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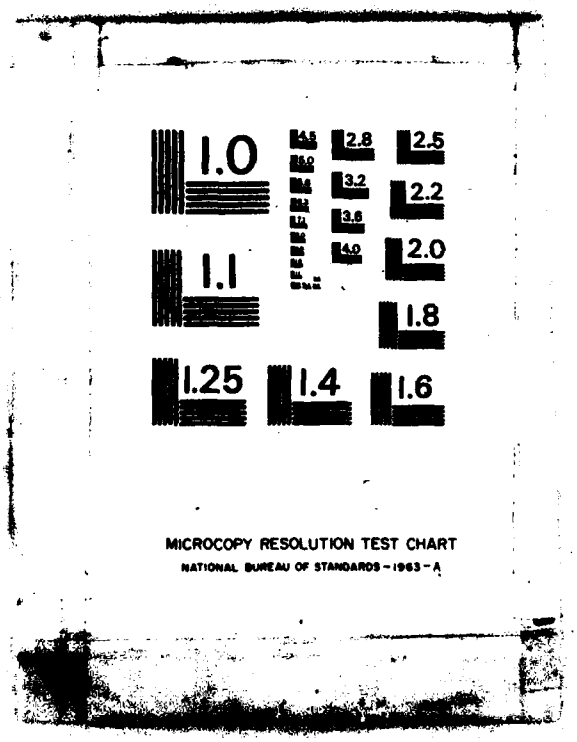
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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)
During this period the two investigators produced seven papers with titles including, "Painleve Property and Multicomponent Isospectral Deformation Equations," "Algebraic Reductions of Scalar Three-Dimensional Systems," "Topological and Algebraic Structure of Linear Problems Associated With Completely Integrable Systems," and "Laws of Composition of Backlund Transformations and the Universal Form of Completely Integrable Systems in Dimensions Two and Three." This report summarizes progress in these areas during this period supported by the grant.

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Grant No. AFOSR-81-0190
June 30, 1982 - June 29, 1983
Principal Investigator: Lipman Bers

Interim Scientific Report
Nonlinear Partial Differential Equations and Related Problems of Padé Approximations

by D.V. Chudnovsky and G.V. Chudnovsky

I. During 1982 - 1983 the investigators carried out the program of their Research Proposal on the study of two and three dimensional classical and quantum lattice and continuous physical systems. One of the main results was the description of classes of two and three dimensional matrix completely integrable systems in terms of universal rules of composition of Bäcklund transformations and symmetry structures arising from infinite-dimensional Lie algebras [1], [2], [3], We mention, among our achievements in this framework, the following: a) The description of equivalent lattice and continuous two dimensional multicomponent completely integrable systems (with the representation of Bäcklund transformations using algebraic generalizations of Christoffel's formula [2]; b) The determination of new completely integrable systems (e.g., many-particle systems) in the form of symmetry reductions of higher-dimensional ones [1], [3]; c) The discovery of new sequences of non-local (time-dependent) conservation laws for classical completely integrable systems of Korteweg-de Vries and nonlinear Schrödinger

type that have a complex algebra of commutation relations [1], [3]. The "Painlevé property" for classes of completely integrable three dimensional systems was establsihed in [4] and was applied to the determination of the pole dynamics of meromorphic solutions.

II. The investigators achieved significant progress in general theory of Padé approximations to solutions of differential equations and in the application of Padé approximations to various problems of algebra and number theory [7]. The main instrument used in the investigation is the application of methods of Bäcklund transformations from mathematical physics and their geometric properties to the explicit construction of Padé (rational) approximations to solutions of differential equations, following the investigators' approach. The investigators also made significant progress in the improvement of the measures of diophantine approximations and transcendence for values of classical analytic functions [6], [9], [10]. These results imply solutions of many important diophantine problems: a) The effective determination of integer solutions of diophantine equations [5], [10]; b) The effectivization of the "Roth $2 + \epsilon$ "bound for the measure of irrationality of constants of classical analysis, e.g., sin 1, cosh $\sqrt{2}$, $J_o(1)$ etc. [6]. Our results give solutions to Lang's problems (1965 - 1971).

Of primary importance to the convergence of Padé approximations is the authors' effective solution of Kolchin's problem (1959) on the almost normality of Padé approximations to solutions of arbitrary linear differential equations with rational function coefficients. [8], [10]. This is an outstanding breakthrough, that answers our aim from the Research Proposal, and it

implies important corollaries in number theory, algebra and numerical analysis [10].

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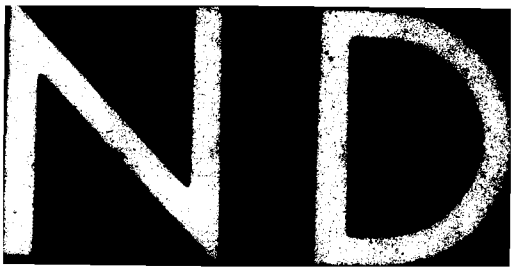
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