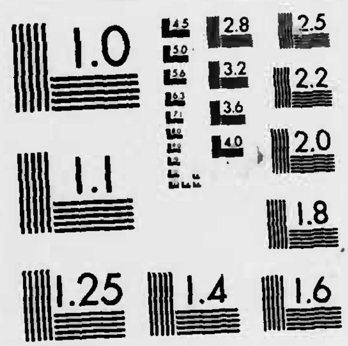
AD-A134 868 PLANE STRESS CRACK-LINE FIELDS FOR CRACK GROWTH IN AN ELASTIC PERFECTLY-P...(U) NORTHWESTERN UNIV EVANSTON IL STRUCTURAL MECHANICS LAB J D ACHENBACH ET AL. 1/1

UNCLASSIFIED 01 SEP 83 NU-SML-TR-83-1 N00014-76-C-0063 F/G 11/9 NL



MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS-1963-A

AD-A134 868

(1)

PLANE STRESS CRACK-LINE FIELDS FOR CRACK GROWTH IN AN ELASTIC PERFECTLY-PLASTIC MATERIAL

by

J. D. Achenbach and Z. L. Li

Department of Civil Engineering
Northwestern University
Evanston, IL. 60201

Office of Naval Research

N00014-76-C-0063

September 1983

NU-SML-TR-No.83-1

DTIC FILE COPY

DTIC ELECTE NOV 22 1983 S D E

Approved for public release; distribution unlimited

83 11 22 068

ABSTRACT: Mode-I crack growth in an elastic perfectly-plastic material under conditions of generalized plane stress has been investigated. In the plastic loading zone, near the plane of the crack, the stresses and strains have been expanded in powers of the distance, y, to the crack line. Substitution of the expansions in the equilibrium equations, the yield condition and the constitutive equations yields a system of simple ordinary differential equations for the coefficients of the expansions. This system is solvable if it is assumed that the cleavage stress is uniform on the crack line. By matching the relevant stress components and particle velocities to the dominant terms of appropriate elastic fields at the elastic-plastic boundary, a complete solution has been obtained for ε_y in the plane of the crack. The solution depends on crack-line position and time, and applies from the propagating crack tip up to the moving elastic-plastic boundary. Numerical results are presented for the edge crack geometry.

Epsilon sub Y

KEY WORDS: crack propagation, Mode-I, elastic-perfectly-plastic behavior, strain on crack line.

DTIC
COPY
INSPECTED
2

Accession For
NTIS GRA&I ☒
DTIC TAB ☐
Unannounced ☐
Justification

By
Distribution/

Availability Codes

Dist	Avail and/or Special
A-1	

Introduction

Quasi-static fields of stress and deformation near the tip of a growing crack in an elastic perfectly-plastic material, have been discussed in considerable detail by Rice [1]. Analytical expressions for the near-tip fields reveal the asymptotic structure of the fields in the immediate vicinity of a moving crack tip. They contain, however, functions that can generally be obtained only by supplemental numerical procedures. The one complete analytical solution, which is for the case of anti-plane strain, has been given by Rice [2].

In a recent paper Achenbach and Dunayevsky [3] considered the case of Mode-I crack growth under plane stress conditions in an elastic perfectly-plastic material. In Ref.[3] it was assumed that the stress components for a centered fan field, which were discussed by Hutchinson [4], and which satisfy the yield condition and the equilibrium equations, are valid up to the elastic-plastic boundary (at least near the crack line). The analytical approach of Ref.[3] then employs expansions of the particle velocities in powers of y (the distance from the plane of the crack), to obtain ordinary differential equations with respect to x for the coefficients in the expansions. Functions of time that enter in integrating these equations were determined by matching the fields in the plastic loading zone to the dominant terms of suitable elastic fields at the elastic-plastic boundary.

In the present paper we reconsider the results of Ref.[3]. Instead of making an a-priori assumption on the stress field, we also write expansions for the stresses near the crack line. Substitution of these expansions into the equilibrium equations and the yield condition produces a simple system of equations for the coefficients. Unfortunately the system is not closed, and additional information is required. The structure of the equations suggests that the cleavage stress on the crack line is uniform. With that assumption, the system of equations can be solved in a simple manner. The resulting expressions for the stresses are consistent with the centered fan field employed in Ref.[3].

