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# DIRECT AND INVERSE DIFFRACTION PROBLEMS FOR QNDE APPLICATIONS

FINAL REPORT

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## 1. Introduction and Statement of Research Objectives

One of the most useful methods of quantitative non-destructive evaluation (QNDE) is based on the scattering of elastic (ultrasonic) waves by flaws. Two related approaches to ultrasonic flaw detection and characterization have been taken. The imaging approach seeks to process the scattered field in such a manner that a visual outline of the flaw is produced on a display. The scattered-field approach attempts to infer geometrical characteristics of a flaw from either the angular dependence of its far-field scattering amplitude at fixed frequency, or from the frequency dependence of its far-field scattering amplitude at fixed angles.

The work that has been completed on this Project, has been concerned with analytical work for the scattering approach to crack-like flaws. Both direct and inverse scattering problems have been considered.

In experimental work on quantitative flaw definition by the ultrasonic pulse method, either the pulse-echo method with one transducer or the pitch catch method with two transducers is used. The transducer(s) may be either in direct contact with the specimen, or transducer(s) and specimen may be immersed in a water bath. Most experimental setups include instrumentation to gate out and spectrum analyze the signal diffracted by a flaw. The raw scattering data generally need to be corrected for transducer transfer functions and other characteristics of the system, which have been obtained on the basis of appropriate calibrations. After processing, amplitudes and phase functions are available as functions of the frequency and the scattering angle. These experimental data can then be directly compared with theoretical results of the kind that have been obtained in the completed research. Inversely, the experimental data can be interpreted with the aid of analytical methods, to characterize the scatterer.

#### 2. Summary of Results

The solution to the direct diffraction problem that is, the computation of the field generated when an ultrasonic wave is diffracted by a known flaw, is a necessary preliminary to the solution of the inverse problem, which is the problem of inferring the geometrical characteristics of an unknown flaw from either the angular dependence of the amplitude of the diffracted far-field at fixed frequency, or from the frequency dependence of the far-field amplitude at fixed angle. In the completed work, analytical methods have been developed to investigate scattering of elastic waves by interior cracks and volume flaws, as well as by surface-breaking cracks. Scattering of timeharmonic signals by flaws has been analyzed on the basis of linearized elasticity theory for homogeneous, isotropic and anisotropic solids.

From the theoretical point of view a flaw crack is a planar surface across which the displacement can be discontinuous. The exact mathematical formulation of the elastodynamic field generated by the presence of a crack is rather complicated. Scattering of an incident wave by a crack is a mixed boundary value problem, whose exact solution satisfies one or more generally singular integral equations for the displacement discontinuities. The solution of the system of integral equations for a crack of finite dimensions requires a substantial amount of numerical analysis, which has been carried out for a number of examples.

To analyze scattering by cavities we have used approximate methods which are valid in the high-frequency range. The appeal of the highfrequency approach is that the probing wavelength is of the same order of magnitude as the length-dimensions of the crack. This gives rise to interference phenomena which can easily be detected.

Papers completed in the course of resarch under Grant DAAG29-80-C-0086 have been published in Technical Journals and in Proceedings of Conferences and Symposia. A list of titles and a compilation of Abstracts are given in Appendices A and B, respectively. In addition, oral presentations have been made at Technical Meetings, Workshops, Symposia and at a Gordon Research Conference.

In brief summary, the completed work has been concerned with methods to compute the fields scattered by cracks and cavities [A.2, A.8, A.9]. It has considered the effect of crack-face roughness in a preliminary manner [A.5]. Considerable emphasis was devoted to scattering by flaws which are located near the surface of a body [A.2, A.6, A.8]. An inverse ray theory was developed to map crack edges on the basis of time of flight information and the geometrical theory of edge-diffraction [A.3, A.4]. In addition, two invited articles of a review nature were prepared [A.1, A.7].

#### 3. Participating Scientific Personnel

Work on the subject Grant was carried out under the direction of Dr. J.D. Achenbach, Walter P. Murphy Professor of Civil Engineering and Applied Mathematics. Also contributing to the research efforts for periods of time have been: Dr. A.N. Norris, Dr.L.M. Keer, Dr. R.J. Brind, R.A. Roberts and W. Lin.

