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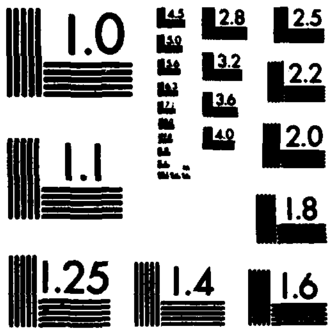
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Lefschetz Center for Dynamical Systems

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Spline-Based Estimation Techniques for Parameters in Elliptic Distributed Systems

by

H. T. Banks, P. L. Daniel, E. S. Armstrong

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Abstract

Parameter and state estimation techniques are discussed for an elliptic system arising in a developmental model for the antenna surface in the Maypole Hoop/Column antenna. A computational algorithm based on spline approximations for the state and elastic parameters is given and numerical results obtained using this algorithm are summarized.

I. Introduction

This report summarizes some of the results from an ongoing Langley Research Center program directed towards developing parameter estimation techniques for flexible systems modeled by partial differential equations with an emphasis on large space structures. The intent of the program is to produce general purpose techniques with a sound theoretical basis which are computationally efficient while contributing to Langley's technology development program in large space antennas [1]. Of the many techniques available for parameter estimation in distributed systems (for example, see [2]) the spline based estimation techniques of [3-8] appear well suited for large space structures applications and are currently being developed to treat this class of problems. Simultaneously, an estimation problem associated with the Maypole (Hoop/Column) antenna [9] is being formulated and will be solved as part of the developmental process. The next section of this paper describes the Hoop/Column antenna and presents the identification problem being considered. The parameter estimation approach is then outlined and discussed in the context of the Hoop/Column application. Subsequent sections include mathematical details of the antenna application and numerical results.

II. The Maypole (Hoop/Column) Antenna

One of the planned activities of NASA's Space Transportation System is the placement in earth orbit of a variety of large space antennas. Potential large space missions for the next two decades will require antennas and structures ranging from 30m to 20km in size. Applications include communications (mobile, trunking, etc.), remote sensing (soil moisture, salinity, etc.), deep space network (orbital relays), astronomy (x-ray observatory, optical array, radio telescope, very long baseline interferometry, etc.), energy, and space platforms.

For the purpose of technology development, the NASA Large Scale Systems Technology(LSST) program office has pinpointed focus missions and identified future requirements for large space antennas for communications, earth sensing, and radio astronomy [1]. In this study, particular emphasis is placed on mesh deployable antennas in the 50-120 meter diameter category. Communication satellites of this size will require a pointing accuracy of 0.035° and surface accuracy of 4-8mm. One such antenna is the Maypole (Hoop/Column) antenna shown for the 100m point-design in Figures 1 and 2. This antenna concept has been selected by the LSST office for development by the Harris Corporation, Melbourne, Florida, under contract to the Langley Research Center [9].

