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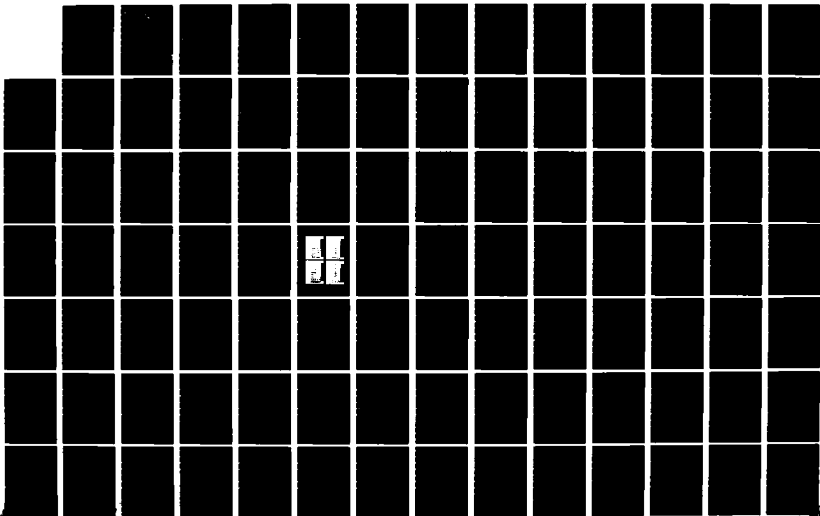

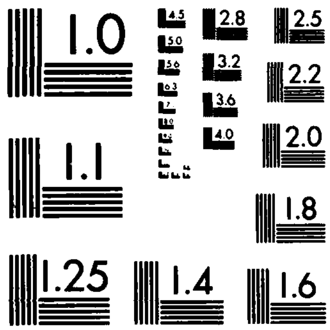

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THE USEFULNESS OF IMPACT DAMPERS FOR SPACE APPLICATIONS

THESIS

AFIT/GA/AA/83M-2

Bruce W. Gibson
1st Lt USAF

DEPARTMENT OF THE AIR FORCE
AIR UNIVERSITY (ATC)
AIR FORCE INSTITUTE OF TECHNOLOGY

Wright-Patterson Air Force Base, Ohio

AFIT/GA/AA/83M-2

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THE USEFULNESS OF IMPACT DAMPERS
FOR SPACE APPLICATIONS

THESIS

Presented to the Faculty of the School of Engineering
of the Air Force Institute of Technology
in Partial Fulfillment of the
Requirements for the Degree of
Master of Science

by

Bruce W. Gibson, B.S.
1st Lt USAF

Graduate Astronautical Engineering

A-1

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Preface

The potential usefulness of the impact damper for space applications was first proposed by Dr. Peter Torvik, who suggested a study of it as a thesis topic. This topic was of interest to me not only as an exercise in basic dynamics and mechanics and as an opportunity to acquire laboratory experience, but also as an opportunity to do basic research in a promising field of vibration control that is not common knowledge.

I would like to thank Dr. Torvik both for the freedom he allowed me in this project, and for his knowledgeable advice, which helped me overcome many frustrating stumbling blocks. Thanks are also due to Captain Wesley Cox, for his assistance with the laboratory equipment, and to Captain Patricia Lawlis, who helped me through the tedious process of learning the UNIX computer operating system. Last, I would like to thank Linda Stoddart of the AFIT Library, who did an outstanding job of obtaining reference material spanning fifty years, often from private laboratories or journals published in Europe, Russia, and Japan.

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Nomenclature

c	-- Viscous damping coefficient for $\dot{\theta}$ of the primary system.
c_m	-- Viscous damping coefficient for $\dot{x}$ of the impacting mass.
c_p	-- Specific heat.
C_s	-- Constant factor in the equation relating s to the system parameters.
c_α	-- Viscous damping coefficient for $\dot{\alpha}$ of the impacting mass; $c_\alpha \cong c_m L$.
C_θ	-- Constant factor in the equation relating θ_{m_r} to the system parameters.
d	-- Total horizontal distance the impacting mass can travel; referred to as the effective gap.
d_s	-- Length of the flex plate.
e	-- Coefficient of restitution.
E	-- Modulus of elasticity of steel.
ΔE	-- Change in total energy.
g	-- Acceleration of gravity.
Δh	-- Change in height.
H_0	-- Magnitude of the total angular momentum about point 0 of both the primary system and the impacting mass.
H_{0_b}	-- Magnitude of the angular momentum of the primary system about point 0.
H_{0_m}	-- Magnitude of the angular momentum of the impacting mass about point 0.

i	--	Number of impacts.
I	--	Moment of inertia of the primary system about its rotation point.
I'	--	Moment of inertia of the cross-sectional area.
I_m	--	Moment of inertia of the impacting mass about the rotation point of the primary system.
I'_B	--	Moment of inertia of the cross-sectional area of the beam in the forced vibration laboratory model.
I'_s	--	Moment of inertia of the cross-sectional area of the flex plate used in the free vibration laboratory model.
k	--	Stiffness constant resisting the angular displacement of the primary system.
k_m	--	Stiffness constant resisting an x displacement of the impacting mass.
L	--	Magnitude of the moment arm used to calculate the angular momentum changes (for the primary system and the impacting mass) due to an impact.
L_b	--	Length of the beam used in the forced vibration laboratory model.
L_m	--	Distance from the rotation point of the impacting mass to the impacting mass.
m	--	Mass of the secondary, or impacting, mass.
M	--	Moment in a beam.
M_s	--	Moment applied to the primary system by the flex plate.
M_T	--	Total mass of the primary system.
O	--	Point about which the primary system rotates.
q	--	Damped frequency of the primary system.
q_m	--	Damped frequency of the impacting mass.

r -- Maximum amplitude of the primary system at time zero.

r_{0m} -- Distance from the point 0 to the center of mass of the impacting mass.

$\bar{r}_{0m}$ -- Vector from the point 0 to the center of mass of the impacting mass.

r_s -- Distance from the point 0 to the point where the flex plate is attached to support the primary system in the free vibration laboratory model.

s -- Constant time rate of change of θ_{m_j} referred to as the damper efficiency.

t -- Time.

t_i -- Time of the ith impact.

$t_i^{(+)}$ -- Time immediately after the ith impact.

$t_i^{(-)}$ -- Time immediately before the ith impact.

t_j -- Time during the jth cycle when the maximum amplitude is reached.

Δt -- Time since the last impact, $\Delta t = t - t_i$.

$\Delta T°$ -- Change in temperature.

ΔT_{max} -- Change in the maximum kinetic energy.

u -- Distance from the center of the flex plate to the surface.

V -- Velocity.

V_m -- Velocity of the impacting mass.

$\bar{V}_m$ -- Vector velocity of the impacting mass.

W_T -- Total weight of the primary system.

x -- Time dependent horizontal position of the impacting mass.

$\dot{x}$ -- Time rate of change of x.

y -- Coordinate along the length of the flex plate.

Greek Characters

α -- Angle between the strings suspending the impacting mass and the vertical. See Figure 24.

δ -- Logarithmic decrement.

ε -- Strain on the surface of the flex plate.

ε_m -- Measured strain on the surface of the flex plate.

ζ -- Viscous damping factor.

θ -- The angular displacement of the centerline of the flex plate from the vertical. Along the flex plate θ is a function of y. At $y = d$ θ is the angular displacement of the primary system from the vertical. See Figure 24.

θ_{m_j} -- Maximum amplitude of the primary system during cycle j.

θ_{m_0} -- Initial maximum amplitude of the primary system.

θ_{m_r} -- Amplitude of the primary system at which the impact damper becomes ineffective, referred to as the residual amplitude.

$\dot{\theta}$ -- Derivative of θ with respect to time.

$\dot{\theta}_{f_{max}}$ -- Final maximum angular velocity of the primary system.

$\dot{\theta}_{0_{max}}$ -- Initial maximum angular velocity of the primary system.

ρ -- Mass per unit length of the beam in the forced vibration laboratory model.

ω_F -- Frequency at which the forced vibration laboratory model is excited.

ω_n -- Natural frequency of the primary system.

Abstract

The usefulness of the impact damper in eliminating vibrations is studied analytically and experimentally. Laboratory models of vibrating systems are constructed to evaluate the performance of the impact damper in reducing or eliminating forced and free vibrations. A computer simulation of a single degree-of-freedom primary system in free vibration employing an impact damper is constructed for the same purpose. Laboratory free vibration results are compared to the computer simulation in order to judge its accuracy.

The computer simulation is employed to determine the impact damper's performance in free vibration as the system's parameters are varied. Two significant measures of the damper's effectiveness are obtained as approximate functions of the system's parameters.

Observations regarding reduction in amplitude and steady state motion were made for the impact damper in forced vibration.

THE USEFULNESS OF IMPACT DAMPERS FOR SPACE APPLICATIONS

I. Introduction

An impact damper, also referred to in the literature as a rattle damper or an acceleration damper, is a simple, passive damping device. It operates by allowing a vibrating primary mass to go through a series of collisions with a secondary mass carried in or on the primary mass. Figure 1 shows one of the simplest models of an impact damper, with a primary mass M free to travel in one dimension only, acted upon by a forcing function F(t), a secondary mass m, a spring of stiffness k, and a dashpot with a damping constant c.

In the simplest case, the motion of the secondary, or impacting, mass m is assumed to be a result of collisions with the primary mass alone, so the impacting mass has a constant velocity between impacts. If F(t) is sinusoidal, then the momentum exchange and the energy dissipation resulting from the impacts usually results in a decrease in the amplitude of motion of the primary mass. If the primary mass is in free vibration (F(t) = 0), then the impacts cause a more rapid decay in the amplitude of the

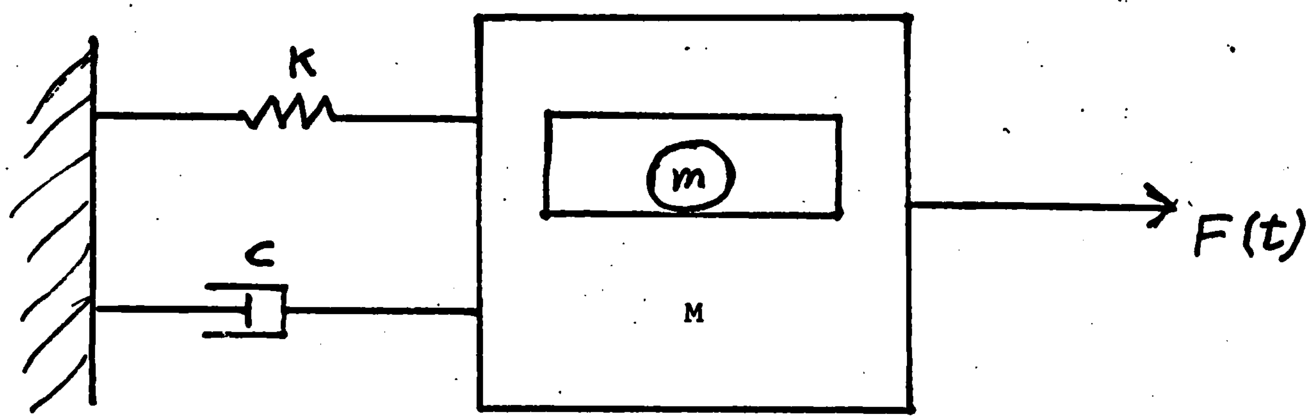


Fig. 1. Simple Impact Damper. The impacting mass m reduces the response of the primary mass M to the forcing function F(t) by impacts with the primary mass.

motion of the primary mass through energy dissipation and momentum exchange.

This simple damper could be of practical use for space applications in eliminating the unwanted vibrations of antennas, telescopes, or any other flexible structure which tends to oscillate about its intended orientation. In the near vacuum of space, external damping forces are essentially zero. Thus, such structures must have internal damping designed into them. If the impact damper provides sufficient, reliable damping without adding prohibitive mass to the total payload, it could be a solution to some oscillation problems.

An extensive literature search turned up much work on the effectiveness of the impact damper in reducing forced vibrations. Contradictory conclusions were identified. Paget (Ref 1) probably did the earliest writing on the impact damper, but the first serious analytical work appears to be that of Lieber and Jenson (Ref 2). In their paper, work and energy considerations were used to solve for the one degree of freedom motion of a primary mass undergoing perfectly inelastic collisions with a secondary mass. These results were used to calculate a damping factor which was verified experimentally through comparison with the damping observed in the free vibration of a beam with an impact damper attached. Their solution predicted the impact damper would be most efficient (do the most work per cycle) if two impacts per cycle occurred with impacts equally spaced in time.

Grubin and Lieber (Ref 3) gave a more straightforward solution of the motion of the system for collisions ranging from perfectly inelastic to perfectly elastic. In Reference (4), it is shown that solutions are possible when stable and symmetric motion is assumed; i.e., that two impacts occur at equal time intervals during the cycle. This is referred to as symmetric, two impact per cycle motion. Such motion has often been assumed, and in Reference (5) was reported to occur when an impact damper was attached to a cantilever beam in forced vibration. Grubin and Lieber (Ref 6) went on to do a stability analysis on this symmetric, two impact per cycle motion. Lieber and Duffy (Ref 7) modeled a cantilever beam with an impact damper as a system composed of four lumped masses and used an electric analog model of this system to study the effects of parametric changes on the dampers' performance.

Feygin (Ref 8) solved and did a stability analysis for the motion of an impact damper similar to that shown in Figure 1, but with the motion of the impacting mass between impacts subjected to dry friction. Masri (Ref 9) started with the assumption of symmetric, two impact per cycle operation and solved for the motion of the system under sinusoidal excitation. He also did a stability analysis to show this motion did exist for a wide range of system parameters, and verified his results experimentally and with both digital and analog computer simulations.

Masri (Ref 10) solved for the motion of a forced system with any number of impacts per cycle. Sadek (Ref 11) assumed two impacts per cycle and used a Fourier series representation of the impacting forces to come to the conclusion that, in general, symmetric impacts do not occur, especially for system parameters leading to the maximum reduction in amplitude of the primary system. He used a laboratory model to verify his results.

Sadek and Mills (Ref 12) solved for the motion of the system in forced vibration with the impact damper affected by gravity, while Sadek and Williams (Ref 13) then provided a stability analysis on these results. Sadek and Thomas (Ref 14) solved for the motion of a system in forced vibration and with the secondary mass attached to a spring and influenced by gravity.

Masri and Sadek have both published several papers on impact dampers with carefully solved equations of motion and stability analyses. The only significant difference in their approach is that Masri, and many other authors, modeled the impacts as being of infinitely short duration, thereby causing a discontinuous change in the velocities, but not the positions, of the two masses. Sadek uses a Fourier series representation of the impact force and treats impacts as being of short, but finite duration. Masri's (Ref 9) solution for the sinusoidally forced system shows symmetric, two impact per cycle motion to be possible for a wide range of system parameters, and that this kind of system gives the maximum reduction in the primary system's amplitude. Sadek's (Ref 11) solution is for a system at a specified ratio of secondary mass to primary mass and at a specified forcing frequency. This solution gives only one value for the gap in which the secondary mass travels which will give symmetry, and this gap does not give maximum reduction in amplitude. Masri (Ref 10) also predicts that the amplitude decreases with an increase in the ratio of the secondary mass to the primary mass, while Sadek (Ref 11) says that there is an optimum value for this ratio, and that increasing it beyond this point increases the system's amplitude.

Roy, Rocke, and Foster (Ref 15) did an analytical and experimental study of the impact damper in the center of a beam in bending vibration, using both a simply supported beam and a beam with both ends clamped. They used both a closed form solution for the motion of the beam between impacts and a discrete mass model of the system to do numerical calculations of the motion of the beam. These numerical solutions were verified with experimental results. All previous researchers did the analytical work assuming a rigid bodied, single degree of freedom primary system.

Dokainish and Elmaraghy (Ref 16) did a computer simulation of an impact damper and produced a series of curves from which damper performance can be predicted for a given set of system parameters. Yamada (Ref 17) solved for the motion of a sinusoidally excited impact damper similar to Figure 1, but with a piecewise linear spring. Other solutions to the forced motion of different impact dampers can be found in References (18), (19), (20), (21), and (22).

Yasuda and Toyoda (Ref 23), considered the usefulness of an impact damper in reducing the free vibration of a lightly damped system. They used experimental results to obtain parametric relations which could be used to solve for the damping.

The purpose of this thesis is to examine the usefulness of the impact damper in reducing both forced and free vibrations. A laboratory model and a computer model of a freely vibrating system with an impact damper were constructed. These models were used to determine damper performance as coefficient of restitution e, mass of impacting mass m, distance between impacting surfaces d, and other parameters were varied. The forced vibration case was examined using a laboratory model consisting of an upright flexible beam with an impact damper on top and a sinusoidal angular displacement applied to the bottom.

II. Analytical Studies

The motion of the free vibration and forced vibration impact damper is considered in this chapter. The analysis is for the types of impact dampers depicted in Figures 2 and 3, which are the types used in the laboratory studies. The motion of the free vibration impact damper of Figure 2 can be followed analytically through any number of impacts. The solution to the motion of the forced vibration impact damper of Figure 3 is not completely described in this thesis, but some useful information is obtained. The equations given in this chapter are derived in detail in Appendix A.

Free Vibration Impact Damper

The primary system of the free vibration impact damper of Figure 2 is the damper assembly, which provides the impacting surfaces, and the beam. The impacting mass is not considered part of the primary system. The angular displacement θ of the primary system can be described as a rotation about an axis perpendicular to the plane of Figure 2 and containing point 0, called the rotation point. Between impacts, the primary system is acted upon by the flex plate, gravity, and viscous damping. The equation

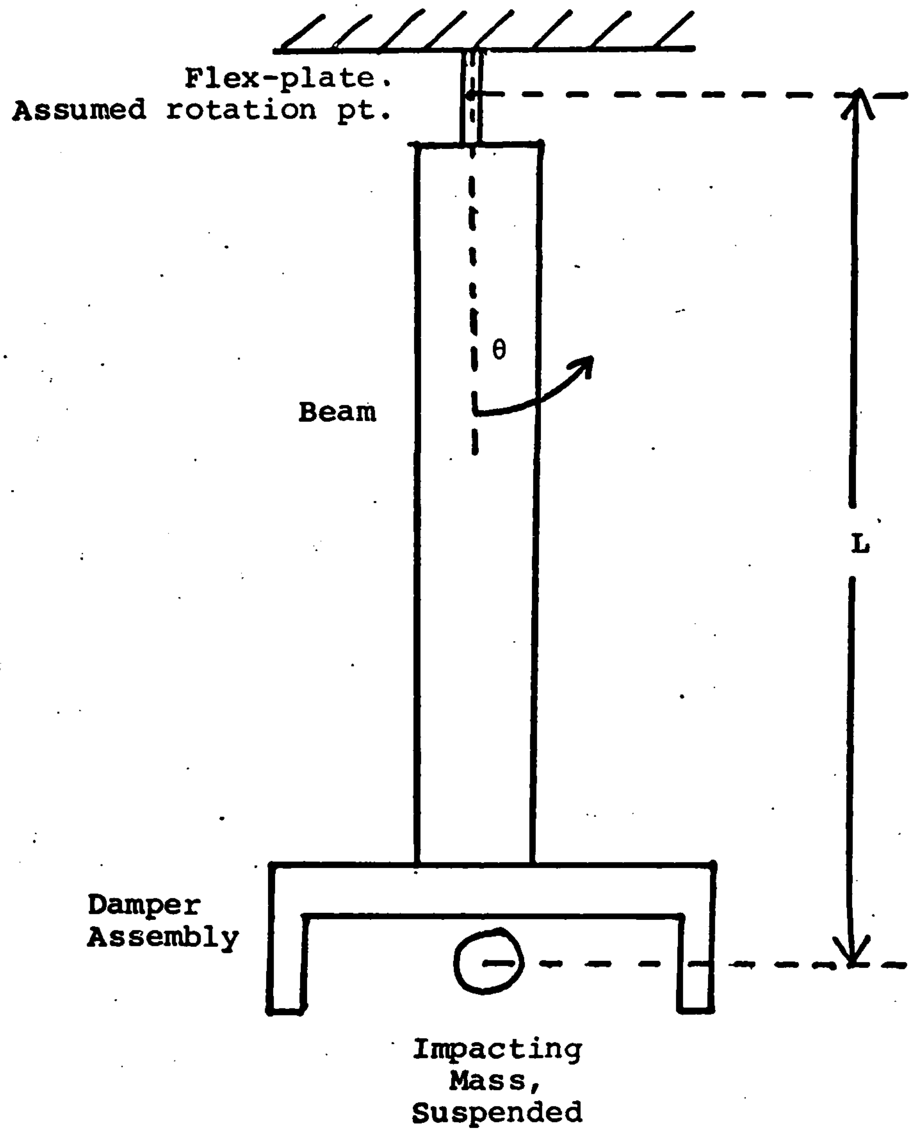


Fig. 2. Free Vibration Impact Damper

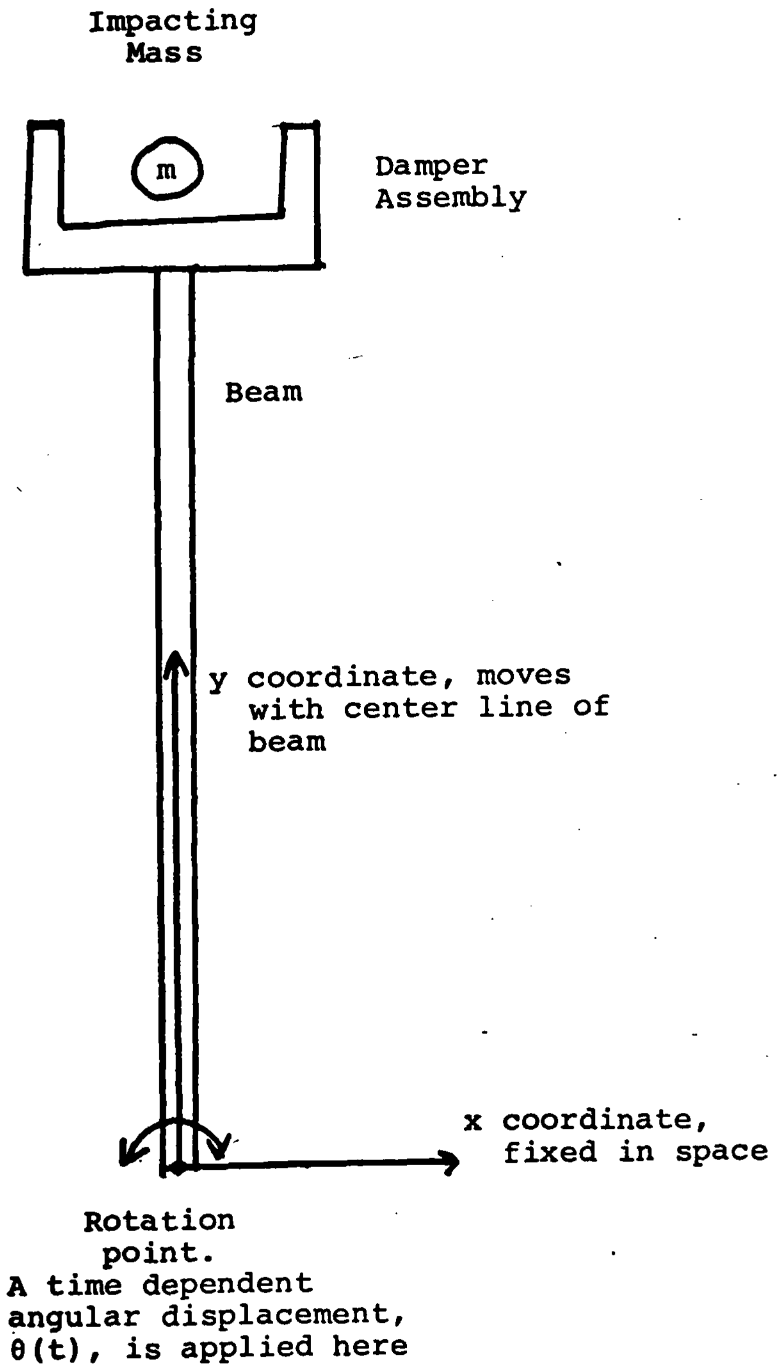


Fig. 3. Forced Vibration Model

of motion of the primary system between impacts is, for small values of θ:

$$I\ddot{\theta} + c\dot{\theta} + \left(W_T r_{0m} + \frac{EI'_s}{d_s}\right)\theta = 0 \tag{1}$$

where:

- I = moment of inertia of the primary system about 0, with units of mass · length^2;
- c = viscous damping constant, with units of force · length · time;
- W_T = total weight of the primary system;
- r_{0m} = distance from 0 to the center of mass of the primary system;
- E = modulus of elasticity of the flex plate material;
- I'_s = moment of inertia of the cross-sectional area of the flex plate, with units of length^4; and
- d_s = length of the flex plate.

This is more conveniently used in the form:

$$I\ddot{\theta} + c\dot{\theta} + k\theta = 0 \tag{2}$$

where:

$$k = W_T r_{0m} + \frac{EI'_s}{d_s}$$

and k has units of force · length. The motion of the primary system between impacts is described by Equation (1) if the rotation point 0 is stationary. This is shown to be approximately true in Appendix A, where point 0 is shown to be located where the center of the flex plate is when it is undeformed.

Solving Equation (2) for θ between impacts i and i+1 gives:

$$\theta(\Delta t) = e^{\left(-\frac{c}{2I}\Delta t\right)} \left[\left(\frac{\dot{\theta}(t_i^{(+)})}{q} + \frac{\theta(t_i^{(+)})c}{2Iq}\right) \sin q\Delta t + \theta(t_i^{(+)}) \cos q\Delta t\right] \quad (3)$$

where Δt is the time since impact i, $\theta(t_i^{(+)})$ and $\dot{\theta}(t_i^{(+)})$ are the angular position and angular velocity after impact i, and

$$q = \sqrt{\frac{k}{I} - \left(\frac{c}{2I}\right)^2}$$

If there is no viscous damping (c=0) then:

$$\theta(\Delta t) = \frac{\dot{\theta}(t_i^{(+)})}{q} \sin q\Delta t + \theta(t_i^{(+)}) \cos q\Delta t \quad (4)$$

and

$$q = \sqrt{\frac{k}{I}}$$

The equation of motion for the suspended impacting mass between collisions is:

$$mL_m^2\ddot{\alpha} + c_\alpha\dot{\alpha} + mgL_m\alpha = 0 \quad (5)$$

where:

m = mass of impacting mass;

L_m = distance of the impacting mass from its rotation point;

α = angular displacement of the strings suspending the impacting mass from the vertical;

c_α = viscous damping constant, with units of force · length · time; and

g = acceleration of gravity.

For small values of α, this motion is almost entirely in the horizontal direction or x direction. Therefore, Equation (3) can be approximated as:

$$m\ddot{x} + c_m\dot{x} + k_m x = 0 \tag{6}$$

where:

$x \cong L_m \alpha$;

$c_m = c_\alpha / L_m^2$; and

$k_m = mg/L_m$.

c_m has units of force · time/length and k_m has units of force/length.

Solving Equation (6) for x between impacts i and i+1 gives:

$$x(\Delta t) = e^{\left(-\frac{c_m}{2m}\Delta t\right)}\left[\left(\frac{\dot{x}(t_i^{(+)})}{q_m} + \frac{x(t_i^{(+)})c_m}{2mq_m}\right)\sin q_m\Delta t + x(t_i^{(+)})\cos q_m\Delta t\right] \tag{7}$$

where $x(t_i^{(+)})$ and $\dot{x}(t_i^{(+)})$ are the position and velocity immediately after impact i, and:

$$q_m = \sqrt{\frac{k_m}{m} - \left(\frac{c_m}{2m}\right)^2}$$

If there is no viscous damping, then:

$$x(\Delta t) = \frac{\dot{x}(t_i^{(+)})}{q_m} \sin q_m \Delta t + x(t_i^{(+)}) \cos q_m \Delta t \qquad (8)$$

and:

$$q_m = \sqrt{\frac{k_m}{m}}$$

Finally, if there is no viscous damping or gravity effects, the impacting mass has constant velocity between impacts, so:

$$x(\Delta t) = x(t_i^{(+)}) + \dot{x}(t_i^{(+)})\Delta t \qquad (9)$$

The initial conditions used in the solutions to Equations (2) and (6) for θ and x during the motion between impact i and impact i+1 are θ, $\dot{\theta}$, x, and $\dot{x}$ evaluated at time $t_i^{(+)}$ (immediately after impact i). If impacts are assumed to be of infinitely short duration (during which θ and x remain unchanged, and $\dot{\theta}$ and $\dot{x}$ change discontinuously), then the initial conditions can be obtained in terms of θ, $\dot{\theta}$, x, and $\dot{x}$ evaluated at time $t_i^{(-)}$ (immediately berore impact i). The positions are given by:

$$\theta(t_i^{(+)}) = \theta(t_i^{(-)}) \tag{10}$$

$$x(t_i^{(+)}) = x(t_i^{(-)}) \tag{11}$$

Angular momentum is conserved across the impact, so:

$$I\dot{\theta}(t_i^{(-)}) + m\dot{x}(t_i^{(-)})L = I\dot{\theta}(t_i^{(+)}) + m\dot{x}(t_i^{(+)})L$$

Also, the velocities across an impact are related by:

$$e[\dot{x}(t_i^{(-)}) - \dot{\theta}(t_i^{(-)})L] = \dot{\theta}(t_i^{(+)})L - \dot{x}(t_i^{(+)})$$

where e is the coefficient of restitution. These two relations give:

$$\dot{x}(t_i^{(+)}) = \frac{L}{I+mL^2}[\dot{\theta}(t_i^{(-)})I(1+e) + \dot{x}(t_i^{(-)})(mL - \frac{Ie}{L})] \tag{12}$$

$$\dot{\theta}(t_i^{(+)}) = \frac{1}{I+mL^2}[\dot{\theta}(t_i^{(-)})(I-mL^2e) + \dot{x}(t_i^{(-)})mL(I+e)] \tag{13}$$

Using Equations (10) to (13), the position and velocity of both the beam and impacting mass are obtained at time $t_i^{(+)}$ (immediately after the ith impact) in terms of the positions and velocities at time $t_i^{(-)}$ (immediately before the ith impact). If the overall system is started with known initial conditions at time $t_0=0$, and if the times of impacts t_1, t_2, t_3... are known, the exact solutions up to time $t_1^{(-)}$ can be solved in terms of the initial conditions at t_0. The initial conditions at $t_1^{(+)}$ can then be solved in terms of the final conditions at $t_1^{(-)}$; these initial conditions can be used to solve for the exact solutions from $t_1^{(+)}$ to $t_2^{(-)}$. Initial conditions and exact solutions can then be obtained from time $t_2^{(+)}$ to $t_3^{(-)}$, and this process can be continued for as many impacts as is desired.

This process assumes that all impact times t_i are known. Actually, *impact time* t_i *must be found* by iteration, using the known solutions after time $t_{i-1}^{(+)}$ and the requirement that the suspended mass must remain between the two stops. The time t_i is then defined as the time when the impacting mass first comes in contact with either stop after time $t_{i-1}^{(+)}$.

This solution technique was used in the two computer programs of Appendix C. The first program, IDEAL, uses Equations (4) and (9) to trace the motion of the primary system and impacting mass in an environment with no viscous damping or gravity. The second program, LABSIM, uses equations (3) and (7) to include viscous damping and gravity effects on the system. Included in Appendix C are the results obtained by running programs IDEAL and LABSIM.

Program IDEAL was run for a variety of values for M, L, e, q, and d, where d is the distance between the impacting surfaces. The results of one of these runs is given in Figure 4. Whenever m, L, and q are greater than zero, e is positive and less than one, and d is positive and less than $2\ \theta_{max}L$ ($d < 2\ \theta_{max}L$ is necessary for impacts to occur), the impact damper will initially bring about a rapid reduction in amplitude. After the maximum amplitude attained by the primary system during each cycle declines to a certain value, the impact damper becomes much less effective, with a much slower reduction in amplitude of the primary system. In the region where the damper is effective, the maximum amplitude attained is observed to decrease approximately linearly with time, and a linear function is fit to the peaks using a least squares method. The decline in maximum amplitude with time is denoted by s, and is a measure of the impact damper's performance. The maximum amplitude attained when the impact damper becomes almost ineffective is denoted θ_{m_r}, and is also a measure of damper performance.

