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APPLICABILITY OF STELLAR RATE FOR ASTRONOMIC POSITION  
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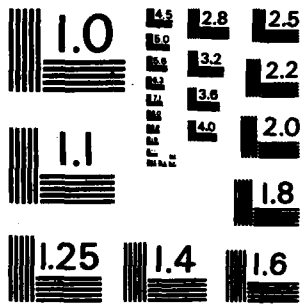
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APPLICABILITY OF STELLAR RATE  
FOR ASTRONOMIC POSITION DETERMINATION

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BIOGRAPHICAL SKETCH

Dr. Angel A. Baldini joined ETL's predecessor organization in 1960. From 1957-1960 he was associated with the Georgetown University Observatory, Washington, D.C. Prior to 1957, he was Professor and Head, Department of Geodesy, La Plata University, Argentina. At ETL, Dr. Baldini worked primarily in the field of astro-geodesy. Since 1974 he has been senior scientist in the Center for Geodesy, Research Institute, ETL. He authored 40 reports and papers since 1963 and presented results thereof at 25 Army, national, and international meetings. He received an Army Research Development achievement award in 1969. He has been a member of the American Geophysical Union since 1960.

ABSTRACT

This paper deals with a method of determining an astronomic position by using the star rate of change in azimuth and zenith distance, with respect to time. The star is observed a number of times. Coefficients of Taylor expansion, up to the third order, can be obtained with a high degree of accuracy as a function of sections of a small great circle of arc.

Methods of application of observed data, time, zenith distance and azimuth are considered. They can be reduced to one arbitrarily chosen star position. A comparison of the results so obtained gives a test of the accuracy of the observation.

It is shown that astronomic refraction problems can be reduced using the method described in this report.

INTRODUCTION

This report deals with the determination of astronomic latitude, longitude and azimuth as well as the instrument constants, in a quite different way than of the classical methods, by using a star rate of change of zenith angle and azimuth with respect to time, and in a way that is independent of refraction problems. The observed times, zenith angles, or azimuth can be reduced to an arbitrary chosen star position, independent of the station coordinates.

A comparison of the results so obtained gives not only a test of the accuracy of the observations, but also the kind of error committed. Independent sets of observations are considered. A closed formula gives accurate values of the difference between zenith angles, which are related to power series according to Taylor's theorem, for determining the station's coordinates.

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### ZENITH DISTANCES RATE AS A FUNCTION OF TIME

Developing in power series according to Taylor's theorem, the following equation is considered:

$$\frac{Z_1 - Z_0}{\theta_1 - \theta_0} = \frac{dZ}{d\theta} + \frac{1}{2} \frac{d^2Z}{d\theta^2} (\theta_1 - \theta_0) + \frac{1}{6} \frac{d^3Z}{d\theta^3} (\theta_1 - \theta_0)^2 + \dots \quad (1)$$

$(\theta_1 - \theta_0)$  must be taken in radius.  $Z_1$  represents the true zenith distances that correspond to the sidereal times  $\theta_1$ , and  $Z_0$  the true zenith distance that corresponds to the sidereal time  $\theta_0$ . The difference between these true zenith distances are known through computations from the closed formula:

$$\cos (Z_1 - Z_0) = \frac{2 \cos S_1 - \cos (\bar{Z}_1 + \bar{Z}_0) (1 - \cos \Delta_1)}{1 + \cos \Delta_1}, \quad (2)$$

in which  $\Delta$  represents the difference in azimuth between the two observations

$$\Delta_1 = A_1 - A_0 = L_1 - L_0$$

$L_1, L_0$  = horizontal readings

$S$  represents the great circle of arc computed from

$$\sin \frac{1}{2} S_1 = \cos \delta \sin \frac{1}{2} (\theta_1 - \theta_0). \quad (3)$$

$\bar{Z}_1$  and  $\bar{Z}_0$  are observed zenith distances with respect to the times  $\theta_1$  and  $\theta_0$ , or derived by computation as can be shown later.