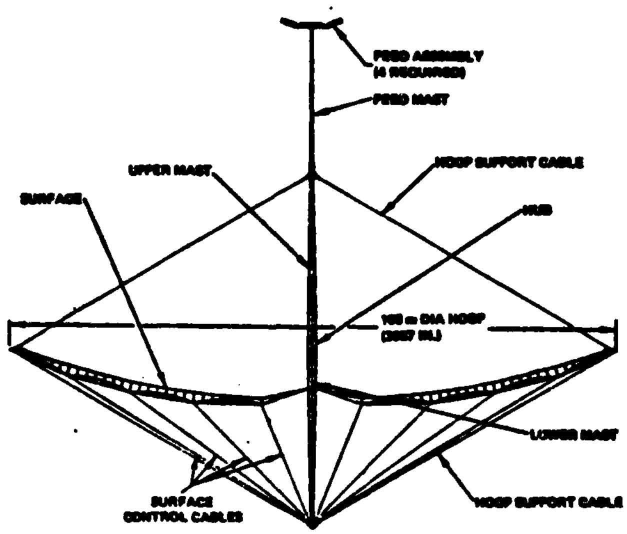

FIGURE 1 Side View of Maypole (Hoop/Column) Antenna

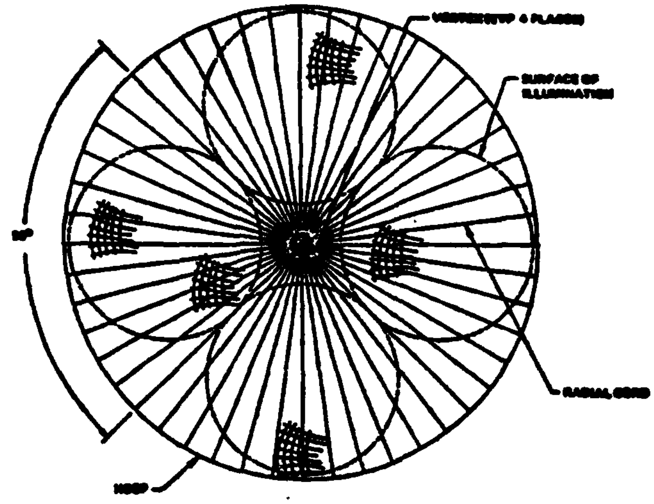

FIGURE 2 Maypole Hoop/Column Antenna Reflector Surface

The Hoop/Column antenna consists of a knitted gold-plated molybdenum wire reflective mesh stretched over a collapsible hoop that supplies the rigidity necessary to maintain a circular outer shape. The mesh grid can be varied to meet a given radio frequency reflectivity requirement. The annular membrane-like reflector surface surrounds a telescoping mast which provides anchoring locations for the mesh center section (Figure 1). The mast also provides anchoring for cables that connect the top end of the mast to the outer hoop and the bottom end of the mast to 48 equally spaced radial graphite cord truss systems woven through the mesh surface [9]. Tensions on the upper (quartz) cables and outer lower (graphite epoxy) cables are counter balanced to provide stiffness to the hoop structure. The inner lower cables produce, through the truss systems, distributed surface loading to control the shape of four circular reflective dishes (Figures 1 and 2) on the mesh surface. Flat, conical, parabolic, or spherical dish surfaces can be produced using this cable drawing technique.

After deployment or after a long period of operation, the reflector surface may require adjustment. Optical sensors are to be located on the upper mast which measure angles of retroreflective targets placed on the truss radial cord edges on the antenna surface. This information can then be processed using a ground-based computer to determine a data set of values of mesh surface location at selected target points. If necessary, a new set of shaping (control) cord tensions can be fed back to the antenna for adjustment.

It is desirable to have an identification procedure which allows one to estimate the antenna mesh shape at arbitrary surface points and the distributed loading from data set observations. It can also be anticipated that environmental stresses and the effects of aging will alter the mesh material properties. The identification procedure must also allow one to address this issue.

The authors are currently developing identification procedures for use in distributed parameter models of the mesh surface. Considering the antenna to be fully deployed and in static equilibrium, we are deriving a mathematical model which describes the antenna surface deviation from a curved equilibrium configuration (for preliminary findings, see [10]). Using a cylindrical coordinate system with the z axis along the mast, it is expected that the resulting model will entail a system of coupled second order linear partial differential equations in two spatial variables. The coefficients of these equations are functions of the material properties of the stretched mesh. The derivation and computer software for this model are still under development. In the meantime, a simpler developmental (prototype) problem has been solved which is descriptive of the original problem and for which the software produced will hopefully be readily extended for use in the more general case.

For the developmental problem, the loading is assumed to be normal to the horizontal plane containing the hoop rim and the mesh surface is assumed to be described by the static two-dimensional stretched membrane equation [11] with variable stiffness (elastic) coefficients and appropriate boundary conditions for the Hoop/Column geometry. Mathematically, in polar coordinates, we have

$$-\frac{1}{r}\frac{\partial}{\partial r}\left[rE(r,\theta)\frac{\partial u}{\partial r}\right]-\frac{1}{r^2}\frac{\partial}{\partial \theta}\left[E(r,\theta)\frac{\partial u}{\partial \theta}\right] = f(r,\theta) \qquad (1)$$

where $u(r,\theta)$ is the vertical displacement of the mesh from the hoop plane, $f(r,\theta)$ is the distributed loading force per unit area, and $E(r,\theta) > 0$ is the distributed stiffness (elastic) coefficient of the mesh surface (force/unit length). Equation (1) is to be solved over the annular region $\Omega = [\varepsilon,R]\times[0,2\pi]$. Appropriate boundary conditions are

$$\begin{aligned} u(\varepsilon,\theta) &= u_0 \\ u(R,\theta) &= 0 \end{aligned} \qquad (2)$$

along with the periodicity requirement

$$u(r,0) = u(r,2\pi)\ , \qquad (3)$$

where R is the radius from the mast center to the circular outer hoop, ε is the radius from the mast to the beginning of the mesh surface (see Figure 2), and u_0 is the coordinate at $r = \varepsilon$ of the mesh surface below the outer hoop plane.