It is important to determine if s and θ_{m_r} are dependent upon parameters other than M, L, e, q, and d. All of the computer simulations of the impact damper began with the primary system displaced in the negative direction 0.1

initial phase angle= .000000000e+00
initial maximum deflection= -.100000001e+00
natural frequency= .250000000e+02 the starting time= .000000000e+00
moment of inertia of the primary mass= .109300001e+00
magnitude of the secondary mass= .200000009e-02 length of the primary system=
2.23000002 the coefficient of restitution= .500000000e+00
secondary masses initial velocity=
.000000000e+00 the gap setting= .250000000e+00
desired number of impacts= 50

mass= .200000009e-02
effective d= .250000000e+00
e= .500000000e+00
x(0)= -.980000049e-01
deltat= .999999978e-02

the time iteration did not converge for i= 45

impact	time	deltat	thetas	x	tdotv	xdot	thetam	dthetam
1	.06769	.067687	.0121	-.098000	2.2958	7.8866	.0926	-.0074
2	.12207	.054383	.0923	.330895	.4363	-2.7636	.0940	.0013
3	.18766	.065595	.0110	.149614	-2.2510	-6.2389	.0907	-.0033
4	.26345	.075782	-.0889	-.323185	.2110	4.0967	.0893	-.0014
5	.41144	.147998	.0709	.283116	-1.1169	-6.0511	.0838	-.0055
6	.50952	.098080	-.0831	-.310375	-.4493	1.7270	.0850	.0012
7	.55843	.048905	-.0453	-.225918	1.7234	4.9869	.0825	-.0026
8	.66214	.103713	.0746	.291290	-.6468	-4.9173	.0789	-.0035
9	.78197	.119824	-.0775	-.297915	.1760	3.2623	.0779	-.0011
10	.86711	.085146	.0470	-.020143	1.5448	3.5436	.0776	-.0002
11	.93917	.072054	.0494	.235188	-1.2663	-6.2654	.0708	-.0069
12	1.02067	.081503	-.0675	-.275461	-.7037	.9689	.0731	.0023
13	1.07296	.052289	-.0448	-.224004	1.3695	4.1810	.0707	-.0024
14	1.18756	.114606	.0580	.254357	-.7959	-4.9939	.0662	-.0046
15	1.29308	.105511	-.0662	-.272560	-.1705	2.1134	.0665	.0004

impact= 1 errmom= .186264515e-07 errvel= .238418579e-06
impact= 2 errmom= .745058060e-08 errvel= -.238418579e-06
impact= 3 errmom= .745058060e-08 errvel= -.953674316e-06
impact= 4 errmom= .000000000e+00 errvel= .476837158e-06
impact= 5 errmom= -.111758709e-07 errvel= -.238418579e-06
impact= 6 errmom= -.372529030e-08 errvel= .238418579e-06
impact= 7 errmom= .298023224e-07 errvel= -.596046448e-06
impact= 8 errmom= -.745058060e-08 errvel= -.238418579e-06
impact= 9 errmom= .372529030e-08 errvel= .238418579e-06
impact= 10 errmom= .166473910e-07 errvel= -.119209290e-06
impact= 11 errmom= -.298023224e-07 errvel= .238418579e-06
impact= 12 errmom= -.111758709e-07 errvel= .238418579e-06
impact= 13 errmom= -.558793545e-08 errvel= .238418579e-06
impact= 14 errmom= -.111758709e-07 errvel= -.238418579e-06
impact= 15 errmom= .000000000e+00 errvel= .476837158e-06

impact	thetam	time	dthetam	dtime
2	.0940	.1295	-.0060	.1295
4	.0893	.3853	-.0047	.2558
7	.0825	.6445	-.0068	.2592
10	.0776	.9039	-.0048	.2594
13	.0707	1.1632	-.0069	.2593

Maximum amplitude (thetam) and time it is attained for each cycle of primary system

Fig. 4. Output from Program IDEAL

```
16    .Ø644   1.423Ø   -.ØØ63    .2598
19    .Ø594   1.6831   -.ØØ5Ø    .26Ø1
22    .Ø526   1.9427   -.ØØ69    .2596
26    .Ø41Ø   2.4622   -.Ø115    .5196
up to the 9
peak the least squares fit to the        peaks is thetam=q+s*t with q=
 .9756416Ø8e-Ø1 and s= -.2296178 6Øe-Ø1 with max error=  .84Ø116292e-Ø3 at peak=
4 and variance=  .214724355e-Ø5


impact thetam  time   dthetam  dtime
26    .Ø41Ø   2.4622   -.Ø59Ø   2.4622
28    .Ø354   2.7238   -.ØØ56    .2616
3Ø    .Ø298   2.9863   -.ØØ57    .2625
32    .Ø239   3.2495   -.ØØ58    .2632
34    .Ø18Ø   3.5136   -.ØØ59    .2641
36    .Ø117   3.7778   -.ØØ63    .2642
38    .ØØ56   4.Ø299   -.ØØ61    .2521
39    .ØØ55   4.2669   -.ØØØ1    .237Ø
4Ø    .ØØ44   4.5122   -.ØØ11    .2453
42    .ØØ25   5.2522   -.ØØ19    .74ØØ
43    .ØØ24   5.7657   -.ØØØ1    .5135
up to the 11
peak the least squares fit to the        peaks is thetam=q+s*t with q=
 .6432Ø1843e-Ø1 and s= -.123949796e-Ø1 with max error=  .9575Ø3542e-Ø2 at peak=
11 and variance=  .354233722e-Ø3
```

Fig. 4--Continued

radians, and with the impacting mass in contact with the positive stop. In order to determine how important these initial conditions were, both the initial position and velocity of the impacting mass was varied. Varying the initial position of the impacting mass through a range of +1" to -1" from the center of the gaps, and giving it a velocity ranging from +2 ft/sec to -2 ft/sec led to a ±4% variation of s from its average value. Doubling the initial amplitude of the system from $\theta_{m_0} = 0.08$ radians to $\theta_{m_0} = 0.16$ radians increased s by 20 percent. None of these changes significantly affected θ_{m_r}. These results were for $I = 0.1893$ slug · ft^2, impact mass = 0.001 slug, L = 2.23', d = 3", and $\omega = 25$ rad/sec.

Initial conditions obviously have an effect on the damper efficiency s. This effect does not justify complicating the analysis of s by considering initial conditions, especially if s is evaluated while keeping θ_{m_0} constant. However, the variation of s with initial amplitude θ_{m_0} suggests that the system's decline in amplitude is not perfectly linear, only approximately so.

An important energy consideration for space operations is that the only energy dissipation will be due to the impacts. Whatever kinetic or potential energy is lost due to these impacts will be converted to heat, which will be distributed between the primary system and the impacting mass. This heat can only be dissipated through radiation, which is a very slow process. Consider a primary system with inertia I about its rotation point and a natural frequency ω_n. If this system's only energy loss is due to impact damping, which reduces the system's amplitude from θ_{m_0} to θ_{m_r}, then the change in total energy equals the change in maximum kinetic energy, so:

$$\Delta E = \Delta T_{max} = \frac{1}{2}I(\theta_{0_{max}})^2 - \frac{1}{2}I(\dot{\theta}_{f_{max}})^2 \qquad (14)$$

where $\theta_{0_{max}}$ is the initial maximum angular velocity, and $\dot{\theta}_{0_{max}}$ is the final maximum angular velocity. From this it is easy to obtain:

$$\Delta E = \frac{1}{2}I\omega_n{}^2(\theta_{m_0}{}^2 - \theta_{m_r}{}^2) \qquad (15)$$

This ΔE is the energy converted to heat.

A worst case example is worked out for the laboratory model's values of m, I and ω_n, assuming all of the heat goes to raise the temperature of the impacting mass. For the smallest mass used, m = 0.000481 slug (about 1/4 ounce), I = 0.188 slug · ft^2, ω_n = 25.9 radians per second, and θ_{m_0} and θ_{m_r} are chosen to be 0.10 radians and zero, respectively. This gives:

$$\Delta E = \frac{1}{2}(0.188)(25.9)^2(0.10)^2$$

$$= 0.631 \text{ lb} \cdot \text{ft}$$

$$= 8.103 \times 10^{-4} \text{ BTU}$$

The temperature of the impacting mass will then be raised by:

$$\Delta T^\circ = \frac{\Delta E}{c_p m} \tag{16}$$

where c_p is the specific heat of the material. If c_p = 0.109 BTU/(lb · °F) (the value for nickel steel at room temperature according to Reference 24), then

$$\Delta T^\circ = 0.048 \text{ °F}$$

While this temperature increase is of no significance, the fact that the impacts convert kinetic energy to heat should be remembered when designing an impact damper, especially if a very small mass is expected to absorb a great deal of energy.

Forced Vibration

The forced vibration impact damper depicted in Figure 3 consists of an upright slender beam with a damper assembly on top and an impacting mass free to move between the stops of the damper assembly. A time dependent angular displacement is applied as a boundary condition to the bottom of the beam. The motion of the primary system, consisting of the beam and damper assembly, was not obtained. However, the motion of a similar system in free vibration, shown in Figure 5, was obtained in Reference (25). From this solution, the natural frequencies can be found.

The undamped, free vibration of the system depicted in Figure 5 is, according to Reference (25):

$$x(y,t) = X(y)e^{i\omega_F t} \tag{17}$$

where:

$$X(y) = C_1 \sin(\beta y) + C_2 \cos(\beta y) + C_3 \sinh(\beta y) + C_4 \cosh(\beta y) \tag{18}$$

and:

$$\beta = \sqrt[4]{\frac{\omega_F{}^2 \rho}{EI'_B}} \tag{19}$$

$$C_2 = -C_4 = C_1 \frac{\sin \beta L_b + \sinh \beta L_b}{\cos \beta L_b + \cosh \beta L_b}$$

$$C_3 = -C_1$$

C_1 is determined by the initial conditions, while β is solved for using:

$$1 + \frac{1}{(\cos \beta L_b)(\cosh \beta L_b)} + \beta\frac{M}{\rho}(\tanh \beta L_b - \tan \beta L_b) = 0 \tag{20}$$

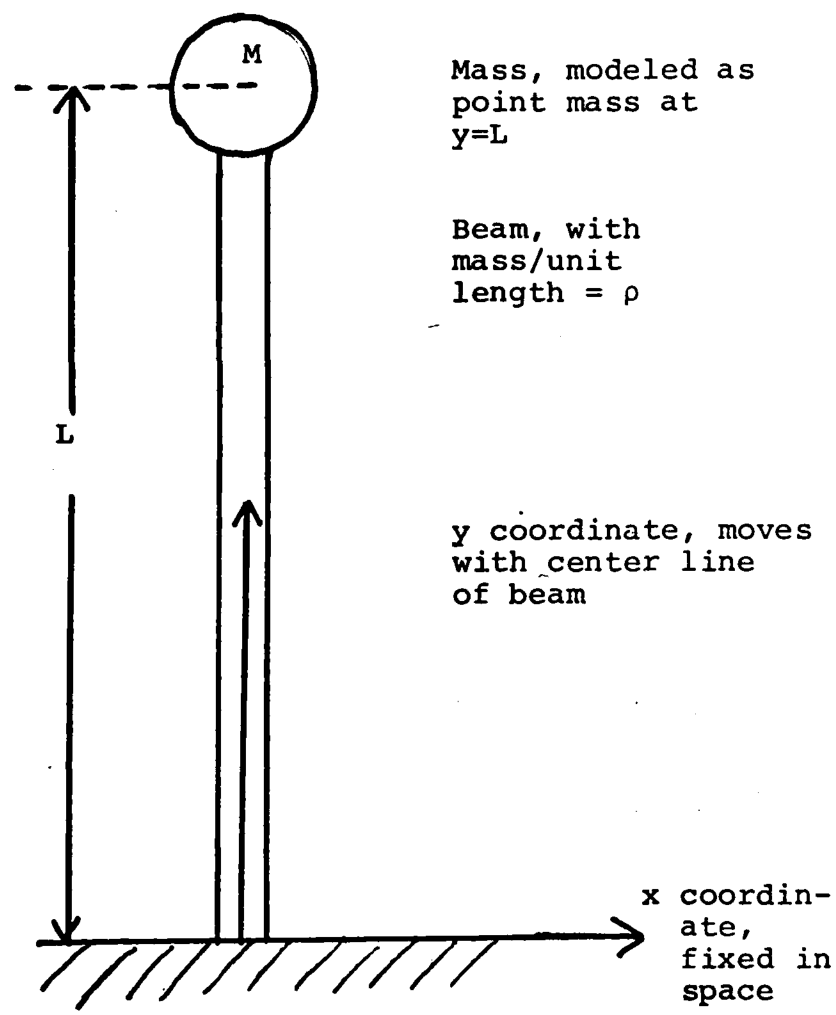


Fig. 5. Model of Free Vibration Problem Solved in Reference 25

ρ is the mass per unit length along the flexible beam, M is the total mass of the damper assembly at the top of the beam, and ω_F is the natural frequency of this system. Once β is obtained from Equation (20), ω_F can be found using Equation (19).

III. Laboratory Models

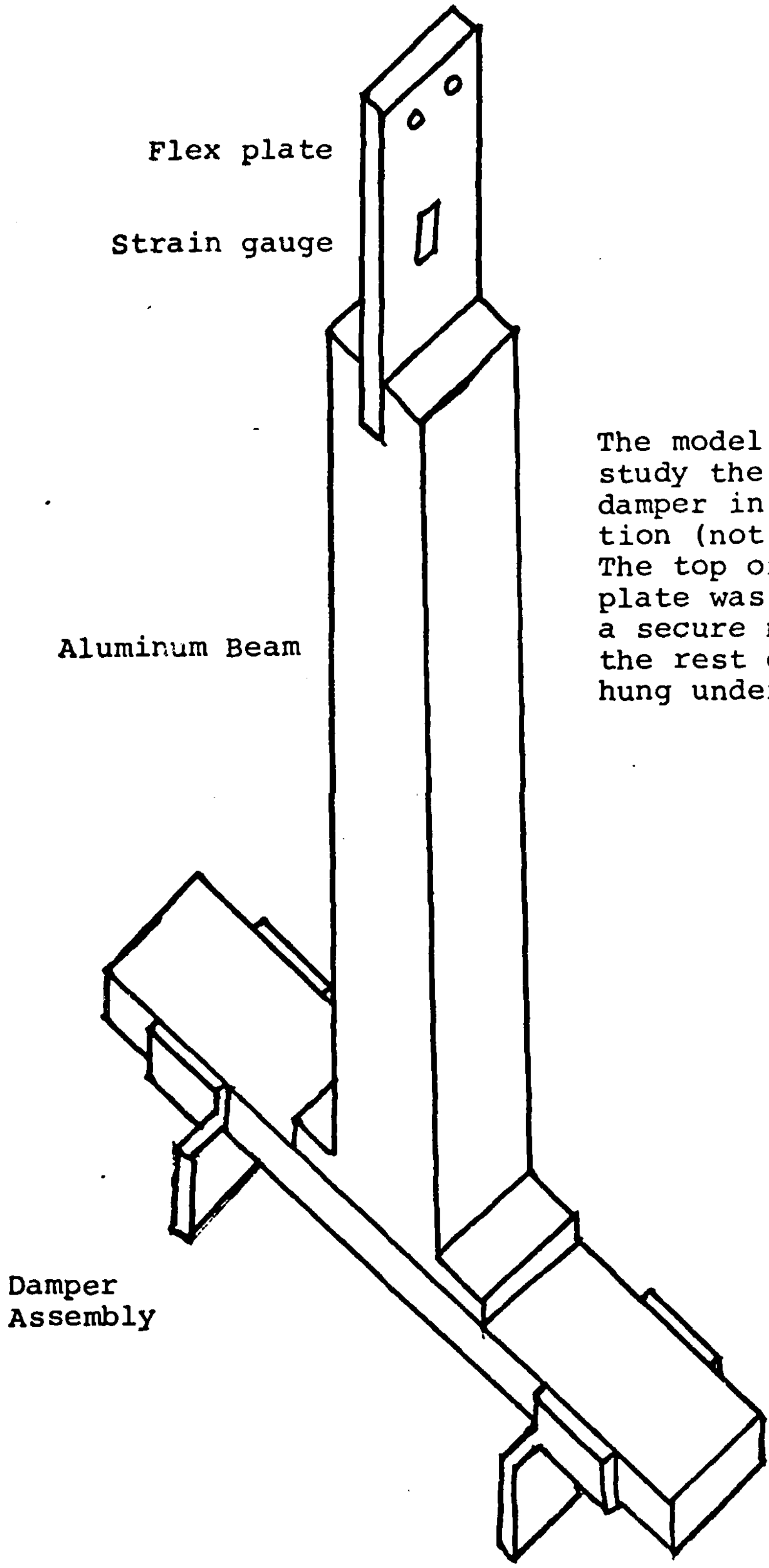
Two impact damping models were designed and constructed to experimentally study the performance of the damper in forced and free vibration. The equations of Chapter II were derived to apply to the models depicted in Figures 6 and 7. The free vibration model of Figure 6 was used to verify the analysis of Chapter II. The forced vibration model of Figure 7 was used to take measurements and make observations on its motion. Details on both of these models, the measurement equipment and techniques, and the conversion of the measurements to actual displacements are given in Appendix B.

Free Vibration Model

The laboratory model of Figure 6 consisted of an aluminum beam suspended by a short, flexible piece of steel acting as a flex plate, with the damper assembly mounted on the bottom. The impacting mass is suspended from a point above the entire system to minimize friction forces. The quantities needed to evaluate the motion using Equations (3) and (7) are:

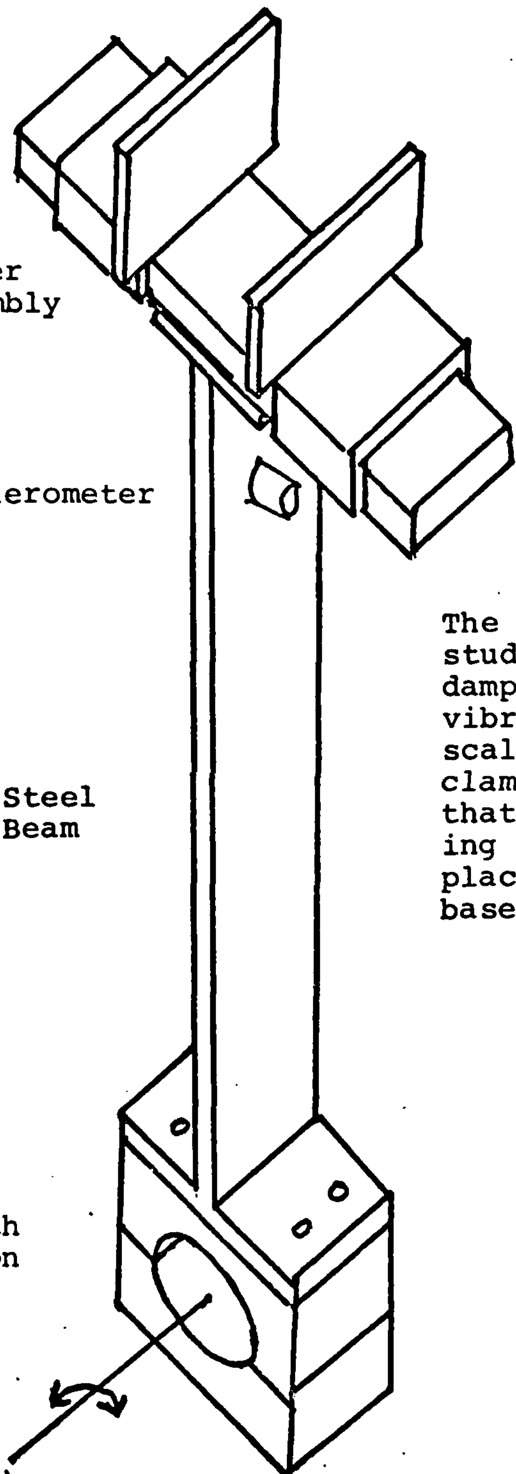
$$I = 0.188 \text{ slug} \cdot \text{ft}^2$$

$$c = 0.02 \text{ lb} \cdot \text{ft} \cdot \text{sec}$$



The model used to study the impact damper in free vibration (not to scale). The top of the flex plate was attached to a secure mount, while the rest of assembly hung underneath.

Fig. 6. Laboratory Model of Free Vibration Impact Damper



The model used to study the impact damper in forced vibration (not to scale). The mount clamped onto a pin that rocked, applying the angular displacement at the base.

Fig. 7. Laboratory Model of Forced Vibration Impact Damper

$$k = W_T \, r_{0m} + \frac{EI'_s}{d_s}$$

$$= 1565 \text{ lb} \cdot \text{in}$$

$$W_T = 2.91 \text{ lb}$$

$$r_{0m} = 15.765"$$

$$E = 28 \times 10^6 \text{ lb} \cdot \text{in}^2$$

$$I'_s = 1/6144 \text{ in}^4$$

$$d_s = 0.25'$$

$$q = \sqrt{\frac{k}{I} - \left(\frac{c}{2I}\right)^2}$$

$$= 26.34 \text{ rad/sec}$$

$$m = 0.000481 \text{ slug}, \; 0.00149 \text{ slug}, \; 0.00503 \text{ slug}$$

$$c_m = 0.0000217, \; 0.0000235, \; 0.0000267 \text{ lb} \cdot \text{sec/ft}$$

$$k_m = mg/L_m$$

$$= 0.00221, \; 0.00685, \; 0.0231 \text{ lb/ft}$$

$$g = 32.174 \text{ ft/sec}^2$$

$$L_m = 7'$$

$$q_m = \sqrt{\frac{k_m}{m} - \left(\frac{c_m}{2m}\right)^2}$$

$$= 2.14 \text{ rad/sec}$$

Quantities needed to obtain the initial conditions after each impact using Equations (10) through (13) are:

$$L = 2.21'$$

$$e = 0.4 - 0.5$$

The measurement of these quantities is discussed in Appendix B, but some comments are in order here. The values of I, W_T, r_{0m}, d_s, I'_s, m, L_m and L were measured, weighed, and calculated to an acceptable degree of accuracy with little uncertainty. E was obtained from Reference 26. c and c_m were obtained by measuring the reduction in amplitude of the freely oscillating primary system and impacting mass after a known number of cycles; c and c_m could then be calculated by the logarithmic decrement method. The value of c obtained in this manner varied from 0.01 to 0.04 lbs · sec/ft, c = 0.02 was taken as the approximate value. The value obtained for c_m did not vary significantly for different tests, but the impacting mass was traveling much slower when these measurements were made than when it was used in the impact damper. Since damping forces are not always directly proportional to velocity, as Equation (6) treats them, this could be a source of error. e was obtained by allowing each of the impacting masses to swing as a pendulum a known distance and strike the impacting surface of the primary system, and then measuring the recoil of the primary system and the impacting mass. These known quantities and measurements were converted into velocities for the primary system and impacting mass both before and after the impact, from which e was determined. 0.4 and 0.5 are the upper and lower values of e obtained. Finally, while $q = 26.34$ rad/sec was the calculated value for the damped frequency of the system, the actual frequency of the system was measured as 25.9 rad/sec.

The motion of the system was measured using two SR-4 type AD-7 strain gages, centered on opposite sides of the flex plate. These gages were connected to a Q-amp in a Type 535A oscilloscope. The strain ε obtained in this manner could be converted to radians of displacement of the primary system using:

$$\theta = 4.80 \times 10^{-5} \varepsilon \tag{21}$$

where ε is measured in micro inches per inch. This relation is obtained in Appendix B, and is only valid for small values of θ, where the relation between ε and θ is linear.

Photographs of the oscilloscope trace were made to obtain the beam deflection as a function of time. Some of these photographs are shown in Figure 8. The actual distance between the impacting surfaces was 3" in this figure, but the actual distance the impacting mass could travel between the impacting surfaces was 3" minus the diameter of the impacting mass. The diameter of the 0.000481 slug mass was 15/32", the 0.00149 slug mass was

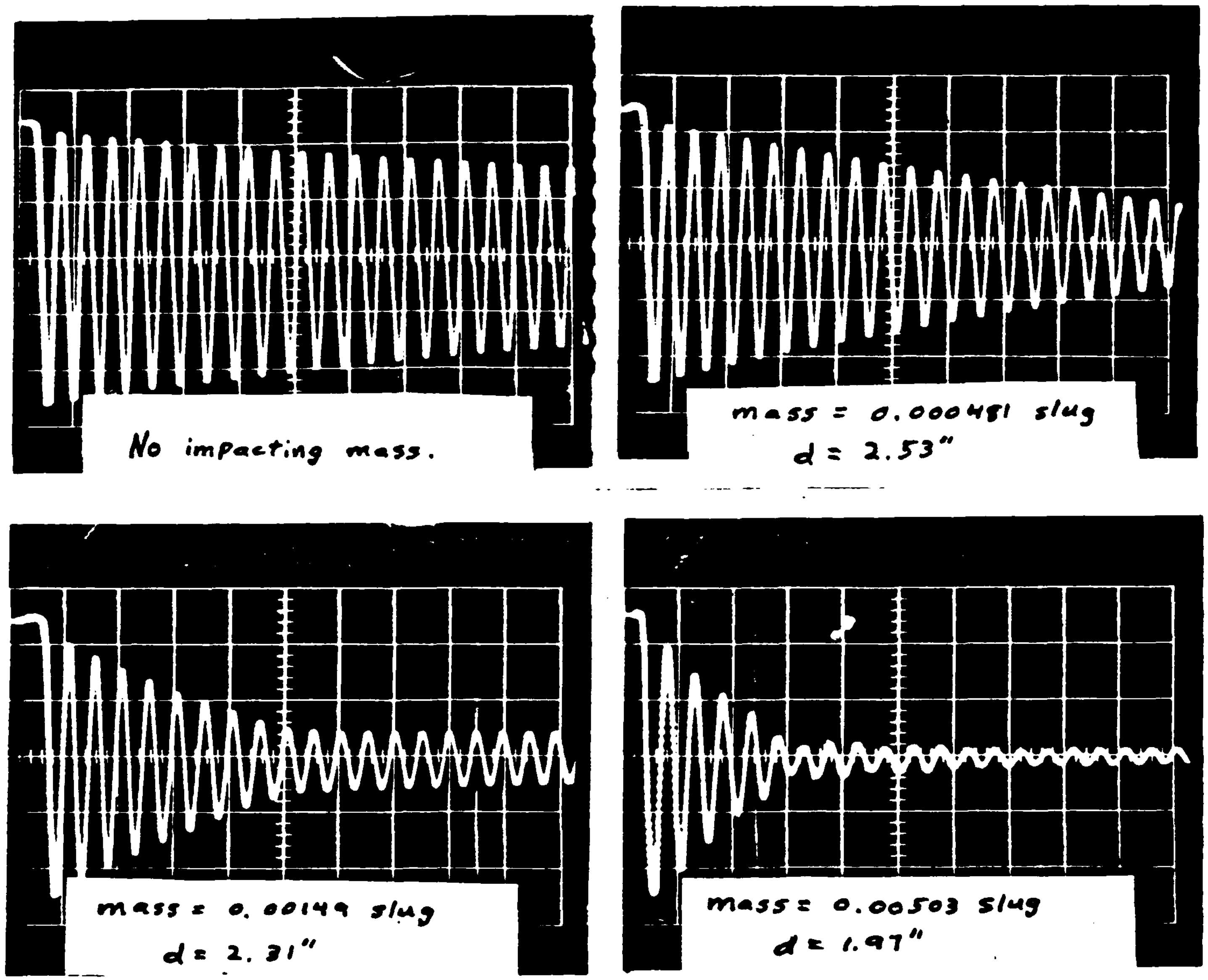


Fig. 8. Oscilloscope Traces Showing Strain Versus Time. Vertical axis is 1000 u"/" per division; horizontal axis is 0.2 sec/div.

11/16", and the 0.00503 slug mass was 1-1/32". d is the actual horizontal distance between impacting surfaces that the impacting mass can travel.

It can be seen from Figure 8 that when the impact damper is operating, the maximum angular displacement reached by the primary system decreases linearly with time. This is not the constant damping of an exponentially decaying system. The usual measures of damping, such as logarithmic decrement, will not be independent of the amplitude of the system. It is also apparent that the impact damper ceases to be effective after the maximum amplitude obtained during a cycle decreases to a certain value. This value is denoted θ_{m_r}, and is referred to as the residual amplitude. When the damper is effective, $\theta_{m_j} > \theta_{m_r}$, where θ_{m_j} is the maximum amplitude attained by the primary system during cycle j. This implies that the maximum angular displacement attained during the jth cycle can be given by:

$$\theta_{m_j} = r - st_j \tag{22}$$

where θ_{m_j} is the maximum angular deflection attained during cycle j; t_j is the time at which the system reaches maximum amplitude during cycle j; r would be the amplitude θ_{m_0} of the system if it were started with zero velocity at $t_j = t_0 = 0$; and s is the rate at which the system's maximum amplitude decreases with time, which will be referred to as the damper efficiency.

Forced Vibration Model

The laboratory model of a continuous system in forced vibration, depicted in Figure 7, consisted of a flexible, upright steel beam, with a time dependent angular displacement applied to the base, and a damper assembly on top. The quantities needed to obtain the natural frequency from equations (16) and (17) are:

$$L_b = 18.5''$$

$$M = 0.01382 \text{ slug}$$

$$\rho = 0.00110 \text{ slug/in}$$

$$E = 28 \times 10^6 \text{ lb} \cdot \text{in}^2$$

$$I'_B = \frac{1}{6144} \text{ in}^4$$

For these values, Equation (17) gives the first two values of βL being 1.345 and 4.071. Using this, Equation (16) gives the first two natural frequencies as being 37.28 and 341.7 rad/sec.

A model MB 303 accelerometer was mounted 1" below the damper assembly. The accelerometer signal was amplified using a model 2614B amplifier powered by an Endevco Mode 2621 power supply; this signal was then recorded using a Honeywell Model 2106 visicorder. High frequency noise required sending the amplified signal through a simple low pass filter before it reached the visicorder. Details of this filter are given in Appendix C.

IV. Correlation of Analytical and Laboratory Results

This chapter compares the computed motion of the free vibration impact damper to the motion measured in the laboratory. Possible sources of errors in both laboratory measurements and in the attempt to preduct the motion of a real system using LABSIM and the equations of Chapter III are discussed.

The program LABSIM, explained in Chapter II, was used with the measured physical quantities of the laboratory free vibration model, given in Chapter III. Table 1 compares values of s and θ_{m_r} obtained from LABSIM for e = 0.4 and 0.5 with the measured laboratory values of s and θ_{m_r} for the different gap settings and impacting masses. 0.4 and 0.5 were the minimum and maximum values measured for the coefficient of restitution e. As can be seen the measured value of damper efficiency s is never more than 14 percent greater than the largest value of s computed, or 5 percent less than the smallest value computed. However, the measured s is not consistent in falling between, above, or below the computed values of s. The measured value of θ_{m_r} may be more than twice the nearest computed value. The rest of this chapter considers possible reasons for these imperfect correlations.

TABLE 1

COMPARISON OF COMPUTER SIMULATION AND LABORATORY RESULTS FOR s and θ_{m_r}

mass (slugs)	d	Computer e = 0.4 s	Computer e = 0.4 θ_{m_r}	Computer e = 0.5 s	Computer e = 0.5 θ_{m_r}	Laboratory s	Laboratory θ_{m_r}
0.000481	3.53"	0.0115		0.0102	0.0107	0.0113	0.010
0.00149	1.312"	0.0134	0.0040	0.0110	0.0032	0.0152	0.0082
	2.312"	0.0199	0.0071	0.0176	0.0048	0.021	0.0137
	3.312"	0.027	0.0105	0.023	0.0075	0.028	0.0184
0.00503	0.969"	0.029		0.025	0.0016	0.024	0.0042
	1.969"	0.043	0.0033	0.037	0.0033	0.038	0.0063
	2.969"	0.061	0.0057	0.052	0.0026	0.056	0.0130

Notes

s measured in radians/sec.

θ_{m_r} measured in radians.

The actual distances between impacting surfaces in the laboratory were set at 2", 3", and 4"; the values given here for the gap d are these distances minus the impacting mass's diameter.

Sources of errors in obtaining the laboratory results fall into two categories: (1) errors in measuring the physical parameters of the system, and (2) errors in measuring the resulting output. Of the physical parameters measured, c and c_m are the most uncertain, for the reasons discussed in Chapter III. In measuring the mass of the impacting masses, the 0.00005 slug mass of the line supporting them was neglected. This certainly increased the effective mass of the impacting masses by some small amount. Measured lengths under six inches could have up to 1/32" error in them, measured lengths over six feet could have up to one inch error in them. In calculating I, the distributed mass of the damper assembly was modeled as a point mass 26" from the rotation point. These are only some of the error sources, most of which can be assumed to be small. With the exception of c, c_m, and e, a qualitative estimate of the errors in the values of the physical parameters given in Chapter III would be that errors are ±5 percent of the quantities given, or less.

Some of the error sources involved in measuring the results of a laboratory test are simple: the strain gage used was accurate to within ±2 percent, while the traveling microscope used in measuring the photographed oscilloscope trace had a small amount of play in the adjustment, causing errors of approximately 0.1 percent or less. Human judgement provided another error source;

in particular, in measuring the photographed oscilloscope trace one had to decide where to take a measurement from on an often fuzzy trace edge. A qualitative estimate of the errors in measuring s and θ_{m_r} is that the smallest values of s and θ_{m_r} may be in error by as much as ±20%, with most values of s and θ_{m_r} being accurate to within ±5% or less.

The computer model of the impact damper, LABSIM, gives the correct values for s and θ_{m_r} for the numbers it is given and the operations it performs. Errors caused by limitations in the accuracy of single precision FORTRAN would be insignificant (less than 1 percent error) for the numbers and operations employed. The only reason LABSIM would not give the motion of the laboratory model of the free vibration impact damper would be if the equations of motion, or their solutions, for this system are in error.

