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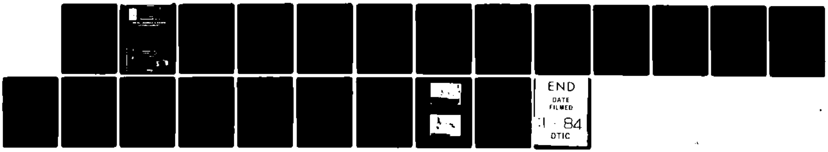

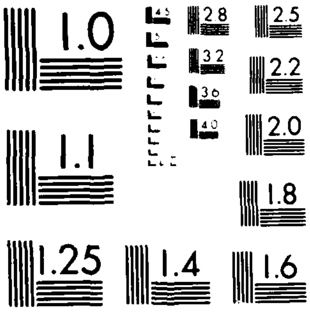

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ROYAL SIGNALS & RADAR ESTABLISHMENT

ZOOMING AND SMOOTHING IMAGES USING CONVOLUTION PROCEDURES

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SUMMARY

A procedure is described which enables images to be digitally zoomed, smoothed and differentiated. It uses a least squares polynomial fitting technique. This method reduces to the convolution of an array of samples from the picture with a corresponding array of integers. Several such integer arrays are calculated for a variety of array sizes and polynomial degree.

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ZOOMING AND SMOOTHING IMAGES USING CONVOLUTION PROCEDURES

H M Lamberton and J F Sherlock

CONTENTS

1 INTRODUCTION

Current interest in image processing depends largely on the fact that an image can be represented as a two dimensional array of samples. This matrix can then be easily processed by a digital computer. A significant amount of processing can be performed by making an estimate of the shape of the energy surface from which the samples are derived. The surface can be used, for example, to smooth or differentiate the image or to interpolate between the samples from which the surface has been estimated. A magnified image can be produced by displaying both the original picture points and regularly interpolated points at the spacing of the samples.

In general, any linear operator acting on a set of samples can be evaluated at a particular point by operating on a polynomial approximation to the function which produced the samples. This method of evaluation is a well established technique in numerical analysis. The evaluation is designed to consist of finding the sum of products of a set of coefficients with the set of samples. The process can additionally use the values of the n^{th} order differential coefficients at the sample points if these are known. Usually in image processing they are not. The advantage of this general method is that the values of the coefficients are determined only by the number of samples, the degree of the approximating polynomial and the point at which evaluation is required. Once they have been calculated they can be used for any set of sample values. There is no need to find the equation of the approximating polynomial. The problem therefore reduces to finding the values of these coefficients.

A variety of methods is available for finding the coefficients. Provided that the same criterion of goodness of fit of the approximating polynomial is used in each method, the values of the coefficients are unchanged. One criterion is that the polynomial should fit the samples exactly. The main criticism of this is that any real system introduces noise into the samples. This will appear in the coefficients. A popular alternative method is that of least squares fitting. However, it should be remembered that it is merely a postulate that minimising the sum of the squares of the deviations actually produces a better fit than alternative methods.

The application of this technique to image processing only involves the extension of the method to an approximating polynomial surface rather than a line. The set of coefficients can then be conveniently represented as a two dimensional matrix. By the use of a suitable divisor the matrix can be made to hold only integers. This matrix is convolved with a corresponding set of samples from the image in order to find the value of the operator at some point within the area covered by the sample set. This is usually the centre point though in the case of interpolation the operator may be evaluated at interstitial points other than the centre if, for example, the centre point is already known.

Several such masks of coefficients have been published. These generally apply to smoothing or differentiation using a quadratic approximating polynomial and a 3 x 3 matrix of samples. This report describes our examination of the technique particularly as a method of interpolation. Coefficient masks have been evaluated for various sizes up to 7 x 7, for polynomials up to 3rd degree and for several interstitial positions. The extent to which the coefficients can be approximated without the introduction of significant error has also been investigated.