To illustrate the analytical results the plastic strains just ahead of a moving crack tip have been computed for an edge crack geometry. The results are particularly suited for use in conjunction with a critical strain criterion.

Governing Equations

The geometry that is being considered in this paper is shown in Fig. 1. The x_3-axis of a stationary coordinate system is parallel to the crack front, and x_1 points in the direction of crack growth. The position of the crack tip is defined by $x_1 = a(t)$. A moving coordinate system, x,y,z is centered at the crack tip, with its axes parallel to the x_1, x_2 and x_3 axes. Relative to the moving coordinate system we also define polar coordinates r, θ, with $\theta = 0$ coinciding with the positive x direction.

In the moving coordinate system the equilibrium equations are

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} = 0 \ , \quad \frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_y}{\partial y} = 0 \qquad \text{(1a,b)}$$

We consider a state of generalized plane stress, hence σ_z, σ_{xz} and σ_{yz} vanish identically. The Huber-Mises yield criterion may then be written

$$\sigma_x^2 + \sigma_y^2 - \sigma_x \sigma_y + 3\tau_{xy}^2 = 3k^2 \qquad \text{(2)}$$

where k is the yield stress in pure shear. The strain rates are

$$\dot{\varepsilon}_x = \frac{\partial \dot{u}}{\partial x} \ , \quad \dot{\varepsilon}_y = \frac{\partial \dot{v}}{\partial y} \ , \quad \dot{\varepsilon}_{xy} = \frac{1}{2}\left(\frac{\partial \dot{u}}{\partial y} + \frac{\partial \dot{v}}{\partial x}\right) \qquad \text{(3a,b,c)}$$

In the moving coordinate system the material time derivative is

$$(\dot{\ }) = \partial_t - \dot{a}\partial_x \qquad \text{(4)}$$

where $\dot{a} = da/dt$ is the speed of the crack tip. The strain rates are related to the stresses and stress rates by

$$\frac{\partial \dot{u}}{\partial x} = \frac{1}{E}(\dot{\sigma}_x - \nu\dot{\sigma}_y) + \frac{1}{3}\dot{\Lambda}(2\sigma_x - \sigma_y) \tag{5}$$

$$\frac{\partial \dot{v}}{\partial y} = \frac{1}{E}(\dot{\sigma}_y - \nu\dot{\sigma}_x) + \frac{1}{3}\dot{\Lambda}(2\sigma_y - \sigma_x) \tag{6}$$

$$\frac{1}{2}\left(\frac{\partial \dot{u}}{\partial y} + \frac{\partial \dot{v}}{\partial x}\right) = \frac{1+\nu}{E}\;\dot{\tau}_{xy} + \dot{\Lambda}\tau_{xy} \tag{7}$$

where E and ν are Young's modulus and Poisson's ratio, respectively and $\dot{\Lambda}$ is a positive function of time and the spatial coordinates.

<u>Solution along the Crack Line</u>

In this paper we are interested in solutions along the crack line $y = 0$, $0 < x \leq x_p$, where $x = x_p$ defines the elastic-plastic boundary. Such solutions can be obtained by considering expansions with respect to y in the region $y/x << 1$:

$$\sigma_x = p_o(x,t) + p_2(x,t)y^2 + p_4(x,t)y^4 + \ldots \tag{8}$$

$$\sigma_y = q_o(x,t) + q_2(x,t)y^2 + q_4(x,t)y^4 + \ldots \tag{9}$$

$$\tau_{xy} = s_1(x,t)y + s_3(x,t)y^3 + \ldots \tag{10}$$

$$\dot{u} = \dot{u}_o(x,t) + \dot{u}_2(x,t)y^2 + \ldots \tag{11}$$

$$\dot{v} = \dot{v}_1(x,t)y + \dot{v}_3(x,t)y^3 + \ldots \tag{12}$$

$$\dot{\Lambda} = \dot{\Lambda}_o(x,t) + \dot{\Lambda}_2(x,t)y^2 + \ldots \tag{13}$$

Here we have taken into account that σ_x, σ_y, u and $\dot{\Lambda}$ are symmetric with respect to $y = 0$, while σ_{xy}, $\dot{v}$ are antisymmetric. Substition of

