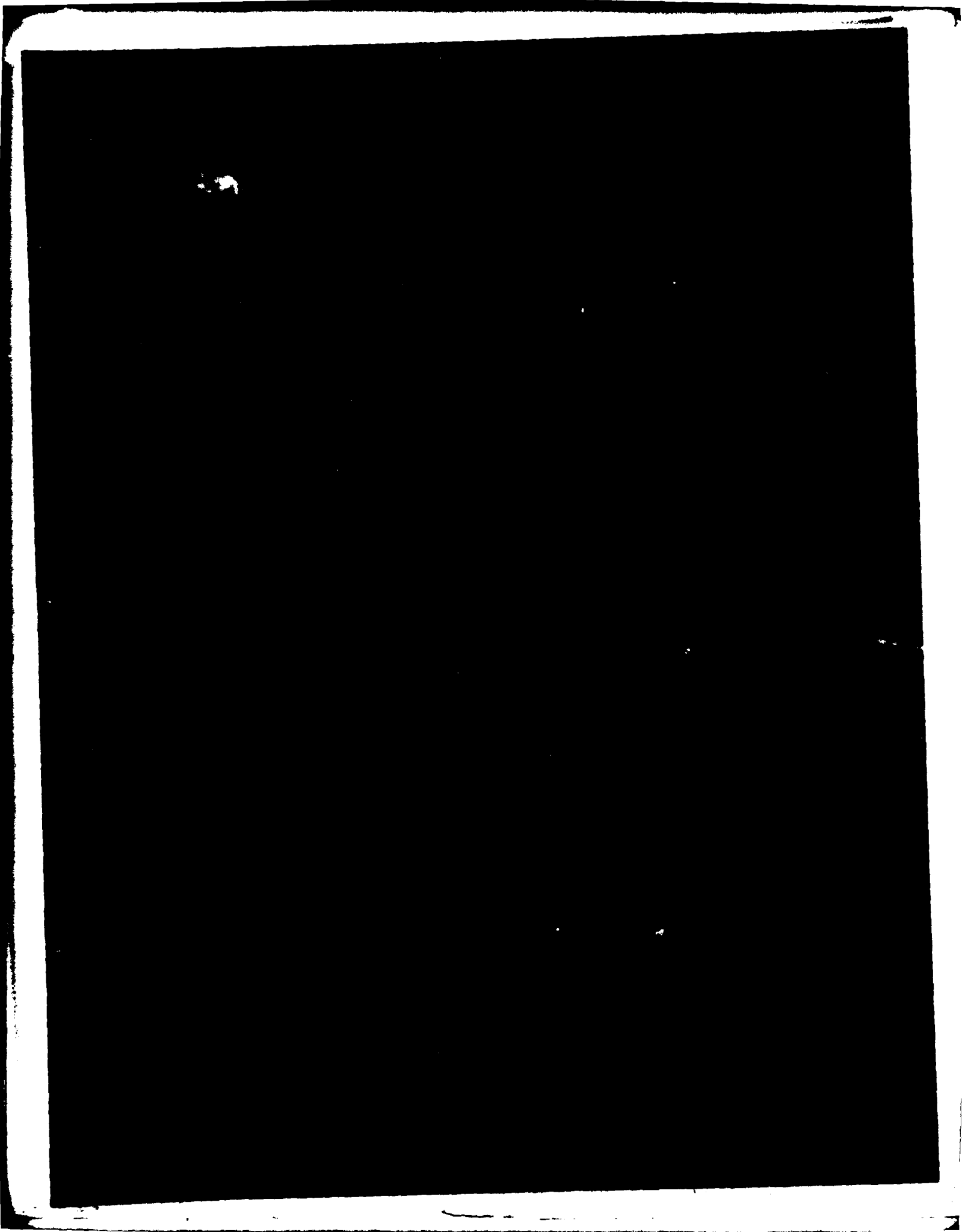
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SOME ALTERNATIVES TO BAYES' RULE

BY

PERSI DIACONIS and SANDY ZABELL

TECHNICAL REPORT NO. 339

NOVEMBER 29, 1983

Prepared Under Contract

N00014-76-C-0475 (NR-042-267)

For the Office of Naval Research

Herbert Solomon, Project Director

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SOME ALTERNATIVES TO BAYES' RULE

Persi Diaconis

and

Sandy Zabell

ABSTRACT

We review Bayes' rule, Jeffrey's rule, and Dempster's rule as methods of revising probability judgments based on new evidence.

