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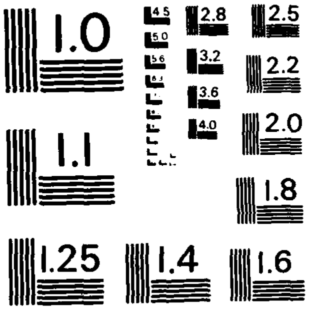

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# Application of a feature selection technique to samples of high resolution synthetic aperture radar imagery

Richard A. Hevenor

July 1983

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U.S. ARMY CORPS OF ENGINEERS
ENGINEER TOPOGRAPHIC LABORATORIES
FORT BELVOIR, VIRGINIA 22060

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A feature selection technique was applied to samples of synthetic aperture radar imagery. This technique was applied to four classes of terrain features on selected samples of radar imagery. The four classes considered were forests, cities, agricultural fields, and water. A feature vector was computed from samples of each class. A linear transformation was utilized to develop a new feature vector of reduced dimensionality. This transformation chooses those features that are most effective for performing class separability.

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## PREFACE

The authority for performing the work described in this research note is contained in Project 4A161102B52C, "Research in Geodetic, Cartographic, and Geographic Sciences."

The work described in this research note represents an application of a standard feature selection technique to samples of high resolution synthetic aperture radar imagery. The task was performed under the supervision of Dr. Frederick W. Rohde, Team Leader, Center for Physical Sciences and Mr. Melvin Crowell, Jr., Director, Research Institute.

COL Edward K. Wintz, CE, was the Commander and Director and Mr. Robert P. Macchia was the Technical Director of the Engineer Topographic Laboratories during the study period.

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# ILLUSTRATIONS

# APPLICATION OF A FEATURE SELECTION TECHNIQUE TO SAMPLES OF HIGH RESOLUTION SYNTHETIC APERTURE RADAR IMAGERY

## INTRODUCTION

The purpose of this research note is to show the application of a feature selection technique to samples of synthetic aperture radar imagery and to present some preliminary results. In the past, feature selection techniques have been applied to data computed from digitized aerial photography. However, it appears that no one has as yet applied feature selection techniques to high resolution radar imagery. In order to perform classification of terrain features using radar imagery, feature selection is an important initial step. Feature selection consists of choosing those features that are most effective for showing class separability and for performing a reduction in the dimensionality of the feature vector. The following sections will present a discussion of the feature selection technique, along with its application to selected samples of radar imagery.

## METHODOLOGY

The application of pattern recognition techniques is accomplished usually in two steps, namely, feature selection and classifier design. The feature selection process that precedes the classification process consists of techniques applicable to one class or to multiple classes. The feature selection technique to be discussed here is applicable to multiple classes. It provides the capability of reducing the number of components of the original feature vector in such a way that the resulting components are optimized to show class separability. The feature selection technique comes from the field of discriminant analysis of statistics and is independent of the probability density functions of the feature vector data. The feature selection operation can be expressed as a linear transformation of the following form:

$$Y = AX \tag{1}$$

where X is the original feature vector with dimensionality n x 1; A is the transformation matrix of dimensionality m x n, where m is less than n; and Y is the transformed feature vector with dimensionality m x 1. The feature selection problem is now reduced to determining the matrix A. In order to calculate A, use is made of the within-class and between-class scatter matrices. A within-class scatter matrix shows the scatter of samples around their class expected vector and can be expressed by

$$S_w = \sum_{i=1}^{N} P(\omega_i)C_i \tag{2}$$

where $S_w$ is the within-class scatter matrix, $P(\omega_i)$ is the a priori probability of the $i^{th}$ class, $C_i$ is the covariance matrix of the $i^{th}$ class, and N is the total number of classes. A between-class scatter matrix can be defined in many ways; however, the following definition was the one utilized here:

$$S_b = C_1 + C_2 + (M_1 - M_2)(M_1 - M_2)^T \tag{3}$$

where $S_b$ is the between-class scatter matrix, $C_1$ is the covariance matrix for class 1, $C_2$ is the covariance matrix for class 2, $M_1$ is the mean vector for class 1, $M_2$ is the mean vector for class 2, and T means transpose. This definition of the between-class scatter matrix is valid only for the case where N (the number of classes) is equal to two. In order to have criteria for class separability, a number must be derived from the scatter matrices. This number should increase when the distances between points belonging to different classes are increasing or when the distances between points belonging to the same class are decreasing. One criterion is the use of $J_1$, which can be defined as follows:

$$J_1 = \text{trace}\,(S_{2m}^{-1} S_{1m}) \tag{4}$$

where

$$S_{2m} = AS_wA^T \text{ and } S_{1m} = AS_bA^T$$

The feature selection problem now requires that we select the particular transformation matrix A, which maximizes the value of $J_1$. Fukunaga[1] shows that A is made up of the normalized eigenvectors of the matrix $S_w^{-1} S_b$.

$$A^T = [\phi_1 \; \phi_2 \ldots\ldots \phi_m] \tag{5}$$

where $\phi_1$ is the eigenvector associated with the largest eigenvalue, $\phi_2$ is the eigenvector associated with the second largest eigenvalue, etc. Once the matrix A is computed from (5), the new feature vector Y can be computed for each point in each class.

## INVESTIGATION

The feature selection technique was applied to samples of high resolution synthetic aperture radar imagery taken over the Huntsville, Alabama, area with the APD–10 radar system. Sections of the radar imagery were digitized and stored on a digital disk unit. A Lexidata system 3400 display processor was used to display the images on a cathode ray tube and to take 100 samples for each of four terrain classes from the imagery. Each sample consisted of a 32 by 32 pixel element window located within a section of one particular terrain class. The four classes considered were cities (combination of commercial and residential structures, DLMS category #504 FIC 301 and #505 FIC 401), fields (agriculture used primarily for crop and pasture land, DLMS category #510 FIC 950), water (rivers with smooth fresh water, DLMS category #510 FIC 940) and fresh water subject to ice (lakes and reservoirs, DLMS category #510 FIC 943), and forests (mixed trees, deciduous and evergreens, DLMS category #510 FIC 954). A feature vector consisting of 13 components was computed for each sample. These 13 components were made up of the first- and second-order gray level histogram statistics computed from each sample window. The explicit equations used for the 13 components of the original feature vector are shown in appendix A. A computer program was written for the Hewlett-Packard 1000 computer to calculate $J_1$ as a function of displacement in x and y. This computer program was also used to calculate the transformation matrix A. A listing of this program is provided in appendix B. A second computer program was written to calculate the new feature vector Y for the four hundred samples taken from the radar imagery. A listing for this second computer program is given in appendix C.

---

[1] Keinosuke Fukunaga, *Introduction to Statistical Pattern Recognition*, Academic Press, 1972.

## RESULTS

In this section some results of numerical calculations are presented. Table 1 shows the results of calculating the value of $J_1$ for various values of displacement DX and DY for the two classes of forests and cities. The a priori probabilities for the two classes were assumed equal to 0.5.[2] In table 1 the largest value of $J_1$ is 29.8525, which is associated with a DX of −3 and a DY of 4. The significance of a maximum value of $J_1$ occurring at these particular values of displacement is not understood at this time. The values of the displacement DX and DY associated with the largest value of $J_1$ were utilized to compute a new feature vector Y with two dimensions.

Figure 1 shows a plot of two second-order statistical components for the forest and city samples. The triangles are the results from city calculations; and t x's are the results from forest calculations. This figure clearly shows the neces or feature selection because the original data for forests and cities is not sepa . Figure 2 presents the results after the transformation Y = AX is applied e original feature vector X. In this figure the x's again represent calculatio m forests. We now see that the data from forests is clustered and almost totally ed from the data for cities, represented by the triangles. Even though a ints still overlap, the improvement is very dramatic. The transformation has succeeded in reducing the dimensionality of the original feature vector from 13 to 2 and also in separating the clusters of data for the two classes.

