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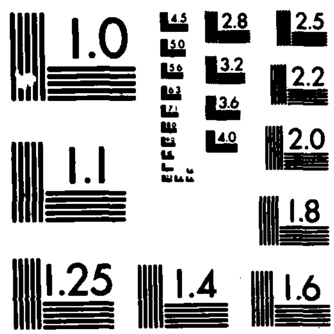

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TECHNICAL REPORT ARPAD-TR-83003

DESIGN CONSIDERATIONS OF PLATENS FOR VIBRATION TESTING

LOUIE JACKSON LIPP

NOVEMBER 1983

U.S. ARMY ARMAMENT RESEARCH AND DEVELOPMENT CENTER

PRODUCT ASSURANCE DIRECTORATE

DOVER, NEW JERSEY

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Vibration testing, particularly when performed in reliability test chambers, is very costly. Discussed in this report are design considerations for a vibration test platen which will accept several test specimens for simultaneous testing. Well designed platens will provide more meaningful results and reduce test time and costs through maximum utilization of test hardware. Specific points covered are:

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20. ABSTRACT (cont)

Bending modes at the platen's frequency

Determining the resonant frequency of a test platen

Determining the damping energy of a platen design

Obtaining signal feedback for vibration control purposes

Deriving equations for resonant frequencies and damping energies

CONTENTS

INTRODUCTION

For more economical vibration testing of systems or components, it is advisable to design a large test platen to allow attachment of numerous items to the shaker. Often, when performing reliability demonstrations in accordance to MIL-STD-781C (ref 1) or environmental testing in accordance with MIL-STD-810C (ref 2), expensive testing time can be reduced if many items are tested simultaneously on one platen. However, serious difficulties arise if this test platen resonates at a frequency that falls within that of the test plan, causing some components to be subjected to much higher vibration amplitudes than others. Also one corner of the platen may resonate at a different frequency from other corners (ref 3). This variation in frequencies was encountered during reliability testing of the hybrid collective protection equipment. The platen, onto which the test specimens were mounted, resonated at each corner at a different frequency, thereby causing uneven vibration loads into items being tested.

Naturally, if at a certain frequency, one test specimen is more suspect to vibration fatigue than another, the test may provide erroneous failure data for test analysis. A problem such as this can defeat the entire purpose of the test and cause confusion as to the correct classification of a vibration-caused failure (ref 4). If fixture resonance is suspect, the failure would be classified as non-relevant and would not be included in reliability calculations.

BENDING MODES OF A RESONATING PLATEN

The Spring-Mass System

When any structural system vibrates such that the mass (or inertia) force is identical to the spring (or stiffness) force, the system will reach what is known as its natural frequency. On a complex structure, this can occur at any number of frequencies when different components combine or separate into different subsystems, each with its own natural frequency. Experience has shown, however, that when a test platen is loaded with many items to be vibrated on an electrodynamic shaker, the corners will go into resonance at one or possibly two lines of bending. These lines of bending occur where there is sudden increase in platen stiffness.

Sudden Stiffness Caused By Change in Area Moment of Inertia

Usually test platens are made with a constant thickness. Where the platen is bolted onto the shaker head, there is a stepped increase in area moment of inertia. The portion of a platen which extends towards a corner from a 45-degree line, starting from one edge of the platen and tangent to the point of contact of the shaker head, will be the most likely portion to first go into resonance. This cantilever effect is illustrated in figure 1. All four corners will go into

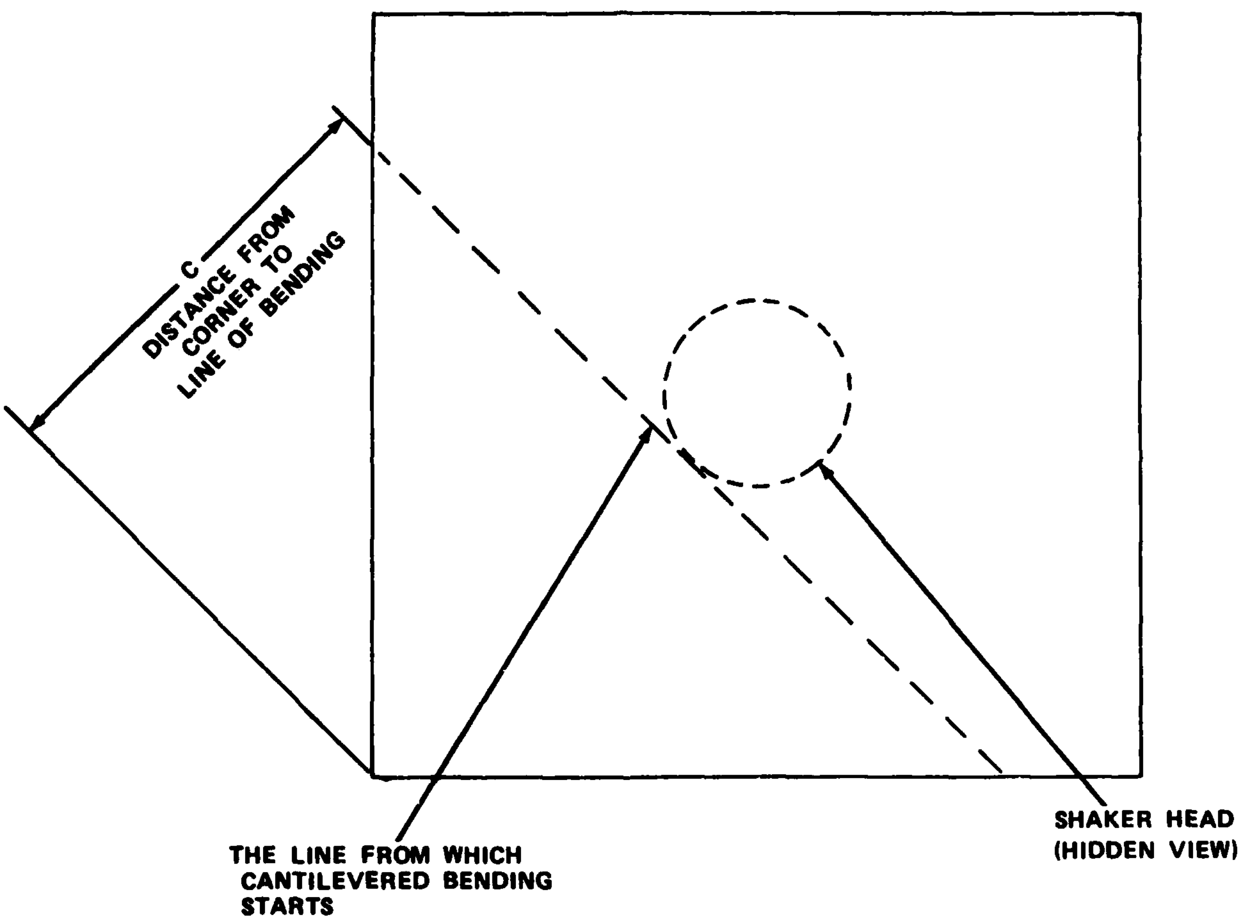

Figure 1. Plan view of test platen illustrating the line of bending starting at the shaker head

resonance along their respective lines. The corner with the heaviest specimen loading will have the lowest natural frequency. Likewise, the corner with the lightest specimen loading will have the highest natural frequency. The other two corners will fall somewhere in between.

Stiffness Caused By Even Platen Loading

When sufficient test specimens are loaded into each corner to equalize the effective mass of each corner, all four corners will have the same natural frequency. This is most probable when all test specimens are identical and laid out identically with respect to the corners. This forces the bending to take place along lines connecting the midpoints of adjacent edges of the platen as shown in figure 2.

In this mode of bending, the natural frequency of a platen is somewhat higher than in the mode illustrated in figure 1. This is because the distance C from the line of bending to the corner is shorter. (The equations found in the next section will show this.) Therefore, it is expedient to calculate the natural frequency for both lines of bending since at any time during a test, a specimen may have to be removed for investigation, thus destroying the symmetrical mass loading of the corners.

DETERMINING THE NATURAL FREQUENCY OF A PLATEN'S CORNERS

General Platen Design Considerations

Several limiting restraints must be considered in designing a vibration platen with a natural frequency higher than the frequency spectrum required by a test plan. The most obvious restraints are:

1. The maximum size allowed in the environmental chamber.

2. The minimum size required to hold the number of test specimens required by the reliability test plan.

3. The total dead weight load on the shaker's voice coil.

4. The higher the dead weight load, the less sinusoidal displacement the shaker is capable of delivering.

The final point is the single, most limiting factor in designing a platen with a natural frequency higher than the frequency spectrum required on a test plan. The shaker is protected from damaging extremes in displacement by limit switches which automatically turn off power to the shaker.