1. INTRODUCTION

There are several different approaches to what might be called "the mathematics of changing one's mind." The most frequently discussed method, Bayes' rule, changes a prior or initial probability P to a posterior or final probability P^* , based on the occurrence of an event E . It specifies that for any event A :

$$\text{BAYES' RULE} \quad P^*(A) = P(A \text{ and } E)/P(E) .$$

Bayes' rule is not (at least directly) applicable if

- New information does not arrive in the form "event E occurred" (e.g., the murderer was a woman), but instead in the form, "the odds on E have changed" (e.g., the murderer was likely to have been a woman). This is sometimes called the problem of probable knowledge.
- Even if " E occurs," we may not have thought about E beforehand. Thus we will not have previously assessed either $P(A \text{ and } E)$ or $P(E)$, and will therefore be unable to make direct use of Bayes' rule. We will call this the problem of unanticipated knowledge.

→ ^{discuss} This review focuses on two proposed alternatives to Bayes' rule for revising probability assessments in the face of new information: Richard Jeffrey's rule of conditioning and Arthur Dempster's rule of combination. Section 2 describes Jeffrey's rule. Section 3 describes upper and lower probabilities and Dempster's rule for their combination. Section 4 shows that the two rules are in fact closely connected: Jeffrey's rule is the additive version of Dempster's rule in those situations where the two rules are comparable.

Our presentation is intended as an introduction to a growing and already sizeable literature. It proceeds mainly by a series of examples. For references to the literature on the limitations of Bayes' rule and for further information on Jeffrey's rule see Diaconis and Zabell (1982); for further references on upper and lower probability and Dempster's rule see Shafer (1976, 1982).

2. JEFFREY'S RULE OF CONDITIONING

While the mathematics of Bayes' rule presupposes some given event E , Jeffrey's rule assumes the existence of a partition $\{E_1, E_2, \dots, E_n\}$ on which new probabilities $P^*(E_i)$ are given (the elements of a partition are by definition, mutually exclusive and exhaustive). It specifies that for any event A :

$$\text{JEFFREY'S RULE} \quad P^*(A) = \sum_{i=1}^n P(A|E_i) P^*(E_i) .$$

Jeffrey's rule is mathematically equivalent to, and hence applicable only if it is judged that the "J-condition"

$$(J) \quad P^*(A|E_i) = P(A|E_i)$$



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holds for all A and i . The J-condition can be interpreted as stating that the only impact of the new evidence was to change the probabilities on the elements of the partition; given an element of the partition, the new and old probabilities agree.

Example 1 (Uncertain Perception). Suppose we are about to hear one of two recordings of Shakespeare on the radio, to be read by either Olivier (E) or Gielgud (E^c), but we are uncertain as to which, and we have a prior with mass $\frac{1}{2}$ on Olivier and $\frac{1}{2}$ on Gielgud. After hearing the recording, one might judge it fairly likely, but by no means certain, to be by Olivier. The change in belief takes place by direct recognition of the voice. If the only impact of hearing the recording is to change the odds on Olivier and Gielgud, in the sense that for any A , $P^*(A|E) = P(A|E)$ and $P^*(A|E^c) = P(A|E)$, then after assessing $P^*(E)$ we may proceed to apply Jeffrey's rule. (Of course, the former might well not be the case; for example the quality of the recording might convey additional information as to its date or manufacture.)

Richard Jeffrey has argued that examples of this type are the norm: "it is rarely or never that there is a proposition for which the direct effect of an observation is to change the observer's degree of belief in a proposition to 1" (Jeffrey (1962), p. 171).