2 THEORY

Assume that a two dimensional image I can be expressed exactly as an infinite series.

$$I = \sum_{K=0}^{\infty} \sum_{J=0}^{\infty} a_{JK} x^J y^K \qquad (1)$$

If we consider a small region of that image I(x,y) covered by a mask of side L centred on x = y = 0, figure 1, then two dimensional surface fitting can be used to obtain a best fit polynomial approximation of some chosen order m,n in x,y to that region. Provided that $(m+1)\cdot(n+1)<L^2$, the samples covered by the mask, then the polynomial will only approximately fit the samples so that a smoothed image will be generated. The fitted surface can be readily differentiated to provide gradient data and, as shown below, it can be used to interpolate intermediate points thus providing image magnification. Various criteria can be adopted to optimise the surface, but a least squares approach (either normal equations or orthogonal polynomials) is straightforward and is applicable to situations where the noise is additive. The samples comprising the image are assumed to be equispaced. In general the polynomial degrees in x and y will be equal (n) and relatively low ($n \lesssim 6$) and the mask size will also be small ($L < 9$). Thus the problem should be suitable for a normal equations approach and should not be ill-conditioned. The processed image point $\bar{z}_{xy}$ corresponding to the observed image point z at x,y is given by

$$\bar{z}_{xy} = K_{00} + K_{01}\, y + K_{02}\, y^2 \,..\, + K_{0n}\, y^n + K_{10}\, x + K_{11}\, yx \,..$$

$$+ K_{n0}\, x^n \qquad (2)$$

$$= \sum_{J=0}^{N} \sum_{K=0}^{N-J} K_{JK}\, x^J y^K$$

The problem is to determine the vector of polynomial coefficients K_{JK} in a form $K_{JK} = |H| \times \underline{Z}$ where $|H|$ is a matrix whose coefficients are z independent and the vector $\underline{Z}$(LL elements) is comprised of the pixel intensity values of the image covered by the mask (of side L). The smoothed value $\bar{z}_{00}$ at the centre of the mask x = y = 0 is given by K_{00}. In this case the matrix $|H|$ reduces to a vector $\underline{H}$ comprised (after suitable normalisation) of a set of integers. Thus $\bar{z}_{00}$ is given by a discrete convolution of $\underline{H}$ with the image and the elements of $\underline{H}$ are termed "convolute integers". The least square polynomial filtering of two dimensional images in this way has been suggested by Jannson (1972) and implemented by Edwards (1979). This approach is analogous to the well known use of convolute integers as digital smoothing (low pass) filters for one dimensional data streams. It is worth noting however, that the technique is one of several which are aimed at evaluating linear operators operating on a range of functions. In addition to samples of the function, which in this case form the image I, various derivatives may also be used.

To fit a surface by least squares using the normal equations method the sum of squares of residuals S is minimised.

$$S = \sum (\bar{z}-z)^2 \qquad (3)$$

$$S = \sum_C \sum_R \left(\left(\sum_{J=0}^{N} \sum_{K=0}^{N-J} K_{JK}\, x_C^J\, y_R^K\right) - z_{CR} \right)^2$$

The summations over rows and columns covers the mask, running from -(L-1)/2 to +(L-1)/2 in each direction. The partial derivatives $\partial S/\partial K_{JK}$ with respect to each polynomial coefficient are set to zero to yield the normal equations

$$K_0 L^2 + K_{01} \sum \sum y + \dots K_{n0} \sum \sum x^n - \sum \sum z_{CR} = 0$$

$$K_0 \sum \sum y + K_{01} \sum \sum y^2 + \dots K_{n0} \sum \sum y\, x^n - \sum \sum y\, z_{CR} = 0$$

$$\vdots \qquad (4)$$

$$K_0 \sum \sum x^n + K_{01} \sum \sum y x^n + \dots K_{n0} \sum \sum x^{2n} - \sum \sum x^n z_{CR} = 0$$

or in matrix notation

$$|A|\, \underline{K} = \underline{B}\,. \qquad (5)$$

$|A|$ is a symmetric matrix of dimension (M x M) where $M = \sum_{1}^{N+1}$ A is the number of polynomial coefficients (6 for a quadratic, 10 for a cubic etc), and $\underline{K}$ and $\underline{B}$ are column matrices of dimension (M, 1).

$$|A| = \begin{vmatrix} L^2 & \sum\sum y_R & \sum\sum y_R^2 \cdots & \sum\sum x_c^n \\ \vdots & & & \\ \sum\sum x_C^n & \sum\sum y_R x_c^n & \sum\sum y_R^2 x_c^n & \sum\sum x^{2n} \end{vmatrix}$$

$$\underline{B} = \begin{vmatrix} \sum \sum z_{CR} \\ \sum \sum z_{CR} y_R \\ \vdots \\ \sum \sum z_{CR} x_C^n \end{vmatrix} \qquad \underline{K} = \begin{vmatrix} K_0 \\ K_1 \\ \vdots \\ K_M \end{vmatrix} \tag{6}$$