The equations of motion of the primary system (Equation 2) and the impacting mass (Equation 6) employed in LABSIM are based upon the assumption that restoring moments and forces ($k\theta$ and $k_m x$) are proportional to displacement, and damping moments and forces ($c\dot{\theta}$ and $c_m\dot{x}$) are proportional to velocities. It is commonly accepted that spring forces on a body and gravity forces on a pendulum restricted to small displacements are both approximately proportional to displacement. Damping forces are not as well understood, and resistance to motion is often assumed to be independent of all but the direction of motion (dry friction) or to be proportional to velocity squared (aerodynamic drag). Thus, the assumed damping forces in Equations (2) and (6) may differ from the actual damping of the laboratory model.

A certain source of error in the attempt to model a real system using LABSIM lies in the small angle approximations made in the derivation of Equations (2) and (6). Other differences between the LABSIM model and reality are the assumption that impacts are of infinitely short duration, or that an impact occurs whenever an iteration puts the impacting mass within 0.000001 feet of a stop. Finally, it is unlikely that Equations (2) and (6) take into account all of the forces acting upon the primary system and the impacting mass. It would be very difficult to quantify all of these sources of error, or to say if these errors add up or cancel out over many cycles.

In view of the errors mentioned, the agreement between the computed and measured values of s seems acceptable. Which of these errors causes the differences between the computed and measured values of θ_{m_r} is unknown. While LABSIM does a poor job of predicting θ_{m_r}, it is reasonable to assume that the errors in LABSIM do not significantly favor one set of system parameters over another. Therefore, for an impact damper with equations of motion similar to Equations (2) and (6), LABSIM should be able to predict how changes in system parameters will affect s and θ_{m_r}, even if it does not reach a correct actual value of θ_{m_r}.

V. Results and Discussion

Free Vibration

All of the results discussed here were obtained by using the computer model of the impact damper IDEAL, the model with no gravity or viscous damping. The system modeled had the parameters:

θ_{m_0} = initial amplitude = 0.10 radian;

I = moment of inertia = 0.1893 slug · ft^2;

L = impact moment arm = 2.23 ft;

d = gap setting = 0.25 ft;

m = impact mass = 0.001 slug;

e = coefficient of restitution = 0.5; and

ω_n = natural frequency = 25 radians/sec.

The parameters θ_{m_0} and I were kept constant, as was the flex - plate stiffness; all of the other parameters were varied one at a time. The effect that changes in the last five parameters had on the damper efficiency, s, and the residual amplitude, θ_{m_r} are shown graphically.

Figure 9, which plots damper efficiency s for different impact moment arms L, was made by varying L from 1 ft to 3-1/4 ft, in quarter foot increments, while holding all other parameters at the values given in the preceding paragraph. Figure 10, which gives s versus d, was obtained

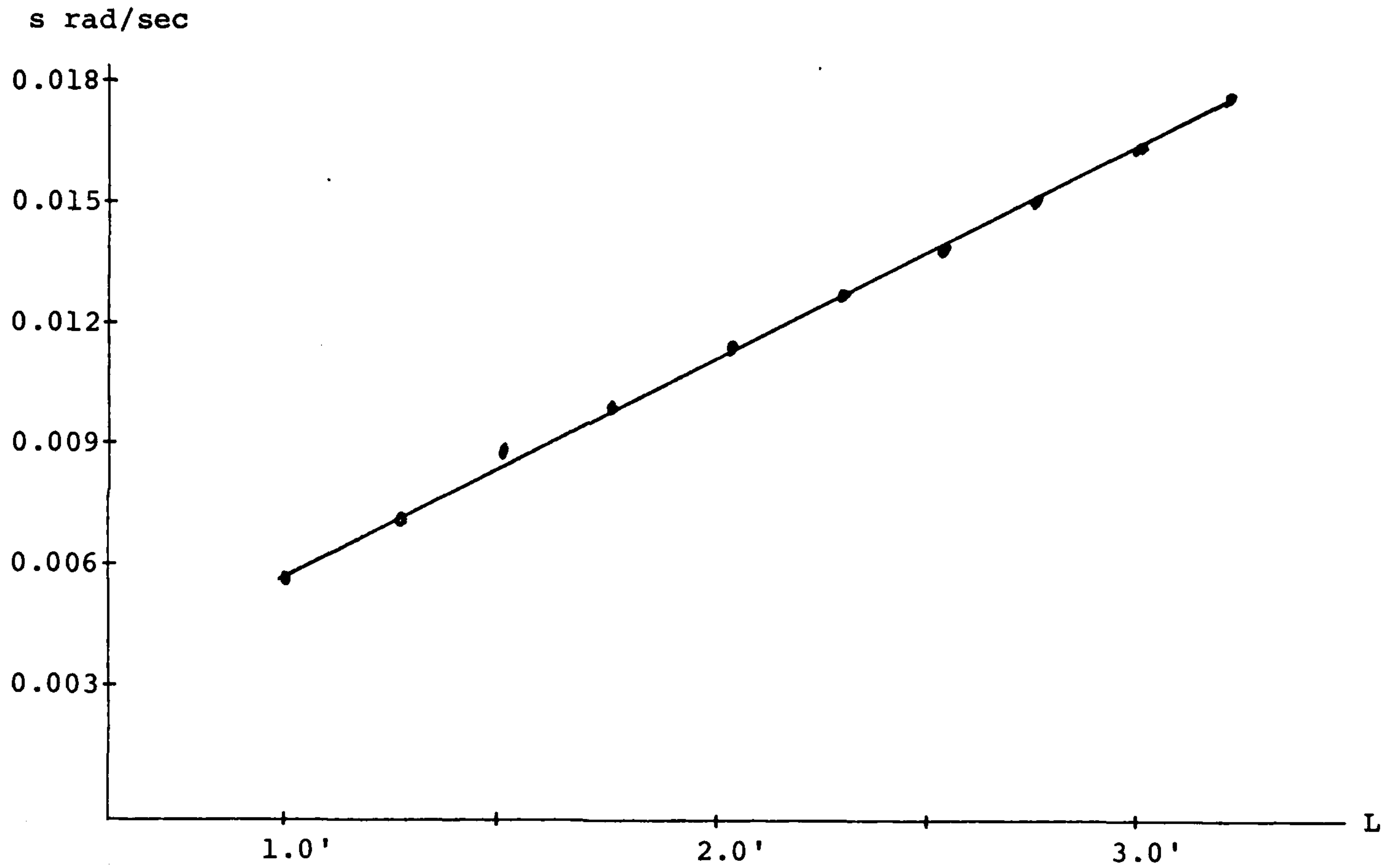


Fig. 9. Damper Efficiency for Different Impact Moment Arms, with m = 0.001 Slug, I = 0.1893 Slug · ft^2, d = 3", e = 0.5, and ω_n = 25 rad/sec

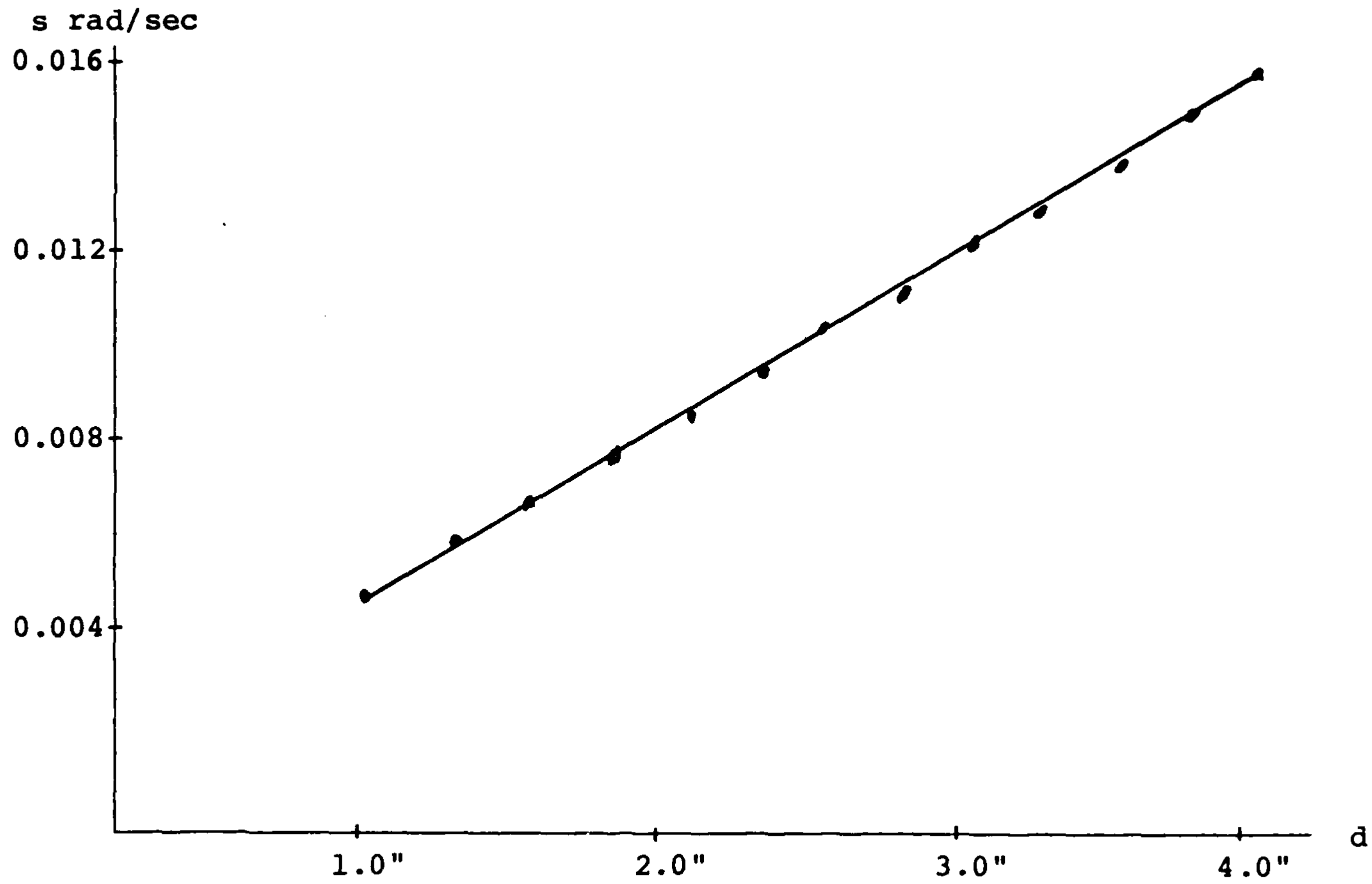


Fig. 10. Damper Efficiency for Different Values of Gap with I = 0.1893 slug · ft^2, m = 0.001 slug, L = 2.23', e = 0.5 and ω_n = 25 rad/sec

similarly, by varying the gap setting d from 1 inch to 4 inches in quarter inch increments, while all other parameters were held constant. The same technique was used in obtaining Figure 11, s versus coefficient of restitution e, and Figure 12, s versus the natural frequency ω_n. The data for Figure 13 was obtained in the same manner as for the parameters of Figures 9 to 12, but the plot was made a little different. It was found that the damper efficiency s is approximately proportional to the quantity:

$$\frac{m}{I + mL^2}$$

The data for Figure 13 was obtained by varying m from 0.001 slug to 0.005 slug in increments of 0.005 slug. The impact moment arm L was kept at 2.23 ft and the moment of inertia I was kept at 0.1893 slug · ft^2.

The approximate linearity of Figures 9, 10, 12, and 13 and the linearity of Figure 11 for $e \geq 0.3$ make the analysis of s a simple matter. Since s appears to be directly proportional to the impact moment arm L, the gap, d, one minus the coefficient of restitution, 1 - e (for e greater than or equal to 0.3), the natural frequency ω_n, and the mass over the total inertia, $m/(I+mL^2)$, the following relation can be written for e greater than or equal to 0.3:

$$s = c_s \left[\frac{mLd(1-e)}{I+mL^2}\right] \omega_n \tag{23}$$

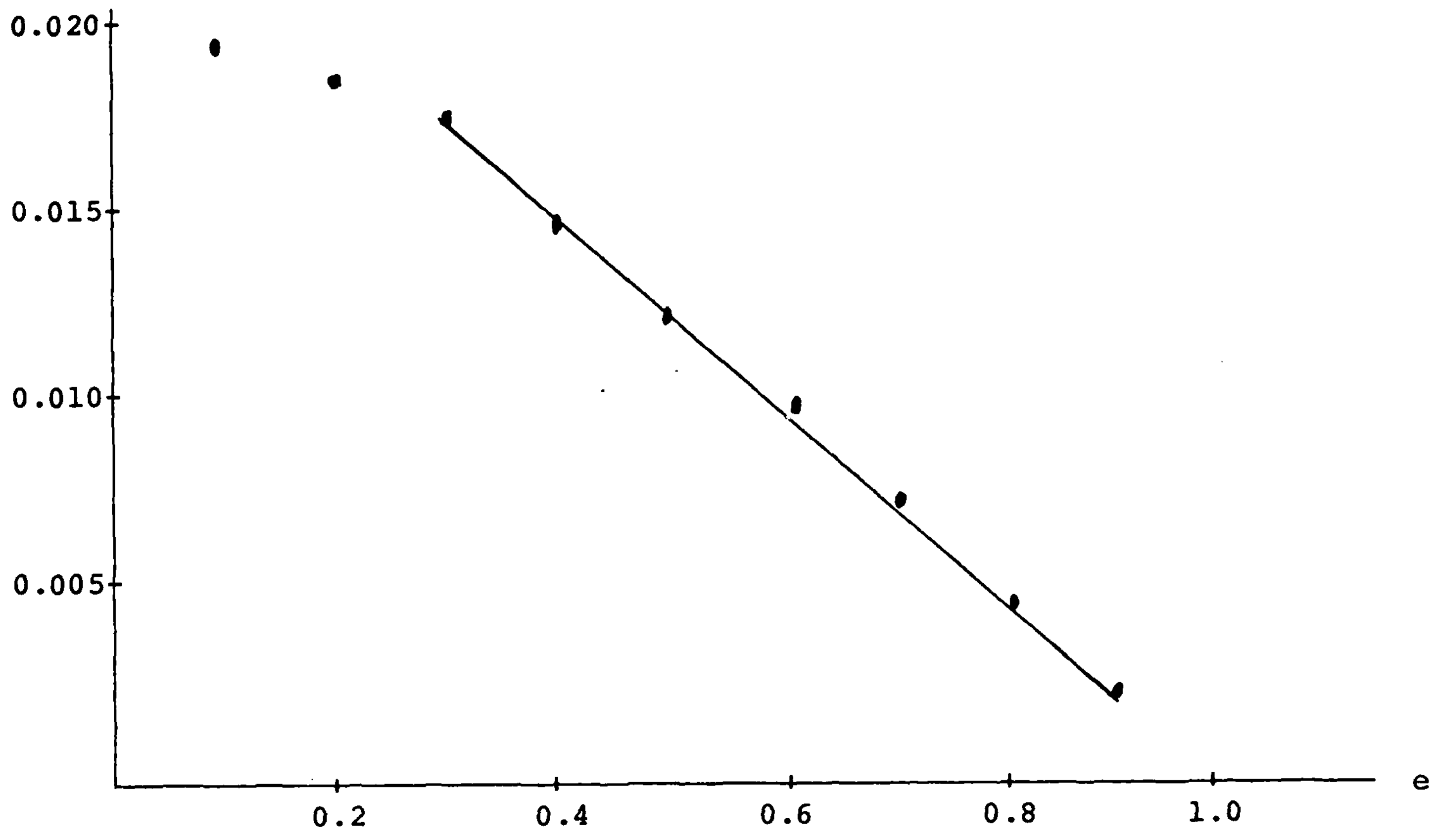


Fig. 11. Damper Efficiency for Different Values of the Coefficient of Restitution, with I = 0.1893 slug · ft^2, m = 0.001 slug, L = 2.23', d = 3", and ω_n = 25 rad/sec

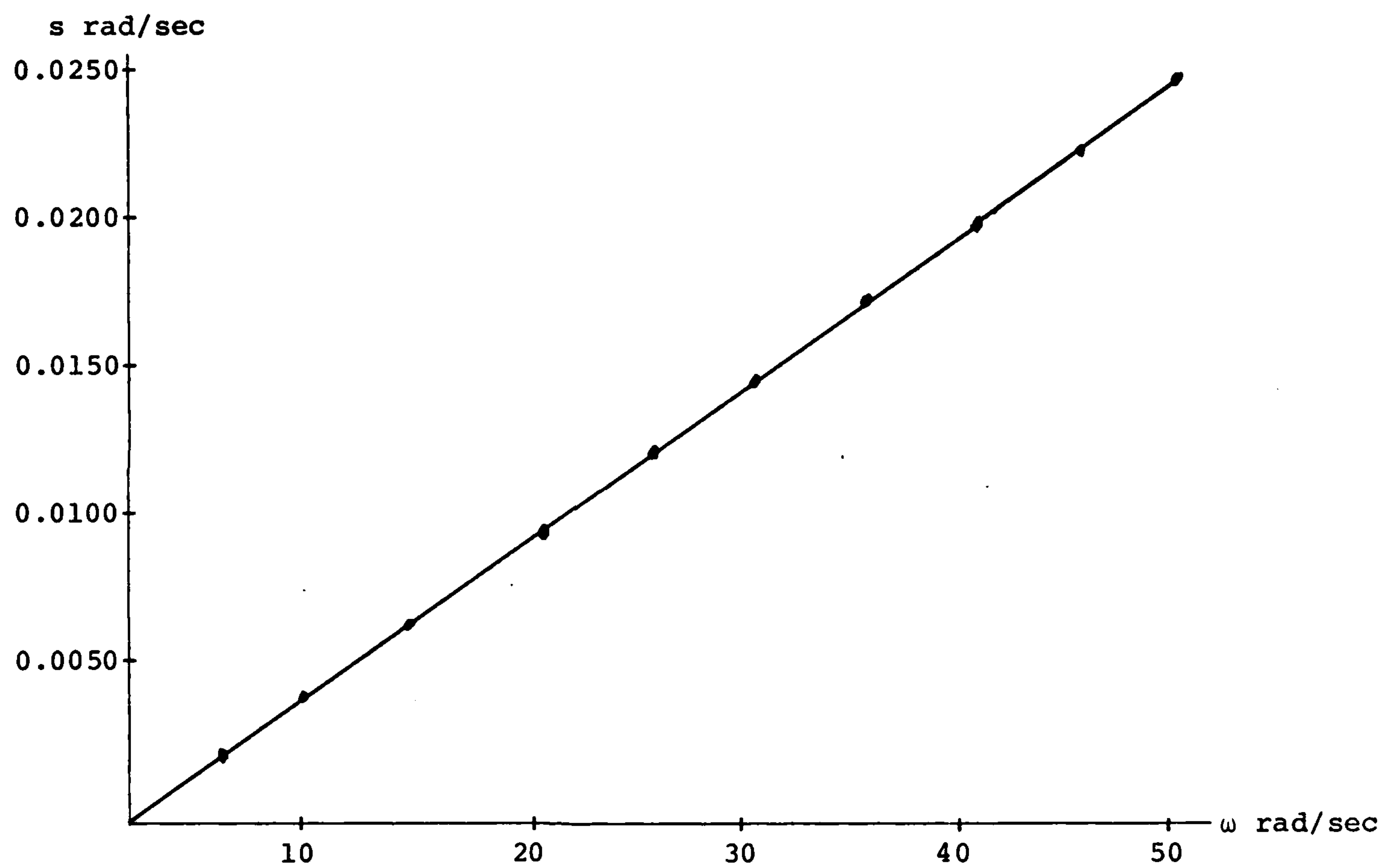


Fig. 12. Damper Efficiency for Different Values of Natural Frequency with $m = 0.001$ slug, $I = 0.1893$ slug · ft^2, $L = 2.23'$, $d = 3''$, and $e = 0.5$.

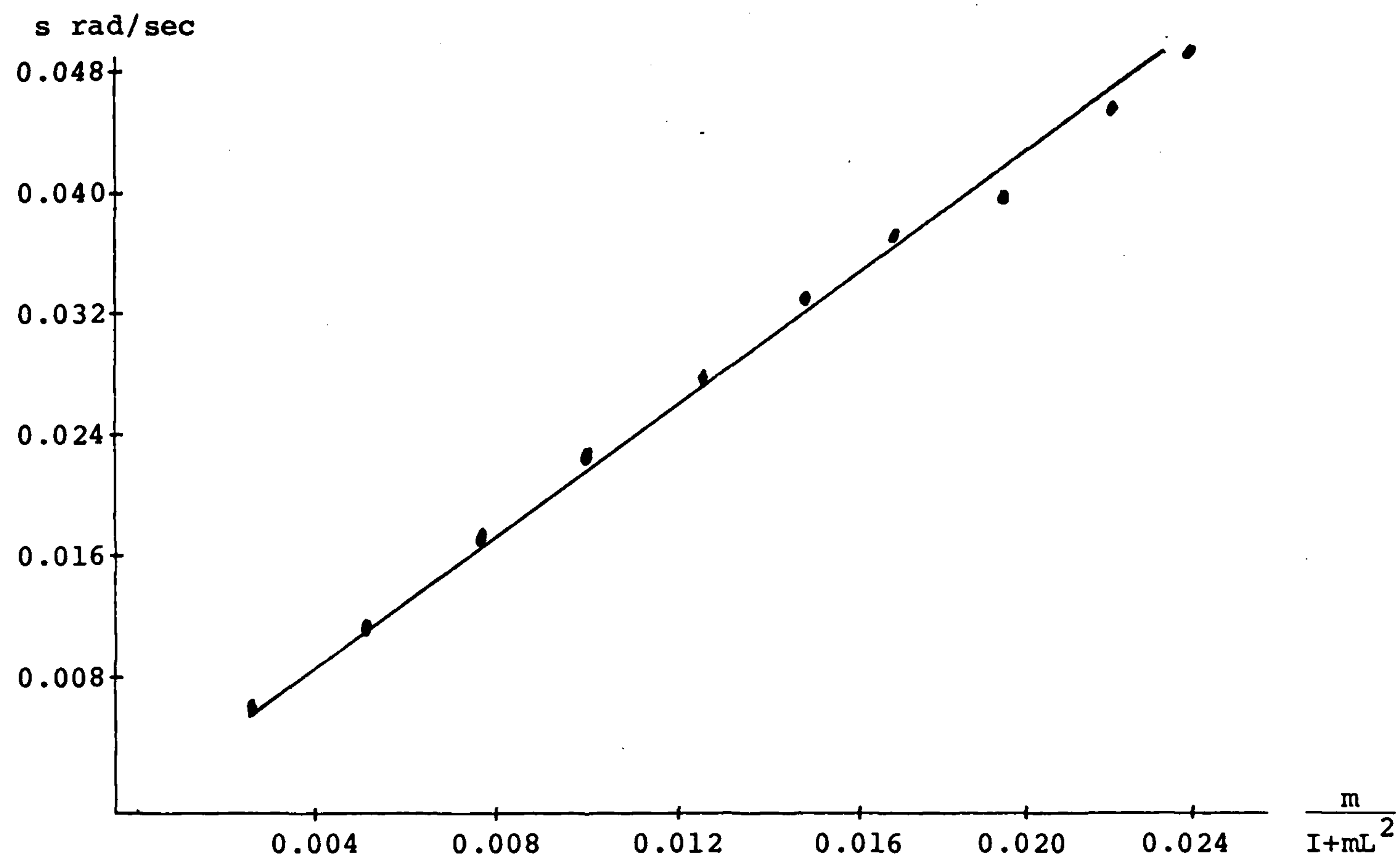


Fig. 13. Damper Efficiency for Different Values of $m/I+mL^2$, with $I = 0.1893$ slug · ft^2, $L = 2.23'$, $d = 3''$, $e = 0.5$, and $\omega_n = 25$ rad/sec

It is interesting to point out that the damper efficiency improves as the coefficient of restitution e decreases, even for e less than 0.3, as Figure 11 shows. This is not surprising, since low coefficients of restitution result in a greater energy loss. However, according to References (7), (8), (10), (16), and (20), for an impact damper used in forced vibration, optimal damper efficiency results by choosing e as close to unity as possible. The difference between the two systems is that for the forced vibration in steady state motion, the forcing function must provide a constant energy input equal to the energy loss, through impact or any other mechanism. In free vibration, the system starts with a certain total energy and any energy lost is not restored. This reduces the possible motion. This illustrates the fact that parameters given in the literature which optimize the impact damper's performance in forced vibration do not, in general, optimize the impact damper's performance in free vibration.

A useful feature of Equation (23) is that the term in brackets is dimensionless. This requires that c_s be dimensionless. If both s and ω_n are measured in radians per second, the solution of Equation (23) for c_s, taken as the average solution for c_s over the full range of parameter variations, is:

$$0.33 = \frac{s(I+mL^2)}{mLd(1-e)\omega_n} \quad (24)$$

Stated another way, the damper efficiency s is related to its parameters by the relation:

$$s = 0.33 \frac{mLd(1-e)\omega_n}{I+mL^2} \quad (25)$$

This relation is only good for $e \geq 0.3$, and may not apply for parameters outside the range of those used here to obtain Equation (25). Within these constraints, Equation (25) should allow one to choose a damper efficiency s.

Figures 14 through 18 are plots of θ_{m_r} versus L, d, e, ω_n, and $m/(I+mL^2)$, obtained in the same manner as Figures 5 through 9. These graphs illustrate that the relationship of the residual amplitude θ_{m_r} to the parameters varied is complicated. Attempts were made to solve for θ_{m_r} as a function of L/d, but these results were considerably more erratic than those of Figure 14, where L is varied and d kept constant; and Figure 15, where d is varied and L kept constant.

By plotting the results given in Figure 14 on logarithmic graph paper (also called log-log graph paper), the graph of Figure 19 was obtained. This graph implies that θ_{m_r} can be approximated as:

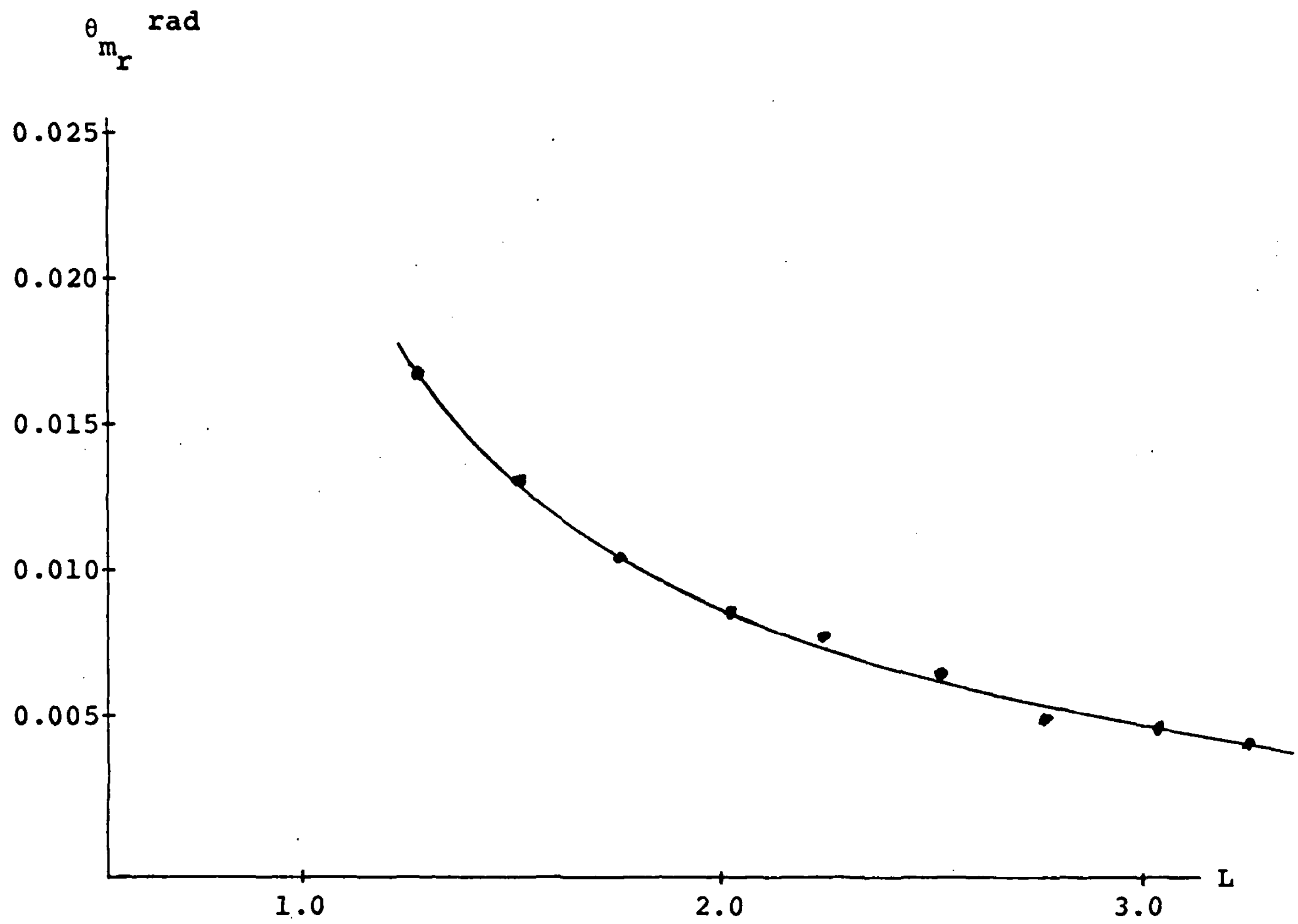


Fig. 14. Residual Amplitude for Different Impact Moment Arms, with $m = 0.001$ slug, $I = 0.1893$ slug $\cdot$ ft^2, $d = 3"$, $e = 0.5$, and $\omega_n = 25$ rad/sec

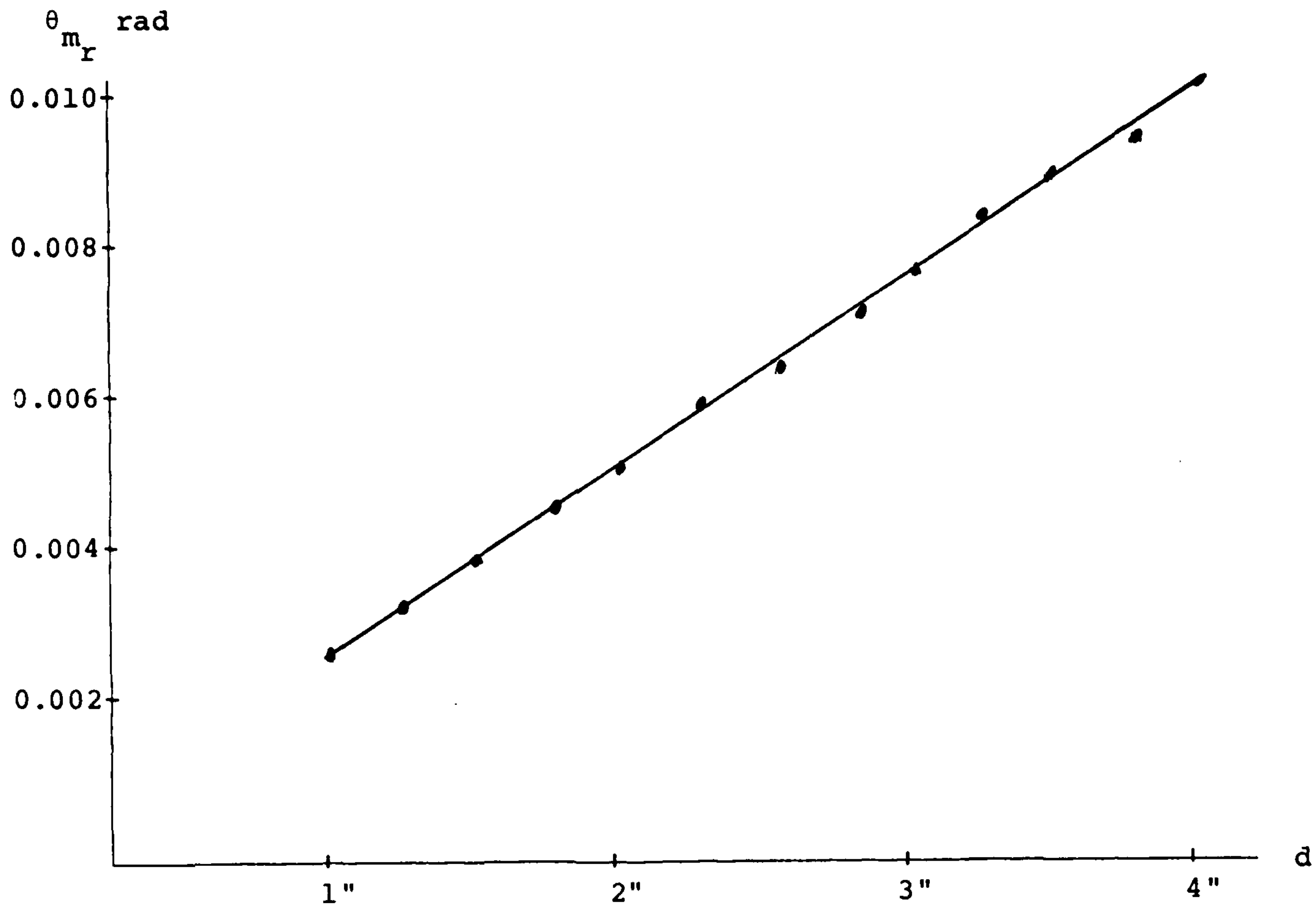


Fig. 15. Residual Amplitude for Different Gap Settings, with $m = 0.001$ slug, $I = 0.1893$ slug $\cdot$ ft^2, $L = 2.23'$, $e = 0.5$ and $\omega_n = 25$ rad/sec