Figure 3 shows the results of the transformation for the two classes of cities and fields. The x's represent the data from cities. Good separation is obtained for the two classes as only a few points are overlapping. Figure 4 shows the results of calculations for the two classes of cities and water. The x's represent the data from cities. In this particular case the two classes are totally separated as well as being clustered fairly well. Figure 5 presents the results for forests and water. The x's represent the data for forests. Again the two classes have clearly been separated by the transformation. Figure 6 shows the results for fields and water, with the data for fields represented by the x's. These two classes have also been totally separated. Figure 7 shows the results for forests and fields with the data for forests represented by the x's. The two classes are not well separated and other features may have to be investigated to obtain better separation. Another possibility would be to try using nonlinear feature selection techniques for these two classes.

---

[2] This assumption of equal probability for cities and forests appeared to be appropriate for the area where the samples were taken.

TABLE 1. Values of $J_1$ for first and second order histogram statistics

| DX | DY | $J_1$ |
|---|---|---|
| 1 | 0 | 22.1485 |
| 2 | 0 | 19.8078 |
| 3 | 0 | 19.4333 |
| 4 | 0 | 19.3099 |
| 5 | 0 | 19.7508 |
| 6 | 0 | 20.3677 |
| 7 | 0 | 19.8529 |
| 0 | 1 | 21.0314 |
| 0 | 2 | 21.7613 |
| 0 | 3 | 21.3524 |
| 0 | 4 | 21.7343 |
| 0 | 5 | 22.3144 |
| 0 | 6 | 23.1462 |
| 0 | 7 | 23.5903 |
| 1 | 1 | 20.6771 |
| 1 | 2 | 20.4869 |
| 2 | 2 | 24.3979 |
| 3 | 2 | 23.9334 |
| 4 | 2 | 21.5577 |
| 3 | 3 | 25.7921 |
| − 1 | 1 | 20.2745 |
| − 2 | 2 | 24.4373 |
| − 3 | 2 | 25.0272 |
| − 3 | 4 | 29.8525 |
| 4 | 4 | 22.7131 |

## CONCLUSIONS

1. The feature selection technique discussed in this report appears to be a powerful tool for the application of pattern recognition to high resolution synthetic aperture radar imagery.

2. In order to separate forests and agricultural fields it appears that the first and second order histogram statistics combined with a linear feature selection technique may not be sufficient as a feature vector.

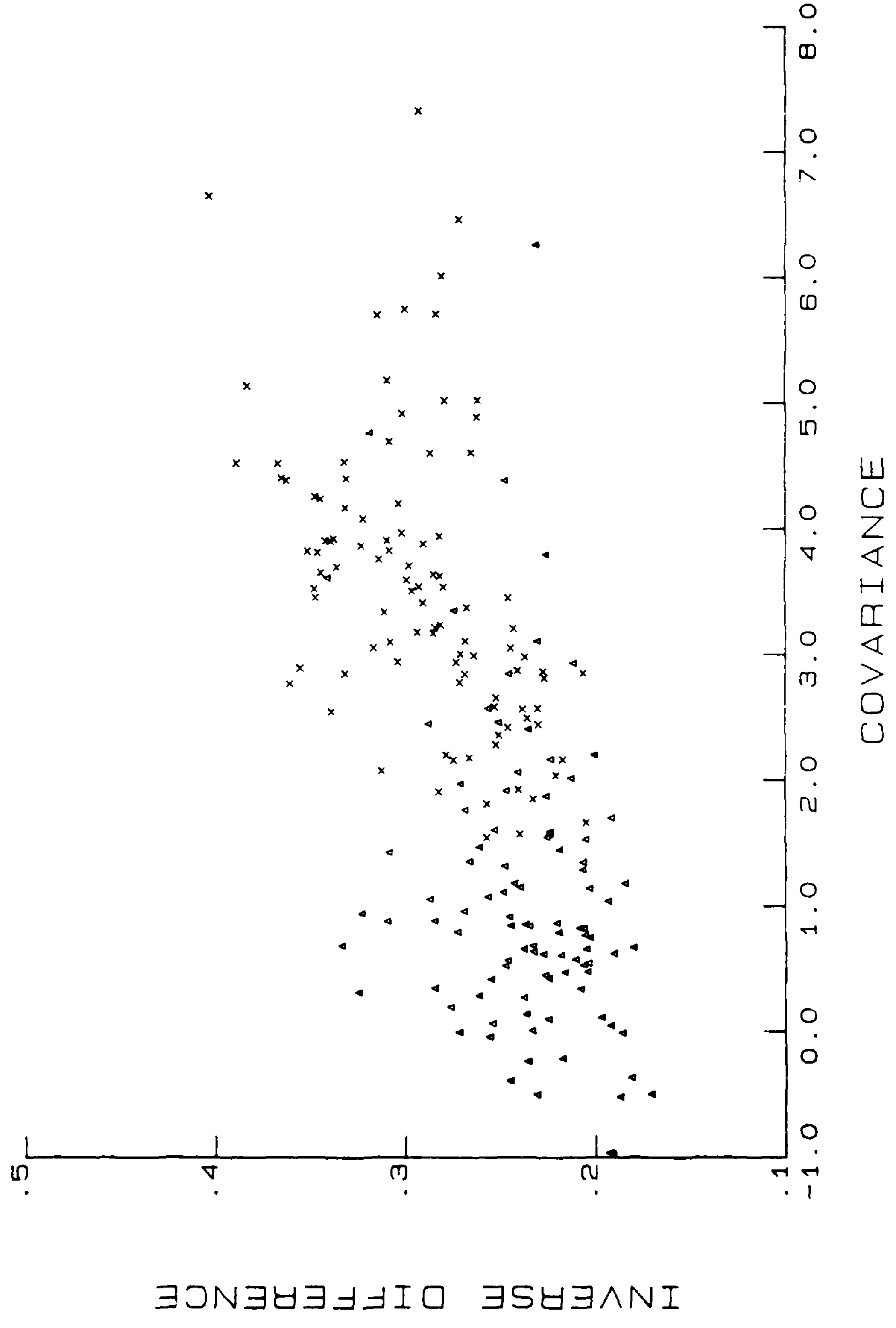

FIGURE 1. Forest and city samples DX = -3 DY = 4.