(8)-(10) into (1a,b) and collecting terms of the same order in y yields

$$\frac{\partial p_o}{\partial x} + s_1 = 0 \tag{14}$$

$$\frac{\partial p_2}{\partial x} + 3s_3 = 0 \tag{15}$$

$$\frac{\partial s_1}{\partial x} + 2q_2 = 0 \tag{16}$$

$$\frac{\partial s_3}{\partial x} + 4q_4 = 0 \tag{17}$$

Substitution of (8)-(10) into the yield condition (2) yields by the same procedure

$$p_o^2 + q_o^2 - p_o q_o = 3k^2 \tag{18}$$

$$(2p_o - q_o)p_2 + (2q_o - p_o)q_2 + 3s_1^2 = 0 \tag{19}$$

$$p_2^2 + (2p_o - q_o)p_4 + q_2^2 + (2q_o - p_o)q_4 - p_2 q_2 + 6s_1 s_3 = 0 \tag{20}$$

In the same manner we obtain by using (8)-(13) in (5)-(7)

$$\frac{\partial \dot{u}_o}{\partial x} = \frac{1}{E}\left(\frac{\partial p_o}{\partial t} - \nu\frac{\partial q_o}{\partial t}\right) - \frac{\dot{a}}{E}\left(\frac{\partial p_o}{\partial x} - \nu\frac{\partial q_o}{\partial x}\right) + \frac{\dot{\Lambda}_o}{3}(2p_o - q_o) \tag{21}$$

$$\frac{\partial \dot{u}_2}{\partial x} = \frac{1}{E}\left(\frac{\partial p_2}{\partial t} - \nu\frac{\partial q_2}{\partial t}\right) - \frac{\dot{a}}{E}\left(\frac{\partial p_2}{\partial x} - \nu\frac{\partial q_2}{\partial x}\right) + \frac{\dot{\Lambda}_o}{3}(2p_2 - q_2) + \frac{\dot{\Lambda}_2}{3}(2p_o - q_o) \tag{22}$$

$$\dot{v}_1 = \frac{1}{E}\left(\frac{\partial q_o}{\partial t} - \nu\frac{\partial p_o}{\partial t}\right) - \frac{\dot{a}}{E}\left(\frac{\partial q_o}{\partial x} - \nu\frac{\partial p_o}{\partial x}\right) + \frac{\dot{\Lambda}_o}{3}(2q_o - p_o) \tag{23}$$

$$3\dot{v}_3 = \frac{1}{E}\left(\frac{\partial q_2}{\partial t} - \nu\,\frac{\partial p_2}{\partial t}\right) - \frac{\dot{a}}{E}\left(\frac{\partial q_2}{\partial x} - \nu\,\frac{\partial p_2}{\partial x}\right) + \frac{1}{3}\dot{\Lambda}_o(2q_2 - p_2) + \frac{1}{3}\dot{\Lambda}_2(2q_o - p_o) \tag{24}$$

$$\dot{u}_2 + \frac{1}{2}\,\frac{\partial \dot{v}_1}{\partial x} = \frac{1+\nu}{E}\,\left(\frac{\partial s_1}{\partial t} - \dot{a}\,\frac{\partial s_1}{\partial x}\right) + \dot{\Lambda}_o s_1 \tag{25}$$

where (4) has also been used.

At this stage we have 14 unknowns and 12 equations. It turns out that this system of equations can be solved provided that one assumption is made. We assume that σ_y is constant on the crack line in the plastic loading zone. Hence, by the use of Eq.(9):

$$q_o = \text{constant} \tag{26}$$

It will be shown later that the assumption is consistent with the stress components for a centered fan field which have been considered by Hutchinson [4], and which are:

$$\sigma_x = k\cos^3\theta, \quad \sigma_y = k(2\cos^3\theta + 3\sin^2\theta\cos\theta), \quad \sigma_{xy} = -k\sin^3\theta \tag{27a,b,c}$$

