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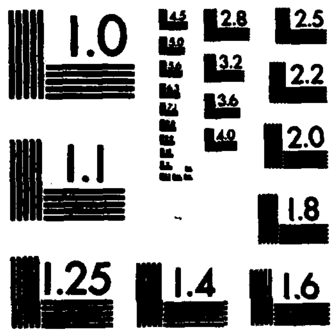

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Solitary Waves in Water Colliding Head-on

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A numerical investigation of head-on collisions of solitary waves in water shows that they reform to be solitary waves almost as before the collision, but not quite. A little energy is deposited in a dispersive wave train generated during the collision. Hence, solitary waves in water are not exactly solitons. The calculations further indicate a transition where the water at the crest attempts to fly off the surface when an identical pair of incoming waves have an amplitude/depth ratio exceeding approximately 0.45.

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SOLITARY WAVES IN WATER COLLIDING HEAD-ON

When solitary waves in water collide head-on, a brief but nonlinear interaction occurs, after which the solitary waves reform. If exactly unchanged in form, the use of the term "soliton," coined by Zabusky and Kruskal[1], is appropriate. Solitons are solutions of many model equations that mathematically describe slightly nonlinear waves in many media. The first equation shown to possess soliton properties is the Korteweg-deVries Equation[2], originally derived to describe the evolution of water waves. Like Ref. 2, this report deals with water waves. It addresses the question of how closely solitary waves following a head-on collision resemble solitons.

Camfield and Street[3] and Maxworthy[4] studied these collisions experimentally, by reflecting a solitary wave from a rigid wall or by running two solitary waves into each other. They find that the maximum amplitude during the collision is larger than the sum of the initial amplitudes, by an amount that increases with increasing amplitude. Maxworthy reports that a new phenomenon occurs when the amplitudes of the colliding waves become sufficiently large. The water near the crest attempts to lift off the body of water and subsequently collapses back into the rest of the water.[4] For a pair of equal-amplitude solitary waves meeting inside the wavetank, the transition occurs when the solitary waves start with amplitudes near 0.5. When a wave reflects from a wall, the effect is deferred to larger amplitudes, probably because of cohesive forces between the water and the wall.

The theoretical models of colliding solitary waves take the water to be inviscid and incompressible, and the flows to be irrotational. Chan and Street[5] applied a modified marker-in-cell numerical method and report "In a

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key test of the method, a wave was reflected from the wall and returned to its initial position for the case $H_0/d = 0.2$. [H_0 is the initial solitary wave's elevation above still water depth d.] The reflected wave had exactly the same surface profile as the incident wave."

Analysis has been applied to this problem, most recently to third order in H_0/d by Su and Mirie.[6] They find that to this order the waves asymptote to solitary waves identical in form to the initial waves, though slightly retarded in location. They also predict, however, the existence of a dispersive wave train generated during the collision. The dispersive wave train has some energy (not necessarily at third order) that has to come at the expense of the solitary waves. Consequently, the existence of a dispersive wave train guarantees that the outgoing solitary waves are less energetic than the incoming solitary waves. In short, the solitary waves are not exactly solitons.

Fenton and Rienecker[7] recently reported the application of an accurate numerical method[8] to colliding solitary waves. They report substantial changes in solitary wave amplitudes after collision, and cast doubts on the analysis of Su and Mirie. They do not find evidence of a dispersive wave train through the admittedly brief duration of their numerical experiments.

This report describes the application of the "unified waves model"[9] to head-on solitary wave collisions. This model uses finite difference techniques for propagation in one horizontal dimension. It is capable of treating both irrotational and rotational waves in channels of gradually varying depth and breadth, with ambient steady currents present, and with forced or free waves. Model results have been compared with results from laboratory experiments and with those from other numerical and analytical theories. To date, the comparisons with theory show the model to be very accurate; some features seen in experiments are seen in the unified waves model, but not in other models. Features of the model are summarized below.

1. The model uses exact prognostic equations in conservation form.[10] These appear not to have been used in wave modeling before now. An integral of the momentum equation is an extension of Kelvin's circulation theorem.

2. Higher order expansions than used in other numerical models connect the velocity variables that appear in the governing equations. This allows the model a) to incorporate long wave theory exactly, b) to include both shallow water and essentially deep water waves in the same model, and c) to represent fairly long nonlinear waves to one order better than Boussinesq -- hence the name "unified waves model."