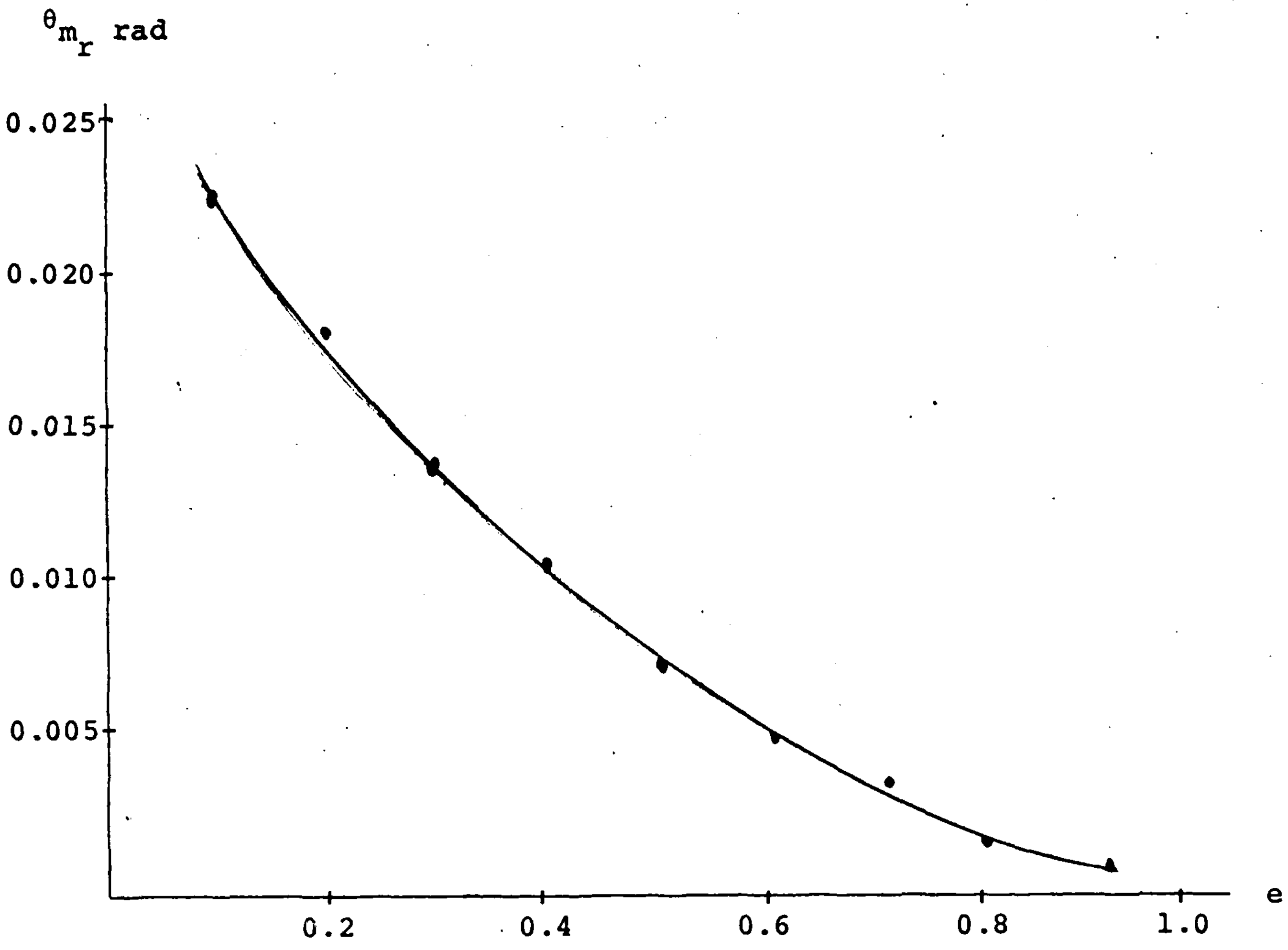


Fig. 16. Residual Amplitudes for Different Coefficients of Restitution with $m = 0.001$ slug, $I = 0.1893$ slug $\cdot$ ft^2, $L = 2.23'$, $d = 3''$, and $\omega_n = 25$ rad/sec

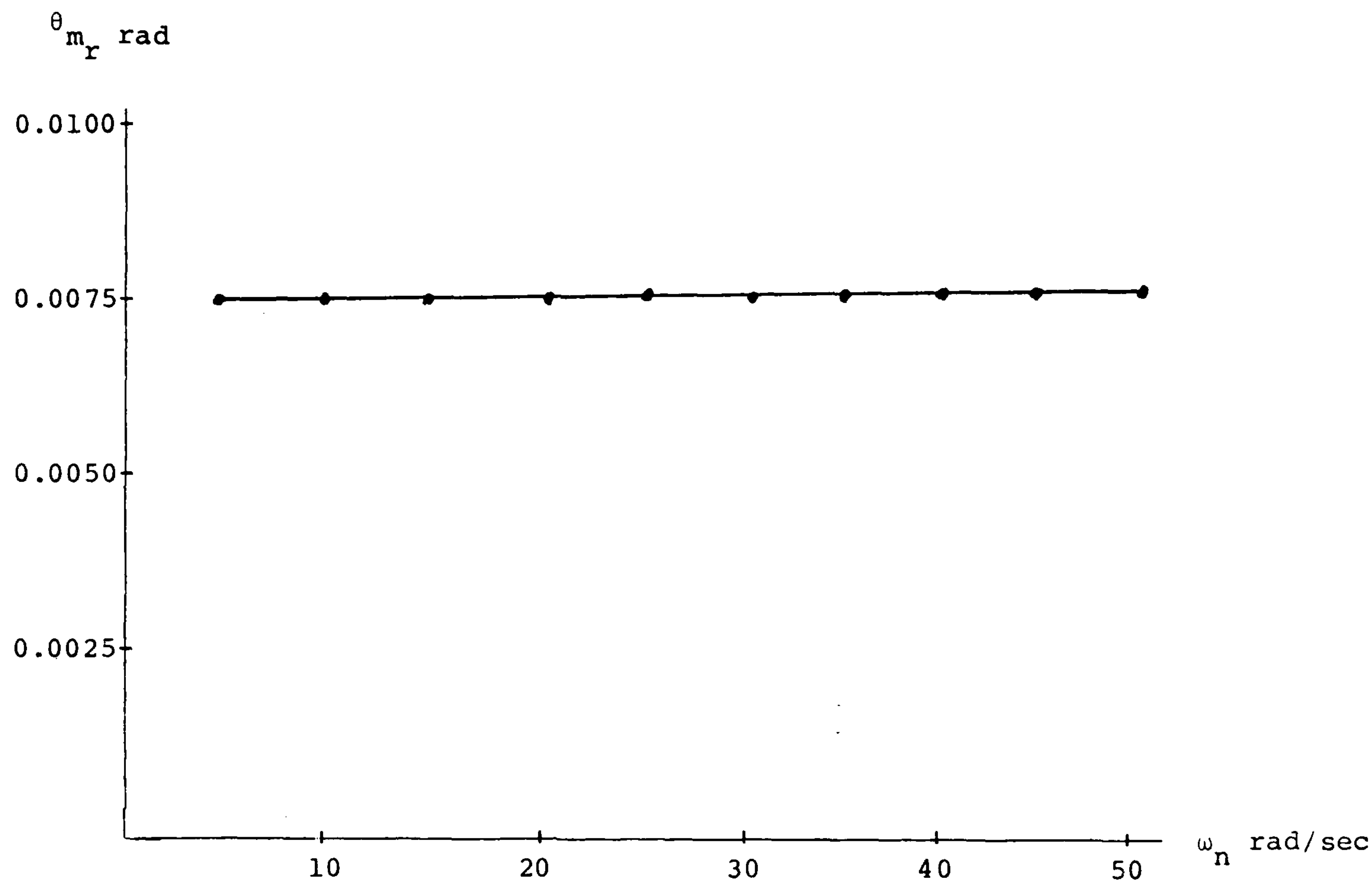


Fig. 17. Residual Amplitude for Different Values of Natural Frequency, with m = 0.001 slug, I = 0.1893 slug · ft^2, L = 2.23', d = 3", and e = 0.5

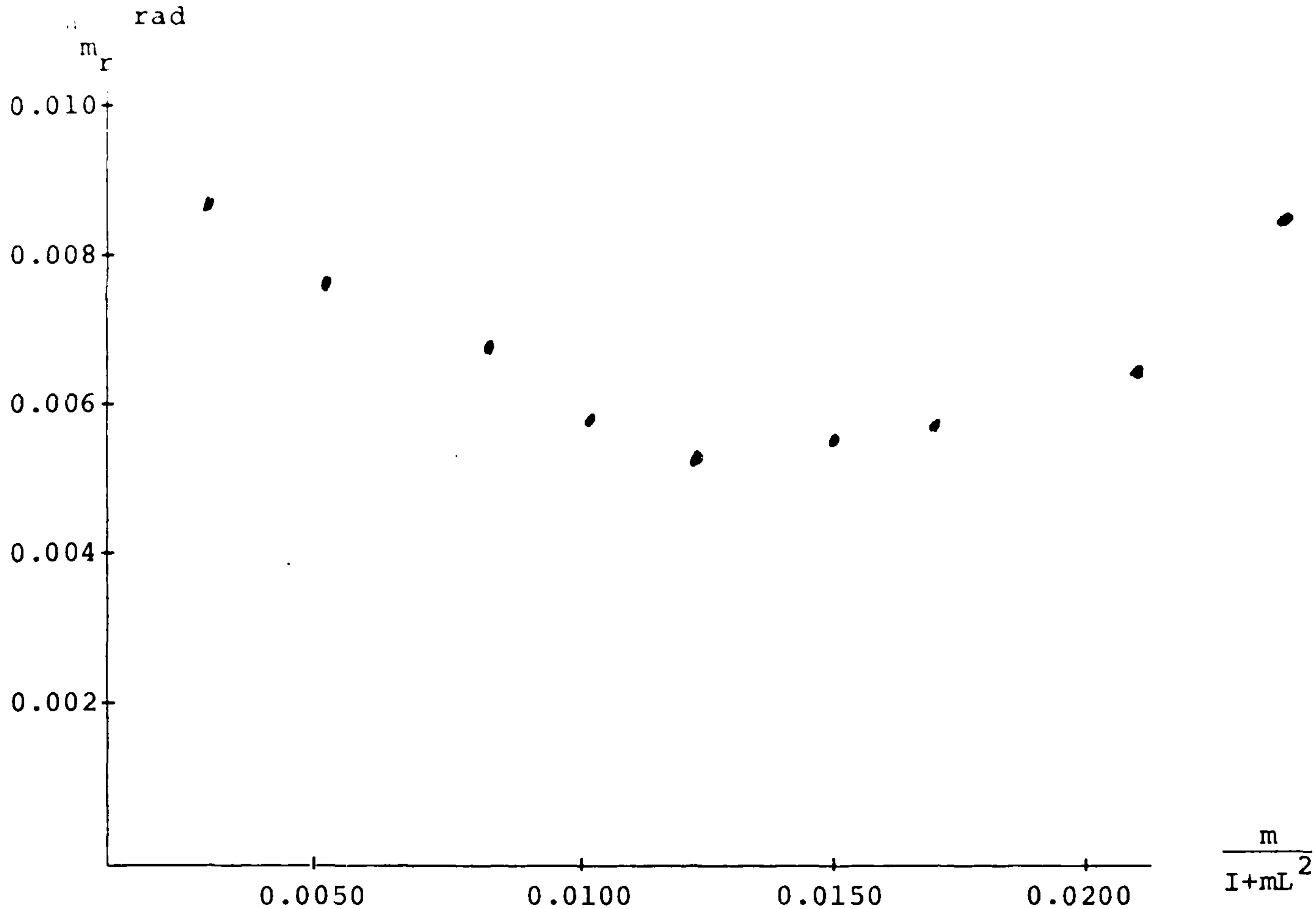


Fig. 18. Residual Amplitude for Different Values of $m/I+mL^2$, with $I = 0.1893$ slug · ft^2, $L = 2.23'$, $d = 3"$, $e = 0.5$, and $\omega_n = 25$ rad/sec

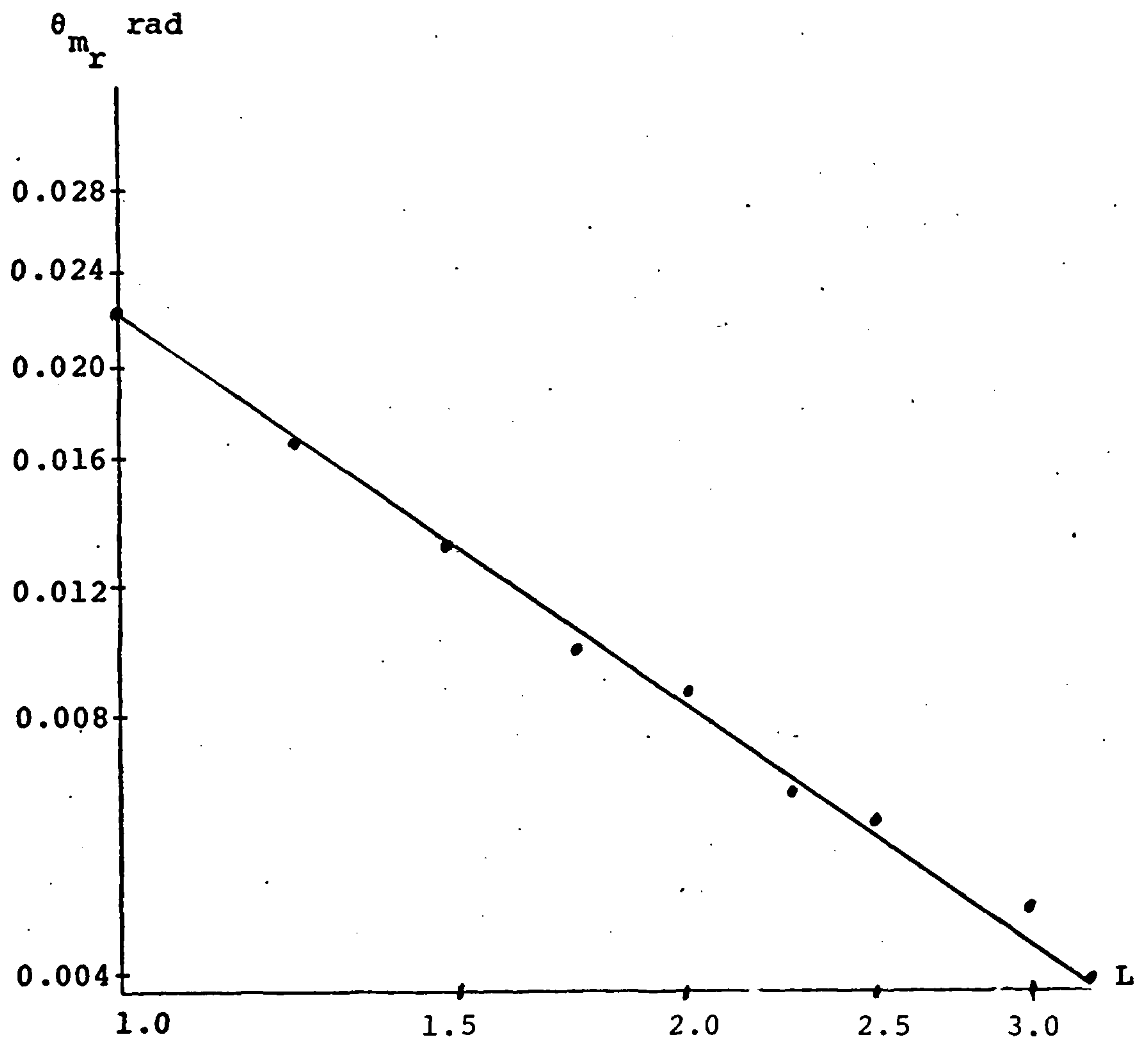


Fig. 19. Residual amplitude for Different Values of the Impact Moment Arm on a Base 10 Logarithmic Scale. $m = 0.001$ slug, $I = 0.1893$ slug $\cdot$ ft^2, $d = 3"$, $e = 0.5$, and $\omega_n = 25$ rad/sec

$$\theta_{m_r} = c_1 + c_2 \left(\frac{L}{1\ \text{ft}}\right)^{-1.35}$$

where L, measured in feet, is divided by one foot to non-dimensionalize it. If we assume that as L approaches infinity, θ_{m_r} approaches zero, or some negligible value, this relation becomes:

$$\theta_{m_r} = 0.022 \left(\frac{L}{1\ \text{ft}}\right)^{-1.35} \tag{26}$$

It is emphasized here that this is an approximation to a function that is not well understood, and this approximation is obviously invalid for small values of L.

Figure 15 shows that, with the exception of one data point, θ_{m_r} is almost exactly linear in the gap d. This one bad data point can easily be explained. The amplitude where the impact damper ceases to function was not always precisely defined in the computer simulation. In many cases there was a sharp transition from where the amplitude declined linearly to where the damping action ceased. However, in some cases the damper transition from effective to ineffective operation took place over one or two cycles, making the determination of θ_{m_r} something of a judgement call. For this reason, occasional variations from what appears to be an otherwise well-defined trend can be expected. In the case of θ_{m_r} versus d, it can be stated with confidence that θ_{m_r} is linear in d.

Figure 16, which is a graph of residual amplitude θ_{m_r} versus coefficient of restitution e, clearly shows that a high value of e is useful in minimizing θ_{m_r}. Unfortunately, a low value of e is desired to maximize damper efficiency s. A quadratic function was fit to the points of Figure 16, using the least squares method to minimize errors. This quadratic function is:

$$\theta_{m_r}(e) = 0.02679 - 0.05132e + 0.0248e^2 \quad (27)$$

This curve comes very close to all of the points plotted, but it is only an approximation to an unknown function relating θ_{m_r} to e.

The plot of θ_{m_r} versus natural frequency ω_n, shown in Figure 17, shows θ_{m_r} to be unaffected by ω_n. By examining the equations of Appendix A upon which the computer simulation is based, it is seen that an increase in ω_n causes a proportional increase in the rate at which the system operates, but does not otherwise affect how it operates. Therefore, it is logical that damper efficiency s would be proportional to ω_n, but that θ_{m_r} would be unaffected by ω_n.

In evaluating Figure 18, the plot of θ_{m_r} versus $m/(I+mL^2)$, it is difficult to envision what occurs as m becomes very large, driving $m/(I+mL^2)$ to the value of $1/L^2$.

In the range of masses varied θ_{m_r} does not change as dramatically as it did when L, d and e were varied. Also, Figure 18 does not suggest a function with which to approximate the θ_{m_r} dependence on $m/(I+mL^2)$. For these reasons, it is noted that there is some dependence of θ_{m_r} upon $m/(I+mL^2)$, but no approximate relationship is given.

Putting together the results of Equations (26) and (27), and using the linearity of θ_{m_r} in d, the residual amplitude can be related to these parameters with:

$$\theta_{m_r} = C_\theta \left[\frac{(1 - 1.914e + 0.925e^2)d}{(L/1\ ft)^{1.35}}\right] \quad (28)$$

For Equation (28) to be in radians, C_θ must have the dimension of 1/ft. This relation is approximate, and totally neglects θ_{m_r}'s dependence upon m. Solving for C_θ for a variety of values of e, d, and L, with L and d measured in feet, the average value of C_θ is $C_\theta = 0.33$ /ft. This given value varied by - 25 percent to +15 percent when calculating it for varied L. This emphasizes that the following relationship gives a very approximate value for θ_{m_r}:

$$\theta_{m_r} = \frac{0.33}{ft}\left[\frac{(1 - 1.914e + 0.925e^2)d}{(L/1\ ft)^{1.35}}\right] \quad (29)$$

Equation (29) is only approximately valid for the range of L, d, and e varied, with m = 0.001 slug. The approximation becomes more uncertain as m is varied, and using a different value for m probably warrants a recalculation of C_θ. C_θ is also only correct if L and d are measured in feet, though it could easily be recalculated for other units.

Forced Vibration

The motion of the beam in forced vibration was measured with an acclerometer mounted just below the damper assembly. The output of the accelerometer had to have high frequency noise filtered out before the visicorder trace of this output would become readable. This noise was serious encugh to drown out the sinusoidal signal expected when the beam was excited without the impact damper. The noise problem was even more serious when the impact damper was in place. Presumably, the accelerometer was picking up the high frequency beam vibrations that caused acoustical noise.

A hand tracing of the filtered accelerometer output is shown in Figure 20, for the motion without the impacting mass, and in Figure 21, for the motion with a 0.00149 slug impacting mass and a total gap of two inches. The actual visicorder traces are given in Appendix B. Figure 21 suggests that there is no simple steady-state motion present. This suspicion was confirmed by the sound of the irregularly occurring impacts. Other than this, the visicorder traces were of little use in a quantitative analysis of the forced system.

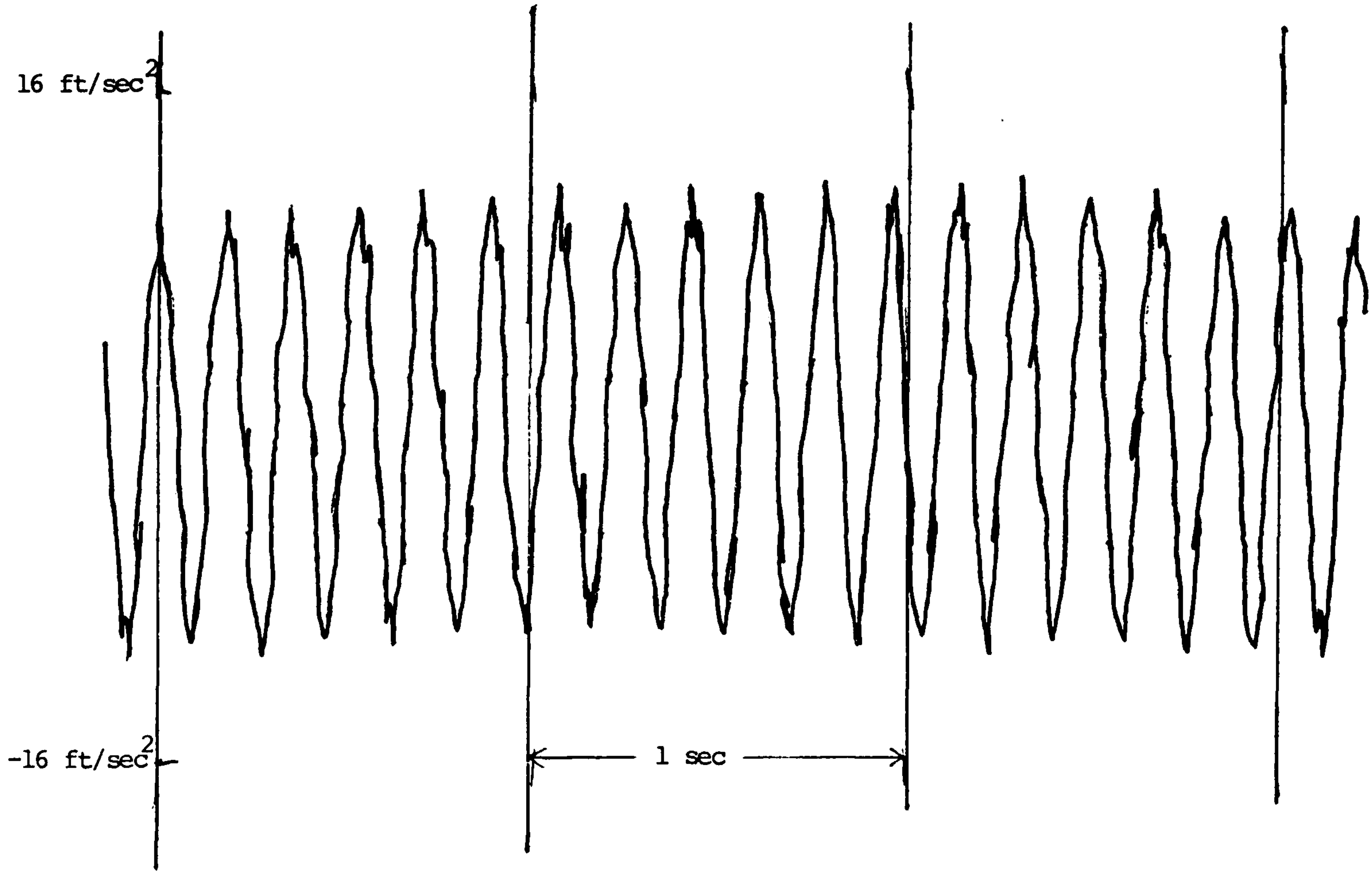


Fig. 20. Trace of Visicorder Output for Forced Vibration Model with no Impacting Mass

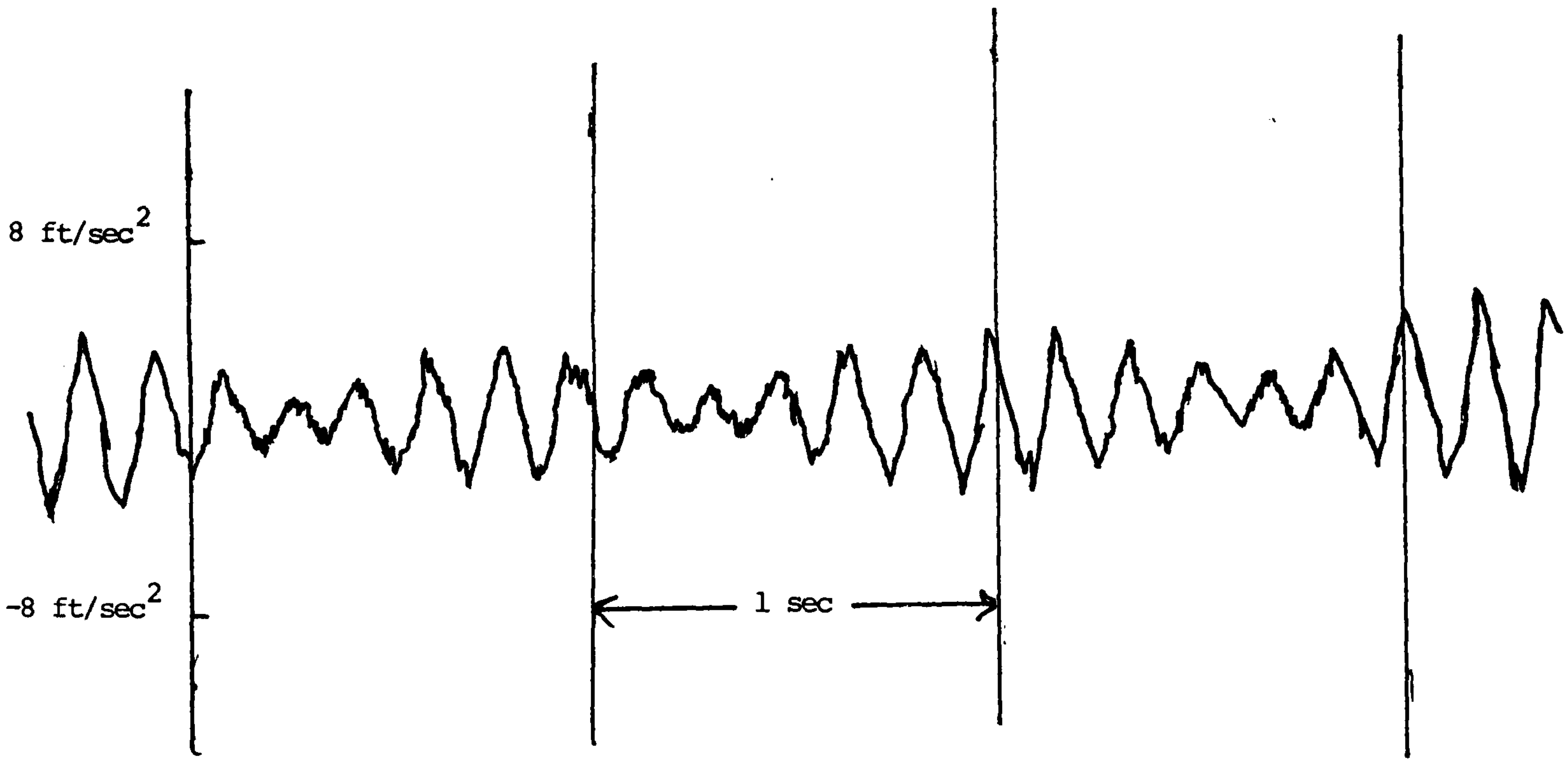


Fig. 21. Trace of Visicorder Output for Force Vibration Impact Damper, Using a 0.00149 Slug Impacting Mass and a 1.31" Gap

Simply observing and listening to the forced system with the damper operating gave some valuable information. The motion of the system was not enough to initiate and sustain impacts unless the system was forced near its first resonant frequency, or unless the gap setting was very small. Higher resonant frequencies caused very low amplitude vibrations. No attempt was made to judge the damper's effectiveness at these resonant frequencies.

At frequencies near the first resonance and gaps greater than one inch the 0.00149 slug mass did reduce the amplitude of the system by a factor of at least two, but it did not approach any detectable steady state operation. For very small gap settings there was a possible steady state reached, but no detectable reduction in amplitude. Using the 0.00503 slug mass at any gap large enough for it to affect the motion of the system led to a very erratic motion of the system with no evident steady state operating state, and no significant sustained reduction in amplitude.

VI. Conclusions and Recommendations

The impact damper shows considerable potential in reducing the free vibration of long, lightly damped structures. It is especially promising for long structures since the effectiveness of the damper increases, and the amplitude at which the damper becomes ineffective decreases, as the damper is moved farther from the rotation point. For a coefficient of restitution of 0.3 or greater, Equation (24) gives a good estimate of the rate at which the oscillations will be reduced, while Equation (29) gives an estimate of the amplitude where the damper becomes ineffective. Even in the ineffective region, where impacts occur sporadically, each impact converts some of the kinetic energy of the system to heat, so only after all impacts cease does the damper become totally ineffective.

A problem with the impact damper is the impacting mass's need for room to travel and stops to impact against. If this cannot be designed into or added onto the structure, without an unacceptable gain in weight or loss of structural strength, the impact damper should not be used. If structural strength is a problem, using a damper with a very low coefficient of restitution might be a solution. While Equation (6) does not hold for e less than 0.3,

Figure 11 indicates that damper efficiency would still be higher than for any e greater than 0.3. However, Figure 16 indicates that the residual amplitude also will be high.

The residual amplitude of the damper can be eliminated using other damping techniques, or, to generalize an idea proposed in Reference (23), by putting two dampers in parallel. One damper would be designed to quickly reduce large amplitude oscillations, while the other would continue to reduce oscillations to a smaller value at a slower rate after the fast damper became ineffective. Another possibility is a system which reduces the gap as the amplitude of the primary system decreases. Maximum damper efficiency could be obtained keeping the gap as large as possible without the damper becoming ineffective; i.e., keep the gap just small enough so impacts are sustained.

Another potentially useful variation on the basic design of the impact damper would be to let the impacting mass travel in two or three dimensions, impacting against a ring or sphere enclosing it. This could be of use in damping out oscillations about more than one axis. This same problem could be attacked by orienting one-dimensional impact dampers along all possible rotation planes, but this could get into weight problems.

Other variations on the impact damper in free vibration would be to replace the impacting mass with many masses or a liquid. Or, the hard stops could be replaced by springs or dashpots. Many of these variations have been studied for the impact damper in forced vibration, but, as has been noted, what maximizes the efficiency of the forced damper does not necessarily maximize the efficiency of the free damper. The basic system studied here shows enough potential to warrant further study.

With the exception of Reference (5), all references on the subject agreed that the impact damper shows great promise in eliminating forced vibrations. (Reference (5) studied impact dampers solely for the purpose of eliminating vibrations in ship's hulls.) The results of this study shows that the impact damper can provide some damping of structures in forced vibration near resonance. However, this damping was sensitive to system parameters, and no steady-state motion was found.

The forced motion of the impact damper needs further study, not only to resolve differences in theories, but also in the laboratory. Laboratory models should be designed not only with the objective of simulating structures of interest, but also with knowledge of the limits of measuring equipment. Many instruments are poorly equipped to handle vibrations of 30 to 35 radians per second.

In summary, the impact damper did not show itself to be effective or predictable in reducing or eliminating force vibrations. However, the impact damper was both effective and predictable in reducing the free vibration of a structure to a certain value. A comparatively small mass can greatly reduce the amplitude of a much larger vibrating structure, in many cases only taking a few cycles to do so. The impact damper's results in reducing free vibrations not only warrants further study, but also warrants careful, cautious consideration for use in current, applicable vibration problems.