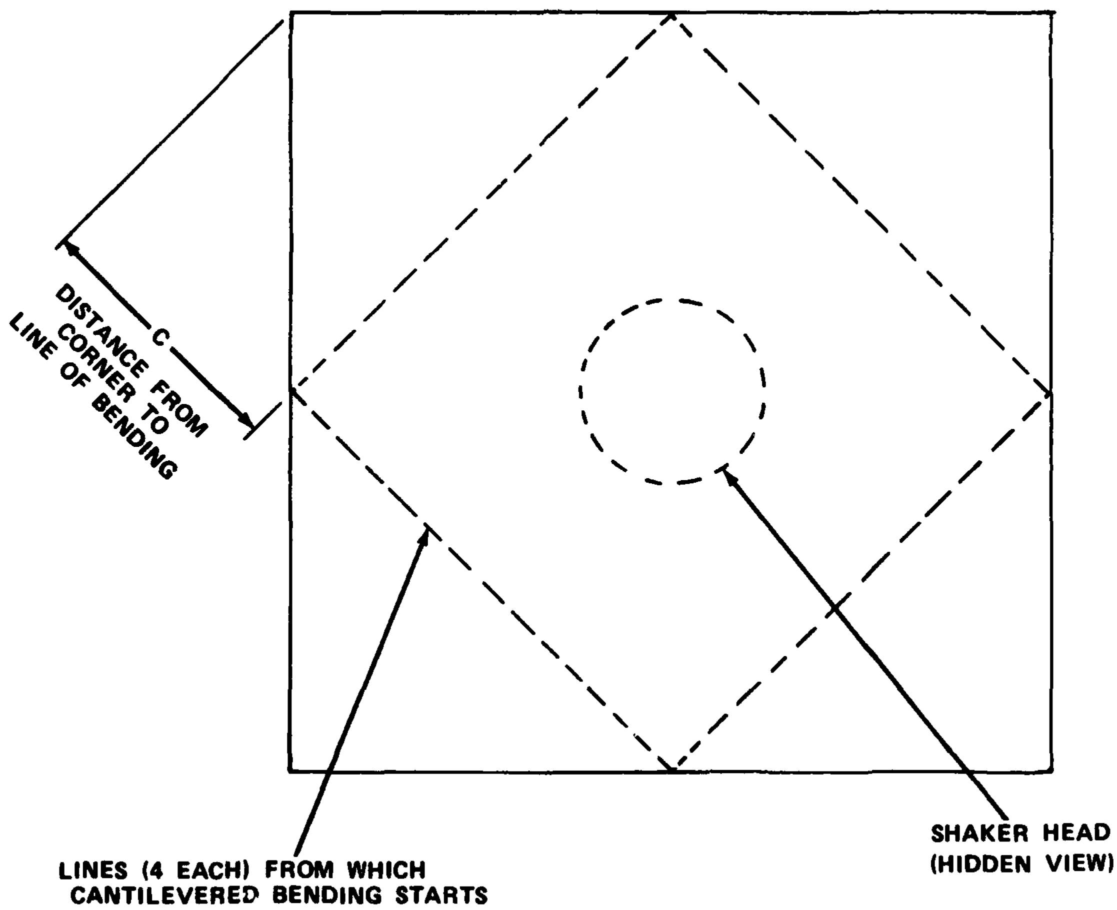

Figure 2. Plan view of test platen illustrating the lines of bending when all four corners resonate at the same frequency

Because of these limitations, particularly the last one, consideration must be given to the total weight of all components making up the dead weight on the voice coil. This includes not only the test specimens, but also any adapters, power cables, bolts, and, of course, the platen. Since the platen is usually the single greatest contributor to the load on the shaker head, it is ordinarily made from a plate of low density material. However, the combined masses of all of these items contribute to determining the natural frequency of a platen.

The Natural Frequency of a Single Plate Platen

The most common platen in use is made from a metal plate of a single material. The natural frequency of such a platen is determined in equation 1. (Experimentally, at Smith Industries in Clearwater, Florida, equation 1 was found to be accurate to within 0.5% in determining the natural frequency of an unloaded corner. The equation's accuracy will largely depend upon how much the loads of the non-corner test specimens vary from the corner-most specimen. However, the load created by corner items is the most critical in determining the system's natural frequency.)

$$f_N = \frac{5.522}{C^2}\sqrt{\frac{E\ h^3}{\frac{W}{A} + \rho h}} \tag{1}$$

NOTE: Equation 1 resulted from combining equations A-10, A-14, and A-36 in the appendix.

where

f_N = natural frequency, hertz

A = area displaced by the corner-most test specimen, in.^{2}

C = distance extending from the midpoint of the line of bending to the corner of the platen (figs. 1 and 2), in.

ρ = density of the material from which the platen is manufactured, lb/in.^{3}

E = modulus of elasticity, lb/in.^{2}

h = thickness of platen, in.

W = weight of corner-most test specimen mounted on platen, lb

As shown by equation 1 and figures 1 and 2, if the resonant frequency increased to where it exceeds the highest required in the test plan, then attention must be given to:

1. Keeping the distance C from the platen corner to the shaker head or other reinforcement as short as possible. This is usually done by making the platen as small as possible.

2. Making the platen thickness h as great as possible and yet not overload the voice coil's displacement limits.

3. Placing the test specimens with the least weight per given area W/A at the corners of the platen and the heaviest ones towards the center.

4. Keeping the ratio of modulus of elasticity to density of the test platen E/ρ as high as possible.

This requires accurate planning and layout of the test platen optimize these four points and yet be within the overall weight limitations of shaker.

There are times, however, when none of these parameters can be optimized to prevent the platen's natural frequency from falling within the the test plan's required frequency band. An added disadvantage is the fact that aluminum, because of its low weight and reasonably favorable E/ρ ratio, is the most frequently used test platen material. The undesirable feature of aluminum is its poor vibration damping characteristics as shown in table 1. If and when a platen's corner goes into resonance, the applitude ratio can be expected to exceed, 10 to 1, the input at the shaker head.

Determining the Natural Frequency with Structural Damping in the Platen

As previously stated, aluminum is the preferred material for test platens because of its low density, machinability, and ductility. However, as shown in table 1, aluminum's specific damping capacity is low. Nodular iron, on the other hand, is preferred by machinery engineers because of its compromise of relatively high strength, machinability, and ductility with moderately good damping characteristics (ref 5). Nodular iron's density and E/ρ ratio make it marginal for use on electrodynamic shakers. However, a platen could be designed that would utilize two materials (for example, alumnium and nodular iron) and synergitically gain the desirable characteristics of both.

The Sandwich, Three Plate, Two-Material Platen

When a beam bends, the farther away its molecules are from the neutral axis, the greater the strain. If the beam is vibrating, it is these outer molecules that have the greatest amount of kinetic energy and, therefore, the greatest potential for damping. If an aluminum plate (or a plate of some other low density material) can be sandwiched between two relatively thin plates of high damping material, and if the plate is excited at its natural frequency, the increase in amplitude of a resonating corner should be minimal. Just as it was possible to calculate the natural frequency of a platen made of a single material, it is possible to calculate the natural frequency of a sandwich design. It can be done using the following equation:

$$f_N = \frac{5.522}{C^2}\sqrt{\frac{h_1{}^3E_1 + 8h_2{}^3\ E_2 + 6h_1{}^2h_2E_2 + 12\ h_1h_2{}^2E_2}{\frac{W}{A} + \rho_1h_1 + 2\ \rho_2h_2}} \tag{2}$$

NOTE: Equation 2 was derived by combining equations A-10, A-19, and A-36 from the appendix.

f_N = natural frequency, hertz

ρ_1 = density of inner plate, $lb/in.^3$

ρ_2 = density of outer plates, $lb/in.^3$

E_1 = modulus of elasticity of inner plate, $lb/in.^2$

E_2 = modulus of elasticity of outer plates, $lb/in.^2$

h_1 = thickness of inner plate, in.

C = distance extending from midpoint of the line of bending to the corner of the platen (figs. 1 and 2), in.

h_2 = thickness of a single outer plate (assuming that each outer plate is the same thickness), in.

W = weight of the corner-most test specimen, lb

A = area displaced by the corner-most test specimen, $in.^2$

The Two Plate, Two-Material Platen

In some designs, preference can be given to platens with the same thin plate of high hysteretic damping material bolted semi-permanently to the shaker head onto which a custom made, thick aluminum plate can be bolted. The top plate of high damping material is not used. This also provides good damping characteristics (but not so good as the sandwich type) and cuts platen manufacturing and setup costs. However, it must be remembered that two materials mounted in this way may curve at the corners if the test program includes great temperature changes. This would be caused by the differences in the two materials' temperature coefficients of expansion. (However, it will probably not be a problem if the plate of high specific damping capacity is kept relatively thin as compared with the aluminum plate.) The formula for the natural frequency of this configuration is:

$$f_n = \frac{5.522}{C^2}\sqrt{\frac{E_1\left[h_1^3 + 12h_1\left(N - \frac{h_1}{2}\right)^2 + \frac{E_2}{E_1}h_2^3 + 12\frac{E_2h_2}{E_1}\left(h_1 - N + \frac{h_2}{2}\right)^2\right]}{\frac{W}{A} + \rho_1h_1 + \rho_2h_2}} \qquad (3)$$

The value $N = \dfrac{E_1h_1^2 + 2E_2h_1h_2 + E_2h_2^2}{2E_1h_1 + 2E_2h_2}$ which is the distance from the top surface of the platen to the neutral axis, in.

NOTE: Equation 3 is a combination of equations A-10, A-21, A-23, and A-36 from the appendix.

where

f_N = natural frequency, hertz

ρ_1 = density of upper plate, lb/in.^{3}

ρ_2 = density of lower plate, lb/in.^{3}

E_1 = modulus of elasticity of upper plate, lb/in.^{2}

E_2 = modulus of elasticity of lower plate, lb/in.^{2}

h_1 = thickness of upper plate, in.

h_2 = thickness of lower plate, in.

C = distance extending from the midpoint of the line of bending to the corner of the platen (figs. 1 and 2), in.

W = weight of the corner-most test specimen, lb

A = area displaced by the corner-most test specimen, in.^{2}

The reason for equation 3's being more complex than equations 1 and 2 is that the neutral axis of bending is not at the geometric center of the platen's cross-sectional thickness.

TOTAL DAMPING ENERGY

Background on Damping

Damping is the energy dissipating quality of a material. This dissipation occurs when some of the kinetic energy of a vibrating system is converted to heat energy. When a structure vibrates at its natural frequency, the mass (or inertia) force is in dynamic equilibrium with the spring (or stiffness) force and 90 degrees out of phase with the exciting force. Therefore, the only structural force available to oppose the exciting force is that of the damping force. If the material has little damping, the sinusoidal velocity must be high to keep the structure in dynamic equilibrium, which also causes the displacement to increase. If the damping qualities are too minute to keep the displacement low and the excitation remains in the natural frequency range too long, the structure will fail from fatigue.