3. The model employs a numerical method, pure leapfrog, that gives no unwanted numerical diffusion. The time-stepping procedures are simple enough to analyze in some detail and to implement efficiently on vector computers.

4. The model can take a time step equal to a space step (in nondimensional units in which the linear long wave speed is unity). This permits efficient machine computations, unlike methods developed for the Korteweg-deVries Equation. Moreover, this procedure removes any spurious numerical dispersion at the lowest order.

5. Finally, the diagnostic equations are cast in a form such that only tridiagonal matrix equations need to be solved. A very fast, fully vectorized algorithm is then used to invert the matrices.

The first three columns of Table 1 list properties of the solitary waves used here. These are obtained by running an initial disturbance that resembles a solitary wave through the computational channel, allowing the solitary wave to outrun the rest of the disturbance, and extracting the solitary wave from the rest to use as initial conditions for runs involving collisions. The solitary wave properties given in Table 1 are very close to those of exact solitary waves, as derived by Longuet-Higgins and Fenton.[11] The accuracy is about that of a fourth order expansion,[12] though the formal accuracy is limited to second order in the treatment of the diagnostic equations. Perhaps the unwarranted accuracies are due to retaining exact prognosis thoughout; perhaps they are fortuitous.

Figure 1 provides a global view of the collision of two solitary waves, each having an initial amplitude of 0.386. The solitary waves appear to survive the collision intact. Weak additional disturbances are visible in the region between the solitary waves after the collision, however.

Figure 2 employs the same data as Fig. 1, but increases amplitudes 10-fold, and clips the profiles at 0.02. Figure 2 brings out some details of the additional disturbance. Clearly, it is a dispersive wave train generated by the collision, consistent with the prediction of Su and Mirie.

Figure 3 plots the amplitude and phase of the wave crest that moves left to right in Figure 1. The behavior is characteristic of all runs. After a brief transient the crest reaches a constant value, merges with the other solitary wave (at $t \cong 100$), where the amplitude more than doubles, separates, reaches a minimum amplitude (just before $t = 120$), and slowly reforms to a solitary wave slightly less high than before the colliion.

The phase curve increases at first, indicating an accelerating crest until the two crests merge at $x = 200$. The crest remains in this location for a while, and, upon the reemergence of two crests, accelerates until $t \cong 140$, after which a crest decelerates. Because the asymptotic outgoing waves travel a little slower than the incoming ones, the phase lag will increase as time goes on. By extrapolating the phase line shown in Fig.3 from the region between $t = 210$ to $t = 268$ back to the collision time $t = 56.675$ one obtains an estimate of the phase lag. This turns out to be $\delta x = 0.478$.

Numerical tests were performed to determine: 1) Can truncation error be generating the dispensive wave train, and so be responsible for the slightly non-soliton behavior? 2) Is the particular second order formulation chosen accidently giving results that are not common to other formulations accurate to the same order? 3) Can coding errors be introducing artificial diffusive or anti-diffusive effects? 4) Are the quantities that should be conserved in the calculations actually conserved?

Runs at different resolutions Δx were performed to answer question (1). Specifically, for head-on collisions between solitary waves having initial amplitudes of 0.386 were run with three resolutions, $\Delta x = 1/4$, $\Delta x = 1/8$, and $\Delta x = 1/16$, each with $\Delta t = \Delta x$. The results differ slightly, particularly in the phases of the crests and troughs of the dispersive wave train. Yet if truncation error is responsible for the dispersive wave train, one expects amplitudes to be proportional to $(\Delta x)^2$, or to vary by a factor the order of 16 between the $\Delta x = 1/4$ and $\Delta x = 1/16$ runs. The observed amplitude variation is only a few percent. Consequently, we can rule out truncation error as being responsible for the existence of the dispersive wave train.

The unified waves model is flexible in evaluating high-order linear dispersive terms. For problems for which nonlinear and dispersive effects are in approximate balance, such as the problem at hand, the theory is probably accurate only to second order, no matter how accurately the dispersive terms have been taken into account. Runs with fourth order dispersive terms included agree very well with runs that drop dispersive terms beyond second order. Consequently, question (2) above is answered, in that at least two partially independent second-order formulations give the same results.