Example 2 (Unanticipated Knowledge). Suppose we are thinking about three possible trials of a new surgical procedure. Under the usual circumstances a probability assignment P is made on the eight possible outcomes $\Omega = \{000, 001, 010, 100, 011, 101, 110, 111\}$ where 1 denotes a successful outcome, 0 not. Suppose a colleague informs us that another hospital had performed this type of operation 100 times, with 80 successful outcomes. This is clearly relevant information and we will obviously want to revise our opinion. The information cannot be put in terms of the occurrence of an

event in the original eight point space Ω and Bayes' rule is not directly available.

Diaconis and Zabell (1982) discuss four possible approaches to the problem of forming P^* - complete reassessment, retrospective conditioning, the use of exchangeability, and Jeffrey's rule. We review here the use of Jeffrey's rule, as an example illustrating how natural partitions $\{E_i\}$ can arise.

Suppose that the original probability P was exchangeable, that is, $P(001) = P(010) = P(100)$, $P(110) = P(101) = P(011)$. In the situation described, the colleague's report says nothing about the order of the trials and we may thus require the new P^* to remain exchangeable. Consider the partition $\{E_0, E_1, E_2, E_3\}$, where E_i is the set of outcomes with i ones: $E_0 = \{000\}$, $E_1 = \{001, 010, 100\}$, $E_2 = \{110, 101, 011\}$, $E_3 = \{111\}$. The exchangeability of both P and P^* is equivalent to Jeffrey's condition:

$$P(A|E_i) = P^*(A|E_i) ,$$

and so, to complete the assignment of P^* , we need only undertake an assessment of $P^*(E_i)$. Then P^* is determined by Jeffrey's rule: for any set A

$$P^*(A) = \sum_{i=1}^n P(A|E_i) P^*(E_i) .$$

Example 3 (Bayes' Rule). If (1) the partition consists of a set E and its complement E^c , and (2) if $P^*(E^c) = 0$, then Jeffrey's rule reduces to Bayes' rule $P^*(A) = P(A|E)$.

Diaconis and Zabell (1982) link the J-condition with the statistical concept of sufficiency, and show that Jeffrey's rule gives the closest probability P^* to P with prescribed values $P^*(E_i)$. Our 1982 paper also

gives continuous versions of the rule, and an analysis of Jeffrey's rule when two or more sources of evidence are considered simultaneously.

3. UPPER AND LOWER PROBABILITIES, AND DEMPSTER'S RULE OF COMBINATION

We begin our discussion with an example drawn from Diaconis (1978).

It concerns the well known problem of the three prisoners:

Of three prisoners, a, b, and c, two are to be executed but a does not know which. He therefore says to the jailer, 'Since either b or c is certainly going to be executed, you will give me no information about my own chances if you give me the name of one man, either b or c, who is going to be executed.' Accepting this argument, the jailer truthfully replies, 'b will be executed.' Thereupon a feels happier because before the jailer replied, his own chance of execution was two-thirds, but afterward there are only two people, himself and c who could be the one not executed, and so his chance of execution is one-half.

Is a justified in believing that his chances of escaping execution have improved? Consider the set of possible outcomes

$$S = \{(\underline{a}, \underline{b}) (\underline{a}, \underline{c}) (\underline{b}, \underline{c}) (\underline{c}, \underline{b})\}$$

where, for example, $(\underline{a}, \underline{b})$ means a will live and the jailer answers b.

In the classical Bayesian solution of this problem (see, e.g., Gardner (1961), Chapter 19), a, b, and c are assumed equally likely to be pardoned and if a is to be set free, it is assumed that the jailer will answer by choosing b or c with probability $\frac{1}{2}$. These assumptions translate into the probability

P on S with $P(\underline{a}, \underline{b}) = P(\underline{a}, \underline{c}) = \frac{1}{6}$, $P(\underline{b}, \underline{c}) = P(\underline{c}, \underline{b}) = \frac{1}{3}$, and Bayes' rule gives

$$P(\underline{a} \text{ lives} | \text{jailer says } \underline{b}) = \frac{P(\underline{a}, \underline{b})}{P(\underline{a}, \underline{b}) + P(\underline{c}, \underline{b})} = \frac{1}{3},$$

i.e., a's chances have not improved.