To obtain a higher degree of accuracy in evaluating  $(Z_1 - Z_0)$  we may choose a star whose declination is greater than that of the observer's latitude. Observations taken before and after the star maximum elongation angles  $\Delta$  can be small, on the order of few minutes of arc, while the changes in zenith distances have maximum variations; or we may observe two stars with small differences in azimuth, so the term  $(1 - \cos \Delta)$  will be small. The sum of the zenith distances may not be far from  $90^\circ$ , so that the second term in the numerator of equation (2) becomes small, and therefore corrections due to refraction and index error can be disregarded. Table 1 shows the influence of an error of six minutes of arc in the sum of the two zenith distances.

Table 1.

$\Delta$	$Z_1 + Z_0$	$\cos (Z_1 + Z_0)(1 - \cos \Delta)$
6'	120°	-0.00000 076
6'	120° 06'	76
12'	120°	-0.00000 305
12'	120° 06'	306
18'	120	-0.00000 685
18'	120 06'	687

Solution by Pairs of Observations

Any two observations provide enough information to obtain the rate  $\frac{dZ}{dt}$ . In this case we have

$$\frac{Z_2 - Z_1}{\theta_2 - \theta_1} = \frac{dZ}{dt} + 1/24 \left( \frac{\pi}{180} \right)^2 [(\theta_2 - \theta_1)^0]^2 \frac{d^3Z}{dt^3}. \quad (4)$$

The second term of this equation is small and the value of  $(\theta_2 - \theta_1)^0$  is expressed in degrees and fraction of it.

We have found for the differential coefficients:

$$\frac{d^3Z}{dt^3} = - \frac{dZ}{dt} \left[ 1 + 3 \cot Z \frac{d^2Z}{dt^2} - \left( \frac{dZ}{dt} \right)^2 \right] \quad (5)$$

and

$$\frac{d^2Z}{dt^2} = \frac{\cos Z (\sin^2 \phi + \sin^2 \delta) - \sin \phi \sin \delta (1 + \cos^2 Z)}{\sin^3 Z}. \quad (6)$$

Inserting equation (5) into equation (4) we get

$$\frac{Z_2 - Z_1}{\theta_2 - \theta_1} = \frac{dZ}{dt} \left\{ 1 - 1/24 \left( \frac{\pi (\theta_2 - \theta_1)^0}{180} \right)^2 \left( 1 + 3 \cot Z \frac{d^2Z}{dt^2} - \left( \frac{dZ}{dt} \right)^2 \right) \right\}. \quad (7)$$

In this equation for the zenith distance we use for Z the value which results from

$$Z = \frac{Z_1 + Z_2}{2} - 1/8 \frac{\pi}{180} (\theta_2 - \theta_1)^2 \frac{d^2Z}{dt^2}. \quad (8)$$

Expressing  $(\theta_2 - \theta_1)$  in degrees and fractions of it, we have

$$Z = \frac{Z_1 + Z_2}{2} - 7.85 (\theta_2 - \theta_1)^2 \frac{d^2Z}{dt^2}. \quad (9)$$

The differential coefficient  $\frac{d^2Z}{dt^2}$  is very small, zero at the maximum elongation, so we can compute it from equation (5) using

$$Z = \frac{Z_1 + Z_2}{2}.$$

For abbreviation let

$$P = 1/24 \left[ \frac{\pi}{180} (\theta_2 - \theta_1)^0 \right]^2 \quad (10)$$

$$Q = 1 + 3 \cot Z \frac{d^2Z}{dt^2}.$$

so equation (6) is rewritten as follows:

$$\frac{Z_2 - Z_1}{\theta_2 - \theta_1} = \frac{dZ}{dt} \left[ 1 - P \left( Q - \left( \frac{dZ}{dt} \right)^2 \right) \right], \quad (11)$$

and because of the size of P, we use the approximate value

$$\frac{dZ}{dt} = \frac{Z_2 - Z_1}{\theta_2 - \theta_1} \left( \frac{1}{1 - P \left[ Q - (Z_2 - Z_1)^2 / (\theta_2 - \theta_1)^2 \right]} \right), \quad (12)$$

in the brackets of equation (11), and derive the true value of the differential coefficient  $\frac{dZ}{dt}$ .