The formulation and numerical methods introduce no numerical diffusion or antidiffusion. To check whether any coding errors introduce diffusion, a run that involved time reversal was performed. For this run $\Delta x = \Delta t = 1/8$, and the initial solitary wave started near $x = 131$, bounced off a wall at $x = 200$, and returned to $x = 152$ over $\delta t = 100$ (800 time steps). Time was then reversed and the run continued for another 800 time steps ($\delta t = -100$). Results are compared at time steps 50 and 1550. With a perfect computer, the fields at these time levels should be identical. The elevation and surface velocity at fixed grid points differ slightly, but by no more than 0.000024 in elevation and 0.000061 in surface velocity. These deviations are consistent with roundoff error. Consequently, diffusive coding errors are absent (or result in undetectably tiny effects).

The final question, that of proper conservation, is a bit complex. The formulation and numerical implementation are designed to conserve two linear field quantities:

$$\int \eta dx = \text{Constant} + \text{Mass injection from ends} \qquad (1)$$

$$\int (u_s + \eta' v_s)dx = \text{Constant} + \text{Momentum injection from ends} \qquad (2)$$

In (1-2) η is the elevation above still water level, u_s and v_s are the components of horizontal and vertical velocities of the fluid at the free surface, and η' is the surface slope. The integrals span the computational channel, left to right. Eq. (1) is the obvious conservation of mass; eq. (2) is an extension of Kelvin's circulation thorem[10]. The numerical methods are designed to satisfy (1) and (2) exactly, and constant monitoring of both sides of (1) and (2) show that the methods work perfectly to the four significant figures monitored. The energy, however, which the physics demands be conserved, is not conserved exactly by the unified waves model. The calculations add some energy to the computational domain during a solitary wave collision, and remove most, though not all, of the energy as time goes on. The net energy addition leaves some uncertainty about the change of solitary wave amplitudes. If one takes away the artificially introduced energy from the emerging solitary waves, they would have slightly smaller amplitudes. Using interpolations from Table 5 of the paper by Longuet-Higgins and Fenton[11] one can relate solitary wave amplitude and total energies. I regard the failure of the unified waves model to exactly conserve energy as introducing an uncertainty in the amplitude of the emerging solitary waves, but not in the general conclusion, to be drawn, that the colliding solitary waves are not quite solitons.

Some quantitative results of the collisions of two solitary waves with themselves and with each other are given Table 1. One property of solitary wave collisions not reported in other theoretical works is the maximum vertical acceleration at the surface. This exceeds that of gravity in the calculations of the 0.4726 collision. This would imply negative pressures inside the fluid near the surface, and the fluid there would attempt to "fly off". More precisely, it would be left behind by the faster falling fluid below it. Thus, new phenomena not describable by the unified waves model are starting when initial solitary wave amplitudes are less than 0.4726. Presumably, these new phenomena are those found experimentally by Maxworthy.[4] A special set of calculations was performed to identify the point at which the maximum vertical deceleration matches g. The initial conditions involved solitary waves outrunning a residual disturbance. Calculations with $\Delta x = \Delta t = 1/16$ were run in a computational domain of 3200 grid points through the time that the crest reached its maximum. The calculations indicate that the transition should occur at initial amplitudes of 0.45.

The other results of Table 1 agree well with the results of at least one previous investigation. The behavior during the collision, as measured by the comparison of runup with Fenton and Rienecker, and with experiments, is comforting, because one cannot guarantee good performance from the unified waves model for the extreme amplitudes encountered during the collision. The substantial decrease in wave amplitude at the early times examined by Fenton and Rienecker is confirmed.

The eventual relaxation to solitary waves only slightly less energetic than initally, and, especially, the strong dependence of Δa on amplitude (about quartic) are consistent with the null result $\Delta a = 0$ given from the third order theory of Su and Mirie. These small changes must be real, however, as Figure 2 and energy conservation make clear. Thus, the term "soliton" is a little loose in describing solitary waves in water, for they are not quite the same after colliding head-on, though almost.

Table 1 -- Properties of colliding solitary waves.

