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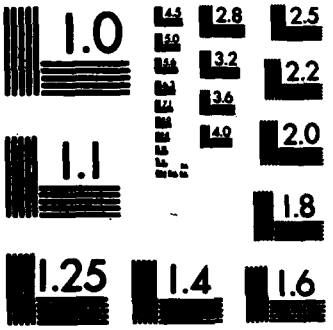

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ESTIMATION FOR INFINITE DIMENSIONAL ORNSTEIN-UHLENBECK PROCESSES

by

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ABSTRACT

The maximum likelihood estimator for parameters in the generating operator of an infinite dimensional Ornstein-Uhlenbeck process is shown to be consistent and asymptotically normal. The generating operator of the process is assumed to be in the form of a finite linear combination of fixed commuting dissipating operators and the coefficients in the linear combination represent the unknown parameters.

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1. Introduction.

Infinite dimensional Ornstein-Uhlenbeck processes have recently been of interest as models arising in a wide variety of physical phenomena: quantum mechanics [3], scattering theory [10], neural response [11], stochastic control [5], chemical reaction problems [5, 12] and as limiting cases in infinite particle systems [6]. From the point of view of applications it is important to have a statistical theory for the estimation of unknown parameters in such models. Despite the presence of a considerable literature (see the survey in [2, Ch. 9]) on estimation for finite dimensional diffusion processes, estimation for infinite dimensional diffusion processes has received little attention. The recent paper of Bagchi and Borkar [1] appears to be the first to address such problems.

In the present paper we study the asymptotic behavior of the maximum likelihood estimator for parameters in the generating operator of an infinite dimensional Ornstein-Uhlenbeck process. The parameters are coefficients in a finite linear combination of known operators which are assumed to commute. The estimator is shown to be consistent and asymptotically normal. Our approach is quite different from [1].

2. Preliminaries.

The basic theory to be used in this work is the generalization of Itô's stochastic calculus to abstract Wiener spaces due to Kuo [7]. Let B denote a real separable Banach space with norm $\|\cdot\|$. It is known [9] that each Gaussian measure on the Borel sets of B can be induced from the canonical Gaussian cylinder set measure on

a separable Hilbert space H contained in B, through the injection i of H into B. The triple (i, H, B) is known as an abstract Wiener space. Denote the inner product on H by $\langle \cdot, \cdot \rangle$ and the norm by $|\cdot|$. The pairing between B and B^* is denoted $(\cdot, \cdot)$. As in [7] assume the following condition on (i, H, B): There exists a sequence Q_n of finite dimensional projections on B such that (1) $Q_n(B) \subset$ range (i^*), (2) Q_n converges strongly to the identity in both B and H.

Let W_t, $t \geq 0$ denote the B-valued Wiener process derived from (i, H, B), see [9]. Suppose that X_t, $t \geq 0$ is a B-valued Ornstein-Uhlenbeck process satisfying the following stochastic integral equation

$$X_t = x_0 + \int_0^t A(X_s)ds + W_t, \quad t \geq 0, \tag{2.1}$$

where $x_0 \in B$ and $A: B \to H$ is a bounded linear operator. By [7, Theorem 5.1] (2.1) has a unique, non-anticipating, continuous solution. Let μ_W^T and μ_X^T denote the measures induced on $C([0, T], B)$ by $(x_0 + W_t, t \in [0, T])$ and $(X_t, t \in [0, T])$ respectively. From [8], μ_X^T and μ_W^T are equivalent and

$$\frac{d\mu_X^T}{d\mu_W^T}(X) = \exp\left[\int_0^T (A(X_t), dX_t) - \frac{1}{2} \int_0^T |A(X_t)|^2 dt \right], \tag{2.2}$$

where the stochastic integral in this expression is defined in [7]. The true generating operator A of the observed process will be denoted A_0. Since the maximum likelihood estimator of A_0 is not defined in general it is necessary to restrict the family of possible generators. Assume that A_0 can be expressed uniquely in the form