We further assume that the distributed loading along with a data set of vertical displacements, $u_m(r_i,\theta_j)$, at selected points (r_i,θ_j) on the mesh surface is known. Given this information, the developmental problem is to estimate the material properties of the mesh as represented by $E(r,\theta)$ and produce estimates of the surface represented by $u(r,\theta)$ at arbitrary (r,θ) points within Ω. The procedure applied to solve this problem is discussed in the next section.

III. The Parameter Estimation Approximation Scheme

The first two authors and their colleagues have derived techniques for approximating the solutions to systems identification and control problems involving delay equation models and partial differential equation models in one spatial variable and have used them in a variety of applications [12, 13]. The Hoop/Column application requires an extension of the theory and numerical algorithms to elliptic distributed systems in several spatial variables. The approach, when specialized to the parameter estimation problem, may be summarized as follows. (1) Select a distributed parameter formulation containing unknown parameters for a specific system. (2) Mathematically "project" the formulation down onto a finite dimensional subspace through some approximation procedure such as finite differences, finite elements, etc. (3) Solve the parameter estimation problem within the finite dimensional subspace obtaining a parameter estimate dependent upon the order of the approximation embodied in the subspace. (4) Successively increase the order of the approximation and, in each case, solve the parameter estimation problem so as to construct a sequence of parameter estimates ordered with increasing refinement of the approximation scheme. (5) Seek a mathematical theory which provides conditions under which the sequence of approximate solutions approaches the distributed solution as the subspace dimension increases with a convergent underlying sequence of parameter estimates.

In applying this approach to the developmental problem described in this report, the stiffness function is parametrized in terms of cubic splines; this converts the estimation of $E(r,\theta)$ into a finite dimensional parameter estimation problem. After writing the energy functional generic to the membrane equation, we use the Galerkin procedure to project the distributed formulation onto a finite dimensional state subspace spanned by tensor products of linear spline functions defined over Ω. The approximate displacement thus obtained is expressible in terms of the spline basis functions. The Galerkin procedure in this case yields algebraic equations which define the displacement approximation coordinates in terms of the unknown parameters. In order to solve the approximating parameter estimation problem, the parameters defining $E(r,\theta)$ are chosen so that a least squares measure of the fit error between the observed and predicted (by the approximate state) data set is minimized. Finally, following steps (4) and (5) an algorithm is constructed to determine the order of the linear spline approximation above which little or no further improvement is obtained in the unknown parameters as one increases the dimension of the subspaces.

Following this procedure, a one-dimensional version of the developmental problem has been solved [14] in which the schemes proposed were successfully tested. We have subsequently considered the two dimensional case and details of these investigations are presented in the following sections.

IV. Finite Dimensional Approximations

We choose a Galerkin procedure [15, 16] with linear spline basis functions to perform the finite dimensional approximation for the developmental problem in which $E(r,\theta)$ and $u(r,\theta)$ of equation (1) are to be estimated. The boundary conditions (2) are first converted to homogeneous form by introducing the new dependent variable

$$y(r,\theta) = u(r,\theta) - \left(\frac{r-R}{\varepsilon-R}\right) u_0 \tag{4}$$

whereby equation (1) becomes

$$-\frac{1}{r}\frac{\partial}{\partial r}\left[rE(r,\theta)\frac{\partial y}{\partial r}\right] - \frac{1}{r^2}\frac{\partial}{\partial \theta}\left[E(r,\theta)\frac{\partial y}{\partial \theta}\right] = f(r,\theta) + \frac{1}{r}\frac{\partial}{\partial r}\left[\frac{rE(r,\theta)u_0}{\varepsilon - R}\right] \tag{5}$$

with boundary conditions

$$\begin{aligned} y(\varepsilon,\theta) &= 0 \\ y(R,\theta) &= 0 \\ y(r,0) &= y(r,2\pi) \ . \end{aligned} \tag{6}$$

Following the standard formulation (see [15, 16]) for the weak or variational form of (5), the energy functional $\mathcal{E}$ associated with (5) is

$$\mathcal{E}(z) = \int_0^{2\pi}\int_\varepsilon^R \left\{\frac{1}{2} E(r,\theta)\nabla z \cdot \nabla z - \tilde{f}(r,\theta)z\right\} r\,dr\,d\theta \ , \tag{7}$$

where ∇ is the gradient in polar coordinates which, <u>in the form used here</u>, is equivalent to

$$\left(\frac{\partial}{\partial r}, \frac{1}{r}\frac{\partial}{\partial \theta}\right)^T . \tag{8}$$