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Appendix A

Derivation of Equations

Introduction

The equations of motion and their solutions for the free vibration of a simple one degree of freedom system are well known. The laboratory model, depicted in Figure 22, used to study the affect of the impact damper on free vibration is described by, for the period between impacts, the equations of motion solved for later in this appendix. The model depicted in Figure 23 is a continuous system equivalent to a flexible beam with a lumped mass attached to one end. The free vibration of this type of structure was solved in Reference (25), and is given in Chapter II of this thesis. These solutions can be evaluated if the position and velocity of the system are known for some specified time t, and are used in the computer models of impact dampers in Appendix C.

Position and Velocity Relations

It is assumed that an impact can be modeled as being of infinitely short duration. During this kind of impact, for the system of Figure 24, the position $\theta(t)$ of the beam, and $x(t)$ of the impacting mass remain unchanged,

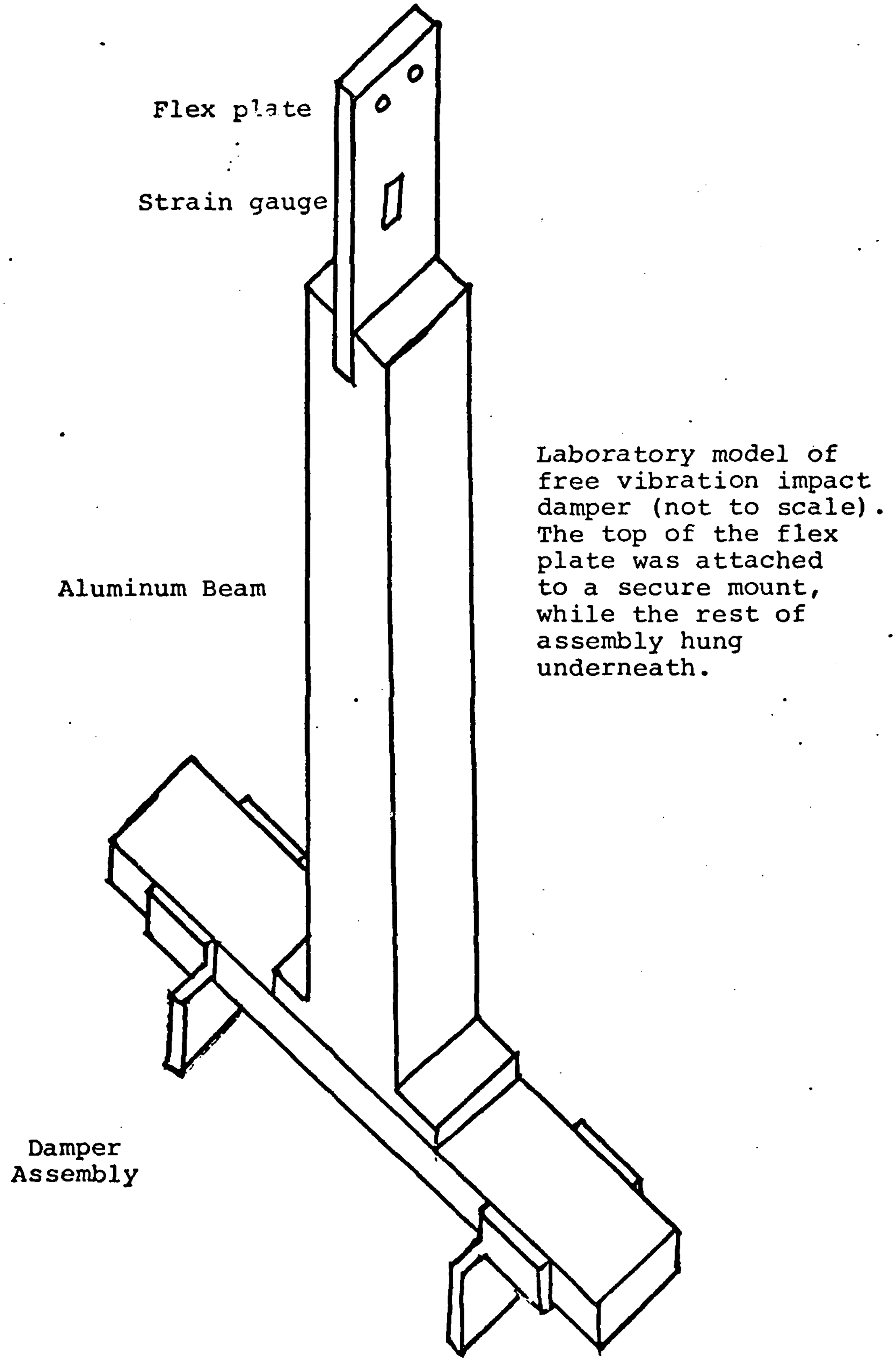
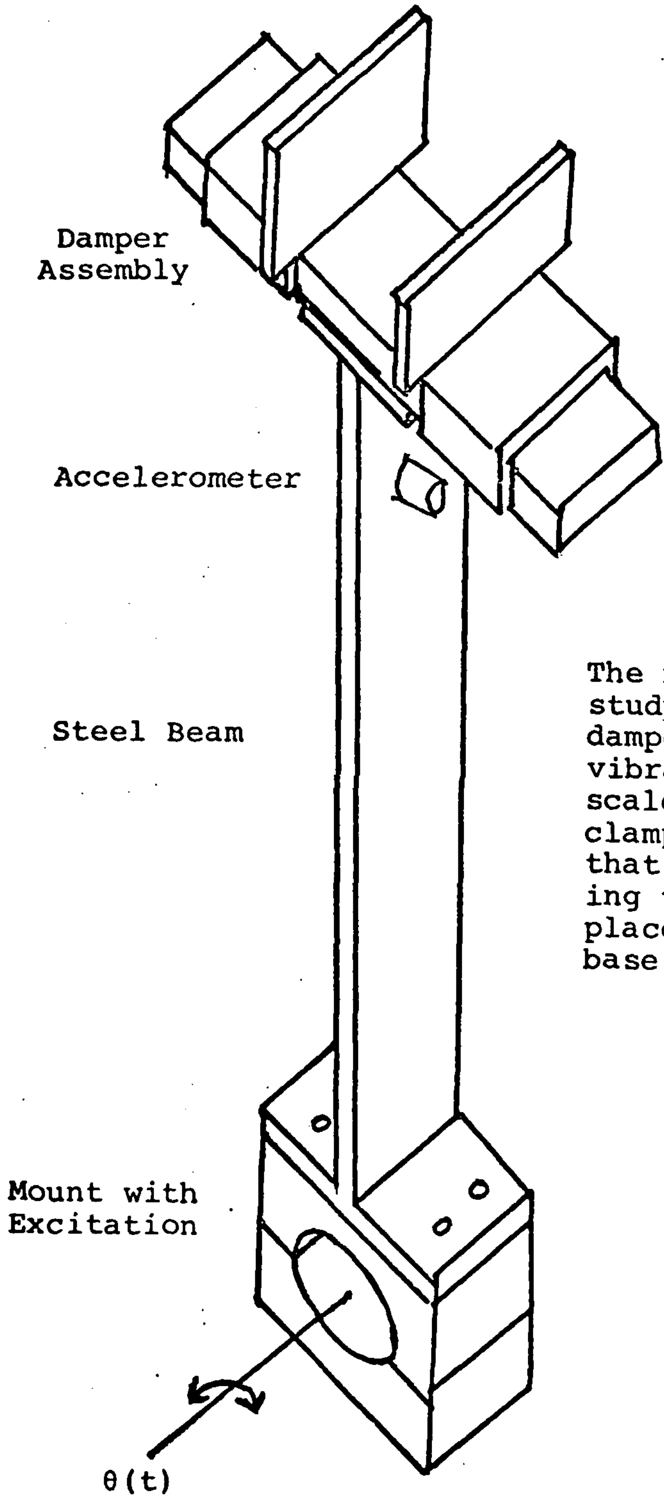


Fig. 22. Model of Impact Damper Used in Free Vibration Tests



The model used to study the impact damper in forced vibration (not to scale). The mount clamped onto a pin that rocked, applying the angular displacement at the base.

Fig. 23. Laboratory Model of Forced Vibration Impact Damper

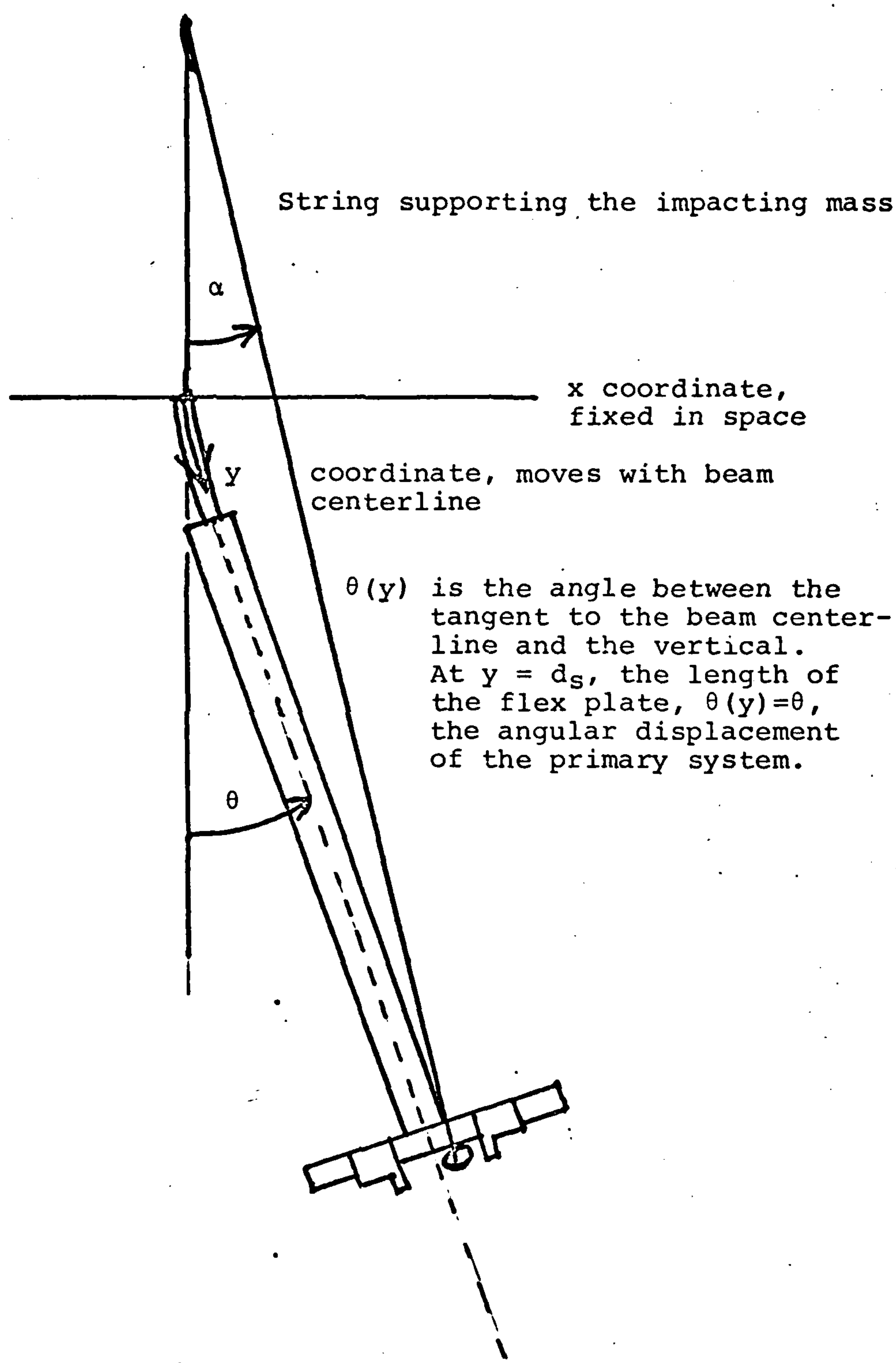


Fig. 24. Coordinates and Angles Used in Deriving the Equations of Motion of the Free Vibration Impact Damper

while their velocities are changed instantaneously from $\dot{\theta}(t_i^{(-)})$ and $\dot{x}(t_i^{(-)}$ to $\dot{\theta}(t_i^{(+)})$ and $\dot{x}(t_i^{(+)})$. While an actual impact is of finite duration, this time is so small compared to the periods of the beam and suspended mass that this assumption is justified. This causes half of the initial conditions to fall out immediately.

$$x(t_i^{(+)}) = x(t_i^{(-)})$$

$$\theta(t_i^{(+)}) = \theta(t_i^{(-)})$$

The remaining two initial conditions can be obtained by the conservation of angular momentum and the velocity condition:

$$e(\dot{x}(t_i^{(-)}) - \dot{\theta}(t_i^{(-)})L) = \dot{\theta}(t_i^{(+)})L - \dot{x}(t_i^{(+)}) \tag{30}$$

Conservation of angular momentum requires that in the absence of external torques the total angular momentum of a system about a fixed point 0 remains constant. There are external torques on this system, so the magnitude of the total angular momentum about point 0 is time dependent, or:

$$H_0 = H_0(t)$$

However, for an impact of infinitely short duration, there is only a momentum exchange within the total system, so:

$$H_0(t_i^{(-)}) = H_0(t_i^{(+)}) \tag{31}$$

It is convenient to take the fixed point 0 to be the point about which the beam rotates. This point is approximately fixed for small angular deflections θ. This will be shown to be true when the motion of the beam is solved later in this appendix. Strictly speaking, if the suspended mass is taken to be a point mass, then the magnitude of its angular momentum about 0 is:

$$H_{0m} = |\bar{r}_{0m} \times m\bar{V}_m| \tag{32}$$

where $\bar{r}_{0m}$ is the vector from point 0 to the mass, and $\bar{V}_m$ is the vector velocity of the mass. Both $\bar{r}_{0m}$ and $\bar{V}_m$ vary with the angular deflection α of the mass, but if α is kept small, then $\bar{r}_{0m}$ and $\bar{V}_m$ are approximately perpendicular. For an infinitely short impact, $\bar{r}_{0m}$ will remain constant in magnitude, so:

$$H_{0m} = mV_m r_{0m} \tag{33}$$

Two further useful substitutions are obtained by noting that for small α's, the vector $\bar{V}_m$ is essentially aligned with the x axis, and the value of r_{0m} is essentially constant; so, writing:

$$V_m \cong \dot{x}(t)$$

and

$$r_{0m} = L$$

the angular momentum of the mass becomes:

$$H_{0m} = m\dot{x}(t)L \tag{34}$$

The substitution for r_{0m} was made to make later equations more readable.

The magnitude of the angular momentum about point 0 of a beam with moment of inertia I is simply:

$$H_{0b} = I\dot{\theta}(t) \tag{35}$$

Conserving angular momentum across impact i results in:

$$I\dot{\theta}(t_i^{(-)}) + m\dot{x}(t_i^{(-)})L = I\dot{\theta}(t_i^{(+)}) + m\dot{x}(t_i^{(+)})L \tag{36}$$

The velocity relations can be rewritten as:

$$\dot{\theta}(t_i^{(+)}) = \frac{1}{L}\{\dot{x}(t_i^{(+)}) + e[\dot{x}(t_i^{(-)}) - \dot{\theta}(t_i^{(-)})L]\} \tag{37}$$

and

$$\dot{x}(t_i^{(+)}) = \dot{\theta}(t_i^{(+)})L - e[\dot{x}(t_i^{(-)}) - \dot{\theta}(t_i^{(-)})L] \tag{38}$$

Using the Equation (37) in Equation (36) and rearranging gives:

$$\dot{x}(t_i^{(+)}) = \frac{L}{I+mL^2} [\dot{\theta}(t_i^{(-)}) I(1+e) + \dot{x}(t_i^{(-)})(mL - \frac{Ie}{L})] \quad (39)$$

Similarly, using Equation (38) in Equation (36) gives:

$$\dot{\theta}(t_i^{(+)}) = \frac{1}{I+mL^2} [\dot{\theta}(t_i^{(-)})(I - mL^2 e) + \dot{x}(t_i^{(-)}) mL(1+e)] \quad (40)$$

These relations, along with:

$$\theta(t_i^{(+)}) = \theta(t_i^{(-)}) \quad (41)$$

$$x(t_i^{(+)}) = x(t_i^{(-)}) \quad (42)$$

give the initial conditions for the motion between impacts i and i+1 in terms of the final conditions between i-1 and i.

Motion of the Vibrating Beam

The equation of motion for a rigid beam rotating about a fixed point 0 can be obtained from the relation:

$$\Sigma \text{ Moments} = I\ddot{\theta} \quad (43)$$

The beam in question is suspended by a flex plate of length d_s, shown in Figure 25. This spring can be said to apply a moment of magnitude M at the top of the beam. This moment can be obtained from an angle θ from the rest (undeformed) position of the spring steel using the relation:

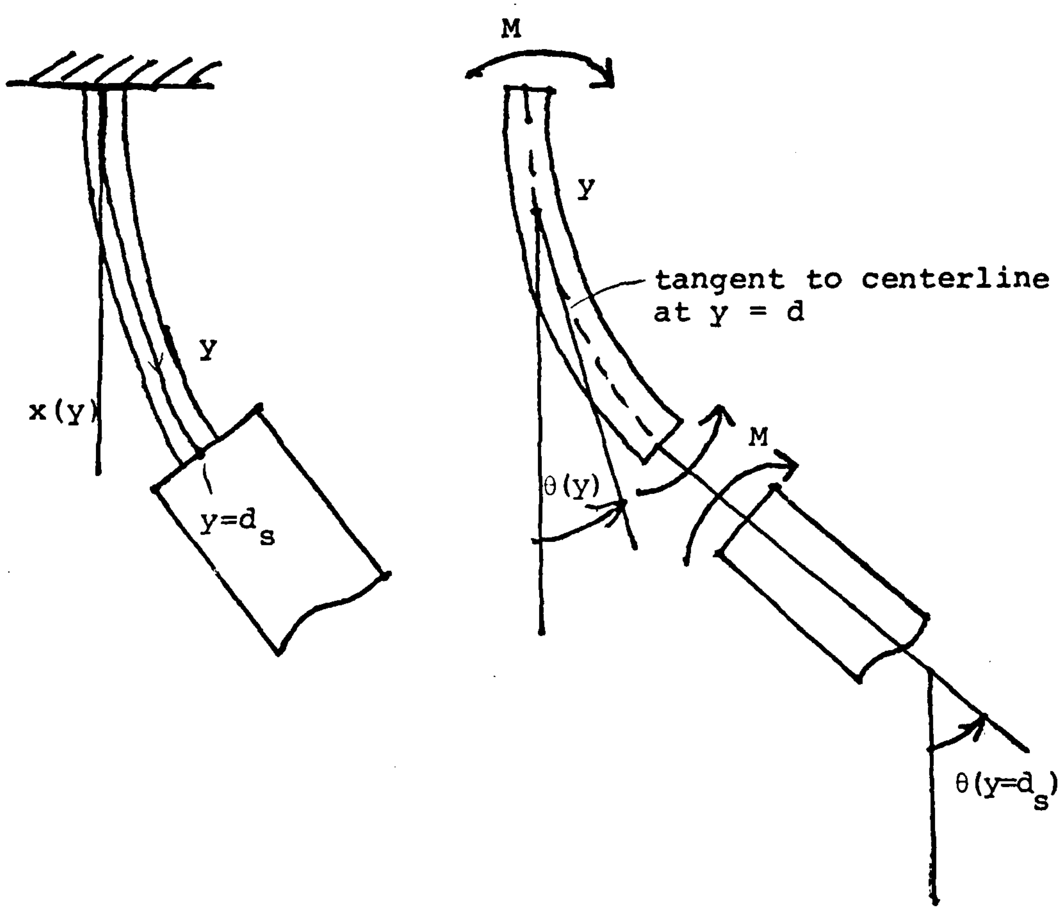


Fig. 25. Free Body Diagram and Coordinates used in Deriving Equations of Motion of the Free Vibration Impact Damper

$$\frac{d^2x}{dy^2} = \frac{d\theta}{dy} = \frac{M(y)}{EI'} \qquad (44)$$

where M(y) is the moment of distance y along the spring, and the x and y coordinates have been reversed from Reference (26). This equation assumes small angles θ, and so assumes small deflection in the x direction of the beam.

The free body diagram shown in Figure 25 shows that the beam moment is approximately independent of y, so integrating Equation (44) once gives:

$$\theta(y) = \frac{My}{EI_s}, \qquad (45)$$

and integrating once again gives:

$$x(y) = \frac{1}{2}\frac{My^2}{EI_s}, \qquad (46)$$

At $y = d_s$, $\theta(y)$ is the angular displacement of the primary system. Evaluating θ and x at $y = d_s$ gives:

$$\theta(d_s) = \frac{Md_s}{EI_s}, \qquad (47)$$

$$x(d_s) = \frac{1}{2}\frac{Md_s^2}{EI_s}, = \frac{1}{2}\theta d_s \qquad (48)$$

If the aluminum beam oscillates about a fixed point 0 a distance r_s from the top of the beam, then for small values of θ:

$$r_s\ \theta \cong r_s(\sin\theta) = x(d_s) = \frac{1}{2}\theta d_s \qquad (49)$$

so $r_s = \frac{1}{2}d_s$, and the beam rotates about a point located at the center of the spring's undeformed position.

The moment M_s applied to the beam by the spring is opposite in sign to the moment at the end of the spring. Using Equation (47), this gives:

$$M_s = \frac{EI_s'\theta(d_s)}{d_s} = -\frac{EI_s'\theta}{d_s} \qquad (50)$$

$\theta = \theta(d_s)$ is the deflection from vertical of the beam. Gravity also causes a moment M_g:

$$M_g = -W_T\ r_{c.g.}\ \sin\theta \cong -W_T\ r_{c.g}\theta \qquad (51)$$

where W_T is the total weight of the primary system, and $r_{c.g}$ is the distance from the center of gravity of the primary system to the rotation point 0. Also, there is a moment due to damping that resists the motion of the system. This damping moment M_d is assumed to be proportional to the angular velocity, so it can be defined as:

$$M_d = c\dot{\theta} \qquad (52)$$

The equation of motion of the system then becomes:

$$I\ddot{\theta} = -c\dot{\theta} - W_T\ r_{c.g}\ \theta - \frac{EI_s'\theta}{d_s} \qquad (53)$$

or:

$$I\ddot{\theta} + c\dot{\theta} + k\theta = 0 \qquad (54)$$

where

$$k = W_T \, r_{c.g.} + \frac{EI_s'}{d_s} \qquad (55)$$

By Reference (27), the solution to $\theta(t)$ can be written:

$$\theta(t) = e^{(-\frac{c}{2I}t)} \, (A \sin qt + B \cos qt)$$

where A and B are constants depending on initial conditions, and:

$$q = \sqrt{\frac{k}{I} - \left(\frac{c}{2I}\right)^2} \qquad (56)$$

Since we are interested in the motion of the system between impact i at time t_i and impact i+1 at time t_{i+1}, the solution is more conveniently written in terms of time $t=t_i+\Delta t$ where Δt ranges from 0 to $t_{i+1}-t_i$. In this case:

$$\theta(\Delta t) = e^{(-\frac{c}{2I}\Delta t)} \, (A \sin q\Delta t + B \cos q\Delta t) \qquad (57)$$

If the angular position and velocity of the primary system and the velocity of the impacting mass are known immediately before impact i at time $t_i^{(-)}$, then $\theta(t_i^{(+)})$ and $\dot{\theta}(t_i^{(+)})$ are given by Equations (40) and (41).

Using these, the constants A and B can be solved, and the position and velocity of the system at time Δt as Δt ranges from t_i to t_{i+1} is:

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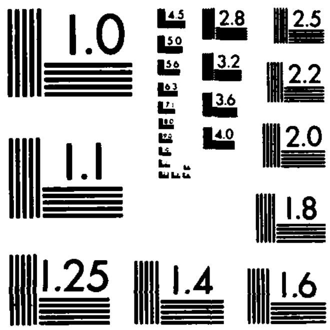


MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS-1963-A

$$\theta(\Delta t) = e^{\left(-\frac{c}{2I}\Delta t\right)}\left[\left(\frac{\dot{\theta}(t_i^{(+)})}{q} + \frac{\theta(t_i^{(+)})c}{2Iq}\right)\sin q\Delta t + \theta(t_i^{(+)})\cos q\Delta t\right] \quad (58)$$

$$\dot{\theta}(\Delta t) = \left(-\frac{c}{2I}\right)e^{\left(-\frac{c}{2I}\Delta t\right)}\left[\left(\frac{\dot{\theta}(t_i^{(+)})}{q} + \frac{\theta(t_i^{(+)})c}{2Iq}\right)\sin q\Delta t + \theta(t_i^{(+)})\cos q\Delta t\right] + e^{\left(-\frac{c}{2I}\Delta t\right)}\left[\left(\dot{\theta}(t_i^{(+)}) + \frac{\theta(t_i^{(+)})c}{2I}\right)\cos q\Delta t - q\theta(t_i^{(-)})\sin q\Delta t\right] \quad (59)$$

If the system is undamped these equations become:

$$\theta(\Delta t) = \frac{\dot{\theta}(t_i^{(+)})}{q}\sin q\Delta t + \theta(t_i^{(+)})\cos q\Delta t \quad (60)$$

$$\dot{\theta}(\Delta t) = \dot{\theta}(t_i^{(+)})\cos q\Delta t - q\theta(t_i^{(+)})\sin q\Delta t \quad (61)$$

Motion of the Secondary System

The motion of the secondary system, which in the laboratory was simply a steel sphere suspended by nylon thread, is most easily obtained by:

$$\Sigma \text{ Moments} = I_m\ddot{\alpha} \quad (62)$$

In this case, the inertia I_m is simply:

$$I_m = mL_m^2 \quad (63)$$

The only moments are from the α component of gravity:

$$M_g = -mgL_m \sin \alpha \cong - mgL_m\alpha \tag{64}$$

and the moment due to damping:

$$M_d = - c_\alpha\dot{\alpha} \tag{65}$$

so:

$$mL_m{}^2\ddot{\alpha} = - c_\alpha\dot{\alpha} - mgL_m\alpha \tag{66}$$

In order to conveniently go from treating the mass as a damped pendulum to treating the mass as being free from all forces except the impacts, it is helpful to note that:

$$x = L_m \sin_\alpha \cong L_m\alpha \tag{67}$$

$$\dot{x} = \dot{\alpha}L_m \cos \alpha \cong \dot{\alpha}L_m \tag{68}$$

$$\ddot{x} = \ddot{\alpha}L_m \cos \alpha - \dot{\alpha}^2L_m \sin \alpha \cong \ddot{\alpha}L_m \tag{69}$$

This implies that for small values of α, little accuracy is lost by assuming all motion is in the x direction, so Equation (66) can be written:

$$mL_m\ddot{x} = - c_\alpha\frac{\dot{x}}{L_m} - mgx$$

or

$$m\ddot{x} + c_m\dot{x} + k_m x = 0 \tag{70}$$

where:

$$c_m = \frac{c_\alpha}{L_m^2} \quad (71)$$

$$k_m = \frac{mg}{L_m} \quad (72)$$

As was done for the primary system, x can be solved for according to Reference (27), giving:

$$x(t) = e^{(-\frac{c_m}{2m}\Delta t)}(A_m \sin q_m t + B_m \cos q_m t)$$

where

$$q_m = \sqrt{\frac{k_m}{m} - (\frac{c_m}{2m})^2} \quad (73)$$

and, writing this in terms of Δt, which ranges from time t_i to time t_{i+1}:

$$x(\Delta t) = e^{(-\frac{c_m}{2m}\Delta t)}(A_m \sin q_m\Delta t + B_m \cos q_m\Delta t) \quad (74)$$

A_m and B_m can be solved here using Equations (39) and (42) the same way A and B were solved earlier for the beam. Doing this, the position and velocity of the impacting mass becomes:

$$x(\Delta t) = e^{(-\frac{c_m}{2m}\Delta t)}\left[\left(\frac{\dot{x}(t_i^{(+)})}{q_m} + \frac{x(t_i^{(+)})c_m}{2mq_m}\right)\sin q_m\Delta t + x(t_i^{(+)})\cos q_m\Delta t\right] \quad (75)$$

$$\dot{x}(\Delta t) = -\frac{c_m}{2m} e^{\left(-\frac{c_m}{2m}\Delta t\right)} \left[\left(\frac{\dot{x}(t_i^{(+)})}{q_m} + \frac{x(t_i^{(+)})c_m}{2mq_m}\right) \sin q_m\Delta t + x(t_i^{(+)}) \cos q_m\Delta t\right]$$

$$+ e^{-\left(\frac{c_m}{2m}\Delta t\right)} \left[\left(\dot{x}(t_i^{(+)}) + \frac{x(t_i^{(+)})c_m}{2m}\right) \cos q_m\Delta t\right] - x(t_i^{(+)})q_m \sin q_m\Delta t \quad (76)$$

In the undamped case these equations become:

$$x(\Delta t) = \frac{\dot{x}(t_i^{(+)})}{q_m} \sin q_m\Delta t + x(t_i^{(+)}) \cos q_m\Delta t \quad (77)$$

$$\dot{x}(\Delta t) = \dot{x}(t_i^{(+)}) \cos q_m\Delta t - x(t_i^{(+)})\, q_m \sin q_m\Delta t \quad (78)$$

and if the motion of the mass depends only upon the impacts, i.e., no gravity or damping forces:

$$x(\Delta t) = x(t_i^{(+)}) + \dot{x}(t_i^{(+)})\Delta t \quad (79)$$

$$\dot{x}(\Delta t) = \dot{x}(t_i^{(+)}) = \text{constant} \quad (80)$$

Appendix B

Laboratory Models

Introduction

This appendix describes in detail the laboratory models and equipment used. The conversions from measured quantities to actual displacements are developed, as well as the methods used to indirectly measure some of the systems parameters.

Free Vibration Model

The model used for the free vibration experiments is depicted in Figure 22 with the important dimensions, masses, and properties given in Figure 26. The 1/8" x 1" x 4-1/2" steel beam used as a flex-plate (note that only 3" of its length was free to bend as a spring) was attached by screws to a support depicted in Figure 27. This support was bolted onto a 1-1/2" x 40" x 42" steel plate which was itself bolted to 12" x 12" I-beams which extended up from the building's foundation.