(1) Amplitude	(2) Speed	(3) Mass	(4) Runup	(5) Accel.	(6) $(\Delta x)_{min}$	(7) $(\Delta a)_{min}$	(8) Phase	(9) $(\Delta a)_{final}$
0.1976	1.0933	1.0888	0.4189	0.13	7.7	0.0035	0.276	0.0001
	{1.0933}	{1.0905}	(0.4205)				(0.275)	
			[0.4206]			[0.0031]	[0.39]	
0.2933	1.1348	1.3519	0.6449	0.28	6.3	0.0109	0.351	0.0004
	{1.1347}	{1.3540}	(0.6485)				(0.346)	
			[0.6465]			[0.0101]	[0.55]	
0.3858	1.1726	1.5683	0.8927	0.55	5.6	0.0235	0.408	0.0009
	{1.1725}	{1.5702}	(.08890)				(0.409)	to
			[0.8905]			[0.0230]	[0.72]	0.0011
0.4726	1.2062	1.7408	1.2107	1.4	5.4	0.0431	0.492	0.0018
	{1.2058}	{1.7433}	(1.1361)				(0.465)	to
			[1.1626]			[0.0422]		0.0027

All entries are in units $g = H_o = 1$. All computations that generate the data here take $\Delta x = \Delta t = 1/8$. The first three columns list solitary wave properties that have evolved to a constant value prior to collision. The figures in brackets use expansions or table interpoltions from Longuet-Higgins and Fenton.[11] Solitary wave mass is $\int_{-\infty}^{\infty} \eta dx$.

Columns 4 and 5 are measured at the time of maximum elevation. "Runup" is the elevation above still water level and "Accel." is the magnitude of the vertical acceleration of the fluid particle at the crest. Here, and in the other entries, the numbers in ordinary parenthases are from the third order analytical results of Su and Mirie,[6] and the numbers in square parentheses are interpolations of the numerical results of Fenton and Rienecker.[7]

Columns 6 and 7 indicate how far away from the center of the collision the minimum amplitude occurs (see Fig. 3), and by how much this minimum amplitude is less than the initial amplitude. Fenton and Rienecker report their numbers as a final change of amplitude, but it appears that their computational domain does not exceed Δx_{min} by very much (see their Fig. 4). Entries to columns 8 and 9 come from analyzing results at very late times. The phase is the time-lag of an outgoing wave extrapolated from late times to the collision time. The range of amplitude changes is set down to account for the failure of the numerical calculations to properly conserve energy. The smaller number is the computed amplitude; the larger number takes away from the solitary waves all of the energy added by the numerics, using energy-amplitude curves of Longuet-Higgins and Fenton.[11]

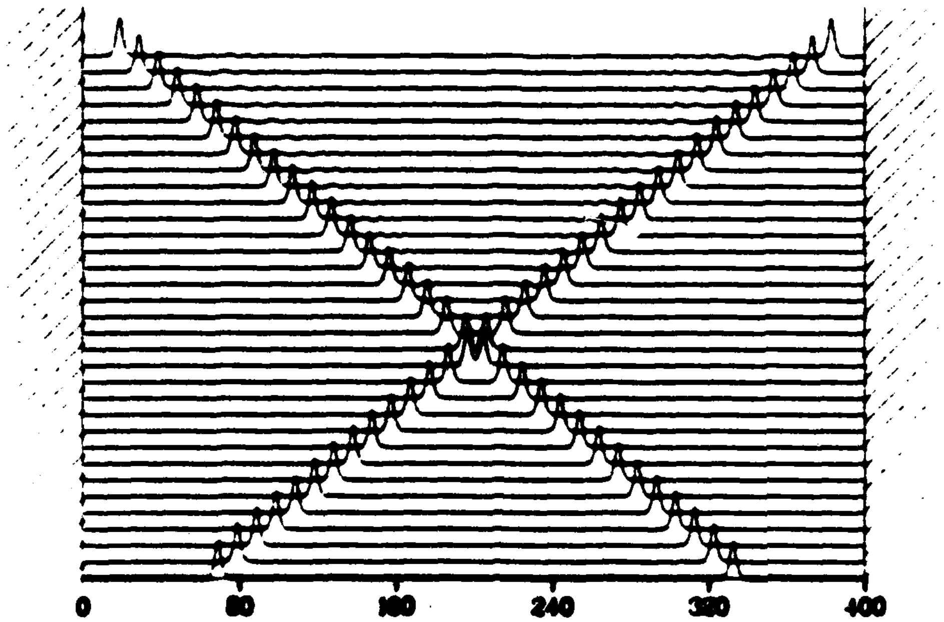

Figure 1 -- Head-on collision of an identical pair of solitary waves of amplitude 0.386. Computations are carried out in a computational channel of 3200 equally-spaced grid points using $\Delta x = \Delta t = 1/8$. The profiles shown are equally-spaced in time, starting at the bottom, with the vertical spacing between profiles set so that disturbances that propagate at $c_{dimensional} = \sqrt{gH_o}$ would be aligned along $\pm 45^{\circ}$ in the figure.