The function $\tilde{f}$ is given by

$$\tilde{f}(r,\theta) = f(r,\theta) + \frac{1}{r}\frac{\partial}{\partial r}\left(\frac{rE(r,\theta)u_0}{\varepsilon - R}\right) \tag{9}$$

and the vertical displacement $z(r,\theta)$ of the mesh surface away from the hoop equilibrium plane is a function satisfying the boundary conditions (6) and possessing first derivatives on Ω in the mean square sense (we denote this by $z \in H^1_{0,per}(\Omega) \equiv Z$). The first variation $\delta\mathcal{E}$ of $\mathcal{E}$ about the function $y(r,\theta)$ is given by

$$\begin{aligned} \delta\mathcal{E}(y;v) &= \int_0^{2\pi}\int_\varepsilon^R \left\{E(r,\theta)\nabla y \cdot \nabla v - \tilde{f}(r,\theta)v\right\} r\,dr\,d\theta \\ &= \int_0^{2\pi}\int_\varepsilon^R \left\{E(r,\theta)\nabla y \cdot \nabla v - [f(r,\theta)v + E(r,\theta)\tilde{k} \cdot \nabla v]\right\} r\,dr\,d\theta \end{aligned} \tag{10}$$

where

$$\tilde{k} = \begin{pmatrix} \hat{k} \\ 0 \end{pmatrix} = \begin{pmatrix} \frac{u_0}{R-\varepsilon} \\ 0 \end{pmatrix} \tag{11}$$

and v is an arbitrary function in $Z = H^1_{0,per}(\Omega)$.

Given a finite dimensional subspace $\hat{Z}$ of Z, the Galerkin procedure defines the approximation $\hat{y}$ as the solution in $\hat{Z}$ of

$$\int_0^{2\pi} \int_\varepsilon^R \left\{ E(r,\theta)\nabla\hat{y} \cdot \nabla\hat{v} \right\} rdrd\theta = \int_0^{2\pi} \int_\varepsilon^R \left\{ f(r,\theta)\hat{v} + E(r,\theta)\tilde{k} \cdot \nabla\hat{v} \right\} rdrd\theta \tag{12}$$

for all $\hat{v} \in \hat{Z}$.

For computational efficiency, the basis functions used for the representations of $\hat{y}$ in (12) are taken as tensor products of linear B-splines [15, p. 27; 16, p. 100]. Thus $\hat{v}$ and $\hat{y}$ are in the space spanned by $v^{M,N}_{ij}$, where

$$v^{M,N}_{ij}(r,\theta) = \alpha^M_i(r)\beta^N_j(\theta), \quad (i = 1,\dots,M-1; \quad j = 1, \dots, N), \tag{13}$$

where $\alpha^M_i = \alpha^M_i(r)$, $(i = 1,\dots,M-1)$, and $\beta^N_j = \beta^N_j(\theta)$, $(j = 1,\dots,N-1)$, are standard linear B-splines with knots uniformly spaced over $[\varepsilon,R]$ and $[0,2\pi]$, respectively. The elements $\{\alpha^M_i\}$ are modified to satisfy homogenous boundary conditions while β^N_N has been altered to satisfy periodic boundary conditions.

For $y^{M,N}(r,\theta)$ within the subspace spanned by $v^{M,N}_{ij}$ we can write

$$y^{M,N}(r,\theta) = \sum_{i=1}^{M-1} \sum_{j=1}^{N} \alpha^M_i(r) w^{M,N}_{ij} \beta^N_j(\theta) . \tag{14}$$

Replacing $y(r,\theta)$ in (12) by $y^{M,N}(r,\theta)$ from (14) and successively setting $v(r,\theta) = v^{M,N}_{ij}(r,\theta)$ for $i = 1,\dots,M-1$ and $j = 1,\dots,N$ leads to a set of linear algebraic equations for the $w^{M,N}_{ij}$ coordinates [17]. For large M and N, these equations may require sparse matrix techniques. We can avoid sparse matrix methods by imposing a separability condition on $E(r,\theta)$:

$$E(r,\theta) = E_1(r)E_2(\theta) . \tag{15}$$

As shown in [17], condition (15) reduces the $w^{M,N}_{ij}$ calculation to the solution of the matrix equation

$$\tilde{B}^M W^{MN} \tilde{A}^N + \tilde{D}^M W^{MN} \tilde{C}^N = \tilde{E}^{MN} \tag{16}$$

with

$$W^{MN} = (w_{ij}^{MN}) \tag{17}$$

$$\tilde{A}^N = \left(\int_0^{2\pi} E_2(\theta)\beta_j^N(\theta)\beta_q^N(\theta)d\theta \right) \tag{18}$$