$\sum_{j=1}^{k} \alpha_j A_j$, where $A_1, \ldots, A_k$ are given bounded linear operators mapping B into H and $\alpha_1, \ldots, \alpha_k$ are real constants. Let $\alpha_1^0, \ldots, \alpha_k^0$ denote the true values of the coefficients $\alpha_1, \ldots, \alpha_n$ and write them in a column vector as α^0. The maximum likelihood estimator $\hat{\alpha}_T$ of α^0 can be derived from (2.2) [cf. 4] and is given by

$$\hat{\alpha}_T = C^{-1}\rho,$$

where

$$\rho_j = \int_0^T (A_j(X_t), dX_t),$$

$$c_{ij} = \int_0^T \langle A_i(X_t), A_j(X_t) \rangle dt.$$

3. Results.

An operator $A:B \to H$ is said to be dissipating if $\langle Ax, x \rangle \leq 0$, for all $x \in H$, and symmetric if $\langle Ax, y \rangle = \langle x, Ay \rangle$, for all $x, y \in H$. Assume that A_0 is dissipating and symmetric. Since i is a compact operator, see [15], $i^*A_0^*: H \to H$ is compact. Thus $i^*A_0^*$ has a spectrum consisting of countably many eigenvalues and since $i^*A_0^*$ is symmetric, the eigenvectors belonging to district eigenvalues are orthogonal. Let the (strictly negative) eigenvalues of $i^*A_0^*$ be denoted $\{-\lambda_n, n \geq 1\}$, where $\lambda_n > 0$ and each eigenvalue is counted according to its multiplicity. Let corresponding orthonormal eigenvectors be denoted $\{e_n, n \geq 1\}$. Assume that $\{e_n, n \geq 1\}$ is a CONS for H. The following condition is to be imposed on operators $A:B \to H$.

(C1) $$\sum_{n=1}^{\infty} \frac{|i^*A^*(e_n)|^2}{\lambda_n} < \infty.$$

Theorem 3.1. Suppose that the operators $A_1, \ldots, A_k$ commute and satisfy condition (C1). Then

(a) $\hat{\alpha}_T \xrightarrow{P} \alpha^0$ as $T \to \infty$

(b) $T^{\frac{1}{2}}(\alpha_T - \alpha^0) \xrightarrow{D} N(0, V^{-1})$ as $T \to \infty$, where V is the $k \times k$ matrix $(V_{j\ell})$ given by

$$V_{j\ell} = \frac{1}{2} \sum_{n=1}^{\infty} \frac{\langle i^* A_j^* e_n, i^* A_\ell^* e_n \rangle}{\lambda_n}.$$

This result will be proved through a series of Lemmata for which the conditions of the Theorem are assumed to hold.

Lemma 3.2. $\frac{1}{T} c_{j\ell} \xrightarrow{P} V_{j\ell}$ as $T \to \infty$.

Proof. Fix $n \geq 1$. Define processes $Y_t^j = \langle e_n, A_j(X_t) \rangle$, $W_t^j = \langle e_n, A_j(W_t) \rangle$ for $j = 1, 2$. Then $W_t = \begin{pmatrix} W_t^1 \\ W_t^2 \end{pmatrix}$ is a two-dimensional Wiener process with covariance $(s \wedge t)\,\Sigma$, where $\Sigma = (\sigma_{j\ell})$ is a 2×2 matrix with $\sigma_{j\ell} = \langle i^* A_j^* e_n, i^* A_\ell^* e_n \rangle$. Let $\phi_j = A_j^*(e_n)$. Then Y_t^j satisfies the stochastic differential equation

$$dY_t^j = \phi_j(A_0(X_t))dt + dW_t^j, \quad t \geq 0$$

$$Y_0^j = \phi_j(x_0).$$

But $\phi_j(A_0(X_t)) = (A_0^* i^* \phi_j, X_t) = (A_0^* i^* A_j^* e_n, X_t)$

$= (A_j^* i^* A_0^* e_n, X_t)$ since A_0 and A_j commute

$= -\lambda_n (A_j^* e_n, X_t)$

$= -\lambda_n Y_t^j$.

Thus $Y_t = \begin{pmatrix} Y_t^1 \\ Y_t^2 \end{pmatrix}$ satisfies $dY_t = -\lambda_n Y_t dt + dW_t$.