#### Solution for Any Number of Observations

Consider a set of observations made with a theodolite in a direct position, and a second set with a theodolite in reverse position.

Let

- i = number of observations in direct position
- j = number of observations in reverse position
- $\theta_1, Z_1$  = time and zenith distance in direct position
- $\theta_j, Z_j$  = time and zenith distance in reverse position
- $\theta_0, Z_0$  = a reference time and zenith distance
- $\tau_1 = \theta_1 - \theta_0$
- $\tau_j = \theta_j - \theta_0$

Two sets of equations can be formed where the differential coefficients refers to the time  $\theta_0$  as follows:

$$\Delta Z_1 = Z_1 - Z_0 = \frac{dZ}{dt} \tau_1 + \frac{1}{2} \frac{d^2Z}{dt^2} \tau_1^2 + \frac{1}{6} \frac{d^3Z}{dt^3} \tau_1^3 + \dots \quad (13)$$

$$\Delta Z_j = Z_j - Z_0 = \frac{dZ}{dt} \tau_j + \frac{1}{2} \frac{d^2Z}{dt^2} \tau_j^2 + \frac{1}{6} \frac{d^3Z}{dt^3} \tau_j^3 + \dots$$

We eliminate the second differential coefficient by postulating that

$$\sum \tau_j^2 - \sum \tau_1^2 = 0. \quad (14)$$

Consider an equal number of observations in each set, that is

$$i = j = n.$$

From the sum of each set, we obtain by subtraction of equation (13):

$$\sum \Delta Z_j - \sum \Delta Z_1 = \frac{dZ}{dt} (\sum \tau_j - \sum \tau_1) + \frac{1}{6} \frac{d^3Z}{dt^3} (\sum \tau_j^3 - \sum \tau_1^3). \quad (15)$$

To satisfy equation (14) the value of  $\theta_0$  must be known. To obtain it we proceed as follows:

Let  $\bar{\theta}_0$  be an approximate value of  $\theta_0$ . We may select the mean value between the last observation of the first set and the first of the second set, so

$$\bar{\theta}_0 = \frac{\theta_n + \theta_1}{2}, \quad (16)$$

and let  $x$  be the amount of correction to obtain  $\theta_0$ , therefore

$$\theta_0 = \bar{\theta}_0 + x \quad (17)$$

from which equation (13) becomes

$$\sum(\theta_j - \bar{\theta}_0)^2 - \sum(\theta_1 - \bar{\theta}_0)^2 - 2x [\sum(\theta_j - \bar{\theta}_0) - \sum(\theta_1 - \bar{\theta}_0)] = 0 \quad (18)$$

and  $x$  is computed from

$$x = \frac{\sum(\theta_j - \bar{\theta}_0)^2 - \sum(\theta_1 - \bar{\theta}_0)^2}{2[\sum(\theta_j - \bar{\theta}_0) - \sum(\theta_1 - \bar{\theta}_0)]}. \quad (19)$$

Adding this value of  $x$  to  $\bar{\theta}_0$ , we obtain the desired value  $\theta_0$ . The  $\Delta Z$ 's are computed according to equation (2).