The mask points are evenly spaced and if the mask is centralised on (0,0) the structure of the symmetric matrix $|A|$ is very sparse since all elements containing odd powers in each axis are zero. Premultiplying by the inverse of $|A|$ gives, provided A is not singular,

$$\underline{K} = |A|^{-1} \underline{B} \tag{7}$$

Examination of the column matrix B shows that it can be regarded as a matrix product

$$\underline{B} = |C| \, \underline{Z} \tag{8}$$

where $|C|$ has dimensions ($M \times L^2$) and $\underline{Z}$ (dimension $L^2 \times 1$) is the column matrix of the observed pixel intensities under the mask

$$|C| = \begin{array}{c c} & \xleftarrow{\quad} L^2 \xrightarrow{\quad} \\ \begin{array}{c} \uparrow \\ M \\ \downarrow \end{array} & \begin{vmatrix} x^o_{-(L-1)/2}\; y^o_{-(L-1)/2}, \;\ldots\ldots\ldots,\; x^o_{(L-1)/2}\; y^o_{(L-1)/2} \\ x^1_{-(L-1)/2}\; y^o_{-(L-1)/2} \\ \vdots \\ x^n_{-(L-1)/2}\; y^o_{-(L-1)/2}, \;\ldots\ldots\ldots,\; x^n_{(L-1)/2}\; y^o_{(L-1)/2} \end{vmatrix} \end{array}$$

Thus we can now write, as was our original objective,

$$\underline{K} = |A|^{-1} |C| \, \underline{Z} = |H| \, \underline{Z} \tag{9}$$

Recalling equation 2 we see that the scalar value $\bar{z}_{xy}$ is given by the product

$$\bar{z}_{xy} = \underline{E}\ \underline{K} \tag{10}$$

where $\underline{E}$ is a row matrix whose elements are derived from the values of the variables x, y associated with the polynomial coefficients for any desired position under the mask.

The row matrix $\underline{E}$ can be differentiated with respect to x or y to obtain any desired higher order derivative of the least squares surface at any position under the mask. Examples of $\underline{E}'$ are

a) Cubic, 0, 0 derivative at x, y

$$\underline{E}' = 1,\ y,\ y^2,\ y^3,\ x,\ xy,\ xy^2,\ x^2,\ x^2y,\ x^3$$

b) Cubic, 1, 0 derivative ($\partial/\partial x$) at x, y

$$\underline{E}' = 0,\ 0,\ 0,\ 0,\ 1,\ y,\ y^2,\ 2x,\ 2xy,\ 3x^2$$

Note that for the trivial case of a smoothed value or derivative at the centre of the mask ($x = y = 0$) all terms but one in $\underline{E}$ disappear. Thus we may write for the value of $\underline{z}$ at x, y

$$\bar{z}_{xy} = \underline{E}\ |A^{-1}|\,|C|\ \underline{Z} = \underline{F}_{xy}\ \underline{Z} \tag{11}$$

or more generally for a derivative of order ℓ, m in x and y

$$\bar{z}_{xy}^{\ell m} = \underline{F}_{xy}^{\ell m}\ \underline{Z} \tag{12}$$

The row matrix $\underline{F}$ has dimension 1 x LL and can be normalised by multiplication by an integer N to contain only integral values. $\underline{F}$ can then be redimensioned to give an L x L array which can be convolved with the image to give the desired smoothed value or derivative.

$$\bar{z}_{xy}^{\ell m} = \sum_{R}\sum_{C} \frac{F_{RC}^{xy\ell m}\ z_{RC}}{N} \tag{13}$$

The array F consists of a set of integers $F_{RC}^{xy\ell m}$, specific to the polynomial degree N, the mask size L, the x, y position and the desired derivative. Once this array has been determined convolution of the array with any image results in least squares image filtering in accordance with the above constraints. The extension of the convolute integer technique given above to calculate non-central smoothed values allows end effects to be minimised and allows the generation of intermediate points.