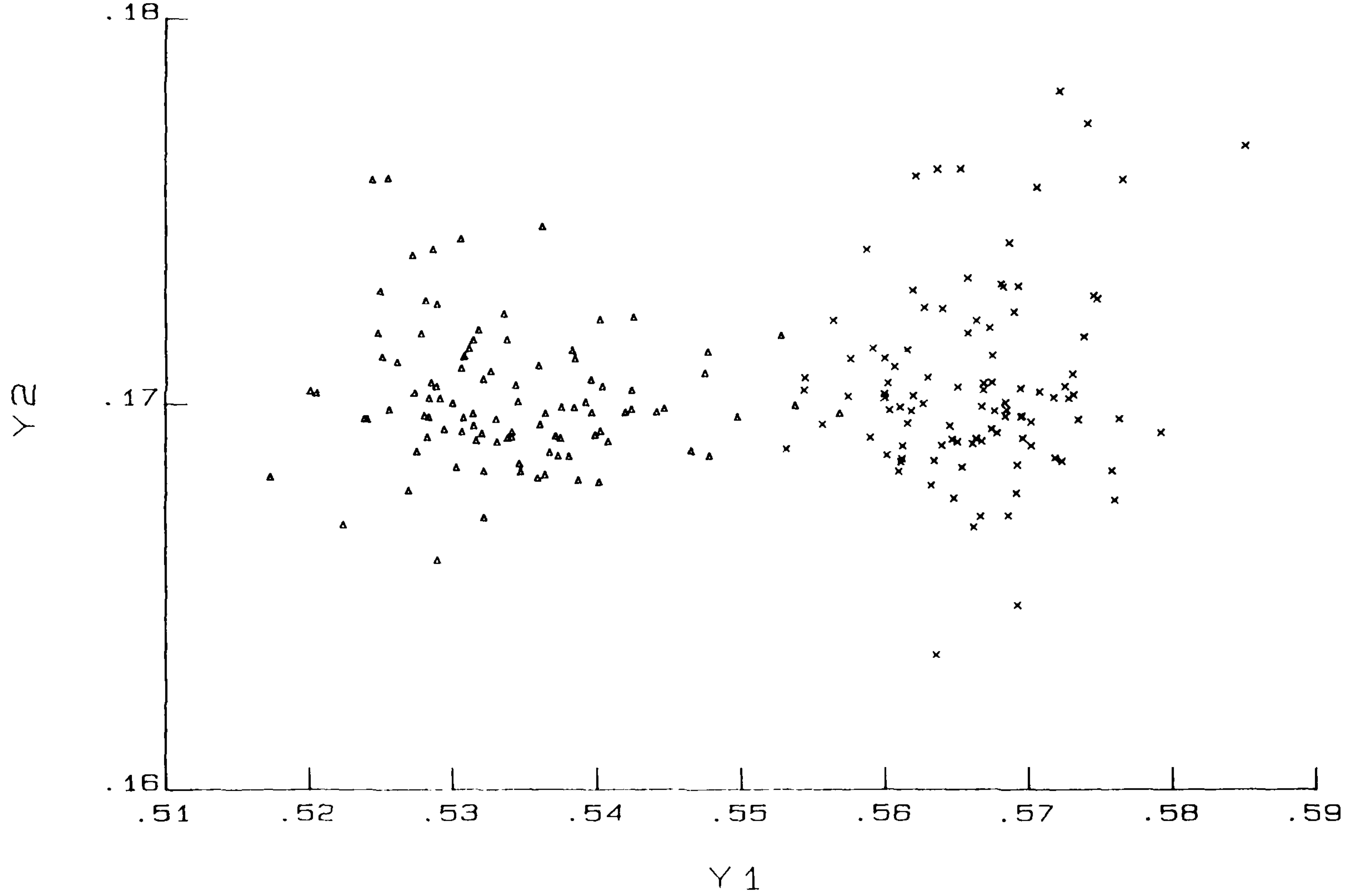

FIGURE 2. Forests and cities DX = –3 DY = 4.

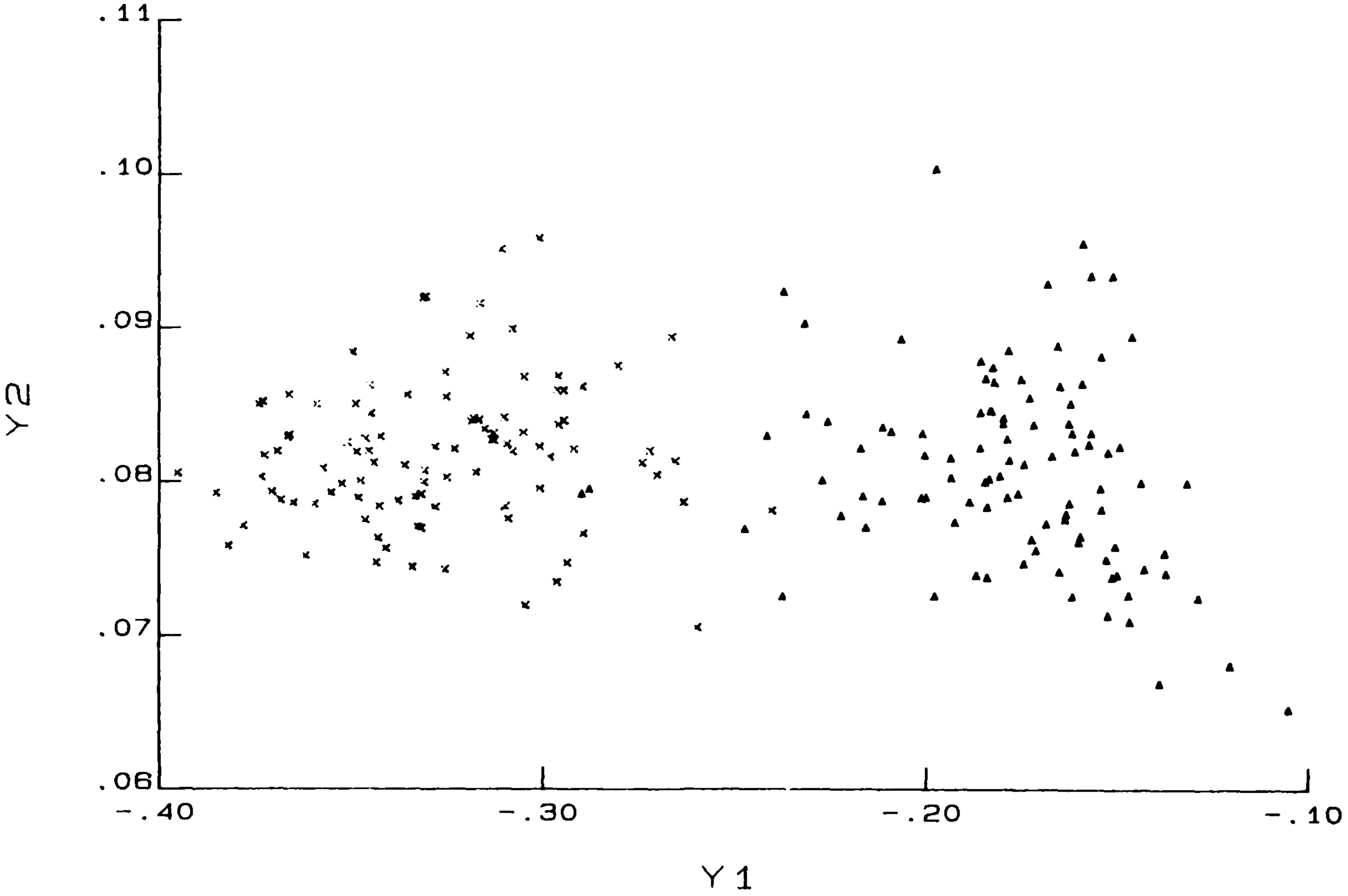

FIGURE 3. Cities and fields DX = 2 DY = 4.