#### Evaluation of the Second Differential Equation $d^2Z/dt^2$ .

Consider the total number of observations of each group to be

$$i = j = 1, 2, \dots, n.$$

For the sum of each group of equations (13), two equations are fixed as follows:

$$\sum \frac{(z_i - z_0)}{\tau_i} = n \frac{dZ}{dt} + \frac{1}{2} \frac{d^2Z}{dt^2} \sum \tau_i + \frac{1}{6} \frac{d^3Z}{dt^3} \sum \tau_i^2 \quad (20)$$

$$\sum \frac{(z_j - z_0)}{\tau_j} = n \frac{dZ}{dt} + \frac{1}{2} \frac{d^2Z}{dt^2} \sum \tau_j + \frac{1}{6} \frac{d^3Z}{dt^3} \sum \tau_j^2.$$

Under the condition established in equation (14) we obtain for the second differential equation

$$\frac{1}{2} \frac{d^2Z}{dt^2} = \left[ \frac{\sum (z_j - z_0)}{\tau_j} - \frac{\sum (z_i - z_0)}{\tau_i} \right] \frac{180}{(\sum \tau_j - \sum \tau_i)^2}. \quad (21)$$



Let the parameters M and N be defined as follows:

$$M = \frac{1}{n} \sum \frac{(z_i - z_o)}{\tau_i} \quad (22)$$

$$N = \frac{1}{n} \sum \frac{(z_j - z_o)}{\tau_j} .$$

Adding the two equations (15), dividing by n and on account of equation (17) we obtain

$$\frac{M+N}{2} = \frac{dZ}{dt} + \frac{1}{4n} \frac{d^2Z}{dt^2} \sum (\tau_j + \tau_i) + \frac{1}{12n} \frac{d^3Z}{dt^3} (\sum \tau_j^2 + \sum \tau_i^2) . \quad (23)$$

The second term in the left hand of this equation is very small and the third term will also be small. To evaluate the first differential we must evaluate the third coefficient, which we found is

$$\frac{d^3Z}{dt^3} = - \frac{dZ}{dt} [1 + 3 \cot Z \frac{d^2Z}{dt^2} - (\frac{dZ}{dt})^2] . \quad (24)$$

Let

$$C = \frac{1}{4n} \frac{d^2Z}{dt^2} \sum (\tau_j + \tau_i)^2 \frac{\pi}{180} \quad (25)$$

in which the  $\tau$ 's are expressed in degrees and fractions of it, and let

$$P = 1 + 3 \cot Z \frac{d^2Z}{dt^2} \quad (26)$$

$$Q = \frac{M+N}{2} - C ,$$

where  $d^2Z/dt^2$  is derived from equation (16). Inserting these values into equation (18) we obtain for evaluating the first differential equation:

$$\frac{dZ}{dt} = \frac{Q}{1 - \frac{1}{6} (P-Q^2) \sum (\tau_j^2 + \tau_i^2) (\frac{\pi}{180})^2} . \quad (27)$$

#### Evaluation of Latitude and Azimuth

Two procedures can be used, as follows:

Procedure 1. Latitude can be computed through the equation:

$$\sin \phi = \cos Z_m \sin \delta_m \pm \sin Z_m \sqrt{\cos^2 \delta_m - (dZ/dt)^2} . \quad (28)$$

The plus sign is used when the star's azimuth increases, otherwise it is negative. The value of  $Z_m$  is derived as follows:

Let  $Z_i$  be the zenith distances of the first set and  $Z_j$  those of the second set. Let  $R_i$  and  $R_j$  be their corrections due to refraction errors and let  $\epsilon$  be the index error and  $P$  the observer personal equation. The first set gives:

$$Z_i = \bar{Z}_i + R_i + (\epsilon + P) \quad (29)$$

The second set gives:

$$Z_j = \bar{Z}_j + R_j - (\epsilon + P) \quad (30)$$

The  $\bar{Z}$  means observed values. The value of  $Z_m$  is obtained by adding to each  $Z_i$  the computed value  $\Delta Z_i$  or  $\Delta Z_j$ , we get

$$n Z_m = \sum (\bar{Z}_i + \Delta Z_i) + \sum R_i + n(\epsilon + P) \quad (31)$$

$$n Z_m = \sum (\bar{Z}_j + \Delta Z_j) + \sum R_j - n(\epsilon + P)$$

from which we obtain

$$Z_m = \frac{1}{2n} [\sum (\bar{Z}_i + \Delta Z_i) + \sum (\bar{Z}_j + \Delta Z_j) + \sum (R_i + R_j)]. \quad (32)$$

After the latitude has been obtained from equation (28), the azimuth, which is for the time  $\theta_0$ , is obtained from

$$\sin A = \frac{dZ}{dt} \sec \phi. \quad (33)$$

Procedure 2. By observing a second star on the other side of the meridian the azimuth is determined independently of the latitude.