We will discuss three ways to model this problem using upper and lower probabilities P_* , P^* . Upper and lower probabilities are functions defined on the subsets of a set S satisfying

$$C1 \quad P_*(\phi) = 0, \quad P_*(S) = 1,$$

$$C2 \quad P^*(A) = 1 - P_*(A^c),$$

and the inequalities

$$C3 \quad P_*(A_1 \cup \dots \cup A_n) \geq \sum P_*(A_i) - \sum_{i < j} P_*(A_i \cap A_j) \\ + \dots + (-1)^{n+1} P_*(A_1 \cap \dots \cap A_n).$$

Conditions C1, C2, and C3 will be motivated later on. For the present we note that the definitions imply $P_* \leq P^*$, so that the upper-lower pair (P_*, P^*) may be thought of as bounds on some "true probability" P, with $P_* \leq P \leq P^*$. A simple example is the vacuous upper-lower pair

$$P_*(A) = 0 \text{ if } A \subsetneq S, \quad P_*(S) = 1.$$

The vacuous pair is often suggested as a way of quantifying a state of "no knowledge."

Arthur Dempster has suggested that, given the occurrence of an event E, the appropriate way of modifying an upper-lower pair to a new upper-lower pair incorporating the new information is via:

DEMPSTER'S RULE $P^*(A|E) = P^*(A \cap E)/P^*(E)$.

A motivation for Dempster's rule will also be given later. First we return to the three prisoner problem and show how it may be analyzed using different upper-lower pairs and Dempster's rule.

Model 1. Suppose that prisoner a models his (lack of) knowledge by putting the vacuous upper-lower pair on the four-point set S . Then the definitions imply $P^*(\underline{a} \text{ will live} | \text{jailer says } \underline{b}) = 1$, $P_*(\underline{a} \text{ will live} | \text{jailer says } \underline{b}) = 0$. Thus, with no assumptions on the problem, the jailer's information does not reduce his uncertainty, and the conditional upper-lower pair remains vacuous.

Model 2. Suppose that a assumes that the initial decision as to who will live is made at random, but assumes nothing about how the jailer will act except that he will tell the truth. One way to model this is to consider the space $L = \{a, b, c\}$; the probability P on L corresponding to the random choice of who will live, i.e. $P(\underline{a}) = P(\underline{b}) = P(\underline{c}) = \frac{1}{3}$; and the multivalued map Γ , from L to the subsets of S , given by

$$\Gamma(\underline{a}) = (\underline{a}, \underline{b}) \cup (\underline{a}, \underline{c}), \quad \Gamma(\underline{b}) = (\underline{b}, \underline{c}), \quad \Gamma(\underline{c}) = (\underline{c}, \underline{b}) .$$

Thus Γ delineates the possible outcomes when a, b, or c are pardoned.

Dempster has described how an upper-lower pair can be constructed on S whenever a set L , probability P on L , and multivalued map $\Gamma: L \rightarrow \text{subsets of } S$ are given. Define

$$P^*(A) = P\{\ell \in L: \Gamma(\ell) \cap A \neq \emptyset\} , \quad \text{and} \quad P_*(A) = P\{\ell: \Gamma(\ell) \subset A\} .$$

P^* and P_* represent the largest and smallest probabilities that can be assigned to A consistent with Γ and P .

The French mathematician Gustave Choquet (1953) proved the following important result.

Theorem. Every upper-lower pair constructed in this way from a multivalued map satisfies conditions C1, C2, and C3. Conversely, given an upper-lower pair satisfying C1, C2, and C3, there exists a set L , a probability P on L , and a multivalued map $\Gamma: L \rightarrow$ subsets of S which realizes the upper-lower pair.