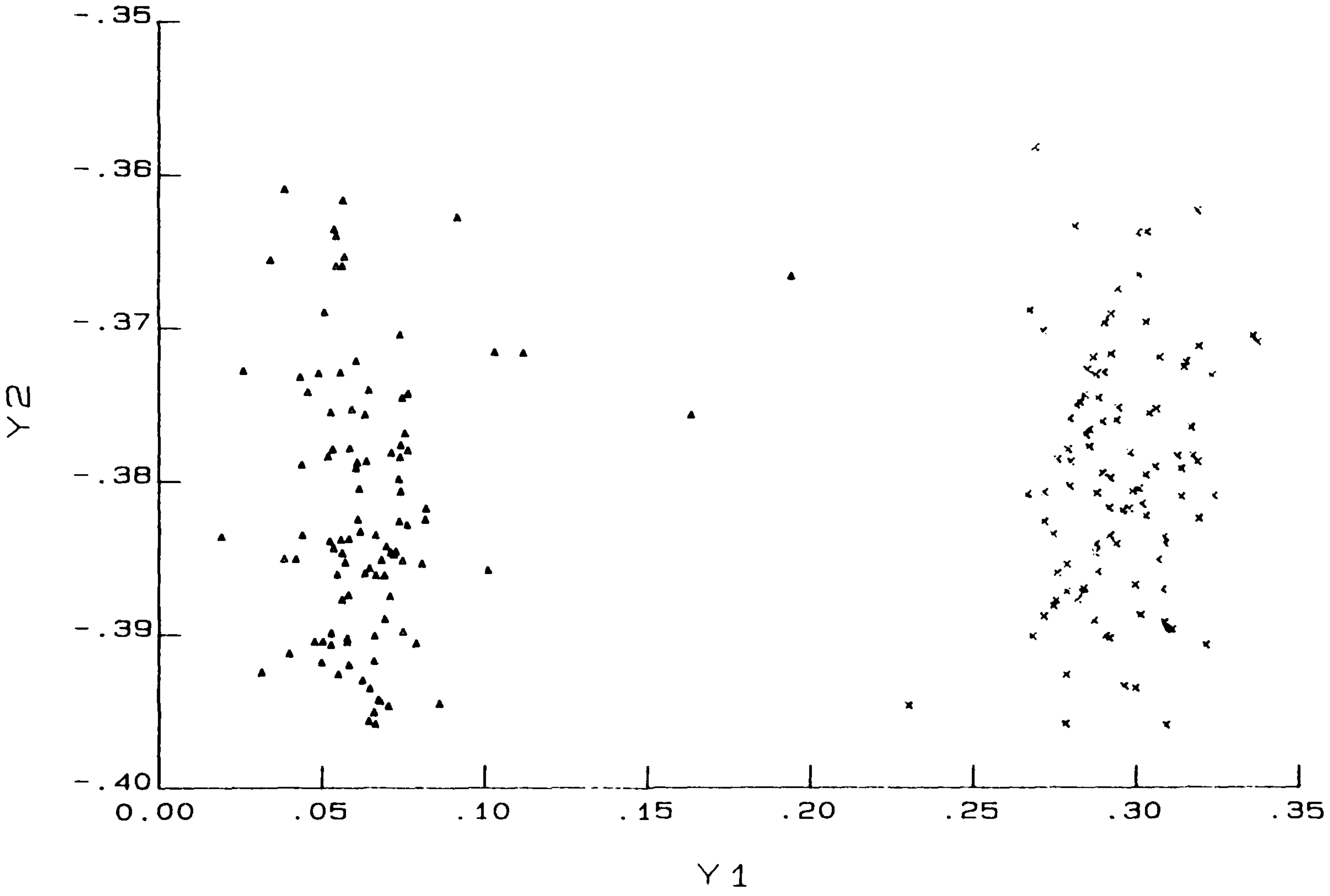

FIGURE 4. Cities and water DX = 1 DY = 0.

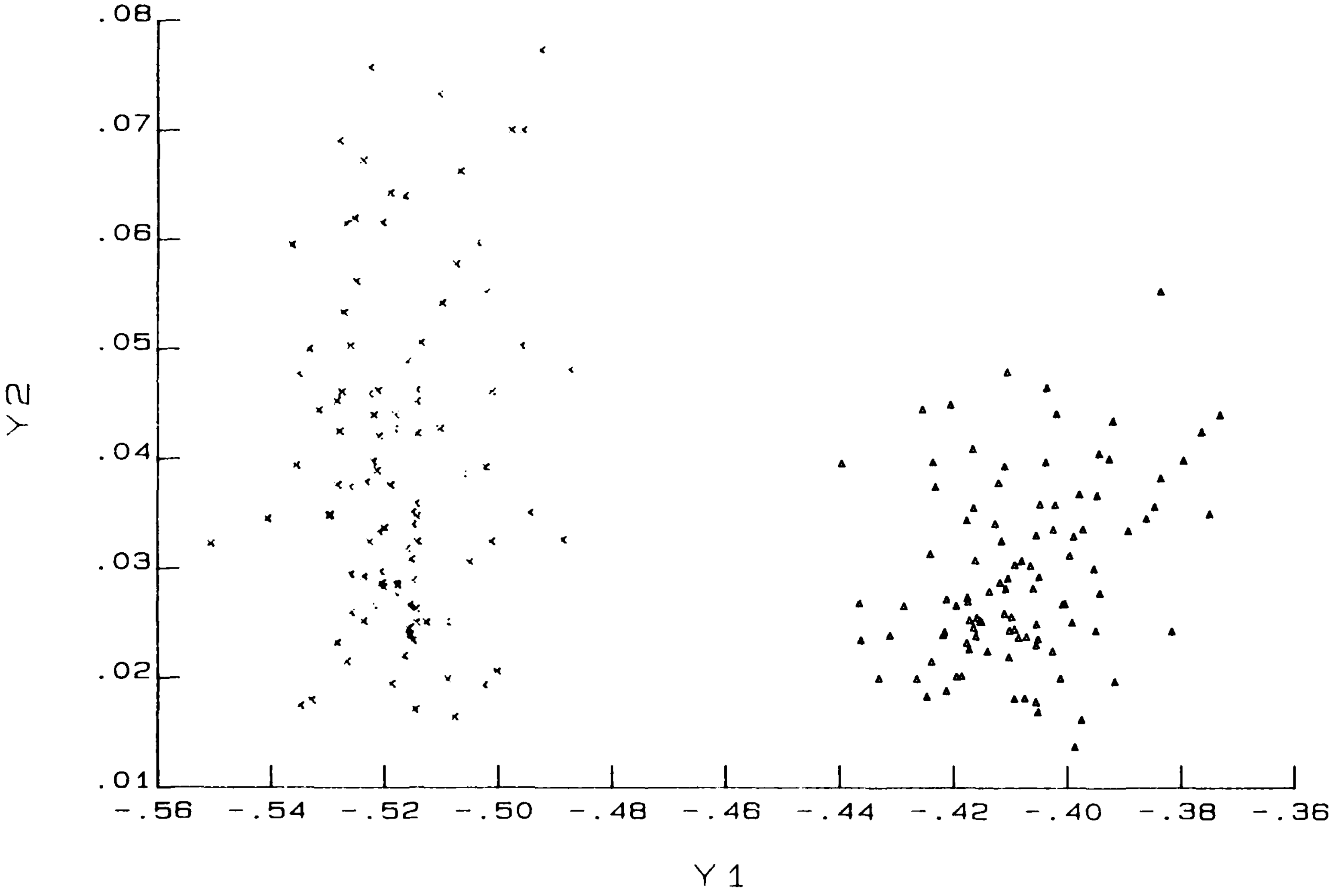

FIGURE 5. Forest and water DX = 1 DY = 0.