#### 4. Advanced Degrees

Dr. A.N. Norris was employed (part-time) as a Research Assistant on the subject Grant. His work resulted in a Ph.D. Dissertation entitled "Ray Methods for Inverse Problems of Elastic Wave Scattering." Dr. Norris is now employed by Exxon Research & Engineering, in New Jersey. His degree was granted in August 1981.

Ronald Roberts was also employed (part-time) as a Research Assistant. His work resulted in a Master's Thesis, entitled "Effects of Sinusoidal Crack-Face Perturbation on Scattering of Elastic Waves." He is continuing his studies for the Ph.D. degree.

#### APPENDIX A

## Titles of Completed Papers

The following papers have been published in Technical Journals

and Proceedings of Conferences

- A.1 J. D. Achenbach, "Direct and Inverse Methods for Scattering by Cracks in the High-Frequency Range," ARO Report 81-1. Transactions of the 26th Conference of Army Mathematicians.
- A.2 J.D. Achenbach and A.N. Norris, "Scattering by Surface-Breaking Cracks," Proceedings 13th Symposium on Nondestructive Evaluation, 1981, 21-23, April, San Antonio, Texas.
- A.3 A.N. Norris and J.D. Achenbach, "Inversion of First-Arrival Crack-Scattering Data," in <u>Elastic Wave Scattering and Propagation</u> (eds. V.K. Varadan and V.V. Varadan), Ann Arbor Science Publishers, Ann Arbor, 1982.
- A.4 J.D. Achenbach, A.N. Norris and K. Viswanathan, "Mapping of Crack Edges by Seismic Methods," <u>Bulletin Seism. Soc. Am.</u>, <u>72</u>, pp. 779-792 (1982).
- A.5 R.A. Roberts, R.J. Brind and J.D. Achenbach, "Effects of Sinusoidal Crack-Face Perturbations on Scattering of Elastic Waves," J. of Nondestructive Evaluation, 2, pp. 153-159 (1981).
- A.6 J.D. Achenbach and A.N. Norris, "Excitation of Surface Waves by Compact Sub-Surface Sources," <u>Proc. Roy. Soc</u>. (London) <u>A382</u>, pp. 411-428 (1982).
- A.7 J.D. Achenbach and A.K. Gautesen, "Edge Diffraction in Acoustics and Elastodynamics," to appear in <u>Handbook on Acoustic, Electro-</u> <u>magnetic and Elastic Wave Scattering--Theory and Experiment,</u> North-Holland Publishing Company.
- A.8 J.D. Achenbach, W. Lin and L.M. Keer, "Surface Waves due to Scattering by a Near-Surface Parallel Crack," <u>IEEE Transactions</u> on Sonics and Ultrasonics, Vol. SU-30, 4, pp. 270-276 (1983).
- A.9 R.J. Brind, J.D. Achenbach and J.E. Gubernatis, "High Frequency Scattering of Elastic Waves from Cylindrical Cavities," to appear in WAVE MOTION.

## APPENDIX B

#### Abstracts of Completed Papers

A.1 J.D. Achenbach, "Direct and Inverse Methods for Scattering by Cracks in the High-Frequency Range," ARO Report 81-1. Transactions of the 26th Conference of Army Mathematicians.

An important method in quantitative non-destructive evaluation of materials (QNDE) is based on scattering of ultrasonic waves by cracks. The presence of a flaw is relatively easy to detect. The determination of its size, shape and orientation from the scattered field poses a challenging inverse scattering problem. In recent years several analytical methods have been developed to investigate scattering of elastic waves by interior cracks and surface-breaking cracks, in both the high- and the low-frequency domains. The appeal of the high-frequency approach is that the probing wavelength is of the same order of magnitude as the lengthdimensions of the crack. This gives rise to interference phenomena which can easily be detected. In this paper we discuss approximate methods for the solution of the direct scattering problem in the high-frequency domain, which show good agreement with experimental results. The simple analytical solutions to the direct problem suggest the application of Fourier-type integrals to solve the inverse problem. The application of two kinds of inversion integrals to far-field high-frequency scattering data from flat cracks has been discussed briefly.