3 CALCULATION OF MASKS

The convolute integer masks $F_{RC}^{xy\ell m}$ have been computed using a Hewlett Packard 9825B desktop computer. The calculation, for a given input surface degree and mask size, proceeds by constructing the matrix A (Eqn 6) and its inverse. The matrix C is then formed and the product $A^{-1}C$ calculated. The row matrix E is calculated for input x,y position and ℓ,m derivative order. The co-ordinate system used is shown in Figure 1, the mask elements run from (-(L-1)/2, -(L-1)/2) in the top left hand corner. The convolution matrix F is then calculated and all the elements are rendered into integer form by finding the lowest common integer multiplier K, due regard being taken of the significant digit range of the computer. Since only a limited number of masks are required little effort has been expended towards reducing the computation time which can be high (~ 2 hrs for large, high order, off axis masks). Since the mask points are assumed to be equispaced and symmetric about the origin the matrix A is sparse and some factorisation could be undertaken. However the major time factor is in the computation of the multiplier K which may be very large ($K \sim 10^5$).

4 SMOOTHING MASKS

Central point smoothing convolute integers have been calculated for 3 x 3, 5 x 5 and 7 x 7 masks, and are presented in Table 1. The masks for n and n + 1 degree (n = 0, 2, 4 etc) are identical. There is a high (8-fold) degree of symmetry in these masks and for the 7 x 7 mask only one octant is presented. The case of n = 0 (fitting a plane to the data) corresponds to direct unweighted neighbourhood averaging. For a given mask size the sets of integers represent low pass filters with cut off frequencies increasing with increasing polynomial degree. The masks have been tested against artificial surfaces of given degree. In Figure 2 a quartic surface, is evaluated. Discrete convolution of this surface with a 5 x 5 quartic smoothing mask yields the correct central value. The least squares fit is exact since the data is noise free; a quartic polynomial is being fitted to a quartic surface. Smoothing a quartic surface with a lower order polynomial results in this case with a poor fit.

5 ROUNDING ERRORS

Some of the values in the higher degree masks can be quite large. It is relevant, therefore, to examine the extent to which they can be approximated. The test of figure 2 can be used to gain such an indication for both coefficients and samples.

The chosen quartic gives values in the range -680 to 988. Rounding one decimal place gives a range from -680 to 990. Assume that this corresponds to a picture with grey values in the range 0 - 255. The centre point of the sample mask which has the value 10 becomes equal to a picture point of brightness 105.36. Then, using the smoothing mask b iii) of table 1

$$\frac{1}{122.5} \times \left\{ \begin{bmatrix} -68 & -10 & 1 & 16 & 99 \\ -53 & -8 & 1 & 10 & 71 \\ -36 & -5 & 1 & 7 & 53 \\ -22 & -5 & 2 & 10 & 47 \\ -16 & -8 & 5 & 24 & 61 \end{bmatrix} \times \begin{bmatrix} \text{Standard} \\ \text{mask} \end{bmatrix} \right\} = 10.22$$

This represents an error of approximately 2% in the centre value but gives a centre point brightness of 105.39. It is evident that this is an insignificant error.

Rounding up each entry in the standard mask by one digit in addition to rounding up the samples gives a centre point value of 10.20. This will again lead to an insignificant change in the centre point brightness.

6 INTERSTITIAL POINT GENERATING MASKS

In order to magnify (zoom) a picture it is necessary to generate interstitial points. Figure 2 shows the points required for true 2 and 3 times magnification for odd and even sided masks. The interstitial point generating convolute integers required for a two times magnification have been calculated for 2 x 2, 3 x 3, 4 x 4 and 5 x 5 maks for various degree and are presented in Table 3. The masks for the 0, -0.5 position are obtained by rotation of the -0.5, 0 masks. Several of the masks required for single pass three and four times magnification are also included.

Stepwise convolution of a digital image with a set of masks of chosen polynomial degree will produce a smoothed magnified image whose pixel values lie on the least squares fitted surface. To generate the three new points required for a two times magnified image it is necessary to carry out three convolution operations for each original picture point. This will yield a picture comprised of 75% new smoothed picture point and 25% original unaltered points. The smoothed values of the original points (0, 0 for an odd-sided mask and -0.5, -0.5 for an even-sided mask) may also be calculated and a comparison of the original and smoothed values provides a method of statistically assessing the goodness of fit of the surface.

Centre points for all mask positions have been calculated for six pictures using 3 x 3 and 5 x 5 masks and up to 5th degree polynomial surfaces. The results are shown in Table 4. This gives the standard deviation of all calculated centre points from the actual centre point value. As might be expected, the higher the degree of the approximating surface the lower the standard deviation. In only one picture does a second degree polynomial produce a standard deviation in excess of five grey levels. If the picture is quantised to 256 levels this implies that in approximately 60% of the mask positions the differences between the actual grey value and the calculated value would not be visible.