Let

$$W = \left(\frac{dZ}{dt}\right)_w \text{ the rate of the west star}$$

$$E = \left(\frac{dZ}{dt}\right)_e \text{ the rate of the east star}$$

$$L_w, L_e = \text{theodolite horizontal readings}$$

The azimuth is derived from

$$\tan \frac{1}{2} (A_w + A_e) = \frac{W+E}{W-E} \tan \frac{1}{2} (L_w - L_e). \quad (34)$$

Let

$$\frac{1}{2} (A_w + A_e) = x \quad (35)$$

$$\frac{1}{2} (L_w - L_e) = y$$

then the star's azimuth are

$$\begin{aligned} A_w &= x+y \\ A_e &= x-y \end{aligned} \quad (36)$$

### Star's Hour Angle

The star's hour angle  $t$ , is computed using the equation:

$$\tan t = \frac{dZ/dt}{A-B \sin \delta}, \quad (37)$$

when the denominator (absolute value) is greater than the numerator, otherwise from

$$\begin{aligned} \tan \gamma &= \frac{A-B \sin \delta}{dZ/dt}, \\ t &= 90^\circ - \gamma. \end{aligned}$$

The meanings of  $A$  and  $B$  are:

$$\begin{aligned} A &= \cos^2 \delta \cot Z \\ B &= \pm \sqrt{\cos^2 \delta - (dZ/dt)^2} \end{aligned} \quad (38)$$

where the positive sign corresponds when the star's azimuth (horizontal readings) increases and negative when it decreases.

### COLLIMATION CONSTANT AND ITS EVALUATION FROM A STAR OBSERVATION

The line of sight of the instrument is in general not perpendicular to the horizontal axis of the instrument, but forms an angle with it of  $90^\circ + c$  where  $c$  is a quantity called collimation to be evaluated. Consider a diagram of a theodolite as shown in Figure 1. Let  $C$  represent the center of the horizontal circle. Let  $NS$  be the meridian line through  $C$ . Let the telescope be pointed to the star  $A$ , and let  $CA'$  be its horizontal projection. This direction  $CA'$  has a reading  $L_s$  in the horizontal circle. Let  $B_0$  be the origin of the horizontal scale reading of the theodolite and let  $B_1$  be its symmetrical point. Then the angle formed by the line  $B_0CB_1$  with respect to the meridian line  $NCS$  is the azimuth of  $B_0B_1$ , that is  $SCB_0$ . Let this angle be represented by  $B$ , so the star's azimuth  $A_s$ , is  $A_s = L_s + B$  which represents the value when no collimation exists but always a small collimation error exists so we must add to  $L_s$  a value  $\delta A$ . This correction is well known, therefore the star's azimuth is:

$$A_s = L_s + B \pm c \operatorname{cosec} Z \quad (39)$$

where the plus sign is assumed when the theodolite is in a direct position and negative sign when the instrument is in reverse position.