Using Equation (50), with $E = 28 \times 10^6$ lb $\cdot$ in^2 and $I = 1/6144$ in^4, the moment applied by the spring onto the aluminum beam was calculated as 1519θ lb $\cdot$ in. The moment of inertia of the beam and damper assembly was calculated by modeling the damper assembly as a point mass

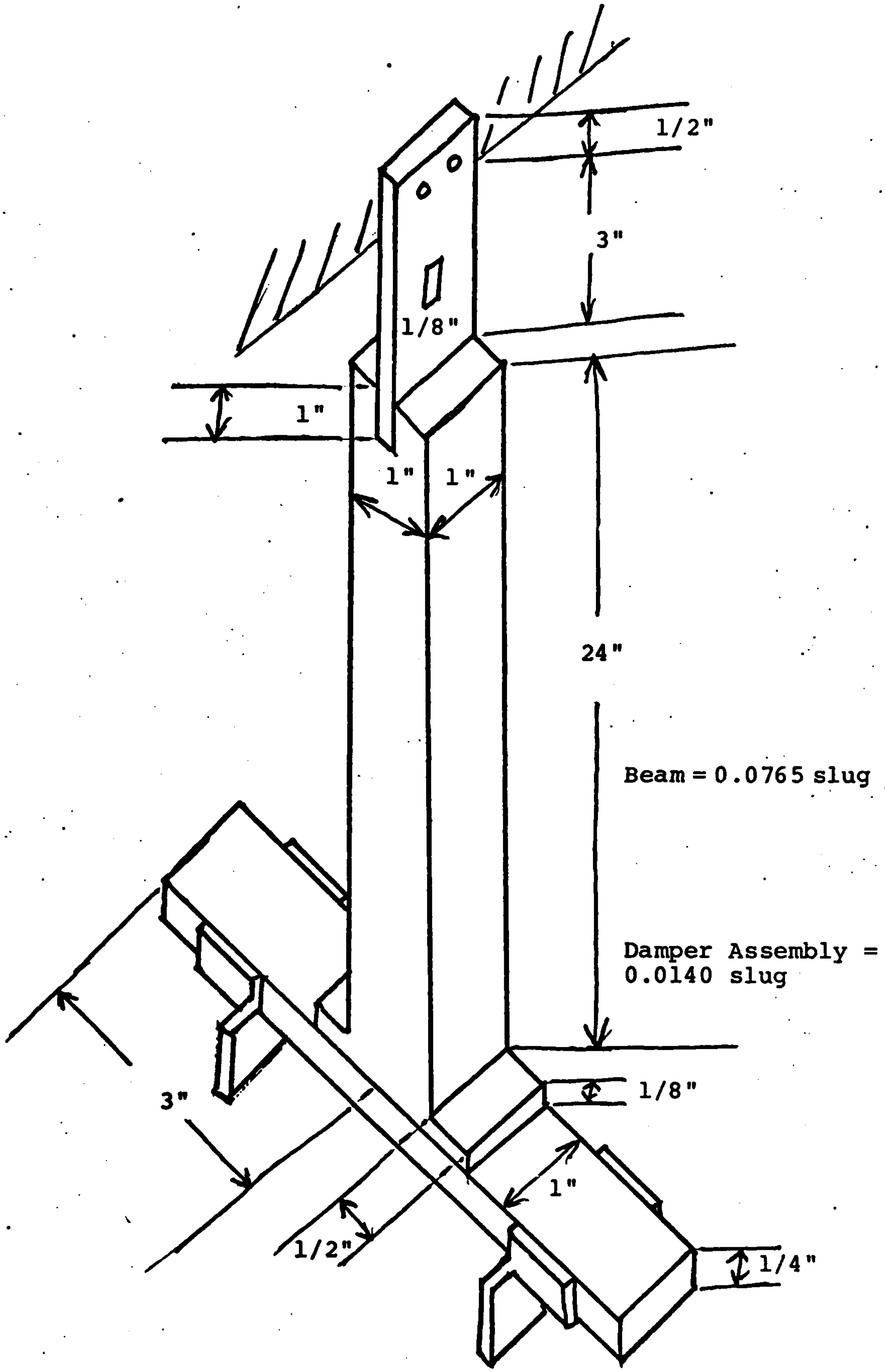


Fig. 26. Free Vibration Impact Damper Used in Laboratory with Important Parameters

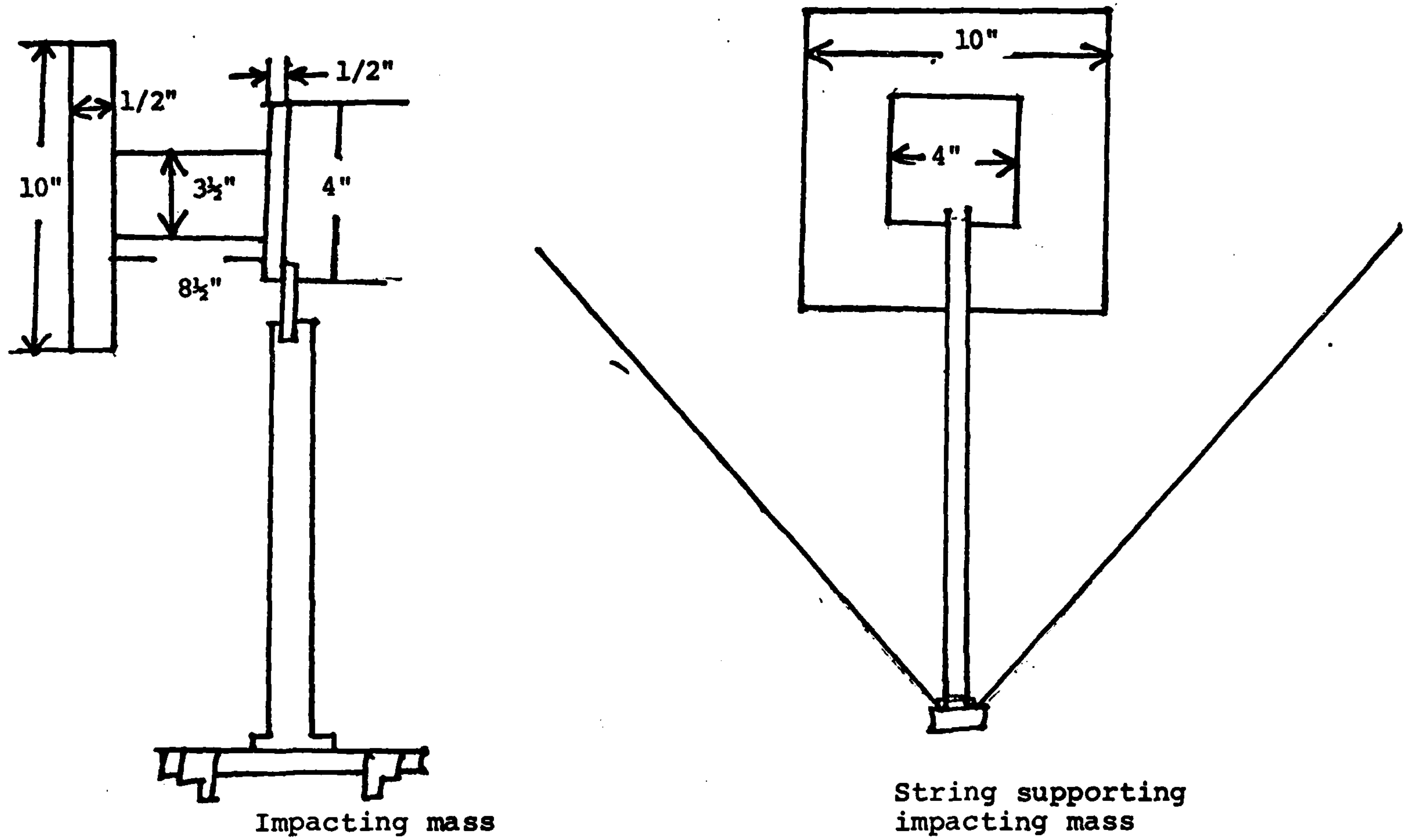


Fig. 27. Support for the Free Vibration Laboratory Model. The 10" x 10" portion of the mount was bolted to a 1½" x 40" x 42" plate supported by 12" by 12" I-beams.

26" from the assumed rotation point; this gave I = 0.1882 slug · ft^2. The moment of inertia was then measured by hanging the beam and damper assembly, minus the steel spring, from 1.5" of nylon fishing line and timing it through a number of cycles as it swung as a pendulum. Neglecting any damping, the equation of motion of this system is:

$$I\ddot{\theta} + rM_T g\theta = 0 \quad (81)$$

where r is the distance from the rotation point to the center of gravity of the assembly, and M_T is the total mass of the assembly. From this, the natural frequency of the system is:

$$\omega_n = \sqrt{\frac{rM_T g}{I}} \quad (82)$$

ω_n was measured as 4.45 rad/sec and M is 0.08983 slug, r was calculated to be 15.4545", and g was taken as 32.174 ft/sec^2. I can then be solved for using:

$$I = \frac{rM_T g}{\omega_n^2} \quad (83)$$

This gave I = 0.1880 slug · ft^2. This value of I was used in all calculations.

Using $\omega_n = \sqrt{k/I}$, Equation (55), and $r_{c.g.} = 15.7648$, the natural frequency of the beam, was calculated to be

26.34 rad/sec. When using a damping factor $c \cong 0.02$, which will be justified later in this appendix, the damped frequency was essentially the same. The actual frequency observed was measured as 25.9 rad/sec. The difference was assumed to be the result of small inaccuracies in the measured quantities. For the purpose of the computer simulation, the measured values of inertia and frequency were used.

The damper assembly consisted of an aluminum bar, 1/4" x 1" x 6", attached to the bottom of the 1" x 1" x 24" aluminum beam. The stops were made of steel and could be attached anywhere along the aluminum bar, and were mounted as shown in Figure 28. The steel balls used as impacting masses were hung by nylon fishing lines attached to points 84" above the damper assembly, with one attachment 46" to the right of the damper assembly and the other 50" to the left. The mass was hung as a pendulum to minimize forces other than impact.

The motion of the beam was measured with two SR-4 Type AD-7, Lot #B-32, strain gages attached to the steel spring, centered on either side. These strain gages had a gage factor of 1.96 ± 2 percent. The strain gages were connected to a Q-amp, serial number 002578 which was installed in a Type 535A oscilloscope. The oscilloscope trace was photographed using an oscilloscope camera C-12. The peak-to-peak amplitudes on the photograph were then

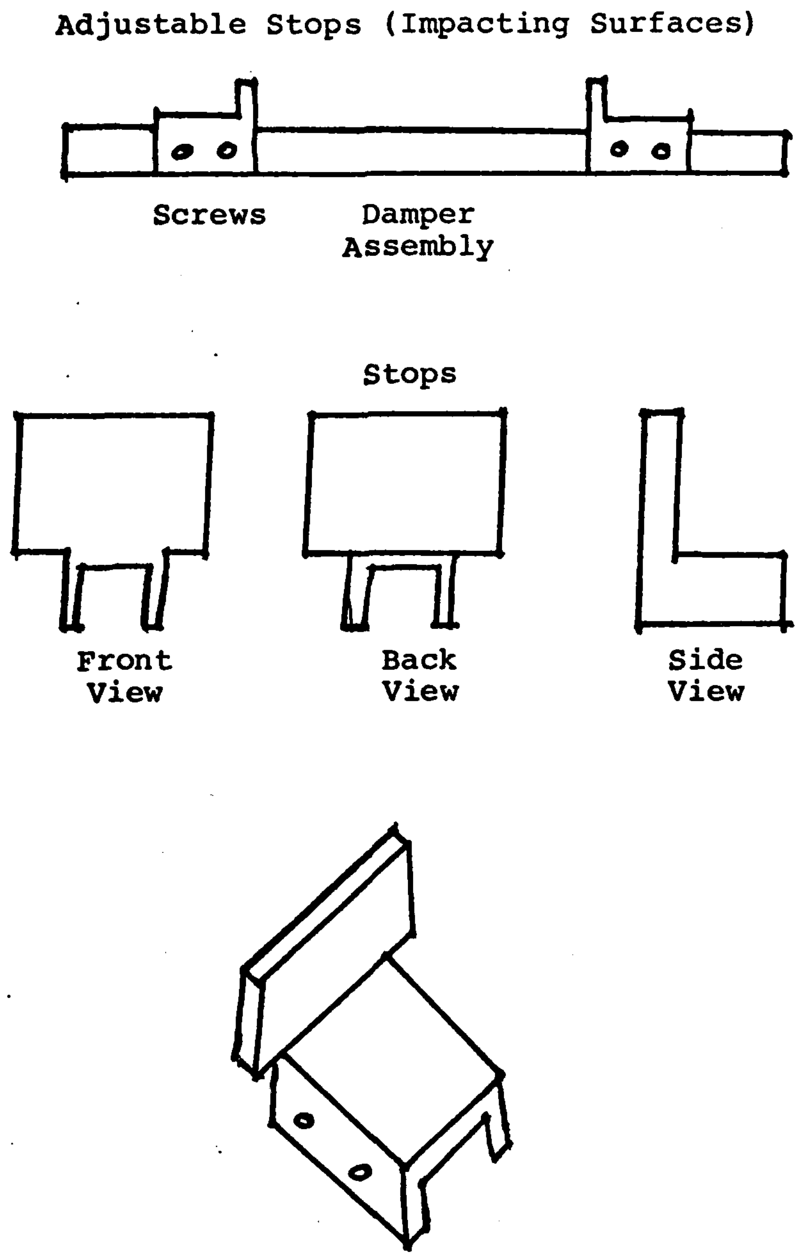


Figure 28. Details of Damper Assembly and Impacting Surfaces

measured in inches by a traveling microscope. These measurements were divided by the measured division size on the photograph to give the amplitudes in scope divisions. The number of scope divisions was multiplied by the Q-amp setting to give the total strain of the two strain gages had they had a gage factor of two. Since the gage factor was 1.96, and only the strain on one side of the steel spring was desired, this measured strain ε_m was converted to the actual strain ε using:

$$\varepsilon = \frac{1}{2}\left(\frac{2.0}{1.96}\right) \varepsilon_m \qquad (84)$$

This strain can be converted into radians of displacement using:

$$\varepsilon = -\frac{Mu}{EI_s} \qquad (85)$$

from Reference (26), where u = 1/16" is the distance from the neutral surface of the spring, and M is the moment calculated as 1519θ lb · in. This gives

$$\varepsilon = 0.02083\theta$$

$$\varepsilon = (20{,}830\ \mu''/'')\theta \qquad (86)$$

So, for an ε given by Equation (86), the angular displacement θ of the beam would be:

$$\theta = \frac{\varepsilon}{20,830} = (4.80 \times 10^{-5})\varepsilon \qquad (87)$$

where ε is given in micro-inches/inch, and θ is in radians.

Measurements were made by manually deflecting the beam, or primary system, until the oscilloscope trace was at the desired position on the screen. The oscilloscope was then set to make only one sweep when triggered. The sweep rate for most measurements was 0.5 sec/div or 0.2 sec/div. The oscilloscope camera was then fastened into position and the lens opened. In rapid succession the primary system was released and the oscilloscope was triggered, so the camera photographed one oscilloscope trace. The amplitude of the photographed cycles was measured in inches using a traveling microscope. These amplitudes were converted to scope divisions by dividing by the measured division width, then converted to strain by multiplying by the Q-amp strain setting. Equation (82) was then used to get the maximum angular displacement per cycle. s was obtained from these displacements using a linear least squares fit.

The natural damping of the beam assembly was measured by deflecting the beam a desired amount and allowing it to vibrate freely with no impacting mass in place. The photographed oscilloscope trace was then used to measure its damping using the log decrement method. From this, the damping factor c of Equation (54) was calculated using:

$$\frac{c}{I} = 2\zeta\omega_n \tag{88}$$

where

$$\zeta = \frac{\delta}{\sqrt{(2\pi)^2 + \delta^2}} \tag{89}$$

and

$$\delta = \frac{1}{j-1} \ln\left(\frac{A_1}{A_j}\right) \tag{90}$$

While the value of c obtained this way was always small, it was not constant. On a given day, ζ appeared to be a linear function of the initial displacement, but this linear function changed from day to day. This is illustrated in Figure 29. For this reason, $\zeta = 0.002$ was taken as giving a good average value for the amplitude at which most measurements were taken with the impact damper operating. This ζ gave $c \cong 0.02$ lb · ft · sec. This value of c was used in the computer simulation of the laboratory model.

In order to measure both the damping on the impacting mass, and the coefficient of restitution e between the mass and the stops, the position of the impacting mass had to be measured without interfering with its motion. This was done by mounting two pieces of white poster board 8" to the right of the beam assembly, facing the beam assembly. Horizontal and vertical lines were drawn at 1" intervals

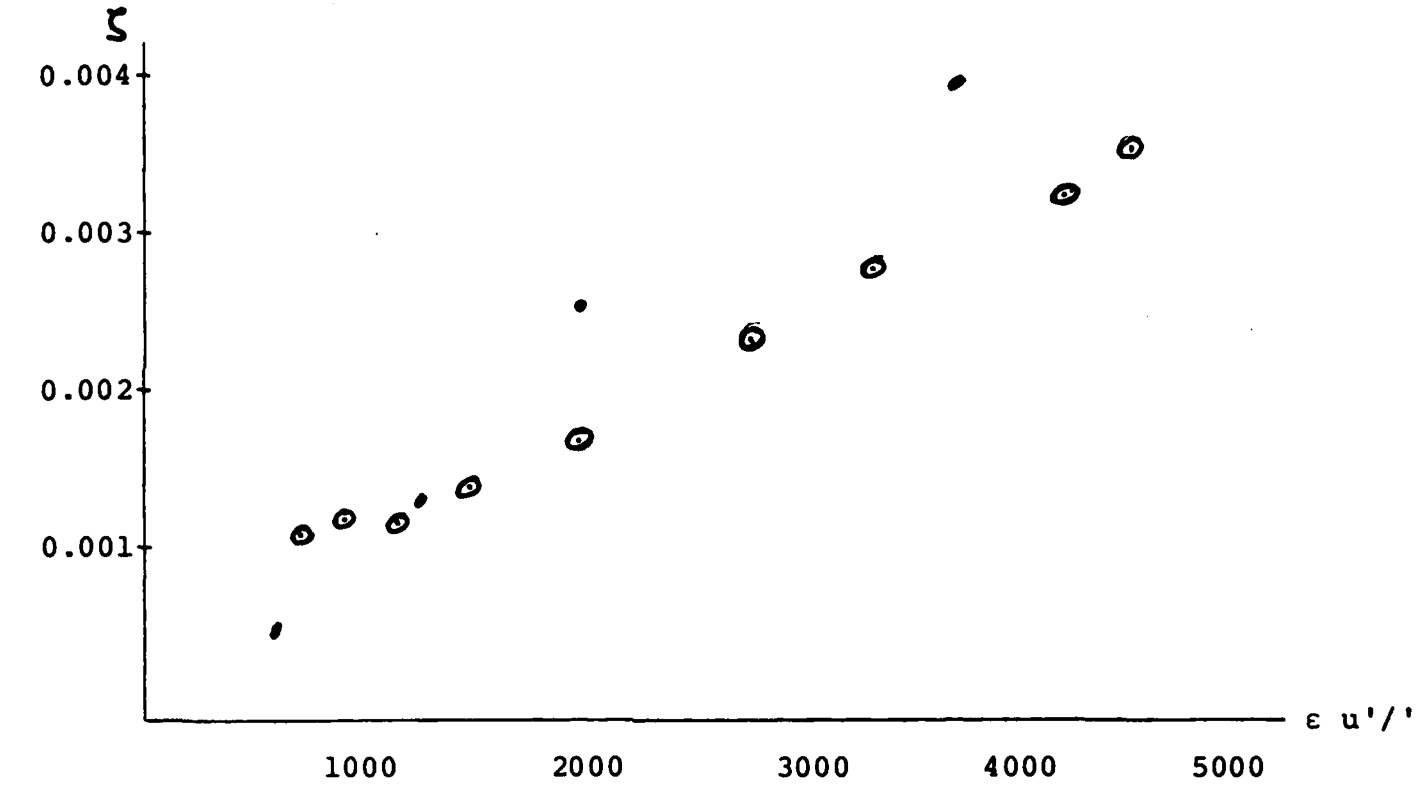


Fig. 29. Measured Values of δ for Different Values of Strain

· = 27 January 1983 results;
⊙ = 1 February 1983 results.

across the poster board. Twenty-six feet to the left of the beam assembly, a lamp was pointed towards the assembly. With all other room lights dimmed, the impacting mass cast a sharp shadow upon the poster board. The impacting mass was removed from between the stops, and one of the stops was placed on the end of the damper beam, facing out. By watching the impacting mass's shadow on the poster board, the mass could be released from a known position and strike the stop at a known position. Assuming negligible damping, the velocity of the mass before impact can be calculated using:

$$mg\Delta h = \frac{1}{2}mV^2 \tag{91}$$

where Δh is the difference between the mass's height at release and its height at impact. The distance that the beam travels due to the impact can be determined from the oscilloscope trace, this maximum angular deflection times the natural frequency of the beam gave its maximum angular velocity, which occurred immediately after the impact. The velocity of the impacting mass after impact was calculated by using conservation of momentum, Equation (36). The coefficient of restitution, then, becomes:

$$e = -\frac{\dot{\theta}^{(+)}L - V^{(+)}}{\dot{\theta}^{(-)}L - V^{(-)}} \tag{92}$$

where (+) implies immediately after the impact, (-) implies immediately before the impact, L is the distance from the rotation point of the beam to the impact height, and $\dot{\theta}^{(-)} = 0$. Using this method the coefficient of restitution for the steel balls striking the steel stop was found to be between 0.40 and 0.50, as is seen in Table 2.

TABLE 2

QUANTITIES FOR CALCULATION OF COEFFICIENT OF RESTITUTION

M	Δh	$V^{(-)}$	ε	θ_m	$\dot{\theta}^{(+)}$	$V^{(+)}$	e
0.000481	11-3/4	7.94	100	0.00254	0.0657	-3.80	0.50
0.00149	11-1/2	7.85	280	0.00711	0.1841	-2.77	0.40
0.00503	11-3/8	7.81	900	0.0228	0.592	-2.31	0.46

Data from which coefficient of restitution e is calculated. m is in slugs, Δh is in inches, $V^{(-)}$ and $V^{(+)}$ are in feet/sec, ε is in micro-inches/inch, θ_m is in radians, and $\dot{\theta}^{(+)} = \theta_m \omega_n$ is in radians/sec.

The same lamp and poster board arrangement was used to measure the damping factor on the impacting mass, but without the beam assembly in place. The mass was displaced to a known position and released. After a known number of oscillations, its maximum amplitude was noted and its damping c was solved using:

$$\frac{c}{m} = 2\zeta\omega_n \qquad (93)$$

where:

$$\zeta = \frac{\delta}{\sqrt{(2\pi)^2 + \delta^2}}$$

$$\delta = \frac{1}{j-1} \ln \left(\frac{A_1}{A_j} \right)$$

$$\omega_n = \sqrt{\frac{g}{L}} \qquad (94)$$

and L = 84" is the difference in height from where the mass is at rest and the points from which it is suspended. The damping measured in this way had an amplitude dependence; ζ increased with amplitude. The values of c were calculated for low amplitudes to get the best correlation with the position of the mass when it was used for impact damping. The data and resulting values of c obtained are shown in Table 3.

TABLE 3

QUANTITIES FOR CALCULATION OF VISCOUS DAMPING FACTOR

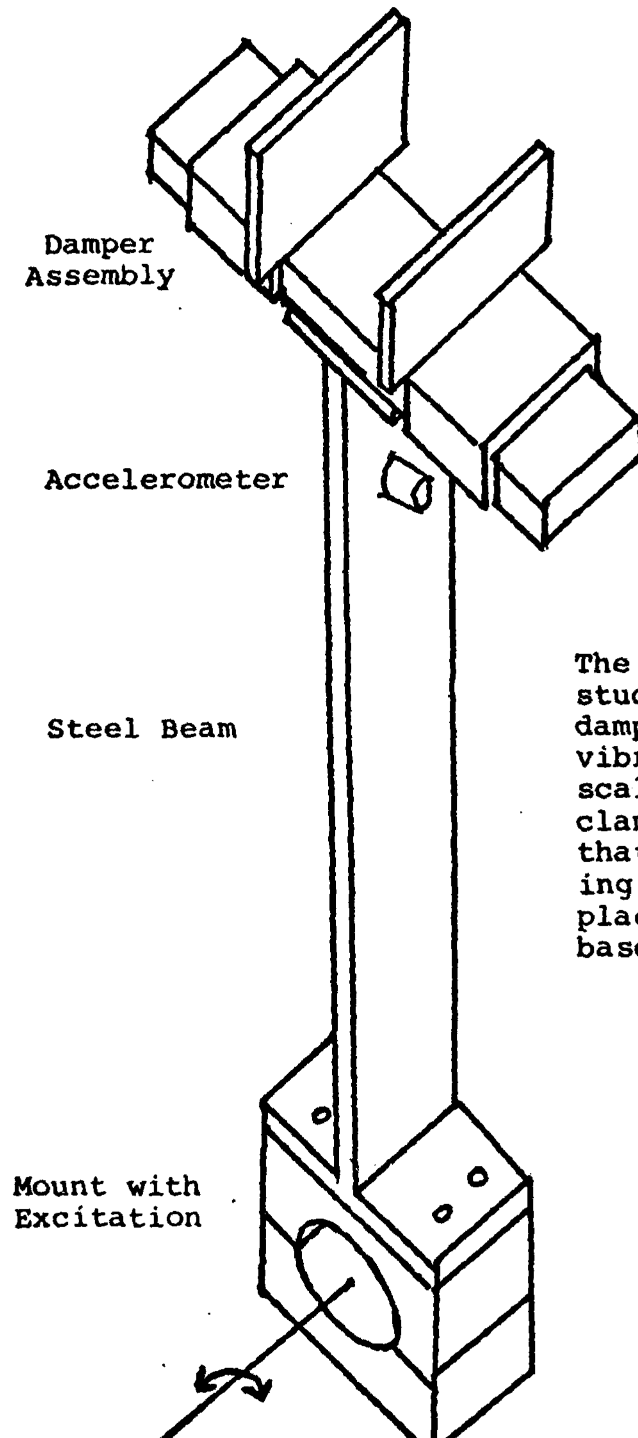
m	A_1/A_j	j	γ	δ	c_m
0.00481	4	22	0.0660	0.0105	0.0000217
0.00149	4	61	0.0231	0.00368	0.0000235
0.00503	4	179	0.00779	0.00124	0.0000267

Data from which the viscous damping coefficient of the impacting mass is calculated. m is in slugs, and c_m is in lb · sec/ft.

Forced Vibration Model

The model used for the forced vibration experiments is depicted in Figure 30 with the important dimensions, masses, and properties given in Figure 31. The first resonant frequency was calculated according to Reference (25), where the motion of a beam in free vibration with one end clamped and a point mass at the other end was solved. The beam was mounted upon a block clamped upon the pin of a pin-tree beam test apparatus. This test apparatus could rotate +24° to -24° at a frequency range of 0 to 83.3 Hz (0 to 5000 RPM). For computational purposes the beam was treated as if its end extended to the center of the pin, where a known sinusoidal angular displacement was applied.

The damper assembly was simply an aluminum bar, 1/4" x 1" x 5" mounted on top of the vibrating beam, with the stops mounted as shown in Figure 28. The stops were made of steel and could be attached anywhere on the aluminum bar. The steel ball used as the impacting mass was suspended by two pieces of fishing line attached to points 93" above and 74" to one side and 90" above and 75" to the other side of the damper assembly. The impacting mass was suspended in order to minimize friction forces on it, so its motion could be treated as resulting entirely from its impacts with the stops. When the mass was between the damper stops it could displace from its rest position a



The model used to study the impact damper in forced vibration (not to scale). The mount clamped onto a pin that rocked, applying the angular displacement at the base.

Fig. 30. Laboratory Model of Forced Vibration Impact Damper

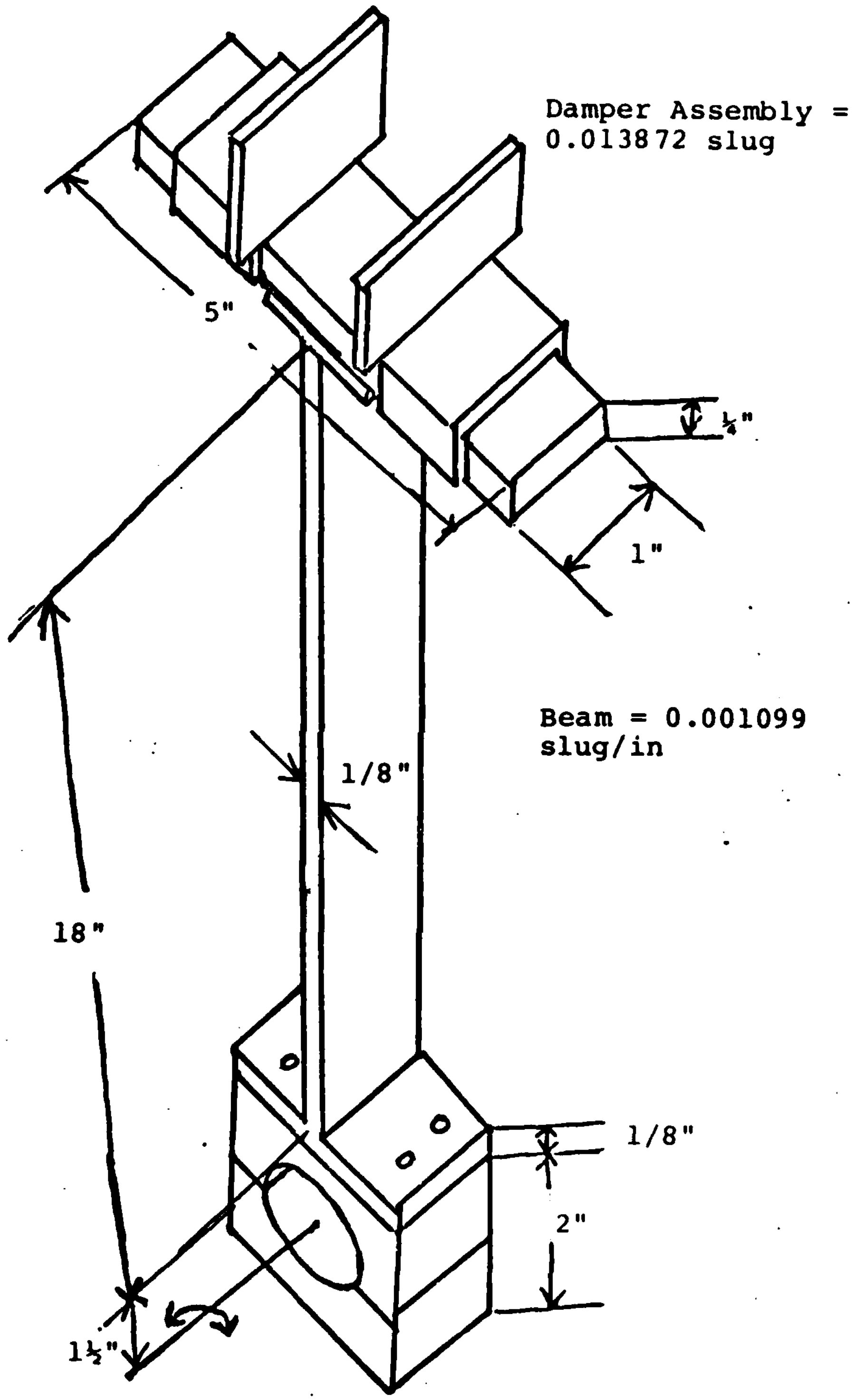


Fig. 31. Laboratory Model of Forced Vibration Impact Damper with Important Parameters

few inches at best, so its velocity due to its pendulum motion was very small compared to the velocity imparted to it by the impacts.

An accelerometer, Model MB 303, serial # 149235, was mounted on the beam 1" below the bottom of the damper assembly. The acclerometer signal was amplified using a model 2614B amplifier powered by an Endevco Model 2621 power supply. High frequency noise was filtered out using the low-pass filter of Figure 32, and the signal was then recorded using a Honeywell visicorder oscillograph Model 2106 with an M-1000 galvanometer. The output was also used with a universal counter timer, Model 726C, to accurately determine the frequency of the system. The resulting visicorder output for the forced vibration model without and with the impacting mass is shown in Figures 33 and 34, respectively.

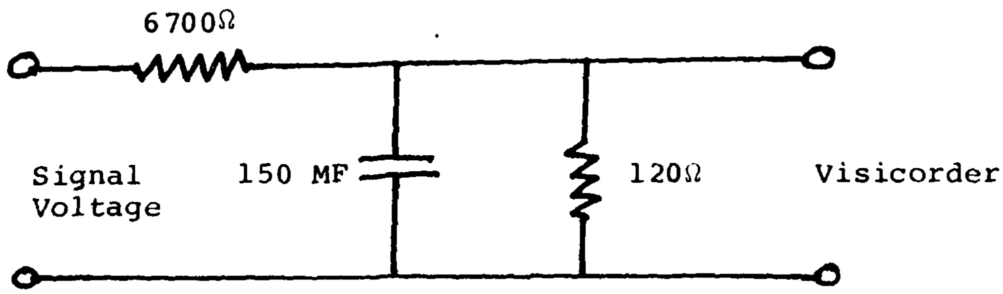


Fig. 32. Low-Pass Filter Used to Filter the Amplified Accelerometer Output Before Inputting it Into the Visicorder

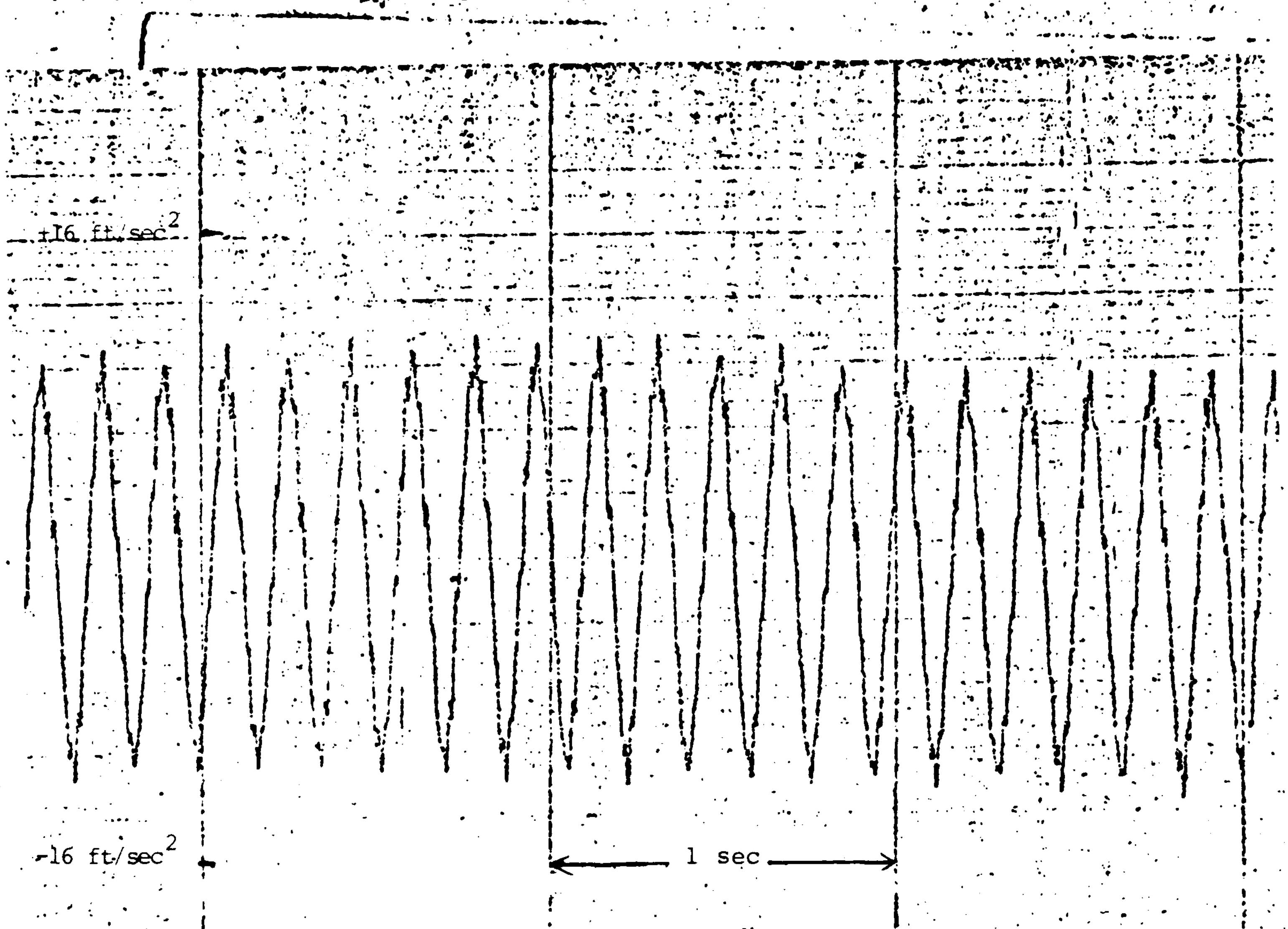


Fig. 33. Visicorder Trace of Acceleration Versus Time of Forced Vibration Laboratory Model with No Impacting Mass. 1 inch on the vertical axis represents approximately 8 ft/sec^2.

