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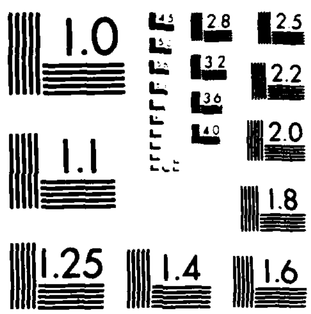

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

During the period of this grant the investigators addressed nonlinear resonance and interactions involving systems of nonlinear ordinary differential equations with slowly varying coefficients. More specifically, they investigated passage through resonance, sustained resonance as in roll resonance, particle accelerators or free electron lasers, global adiabatic invariants and weakly nonlinear wave interactions. Approaches used included matching asymptotic expansions, multiple scale expansions, higher order averaging, the Melnikov method and near identity canonical transformations to obtain global adiabatic invariants.

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AFOSR-TR- 83-0944

FINAL SCIENTIFIC REPORT

AFOSR [redacted] 80-0175

J. Kevorkian, Principal Investigator

In this final report we survey the research supported by the AFOSR over several years on various aspects of nonlinear resonance and interactions, and indicate some of the applications. Continued research in these areas is now being supported by other agencies. Nonlinear resonance is an area of research where a basic mathematical model describes several totally unrelated physical phenomena. Thus, a fundamental understanding of the model can provide far reaching results.

We consider systems of weakly nonlinear ordinary differential equations with slowly varying coefficients (slowness being measured relative to some basic period of oscillation). Resonance refers to a critical relation between two or more slowly varying frequencies whereby the associated modal amplitudes increase anomalously. In general, the frequencies pass through this critical condition in a finite time after which the growth rate of the corresponding amplitudes subsides. Sustained resonance is the unusual circumstance where the critical condition persists for a long time.

1) Passage Through Resonance

In [1] - [2], the author studied the simple mathematical model of a linear oscillator with given slowly varying frequency forced by a constant frequency in order to establish a solution technique which remains valid through resonance passage. This technique consists of constructing and matching three separate multiple scale expansions describing the solution before, during and after resonance. The basic ideas of matching asymptotic expansions and multiple scale expansions are discussed in [3].

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2) Sustained Resonance: Roll Resonance, Particle Accelerators or Free Electron Lasers

These ideas were later applied in [4] to the problem of reentry roll resonance. Roll resonance occurs in spinning reentry vehicles with small aerodynamic and mass asymmetrics. For a large set of initial conditions the slowly varying pitch/yaw rate becomes equal to the roll rate at which time the roll rate undergoes some moderate perturbations; thus, the term "roll resonance." In nearly all cases, the ensuing buildup in roll rate ceases after a short time. However, for a very small subset of initial conditions, the buildup persists indefinitely and this phenomenon was discussed in [5], where it was shown that the essential behavior hinges on solving the following "pendulum" equation with slowly varying coefficients.

$$\frac{d\theta^2}{dt^2} - a^2(\tilde{t})\sin\theta = - b^2(\tilde{t}); \ \tilde{t} = \varepsilon t, \ 0<\varepsilon<<1 \qquad (1)$$

Sustained resonance corresponds to capture by a slowly varying center in the phase plane.

Recently, there have been four independent studies, [6] - [10], using different approaches that have further refined and provided rigorous results of the above mathematical phenomenon. In [6], near-identity transformations are used to derive a more accurate version of Eq. (1) as well as sharper necessary conditions for sustained resonance. The corresponding energy bounds are derived in [7]. In [8], a general version of Eq. (1) is analyzed using matched asymptotic expansions to derive to the slowly varying energy bounds for capture. Finally, the authors of [9] and [10] transform the model equations of [5] to a form suitable for analysis by higher-order averaging, and the Melnikov method (cf. [11]).

As pointed out by the author in [12], Eq. (1) also occurs in the study of particle accelerators. An early reference to this equation appears in [13]. More recently, in [14], Eq. (1) is also derived in describing the motion of electron beams near synchronous energy in free electron lasers.

In the physics of free electron lasers one studies the motion of an electron beam (assuming no electrostatic interaction between electrons) in a cylindrically symmetric magnetic field with slow axial variations. A primary goal of such research is to determine the influence of the external "wiggler" field upon the motion and resulting radiation of the electron beam. In the absence of interactions, the motion of each electron may be approximated by a slowly varying Hamiltonian of one degree of freedom with the electron energy as the canonical momentum, the phase angle as the coordinate, and the axial distance as the independent variable (Eq. (2.15) of [14]). Near the synchronous energy, Hamilton's differential equations reduce to (1)!

In the context of free electron lasers, one is interested in the design of the external field to achieve a particular goal. For example, as discussed in [14], one objective consists of a 5 phase program involving

i) capture of the electrons into a stationary bucket with a resonant energy equal to the mean electron energy,

ii) increasing the average phase angle to a positive value so that the electrons can be placed in the center of a moving bucket,

iii) decelerating the electrons,

iv) decreasing the average phase angle of the electrons, and

v) decapture.

The purpose of this program is to obtain a minimum increase in the energy spread while reducing the average electron energy. As pointed out by the authors of [14], a detailed analysis for this program is missing and one must rely on qualitative estimates for both the number of oscillations required for the process, and for its stability. Work is in progress to identify and model the various weak perturbations that must be taken into account in order to derive an accurate quantitative description of free electron laser dynamics.

3) Global Adiabatic Invariants

In [15] and [16] the author discussed the detailed structure of the solution for weakly nonlinear two degree of freedom systems passing slowly through an internal resonance. The work in [15] focuses on the problem of two oscillators with a weak nonlinear coupling, a well known model (for the autonomous case) in many applications, e.g., celestial mechanics [17], and hydrodynamics [18]. In order to calculate a solution which remains uniformly valid through resonance, two generalized multiple-variable expansions are constructed and matched with an interior expansion valid during resonance. The dominant effect of passage through resonance is the excitation of an oscillation of order $\varepsilon^{\frac{1}{2}}$ not present before resonance where ε measures the nonlinearity. Corresponding to this, the actions of the individual modes change by $O(\varepsilon)$ after resonance.

In [16], a more general Hamiltonian system with two degrees of freedom is used to derive a global adiabatic invariant (i.e., one which remains constant through resonance) using near identity canonical transformations. This invariant turns out to be a generalization of the formal integral that one can construct for the autonomous case and also confirms the results of [15]. It is tailored for a particular resonance and its constancy is verified both by a direct substitution and numerical integration of the exact equations. In isolating the global adiabatic invariant the Hamiltonian is reduced to one degree of freedom and this problem is solved using the technique in [15].

In a recently completed study [19], we have identified classes of Hamiltonian and non-Hamiltonian systems with infinite degrees of freedom passing through resonance and for which global adiabatic invariants exist. Such systems arise in various physical contexts including the modal representation for wave-guides with slowly varying boundaries.

4) Weakly Nonlinear Wave Interactions

Wave propagation problems in the infinite or semi-infinite domain are generally not solvable by discrete modal representations. Although the inverse scattering technique provides a powerful tool for exact solutions in a class of nonlinear initial-value problems, it does not apply for signaling problems or when the medium has variable properties.

The multiple variable method has proven quite successful in treating such problems in the weakly nonlinear case and there have been numerous contributions to this area of research since the work reported in [20]. A survey of the technique is given in [3] and work focusing on fluid mechanics applications can be found in [21] and [22]. Research in this area with particular emphasis or media with slowly varying properties is continuing.

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