$$\tilde{B}^M = \left(\int_\varepsilon^R E_1(r)\left[\frac{d}{dr}\, \alpha_i^M(r)\right]\left[\frac{d}{dr}\, \alpha_p^M(r)\right] r dr \right) \tag{19}$$

$$\tilde{C}^N = \left(\int_0^{2\pi} E_2(\theta)\left[\frac{d}{d\theta}\, \beta_j^N(\theta)\right]\left[\frac{d}{d\theta}\, \beta_q^N(\theta)\right] d\theta \right) \tag{20}$$

$$\tilde{D}^M = \left(\int_\varepsilon^R E_1(r)\, \frac{\alpha_i^M(r)\ \alpha_p^M(r)}{r}\, dr \right) \tag{21}$$

and

$$\tilde{E}^{MN} = \left(\int_0^{2\pi} \int_\varepsilon^R f(r,\theta)\alpha_i^M(r)\ \beta_j^N(\theta) r dr d\theta \right.$$

$$\left. + \int_0^{2\pi} \int_\varepsilon^R E(r,\theta)\hat{k}\beta_j^N(\theta)\left[\frac{d}{dr}\, \alpha_i^M(r)\right] r dr d\theta \right), \tag{22}$$

where, in (17)-(22), $i,p = 1,\ldots,M-1$ and $j,q = 1,\ldots,N$. The coefficient matrices of W^{MN} in (16) have numerically attractive properties: (i) all are symmetric, (ii) $\tilde{B}^M$, $\tilde{C}^N$, and $\tilde{D}^M$ are banded (tri-diagonal), and (iii) $\tilde{A}^N$ and $\tilde{D}^M$ are positive definite.

Research to construct a numerical algorithm for solving (16) which utilizes these properties is planned. At present, (16) is rewritten in the equivalent form

$$[(\tilde{D}^M)^{-1}\tilde{B}^M]W^{MN} + W^{MN}[\tilde{C}^N(\tilde{A}^N)^{-1}] = (\tilde{D}^M)^{-1}\tilde{E}^{MN}(\tilde{A}^N)^{-1} \tag{23}$$

and solved by the Bartels-Stewart algorithm [18].

In order to estimate, via a numerical scheme, the functional coefficients E_1 and E_2, we must further parametrize these functions so that identification is performed over a finite-dimensional (instead of an infinite-dimensional) parameter set. To this end, we let

$$E_1(r) = \sum_{k=1}^{M_1} \nu_k \lambda_k(r) \tag{24}$$

$$E_2(\theta) = \sum_{j=1}^{N_1} \delta_j \mu_j(\theta) \tag{25}$$

where ν_k and δ_j are scalar parameters and λ_k and μ_j are cubic B-spline functions defined [15, p. 61] over $[\varepsilon,R]$ and $[0,2\pi]$, respectively, whose orders are independent of M and N. The functions have been modified so that μ_j and its derivatives satisfy periodic boundary conditions. Substitution of (24), (25) into the matrices $\tilde{A}^N$ through $\tilde{E}^{MN}$ greatly simplifies our computational efforts since many components of these matrices may be computed one time only in advance of the estimation process.

We turn next to implementation of the parameter estimation scheme, i.e., the numerical determination of ν_k, $k=1,\ldots,M_1$, and δ_j, $j=1,\ldots,N_1$, that appear in (24) and (25) and correspond to "optimal" values of the parameters E_1 and E_2.

V. The Parameter Estimation Algorithm

Appealing to the ideas found in previous sections, we may now detail an algorithm for estimating the coefficients ν_k, $k=1,\ldots,M_1$, and δ_j, $j=1,\ldots,N_1$, for $E(r,\theta)$ that provide the "best fit" between approximations for the state u and observed data u_m obtained from various sample points on the surface. We may equivalently consider data for y by making the transformation

$$y_m(r_i,\theta_j) = u_m(r_i,\theta_j) - \left(\frac{r_i-R}{\varepsilon-R}\right)u_0 \tag{26}$$

for $i=1,\ldots,L_r$, and $j=1,\ldots,L_\theta$.

We organize the parameter estimation algorithm into the following steps.