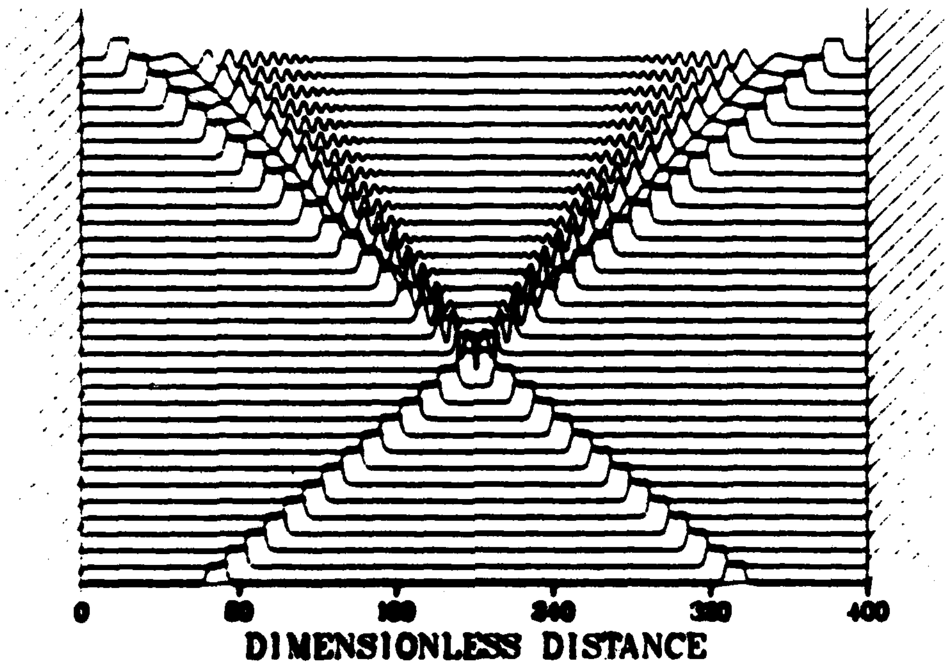

Figure 2 -- The same collision as depicted in Fig. 1, but with output data post-processed to amplify by a factor of 10, and clip wave elevation above 0.02.