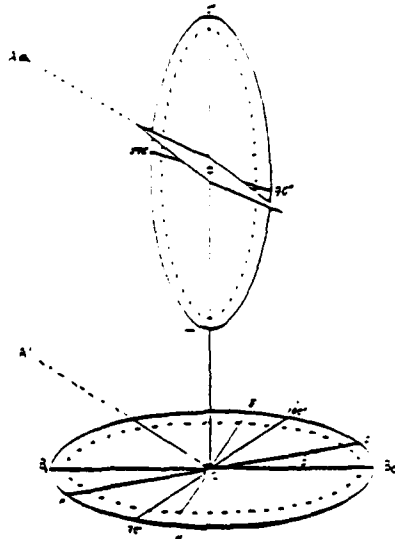


Figure 1. Diagram of a Theodolite

The collimation can be determined by selecting any terrestrial object that presents some well defined point, and at a far distance so that the stellar focus of the telescope need not be changed to obtain a good definition of the point. From the horizontal readings of the theodolite in a direct and a reverse position the collimation can be obtained. But it appears that this collimation should not be the same when it is derived from a star's observations. We think that it is more appropriate to obtain the collimation constant by observing a moving object, a star, which is related to the observer's behavior in observing the stars for latitude and longitude determination. Instead of a distant terrestrial point, we substitute it by choosing a circumpolar star. The rate of change of azimuth as function of time is used to evaluate the collimation constant. The star is observed a number of times in a direct and reverse instrument position. The time when the star image crosses the central vertical wire is recorded as well as the horizontal readings of the theodolite. Let  $L_1$  be the horizontal scale readings and  $T_1$  be the corresponding times for a direct instrument position and  $L_j, T_j$  the values that corresponds to the reverse instrument position, where

$$j = 1, 2, 3, \dots, n.$$

Let  $T_0$  be an arbitrary time chosen between the two sets of observations, and let us reduce all observations, to the time  $T_0$  through the Taylor's theorem, expressing all quantities in radians and omitting the terms higher than the third.

The amount  $\Delta A_1$  to be added to each  $L_1$  to reduce it to the value that would correspond to the time  $T_0$ , is

$$\Delta A_1 = \frac{\partial A}{\partial t} \sigma_1 + 1/2 \frac{\partial^2 A}{\partial t^2} \sigma_1^2 + 1/6 \frac{\partial^3 A}{\partial t^3} \sigma_1^3 \quad (40)$$

where

$$\sigma_1 = T_1 - T_0 .$$

The amount  $\Delta A_j$  to be added to the readings  $L_j$ , to reduce them also to the time  $T_0$ , is

$$\Delta A_j = \frac{dA}{dt} \sigma_j + 1/2 \frac{d^2 A}{dt^2} \sigma_j^2 + 1/6 \frac{d^3 A}{dt^3} \sigma_j^3 \quad (41)$$

$$\sigma_j = T_j - T_0 .$$

The star's azimuth to the time  $T_0$  is

$$A_s = L_1 + \beta + \Delta A_1 + c \operatorname{cosec} Z_1 \quad (42)$$

and

$$A_s = L_j + \beta + 180 + \Delta A_j - c \operatorname{cosec} Z_j$$

from which the collimation error results as

$$c = \frac{L_j + 180 - L_1 + \Delta A_j - \Delta A_1}{\operatorname{cosec} Z_1 + \operatorname{cosec} Z_j} . \quad (43)$$

We do not give a detailed derivation of Taylor's coefficients, which are to be used in equations (33) and (34), and give only its final results. We find that such coefficients can be evaluated from:

$$\frac{dZ}{dt} = \pm \operatorname{cosec} Z \sqrt{2 \cos Z \sin \delta \sin \phi + \sin^2 Z - \sin^2 \phi - \sin^2 \delta} \quad \begin{matrix} +W \\ -E \end{matrix} \quad (44)$$

$$\frac{d^2 Z}{dt^2} = \cot Z \left[ 1 - \left( \frac{dZ}{dt} \right)^2 \right] - \sin \phi \sin \delta \operatorname{cosec} \delta Z \quad (45)$$

$$\frac{dA}{dt} = \frac{\sin \phi - \cos z \sin \delta}{\sin^2 z} \quad (46)$$

$$\frac{d^2 A}{dt^2} = \left( \sin \delta - 2 \frac{dA}{dt} \cos z \right) \frac{dZ}{dt} \operatorname{cosec} Z \quad (47)$$

$$\frac{d^3 A}{dt^3} = \frac{d^2 z}{dt^2} \left( \frac{\sin \phi}{\sin z} - 2 \cot z \frac{dA}{dt} \right) + 2 \left( \frac{dz}{dt} \right)^2 \frac{dA}{dt} - 3 \cot z \frac{dZ}{dt} \frac{d^2 A}{dt^2} \quad (48)$$