If q_o = constant, it follows from Eq.(18) that p_o = constant. On the basis of p_o = constant, it follows from Eq.(14) that $s_1 = 0$, and subsequently from Eq.(16) that $q_2 = 0$. Substitution of these results in Eq.(19) yields $q_o = 2p_o$, and Eq.(18) then gives $p_o = k$ and $q_o = 2k$. Equation (20) subsequently yields $q_4 = -p_2^2/3k$. Substitution of the latter result in Eq.(17) and then in Eq.(15) yields

$$\frac{\partial^2 p_2}{\partial x^2} + \frac{4}{k}\,p_2^2 = 0 \tag{28}$$

The solution to Eq.(28) which satisfies the condition that the stresses are multivalued at the crack tip is

$$p_2 = -\frac{3}{2}\frac{k}{x^2} \tag{29}$$

In summary, it has been shown that the equilibrium equations and the yield condition are satisfied by

$$\sigma_x = k[1 - \frac{3}{2}(\frac{y}{x})^2] + 0(\frac{y}{x})^4 \tag{30}$$

$$\sigma_y = 2k + 0(\frac{y}{x})^4 \tag{31}$$

$$\tau_{xy} = -k(\frac{y}{x})^3 + 0(\frac{y}{x})^5 \tag{32}$$

It is noted that Eqs.(30)-(31) are indeed expansions with respect to $\theta = y/x$ of Eqs.(27a,b,c).

In the next step we substitute Eqs.(30)-(32) into (21)-(25) and collect terms of the same order. The result is

$$\frac{\partial \dot{u}_o}{\partial x} = 0 \ , \ \frac{\partial \dot{u}_2}{\partial x} = -3\frac{k}{E}\frac{\dot{a}}{x^3} - \frac{1}{x^2}\dot{\Lambda}_o k \tag{33}$$

$$\dot{v}_1 = \dot{\Lambda}_o k \tag{34}$$

$$2\dot{u}_2 + \frac{\partial \dot{v}_1}{\partial x} = 0 \tag{35}$$

By combining (33),(34) and (35) we obtain

$$\frac{1}{2}\frac{\partial^2 \dot{v}_1}{\partial x} - \frac{\dot{v}_1}{x^2} = 3\frac{k}{E}\frac{\dot{a}}{x^3} \tag{36}$$

The general solution to Eq.(36) is

$$\dot{v}_1 = \frac{k}{E}\{-2\frac{\dot{a}}{x}\ell n(\frac{x}{x_p}) + \frac{B(t)}{x} + C(t)x^2\} \ , \tag{37}$$

where x_p defines the x-coordinate of the elastic-plastic boundary on the crack line. The functions B(t) and C(t) can be obtained from continuity conditions at the elastic-plastic boundary.

For small values of θ (i.e., $y/x << 1$) the field in the plastic loading zone will be matched at the elastic-plastic boundary to the dominant terms of a corresponding elastic field. For the elastic field we do, however, not take the field for a crack, but rather that for a notch with $\frac{1}{2}\rho$ as radius of curvature at its tip. In polar coordinates R,ψ, the appropriate Mode-I stress fields are given by Creager and Paris [5] as

$$\sigma_x = \left(\frac{1}{2\pi R}\right)^{\frac{1}{2}} K_I\{\cos\tfrac{1}{2}\psi\ [1-\sin\tfrac{1}{2}\psi\ \sin\tfrac{3}{2}\psi] - \frac{\rho}{2R}\cos\tfrac{3}{2}\psi\} \tag{38}$$

$$\sigma_y = \left(\frac{1}{2\pi R}\right)^{\frac{1}{2}} K_I\{\cos\tfrac{1}{2}\psi\ [1+\sin\tfrac{1}{2}\psi\ \sin\tfrac{3}{2}\psi] + \frac{\rho}{2R}\cos\tfrac{3}{2}\psi\} \tag{39}$$

$$\tau_{xy} = \left(\frac{1}{2\pi R}\right)^{\frac{1}{2}} K_I\{\sin\tfrac{1}{2}\psi\ \cos\tfrac{1}{2}\psi\cos\tfrac{3}{2}\psi - \frac{\rho}{2R}\sin\tfrac{3}{2}\psi\} \tag{40}$$