Let

$$P = \begin{pmatrix} 1 & -\frac{\sigma_{12}}{\sigma_{22}} \\ 0 & 1 \end{pmatrix}, \quad Z_t = PW_t, \quad U_t = PY_t.$$

Then (Z_t) has diagonal covariance $(s\Delta t)P\Sigma P'$ where

$$P\Sigma P' = \begin{pmatrix} \sigma_{11} - \dfrac{\sigma_{12}^2}{\sigma_{22}} & 0 \\ 0 & \sigma_{22} \end{pmatrix}.$$

(U_t) satifies $dU_t = -\lambda_n U_t dt + dZ_t$.

Now $Y_t = P^{-1}U_t = \begin{pmatrix} U_t^1 + \dfrac{\sigma_{12}}{\sigma_{22}} U_t^1 \\ U_t^2 \end{pmatrix}$, so that

$$Y_t^1 Y_t^2 = U_t^1 U_t^2 + \frac{\sigma_{12}}{\sigma_{22}} [U_t^2]^2$$

and it follows from the independence of U^1 and U^2 and the ergodic theorem that

$$\frac{1}{T}\int_0^T Y_t^1 Y_t^2 dt \to \frac{\sigma_{12}}{2\lambda_n}, \text{ a.s. as } T \to \infty.$$

i.e.

$$\frac{1}{T}\int_0^T \langle e_n, A_1(X_t)\rangle \langle e_n, A_2(X_t)\rangle dt \to \frac{\langle i^*A_1^*e_n, i^*A_2^*e_n\rangle}{2\lambda_n},$$

a.s. as $T \to \infty$. Since

$$c_{12} = \int_0^T [\sum_{n=1}^{\infty} \langle e_n, A_1(X_t)\rangle \langle e_n, A_2(X_t)\rangle] dt, \text{ a.s.},$$

the proof of the Lemma will be complete if we show that

$$\frac{1}{T}\sum_{n=m}^{\infty} \int_0^T \langle e_n, A_1(X_t)\rangle^2 dt \xrightarrow{L^1} 0, \text{ as } m \to \infty,$$

uniformly in T. Now, since $\langle e_n, A_1(X_t)\rangle$ is a one dimensional Ornstein-Uhlenbeck process it is easily checked that

$$E\langle e_n, A_1(X_t)\rangle = e^{-\lambda_n t}\langle e_n, A_1(x_0)\rangle,$$

$$\mathrm{Var}\langle e_n, A_1(X_t)\rangle = \frac{|i^*A_1^*(e_n)|^2}{2\lambda_n}(1-e^{-2\lambda_n t})$$

Thus

$$\frac{1}{T}\int_0^T E\langle e_n, A_1(X_t)\rangle^2 dt \le \langle e_n, A_1(x_0)\rangle^2 + \frac{|i^*A_1^*(e_n)|^2}{2\lambda_n},$$

for all $T > 0$. The proof is completed using the fact that $A_1(x_0) \in H$ and condition (C1). □

Lemma 3.3. Let $A:B \to H$ be a bounded linear operator which commutes with A_0 and satifies condition (C1). Then

$$R_T \equiv \frac{1}{\sqrt{T}}\int_0^T (A(X_t), dW_t) \xrightarrow{\mathcal{D}} N(0, D) \text{ as } T \to \infty,$$

where $D = \frac{1}{2}\sum_{n=1}^{\infty} \frac{|i^*A^*(e_n)|^2}{\lambda_n}$.

Corollary 3.4. $\frac{1}{T}\int_0^T (A_j(X_t), dW_t) \xrightarrow{p} 0$ as $T \to \infty$.

Proof of Lemma 3.3. Let $\pi_m:H \to H$ denote the projection onto $sp(e_1, \ldots, e_m)$. Denote

$$R_T^m = \frac{1}{\sqrt{T}}\int_0^T (\pi_m A(X_t), dW_t)$$

and let $f_n = \lambda_n^{-1}A_0^*(e_n)$. Note that $i^*(f_n) = e_n$. Thus

$$R_T^m = \frac{1}{\sqrt{T}}\sum_{n=1}^{m}\int_0^T \langle A(X_t), e_n\rangle dW_t^n,$$

where $W_t^n = \langle f_n, W_t\rangle$. The $(W_t^n, t \ge 0)$, $n=1, \ldots, m$ are independent standard Wiener processes. By the proof of Lemma 3.2

$$\frac{1}{T}\int_0^T \langle A(X_t), e_n\rangle^2 dt \to \frac{|i^*A^*(e_n)|^2}{2\lambda_n} \text{ a.s.}$$

as $T \to \infty$, for $n = 1, \ldots, m$. Thus, by Kutoyant's Central Limit Theorem for stochastic integrals [2, p. 405] it follows that

$$R_T^m \xrightarrow{\mathcal{D}} N(0, D^m) \text{ as } T \to \infty, \text{ where}$$