EXAMPLES OF COMPUTATION

To illustrate the process of reduction using the technique shown in this report we consider the following observations taken on February 22, 1955, in a place:

$$\lambda = 6^h 47^m 45^s.91 \quad \text{West}$$

$$\phi = 26^\circ 37' 22'' \quad \text{South}$$

with a theodolite Zeiss 1". The time was noted by a sidereal clock whose rate was so small as not to require notice. The Star's apparent place,  $\alpha$  Pictoris, is:

$$\alpha = 6^h 47^m 45^s.91$$

$$\delta = -61^\circ 53' 50''.8$$

Tables 2 and 3 show: first column the sidereal time of observation; second column the zenith distance after correction for refraction; the third column are the horizontal readings. The Taylor's coefficients shown on each table were computed according to equations (37) through (41), from which the corrections I, II and III were made to reduce each observation to the chosen time  $T_0$ , according to equations (33) and (34). The last column indicates the reduced azimuth with the plus or minus of collimation coefficient according to the instrument position. The collimation was computed according to equation (36). The resulting value is  $C = 7''.83$ .

Table 4 shows the reduced zenith angles to a common time  $T_0$ . The first column indicates the number of observations for the direct and reverse instrument position. The second gives the values of  $(Z - Z_0)$  computed according to equation (2). The third column gives the reduced zenith angles from direct observations and in column four from reverse instrument position. From the mean values of these two columns the index error was computed which was used to compute the final zenith angle reduced to the time  $T_0$ .

Table 2. Telescope Direct Position.

1	2	3	4			5 = 3-4+C cosec Z	
Sidereal Time	Zenith Angle	Horizontal Angle	Corrections			Reduced Azimuth to $T_0 = 11^h 55^m 08^s.56$	
			I	II	III	I+II+III	
11 <sup>h</sup> 49 <sup>m</sup> 06 <sup>s</sup> .34	60°02'15"	31°46'31"	2'02".03	-33".20	-07".23	1'28".6	31°45'24".4 ± 1.1540
50 36.6	12 54	46 16	1 31.64	-18.72	-0.10	1 12.8	3.2 ± 1.152
51 05.2	16 15	45 10	1 22.03	-14.99	-0.07	1 06.9	3.2 ± 1.152
51 34.6	19 41	46 03	1 12.10	-11.59	-0.05	1 00.5	2.5 ± 1.151
52 16.0	24 35	45 52	0 58.15	-7.54	-0.02	0 50.6	1.4 ± 1.150
52 51.9	28 49	45 43	0 46.06	-4.72	-0.01	0 41.3	1.7 ± 1.150
53 20.2	32 10	45 35	0 36.52	-2.97	-0.01	0 33.5	1.5 ± 1.149
54 24.5	39 43	45 15	1 14.86	-0.49	0	0 14.4	0.6 ± 1.147
11 55 08.5	60 44 55	31 45 02	0	0	0	0	31 45 20.0 ± 1.1460
							31°45'02".0 ± 1.1500
12 <sup>h</sup> 04 <sup>m</sup> 26 <sup>s</sup> .70	--	31°45'02.0	-3'07".80	-1'18".63	0".84	-4'25.59	31°40'36".41 ± 1.1500
12 07 38.529	--	31 45 02.0	-4 12.66	-2 22.32	2.04	-6 73.0	31 38 29.06 ± 1.1500