A.2 J.D. Achenbach and A.N. Norris, "Scattering by Surface-Breaking Cracks," Proceedings 13th Symposium on Nondestructive Evaluation, 1981, 21-23, April, San Antonio, Texas

In this paper we discuss the application of elastodynamic ray theory to the scattering of bulk waves by surface-breaking cracks. Because of the presence of both a reflecting free surface and a crack, the number of possible ray paths is larger than for an interior crack. For L-L scattering by an interior crack generally only two rays need be considered. Mode conversion at the free surface and scattering by the interior edge of the surface-breaking crack may generate as many as nine relevant ray paths. The application of ray theory to the three-dimensional case of scattering of incident longitudinal waves by a surface breaking crack has been examined for the case that only the longitudinal signals are measured in the scattered field. An inverse method which is based on accurate information of the arrival time of the very first diffracted signal has been discussed.

A.3 A.N. Norris and J.D. Achenbach, "Inversion of First-Arrival Crack-Scattering Data," in <u>Elastic Wave Scattering and Propagation</u> (eds. V.K. Varadan and V.V. Varadan), Ann Arbor Science Publishers, Ann Arbor, 1982.

The problem of locating cracks and determining their extent is of fundamental concern in the non-destructive evaluation of materials. In this paper we consider the inverse problem of determining the geometrical characteristics of a crack from the diffraction of an incident wave by the crack-edge. It is assumed that the material is homogeneous and isotropic. By the use of elastodynamic ray theory we have obtained an inversion procedure which uses only the time delay of the first received diffracted signal as input. For a set of observations the method yields the position of a "flashpoint" on the crack edge. Several flashpoints constitute a segment of the crack edge from which we can infer the plane of the crack. Each observation in the time domain contains information on a second flashpoint on the crack edge. The time delay for this flashpoint can be obtained from the periodicity of the high-frequency amplitude spectrum. The inversion procedure is then again applied to yield another segment of the crack edge. In general, it may be expected that the two segments will adequately define the size of the crack.

A.4 J.D. Achenbach, A.N. Norris and K. Viswanathan, "Mapping of Crack Edges by Seismic Methods," <u>Bulletin Seism. Soc. Am.</u>, <u>72</u>, pp. 779-792 (1982).

The inverse problem of diffraction of elastic waves by the edge of a large crack has been investigated on the basis of elastodynamic ray theory and the geometrical theory of diffraction. Two methods are discussed for the mapping of the edge of a crack-like flaw in an elastic medium. The methods require as input data the arrival times of diffracted ultrasonic signals. The first method maps flash points on the crack edge by a process of triangulation with the source and receiver as given vertices of the triangle. By the use of arrival times at neighboring positions of the source and/or the receiver, the directions of signal propagation, which determine the triangle, can be computed. This inverse mapping is global in the sense that no a-priori knowledge of the location of the crack edge is necessary. The second method is a local edge mapping which determines planes relative to a known point close to the crack edge. Each plane contains a flash point. The envelope of the planes maps an approximation to the crack edge. The errors due to inaccuracies in the input data and in the computational procedure have been illustrated by specific examples.

A.5 R.A. Roberts, R.J. Brind and J.D. Achenbach, "Effects of Sinusoidal Crack-Face Perturbations on Scattering of Elastic Waves," J. of Nondestructive Evaluation, 2, pp. 153-159, (1981).

The scattering of a plane longitudinal wave from a two-dimensional crack, with a sinusoidal surface perturbation whose amplitude and wavelength are much smaller than the length of the crack, is investigated. The amplitude of the cylindrical body waves in the far-field are calculated from a Kirchhoff approximation that utilizes the solution to the reflection from the sinusoidal surface profile of a semi-infinite solid. The results are compared to those for a flat crack, and conditions for significant differences of the amplitude as a function of the angle of observation are discussed. Characteristics changes in the scattered field produced by profiles with different amplitudes and periods are explained.

A.6 J.D. Achenbach and A.N. Norris, "Excitation of Surface Waves by Compact Sub-surface Sources," <u>Proc. Roy. Soc</u>. (London) <u>A382</u>, pp. 411-428 (1982).