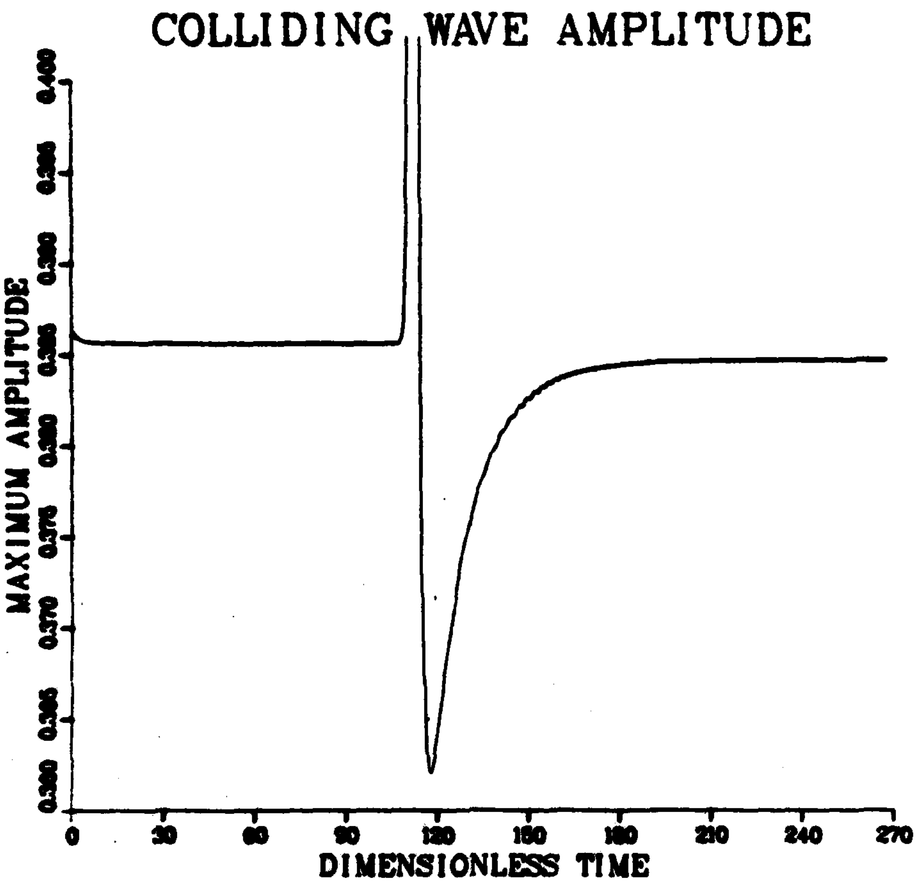

Figure 3 -- Time history of a crest. (a) Amplitude: The crest amplitude goes far off scale during the collision (to 0.893). The small wiggles in the vicinity of time = 150 are not an artifact of the machine-drawn graphics, but may be a numerical artifact; other second order versions do not generate them. All versions asymptote to amplitudes less than initial by 0.0009.

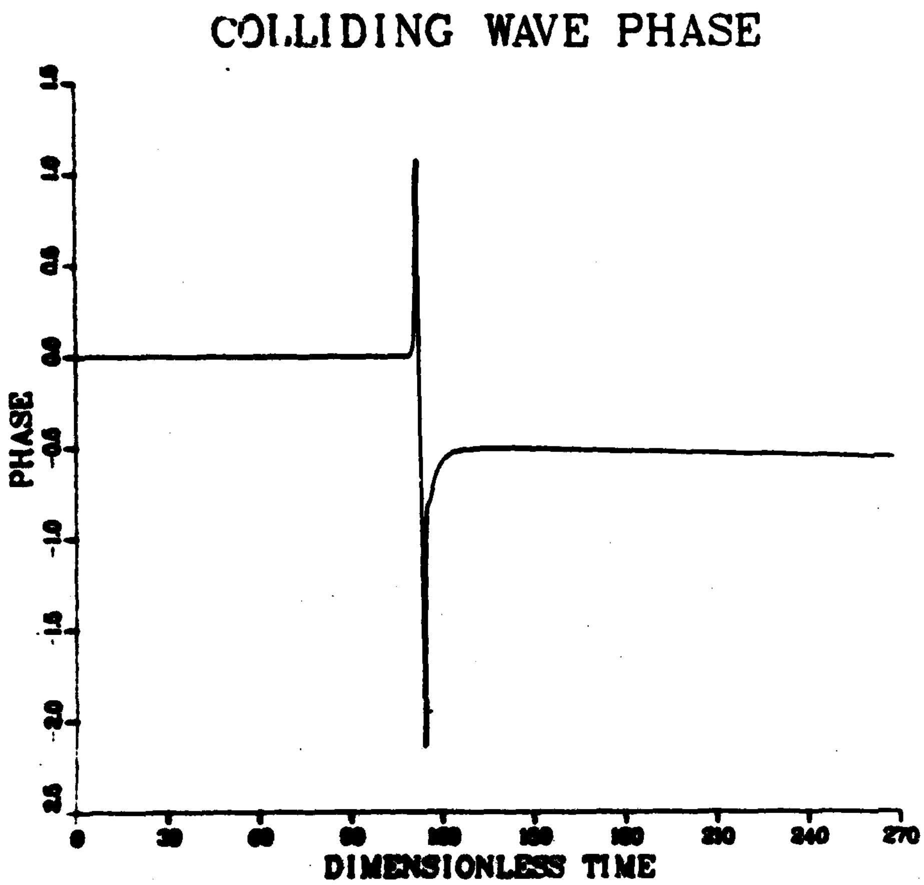

Figure 3 -- Time history of a crest. (b) Phase: The data are analyzed to give the speed and a reference location of an incoming solitary wave. A tracer moving with this speed and lined up at the reference location is defined. The plot shows the difference between the position of a wave crest and the position of the tracer. Between the times of maximum and minimum on the curve the crests are merged at x = 200.

ACKNOWLEDGMENT

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