The damping coefficient is easy to measure experimentally in a completed structure, but is very difficult to determine theoretically in the design stage of a complex structure, such as an entire automobile. A platen, however, is relatively simple and it is possible to derive equations of damping energy for such a structure. The final units are in.-lb/cycle for the entire resonating corner of the platen and, as such, are useful in comparing one design with another. Furthermore, experimental data can be directly compared with calculated data by measuring the energy per cycle into a platen corner at resonance and comparing it with equations provided in this report.

Hysteresis Damping

Materials have two types of damping qualities, viscoelastic and hysteretic. The former will not be discussed because it pertains mostly to polymetric materials. Hysteretic damping occurs where the stress-strain curve on loading does not coincide with the stress-strain curve on unloading. The area between the two curves represents the amount of kinetic energy that has been dissipated as heat energy (ref 6). A hysteresis loop which illustrates this energy dissipation is shown in figure 3.

Specific Damping Capacity Values

Specific damping capacities (%-per-cycle) of many materials for given shear stresses are contained in table 1. Values in the specific damping capacity column are to be used where the symbol Ψ appears in the formulas to calculate the relative damping capacity in platens. The validity of applying values obtained from shear-stress tests to an obvious bending stress application, is supported by the intricate test methodology of collecting the data.

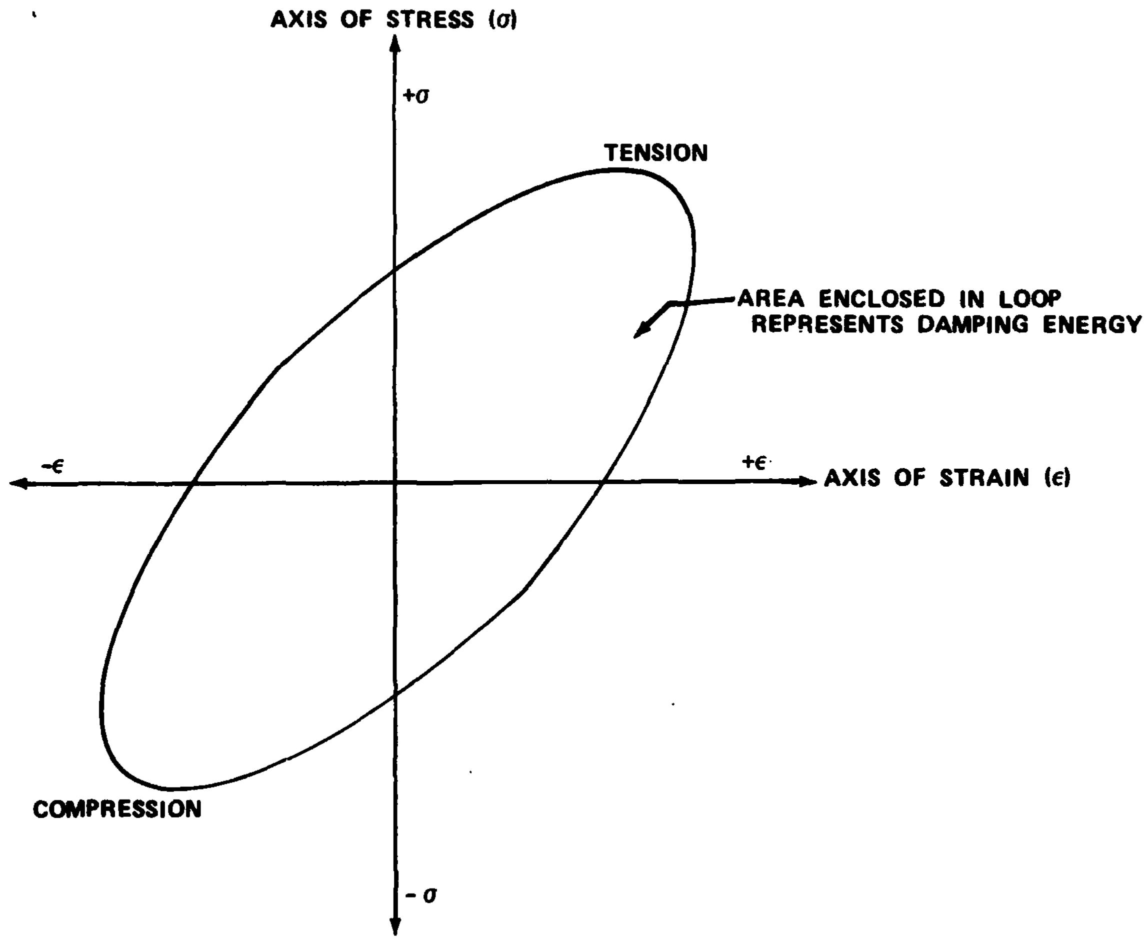

Figure 3. Typical stress-strain hysteresis loop for a material experiencing cyclic stress

Table 1.

THE SPECIFIC DAMPING CAPACITY OF COMMERCIAL ALLOYS AT ROOM TEMPERATURE

The specific damping capacity which is normally measured on solid cylinders stressed in torsion is defined as the ratio of the vibrational strain energy dissipated during one cycle of vibration to the vibrational strain energy at the beginning of the cycle.

Alloy	*Composition %*	*Specific damping capacity %*	*Surface shear stress* (lb/in²)
Mn–Cu (quenched from 850°C aged 2 hr at 425°C)	90% Mn–10% Cu	21	5,000
	85% Mn–15% Cu	27	5,000
	80% Mn–20% Cu	22	5,000
	70% Mn–30% Cu	42	5,000
	60% Mn–40% Cu	42	5,000
	50% Mn–50% Cu	33	5,000
Ni–Ti (Nitinol)	55% Ni–45% Ti	26	10,000
T–D Nickel	2·5% Thoria	10·7	10,000
Mallory No-chat	—	9·4	10,000
Cast Irons			
High carbon inoculated flake iron	2·5% C, 1·9% Si, 1·0% Mn, 20·7% Ni, 1·9% Cr, 0·13% P	19·3	5,000
Spun cast iron	3·54% C, 3·39% G.C., 1·9% Si, 0·4% Mn, 0·38% P	10·8	5,000
Non-inoculated flake iron	3·3% C, 2·2% Si, 0·5% Mn, 0·14% P, 0·03% S	8·5	5,000
Inoculated flake iron	3·3% C, 2·2% Si, 0·5% Mn, 0·14% P, 0·03% S	7·3	5,000
Austenitic flake graphite	2·5% C, 1·9% Si, 1% Mn, 20·7% Ni, 1·9% Cr, 0·03% P, 0·03% S	7·1	5,000
Alloyed flake graphite	3·14% C, 2% Si, 0·6% Mn, 0·7% Ni, 0·4% Mo, 0·14% P, 0·03% S	5·3	5,000
Nickel–copper austenitic flake	2·35% C, 1·9% Si, 1·25% Mn, 15·2% Ni, 7·3% Cu, 2% Cr, 0·03% P, 0·04% S	3·9	5,000
Undercooled flake graphite titanium/CO_2 treated	3·27% C, 2·2% Si, 0·6% Mn, 0·35% Ti, 0·14% P, 0·03% S	3·9	5,000
Annealed ferritic nodular	3·7% C, 1·8% Si, 0·4% Mn, 0·76% Ni, 0·06% Mg, 0·03% P, 0·01% S, <0·003% Ce	2·8	5,000
Pearlitic malleable	B.S. 3333/1961 Grade B.33/4	1·5	5,000
Blackheart malleable	B.S. 310 1958 Grade B.22/14	1·5	5,000
As cast pearlitic nodular	3·66% C, 1·8% Si, 0·4% Mn, 0·76% Ni, 0·06% Mg, 0·03% P, 0·01% S, <0·003% Ce	1·4	5,000
Steels			
E.N.3	0·17% C mild steel, normalized	1·5	5,000
B.S.1407 (silver steel)	Spherodized	0·8	5,000
B.S.1407 (silver steel)	Water quenched 800°C	0·5	5,000
B.S.1407 (silver steel)	Water quenched 800°C aged 100°C 1½ hr	0·2	5,000
E.N.23/T	3% Ni, 1% Cr, 0·3% C	0·8	5,000
E.N.12/Q	1% Ni, 0·4% C	0·3	5,000
E.N.19/U	1% Cr, 0·3% Mo, 0·4% C	0·15	5,000
B.S. S62	12% Cr, 0·2% C. Quenched tempered to 225 B.H.N.	3·8	5,000
E.N.58B	18% Cr, 8% Ni, 0·6% Ti, 0·1% C. Solution treated 1050°C water quenched	1·8	5,000
N.M.C.	0·62% C, 3·86% Cr, 8·6% Ni, 7·3% Mn. Solution treated 1050°C, water quenched	0·7	5,000
E.N. 58A	18% Cr, 8% Ni, 0·1% C. Solution treated 1050°C, water quenched	0·3	5,000
Copper Alloys			
Hidurel 6	As cast	1·35	5,000
Gunmetal	88% Cu, 10% Zn, 2% Sn	1·0	5,000
Brass (B.S. 265)	As extruded	0·4	5,000
Hidurel 5	As cast	0·4	5,000
Hidurel 7	As cast	0·25	5,000
High tensile brass	As cast	0·25	5,000
Novoston	—	0·25	5,000
Aluminium Alloys			
Duralumin (H.E. 14)	—	0·25	5,000
RR57 (DTD 5004 W.P.)	—	0·20	5,000
RR58 (DTD 5014 W.P.)	—	0·10	5,000
Hiduminium 100 (S.A.P.)	—	5·0	5,000
Magnesium Alloys			
DTD 5005	Mg/Zn/Zr/Th	7·4	3,000
B.S. 1278	Mg/Zn/Mn	1·6	3,000
DTD 721A	Mg/Zn/Zr	0·65	3,000
Magnesium Elektron	Mg/Ag/Zr	0·4	3,000
M.S.R. Alloy			

Reference: D. Birchon, *Engineering Materials and Design*, Sept., Oct., 1964. By permission.