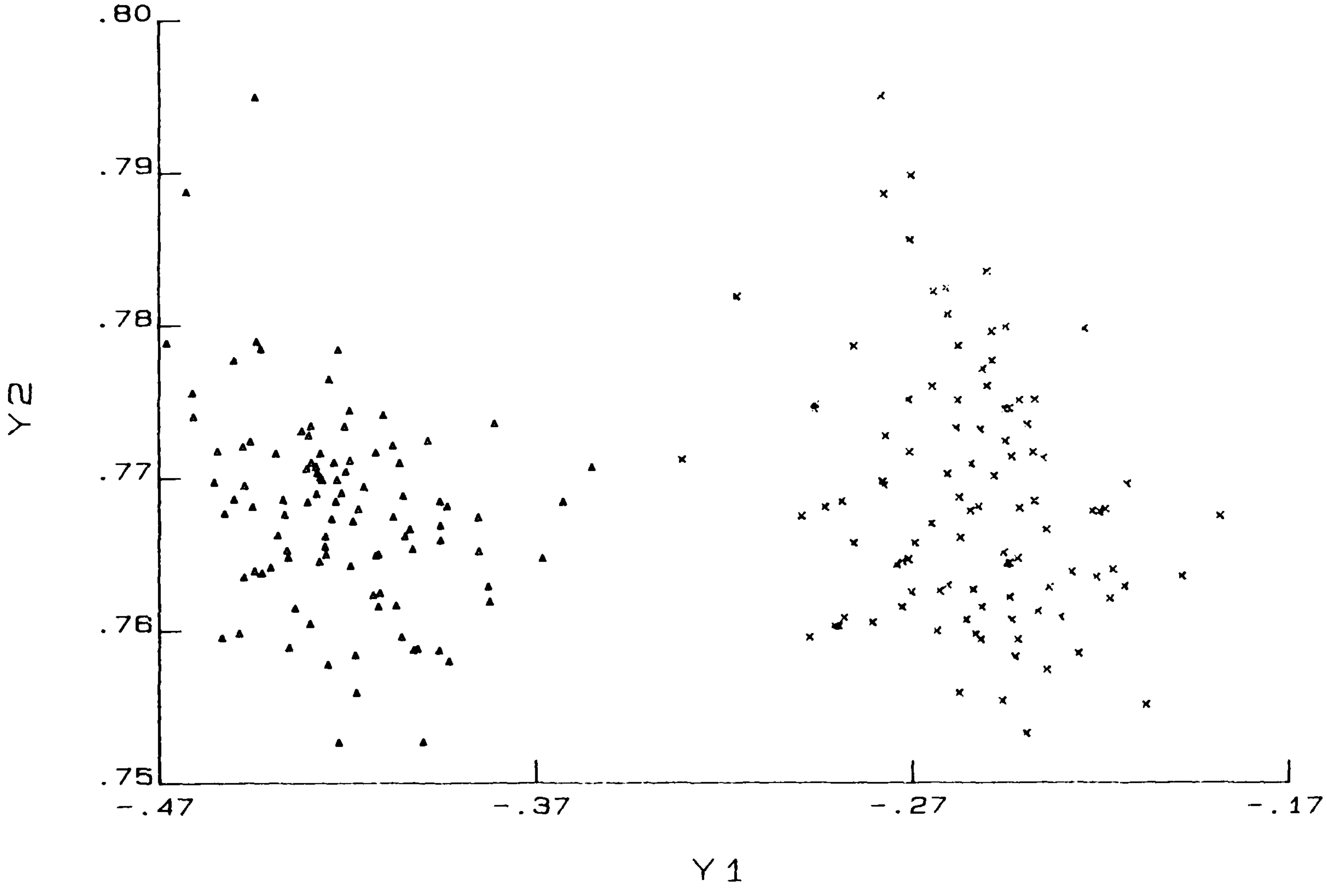

FIGURE 6. Fields and water DX = 1 DY = 0.

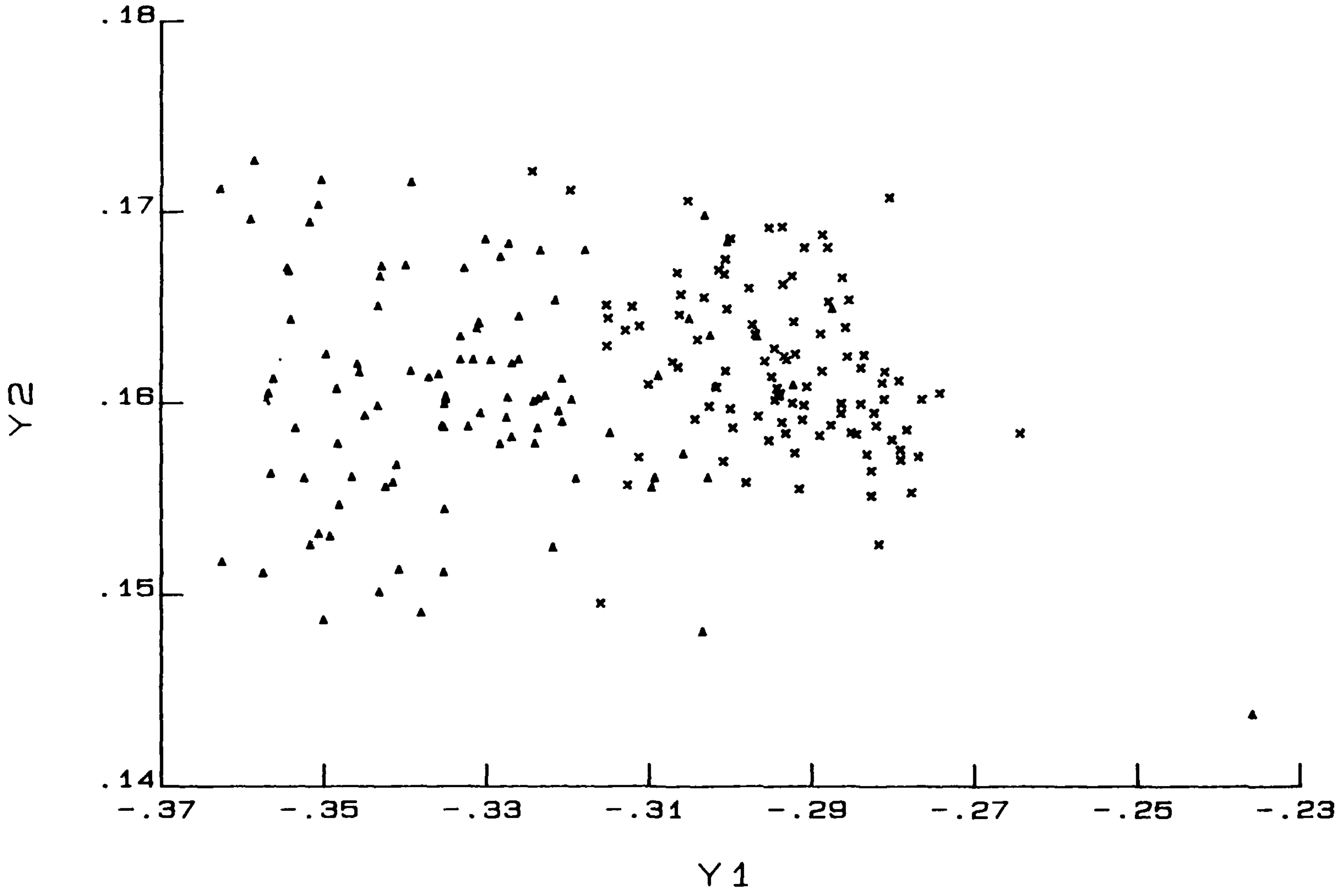

FIGURE 7. Forests and fields DX = 2 DY = 0.

/

# APPENDIX A

## FEATURE VECTOR COMPONENTS

The following first and second order histogram measures were used to construct a thirteen dimensional feature vector:

Mean $$\overline{b} = \sum_{b=o}^{L-1} bP(b) = x_1$$

Variance $$\sigma_b^2 = \sum_{b=o}^{L-1} (b-\overline{b})^2 P(b) = x_2$$

Skewness $$b_S = \frac{1}{\sigma_b^3} \sum_{b=o}^{L-1} (b-\overline{b})^3 P(b) = x_3$$

Kurtosis $$b_K = \frac{1}{\sigma_b^4} \sum_{b=o}^{L-1} (b-\overline{b})^4 P(b) - 3 = x_4$$

Energy $$b_N = \sum_{b=o}^{L-1} [P(b)]^2 = x_5$$

Entropy $$b_E = -\sum_{b=o}^{L-1} P(b)\log_2[P(b)] = x_6$$

Autocorrelation $$B_A = \sum_{a=o}^{L-1} \sum_{b=o}^{L-1} abP(a,b) = x_7$$

Covariance $$B_C = \sum_{a=o}^{L-1} \sum_{b=o}^{L-1} (a-\bar{a})(b-\bar{b}) P(a,b) = x_8$$

Inertia $$B_I = \sum_{a=o}^{L-1} \sum_{b=o}^{L-1} (a-b)^2 P(a,b) = x_9$$

Absolute Value $$B_V = \sum_{a=o}^{L-1} \sum_{a=o}^{L-1} |a-b| P(a,b) = x_{10}$$

Inverse Difference $$B_D = \sum_{a=o}^{L-1} \sum_{b=o}^{L-1} \frac{P(a,b)}{1+(a-b)^2} = x_{11}$$

Energy $$B_N = \sum_{a=o}^{L-1} \sum_{b=o}^{L-1} [P(a,b)]^2 = x_{12}$$

Entropy $$B_E = -\sum_{a=o}^{L-1} \sum_{b=o}^{L-1} P(a,b) \log_2 [P(a,b)] = x_{13}$$

where $L$ is the number of grey levels and $P(b)$ and $P(a,b)$ are given below

$$P(b) = \frac{Q(b)}{M}$$

$M$ is the total number of pixels in the sample window. In this case $M$ was equal to 1024. $Q(b)$ is the number of pixels of greytone $b$ which occur in the sample window.

$$P(a,b) = \frac{Q(a,b)}{M}$$

$Q(a,b)$ is the number of times greytone $a$ is located next to greytone $b$ by the displacement $\Delta x$ and $\Delta y$.