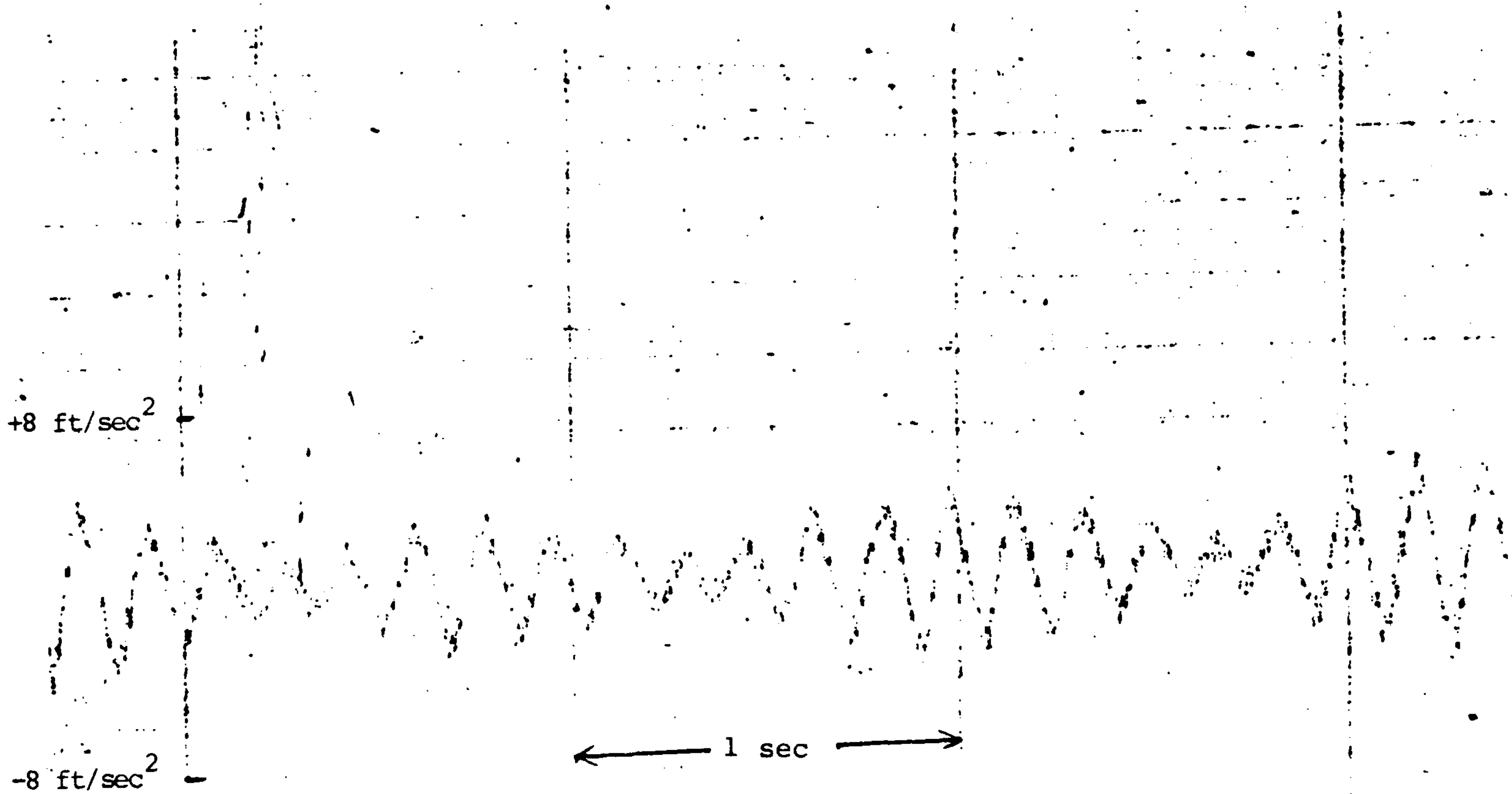


Fig. 34. Visicorder Trace of Acceleration Versus Time of Forced Vibration Laboratory Model. 0.00149 slug impacting mass, 1 inch on the vertical axis represents approximately 8 ft/sec^2; gap = 1.31".

Appendix C

Computer Simulations

The following pages contain two FORTRAN 77 computer programs which solve for the motion of both the primary system and the impacting mass for an impact damper in free vibration. The first program is the ideal case, in which the primary system is undamped except for the impact damping, and the motion of the impacting mass is due entirely to the impacts. The second program assumes a lightly damped impacting mass hanging as a pendulum. Though both programs were written with the laboratory model in mind, they are applicable to any one degree of freedom system in free vibration using a one degree of freedom impact damper.

While comments explaining the programs are inserted in appropriate places, a few additional words are in order. The position of the primary system was put in the form of:

$$\theta_1(\Delta t) = a \sin(\omega\Delta t) + b \cos(\omega\Delta t)$$

and

$$\theta_2(\Delta t) = e^{-\frac{c}{2I}\Delta t}[a \sin(\omega\Delta t) + b \cos(\omega\Delta t)]$$

for the first and second programs, respectively. The numbers associated with parameters in these programs were assigned with units of feet, seconds, and slugs intended.

The only sources of error in the program are the computer round-off errors, and the errors in defining an impact as occurring whenever the impacting mass and a stop were less than 10^{-6} ft from each other. This leads to small errors in position and velocity of the system in impact. In order to judge the seriousness of these errors, as well as to insure the equations and solution approach are correct, the errors in velocities and momentum were calculated after each impact, using Equations (31) and (36). The only other serious source of errors lies in the approximations and assumptions made in the derivation of equations of Appendix A. While the programs will generate output for any magnitude of θ_m, this output will be reasonably correct only if the assumption of small angles is not violated.

```
      program IDEAL
*
*     THIS PROGRAM FOLLOWS THE SYSTEM THROUGH A SERIES OF IMPACTS
*     IN THE "IDEAL" CASE, I.E. WITH NO VISCOUS DAMPING OR
*     GRAVITY AFFECTS.
*
      integer i,m,mm,c,n,j,imperr
      parameter (n=5Ø)
*
      real phi(Ø:n),thetas(1:n),thetav(Ø:n),tdots(1:n),tdotv(Ø:n),
     :     time(Ø:n),vels(1:n),velv(Ø:n),cone,ctwo,a(Ø:n),b(Ø:n),
     :     aa,bb,dtheta,ltheta,inct,ltime,t,tt,tts,at,tat,q,s,
     .     error(Ø:n),maxerr,vari,impt(1:n),
     :     inert,mass,l,e,ll,dd,qone,qtwo,
     :     deltat,il,ir,x(Ø:n),vel(Ø:n),d,thetam(Ø:n),resid,
     :     errmom,errvel,bmass,dt(Ø:n),mtt,mtts,mimpt(1:n)
*
*    The parameters varied are defined below.  Only one set of
*    parameters are varied here.  In practice, "do loops" were
*    used to obtain a large combination of parameters.
*
      mass=Ø.ØØ2
      d=Ø.25
      c=Ø.5
      qone=25.Ø
      l=2.23
*
*   Conditions from any previous run set to zero in the loop to line 1Ø1.
*
      do 1Ø1 j=1,n,1
      thetas(j)=Ø.Ø
      thetav(j)=Ø.Ø
      tdots(j)=Ø.Ø
      tdotv(j)=Ø.Ø
      time(j)=Ø.Ø
      vels(j)=Ø.Ø
      velv(j)=Ø.Ø
      x(j)=Ø.Ø
      thetam(j)=Ø.Ø
1Ø1   continue
*
*    The remaining parameters are defined.
*
      data   phi(Ø), thetav(Ø),  time(Ø), inert,
     :         [illegible](Ø),tdotv(Ø)/
     :         Ø.Ø,-Ø.1,Ø.Ø,Ø.1893,
     :         Ø.Ø,Ø.Ø/
      print*, ' '
      print*, ' '
      print*, ' '
      print*,'initial phase angle=', phi(Ø),'initial
     :  maximum deflection=', thetav(Ø),'natural frequency=', qone,
     :  'the starting time=',time(Ø),'moment of inertia of the
     :  primary mass=',inert,'magnitude of the secondary mass=',
     :  mass,'length of the primary system=',l,'the coefficient
     :  of re[illegible]',c,'[illegible] and [illegible]
     :  initial velocity=',v[illegible]),'the gap setting=',d,'desired
     :  number of impacts=',n
      qtwo=Ø.Ø
*
*    The impacting mass is given an initial position next to the stop
*    opposite the direction of the initial deflection of the system.
*
      x(Ø)=thetav(Ø)*l+(d/2.Ø)
```

```
      print*, ' '
      print*, ' '
      print*, ' '
      print*, 'mass=',mass
      print*, 'effective d=',d
      deltat=Ø.Ø1
      print*, 'e=',e
      print*, 'x(Ø)=',x(Ø)
      print*, 'deltat=',deltat
      print*, ' '
      print*, ' '
      print*, ' '
*
*     The loop to line 9ØØ solves for the motion of the system through
*     n impacts.
*
      do 9ØØ i=1,n,1
       inct=Ø.Ø1
       deltat=Ø.Ø1
*
*     Here the motion of the system is solved for after the known time
*     of impact.
*
        a(i-1)=tdotv(i-1)/qone
        b(i-1)=thetav(i-1)
        thetam(i-1)=sqrt(a(i-1)*a(i-1)+b(i-1)*b(i-1))
        aa=(velv(i-1))
        bb=x(i-1)
*
*    The loop to line 5ØØ iterates to the time of the next impact using
*    the requirement that the impacting mass must remain between the stops.
*    An impact is considered to have ocurred whenever the impacting mass
*    comes within Ø.ØØØØØ1 feet of either stop.
*
        do 6ØØ c=1,5ØØ,1
         time(i)=time(i-1)+deltat
         thetas(i)=(a(i-1)*sin(qone*deltat)+b(i-1)*cos(qone*deltat))
          x(i)=(aa*deltat)+bb
         il=thetas(i)*l-(d/2)
         ir=il+d
         if (abs(x(i)-il) .LE. Ø.ØØØØØ1) then
          goto 6Ø1
         elseif (abs(x(i)-ir) .LE. Ø.ØØØØØ1) then
          goto 6Ø1
         elseif ((x(i)-il) .LE. Ø.Ø) then
          inct=Ø.5*inct
          deltat=deltat-inct
          goto 5ØØ
         elseif ((x(i)-ir) .GE. Ø.Ø) then
          inct=Ø.5*inct
          deltat=deltat-inct
          goto 5ØØ
         else
          deltat=deltat+inct
          goto 5ØØ
         endif
5ØØ     continue
6ØØ     continue
6Ø1     continue
        if (c .ge. 5ØØ) then
         goto 993
        endif
        if (c .LE. 2) then
         goto 6ØØ
```

```
         endif
 800     continue
*
*       The conditions immediately before the time of impact just iterated
*       to are solved for.
*
          tdots(i)=qone*(a(i-1)*cos(qone*deltat)-b(i-1)*sin(qone*deltat))
          thetav(i)=thetas(i)
            vels(i)=velv(i-1)
          tdotv(i)=(1.0/(inert+mass*l*l))*((tdots(i)*(inert-mass*l*l*e)
     :             +vels(i)*mass*l*(1.0+e)))
          velv(i)=(1/(inert+mass*l*l))*(tdots(i)
     :          *inert*(1.0+e)+
     :          vels(i)*(mass*l-(inert*e/l)))
 900      continue
          goto 994
 993      continue
          print '(a,i3)','the time iteration did not converge for i=',i
 994      continue
*
*       Here the motion of the system is given for the first 15 impacts.
*
          print *,'impact   time   deltat  thetas     x      tdotv   xdot
     :     thetam dthetam'
          do 950 j=1,15,1
           print '(i7,f8.5,f8.6,f8.4,f9.6,f9.4,f9.4,f7.4,f9.4)',j,
     :     time(j),time(j)-time(j-1),thetas(j),x(j),tdotv(j),velv(j),
     :     thetam(j),thetam(j)-thetam(j-1)
 950      continue
          print*, ' '
          print*, ' '
          print*, ' '
*
*       The errors in position and velocity across the first 15 impacts are
*       given below.
*
          do 970 j=1,15,1
           errmom=inert*(tdotv(j)-tdots(j))+mass*l*(velv(j)-vels(j))
           errvel=e*(vels(j)-tdots(j)*l)-(tdotv(j)*l-velv(j))
           print*, 'impact=',j,'errmom=',errmom,'errvel=',errvel
 970       continue
          print*, ' '
          print*, ' '
          print*, ' '
           print*, 'impact thetam   time   dthetam    dtime'
           ltheta=0.1
           ltime=0.0
           deltat=0.0
           m=1
           tt=0.0
           tts=0.0
           mtt=0.0
           mtts=0.0
           at=0.0
           tat=0.0
           error(0)=0.1
           zeta=0.0
           tzeta=0.0
           dt(0)=0.0
           mm=1
*
*       In the loop to line 980 the maximum positive amplitude the reaches
*       during each cycle is obtained from the motion of the system already
*       obtained.  Information needed to do a least squares to these
```

```
*     amplitudes is also obtained.  The loop also looks ahead to see
*     when the system goes through a cycle without an impact on the
*     assumption that this when the damper becomes inoperative.
*     The loop is exited before the damper becomes inoperative.
*
         do 980 j=1,n-1,1
          if (b(j) .eq. 0.0) then
           goto 981
          endif
          deltat=(atan(a(j)/b(j)))/qone
          if (deltat .le. 0.0) then
           deltat=deltat+(3.1415927/qone)
          endif
          if ((b(j)*cos(qone*deltat)) .le. 0.0) then
           goto 979
          elseif ((time(j+12)-time(j+11)) .ge. 0.25) then
           goto 981
          endif
          if ((time(j)+deltat) .le. time(j+1)) then
           if (deltat .ge. 0.0) then
            dtheta=thetam(j)-ltheta
            t=time(j)+deltat
            dt(j)=t-ltime
            print '(i3,f9.4,f9.4,f9.4,f9.4)',j,thetam(j),t,
     1               dtheta,dt(j)
            resid=j
            ltheta=thetam(j)
            ltime=t
            impt(m)=t
            mimpt(mm)=t
            tt=tt+t
            tts=tts+t*t
            at=at+thetam(j)
            tat=tat+t*thetam(j)
            error(m)=thetam(j)
            m=m+1
           endif
          endif
979      continue

980      continue
981      continue
         m=m-1
         if (tts .eq. 0.0) then
          goto 983
         elseif (m .eq. 0) then
          goto 983
         elseif (tts .eq. tt*tt) then
          goto 983
         endif
*
*     A least squares approximation is fit to the maximum amplitudes
*     below.  The maximum deviation from this approximation is
*     obtained, as well as the variance.
*
         s=(((1.0*m)*tat)-(at*tt))/(((1.0*m)*tts)-(tt*tt))
         q=(at-(s*tt))/(1.0*m)
         maxerr=0.0
         vari=0.0
         do 982 j=1,m,1
          error(j)=error(j)-q-(s*impt(j))
          if (abs(error(j)) .ge. maxerr) then
           maxerr=error(j)
           imperr=j
```

```
           endif
           vari=vari+error(j)*error(j)
982       continue
          print*, 'up to the',m,'peak the least squares fit to the
     :  peaks is thetam=q+s*t with q=',q,'and s=',s,'with max error=',
     :  maxerr,'at peak=',imperr,'and variance=',vari
983     continue
          print*, ' '
          print*, ' '
          print*, ' '

          if (resid .ge. n) then
           goto 1002
          endif
          if (resid+25 .ge. n) then
           goto 1951
          endif
        print*, ' '
*
*
*     Here the conditions at the impacts beginning where the damper was
*     assumed to become ineffective are given.  This gives the residual
*     amplitude. After that the first 15 cycles of the ineffectively damped
*     portion of the system are given.
*
        print *,'impact   time   deltat  thetas     x      tdotv   xdot
     :   thetam dthetam'
        do 1950 j=resid,resid+25,1
         print '(i7,f8.5,f8.6,f8.4,f9.6,f9.4,f9.4,f7.4,f9.4)',j,
     :   time(j),time(j)-time(j-1),thetas(j),x(j),tdotv(j),velv(j),
     :   thetam(j),thetam(j)-thetam(j-1)
1950    continue
1951    continue
        print*, ' '
         print*, 'impact thetam  time   dthetam  dtime'
         tat=0.0
         ltheta=0.1
         ltime=0.0
         deltat=0.0
         m=1
         tt=0.0
         tts=0.0
         at=0.0
         mtt=0.0
         mtts=0.0
         error(0)=0.1
         zetat=0.0
         tzeta=0.0
         dt(0)=0.0
         mm=1
         error(0)=0.1
         do 1980 j=resid,n,1
          if (b(j) .eq. 0.0) then
           goto 1981
          endif
          deltat=(atan(a(j)/b(j)))/qone

          if (deltat .le. 0.0) then
           deltat=deltat+(3.1415927/qone)
          endif
          if ((b(j)*cos(qone*deltat)) .le. 0.0) then
           goto 1979
          endif
          if ((time(j)+deltat) .le. time(j+1)) then
```

```
            if (deltat .ge. Ø.Ø) then
             dtheta=thetam(j)-ltheta
             t=time(j)+deltat
             dt(j)=t-ltime
             print '(i3,f9.4,f9.4,f9.4,f9.4)',j,thetam(j),t,
     :               dtheta,dt(j)
             ltheta=thetam(j)
             ltime=t
             impt(m)=t
             mimpt(mm)=t
             tt=tt+t
             tts=tts+t*t
             at=at+thetam(j)
             tat=tat+t*thetam(j)
             error(m)=thetam(j)
             m=m+1
             if (m .ge . 16) then
              goto 1981
             endif
            endif
           endif
1979      continue
198Ø      continue
1981      continue
          m=m-1
          if (tts .eq. Ø.Ø) then
           goto 1983
          elseif (m .eq. Ø) then
           goto 1983
          elseif (tts .eq. tt*tt) then
           goto 1983
          endif
          s=(((1.Ø*m)*tat)-(at*tt))/(((1.Ø*m)*tts)-(tt*tt))
          q=(at-(s*tt))/(1.Ø*m)
          maxerr=Ø.Ø
          vari=Ø.Ø
          do 1982 j=1,m,1
           error(j)=error(j)-q-(s*impt(j))
           if (abs(error(j)) .ge. maxerr) then
            maxerr=error(j)
            imperr=j
           endif
           vari=vari+error(j)*error(j)
1982      continue
          print*, 'up to the',m,'peak the least squares fit to the
     :   peaks is theta=q+s*t with q=',q,'and s=',s,'with max error=',
     :   maxerr,'at peak=',imperr,'and variance=',vari
1983    continue
999     continue
1ØØ2    continue
1ØØØ    continue
1ØØ1    continue
        end
```

initial phase angle= .000000000e+00
initial maximum deflection= -.100000001e+00
natural frequency= .250000000e+02 the starting time= .000000000e+00
moment of inercia of the primary mass= .109300001e+00
magnitude of the secondary mass= .200000009e-02 length of the primary system=
2.2300002 the coefficient of restitution= .500000000e+00
secondary masses initial velocity=
.000000000e+00 the gap setting= .250000000e+00
desired number of impacts= 50

mass= .200000009e-02
effective d= .250000000e+00
e= .500000000e+00
x(0)= -.980000049e-01
deltat= .999999970e-02

the time iteration did not converge for i= 45

impact	time	deltat	thetas	x	tdotv	xdot	thetam	dthetam
1	.06769	.067687	.0121	-.098000	2.2958	7.8866	.0926	-.0074
2	.12207	.054383	.0923	.330895	.4363	-2.7636	.0940	.0013
3	.18766	.065595	.0110	.149614	-2.2510	-6.2389	.0907	-.0033
4	.26345	.075782	-.0869	-.323185	.2110	4.0967	.0093	-.0014
5	.41144	.147998	.0709	.283116	-1.1169	-6.0511	.0838	-.0055
6	.50552	.090080	-.0631	-.310375	-.4493	1.7270	.0850	.0012
7	.55643	.040105	-.0453	-.225910	1.7234	4.9869	.0825	-.0026
8	.66214	.103713	.0746	.291290	-.6468	-4.9173	.0789	-.0035
9	.78197	.119824	-.0775	-.297915	.1760	3.2623	.0779	-.0011
10	.86711	.085146	.0470	-.020143	1.5448	3.5436	.0776	-.0002
11	.93917	.072054	.0494	.235188	-1.2663	-6.2654	.0708	-.0069
12	1.02067	.081503	-.0675	-.275461	-.7037	.9688	.0731	.0023
13	1.07296	.052289	-.0440	-.224004	1.3695	4.1010	.0707	-.0024
14	1.18756	.114606	.0580	.254357	-.7959	-4.9939	.0662	-.0046
15	1.29308	.105511	-.0662	-.272560	-.1705	2.1134	.0665	.0004

impact= 1 errmom= .186264515e-07 errvel= .238418579e-06
impact= 2 errmom= .745058060e-08 errvel= -.238418579e-06
impact= 3 errmom= .745058060e-08 errvel= -.953674316e-06
impact= 4 errmom= .000000000e+00 errvel= .476837158e-06
impact= 5 errmom= -.111758709e-07 errvel= -.238418579e-06
impact= 6 errmom= -.372529030e-08 errvel= .238418579e-06
impact= 7 errmom= .298023224e-07 errvel= -.596046448e-06
impact= 8 errmom= -.745058060e-08 errvel= -.238418579e-06
impact= 9 errmom= .372529030e-08 errvel= .238418579e-06
impact= 10 errmom= .186264515e-07 errvel= -.119209290e-06
impact= 11 errmom= -.298023224e-07 errvel= .238418579e-06
impact= 12 errmom= -.111758709e-07 errvel= .238418579e-06
impact= 13 errmom= -.558793545e-08 errvel= .238418579e-06
impact= 14 errmom= -.111758709e-07 errvel= -.238418579e-06
impact= 15 errmom= .000000000e+00 errvel= .476837158e-06

impact	thetam	time	dthetam	dtime
2	.0940	.1295	-.0060	.1295
4	.0893	.3853	-.0047	.2558
7	.0825	.6445	-.0068	.2592
10	.0776	.9039	-.0048	.2594
13	.0707	1.1632	-.0069	.2593

16	.Ø644	1.423Ø	-.ØØ63	.2598
19	.Ø594	1.6831	-.ØØ5Ø	.26Ø1
22	.Ø526	1.9427	-.ØØ69	.2596
26	.Ø41Ø	2.4622	-.Ø115	.5196

up to the 9
peak the least squares fit to the peaks is thetam=q+s*t with q=
.97564160Øe-Ø1 and s= -.22961786Øe-Ø1 with max error= .84Ø116292e-Ø3 at peak=
4 and variance= .214724355e-Ø5

impact	thetam	time	dthetam	dtime
26	.Ø41Ø	2.4622	-.Ø59Ø	2.4622
28	.Ø354	2.7238	-.ØØ56	.2616
3Ø	.Ø298	2.9863	-.ØØ57	.2625
32	.Ø239	3.2495	-.ØØ58	.2632
34	.Ø18Ø	3.5136	-.ØØ59	.2641
36	.Ø117	3.7778	-.ØØ63	.2642
38	.ØØ56	4.Ø299	-.ØØ61	.2521
39	.ØØ55	4.2669	-.ØØØ1	.237Ø
4Ø	.ØØ44	4.5122	-.ØØ11	.2453
42	.ØØ25	5.2522	-.ØØ19	.74ØØ
43	.ØØ24	5.7657	-.ØØØ1	.5135

up to the 11
peak the least squares fit to the peaks is thetam=q+s*t with q=
.643201843e-Ø1 and s= -.123949796e-Ø1 with max error= .9575Ø3542e-Ø2 at peak=
11 and variance= .354233722e-Ø3

```
      program LABSIM
*
*     THIS PROGRAM ATTEMPTS TO SIMULATE THE CONDITIONS EXPERIENCED IN
*     THE LABORATORY WHILE FOLLOWING THE IMPACT THROUGH A SERIES OF
*     IMPACTS.
*
      integer i,m,c,n,j,p,imperr,mm,y,u
      parameter (n=5ØØ)
*
      real phi(Ø:n),thetas(1:n),thetav(Ø:n),tdots(1:n),tdotv(Ø:n),
     :     time(Ø:n),vels(1:n),velv(Ø:n),cone,ctwo,a(Ø:n),b(Ø:n),
     :     aa,bb,dtheta,ltheta,inct,ltime,t,
     :     inert,mass,l,e,ll,dd,qone,qtwo,mtts,impt(1:n),
     :     impt(1:n),tt,tts,at,tat,error(Ø:n),s,q,
     :     maxerr,vari,dt(1:n),mimpt(1:n),
     :     deltat,il,ir,x(Ø:n),vel(Ø:n),d,thetam(Ø:n),
     :     errmom,errvel,dia,resid
        print*, ' '
        print*, ' '
        print*, ' '
*
*    The same parameters varied in the laboratory are varied here. Note
*    that the diameter of the impacting mass is subracted from the actual
*    gap to give an effective gap. Also note that the same damping constant
*    is not used for each mass.
*
       do 1ØØ1 p=1,3,1
        if (p .eq. 1) then
         mass=Ø.ØØØ481
         ctwo=Ø.ØØØØ217
         dia=Ø.469/12.Ø
        elseif (p .eq. 2) then
         mass=Ø.ØØ149
         ctwo=Ø.ØØØØ235
         dia=Ø.6875/12.Ø
        else
         mass=Ø.ØØ5Ø3
         ctwo=Ø.ØØØØ267
         dia=1.Ø31/12.Ø
        endif
        d=3.Ø*(1.Ø/12.Ø)
        d=d-dia
         e=Ø.5
*
*    Previous values of the system position's and velocities are set to zero.
       do 1Ø1 j=1,n,1
       thetas(j)=Ø.Ø
       thetav(j)=Ø.Ø
       tdots(j)=Ø.Ø
       tdotv(j)=Ø.Ø
       time(j)=Ø.Ø
       vels(j)=Ø.Ø
       velv(j)=Ø.Ø
       x(j)=Ø.Ø
       thetam(j)=Ø.Ø
1Ø1    continue
        print*, ' '
        print*, ' '
        print*, ' '
*
*   The remainder of the system's parameters are defined.
*
       data   phi(Ø), thetav(Ø),time(Ø),qone,inert,l,
     :        velv(Ø),m,tdotv(Ø),cone,ll/
```

```
:         Ø.Ø,-Ø.1Ø,Ø.Ø,25.9,Ø.188 ,2.21,
:         Ø.Ø,1,Ø.Ø,Ø.Ø2,7.Ø/
   print*,'initial phase angle=', phi(Ø),'initial
:    maximum deflection=', thetav(Ø),
:    'the starting time=',time(Ø),'moment of inertia of the
:    primary mass=',inert,'magnitude of the secondary mass=',
:    mass,'length of the primary system=',l,'the coefficient
:    of restitution=',e,'secondary masses
:    initial velocity=',vel(Ø),'the gap setting=',d,'desired
:    number of impacts=',n
   qone=25.9
   qtwo=sqrt((32.174/l1)-(ctwo/(2.Ø*mass))*(ctwo/(2.Ø*mass)))
    print*, ' '
    print*, ' '
    print*, ' '
*
*    The initial position of the impacting mass is placed against the stop
*    opposite to the initial deflection.
*
    x(Ø)=thetav(Ø)*l+(d/2.Ø)
    print*, 'mass=',mass
    print*, 'effective d=',d
    deltat=Ø.Ø1
    print*, 'x(Ø)=',x(Ø)
    print*, 'deltat=',deltat
    print*, 'e=',e
    print*, 'qone=',qone
    print*, 'qtwo=',qtwo
*
*    The loop to line 9ØØ solves for the series of impacts.
*
    do 9ØØ i=1,n,1
     inct=Ø.Ø1
     deltat=Ø.Ø1
*
*    The motion is solved for after a known time of impact.
*
     a(i-1)=(tdotv(i-1)/qone+thetav(i-1)*cone/(2.Ø*inert*qone))
     b(i-1)=thetav(i-1)
     thetam(i-1)=sqrt(a(i-1)*a(i-1)+b(i-1)*b(i-1))
     aa=(velv(i-1)/qtwo+(ctwo*x(i-1)/(2.Ø*mass*qtwo)))
     bb=x(i-1)
*
*    The loop to line 6ØØ iterates to the next time of impact.  This
*    iteration uses the requirement that the impacting mass remain
*    between the stops.  An impact is defined as ocurring whenever
*    the impacting mass is within Ø.ØØØØØ1 feet of either stop.
*
     do 6ØØ c=1,5ØØ,1
      time(i)=time(i-1)+deltat
      thetas(i)=(exp((-(cone)/(2.Ø*inert)*deltat)))*
:            (a(i-1)*sin(qone*deltat)+b(i-1)*cos(qone*deltat))
       x(i)=(exp((-(ctwo)/(2.Ø*mass))*deltat))*
:          (aa*sin(qtwo*deltat)+bb*cos(qtwo*deltat))
      il=thetas(i)*l-(d/2)
      ir=il+d
      if (abs(x(i)-il) .LE. Ø.ØØØØØ1) then
       goto 6Ø1
      elseif (abs(x(i)-ir) .LE. Ø.ØØØØØ1) then
       goto 6Ø1
      elseif ((x(i)-il) .LE. Ø.Ø) then
       inct=Ø.5*inct
       deltat=deltat-inct
       goto 5ØØ
```

```
      elseif ((x(i)-ir) .GE. Ø.Ø) then
       inct=Ø.5*inct
       deltat=deltat-inct
       goto 5ØØ
      else
       deltat=deltat+inct
       goto 5ØØ
      endif
5ØØ   continue
6ØØ   continue
6Ø1   continue
      if (c .ge. 5ØØ) then
       goto 993
      endif
      if (c .LE. 2) then
       goto 8ØØ

      endif
8ØØ   continue
*
*     The conditions immediately before the time of impact just iterated
*     to are solved.
*
        tdots(i)=-(cone/(2.Ø*inert))*(exp((-cone/(2.Ø*inert)*deltat)))*
     :           (a(i-1)*sin(qone*deltat)+b(i-1)*cos(qone*deltat))+
     :           (exp((-cone/(2.Ø*inert)*deltat)))*qone*
     :           (a(i-1)*cos(qone*deltat)-b(i-1)*sin(qone*deltat))
     :
        thetav(i)=thetas(i)
        vels(i)=-(ctwo/(2.Ø*mass))*(exp((-ctwo/(2.Ø*mass)*deltat)))*
     :          (aa*sin(qtwo*deltat)+bb*cos(qtwo*deltat))+
     :          (exp((-ctwo/(2.Ø*mass)*deltat)))*qtwo*
     :          (aa*cos(qtwo*deltat)-bb*sin(qtwo*deltat))
        tdotv(i)=(1.Ø/(inert+mass*l*l))*((tdots(i)*(inert-mass*l*l*e)
     :          +vels(i)*mass*l*(1.Ø+e)))
        velv(i)=(l/(inert+mass*l*l))*(tdots(i)
     :       *inert*(1.Ø+e)+
     :       vels(i)*(mass*l-(inert*e/l)))
9ØØ     continue
        goto 994
993     continue
        print '(a,i3)','the time iteration did not converge for i=',i
994     continue
5ØØØ    continue
*
*      Here the system's condition at the first 15 impacts is given.
*
        print *,'impact   time   deltat  thetas     x      tdotv    xdot
     :   thetam  dthetam'
        do 95Ø j=1,15,1
         print '(i7,f8.5,f8.6,f8.4,f9.6,f9.4,f9.4,f7.4,f9.4)',j,
     :   time(j),time(j)-time(j-1),thetas(j),x(j),tdotv(j),velv(j),
     :   thetam(j),thetam(j)-thetam(j-1)
95Ø     continue
951     continue
        print*, ' '
        print*, ' '
        print*, ' '
*
*     The errors in the momentum and velocity across the first 15 impacts
*     is given.  They should be small enough to be assumed neglible.
*
        do 97Ø j=1,15,1
         errmom=inert*(tdotv(j)-tdots(j))+mass*l*(velv(j)-vels(j))
```

```
      errvel=e*(vels(j)-tdots(j)*l)-(tdotv(j)*l-velv(j))
      print*, 'impact=',j,'errmom=',errmom,'errvel=',errvel
 970  continue
     print*, ' '
     print*, ' '
     print*, ' '
      print*, 'impact thetam  time   dthetam  dtime'
      ltheta=0.1
      ltime=0.0
      deltat=0.0
      m=1
      tt=0.0
      tts=0.0
      at=0.0
      tat=0.0
      mtt=0.0
      mtts=0.0
      error(0)=0.1
      dt(0)=0.0
      mm=1
*
*   The following loop takes the series of positions and solves for
*   the peak positive amplitudes.  It also looks ahead to see when the
*   system goes through a complete cycle without an impact to on the that
*   this is when the damper becomes ineffective.  Information needed
*   to perform a least squares fit to these peaks is also obtained.
*
       do 980 j=2,n-2,1
        if (a(j) .eq. 0.0) then
         goto 980
        endif
        phi(j)=atan(b(j)/a(j))
        deltat=(atan(2.0*inert*qone/cone)-phi(j))/qone

        if (deltat .le. 0.0) then
         deltat=deltat+(3.1415927/qone)
        endif
        if ((b(j)*cos(qone*deltat)) .le. 0.0) then
         goto 979
        elseif ((time(j+2)-time(j+1)) .ge. 0.25) then
         goto 981
        endif
        if ((time(j)+deltat) .le. time(j+1)) then
         if (deltat .ge. 0.0) then
          dtheta=thetam(j)-ltheta
          t=time(j)+deltat
          dt(j)=t-ltime
          print '(i3,f9.4,f9.4,f9.4,f9.4)',j,thetam(j),t,
     :          dtheta,dt(j)
          resid=j+2
          ltheta=thetam(j)
          ltime=t
          impt(m)=t
          mimpt(mm)=t
          tt=tt+t
          tts=tts+t*t
          at=at+thetam(j)
          tat=tat+t*thetam(j)
          error(m)=thetam(j)
          m=m+1
         endif
        endif
 979   continue
```

```
98Ø     continue
981     continue

        m=m-1
        if (tts .eq. Ø.Ø) then
         goto 983
        elseif (m .eq. Ø) then
         goto 983
        elseif (tts .eq. tt*tt) then
         goto 983
        endif
*
*     The following equations solve for the least squares fit to
*     maximum amplitude of the cycles just obtained.  The maximum
*     departure from this least squares approximation and the variance
*     is also obtained.
*
        s=(((1.Ø*m)*tat)-(at*tt))/(((1.Ø*m)*tts)-(tt*tt))
        q=(at-(s*tt))/(1.Ø*m)
        maxerr=Ø.Ø
        vari=Ø.Ø
        do 982 j=1,m,1
         error(j)=error(j)-q-(s*impt(j))
         if (abs(error(j)) .ge. maxerr) then
          maxerr=error(j)
          imperr=j
         endif
         vari=vari+error(j)*error(j)
982     continue
        print*, 'up to the',m,'peak the least squares fit to the
     :  peaks is thetam=q+s*t with q=',q,'and s=',s,'with max error=',
     :  maxerr,'at peak=',imperr,'and variance=',vari
        print*, ' '
        print*, ' '


        print*, ' '
2985    continue
983     continue


        if (resid .ge. n) then
         goto 1ØØ2
        elseif
        if (resid+25 .ge. n) then
         goto 1951
        endif
       print*, ' '
*
*     Here the conditions at the impacts beginning where the damper was
*     assumed to become ineffective are given.  This gives the residual
*     amplitude. After that the first 15 cycles of the ineffectively damped
*     portion of the system are given.
*
       print *,'impact   time   deltat  thetas     x      tdotv    xdot
     :   thetam  dthetam'
       do 195Ø j=resid,resid+25,1
        print '(i7,f8.5,f8.6,f8.4,f9.6,f9.4,f9.4,f7.4,f9.4)',j,
     :  time(j),time(j)-time(j-1),thetas(j),x(j),tdotv(j),velv(j),
     :  thetam(j),thetam(j)-thetam(j-1)
195Ø   continue
1951   continue
       print*, ' '
        print*, 'impact thetam  time   dthetam  dtime'
```

```
         tat=Ø.Ø
         ltheta=Ø.1
         ltime=Ø.Ø
         deltat=Ø.Ø
         m=1
         tt=Ø.Ø
         tts=Ø.Ø
         at=Ø.Ø
         mtt=Ø.Ø
         mtts=Ø.Ø
         error(Ø)=Ø.1
         zetat=Ø.Ø
         tzeta=Ø.Ø
         dt(Ø)=Ø.Ø
         mm=1
         error(Ø)=Ø.1
         do 198Ø j=resid,n,1
          if (a(j) .eq. Ø.Ø) then
           goto 198Ø
          endif
          phi(j)=atan(b(j)/a(j))
          deltat=(atan(2.Ø*inert*qone/cone)-phi(j))/qone

          if (deltat .le. Ø.Ø) then
           deltat=deltat+(3.1415927/qone)
          endif
          if ((b(j)*cos(qone*deltat)) .le. Ø.Ø) then
           goto 1979
          endif
          if ((time(j)+deltat) .le. time(j+1)) then
           if (deltat .ge. Ø.Ø) then
            dtheta=thetam(j)-ltheta
            t=time(j)+deltat
            dt(j)=t-ltime
            print '(i3,f9.4,f9.4,f9.4,f9.4)',j,thetam(j),t,
                     dtheta,dt(j)
            ltheta=thetam(j)
            ltime=t
            imps(m)=t
            mimpt(mm)=t
            tt=tt+t
            tts=tts+t*t
            at=at+thetam(j)
            tat=tat+t*thetam(j)
            error(m)=thetam(j)
            m=m+1
             if (m .ge . 16) then
              goto 1981
            endif
           endif
          endif
1979     continue
1980     continue
1981     continue
         m=m-1
         if (tts .eq. Ø.Ø) then
          goto 1983
         elseif (m .eq. Ø) then
          goto 1983
         elseif (tts .eq. tt*tt) then
          goto 1983
         endif
         s=(((1.Ø*m)*tat)-(at*tt))/(((1.Ø*m)*tts)-(tt*tt))
         q=(at-(s*tt))/(1.Ø*m)
```

```
          maxerr=0.0
          vari=0.0
          do 1982 j=1,m,1
           error(j)=error(j)-q-(s*impt(j))
           if (abs(error(j)) .ge. maxerr) then
            maxerr=error(j)
            imperr=j
           endif
           vari=vari+error(j)*error(j)
1982      continue
          print*, 'up to the',m,'peak the least squares fit to the
     :  peaks is thetam=q+s*t with q=',q,'and s=',s,'with max error=',
     :  maxerr,'at peak=',imperr,'and variance=',vari
1983    continue
999     continue
1002    continue
1000    continue
1001    continue
        end
```

```
initial phase angle=  .000000000e+00
initial                        maximum deflection= -.100000001e+00
the starting time=  .000000000e+00
moment of inertia of the                primary mass=  .107999994e+00
magnitude of the secondary mass=  .480999995e-03 length of the primary system=
 2.21000004 the coefficient              of restitution=  .500000000e+00
secondary masses                                    initial velocity=
 .000000000e+00 the gap setting=  .210916668e+00
desired             number of impacts= 500


mass=  .480999995e-03
effective d=  .210916668e+00
x(0)= -.115541667e+00
deltat=  .999999978e-02
e=  .500000000e+00
qone=  .256999996e+02
qtwo=  2.14377642
the time iteration did not converge for i=150
```