Masks such as these have been used to process pictures derived from a TV compatible thermal imager. Processing is performed by moving a window equal to the mask size across and down the picture, one picture point at a time so that all mask size groups of adjacent picture points are eventually covered. For each window position the interstitial points are calculated. The original samples and the interpolated points are displayed using a TVframe store. The processing produces a final image with typically a minimum of x2 magnification though other degrees of magnification could be generated in one step by different masks. Higher magnification can also be obtained by repeated application of the x2 procedure. Figure 4 shows two pictures of a light aircraft. The first shows only the original image data while the second is a x2 digital magnifiction of the first. A 4 x 4 mask was used in order to balance out any mismatch between the two fields of the TV picture. The magnified picture shows clearly the defects in the image generated by the imager while producing a visually acceptable result.

7 CONCLUSIONS AND FUTURE WORK

This memo describes the theory behind the use of convolution techniques for integrating and differentiating digital images and for interpolating between picture points. In particular it describes the application of the technique to the production of a magnified version of an original image. It demonstrates that, computationally, the processing involved is very simple. However, in terms of processing speed the load when processing TV rate data will be very large. Future work should therefore be aimed at developing the technique to use for example, spline functions. This would then allow non-overlapping windows to be used and the computational load to be reduced.

The problem of designing suitable hardware to implement the process in real time at TV rates has not been examined. It seems likely that such a system would offer useful facilities in both military and civil environments. Future work should therefore be aimed at defining and building a demonstration system and at examining the impact of VLSI on such a design.

8 REFERENCES

1 Jansson, P A (1972) - "Least Squares Polynomial Filtering of Images by Convolution" Journal Opt. Soc. America, Feb. p 195.

2 Edwards, T R (1979) - "Two-Dimensional Convolution Integers for Optical Image Data Processing" Proc. of Workshop in Imaging Trackers and Autonomous Acquisition Applications for Missile Guidance. GACIAC PR-80-01, Nov, p 135-154.

TABLE 1

SMOOTHING MASKS

a) <u>3 x 3</u>

	Degree	Mask			Normaliser
i)	0, 1	1	1	1	9
		1	1	1	
		1	1	1	
ii)	2, 3	-1	2	-1	17
		2	5	2	
		-1	2	-1	

b) <u>5 x 5</u>

	Degree	Mask					Normaliser
	0, 1	1	1	1	1	1	25
i)		1	1	1	1	1	
		1	1	1	1	1	
		1	1	1	1	1	
		1	1	1	1	1	
	2, 3	-13	2	7	2	-13	175
		2	17	22	17	2	
ii)		7	22	27	22	7	
		2	17	22	17	2	
		-13	2	7	2	-13	
	4, 5	51	-99	96	-99	51	1225
		-99	-24	246	-24	-99	
		96	246	541	246	96	
iii)		-99	-24	246	-24	-99	
		51	-99	96	-99	51	

c) <u>7 x 7</u>

	Degree	Mask				Normaliser
	0, 1	1	1	1	1	49
i)			1	1	1	
				1	1	
					1	
	2, 3	-7	-2	1	2	147
			3	6	7	
ii)				9	10	
					11	

TABLE 1 (Contd)

SMOOTHING MASKS

	Degree	Mask				Normaliser
	4, 5	206	-174	-24	89	4851
iii)			-279	36	204	
				450	651	
					863	
	6, 7	-94	216	-228	212	4851
iv)			-99	-228	342	
				54	924	
					1895	

TABLE 2

DERIVATIVE MASKS (Central Position)

a) <u>3 x 3</u>

Degree	Derivative	Mask	Normaliser
1, 2	$\frac{\partial}{\partial x}$	$\begin{matrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{matrix}$	6
1, 2	$\frac{\partial}{\partial y}$	$\begin{matrix} -1 & -1 & -1 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{matrix}$	6
2	$\frac{\partial^2}{\partial x^2}$	$\begin{matrix} 1 & -2 & 1 \\ 1 & -2 & 1 \\ 1 & -2 & 1 \end{matrix}$	3
2	$\frac{\partial^2}{\partial x \partial y}$	$\begin{matrix} 1 & 0 & -1 \\ 0 & 0 & 0 \\ -1 & 0 & 1 \end{matrix}$	4

b) <u>5 x 5</u>

Degree	Derivative	Mask	Normaliser
3	$\frac{\partial^3}{\partial x^3}$	$\begin{matrix} 31 & -44 & 0 & 44 & -31 \\ -5 & -62 & 0 & 62 & 5 \\ -17 & -68 & 0 & 68 & 17 \\ -5 & -62 & 0 & 62 & 5 \\ 31 & -44 & 0 & 44 & -31 \end{matrix}$	420