## APPENDIX B.

## Computer Program for Calculating $J_1$ and the Transformation Matrix A

[illegible]

```
FTN4,L
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C THIS PROGRAM PERFORMS FEATURE SELECTION FOR                        C
C MULTIDISTRIBUTIONS BY MAXIMIZING THE VALUE OF                      C
C J1                                                                 C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      PROGRAM FSELE(3,1000)
      DIMENSION IDUT(5),IMAGE(1024),X(100,13),IDCB(144),IFILE(3)
      DIMENSION A(13,2),C(13,13,2),S(13,13),S2(13,13),AMU(13),P(2)
      DIMENSION E(169),B(169),R(169)
      CALL RMPAR(IDUT)
      LU=IDUT(1)
      CALL FRLU(LU)
    1 NC=2
      DO 80 NJ=1,NC
      WRITE(LU,6)
    6 FORMAT("ENTER THE NUMBER OF IMAGE SAMPLES TO BE ANALYZED,LE 100")
      READ(LU,8)NDATA
    8 FORMAT(I3)
      WRITE(LU,10)
   10 FORMAT("ENTER THE FILE NAME FOR THE DATA _")
      READ(LU,12)IFILE
   12 FORMAT(3A2)
      WRITE(LU,15)
   15 FORMAT("DISK LU NUMBER?")
      READ(LU,2101)IDLU
      WRITE(LU,2100)
 2100 FORMAT("ENTER A VALUE FOR IDX")
      READ(LU,2101)IDX
 2101 FORMAT(I2)
      WRITE(LU,2200)
 2200 FORMAT("ENTER A VALUE FOR IDY")
      READ(LU,2101)IDY
      WRITE(6,1400)IDX,IDY
 1400 FORMAT(2X,"IDX=",I2,5X,"IDY=",I2)
      WRITE(6,200)IFILE
  200 FORMAT(1X,3A2)
      CALL OPEN(IDCB,IERR,IFILE,0,0,IDLU)
      IF(IERR.LT.0)GO TO 2000
      GO TO 14
 2000 WRITE(LU,2010)IERR
 2010 FORMAT("OPEN FILE ERROR",I5)
      GO TO 999
   14 ICONT=1
   13 J=1
      DO 16 I=1,8
      CALL READF(IDCB,IERR,IMAGE(J))
   16 J=J+128
      IF(IERR.LT.0)GO TO 3000
      GO TO 18
 3000 ICONT=NDATA
      WRITE(LU,2020)IERR
 2020 FORMAT("READ FILE ERROR",I5)
      GO TO  999
   18 NGRAY=16
      CALL ISCAL(IMAGE,IMAGE,1024,0,15)
      CALL FEVEL(IMAGE,AVG,VAR,SKW,XKT,ENG,ENT,AUT,COV,XII,
     1ABSS,XID,ENY,ENR,NGRAY,IDX,IDY)
```

## APPENDIX B
## (Continued)

```
C
C ---STORE RESULTS IN AN ARRAY------------------------------------------
C
      X(ICONT,1)=AVG
      X(ICONT,2)=VAR
      X(ICONT,3)=SKW
      X(ICONT,4)=AKT
      X(ICONT,5)=ENG
      X(ICONT,6)=ENT
      X(ICONT,7)=AUT
      X(ICONT,8)=COV
      X(ICONT,9)=XIF
      X(ICONT,10)=ABSS
      X(ICONT,11)=XID
      X(ICONT,12)=ENY
      X(ICONT,13)=ENK
      IF(ICONT-NDATA)20,22,22
   20 ICONT=ICONT+1
      GO TO 13
   22 CALL CLOSE(IDCB)
      WRITE(LU,24)NJ,IFILE
   24 FORMAT("NJ=",I1,3X,5A2)
      CALL COVER(NDATA,C,A,13,NJ,X,D3)
      WRITE(6,70)D3
   70 FORMAT(1X," THE INTRASET DISTANCE=",E15.6)
      WRITE(6,140)
  140 FORMAT(2X,"COVARIANCE MATRIX")
      DO 142 K=1,13
      WRITE(6,144)(C(K,M,NJ),M=1,13)
  144 FORMAT(1X,13(F8.4,1X))
  142 CONTINUE
   80 CONTINUE
  150 DO 152 NJ=1,NC
      WRITE(LU,154)NJ
  154 FORMAT("NJ=",I1,2X,"ENTER THE APRIORI PROBABILITY OF THIS CLASS")
      READ(LU,28)P(NJ)
   28 FORMAT(F10.6)
      WRITE(6,156)NJ,P(NJ)
  156 FORMAT(1X,"NJ=",I1,5X,"P(NJ)=",F10.6)
  152 CONTINUE
      DO 90 I=1,NC-1
      DO 90 J=I+1,NC
   90 S2(I,J)=0.0
      DO 72 I=1,NC-1
      DO 72 J=I+1,NC
      DO 72 M=1,13
      S2(I,J)=S2(I,J)+(A(M,I)-A(M,J))**2
   72 CONTINUE
      DO 91 I=1,NC-1
      DO 91 J=I+1,NC
      S2(I,J)=SQRT(S2(I,J))
   91 CONTINUE
      WRITE(6,73)
   73 FORMAT(1X,"THE INTERSET DISTANCES")
      DO 92  I=1,NC-1
      DO 92  J=I+1,NC
      WRITE(6,74)I,J,S2(I,J)
   74 FORMAT(1X,"I=",I1,5X,"J=",I1,5X,"D(I,J)=",E15.8)
   92 CONTINUE
      DO 30 I=1,13
```

# APPENDIX B
## (Continued)

```
      DO 30 J=1,13
      S2(I,J)=0.0
   30 S1(I,J)=0.0
      DO 32 I=1,13
      DO 32 J=1,13
      DO 32 K=1,NC
   32 S2(I,J)=S2(I,J)+C(I,J,K)*P(K)
      DO 34 J=1,13
   34 XMO(J)=0.0
      DO 40 J=1,13
      DO 40 M=1,13
   40 S1(J,M)=C(J,M,1)+C(J,M,2)+(A(J,1)-A(J,2))*(A(M,1)-A(M,2))
      DO 160 J=1,13
      DO 160 I=1,13
      K=I+13*(J-1)
      R(K)=S1(I,J)
      B(K)=S2(I,J)
  160 CONTINUE
      DO 41 I=1,13
   41 XMO(I)=0.0
      CALL NROOT(13,R,B,XMO,EI)
      DO 165 J=1,13
      DO 165 I=1,13
      K=I+13*(J-1)
      S2(I,J)=EI(K)
  165 CONTINUE
      WRITE(6,42)
   42 FORMAT(2X,"EIGENVALUES")
      WRITE(6,46)(XMO(I),I=1,13)
   46 FORMAT(1X,13(E9.3,1X))
      WRITE(6,48)
   48 FORMAT(2X,"EIGENVECTORS")
      DO 50 I=1,13
      WRITE(6,76)(S2(I,J),J=1,13)
   76 FORMAT(1X,13(F8.5,1X))
   50 CONTINUE
   64 WRITE(LU,52)
   52 FORMAT("ENTER A VALUE FOR M.LE.NC")
      READ(LU,4)M
      XJ1=0.0
      DO 54 I=1,M
   54 XJ1=XJ1+XMO(I)
      WRITE(6,56)XJ1,M
   56 FORMAT(2X,"THE VALUE OF J1=",F10.4,5X,"M=",I1)
      CALL TRMAT(S2,S1,13,13,0)
      WRITE(6,58)
   58 FORMAT(2X,"THE TRANSFORMATION MATRIX A")
      DO 60 I=1,M
      WRITE(6,46)(S1(I,J),J=1,13)
   60 CONTINUE
      WRITE(LU,62)
   62 FORMAT("DO YOU WANT TO CHANGE THE VALUE OF M? IF YES TYPE 1 IF
     1NO TYPE 2")
      READ(LU,4)IZ
    4 FORMAT(I1)
      IF(IZ.EQ.1)GO TO 64
      WRITE(LU,82)
   82 FORMAT("DO YOU WANT TO CHANGE THE VALUES OF THE APRIORI PROBABIL
     1ITIES? IF YES TYPE 1,IF NO TYPE 2")
      READ(LU,4)IAZ
```