This is the promised motivation for C1, C2, and C3. Any function P_* satisfying C1, C2, C3 is said to be a capacity of infinite order. The infinite system of inequalities C3 are known as the Block-Marschack inequalities in the psychology of choice; see the article by Batchelder in this volume.

Returning to the three prisoner example, we have for the upper-lower pair P_*, P^* that arises from L, P , and Γ :

$$P^*(\text{jailer says } \underline{b}) = P^*\{(\underline{a}, \underline{b}) \cup (\underline{c}, \underline{b})\} = P\{\underline{a} \cup \underline{c}\} = \frac{2}{3}$$

$$P_*(\text{jailer says } \underline{b}) = 1 - P^*\{\text{jailer does not say } \underline{b}\} = 1 - P^*\{(\underline{a}, \underline{c}) \cup (\underline{b}, \underline{c})\} = \frac{1}{3}.$$

This result is intuitively reasonable: if the jailer said \underline{b} when he truthfully could he would say \underline{b} $\frac{2}{3}$ of the time. If the jailer avoided saying \underline{b} whenever he truthfully could, he would say \underline{b} $\frac{1}{3}$ of the time. Dempster's rule of conditioning then gives

$$P^*(\underline{a} \text{ will live} | \text{jailer says } \underline{b}) = P_*(\underline{a} \text{ will live} | \text{jailer says } \underline{b}) = \frac{1}{2}.$$

Thus, with this set of assumptions \underline{a} is justified in reasoning exactly as described in the original version of the problem. Observe that after Dempster conditioning the two members of the upper-lower pair are actually equal, coalescing to a bona fide probability.

A "lazy Bayesian" could regard the formation of an upper-lower pair based on a multivalued mapping as a way of proceeding without quantifying

belief within the elements of $\Gamma(\ell)$. The calculations result in bounds which would be useful in checking a more refined quantification.

Here is Dempster's motivation for his rule of conditioning, via multivalued mappings. Consider a pair of probability spaces and multivalued mappings:

$$(L_1, P_1) \xrightarrow{\Gamma_1} \mathcal{S}, \quad (L_2, P_2) \xrightarrow{\Gamma_2} \mathcal{S},$$

(where \mathcal{S} denotes the subsets of S). Define a product space $(L_1 \times L_2, P_1 \times P_2)$ and $\Gamma_1 \times \Gamma_2: L_1 \times L_2 \rightarrow \mathcal{S}$ by

$$\Gamma_1 \times \Gamma_2(\ell_1, \ell_2) = \Gamma_1(\ell_1) \cap \Gamma_2(\ell_2).$$

It is easy to show that

- D1 If $\Gamma_1(\ell) \equiv S$ then the upper-lower pair associated with Γ_1 is vacuous and the upper-lower pair associated with the product $\Gamma_1 \times \Gamma_2$ is identical to the upper-lower pair associated with Γ_2 .
- D2 If either the component upper-lower pairs is a probability, then the product is a probability.
- D3 If $\Gamma_1(\ell) \equiv E$, then the product yields Dempster's rule of conditioning.

For further discussion of this motivation for Dempster's rule, see Dempster (1968).

To us, the multivalued mapping approach to upper-lower pairs seems preferable to their direct use and interpretation (as favored by Shafer (1976)).

To discuss Model 2 and the example further, consider the general Bayesian solution to the three Poisson problem: let $\pi_{\underline{a}}$, $\pi_{\underline{b}}$, and $\pi_{\underline{c}}$ be the prior probabilities that \underline{a} , \underline{b} , and \underline{c} are pardoned, and let p be the probability that the jailer names \underline{b} when \underline{a} is pardoned. Then

$$P(\underline{a}, \underline{b}) = \pi_{\underline{a}} p, \quad P(\underline{a}, \underline{c}) = \pi_{\underline{a}} (1-p),$$

$$P(\underline{b}, \underline{c}) = \pi_{\underline{b}}, \quad P(\underline{c}, \underline{b}) = \pi_{\underline{c}},$$

and

$$P(\underline{a} \text{ lives} | \text{jailer says } \underline{b}) = \frac{\pi_{\underline{a}} p}{\pi_{\underline{a}} p + \pi_{\underline{c}}}.$$