TABLE 3

INTERSTITIAL POINT GENERATORS

a) 2 x 2

Degree	Position	Mask	Normaliser
0	all	1 1 1 1	4
1	0, 0	1 1 1 1	4
	-0.5, -0.5	3 1 1 -1	4
	-0.5, 0	2 0 2 0	4
	-0.25,0	3 1 3 1	8
	-0.25,-0.25	2 1 1 0	4
	-0.5, -0.25	5 1 3 -1	8
	0.33, 0.33	1 3 3 7	12
	0.33,-0.5	2 4 -1 1	6

b) 3 x 3

Degree	Position	Mask	Normaliser
0	all	1 1 1 1 1 1 1 1 1	9
1	0, 0	1 1 1 1 1 1 1 1 1	9
	-0.5,-0.5	10 7 4 7 4 1 4 1 -2	36
	-0.5, 0	7 4 1 7 4 1 7 4 1	36
	0, -0.5	7 7 7 4 4 4 1 1 1	36
2	0, 0	-1 2 -1 2 5 2 -1 2 -1	9
	-0.5,-0.5	29 38 -13 38 56 14 -13 14 -19	144
	-0.5, 0	1 10 -11 25 34 13 1 10 -11	72

TABLE 3 Contd

c) 4 x 4

Degree	Position	Mask				Normaliser
0	all	1	1	1	1	16
		1	1	1	1	
		1	1	1	1	
		1	1	1	1	
1	0, 0	As Degree 0				
	-0.5, -0.5	11	9	7	5	80
		9	7	5	3	
		7	5	3	1	
		5	3	1	-1	
	-0.5, 0	4	3	2	1	40
		4	3	2	1	
		4	3	2	1	
		4	3	2	1	
2	0, 0	-3	2	2	-3	32
		2	7	7	2	
		2	7	7	2	
		-3	2	2	-3	
	-0.5, -0.5	7	24	16	-17	200
		24	43	37	6	
		16	37	33	4	
		-12	6	4	-33	
	-0.5, 0	-13	19	11	-37	320
		37	69	61	13	
		37	69	61	13	
		-13	19	11	-37	
3	0, 0	As Degree 2				
	-0.5, -0.5	-23	44	-4	-17	200
		44	93	37	26	
		-4	37	-17	-16	
		-17	26	-16	7	
	-0.5, 0	-4	5	-2	-1	32
		4	11	2	1	
		4	11	2	1	
		-4	-5	-2	-1	
	-0.25, -0.25	-412	466	64	-293	3200
		466	1162	673	324	
		64	673	192	-54	
		-293	324	-54	-102	
	-0.5, -0.25	-1641	2323	-593	-781	12,800
		2363	5311	1699	1127	
		677	3369	-179	-367	
		-1399	1797	-927	29	
	-0.25, 0	-59	61	-1	-33	512
		45	149	71	23	
		45	149	71	23	
		-59	61	-1	-33	

TABLE 3 Contd

c) Contd

Degree	Position	Mask					Normaliser
	0.35, 0.35	-587	-2614	7274	-5823		64,800
		-2614	567	13203	394		
		7274	13203	26127	11146		
		-5823	394	11146	-8467		
	-0.35, -0.5	-8153	12939	1821	-6607		64,800
		12484	28158	15312	8846		
		-2519	9897	-2517	-4861		
		-4562	6756	-3066	872		

d) 5 x 5

Degree	Position	Mask					Normaliser
a	0, 0	See Table 1b					
	-0.5, -0.5	-90	61	47	-27	-56	800
		61	184	152	70	43	
		47	152	112	32	17	
		-27	70	32	-36	-29	
		-56	43	17	-29	10	
	-0.5, 0	-571	314	184	-226	-181	5600
		149	914	664	134	59	
		389	1114	824	254	139	
		149	914	664	134	59	
		-571	-314	184	-226	-181	