1. Select an order of approximation for the cubic spline elements λ_k, $k=1,\ldots,M_1$, and μ_j, $j=1,\ldots,N_1$, used to represent E_1 and E_2. Set $n=1$.
2. Select M and N, the orders of the linear spline basis elements used to represent $u^{M,N}$ (and $y^{M,N}$).
3. Assume a nominal set of values for

$$\nu = (\nu_1, \nu_2, \ldots, \nu_{M_1}) \tag{27}$$

and

$$\delta = (\delta_1, \delta_2, \ldots, \delta_{N_1}) . \tag{28}$$

4. Calculate the coefficient matrices in (23) and solve for $W^{MN}(\nu,\delta)$.
5. Calculate, from (14), $y^{M,N}(r_i,\theta_j;\ \nu,\delta)$ and evaluate

$$J^{MN}(\nu,\delta) = \sum_{i=1}^{L_r} \sum_{j=1}^{L_\theta} [y^{M,N}(r_i,\theta_j;\ \nu,\delta) - y_m(r_i,\theta_j)]^2 . \tag{29}$$

6. Proceed to step 8 if $J^{M,N}(\nu,\delta)$ is sufficiently small. Otherwise, through an optimization procedure, determine a new pair $(\hat{\nu},\hat{\delta})$ which decreases the value of $J^{M,N}$. If no such pair can be found, go to step 8.
7. Set $(\nu,\delta) = (\hat{\nu},\hat{\delta})$ and return to step 4.
8. Preserve the current values of $J^{M,N}$ and the corresponding (ν,δ) pair as the n^{th} entry in a sequence of these pairs, ordered with increasing M and N.
9. Proceed to step 10 if sufficient data has been obtained to analyze the sequences. Otherwise, set $n = n+1$ and return to step 2 with increased M and N. The current values of (ν,δ) will be used as initial values for the next optimization process.
10. From analysis of the numerical sequences, select the (M,N) entry which indicates the best numerical results. The corresponding (ν,δ) pair yields $E(r,\theta)$ which determines the material properties of the antenna mesh. The matrix $W^{MN}(\nu,\delta)$, when used in conjunction with (14), determines an approximation $y^{M,N}$ of the shape of the antenna surface.

A convergence theory for the parameter estimation algorithm may be found in [17]. Numerical results are described in the next section.

VI. Numerical Results

Experimental data for the Hoop/Column antenna is not available at this time. Therefore simulated data is constructed to evaluate the preceeding algorithm.

As shown in Figure 2, the parent reflector has four separate areas of illumination on its surface. Each separate area is assumed to have the same parabolic shape given, for $0 \le \theta \le \frac{\pi}{2}$ and $\varepsilon \le r \le R$, by

$$u^0(r,\theta) = \begin{cases} \frac{u_0(R-r)}{R-\varepsilon}\left[k(\frac{r-\varepsilon}{R})\ q_2(\theta) + 1\right], & 0 \le \theta \le \frac{\pi}{36} \\ \frac{u_0(R-r)}{R-\varepsilon}\left[k(\frac{r-\varepsilon}{R})\ q_1(\theta) + 1\right], & \frac{\pi}{36} \le \theta \le \frac{17\pi}{36} \\ \frac{u_0(R-r)}{R-\varepsilon}\left[k(\frac{r-\varepsilon}{R})\ q_3(\theta) + 1\right], & \frac{17\pi}{36} \le \theta \le \frac{\pi}{2} \end{cases} \tag{30}$$

where

$$q_1(\theta) = \sin\theta + \cos\theta\ , \tag{31}$$

$$q_2(\theta) = \frac{a_2}{6}(\theta - \frac{\pi}{36})^3 + \frac{1}{2}(\theta - \frac{\pi}{36})^2 \frac{d^2q_1}{d\theta^2}(\frac{\pi}{36}) + (\theta - \frac{\pi}{36})\frac{dq_1}{d\theta}(\frac{\pi}{36}) + q_1(\frac{\pi}{36}), \tag{32}$$

$$a_2 = \frac{-279936}{\pi^3}\left\{q_1(0) - \frac{\pi^2}{2592}\frac{d^2q_1}{d\theta^2}(\frac{\pi}{36}) + \frac{\pi}{36}\frac{dq_1}{d\theta}(\frac{\pi}{36}) - q_1(\frac{\pi}{36})\right\}, \tag{33}$$

$$q_3(\theta) = \frac{a_3}{6}(\theta - \frac{17\pi}{36})^3 + \frac{1}{2}(\theta - \frac{17\pi}{36})^2 \frac{d^2q_1}{d\theta^2}(\frac{17\pi}{36}) + (\theta - \frac{17\pi}{36})\frac{dq_1}{d\theta}(\frac{17\pi}{36}) + q_1(\frac{17\pi}{36}), \tag{34}$$

$$a_3 = \frac{279936}{\pi^3}\left\{q_1(\frac{\pi}{2}) - \frac{\pi^2}{2592}\frac{d^2q_1}{d\theta^2}(\frac{17\pi}{36}) - \frac{\pi}{36}\frac{dq_1}{d\theta}(\frac{17\pi}{36}) - q_1(\frac{17\pi}{36})\right\}. \tag{35}$$

The parameter $k > 0$, a stretch factor used to perturb the surface below the conic ($k = 0$) shape, is taken as 0.25.