For Model 2 we have $\pi_{\underline{a}} = \pi_{\underline{b}} = \pi_{\underline{c}} = \frac{1}{3}$, with p remaining a free parameter. This generates a family \mathcal{P} of possible probability measures. This family can be used to define a different kind of upper-lower probability, say U and L , defined by

$$U(A) = \max\{P(A) : P \in \mathcal{P}\} \quad \text{and} \quad L(A) = \min\{P(A) : P \in \mathcal{P}\},$$

In this case, it is easy to check that U and L are exactly the same as those derived via the multivalued map \mathcal{L} . In general, however, upper-lower pairs defined by sups and infs will not be capacities of infinite order, but merely capacities of order 2. Wally and Fine (1979) contains further discussion.

Note here that the conditional probabilities generated by \mathcal{P} range from 0 to $\frac{1}{2}$, while Dempster's rule of conditioning picks out the unique value $\frac{1}{2}$. This is a disturbing result for a Bayesian, since it calls into question both the interpretation and justification of Dempster's rule.

Either Dempster's rule contains further hidden, implicit assumptions, here

responsible for narrowing down the range of possible conditional probabilities to but one, or it operates in a manner verry different from ordinary, Bayesian conditioning, in which case we would wish some further guidance as to its interpretation and meaning. Mere surface plausibility is insufficient, for it is possible to suggest at least one equally plausible alternative to Dempster's rule, namely

$$P_*(A|B) = \frac{P_*(A \text{ and } B)}{P_*(B)} \quad \text{and} \quad P^*(A|B) = 1 - P_*(A^c | B) .$$

This yields a rule of conditioning different from Dempster's, yet the resulting conditional set functions are capacities. In what sense is one of them right? (Note that for this method of conditioning the upper-lower pair for Model 2 of the three prisoner problem yields upper and lower conditional probabilities of 0!)

Model 3. Suppose a knows nothing about the selection process for who will live, but assumes (or is told) that if he lives, the jailer will choose randomly between answering b or c. (Of course, if the jailer knows b is to live, he will answer c, and vice versa.) This problem can be modeled by assuming that three different probability measures are given on the set $W = \{\underline{b}, \underline{c}\}$ of the jailer's possible answers: $P_{\underline{a}}(\underline{b}) = P_{\underline{a}}(\underline{c}) = \frac{1}{2}$; $P_{\underline{b}}(\underline{c}) = 1$; $P_{\underline{c}}(\underline{b}) = 1$. Given the jailer's answer, Chapter 11 of Shafer (1976) proposes a method related to direct use of likelihood for deriving an upper-lower pair on the parameter set $L = \{\underline{a}, \underline{b}, \underline{c}\}$. This yields $P_*(\underline{a} \text{ will live} | \text{jailer says } \underline{b}) = 0$, $P^*(\underline{a} \text{ will live} | \text{jailer says } \underline{b}) = \frac{1}{2}$. In this model, before questioning the jailer, a might have expressed his ignorance by $P_*(\underline{a} \text{ lives}) = 0$, $P^*(\underline{a} \text{ lives}) = 1$. After learning b will die, a can no longer be so optimistic.

Again, the comparison with the Bayesian analysis is instructive. Now $\pi_{\underline{a}}$, $\pi_{\underline{b}}$, $\pi_{\underline{c}}$ are arbitrary and $p = \frac{1}{2}$, so that the resulting conditional probabilities for $P(\underline{a} \text{ will live} | \text{jailer says } \underline{b})$ range from 0 to 1.

Thus while Shafer's method does not suffer in this case from the defect of picking out a unique conditional probability, the range spanned by his resulting upper-lower pair differs markedly from that arising from the Bayesian analysis, again calling into question both the interpretation and justification for the method.