To determine the specific damping capacity, accurate and repeatable test procedures had to be set up to measure the deflection of a vibrating system. Methods, other than torsion, available to measure the required stresses would require long gage lengths and stresses of about 60% of the fatigue limit of many materials. Such test apparatus would be unwieldly and the values would be at stress levels too high for many vibration difficulties, including this one. Because of these limitations, a torsional device was used as a test standard. The values of the specific damping capacity are usable for vibration problems in any stress mode where the stresses are relatively low as compared to the elastic limit (ref 7).

The Single Plate Platen

The equation for the total damping energy of a resonating corner of a platen becomes:

$$D = \frac{\Psi C^6 \left(\frac{W}{A} + \rho h\right)^2}{45\ E\ h^3} \tag{4}$$

NOTE: This equation substitutes the values of V_s from equation A-14 and Z from equation A-10 into equation A-44.

where

D = total damping energy, in.-lb/cycle

Ψ = specific damping capacity, %/cycle

W = weight of corner-most test specimen, lb

C = distance extending from the midpoint of the line of bending to the corner of the platen (figs. 1 and 2), in.

A = area displaced on platen by corner-most test specimen, in.^{2}

ρ = density of plate material, lb/in.^{3}

E = modulus of elasticity, lb/in.^{2}

h = plate thickness, in.

The Three Plate, Two-Material Platen

The platen which has a center plate of a low density ρ_1, low specific damping capacity Ψ material and is sandwiched between two identical plates of high density ρ_2, high specific damping capacity Ψ material, may offer a solution to a serious vibration problem. It is an ideal compromise of total weight on the shaker head, natural frequency, and damping. The equation for this design becomes:

$$D = \frac{\left[2\left(\frac{W}{A} + \rho_1 h_1 + 2\rho_2 h_2\right)\right]^2 C^6 \left[\frac{h_1^3 \Psi_1}{8E_1} + \frac{\left(\frac{h_1}{2} + h_2\right)^3 \Psi_2}{E_2} - \frac{h_1^3 \Psi_2}{8E_2}\right]}{405\left[\frac{h_1^3}{6} + \frac{4h_2^3}{3}\left(\frac{E_2}{E_1}\right) + h_1^2 h_2\left(\frac{E_2}{E_1}\right) + 2h_1 h_2^2\left(\frac{E_2}{E_1}\right)\right]^2} \qquad (5)$$

NOTE: This equation is derived from substituting the values of V_{HG} of equation A-19 and Z of equation A-10 into equation A-45.

where

D = total damping energy, in-lb/cycle

Ψ_1 = specific damping capacity of inner plate material, %/cycle

Ψ_2 = specific damping capacity of outer plate material, %/cycle

W = weight of corner-most test specimen, lb

C = distance extending from the midpoint of the line of bending to the corner of the platen (figs. 1 and 2), in.

A = area displaced on platen by corner-most test specimen, in.^{2}

ρ_1 = density of inner plate material, lb/in.^{3}

ρ_2 = density of material of outer plates, lb/in.^{3}

E_1 = modulus of elasticity of inner material, lb/in.^{2}

E_2 = modulus of elasticity of outer material, lb/in.^{2}

h_1 = thickness of inner plate, in.

h_2 = thickness of a single outer plate (assuming each has the same thickness), in.

The Two Plate, Two-Material Platen

The two plate, two-material platen may be more economical in its use of a material of high density and high specific damping capacity to be used as a permanent mounting fixture for a custom-made specimen mounting plate. It is not quite so efficient in terms of damping per unit weight as the three-plate, two-material design, but may be efficient enough for the convenience it offers. This equation is used to obtain the total damping energy:

$$D = \frac{C^6\left(\frac{W}{A} + \rho_1 h_1 + \rho_2 h_2\right)\left\{\frac{\Psi_1}{E_1}\left[N^3 + (h_1 - N)^3\right] + \frac{\Psi_2}{E_2}\left[(h_2 + h_1 - N)^3 - (h_1 - N)^3\right]\right\}}{810\left[\frac{h_1^3}{6} + 2h_1\left(N - \frac{h_1}{2}\right)^2 + \frac{E_2 h_2^3}{6\ E_1} + 2\frac{E_2}{E_1} h_2 (h_1 - N + \frac{h_2}{2})^2\right]^2} \qquad (6$$

$$\text{The value } N = \frac{E_1 h_1^2 + 2E_2 h_1 h_2 + E_2 h_2^2}{2\ E_1 h_1 + 2\ E_2 h_2}\text{, in.}$$

NOTE: This equation 6 is derived by combining the values of V_{HN} in equation A-23 and Z in equation A-10 with equation A-46.

where

D = total damping energy, in.-lb/cycle

Ψ_1 = specific damping capacity of upper plate material, %/cycle

Ψ_2 = specific damping capacity of lower plate material, %/cycle

C = distance extending from the midpoint of the line of bending to the corner of the platen (figs. 1 and 1), in.

W = weight of corner-most test specimen, lb.

A = area displaced on platen by corner-most test specimen, in.^{2}

ρ_1 = density of upper plate material, lb/in.^{3}

ρ_2 = density of lower plate material, lb/in.^{3}

E_1 = modulus of elasticity of upper plate material, lb/in.^{2}

E_2 = modulus of elasticity of lower plate material, lb/in.^{2}

h_1 = thickness of upper plate, in.

h_2 = thickness of lower plate, in.

N = distance from top surface of upper plate to the neutral axis

After completion of all equations required to calculate the total damping energy of a given platen design, the next consideration is material selection. Generally, in addition to the qualities of density, modulus of elasticity, and specific damping capacity, several other qualities are important. Others are availability, ductility, and machinability. Coefficients of expansion may be important for the two plate, two-material platen undergoing very wide temperature extremes. Ideally, all of the plates would have the same expansion coefficients.

THE FEEDBACK SIGNAL FOR VIBRATION CONTROL PURPOSES

The Feedback Control Problem

The electrodynamic shaker's amplitude of vibration is determined by the amount of current that flows through the voice coil. Since this current is constantly changing direction as well as magnitude in the presence of high magnetic field, it forces the voice coil to move. Since the voice coil has a shaker head attached to it and the platen is attached to the shaker head, the transducer (usually an accelerometer) responsible for the feedback signal can be mounted anywhere, in a non-resonant condition. However, when the natural frequency of the platen's corners is within the frequency spectrum dictated by the test plan, this simplicity will not work.

If, for example, the accelerometer is placed in the center of the platen, the test specimens mounted on the corners will experience loads far in excess of what are required in the test plan. If, on the other hand, the accelerometer is placed at any one corner, the specimens in that corner will be the only ones following the test program. The others will receive either more and less vibration than they are supposed to receive. This will happen even with a platen with good dampening energy characteristics. The answer lies with some type of vibration averaging.

With five accelerometers, one at each corner and one in the middle of the platen, the acceleration of each possible resonating location and the one non-resonating location can be measured. These signals can be summed and divided by five in one of two ways (discussed later). However, the main advantage of this is an averaging of vibration input error. For example, if the test plan calls for an input of 1-g acceleration and at a specific frequency one corner wants to resonate at 3 g's with this input, the actual vibrational acceleration put out by the shaker will be the sum of all locations, which is 1 + 1 + 1 + 1 + 3 = 7, divided by the number of accelerometers, 5, which is an average of 1.4. The controller reduces the shaker's output until it senses 1-g. All corners will get an 0.714-g input except the resonating one which will receive 2.143 g's. This is not what the program specifies, but it is an improvement over 3 g's.

Feedback Averaging Techniques

Modern digital computerized vibration controls may possibly come with interfaces which can average five signals and use the result as the feedback control signal. These are costly, however, and many test laboratories cannot afford such capital expenditures. However, it is possible to use five operational amplifiers to sum the accelerometer signals and a sixth one to divide the summed signal by five to get a single signal back to the main controller. In effect, this becomes an on-line analog computer which is relatively inexpensive and versatile for a number of other instrumentation tasks in a test laboratory.

CONCLUSIONS AND RECOMMENDATIONS

It may not be possible to design a vibration test platen with a natural frequency higher than the frequency spectrum of the test plan, but several methods could be tried to get the natural frequency as high as possible:

1. Keep the sides of the platen as short as possible.

2. Place at the extreme corners the test specimens which have the lightest weight per area of platen displaced. Heavier items should be placed toward the platen's center.

3. Choose a material which has a high modulus:density ratio.

4. Make the platens as thick as possible within the shaker's dead weight limitations for the maximum sinusoidal displacement required.

After all of the above techniques have been tried, then materials with high specific damping capacities should be tried. Since the usual materials with good damping characteristics have high densities as compared with materials with poor damping characteristics, it may be necessary to devise a multiple plate platen. Such a platen would concentrate the low density plates near the neutral axis and at least one, and preferably two, plates of high specific damping capacity in the areas of high bending strains.

If it is known from the natural frequency calculations that the platen will have a corner that will resonate, then the feedback for control purposes should be planned accordingly. The choices are either an analog device which can average the signals from several platen locations, or, if possible, a digital computer that has real time capability to average the signals. An averaged feedback signal will provide the overall optimized vibration load on the test specimens.