Note that the tip of the notch, which is not the tip of the crack nor the elastic-plastic boundary, is a distance $\frac{1}{2}\rho$ from the origin E, as shown in Fig. 1. The center of the elastic field E, whose position is defined by $x_1 = e(t)$, $y_1 = 0$, is located in between the crack tip and the elastic-plastic boundary defined by $x = x_p(t)$. For generalized plane stress, the displacements corresponding to (38)-(40) are

$$u = \left(\frac{R}{2\pi}\right)^{\frac{1}{2}} \frac{1}{2\mu} K_I\{\cos\tfrac{1}{2}\psi\ [\kappa - 1+2\sin^2\tfrac{1}{2}\psi] + \frac{\rho}{R}\cos\tfrac{1}{2}\psi\} \tag{41}$$

$$v = \left(\frac{R}{2\pi}\right)^{\frac{1}{2}} \frac{1}{2\mu} K_I \{\sin\frac{1}{2}\psi \,[\kappa + 1 - 2\cos^2\frac{1}{2}\psi] + \frac{\rho}{R} \sin\frac{1}{2}\psi\} \tag{42}$$

where $\kappa = (3-\nu)/(1+\nu)$.

From the condition that the elastic field should just reach the yield condition at the elastic-plastic boundary, we obtain by the use of (2) and (38-(40)

$$\left[\left(\frac{1}{2\pi R_p}\right)^{\frac{1}{2}} K_I\right]^2 \left[1+3\left(\frac{\rho}{2R_p}\right)^2\right] = 3k^2 \tag{43}$$

where $R = R_p$ at the elastic-plastic boundary, at least for small values of y. Another condition is that σ_x should be continuous at the elastic-plastic boundary on y=0. By the use of (30) and (38) we find

$$k = \left(\frac{1}{2\pi R_p}\right)^{\frac{1}{2}} K_I \left(1 - \frac{\rho}{2R_p}\right) \tag{44}$$

From (43) and (44) it follows that

$$\frac{\rho}{R_p} = \frac{2}{3} \tag{45}$$

Hence (44) yields

$$\left(\frac{1}{2\pi R_p}\right)^{\frac{1}{2}} K_I = \frac{3}{2}k \tag{46}$$

Equations (43) and (46) show why we have taken elastic fields for a notch rather than for a crack. For an elastic crack-tip field the conditions of reaching the yield condition at the elastic-plastic boundary would conflict with the condition of continuity of σ_x, as can be checked by setting $\rho \equiv 0$ in (43) and (44).

Further details of the matching procedure have been given by Achenbach and Dunayevsky [3]. The relevant results are

$$x_E = (1-\gamma)x_p, \text{ and thus } R_p = \gamma x_p , \tag{47,a,b}$$

where $\gamma = 1/\sqrt{2}$.

$$B(t) = B_1\dot{a}(t) + B_2\dot{x}_p(t) \tag{48}$$

$$C(t) = [C_1\dot{a}(t) + C_2\dot{x}_p(t)]/[x_p(t)]^3, \tag{49}$$

where

$$B_1 = \frac{1}{32}\frac{1}{\gamma^2}[\kappa+5 +4\gamma(\kappa+1)](E/\mu) - \frac{2}{3} \tag{50}$$

$$B_2 = \frac{1}{32}\frac{1}{\gamma^2}[\kappa+5 + 2\gamma(\kappa+1)](E/\mu) \tag{51}$$

$$C_1 = \frac{1}{32}\frac{1}{\gamma^2}[-(\kappa+5) + 2\gamma(\kappa+1)](E/\mu) + \frac{2}{3} \tag{52}$$

$$C_2 = \frac{1}{32}\frac{1}{\gamma^2}[-(\kappa+5) + 4\gamma(\kappa+1)](E/\mu) \tag{53}$$

The Strain on the Crack Line

In the plane of the crack we have $\varepsilon_y = v_1$. At the elastic plastic boundary (42) yields for small y

$$(\varepsilon_y)_{PB} = \frac{3}{8}\frac{k}{\mu}(\kappa-\frac{1}{3}) = \frac{2-\nu}{E}k \tag{54}$$

where (46), (47) and $\theta \sim y/x$ have been used. In the stationary coordinate system, (37) can now be integrated to yield the total strain for $t \geq t_p$ as

$$\varepsilon_y(x_1,t) = (\varepsilon_y)_{PB} + \int_{t_p}^{t} \dot{v}_1(x_1,s)ds \tag{55}$$