A method is proposed for determining the amplitude of surface waves produced by a buried source in a half-space. The source is an arbitrary compact region whose outgoing field may be characterized by an analytic function, which we call the "emission" function. Using Green's theorem, the surface wave amplitude is related to the value of the emission function at a complex angle. The method may be used to approximate the surface wave amplitude in cases where the exact emission function is unknown. A cylindrical cavity with a concentrated line source on its circumference is considered. In this case only the full emission function is known. A semi-infinite slit is considered as an example of a non-compact source. The limitations of the present method are noted.

A.7 J.D. Achenbach and A.K. Gautesen, "Edge Diffraction in Acoustics and Elastodynamics," to appear in <u>Handbook on Acoustic, Electro-</u> <u>magnetic and Elastic Wave Scattering--Theory and Experiment</u>, North-Holland Publishing Company.

The geometrical theory of edge diffraction for acoustics and elastodynamics is discussed, with the emphasis placed on elastodynamics.

After a brief summary of the equations governing wave propagation in linearly elastic solids and ideal fluids, the discussion of ray theory starts with a statement of Fermat's principle. The eikonal and transport equations which govern asymptotic solutions to the reduced wave equation are then developed. For homogeneous isotropic

media, the solutions to the eikonal equation and to the leading order transport equation are given. These solutions are used to construct asymptotic solutions to the equations of linear elastodynamics. As an example we discuss the reflection of asymptotic waves from a plane traction-free surface.

The canonical problems for edge diffraction of waves in an elastic solid and in an acoustic medium are discussed and the corresponding theories of edge diffraction are presented. For waves in elastic solids, the generation of diffracted surface-wave rays and their subsequent interaction with crack edges is examined. Ray analysis of diffraction by a penny shaped crack is discussed in some detail. The geometrical theory of diffraction yields discontinuities at shadow boundaries. A uniform asymptotic theory which removes these discontinuities is also presented.

As an illustrative example of the geometrical theory of edge diffraction, we consider in some detail the two-dimensional problem of scattering of a plane wave by a crack. Finally, the roles of boundary and edge conditions in the geometrical theory of diffraction are placed in some perspective by applying the method of matched asymptotic expansions to the elastodynamic edge-diffraction problem.

A.8 J.D. Achenbach, W. Lin and L.M. Keer, "Surface Waves due to Scattering by a Near-Surface Parallel Crack," <u>IEEE Transactions</u> on Sonics and Ultrasonics, Vol. SU-30, <u>4</u>, pp. 270-276 (1983).

Scattering of ultrasonic waves by a crack which is oriented parallel to a free surface is analyzed. Incident longitudinal and transverse body waves, as well as incident Rayleigh surface waves are considered. The presence of a parallel sub-surface crack gives rise to scattered surface waves whose amplitude-spectra show distinct resonance peaks, due to resonance of the layer between the crack and the free surface, particularly when this layer is thin. Scattering of body waves for normal incidence, both from the side of the free surface and from the interior of the solid, is investigated in some detail. The ratio of the amplitude spectra of the surface wave displacements for these two cases provides a measure of the distance d from the crack to the free surface. The length a of the crack can subsequently be obtained from a master curve which plots the first resonance frequency versus d/a. The amplitude spectra for scattered surface waves due to an incident surface wave also show resonance peaks, but these are part of a more complicated pattern of peaks and valleys, and they are therefore more difficult to classify.

A.9 R.J. Brind, J.D. Achenbach and J.E. Gubernatis, "High Frequency Scattering of Elastic Waves from Cylindrical Cavities," to appear in WAVE MOTION 9

The scattering of time-harmonic plane longitudinal elastic waves by smooth convex cylindrical cavities is investigated. The exact solution for a circle is evaluated for wavelengths of the same order as the radius, and the geometrical and physical elastodynamics approximations are shown to be inadequate. The application of Watson's transformation exhibits the various diffraction effects and the relative importance of each is assessed. Excellent approximations for the scattered far-field are obtained with a hybrid method, in which an approximation for the surface field is constructed from the creeping wave contributions and this is then used in an integral representation, the hybrid method to cavities of smooth convex cross-section is presented and applied to the specific case of an ellipse. The predictions of the hybrid method compare well with numerical resul obtained by an eigenfunction expansion method.

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