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APPENDIX

NATURAL FREQUENCY OF A CANTILEVERED RIGHT-TRIANGULAR PLATE

Introduction

Each corner of a shaker's test platen behaves as a cantilevered right triangular plate when it resonates. If loading is equal on all four corners, they will resonate at the same frequency. If the loading is unequal, the corners will not only resonate at different frequencies, but the cantilevered line from which the bending originates may shift as well. Therefore, any generalized equation must be in terms of (1) the distance from the corner of the platen to the line where the bending begins as illustrated in figures 1 and 2 of the text, (2) equivalent mass loading, (3) area moment of inertia, and (4) Young's Modulus of elasticity.

Raleigh Method of Natural Frequency Calculation

The Raleigh Method of natural frequency calculation has been found to produce values which agree closely with experimental findings if accurate assumptions are made of the physical characteristics of the vibrating system. The equation for this method is[A-1]:

$$\omega = \sqrt{g \frac{\int Y dx}{\int Y^2 dx}} \tag{A-1}$$

where

ω = natural frequency, rad/s

Y = equation for the deflection of the vibrating system, in.

g = acceleration due to gravity, which is a constant of 386.4 in./s^2

The deflection is found by solving the fourth order differential equation[A-2]

$$\frac{d^4Y}{dx^4} = \frac{W_T}{EI} \tag{A-2}$$

A-1 H. M. Hansen and Paul F. Citenea, Mechanics of Vibration, John Wiley and Sons, New York, 1952.

A-2 E. P. Popov, Mechanics of Materials, Prentice-Hall, New York, 1952.

where

W_T = is the total loading on the vibrating system, lb/in.

E = Young's Modulus of Elasticity, lb/in.^{2}

I = area moment of inertia, in.^{4}

The primary problem in solving equation A-2 is determining the generalized expressions of W_T and I for a loaded, heterogeneous, right triangular, cantilevered plate. The load will be calculated first.

Calculation of Load

The load contribution on the natural frequency of a vibrating system includes the mass of the system itself and any attached test specimens. On a test platen, this can be a difficult problem because:

1. The mass of the vibrating triangular plate, which is a corner of the platen, increases as a squared function (if the thickness is constant) as the distance increases from a corner to the line of bending.

2. The mass per displaced area of the various mounted test specimens can vary greatly.

The load equation will be developed in two parts, one load for the vibrating triangular plate Wp, and the other for the attached test items W_x. They will be combined to form a final load express W_T.

Mass Load of the Vibrating Triangular Plate

The load shall be calculated in terms of the weight of a right triangular plate whose mass is distributed from the 90-degree corner to the cantilevered edge. The distributed weight is then defined as:

$$Wp = \rho \ a \tag{A-3}$$

where

Wp = distributed load of the plate, lb/in.

ρ = density of the plate material, lb/in.^{3}

a = the cross-sectional area of the triangular plate at distance x from a platen's corner.

Since the width of the triangle varies with the distance from x = 0, then so does the cross-sectional area and the distributed weight. Figure A-1 shows a right triangular plate which has been bisected into two 45-degree triangular plates of thickness h. The equation for the cross-sectional area of a triangular plate cross section is

$$da_1 = dzdy$$

Since

$$dz = 2dx \tag{A-4}$$

then

$$da = 2dx\ dy$$

$$a = 2\int_o^x d\,x \int_{\frac{-h}{2}}^{\frac{h}{2}} dy$$

$$a = 2hx \tag{A-5}$$

By substituting equation A-5 into equation A-3, the distributed weight for the triangular plate is obtained:

$$Wp = 2\ \rho hx \tag{A-6}$$

If the platen is constructed of three plates and two materials such as in a sandwich arrangement, the individual densities, ρ_1 and ρ_2, and thicknesses, h_1 and h_2, of each material are used. In such case, the equation would look like this:

$$Wp = 2x\left(\rho_1 h_1 + 2\rho_2 h_2\right) \tag{A-7}$$

The 2 inside the brackets is for the two outside plates of the same thickness h_2.

The natural frequency of a cantilever is most affected by masses that are farthest from the edge of bending. The greater this mass, the lower the system's natural frequency happens to be. This is because the natural frequency is related to the deflection, and the deflection for a cantilevered system is greatest at the free end. Therefore, to greatly simplify the deflection equation, one can assume that the weight per unit area of the corner-most test item is the same for the entire triangular plate being considered. (Test prudence would dictate, wherever possible, that the lightest items per square inch go at the extreme corners and the heavier items go towards the center of the platen to get the highest natural frequency.) Include the weight of the test item, the adapters, and the boltheads to obtain the most accurate estimate of the cantilever's loading. This total specimen weight will be designated as W_x. Next, calculate the area displaced on the platen by the test item and all associated hardware and designate this area as A. This provides a load pressure of W/A in pounds per square inch for the corner-most position on the test platen. However, what is needed is the equivalent distributed load that varies as the right triangular area expands from the corner to the line of bending. This load increases at the same 2x multiplier as the mass of the plate does (equation A-5) so the equation for the weight distribution of the test items is

$$W_x = 2 \frac{W}{A} x \tag{A-8}$$

The justification for using the corner-most loading for the entire triangular plate is that the heavier test specimens that are closer to the line of bending have a relatively low contribution on the deflection. This assumption will yield a slightly higher calculated natural frequency than will actually be experienced, but the difference should be negligible.

Total Load Equation

The total load equation on a right triangular plate is

$$W_T = W_x + Wp$$

or

$$W_T = 2\left(\frac{W}{A} + \rho_1 h_1 + 2\ \rho_2 h_2\right) x \tag{A-9}$$

Since only the x value is variable, for purposes of simplifying the differential equation, we will make the substitution

$$Z = 2\ \left(\frac{W}{A} + \rho_1 h_1 + 2\ \rho_2 h_2\right) \tag{A-10}$$

One should realize that the subscripts are used only on platens made of one material sandwiched between two others. For a single material platen $Z = 2\left(\frac{W}{A} + \rho h\right)$.

Then

$$W_T = Zx \tag{A-11}$$

Calculation of Area Moment of Inertia

The Single Plate Platen

The area moment of inertia of the cross section varies with x the same as the area does (fig. A-1). The equation for the area moment of inertia of a rectangular section is

$$dI = \frac{h^3}{12} dz \tag{A-12}$$

where dz is the change in width of the cross section.

From equation A-4, the width varies (dz) as the distance from the corner doubles (2dx) or:

$$dz = 2dx$$

therefore

$$I_s = \frac{h^3 x}{6} \qquad \text{(A-13)}$$

To keep the calculations as simple as possible

$$\text{Let } V_s = \frac{h^3}{6} \qquad \text{(A-14)}$$

therefore

$$I_s = V_s x \qquad \text{(A-15)}$$

The three plate, two-material platen increases the difficulty of calculating the area moment of inertia because the area moment of inertia derivations require a complicated "section transformation" using the modulus of elasticity of each material used. This is not considered in the single-plate model.

The technique of section transformation[A-3] used to develop equivalent cross sections is illustrated in figure A-2. Essentially, the outer plates have their width increased by the ratio of the modulus of elasticities of the outer plates over the inner plate:

$$z_T = \frac{E_2}{E^1} z \qquad \text{(A-16)}$$

where

z_T = the transformed width, in.

z = the geometric width, in.

E_2 = modulus of elasticity of the outer material, $lb/in.^2$

E_1 = modulus of elasticity of the inner material, $lb/in.^2$

It has already been established from figure A-1 that

$$z = 2x$$

$$z_T = 2 \frac{E_2}{E_1} x \qquad \text{(A-17)}$$

[A-3] E. P. Popov, Mechanics of Materials, Prentice-Hall, New York, 1952.

This value of z_T can now be used to calculate the area moment of inertia. The equation is now written, in its most general form, using Popov's methods (fig. A-2):

$$I_{HG} = \frac{h_1{}^3\ Y}{6} + 2\left[\frac{h_2{}^3 E_2}{6\ E_1}\ x\right] + 2\left[h_2\left(2\ \frac{E_2}{E_1}\ x\right)\left(\frac{h_1}{2} + \frac{h_2}{2}\right)^2\right]$$

where

I_{HG} = the area moment of inertia of one material sandwiched between two others with the neutral axis of bending the cross-sectional geometric center, in.^{4}

h_1 = thickness of center material, in.

h_2 = thickness of each of the outer plates, in.

$$I_{HG} = \left[\frac{h_1{}^3}{6} + \frac{4\ h_2{}^3}{3}\left(\frac{E_2}{E_1}\right) + h_1{}^2 h_2\left(\frac{E_2}{E_1}\right) + 2\ h_1 h_2{}^2\left(\frac{E_2}{E_1}\right)\right] x \qquad \text{(A-18)}$$

To simplify future calculations, let:

$$V_{HG} = \frac{h_1{}^3}{6} + \frac{4\ h_2{}^3}{3}\left(\frac{E_2}{E_1}\right) + h_1{}^2 h_2\left(\frac{E_2}{E_1}\right) + 2\ h_1 h_2{}^2\left(\frac{E_2}{E_1}\right) \qquad \text{(A-19)}$$

then

$$I_{HG} = V_{HG} x \qquad \text{(A-20)}$$

The Two Plate, Two-Material Platen

Next is an examination of the equations for two plates vibrating together where the neutral axis of bending is not at the cross-sectional geometric center of the two plates. This is likely to occur in the design of a semi-permanent plate with high specific damping characteristics that attaches to a shaker head onto which all future light weight, special purpose platens are to be bolted. This example is illustrated in figure A-3.