$D^m = \frac{1}{2}\sum_{n=1}^{m} \frac{|i^*A^*(e_n)|^2}{\lambda_n}$. By [7, Theorem 3.2],

$$E|R_T - R_T^m|^2 = \frac{1}{T}E\int_0^T |A(X_t) - \pi_m A(X_t)|^2 dt$$

$$= \frac{1}{T}\sum_{n=m+1}^{\infty} \int_0^T E\langle e_n, A(X_t)\rangle^2 dt$$

$$\to 0 \text{ as } m \to \infty, \text{ uniformly in } T > 0,$$

by the proof of Lemma 3.2. In particular $R_T^m \xrightarrow{\mathcal{D}} R_T$ as $m \to \infty$, uniformly in $T > 0$. By the usual method of interchange of limits [13, p. 28] with respect to weak convergence, we conclude that $R_T \xrightarrow{\mathcal{D}} N(0, D)$ as $T \to \infty$. □

Given Lemmas 3.2 and 3.3, the proof of Theorem 3.1 now follows along the lines of the proof given by Brown and Hewitt [4, p. 236] for the one dimensional case.

Remarks

a) Analogous versions of Theorem 3.1 could be obtained for other generalized Ornstein-Uhlenbeck processes considered in the literature [6, 11, 12], provided the appropriate absolute continuity results are available. However, one would need to be very careful in such cases, to avoid the problem of singularity. Two generalized Ornstein-Uhlenbeck processes (driven by the same Wiener process) on a space of distributions can have singular stationary measures, see [16, 17].

b) It is clear from our method of proof that the assumption, $\{e_n, n \geq 1\}$ is complete in H, can be weakened to

$$\text{range } (A_j) \subset \text{span } \{e_n, n \geq 1\}, \text{ for } j = 1, \ldots, k.$$

c) The commutativity assumption in Theorem 3.1 may appear restrictive, but it is often satisfied in applications. For example, in the neural

response model of Walsh [11] it is of interest to estimate parameters β_1, β_2 which represent characteristics of the neuron arising in the following equation for the neuron potential $V(x, t)$:

$$dV = (\beta_1 \frac{\partial^2 V}{\partial x^2} - \beta_2 V)dt + dW,$$

where W is white noise in space-time. V is a $C[0, L]$ valued diffusion process observed over time $[0, T]$. Theorem 3.1 is not applicable, but since the differential operator $\frac{\partial^2}{\partial x^2}$ and the identity operator commute we expect that estimation of β_1, β_2 is still possible. Indeed, one way of doing this is to restrict analysis to the finite dimensional Ornstein-Uhlenbeck process $A^n(t) = (A_0(t), \ldots, A_n(t))$, where $A_k(t)$ can be expressed [11. p. 247] in terms of the observed process V. The unknown parameters β_1, β_2 appear through the eigenvalues λ_k of the separated problem, which in this case are given by $\lambda_k = \beta_2 + \beta_1 \pi^2 k^2 L^2$, $k \geq 0$. The process $A^n(t)$ satisfies the stochastic differential equation

$$dA^n(t) = -FA^n(t)dt + dB^n(t),$$

where $B^n(t)$ is an $(n+1)$-dimensional Wiener process with covariance $(s \wedge t)R$, $R = (\rho_{jk})$ with ρ_{jk} defined in [11, p. 246], F is the diagonal matrix with diagonal elements λ_k, $k = 0, \ldots, n$. The usual methods [2, Ch. 9] of estimation for finite dimensional diffusion processes can be applied to $A^n(t)$. Provided $n \geq 1$, so that β_1, β_2 are identifiable, the maximum likelihood estimator of (β_1, β_2) based on observation of $A^n(t)$ is consistent and asymptotically normal.

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

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