For the complete surface, we define, for $\varepsilon \le r \le R$,

$$\bar{u}(r,\theta) = \begin{cases} u^0(r,\theta) & 0 \le \theta \le \frac{\pi}{2} \\ u^0(r,\theta - \frac{\pi}{2})\ , & \frac{\pi}{2} \le \theta \le \pi \\ u^0(r,\theta - \pi)\ , & \pi \le \theta \le \frac{3\pi}{2} \\ u^0(r,\theta - \frac{3\pi}{2}), & \frac{3\pi}{2} \le \theta \le 2\pi \end{cases} . \tag{36}$$

The cubic polynomial fits (32) and (34) are used to ensure smoothness in θ, in regions near $\theta = \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi$.

It is expected that the mesh will be stiffest near the outer hoop ($r = R$) and around the inner radius ($r = \varepsilon$). For this reason we choose a known value of $E_1(r)$ as

$$\bar{E}_1(r) = 2\hat{\tau} - \hat{\tau}\sin[\pi \frac{(r-\varepsilon)}{(R-\varepsilon)}] \qquad (\varepsilon \le r \le R) \qquad (37)$$

where $\hat{\tau}$ is a constant dependent on the mesh material. The stiffness in the angular direction is expected to be uniform. Thus a known value of $E_2(\theta)$ is taken as

$$\bar{E}_2(\theta) \equiv \hat{\tau}\,. \qquad (38)$$

From data provided in [9], a reasonable value for $\hat{\tau}$ (given in units $\sqrt{N/m}$) is

$$\hat{\tau} = 3.391\ ; \qquad (39)$$

similarly, other parameters are estimated to be $u_0 = -7.5m$, $\varepsilon = 8.235m$, and $R = 50m$.

A 10×24 grid of data points $u_m(r_i, \theta_j)$ is calculated by determining $\bar{u}$ at (r_i, θ_j) where the values of θ_j correspond to data taken along every other radial cord truss system; the 10 values of r_i are spaced uniformly in (ε, R). Distributed loads are obtained by substituting (36) - (38) into (1) and evaluating for $f(r,\theta)$. Equations (24) and (25) are used with $M_1 = N_1 = 4$ to represent $E_1(r)$ and $E_2(\theta)$.

All numerical results presented were computed on the CDC 6600 at Southern Methodist University. We would like to express our sincere appreciation to David Krakosky at S.M.U. who assisted in the preparation of data and graphical display of these results. The optimization scheme employed to minimize J^{MN} is the IMSL version (ZXSSQ) of the Levenberg-Marquardt algorithm [19], where we typically use default values of IMSL parameters.

For each example reported, values of N are specified; in order that N basis elements are used in both r and θ directions, we take $M = N + 1$. Two measures of performance will be given in each case: $(J^{MN})^{1/2}$ (which provides a good measure of state approximation) and $R^{M,N}$ where, using $|\cdot|$ to denote the L_2 norm on $[\varepsilon,R] \times [0,2\pi]$,

$$R^{M,N} = \frac{|E^{MN} - \bar{E}|}{|\bar{E}|} \times 100\%$$

measures the relative error between the "true" parameter $\bar{E}$ and the "optimal" parameter E^{MN} associated with the $(M,N)^{th}$ approximate parameter estimation problem. Throughout we choose a starting guess E^0 for the optimization procedure in the case of $N = 4$; for $N = 8$, 16, 24 and 30, previous converged values of E^{MN} are used as initial guesses. For example, the initial guess for $N = 8$ is $E^{M\tilde{N}}$ where $\tilde{N} = 4$.

<u>Example 1</u>: We estimate $E_2(\theta)$ only, holding $E_1(r)$ fixed at the true value $\bar{E}_1(r)$. The starting guess is $E_2^0(\theta) = 1 + .5\cos\theta$. Our results are summarized below and in Figure 3.

N	$\mathcal{R}^{M,N}$	$(J^{MN})^{1/2}$	CP time (sec.)
4	5.15%	.609	5.41
8	5.72%	.507	5.42
16	5.92%	.514	21.58
24	5.95%	.509	55.69
30	5.96%	.502	172.99

<u>Example 2</u>: We now hold $E_2(\theta)$ fixed, $E_2(\theta) \equiv \bar{E}_2(\theta)$, and estimate $E_1(r)$ from the initial guess of $E_1^0 \equiv 1$. Our findings are outlined below and in Figure 4.