## APPENDIX B
## (Continued)

```
      IF(JAZ.EQ.1)GO TO 150
      WRITE(LU,68)
   68 FORMAT("DO YOU WANT TO CHANGE THE VALUES OF IDX AND IDY? IF YES
     1TYPE 1, IF NO TYPE 2.")
      READ(LU,4)IXZ
      IF(IXZ.EQ.1)GO TO 1
  999 STOP
      END
      SUBROUTINE COVER(KK,COV,AV,N,NI,X,D3)
      DIMENSION X(100,13),C(13,13),A(13),Z(100,13),COV(13,13,2),AV(13
     1,2),VAR(13)
      XK=KK
      DO 50 K=1,N
      A(K)=0.0
      DO 45 J=1,KK
   45 A(K)=A(K)+X(J,K)
   50 A(K)=A(K)/XK
      DO 55 K=1,N
      VAR(K)=0.0
      DO 54 J=1,KK
   54 VAR(K)=VAR(K)+(X(J,K)-A(K))**2
   55 CONTINUE
      DO 60 K=1,N
   60 VAR(K)=VAR(K)/(XK-1.)
      D3=0.0
      DO 65 K=1,N
   65 D3=D3+VAR(K)
      D3=2.*D3
      DO 110 K=1,N
      DO 115 J=1,KK
  115 Z(J,K)=X(J,K)-A(K)
  110 CONTINUE
      DO 120 K=1,N
      DO 125 M=1,N
      C(K,M)=0.0
      DO 130 J=1,KK
  130 C(K,M)=C(K,M)+Z(J,K)*Z(J,M)
      C(K,M)=C(K,M)/XK
  125 CONTINUE
  120 CONTINUE
      DO 135 K=1,N
      DO 135 M=1,N
      AV(K,NI)=A(K)
  135 COV(K,M,NI)=C(K,M)
      RETURN
      END
      SUBROUTINE [illegible]
      DIMENSION A(169),[illegible]
      K=[illegible]
      DO 100 [illegible]
      [illegible]
      DO 100 [illegible]
      [illegible]
      K=[illegible]
  100 B(K)=B(L)
      [illegible]
      CALL [illegible]
      [illegible]
      DO 110 [illegible]
      [illegible]
```

/

## APPENDIX B
### (Continued)

```
110 X(I,J)=1.0/SQRT(ABS(R(I)))
    K=0
    DO 115 J=1,M
    DO 115 I=1,M
    K=K+1
115 R(K)=X(K)*XI(J)
    DO 120 I=1,M
    N2=0
    DO 120 J=1,M
    N1=M*(I-1)
    L=M*(J-1)+1
    X(I)=0.0
    DO 120 K=1,M
    N1=N1+1
    N2=N2+1
120 X(I)=X(I)+B(N1)*G(N2)
    L=0
    DO 130 J=1,M
    DO 130 I=1,J
    N1=I-M
    N2=M*(J-1)
    L=L+1
    A(L)=0.0
    DO 130 K=1,M
    N1=N1+M
    N2=N2+1
130 A(L)=A(L)+X(N1)*B(N2)
    CALL EIGEN(A,X,M,MV)
    L=0
    DO 140 I=1,M
    L=L+I
140 W(I)=A(L)
    DO 150 I=1,M
    N2=0
    DO 150 J=1,M
    N1=I-M
    L=M*(J-1)+I
    A(L)=0.0
    DO 150 K=1,M
    N1=N1+M
    N2=N2+1
150 A(L)=A(L)+F(N1)*X(N2)
    L=0
    K=0
    DO 170 J=1,M
    SUMV=0.0
    DO 160 I=1,M
    L=L+1
160 SUMV=SUMV+A(L)*A(L)
    [illegible]
    DO 170 I=1,M
    K=K+1
170 A(K)=A(K)/SUMV
    [illegible]
    END
    [illegible]
    [illegible]
    [illegible]
    [illegible]
```

# APPENDIX B
## (Continued)

```
      VAR=0.0
      SKW=0.0
      XKT=0.0
      ENG=0.0
      ENT=0.0
      AUT=0.0
      COV=0.0
      XTE=0.0
      ABS5=0.0
      XID=0.0
      ENY=0.0
      ENR=0.0
      DO 10 J=1,32
      DO 10 I=1,32
      M=J-1
   10 IAR(I,J)=IMAGE(I+M*32)
      M=1024
      DO 20 K=1,NGRAY
   20 CONT(K)=0.0
      DO 30 I=1,32
      DO 30 J=1,32
      IR=IAR(I,J)
      CONT(IR+1)=CONT(IR+1)+1.
   30 CONTINUE
      XM=M
      DO 32 K=1,NGRAY
      DO 32 M=1,NGRAY
   32 COUT(K,M)=0.0
      MX=32+IDX
      IF(32-MX)12,14,16
   12 INCR=MX-32
      NXB=1
      NXE=32-INCR
      GO TO 19
   14 NXB=1
      NXE=32
      GO TO 19
   16 INCR=32-MX
      NXB=1+INCR
      NXE=33-NXB
   19 MY=32+IDY
      IF(32-MY)21,23,25
   21 INCR=MY-32
      NYB=1
      NYE=32-INCR
      GO TO 27
   23 NYB=1
      NYE=32
      GO TO 27
   25 INCR=32-MY
      NYB=1+INCR
      NYE=33-NYB
   27 CONTINUE
      DO 35 I=1,NYE
      DO 35 J=1,NXE
      I1=IAR(NYB+I-1,NXB+J-1)
      I2=IAR(NYB+I-1+IDY,NXB+J-1+IDX)
      COUT(I1+1,I2+1)=COUT(I1+1,I2+1)+1.
   35 CONTINUE
      DO 40 K=1,NGRAY
```

## APPENDIX B
## (Continued)

```
40 P(K)=CONT(K)/XM
   DO 45 K=1,NGRAY
45 AVG=AVG+(K-1)*P(K)
   DO 50 K=1,NGRAY
50 VAR=VAR+((K-1-AVG)**2)*P(K)
   STD=SQRT(VAR)
   DO 55 K=1,NGRAY
55 SKW=SKW+((K-1-AVG)**3)*P(K)
   SKW=SKW/STD**3
   DO 60 K=1,NGRAY
60 XKT=XKT+((K-1-AVG)**4)*P(K)
   XKT=(XKT/(STD**4))-3.
   DO 65 K=1,NGRAY
65 ENG=ENG+P(K)*P(K)
   DO 70 K=1,NGRAY
   IF(P(K).EQ.0.0)GO TO 70
   ENT=ENT+3.321929*P(K)*ALOGT(P(K))
70 CONTINUE
   ENT=-ENT
   DO 75 K=1,NGRAY
   DO 75 M=1,NGRAY
75 PP(K,M)=COUT(K,M)/XM
   DO 80 K=1,NGRAY
   DO 80 M=1,NGRAY
   AUT=AUT+(K-1)*(M-1)*PP(K,M)
80 CONTINUE
   AVGK=0.0
   AVGM=0.0
   DO 85 K=1,NGRAY
   DO 85 M=1,NGRAY
   AVGK=AVGK+(K-1)*PP(K,M)
85 AVGM=AVGM+(M-1)*PP(K,M)
   DO 90 K=1,NGRAY
   DO 90 M=1,NGRAY
90 XIE=XIE+(K-M)*(K-M)*PP(K,M)
   DO 95 K=1,NGRAY
   DO 95 M=1,NGRAY
   ABSS=ABSS+(IABS(K-M))*PP(K,M)
95 CONTINUE
   DO 100 K=1,NGRAY
   DO 100 M=1,NGRAY
100 XID=XID+PP(K,M)/(1.0+(K-M)**2)
   DO 105 K=1,NGRAY
   DO 105 M=1,NGRAY
105 ENY=ENY+PP(K,M)*PP(K,M)
   DO 110 K=1,NGRAY
   DO 110 M=1,NGRAY
   IF(PP(K,M).EQ.0.0)GO TO 110
   ENR=ENR+3.321929*PP(K,M)*ALOGT(PP(K,M))
110 CONTINUE
   ENR=-ENR
   DO 115 K=1,NGRAY
   DO 115 M=1,NGRAY
   COV=COV+(K-1-AVGK)*(M-1-AVGM)*PP(K,M)
115 CONTINUE
   RETURN
   END
   END$
```