The first step is to calculate the transformation of the width of the lower plate. This was calculated previously in equation A-17 and is:

$$z_T = 2\,\frac{E_2}{E_1}\,x$$

The equation to find the location of the neutral axis as measured from the top of the platen, using figure A-3d as a basis for the calculation, is

$$N = \frac{2x(h_1)\left(\frac{h_1}{2}\right) + 2\left(\frac{E_2}{E_1}\right)x\ (h_2)(h_1 + \frac{1}{2}\,h_2)}{2x(h_1) + 2\left(\frac{E_2}{E_1}\right)x\ (h_2)}$$

$$N = \frac{E_1 h_1^2 + 2\,E_2 h_1 h_2 + E_2 h_2^2}{2\,E_1 h_1 + 2\,E_2 h_2} \qquad \text{(A-21)}$$

Knowing the neutral axis, we can now write the equation for the area moment of inertia of a two material test platen

$$I_{HN} = \frac{h_1^3}{6}\,x + 2xh_1\left(N - \frac{h_1}{2}\right)^2 + \frac{\frac{E_2}{E_1}\,h_2^3}{6}\,x + 2\,\frac{E_2}{E_1}\,h_2\,x\left(h_1 - N + \frac{h_2}{2}\right)^2 \qquad \text{(A-22)}$$

$$I_{HN} = \left[\frac{h_1^3}{6} + 2\,h_1\left(N - \frac{h_1}{2}\right)^2 + \frac{\frac{E_2}{E_1}\,h_2^3}{6} + 2\,\frac{E_2}{E_1}\,h_2\left(h_1 - N + \frac{h_2}{2}\right)^2\right]x$$

Again, as in the last equation A-18, it is too unwieldy to use in future calculations, so let

$$V_{HN} = \frac{h_1^3}{6} + 2h_1\left(N - \frac{h_1}{2}\right)^2 + \frac{E_2 h_2^3}{6\,E_1} + 2\,\frac{E_2 h_2}{E_1}\left(h_1 - N + \frac{h_2}{2}\right)^2 \qquad \text{(A-23)}$$

Therefore

$$I_{HN} = V_{HN}\,x \qquad \text{(A-24)}$$

This completes the preliminary calculations necessary to calculate the deflection equation for a cantilevered right triangular plate.

Derivation of the Deflection Equation

As stated in equation A-2, the deflection of a beam is:

$$\frac{d^4y}{dx^4} = \frac{W}{EI}$$

The solution to this fourth order differential equation, fortunately, is four easy steps of integration. They are:

$$S = \int W dx + Vo \qquad \text{(A-25)}$$

$$M = \int S dX + Mo \qquad \text{(A-26)}$$

$$\Theta = \int \frac{M}{EI} dx + \Theta o \qquad \text{(A-27)}$$

$$Y = \int \Theta dX + Yo \qquad \text{(A-28)}$$

where

S = shear load in plate, lb

M = moment load in beam, in.-lb

Θ = slope, rad.

Y = deflection, in.

The mathematical work will follow these steps with integration starting from the free end and going to the fixed end as shown in figure A-1.

Shear Calculation

From equation A-25

$$S = \int W dx + Vo$$

From equation A-11

$$W = Zx$$

Then

$$S = \int Zxdx + Vo$$

$$S = \frac{Z}{2} x^2 + Vo$$

at $x = 0$ $S = 0$, then $V_o = 0$

Therefore:

$$S = \frac{Z}{2} x^2 \tag{A-29}$$

Moment Calculation

From equation A-26

$$M = \int Sdx + Mo$$

Substituting the value of S from equation A-29 into the above equation:

$$M = \int \frac{1}{2} Zx^2 \, dx + Mo$$

$$M = \frac{1}{6} Zx^3 + Mo$$

at $x = 0$, $M = 0$, then $Mo = 0$

$$M = \frac{1}{6} Zx^3 \tag{A-30}$$

Slope Calculation

From equation A-27:

$$\Theta = \int \frac{M}{EI} dx + \Theta_o$$

From equations A-15, A-20, and A-24, we know that the area moment of inertia I with a subscript equals a constant V with the same subscript multiplied times the variable x. Therefore, to simplify calculations:

$$I = Vx \tag{A-31}$$

We can now substitute A-30 and A-31 into equation A-27 and get:

$$\Theta = \int \frac{Zx^3}{6\ EVx} dX + \Theta o$$

$$\Theta = \frac{Zx^3}{18\ EV} + \Theta o$$

At x = C (figs. 1 and 2), $\theta = 0$

Then $\theta o = -\frac{ZC^3}{18\ EV}$

Therefore:

$$\theta = \frac{Zx^3}{18\ EV} - \frac{ZC^3}{18\ EV} \qquad \text{(A-32)}$$

Deflection Calculation

From equation A-28:

$Y = \theta\ dx + Yo$

Substituting in the value from equation A-32 we get:

$$Y = \int \left(\frac{Zx^3}{18\ EV} - \frac{ZC^3}{18\ EV} \right) dx + Yo$$

$$Y = \frac{Zx^4}{72\ EV} - \frac{ZC^3x}{18\ EV} + Yo$$

At x = C, (figs. 1 and 2 in main text of this report), Y = 0

Then

$$Yo = \frac{ZC^4}{18\ EV} - \frac{ZC^4}{72\ EV}$$

$$Yo = \frac{ZC^4}{24\ EV}$$

Therefore:

$$Y = \frac{Zx^4}{72\ EV} - \frac{ZC^3x}{18\ EV} + \frac{ZC^4}{24\ EV}$$

$$Y = \frac{Z}{72\ EV} (x^4 - 4C^3x + 3C^4) \qquad \text{(A-33)}$$

It is this deflection equation that will be used to calculate the natural frequency of the test platen.

Calculation of Natural Frequency

Equation 1 provides the Raleigh equation for determining the natural frequency of a vibrating system. It is:

$$\omega = \sqrt{g \frac{\int Ydx}{\int Y^2dx}}$$

This requires an additional integration step of the numerator and an additional integration step after squaring in the denominator. So we will:

let k = Ydx

From equation A-33 and figures 1 and 2 from main text of this report:

$$k = \int_o^C \left[\frac{Z}{72\ EV} \left(x^4 - 4C^3x + 3C^4\right) \right] dx$$

$$k = \left(\frac{Z}{72\ EV} \quad \frac{C^5}{5} - 2C^3C^2 + 3C^4C \right)$$

$$\int Ydx = k = \frac{Z}{72\ EV} \left(\frac{6}{5}\ C^5 \right) \qquad \text{(A-34)}$$

The equation will be left in this form to simplify calculations later.

Now, let:

$$j = \int Y^2dx$$

$$j = \int_o^C \left[\frac{Z}{72\ EV} (x^4 - 4C^3x + 3C^4) \right]^2 dx$$

$$j = \int_o^C \left(\frac{Z}{72\ EV} \right)^2 (x^8 - 8C^3x^5 + 6C^4x^4 + 16C^6x^2 - 24C^7x + 9C^8)dx$$

$$j = \left(\frac{Z}{72\ EV} \right)^2 \left(\frac{C^9}{9} - \frac{8}{6}\ C^3C^6 + \frac{6}{5}\ C^4C^5 + \frac{16}{3}\ C^6C^3 - 12C^7C^2 + 9C^8C \right)$$

$$j = \left(\frac{Z}{72\ EV}\right)^2 \left(\frac{10}{90} - \frac{120}{90} + \frac{108}{90} + \frac{480}{90} - \frac{270}{90}\right) C^9$$

$$j = \int Y^2 dx = \left(\frac{Z}{72\ EV}\right)^2 \frac{104}{45}\ C^9 \qquad \text{(A-35)}$$

Now substitute equations A-34 and A-35 into equation A-2:

$$\omega = \sqrt{g\ \frac{\left(\frac{Z}{72\ EV}\right)\left(\frac{6}{5}\ C^5\right)}{\left(\frac{Z}{72\ EV}\right)^2\left(\frac{10^4}{45}\right)C^9}}$$

To put the final solution into the more convenient units of hertz:

$$\omega = 2\pi\ f_N$$

$$f_N = \frac{\omega}{2\pi}$$

then:

$$f_N = \frac{1}{2\pi}\sqrt{\frac{72\ EV\ (6)(45)(386.4)}{Z(5)(104)C^4}}$$

$$f_N = \frac{1}{2\pi}\sqrt{\frac{14,445\ EV}{Z\ C^4}}$$

$$f_N = \frac{19.128}{C^2}\sqrt{\frac{EV}{Z}} \qquad \text{(A-36)}$$

The main text of this report contains this equation with the values of V and Z (with applicable subscripts) substituted into it and simplified into the forms available for easy use. These equations should provide the fixture designer with reasonably accurate frequency calculations for decision-making purposes.

TOTAL DAMPING ENERGY

How Damping Occurs

When materials with hysteresis damping are cycled, the stress-strain line upon loading does not coincide with the line upon unloading. Figure 3 of the text is a sample of how a hysteresis loop of such a material would look when plotted. The area enclosed inside the loop is the damping or heat energy that is dissipated during cycling. Obviously, the loop would close into a straight line if the material were free of damping. In a resonating corner of a platen, every molecule that is being compressed and pulled is dissipating heat energy. It is this energy that is of interest to the vibration engineer if different designs are to be considered.