TABLE 4

PICTURE SMOOTHING RESULTS

Image	Mask	Degree	σ
1	3 x 3	0, 1	6.14
		2, 3	1.01
	5 x 5	0, 1	8.10
		2, 3	5.12
		4, 5	1.33
2	3 x 3	0, 1	5.77
		2, 3	1.25
	5 x 5	0, 1	7.55
		2, 3	4.88
		4, 5	1.66
3	3 x 3	0, 1	3.43
		2, 3	0.79
	5 x 5	0, 1	4.78
		4, 5	0.99
4	3 x 3	0, 1	4.4
		2, 3	1.18
5	3 x 3	0, 1	5.6
		2, 3	0.89
6	3 x 3	0, 1	4.8
		2, 3	1.25

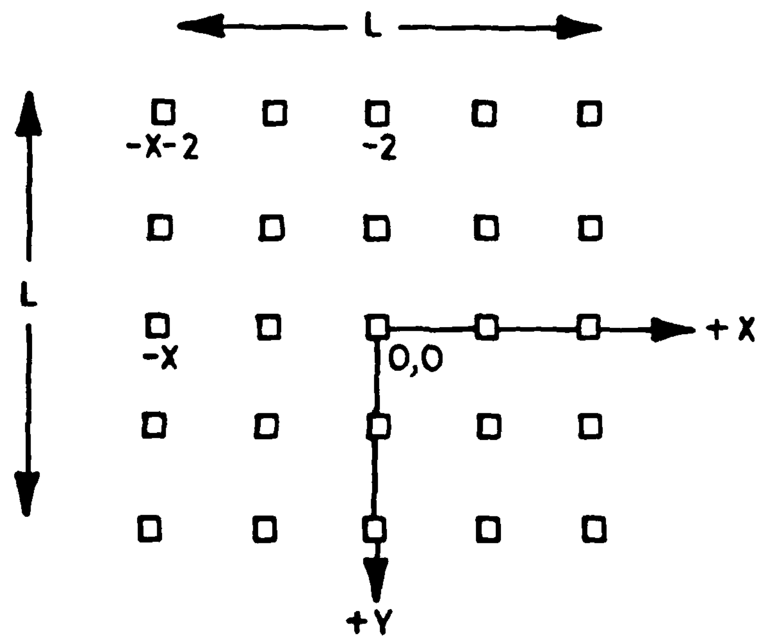

FIG.1. MASK COORDINATE SYSTEM

FIGURE 2

TEST OF 5 x 5 SMOOTHING MASKS

Quartic Surface

$$Z = 10 + 5x + 6y + 12xy - 5x^2 + 2y^2 + 2x^2y + 21xy^2$$
$$+ 54x^3 + y^3 + 6x^4 - 21x^3y + 5x^2y^2 + 4xy^3 + y^4$$

$$\begin{array}{c|ccccc} & -2 \leftarrow & & x & & \rightarrow +2 \\ \hline -2 & -680 & -98 & 14 & 160 & 988 \\ \uparrow & -526 & -75 & 6 & 95 & 714 \\ y & -356 & -48 & 10 & 70 & 528 \\ \downarrow & -224 & -47 & 20 & 103 & 472 \\ +2 & -160 & -78 & 54 & 236 & 612 \end{array} \equiv S$$

Smooth with 4th, 5th degree convolution mask (Table 1b, iii))

$$\frac{1}{1225}\begin{bmatrix} S & * & 51 & -99 & 96 \\ & & & -24 & 246 \\ & & & & 541 \end{bmatrix} = 10 \quad \text{at } x = y = 0$$

Smooth with 2nd, 3rd degree convolution mask (Table 1b, ii))

$$\frac{1}{175}\begin{bmatrix} S & * & -13 & 2 & 7 \\ & & & 17 & 22 \\ & & & & 27 \end{bmatrix} = -24.4 \quad \text{at } x = y = 0$$

Actual Value at $x = y = 0$ is 10.

ORIGINAL

FIG. 3.

2 x DIGITAL ZOOM

FIG. 4.

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Abstract A procedure is described which enables images to be digitally zoomed, smoothed and differentiated. It uses a least squares polynomial fitting technique. This method reduces to the convolution of an array of samples from the picture with a corresponding array of integers. Several such integer arrays are calculated for a variety of array sizes and polynomial degree.

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