N	$\mathcal{R}^{M,N}$	$(J^{MN})^{1/2}$	CP time (sec.)
4	39.38%	.551	12.22
8	4.19%	.418	26.16
16	8.16%	.419	67.11
24	8.85%	.411	116.06
30	8.30%	.401	228.02

<u>Example 3</u>: We estimate both $E_1(r)$ and $E_2(\theta)$. Initial guesses are given by $E_1^0(r) = 5$ and $E_2^0(\theta) = 1 - .25\sin\theta$, respectively. In each case, the first coefficient of E_2 is held fixed.

N	$\mathcal{R}^{M,N}$	$(J^{MN})^{1/2}$	CP time (sec.)
4	42.10%	.551	26.88
8	4.21%	.418	36.24
16	8.17%	.419	74.23
24	8.87%	.411	190.83
30	8.29%	.401	206.02

We note (by comparing values of $\mathcal{R}^{M,N}$ for Examples 1 and 2) that we have greater success here in estimating $E_2(\theta)$ than $E_1(r)$. This disparity appears to be a function of the quantity and placement of sample data. (See for example [17] where, by changing the data, we are able to better estimate E_1 than E_2. In fact, in comparing the results of Example 2 with Table 6 in [17], it is easy to see how a different distribution of data for this example can actually reduce the values of $R^{M,N}$ shown in Example 2 by one-half. Regardless of the differences in estimating E_1 and E_2, however, we remark that in all examples we have been able to successfully estimate the surface shape of the model antenna (as is evidenced by values of J^{MN}, computed using 240 data points).

We have also obtained similar results in the case where random noise (approximately 5% noise level) has been added to the data. These and other findings are summarized in Section VI of [17].

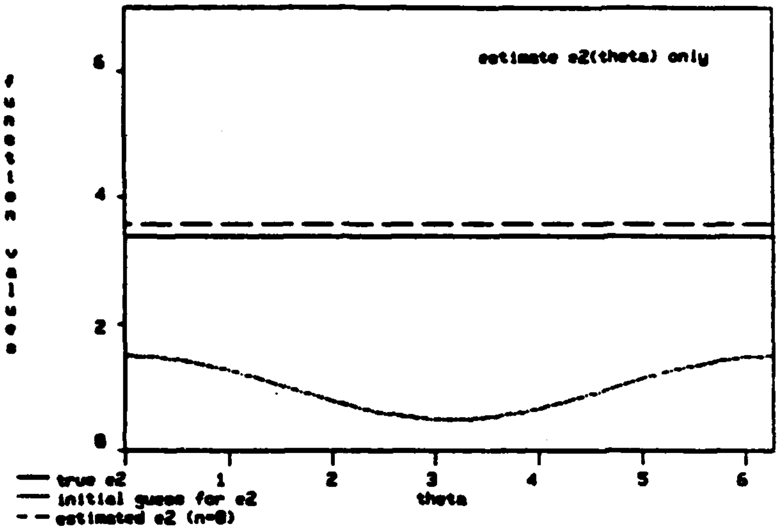

FIGURE 3

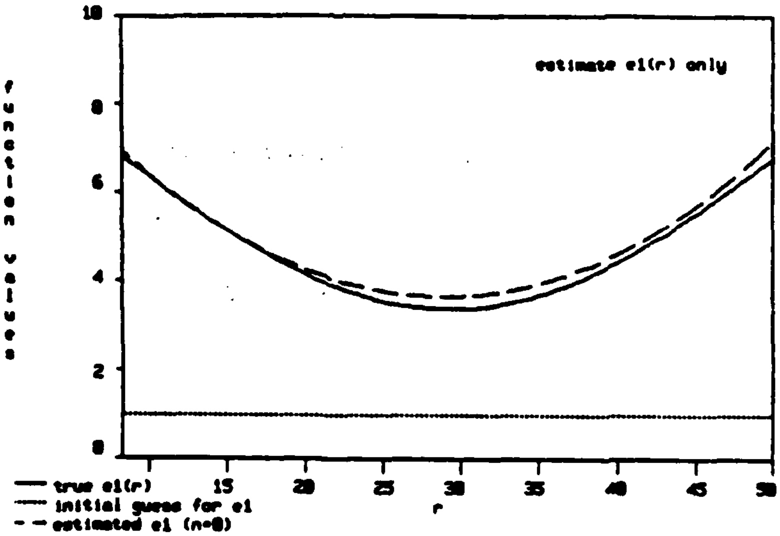

FIGURE 4

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