Column Ψ of table 1 in the main text of this report contains the ratio of vibrational strain energy of one cycle to that energy at the beginning of the cycle. This is called the specific damping capacity. If one could determine the strain energy U in a corner of a platen that has deflected from a neutral-through-one complete cycle and back to neutral again and multiplied it by Ψ, one would have the total damping energy D in the units of inch-pounds per cycle. This is the standard measurement used by vibration engineers to compare designs. It can also be measured on structures already built in a vibration laboratory.[A-4]

Energy Method of Strain Analysis

As cited by Popov, the equation for determining the elastic strain energy in a beam which is being deflected from its neutral axis to a stress level σ (or 25% of a vibration cycle) is:

$$U_i = \int \frac{\sigma^2}{2E}\, dV \qquad \text{(A-37)}$$

Since a vibrating beam will return to its neutral axis and then deflect the opposite direction, the total energy U that is available for dissipation is four times that of a quarter cycle.

$$U = 4\, U_i$$

$$U = \int 2\frac{\sigma^2}{E}\, dV \qquad \text{(A-38)}$$

[A-4] Cyril M. Harris and Charles E. Crede, Shock and Vibration Handbook, second edition, McGraw-Hill, New York, 1976.

where

V = total volume of material being strained, in.^{3}

σ = bending stress, lb/in.^{2}

The total damping energy D then would be:

$$D = \Psi U = \Psi \int 2\frac{\sigma^2}{E}\, dV \tag{A-39}$$

The derivative of the volume dV is the product of the derivatives of:

dx = measured from the corner of the platen to the line of bending (fig. A-2)

dz = the direction 90 degrees from dx and parallel to the line of bending

dy = the up-and-down direction from the neutral axis

Therefore

$$dV = (dx)(dy)(dz)$$

However, it has already been determined that:

$$dz = 2dx$$

then

$$dV = 2dx^2dy \tag{A-40}$$

Finally, as shown by Popov, the formula for bending stress is:

$$\sigma = \frac{My}{I} \tag{A-41}$$

By substituting equations A-40 and A-41 into equation A-39 and simplifying a generalized triple integral equation for a vibrating platen corner:

$$D = \frac{4\psi}{E} \iiint \left(\frac{My}{I}\right)^2 dx^2dy \tag{A-42}$$

This equation can be used to calculate the total damping energy of all three types of platens discussed in the text of the report.

Single Plate Platen

By substituting the value of I from equation A-15 and the value of M from equation A-30 into equation A-42, the damping equation for the single plate platen can be obtained:

$$D = \frac{4\Psi}{E} \iiint \frac{\left(\frac{1}{6}\right)^2 Z^2 x^6 y^2}{V_s^{\,2}\, x^2} dx^2 dy$$

The solution results from integrating the variable y from the neutral axis to ±h/2, then x from zero to x and from zero to C.

$$D = \frac{\Psi Z^2}{9 V_s^{\,2} E} \int_o^C \int_o^x \int_{-\frac{h}{2}}^{\frac{h}{2}} x^4 y^2 dx^2 dy \tag{A-43}$$

$$D = \frac{h^3 \Psi Z^2}{216\ V_s^{\,2}\ E} \int_o^C \int_o^x x^4 dx^2 - \frac{-\ h^3 \Psi Z^2}{216\ V_s^{\,2}\ E} \int_o^C \int_o^x x^4 dx^2$$

$$D = 2\ \frac{h^3 \Psi Z^2}{1080\ V_s^{\,2}\ E} \int_o^C X^5 dx$$

$$D = 2\ \frac{h^3 \Psi Z^2 C^6}{6480\ V_s^{\,2}\ E} = \frac{h^3 \Psi Z^2 C^6}{3240\ V_s^{\,2}\ E} \tag{A-44}$$

It is this equation that appears as equation 4 in the main text of the report with the values of Z and V_s from this appendix substituted into it.

The Three Plate, Two-Material Platen

This platen is handled by a three-part equation:

$$D = D_I + D_O - D_{IO}$$

where

D_I = the damping of the inner plate with its specified material

D_O = the damping of the whole platen, assuming it was made of only the outer specified material

D_{IO} = the damping of the inner plate if it was made of the outer material

By referring to figure A-2d, one can see the source of the dimensions for the following equation (taken from A-43) and the value of the moment of inertia from equation A-20:

$$D = \frac{h_1{}^3\Psi_1 \; Z^2C^6}{3240 \; V_{HG}{}^2 \; E_1} + \frac{\left(\frac{h_1}{2} + h_2\right)^3 \Psi_2 \; Z^2C^6}{405 \; V_{HG}{}^2 \; E_2} - \frac{h_1{}^3\Psi_2 \; Z^2C^6}{3240 \; V_{HG}{}^2 \; E_2}$$

$$D = \frac{Z^2C^6}{405 \; V_{HG}{}^2}\left(\frac{h_1{}^3\Psi_1}{8E_1} + \frac{\left(\frac{h_1}{2} + h_2\right)^3 \Psi_2}{E_2} - \frac{h_1{}^3\Psi_2}{8E_2}\right) \qquad \text{(A-45)}$$

It is this equation that appears as equation 5 in the main text of the report with the values of Z and $V_{HG}{}^2$ from the appendix substituted into it.

The Two Plate, Two-Material Platen

Figures A-3d and A-3e illustrate not only the dimensions of a two plate, two material platen, but also the complexity for problem solution. For example, the limits of integration in equation A-43 for the y direction are from the bottom of the plate -h/2 to the top of the plate h/2. This same integration procedure was applicable for equation A-45. This procedure cannot be used for this problem because of its unsymmetrical geometry. This will require a four-part equation:

$$D = D_{UI} + D_{LI} + D_{LO} - D_{LIO}$$

where

D_{UI} = the damping of the less dense material above the neutral axis

D_{LI} = the damping of the less dense material below the neutral axis

D_{LO} = the damping of the more dense material assuming it was used entirely below the neutral axis

D_{LIO} = the damping of the more dense material assuming it was used from the neutral axis to a distance h_1-N below the neutral axis

Equation A-43 will be the basis for calculating the damping of this system, except the constant in the moment of inertial formula is V_{HN} from equation A-24:

$$D = \frac{\Psi_1\ Z^2}{9V_{HN}^{\ 2}\ E_1}\left[\int_0^C\int_0^x\int_0^N x^4y^2dx^2dy + \int_0^C\int_0^x\int_0^{(h_1-N)} x^4y^2dx^2dy\right]$$

$$+ \frac{\Psi_2\ Z^2}{9V_{HN}^{\ 2}\ E_2}\left[\int_0^C\int_0^x\int_0^{h_2+(h_1-N)} x^4y^2dx^2dy - \int_0^C\int_0^x\int_0^{(h_1-N)} x^4y^2dx^2dy\right]$$

$$D = \frac{\Psi_1\ Z^2}{27V_{HN}^{\ 2}\ E_1}\left[N^3\int_0^C\int_0^x x^4dx^2 + (h_1-N)^3\int_0^C\int_0^x x^4dx^2\right]$$

$$+ \frac{\Psi_2\ Z^2}{27V_{HN}^{\ 2}\ E_2}\left[(h_2+h_1-N)^3\int_0^C\int_0^x x^4dx^2 - (h_1-N)^3\int_0^C\int_0^x x^4dx^2\right]$$

$$D = \frac{\Psi_1 Z^2 C^6}{810V_{HN}^{\ 2}\ E_1}\left[N^3 + (h_1-N)^3\right] + \frac{\Psi_2 Z^2 C^6}{810V_{HN}^{\ 2}\ E_2}\left[(h_2+h_1-N)^3 - (h_1-N)^3\right]$$

$$D = \frac{Z^2\ C^6}{810\ V_{HN}^{\ 2}}\left\{\frac{\Psi_1}{E_1}\left[N^3 + (h_1-N)^3\right] + \frac{\Psi_2}{E_2}\left[(h_2+h_1-N)^3 - (h_1-N)^3\right]\right\} \qquad \text{(A-46)}$$

This equation, with the values of Z, V_{HN}, and N substituted into it is found in the body of the report as equation 6.