# APPENDIX C.

## Computer Program for Calculating the Transformed Feature Vector Y = AX

```
&YAXIR T=00004 IS ON CR00011 USING 00013 BLKS R=0000

FTN4,L
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C   THIS PROGRAM COMPUTES THE TRANSFORMATION                           C
C   Y=AX WHERE A IS COMPUTED FROM ANOTHER                              C
C   PROGRAM AND IS INPUT HERE                                          C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      PROGRAM YAXIR(3,1000)
      DIMENSION LUOT(5),IMAGE(1024),X(100,13),Y(100,2),A(2,13)
      DIMENSION IDCB(144),IFILE(3),AM(2),VAR(2)
      DIMENSION AV(6,6),D(2,2)
      EQUIVALENCE (IMAGE,Y)
      CALL RMPAR(LUOT)
      LU=LUOT(1)
      CALL FRLU(LU)
      NC=2
    4 FORMAT(I1)
      WRITE(LU,6)
    6 FORMAT("ENTER THE VALUE FOR M THE NUMBER OF ROWS IN THE TRANSFOR
     1MATION MATRIX A")
      READ(LU,4)M
      WRITE(LU,8)
    8 FORMAT("ENTER THE TRANSFORMATION MATRIX A")
      DO 10 I=1,M
      DO 10 J=1,13
      WRITE(LU,12)I,J
   12 FORMAT("I=",I2,2X,"J=",I2,"A(I,J)=")
      READ(LU,14)A(I,J)
   14 FORMAT(E9.3)
   10 CONTINUE
      WRITE(6,79)
   79 FORMAT(1X,"THE TRANSFORMATION MATRIX A")
      DO 78 I=1,M
      WRITE(6,77)(A(I,J),J=1,13)
   77 FORMAT(1X,13(E9.3,1X))
   78 CONTINUE
      DO 80 NJ=1,NC
      WRITE(LU,16)
   16 FORMAT("ENTER THE NUMBER OF IMAGE SAMPLES TO BE ANALYZED .LE.100")
      READ(LU,18)NDATA
   18 FORMAT(I3)
      WRITE(LU,20)
   20 FORMAT("ENTER THE FILE NAME FOR THE DATA SET")
      READ(LU,22)IFILE
   22 FORMAT(3A2)
      WRITE(LU,15)
   15 FORMAT("DISK LU NUMBER?")
      READ(LU,2101)IDLU
      WRITE(LU,2100)
 2100 FORMAT("ENTER A VALUE FOR IDX")
      READ(LU,2101)IDX
 2101 FORMAT(I2)
      WRITE(LU,2200)
 2200 FORMAT("ENTER A VALUE FOR IDY")
      READ(LU,2101)IDY
      WRITE(6,1400)IDX,IDY
 1400 FORMAT(2X,"IDX=",I2,5X,"IDY=",I2)
      WRITE(6,200)IFILE
  200 FORMAT(1X,3A2)
```

## APPENDIX C.
## (Continued)

```
      CALL OPEN(IDCB,IERR,IFILE,0,0,-IDLU)
      IF(IERR.LT.0)GO TO 2000
      GO TO 24
 2000 WRITE(LU,2010)IERR
 2010 FORMAT("OPEN FILE ERROR",I5)
      GO TO 999
   24 ICONT=1
      CALL LABIN(IDCB,6)
   13 J=1
      DO 19 I=1,8
      CALL READF(IDCB,IERR,IMAGE(J))
   19 J=J+128
      IF(IERR.LT.0)GO TO 3000
      GO TO 26
 3000 ICONT=NDATA
      WRITE(LU,2020)IERR
 2020 FORMAT("READ FILE ERROR",I5)
      GO TO 999
   26 NGRAY=16
      CALL ISCAL(IMAGE,IMAGE,1024,0,15)
      CALL FEVEC(IMAGE,AVG,VAR,SKW,XKT,ENG,ENT,AUT,COV,XIE,ABSS,XID,
     1ENY,ENR,NGRAY,IDX,IDY)
      X(ICONT,1)=AVG
      X(ICONT,2)=VAR
      X(ICONT,3)=SKW
      X(ICONT,4)=XKT
      X(ICONT,5)=ENG
      X(ICONT,6)=ENT
      X(ICONT,7)=AUT
      X(ICONT,8)=COV
      X(ICONT,9)=XIE
      X(ICONT,10)=ABSS
      X(ICONT,11)=XID
      X(ICONT,12)=ENY
      X(ICONT,13)=ENR
      IF(ICONT-NDATA)28,30,30
   28 ICONT=ICONT+1
      GO TO 13
   30 WRITE(LU,32)NJ,IFILE
   32 FORMAT("NJ=",I1,3X,3A2)
      DO 34 I=1,NDATA
      DO 34 K=1,M
   34 Y(I,K)=0.0
      DO 36 I=1,NDATA
      DO 36 K=1,M
      DO 36 MK=1,13
   36 Y(I,K)=Y(I,K)+A(K,MK)*X(I,MK)
      CALL GPLOT(Y(1,1),Y(1,2),100,LU,NJ)
      DO 50 K=1,M
      AM(K)=0.0
      XK=NDATA
      DO 45 J=1,NDATA
   45 AM(K)=AM(K)+Y(J,K)
   50 AM(K)=AM(K)/XK
      DO 55 K=1,M
      VAR(K)=0.0
      DO 54 I=1,NDATA
   54 VAR(K)=VAR(K)+(Y(I,K)-AM(K))**2
   55 CONTINUE
      DO 60 K=1,M
```

## APPENDIX C.
### (Continued)

```
   60 VAR(K)=VAR(K)/(XK-1.)
      D3=0.0
      DO 65 K=1,M
   65 D3=D3+VAR(K)
      D3=2.*D3
      DO 70 K=1,M
   70 AV(K,NJ)=AM(K)
   80 CONTINUE
      CALL GPLOT(Y(1,1),Y(1,2),-100,-LU,2)
      DO 81 I=1,NC-1
      DO 81 J=I+1,NC
   81 D(I,J)=0.0
      DO 82 I=1,NC-1
      DO 82 J=I+1,NC
      DO 82 K=1,M
   82 D(I,J)=D(I,J)+(AV(K,I)-AV(K,J))**2
      DO 83 I=1,NC-1
      DO 83 J=I+1,NC
   83 D(I,J)=SQRT(D(I,J))
      WRITE(6,87)
   87 FORMAT(1X,"THE INTERSET DISTANCES")
      DO 86 I=1,NC-1
      DO 86 J=I+1,NC
      WRITE(6,84)I,J,D(I,J)
   84 FORMAT(1X,"I=",I1,2X,"J=",I1,2X,"D(I,J)=",E15.8)
   86 CONTINUE
      CALL CLOSE(IDCB)
  999 STOP
      END
      END$
```

FILM