impact	time	deltat	thetas	x	tdotv	xdot	thetam	dthetam
1	.05912	.059122	-.0041	-.114616	2.5324	8.4316	.0979	-.0021
2	.10972	.050594	.0532	.311323	.8051	-1.5881	.0983	.0004
3	.15881	.049095	.0572	.231865	-2.0598	-5.9662	.0972	-.0010
4	.25055	.091842	-.0952	-.315895	.3935	4.3304	.0964	-.0008
5	.38817	.137513	.0602	.282787	-1.2935	-6.5124	.0944	-.0020
6	.47658	.090513	-.0912	-.307081	-.6221	1.2261	.0944	.0000
7	.51742	.038741	-.0693	-.258617	1.6360	4.8082	.0937	-.0007
8	.55312	.035703	.0066	-.086497	2.4067	5.5684	.0933	-.0003
9	.62277	.069641	.0881	.300210	-.7131	-5.1892	.0923	-.0011
10	.73913	.116362	-.0903	-.305087	.3631	3.8401	.0914	-.0009
11	.80337	.064238	.0221	-.056652	2.2773	5.6191	.0907	-.0007
12	.85631	.062947	.0862	.295993	-.6490	-5.0017	.0897	-.0010
13	.98481	.118496	-.0872	-.298257	.4319	3.9748	.0888	-.0009
14	1.12431	.139504	.0697	.259508	-1.3413	-6.4926	.0868	-.0020
15	1.20832	.084607	-.0818	-.286139	-.7575	.7655	.0869	.0001

```
impact= 1 errmom= -.400468707e-07 errvel=  .119209290e-05
impact= 2 errmom= -.232830644e-07 errvel=  .715255737e-06
impact= 3 errmom=  .293366611e-07 errvel= -.715255737e-06
impact= 4 errmom= -.372529030e-08 errvel=  .476837158e-06
impact= 5 errmom=  .149011612e-07 errvel=  .000000000e+00
impact= 6 errmom= -.931322575e-09 errvel= -.238418579e-06
impact= 7 errmom= -.265426934e-07 errvel=  .238418579e-06
impact= 8 errmom= -.555301065e-07 errvel=  .953674316e-06
impact= 9 errmom=  .223517418e-07 errvel= -.476837158e-06
impact= 10 errmom= -.838190317e-08 errvel=  .000000000e+00
impact= 11 errmom= -.471482053e-07 errvel=  .953674316e-06
impact= 12 errmom=  .931322575e-09 errvel= -.238418579e-06
impact= 13 errmom= -.186264515e-08 errvel=  .238418579e-06
impact= 14 errmom=  .931322575e-08 errvel= -.476837158e-06
impact= 15 errmom=  .150921128e-07 errvel= -.715255737e-06
```

impact	thetam	time	dthetam	dtime
2	.0983	.1221	-.0017	.1221
4	.0964	.3658	-.0019	.2437
8	.0933	.6101	-.0031	.2443

11	.0907	.8544	-.0026	.2443
13	.0888	1.0988	-.0019	.2444
17	.0859	1.3431	-.0029	.2443
20	.0834	1.5875	-.0025	.2443
23	.0811	1.8319	-.0023	.2444
26	.0788	2.0763	-.0023	.2445
28	.0769	2.3208	-.0019	.2444
31	.0744	2.5651	-.0025	.2444
34	.0720	2.8096	-.0024	.2445
36	.0702	3.0541	-.0018	.2445
39	.0677	3.2986	-.0025	.2444
41	.0660	3.5431	-.0017	.2445
44	.0635	3.7875	-.0025	.2444
46	.0619	4.0321	-.0016	.2446
49	.0593	4.2763	-.0026	.2443
51	.0578	4.5210	-.0016	.2446
54	.0553	4.7653	-.0025	.2443
56	.0539	5.0100	-.0015	.2447
59	.0515	5.2542	-.0024	.2442
62	.0495	5.4987	-.0020	.2446
65	.0477	5.7434	-.0018	.2446
68	.0458	5.9878	-.0018	.2445
72	.0423	6.4766	-.0035	.4888
74	.0405	6.7213	-.0018	.2446
76	.0388	6.9659	-.0017	.2447
78	.0371	7.2107	-.0017	.2448
80	.0354	7.4555	-.0017	.2448
82	.0337	7.7004	-.0017	.2449
84	.0321	7.9454	-.0016	.2450
86	.0305	8.1900	-.0016	.2451
88	.0289	8.4356	-.0016	.2451
90	.0273	8.6808	-.0016	.2452
92	.0258	8.9261	-.0015	.2452
94	.0242	9.1714	-.0015	.2453
96	.0227	9.4167	-.0015	.2454
98	.0212	9.6621	-.0015	.2454
100	.0196	9.9076	-.0015	.2455
102	.0181	10.1531	-.0015	.2455
104	.0165	10.3986	-.0016	.2455
106	.0150	10.6441	-.0016	.2455
108	.0134	10.8895	-.0016	.2454
110	.0118	11.1346	-.0016	.2451
112	.0101	11.3791	-.0016	.2445
114	.0086	11.6223	-.0016	.2432

up to the 47
peak the least squares fit to the peaks is thetam=q+s*t with q=
.[illegible]e-01 and s= -.7[illegible]15081e-02 with max error= .470635900e-02 at peak=
2 and variance= .216564877e-03

impact	time	deltat	thetas	x	tdotv	xdot	thetam	dthetam
116	11.02[illegible]	.134[illegible]	.0041	-.096373	.1562	1.2656	.0073	-.0006
117	11.97174	.140[illegible]	-.0[illegible]	.090238	-.0467	-.7971	.0071	-.0002
118	12.[illegible]34	.260099	-.0068	-.120500	.0376	.5171	.0070	-.0002
119	12.6[illegible]	.411181	.0009	.107523	-.1672	-.0221	.006[illegible]	-.0004
120	12.9[illegible]69	.[illegible]58067	-.0017	-.109283	-.1641	-.[illegible]344	.0066	.0001
121	12.97[illegible]	.072369	-.0055	-.117641	.0893	.3469	.0065	-.0001
122	12.57[illegible]7	.597589	.0160	.118637	-.0486	-.3317	.0062	-.0002
123	14.187[illegible]	.614369	-.00[illegible]	-.117140	.0722	.4048	.0060	-.0003
124	14.65[illegible]	.464141	-.0058	.092606	-.0052	-.2329	.0058	-.0002
125	15.41[illegible]	.767514	-.0[illegible]31	-.112[illegible]61	.1172	.4[illegible]	.0055	-.0004
126	15.82[illegible]	.409673	-.0030	.098901	-.1090	-.6098	.0052	-.0003

12716.15927	.33Ø2Ø5	-.ØØ13	-.1Ø8324	.1195	.7ØØ5	.ØØ48	-.ØØØ4
12816.47Ø68	.3116Ø9	.ØØ47	.115854	.Ø124	-.3Ø75	.ØØ47	-.ØØØ1
12917.1Ø638	.6355Ø2	-.ØØ36	-.113511	.Ø679	.378Ø	.ØØ45	-.ØØØ2
13Ø17.61Ø48	.51ØØ98	-.ØØ13	.1Ø2575	.1ØØ1	.1693	.ØØ44	-.ØØØ1
13117.69881	.Ø82835	.ØØ42	.114828	-.Ø28Ø	-.1583	.ØØ44	.ØØØØ
13217.74Ø47	.Ø41Ø6Ø	.ØØ11	.1Ø7791	-.1Ø92	-.2729	.ØØ43	.ØØØØ
13318.37861	.638134	.ØØ23	-.1ØØ418	.ØØ72	.43Ø7	.ØØ41	-.ØØØ3
13418.81332	.435215	-.ØØ26	.Ø998ØØ	.ØØ1Ø	.Ø555	.ØØ4Ø	.ØØØØ
13518.97399	.16Ø166	-.ØØ13	.1Ø2675	-.Ø967	-.3126	.ØØ39	-.ØØØ1
13619.56527	.591286	-.ØØØ3	-.1Ø6Ø37	.Ø945	.4661	.ØØ37	-.ØØØ3
13719.97589	.41Ø62Ø	-.ØØ32	.Ø98261	-.Ø34Ø	-.3499	.ØØ35	-.ØØØ1
1382Ø.519Ø3	.543135	-.ØØ15	-.1ØØ774	.Ø751	.4158	.ØØ33	-.ØØØ2
1392Ø.97719	.458164	-.ØØ29	.Ø98959	.Ø346	-.Ø968	.ØØ32	.ØØØØ
14Ø22.235381.258192		.ØØØ1	-.1Ø5289	.Ø767	.2575	.ØØ3Ø	-.ØØØ3
14122.89366	.658272	-.ØØ28	.Ø99267	-.Ø124	-.1724	.ØØ28	-.ØØØ1

impact	thetam	time	dthetam	dtime
116	.ØØ73	11.8626	-.Ø927	11.8626
118	.ØØ7Ø	12.347Ø	-.ØØØ3	.4844
121	.ØØ65	13.Ø758	-.ØØØ5	.7287
123	.ØØ6Ø	14.29[illegible]	-.ØØ[illegible]	1.2147
125	.ØØ55	15.5Ø[illegible]	-.ØØ[illegible]	1.2126
127	.ØØ48	16.23Ø4	-.ØØØ7	.7274
128	.ØØ47	16.4748	-.ØØØ1	.2444
129	.ØØ45	17.2Ø36	-.ØØØ2	.7288
13Ø	.ØØ44	17.6886	-.ØØØ1	.4851
133	.ØØ41	18.4162	-.ØØØ3	.7276
134	.ØØ4Ø	18.9ØØ9	.ØØØØ	.4847
136	.ØØ37	19.6286	-.ØØØ4	.7277
138	.ØØ33	2Ø.5981	-.ØØØ4	.9694
139	.ØØ32	21.ØØ2Ø	.ØØØØ	.4839
14Ø	.ØØ3Ø	22.295Ø	-.ØØØ3	1.2129

up to the 15
peak the least squares fit to the peaks is thetam=q+s*t with q=
.12ØØ3156e-Ø1 and s= -.422393998e-Ø3 with max error= .3985Ø8273e-Ø3 at peak=
15 and variance= .617Ø17292e-Ø6

initial phase angle= .ØØØØØØØØØe+ØØ
initial maximum deflection= -.1ØØØØØØØ1e+ØØ
the starting time= .ØØØØØØØØØe+ØØ
moment of inertia of the primary mass= .1Ø7999994e+ØØ
magnitude of the secondary mass= .149ØØØØØ5e-Ø2 length of the primary system=
2.21ØØØØØ4 the coefficient of restitution= .5ØØØØØØØØe+ØØ
secondary masses initial velocity=
.ØØØØØØØØØe+ØØ the gap setting= .1927Ø8328e+ØØ
desired number of impacts= 5ØØ

mass= .149ØØØØØ5e-Ø2
effective d= .1927Ø8328e+ØØ
x(Ø)= -.12[illegible]e+ØØ
deltat= .9[illegible]9978e-Ø2
e= .5ØØØØØ[illegible]ØØ
qone= .258999956e+Ø2
qtwo= 2.14[illegible]

the time iteration did not converge for i= 71

impact	time	deltat	thetas	x	tdotv	xdot	thetam	dthetam
1	.Ø559[illegible]	.Ø55915	-.Ø124	-.12375[illegible]	2.42Ø5	8.1653	.Ø942	-.ØØ58
2	.1Ø6[illegible]7	.Ø5[illegible]6Ø	.Ø867	.268Ø24	1.Ø877	-.634Ø	.Ø964	.ØØ22
3	.151Ø3	.Ø44651	.Ø733	.25845Ø	-1.5436	-4.8528	.Ø944	-.ØØ2Ø
4	.19738	.Ø46[illegible]7	-.Ø288	.Ø3266Ø	-2.3144	-5.2392	.Ø939	-.ØØØ5

5	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	-.0024
6	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	-.0015
7	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	-.0017
8	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	-.0011
9	.64715	[illegible]	.0039	.251661	-1.1567	[illegible]	[illegible]	-.0043
10	[illegible]	[illegible]	[illegible]	-.271475	[illegible]	.0311	[illegible]	.0010
11	[illegible]	[illegible]	-.0670	-.244320	1.2504	[illegible]	[illegible]	-.0014
12	[illegible]	.028113	-.0179	-.135941	2.0639	4.9796	.0016	-.0008
13	[illegible]	.081712	.0770	.266628	-.5133	-4.3296	.0735	-.0021
14	[illegible]	.122102	-.0761	-.264508	.4111	[illegible]	.0777	-.0018
15	1.13619	.135891	.0645	.238661	-.9468	-5.1487	.0741	-.0037

```
impact= 1 errmom=  .279396772e-07 errvel= -.715255737e-06
impact= 2 errmom=  .111758709e-07 errvel= -.476837158e-06
impact= 3 errmom= -.745058060e-08 errvel=  .357627869e-06
impact= 4 errmom= -.352738425e-07 errvel=  .000000000e+00
impact= 5 errmom=  .223517418e-07 errvel=  .000000000e+00
impact= 6 errmom= -.372529030e-08 errvel=  .000000000e+00
impact= 7 errmom= -.232830644e-08 errvel=  .357627869e-06
impact= 8 errmom=  .372529030e-08 errvel=  .000000000e+00
impact= 9 errmom= -.745058060e-08 errvel=  .000000000e+00
impact= 10 errmom= -.111758709e-07 errvel=  .000000000e+00
impact= 11 errmom=  .931322575e-09 errvel= -.119209290e-06
impact= 12 errmom=  .491272658e-07 errvel=  .000000000e+00
impact= 13 errmom= -.186264515e-08 errvel= -.238418579e-06
impact= 14 errmom=  .372529030e-08 errvel=  .000000000e+00
impact= 15 errmom= -.167638063e-07 errvel= -.238418579e-06
```

impact	thetam	time	dthetam	dtime
2	.0964	.1238	-.0035	.1238
5	.0915	.3699	-.0049	.2462
8	.0872	.6177	-.0043	.2478
12	.0816	.8656	-.0056	.2479
14	.0777	1.1137	-.0039	.2481
17	.0727	1.3617	-.0050	.2481
20	.0684	1.6098	-.0043	.2480
23	.0640	1.8582	-.0044	.2484
26	.0594	2.1066	-.0046	.2484
28	.0562	2.3553	-.0032	.2487
31	.0512	2.6035	-.0050	.2482
33	.0482	2.8525	-.0030	.2490
36	.0434	3.1001	-.0049	.2476
39	.0395	3.3489	-.0039	.2488
43	.0315	3.8459	-.0080	.4970
45	.0279	4.0959	-.0037	.2500
47	.0243	4.3463	-.0036	.2505
49	.0206	4.5973	-.0036	.2509
51	.0170	4.8487	-.0036	.2514
53	.0133	5.1005	-.0037	.2518
55	.0094	5.3519	-.0039	.2514
57	.0056	5.5972	-.0039	.2453

```
up to the 22
peal the least squares fit to the        peaks is thetam=q+s*t with q=
 .957418606e-01 and s= -.164172854e-01 with max error=  .272871181e-02 at peak=
1 and variance=  .372083668e-04
```

impact	time	deltat	thetas	x	tdotv	xdot	thetam	dthetam

59	5.88442	.199719	.0009	-.094315	-.1293	.0390	.0051	.0006
60	7.125271	.240654	-.0024	.090963	-.0981	-.3024	.0045	-.0006
61	7.62555	.500273	-.0036	-.104315	-.0698	-.0549	.0045	.0000
62	7.66062	.035078	-.0043	-.105942	.0280	.1148	.0045	.0000
63	7.67969	.019063	-.0033	-.103666	.0766	.1933	.0044	.0000
64	7.69195	.012266	-.0022	-.101259	.0969	.2289	.0044	.0000
65	7.70070	.000750	-.0013	-.099239	.1090	.2451	.0044	.0000
66	7.70083	.000125	-.0014	-.097233	.1135	.2519	.0044	.0000
67	8.33683	.620007	-.0019	.092121	-.0063	-.4279	.0038	-.0006
68	8.73245	.395616	.0037	-.080246	.0098	.2601	.0037	-.0002
69	9.32574	.593290	-.0032	.089229	-.0311	-.2404	.0034	-.0002
70	9.95603	.630293	.0032	-.089240	-.0269	.0345	.0034	-.0001
71	14.951955	.000020	-.0014	.008827	.0000	.0000	.0000	-.0034
72	.00000	*******	.0000	.000000	.0000	.0000	.0000	.0000
73	.00000	.000000	.0000	.000000	.0000	.0000	.0000	.0000
74	.00000	.000000	.0000	.000000	.0000	.0000	.0000	.0000
75	.00000	.000000	.0000	.000000	.0000	.0000	.0000	.0000
76	.00000	.000000	.0000	.000000	.0000	.0000	.0000	.0000
77	.00000	.000000	.0000	.000000	.0000	.0000	.0000	.0000
78	.00000	.000000	.0000	.000000	.0000	.0000	.0000	.0000
79	.00000	.000000	.0000	.000000	.0000	.0000	.0000	.0000
80	.00000	.000000	.0000	.000000	.0000	.0000	.0000	.0000
81	.00000	.000000	.0000	.000000	.0000	.0000	.0000	.0000
82	.00000	.000000	.0000	.000000	.0000	.0000	.0000	.0000
83	.00000	.000000	.0000	.000000	.0000	.0000	.0000	.0000
84	.00000	.000000	.0000	.000000	.0000	.0000	.0000	.0000

impact	thetam	time	dthetam	dtime
66	.0044	7.7729	-.0956	7.7729
68	.0037	8.7364	-.0007	.9635

up to the 2
peak the least squares fit to the peaks is thetam=q+s*t with q=
.101258373e-01 and s= -.730763643e-03 with max error= .139698386e-08 at peak=
1 and variance= .216040434e-17

initial phase angle= .000000000e+00
initial maximum deflection= -.100000001e+00
the starting time= .000000000e+00
moment of inertia of the primary mass= .107999994e+00
magnitude of the secondary mass= .503000012e-02 length of the primary system=
2.21000004 the coefficient of restitution= .500000000e+00
secondary masses initial velocity=
.000000000e+00 the gap setting= .164083332e+00
desired number of impacts= 500

mass= .503000012e-02
effective d= .164083332e+00
x(0)= -.138553335e+00
deltat= .9[illegible]5978e-02
e= .500000[illegible]
qone= .2559[illegible]996e+02
qtwo= 2.14339324
the time iteration did not converge for i= 41

impact	time	deltat	thetas	x	tdotv	xdot	thetam	dthetam
1	.05079	.050792	-.0254	-.138135	2.0686	7.3171	.0838	-.0162
2	.10[illegible]	.051744	.0717	.240505	1.4068	.0561	.0919	.0002
3	.14178	.039242	.0065	.273208	-.5954	-2.6015	.0895	-.0025
4	.16374	.021962	.0605	.215797	-1.6160	-4.1433	.0868	-.0026
5	.17[illegible]95	.015205	.0320	.152697	-2.0544	-4.7726	.0055	-.0014
6	.19[illegible]32	.015371	-.0013	.079271	-2.2029	-4.9217	.0051	-.0004

7	.26406	.069740	-.0821	-.263509	.0693	3.2036	.0822	-.0029
8	.33110	.067042	.0160	-.046792	1.9733	5.0326	.0779	-.0043
9	.39036	.059285	.0765	.251094	.1063	-2.6491	.0766	-.0013
10	.45261	.062228	.0011	.084533	-1.8462	-4.9173	.0713	-.0053
11	.51827	.065655	-.0706	-.238027	-.2070	2.2220	.0711	-.0002
12	.57799	.059725	-.0098	-.103753	1.6801	4.5869	.0656	-.0055
13	.64971	.071722	.0647	.225099	.1650	-2.1762	.0651	-.0005
14	.71357	.063654	.0011	.004491	-1.5622	-4.1974	.0603	-.0047
15	.78406	.070489	-.0584	-.211164	-.0235	2.4422	.0584	-.0019

impact= 1 errmom= -.521540642e-07 errvel= .953674316e-06
impact= 2 errmom= .000000000e+00 errvel= .238418579e-06
impact= 3 errmom= .745058060e-08 errvel= -.119209290e-06
impact= 4 errmom= .745058060e-08 errvel= .119209290e-06
impact= 5 errmom= .265426934e-07 errvel= -.715255737e-06
impact= 6 errmom= -.570435077e-08 errvel= -.238418579e-06
impact= 7 errmom= -.745058060e-08 errvel= -.238418579e-06
impact= 8 errmom= -.2421438[illegible]e-07 errvel= -.119209290e-06
impact= 9 errmom= .000000000e+00 errvel= .000000000e+00
impact= 10 errmom= -.745058[illegible]e-08 errvel= .476837158e-06
impact= 11 errmom= .000000000e+00 errvel= .000000000e+00
impact= 12 errmom= -.186264515e-07 errvel= .834465027e-06
impact= 13 errmom= .745058060e-08 errvel= .000000000e+00
impact= 14 errmom= .186264515e-07 errvel= -.476837158e-06
impact= 15 errmom= .000000000e+00 errvel= .000000000e+00

impact	thetam	time	dthetam	dtime
2	.0919	.1286	-.0081	.1286
8	.0779	.3837	-.0141	.2551
9	.0766	.3925	-.0013	.0088
12	.0656	.6444	-.0110	.2519
13	.0651	.6535	-.0005	.0091
19	.0480	1.1667	-.0171	.5132
22	.0382	1.4297	-.0098	.2630
25	.0300	1.6933	-.0081	.2636
29	.0097	2.2223	-.0203	.5290

up to the 9
peak the least squares fit to the peaks is thetam=q+s*t with q=
.921930224e-01 and s= -.374944247e-01 with max error= .457199011e-02 at peak=
1 and variance= .374561023e-04

impact	time	deltat	thetas	x	tdotv	xdot	thetam	dthetam
31	2.43332	.183959	.0011	-.079687	.0806	.8660	.0033	-.0041
32	2.60966	.176336	-.0032	.074957	.0010	-.2373	.0045	.0012
33	3.16260	.552939	.0036	-.074203	.0349	.2674	.0030	-.0007
34	3.70678	.544179	.0015	.085272	-.0529	-.3393	.0025	-.0013
35	4.14308	.437099	.0023	-.076060	-.0120	.1675	.0024	-.0001
36	4.91273	.768857	.0007	.033605	-.0349	-.2150	.0015	-.0009
37	5.55019	.637461	.0005	-.001025	.0127	.1773	.0007	-.0008
38	6.28752	.737324	.0005	.083247	.0209	-.0294	.0010	.0003
39	7.53230	1.244784	.0009	-.060047	-.0013	.0284	.0009	-.0001
40	8.86230	1.325999	-.0003	.001184	.0000	-.0105	.0000	-.0001
41	13.862325	.000020	.0005	-.017012	.0000	.0000	.0000	-.0008
42	[illegible]	*******	[illegible]	.000000	.0000	.0000	.0000	.0000
43	.00000	.000000	.0000	.000000	.0000	.0000	.0000	.0000
44	.00000	.000000	.0000	.000000	.0000	.0000	.0000	.0000
45	.00000	.000000	.0000	.000000	.0000	.0000	.0000	.0000

46	.00000	.000000	.0000	.000000	.0000	.0000	.0000	.0000
47	.00000	.000000	.0000	.000000	.0000	.0000	.0000	.0000
48	.00000	.000000	.0000	.000000	.0000	.0000	.0000	.0000
49	.00000	.000000	.0000	.000000	.0000	.0000	.0000	.0000
50	.00000	.000000	.0000	.000000	.0000	.0000	.0000	.0000
51	.00000	.000000	.0000	.000000	.0000	.0000	.0000	.0000
52	.00000	.000000	.0000	.000000	.0000	.0000	.0000	.0000
53	.00000	.000000	.0000	.000000	.0000	.0000	.0000	.0000
54	.00000	.000000	.0000	.000000	.0000	.0000	.0000	.0000
55	.00000	.000000	.0000	.000000	.0000	.0000	.0000	.0000
56	.00000	.000000	.0000	.000000	.0000	.0000	.0000	.0000

impact	thetam	time	dthetam	dtime
31	.0033	2.4812	-.0967	2.4812
32	.0045	2.7011	.0012	.2199
33	.0038	3.1766	-.0007	.4755
37	.0007	5.5817	-.0031	2.4051
38	.0010	6.3252	.0003	.7435
40	.0008	8.9823	-.0001	2.6571

up to the 6
peak the least squares fit to the peaks is thetam=q+s*t with q= .512044970e-02 and s= -.569925818e-03 with max error= .840139110e-03 at peak= 6 and variance= .300157439e-05

Vita

Bruce W. Gibson was born on 12 September 1957 in Tallahassee, Florida. He graduated from Taylor County High School in Perry, Florida in 1975 and attended Duke University. He graduated with a B.A. in Mathematics in May 1979 and received a commission in the USAF through the ROTC program. He entered active duty in November 1979 and was selected for the Air Force Institute of Technology's first Undergraduate Aeronautical Engineer Conversion Program, which ran from August 1980 to March 1981. He received his B.S. in Aeronautical Engineering from AFIT and was selected for continuation in the Graduate Astronautical Engineering Program.

Permanent Address: 500 W. Plantation Road
Perry, Florida 32347

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19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Impact Damper
Acceleration Damper
Damping
Structural Oscillation
Structural Vibration

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

The usefulness of the impact damper in eliminating vibrations is studied analytically and experimentally. Laboratory models of vibrating systems are constructed to evaluate the performance of the impact damper in reducing or eliminating forced and free vibrations. A computer simulation of a single degree-of-freedom primary system in free

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SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered) vibration employing an impact damper is constructed for the same purpose. Laboratory free vibration results are compared to the computer simulation in order to judge its accuracy.

The computer simulation is employed to determine the impact damper's performance in free vibration as the system's parameters are varied. Two significant measures of the damper's effectiveness are obtained as approximate functions of the system's parameters.

Observations regarding reduction in amplitude and steady state motion were made for the impact damper in forced vibration.

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