Dempster (1966) has proposed a different approach to this problem. In general the two methods do not agree, but in this simple example they do, and hence the objection just voiced to Shafer's analysis applies with equal force to Dempster's.

3. RELATIONSHIPS BETWEEN JEFFREY'S RULE AND DEMPSTER'S RULE

Glenn Shafer has observed that Jeffrey's rule and Dempster's rule agree in certain cases. This is an easy consequence of the three properties D1-D3 of Dempster's rule given at the end of the preceding section. To be precise, let P_1 be a probability on a set S , let $\{E_i\}_{i=1}^n$ be a partition of S , and suppose that $P_2(E_i)$ are positive numbers summing to 1. Define multi-valued mappings Γ_i from $L_i \rightarrow$ subsets of S as follows:

$$L_1 = S, \quad \Gamma_1(s) = s,$$

$$L_2 = \{1, \dots, n\}, \quad \Gamma_2(i) = E_i.$$

The product of (P_1, L_1, Γ_1) and (P_2, L_2, Γ_2) combine to give a probability on S because of property C3. Shafer (1981, Section 7) shows that this is precisely the probability given by Jeffrey's rule.

Thus Dempster's rule may be viewed as a generalization of Jeffrey's rule. The difference between them may be summarized as follows:

1. Jeffrey's rule works with ordinary probabilities which have a well understood interpretation in a variety of real world situations.

Dempster's rule works with upper and lower probabilities which presently lack an operational interpretation, objective or subjective.

2. Dempster's rule is a way to pool fairly general types of information. If one is willing to work outside the world of well defined probabilities, upper-lower pairs representing information from very general sources can be combined. An additive approach to the combination of different types of evidence is given in Sections 3, 4, 5 of Diaconis and Zabell (1982). The comparison of the two approaches is instructive: Dempster's rule is based on an intuitive notion of independence; the method using Jeffrey's rule that we suggest is not tied to such independence.

Finally, it is worth considering a problem that neither theory claims to know how to treat. Suppose we have a probability \underline{P} defined on a class \mathcal{F} of subsets of a space S . After observation or reflection we decide that we need to work with a richer collection of sets \mathcal{F}^* , perhaps even a larger basic space S^* . For example, new data may force us to consider outcomes previously thought impossible or unimportant. How should we proceed to extend \underline{P} , changing it as little as possible? Several procedures are available under special circumstances, but any semblance of a general theory is presently lacking.

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REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER 339	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) Some Alternatives to Bayes' Rule		5. TYPE OF REPORT & PERIOD COVERED TECHNICAL REPORT
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) Persi Diaconis and Sandy Zabell		8. CONTRACT OR GRANT NUMBER(s) N00014-76-C-0475
9. PERFORMING ORGANIZATION NAME AND ADDRESS Department of Statistics Stanford University Stanford, CA 94305		10. PROGRAM ELEMENT PROJECT, TASK AREA & WORK UNIT NUMBERS NR-042-267
11. CONTROLLING OFFICE NAME AND ADDRESS Office of Naval Research Statistics & Probability Program Code 411SP		12. REPORT DATE November 29, 1983
		13. NUMBER OF PAGES
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)		15. SECURITY CLASS. (of this report) UNCLASSIFIED
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE
16. DISTRIBUTION STATEMENT (of this Report) APPROVED FOR PUBLIC RELEASE: DISTRIBUTION UNLIMITED.		
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)		
18. SUPPLEMENTARY NOTES Also issued as Technical Report No. 205 under National Science Foundation Grant MCS80-24649, Department of Statistics, Stanford University.		
19. KEY WORDS (Continue on reverse side if necessary and identify by block number) Bayes' rule, Jeffrey's rule, Dempster's rule, probability judgments.		
20. ABSTRACT (Continue on reverse side if necessary and identify by block number) We review Bayes' rule, Jeffrey's rule, and Dempster's rule as methods of revising probability judgments based on new evidence.		

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