$$dA/dt = -0.02246$$

$$d^2A/dt^2 = -0.46299$$

$$d^3A/dt^3 = 0.36527$$

$$dZ/dt = 0.47064$$

$$d^2Z/dt^2 = 0.01707$$

Table 3. Telescope Reverse Position

1	2	3	4			5 = 3-4 + C cosec Z	
Sidereal Time	Zenith Angle	Horizontal Angle	Corrections			Reduced Azimuth to $T_0 = 12^h 09^m 26.90$	
			I	II	III	I+II+III	$T_0 + 12^h 09^m 26.90$
12 <sup>h</sup> 08 <sup>m</sup> 26.90	61°40'40"	211°40'54"	0	0	0	0	211°40'54.0-1.130C
5 42.8	61 59 40	40 06	-47.20	-1.45	0	-48.6	54.5-1.133
6 37.6	62 06 05	39 28	-1 20.87	-4.24	0.01	-1 25.1	53.1-1.132
7 23.7	62 11 30	38 56	-1 49.21	-7.74	0.02	-1 56.9	52.9-1.131
7 58.9	62 15 37	38 33	-2 10.84	-11.11	0.04	-2 21.9	54.4-1.130
8 28.8	62 19 08	38 11	-2 29.21	-14.45	0.06	-2 43.6	54.6-1.129
9 08.6	62 23 48	37 41	-2 53.67	-19.57	0.10	-3 13.1	54.1-1.128
9 56.5	62 29 24	37 06	-3 23.11	-26.77	0.16	-3 49.7	55.7-1.127
12 10 35.6	62 24 00	211 36 34	-3 47.14	-33.48	0.22	-4 20.4	211 40 54.4-1.127
							211°40'54.27-1.130C
11 55 08.6	--	211°40'54.27	5 42.55	-1 16.14	-0.77	-4'25.64	211°45'19.91-1.130C
12 07 38.529	--	211 40 54.27	-1 58.32	-9.08	0.03	-2 07.37	211 38 46.90-1.130C

Direct                      Collimation                      Reverse

11 <sup>h</sup> 55 <sup>m</sup> 08.6	31°45'02.04+1.115C = 211°45'19.91-180-1.1130C	C = 7.84
12 04 26.0	31 40 36.41-1.115C = 211 40 54.27-180-1.1130C	C = 7.83
12 07 38.53	31 38 29.06+1.115C = 211 38 46.90-180-1.1130C	C = 7.83

dA/dt = -0.04097                      dZ/dt = 0.46966  
 d<sup>2</sup>A/dt<sup>2</sup> = -0.44931                      d<sup>2</sup>Z/dt<sup>2</sup> = -0.03117  
 d<sup>3</sup>A/dt<sup>3</sup> = 0.33525

Table 4. Reduced Zenith Angles to  $T_0 = 11^h 55^m 08.6$ .

	AZ	DIRECT Z + ΔZ + ε	REVERSE Z - ΔZ - ε	Z <sub>0</sub>
1	0°42'37.8	60°44'52.8		60°44'59.5
2	32 00.7	54.7		61.4
3	28 38.7	53.7		60.4
4	25 11.0	52.0		58.7
5	20 18.7	53.7		60.4
6	16 05.2	54.2		60.9
7	12 45.3	55.3		62.0
8	5 11.3	54.3		61.0
9	0 0 0	60 44 55.0		61.7
1	-1°06'31.4		60°45'8.7	62.0
2	-1 14 32.3		7.7	61.0
3	-1 20 58.0		7.0	60.3
4	-1 26 22.5		7.5	60.8
5	-1 30 30.4		6.6	59.9
6	-1 34 00.7		7.3	60.6
7	-1 38 40.7		7.3	60.6
8	-1 44 17.7		7.3	60.6
9	-1 48 52.7		60 45 7.3	60 44 60.6
		60°44'53.97	60°45'7.38	60°45'00.7
		53.97 + ε = 67.38 - ε		
		ε = 6.70		

Further sample calculations, not presented here because of space limitations, are available from the author upon request.

END

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