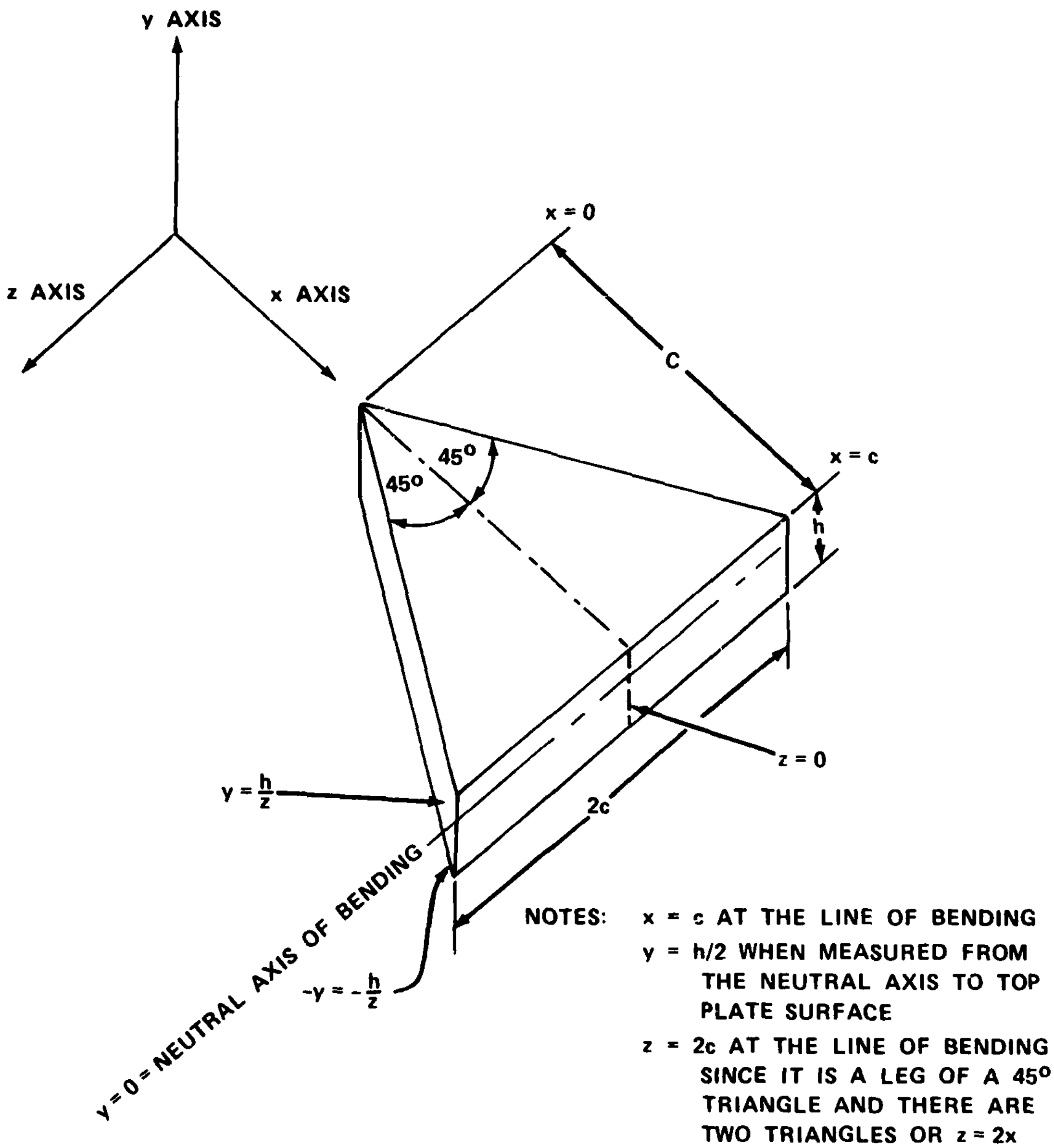

Figure A-1. Illustration of triangular plate to be used to calculate the cross-sectional area at the edge of bending

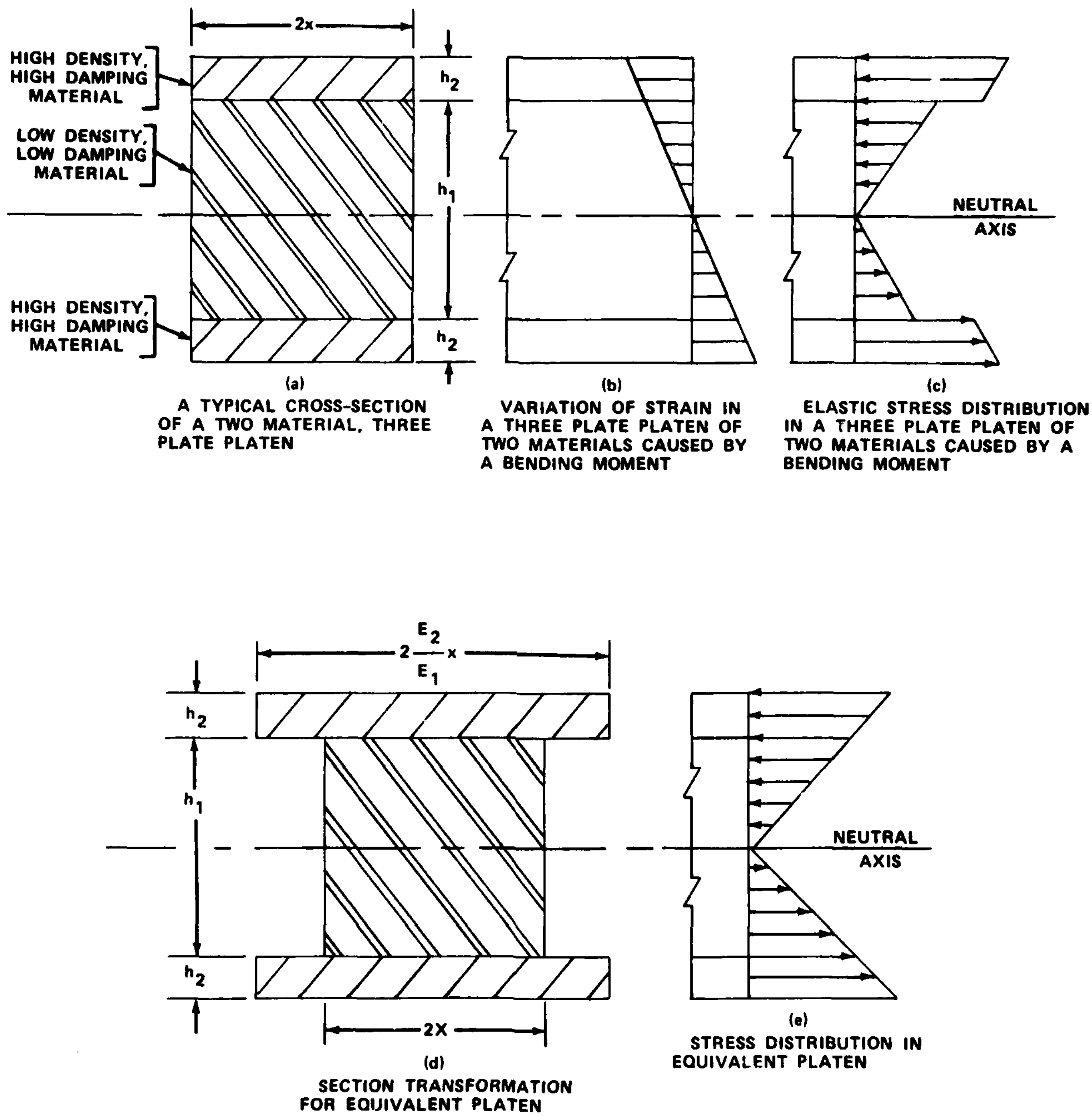

Figure A-2. Sectional transformation of a two material, three plate platen

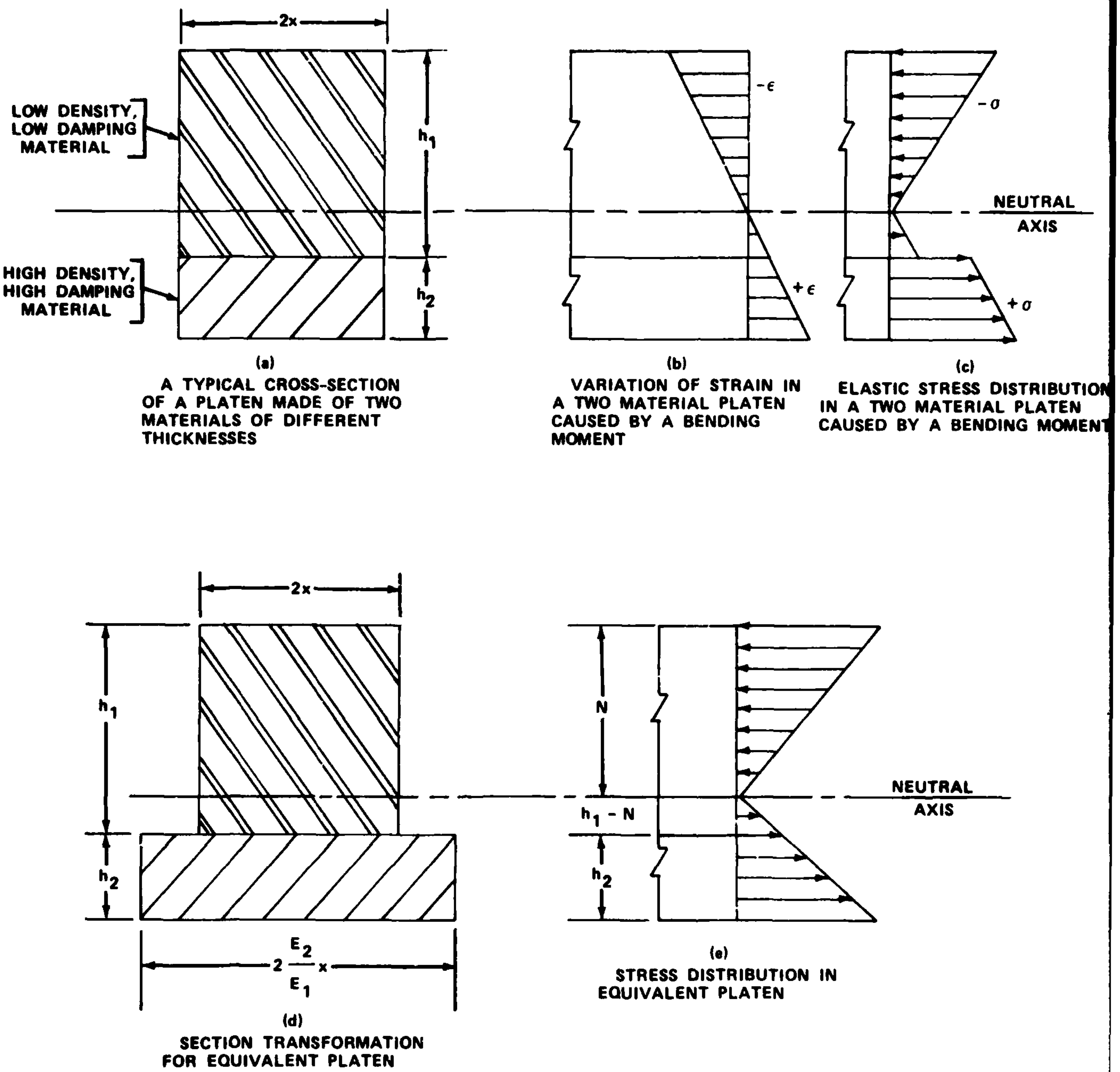

Figure A-3. Sectional transformation of a two material, two plate platen

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