where $\dot{v}_1(x_1,s)$ is obtained from (37) by using the relation $x = x_1 - a(t)$. In (55), t_p is the time that the elastic-plastic boundary arrives at position x_1. Thus, for the propagating crack tip, t_p follows from the equation

$$a(t_p) + x_p(t_p) = x_1 , \tag{56}$$

Here $x_p(t_p)$ is obtained from the stress intensity factor by using (46) and (47b):

$$x_p(t) = \frac{2\sqrt{2}}{9}\frac{1}{\pi}[K_I(t)/k]^2 \tag{57}$$

Equation (37) is also valid for a stationary crack. By setting $\dot{a} \equiv 0$ we obtain from (37) for $t \geq t_p$

$$\dot{v}_1^{sc}(\Delta,t) = \frac{k}{E}[B_2/\Delta + C_2\Delta^2/x_p^3(t)]\,\dot{x}_p(t) , \tag{58}$$

where (48) and (49) have been used and Δ is a distance ahead of the crack tip. The arrival time of the elastic-plastic boundary follows from (56) as

$$a_o + x_p(t_p) = x_1 \tag{59}$$

Let us consider the case that loading starts at time $t = 0$, but that the crack tip does not start to propagate until time $t = t_s$. For a position x_1 which is inside the plastic zone at time $t = t_s$, we find

$$\varepsilon_y(x_1,t_e) = (\varepsilon_y)_{PB} + \int_{t_p}^{t_s} \dot{v}_1^{sc}(x_1,t)ds + \int_{t_s}^{t_e} \dot{v}_1(x_1,s)ds \tag{60}$$

Here t_p follows from (59), and t_e is the time that the crack tip arrives at a small distance Δ from the position x_1 that is being observed. We have

$$x_1 - \Delta = a(t_e) \tag{61}$$

For a position x_1 which is outside the plastic zone at time $t = t_s$,

$$x_1 > a_o + x_p(t_s) \ , \tag{62}$$

the expression (55) holds, where t_p is now defined by (56).

Equations (55) and (60) can be manipulated to yield the singular parts of the strain at $x_1 = a(t)$, plus a bounded integral. The result can be found in Ref.[3]. In the present paper the integrals (55) and (60) have been evaluated numerically.

The result simplifies considerably for the case that all fields are assumed to be time-invariant to an observer traveling with the crack tip. This is the steady-state case when ε_y depends on $x = x_1 - a(t)$ only. Now we have that $\dot{a}$ = constant = c_F, $\dot{x}_p = 0$, and $(\dot{\ }) = -c_F\, d/dx$. The solution to Eq.(37) becomes

$$\frac{dv_1}{dx} = \frac{k}{E}\left\{\frac{2}{x}\,\ell n\left(\frac{x}{x_p}\right) - \frac{B_1}{x} - \frac{C_1 x^2}{x_p^3}\right\} \tag{63}$$

where B_1, C_1 are defined by (50) and (52), and x_p = constant. Equation (56) may be integrated to yield

$$\varepsilon_y(x) = (\varepsilon_y)_{PB} + \frac{k}{E}\left\{\left[\ell n\left(\frac{x}{x_p}\right)\right]^2 - B_1 \ell n\left(\frac{x}{x_p}\right) - \frac{1}{3} C_1 \left[\left(\frac{x}{x_p}\right)^3 - 1\right]\right\} \tag{64}$$

Numerical Results

For a given external load, we presumably know K_I in terms of the crack length a(t). A relation between $x_p(t)$ and a(t) can subsequently be obtained by the use of (57). Hence, in principle, a(t) is the only unknown quantity in (55). An equation for a(t) and $\dot{a}(t)$ can, for example, be obtained from (55) by the use of the critical strain criterion for crack propagation. This criterion stipulates that crack growth will proceed when a critical strain level ε_{cr} is maintained for ε_y in the plane of the crack at a characteristic distance Δ ahead of the crack tip. It appears, however, that it will be very difficult to solve a(t) and $\dot{a}(t)$ from the integral equation that can be extracted from (55).

In the examples that are considered here, we consider an inverse problem, that is, we prescribe the increasing crack length a(t) and the variation of the distant tensile stresses $\sigma(t)$, and we use (55) and (60) to compute the strain at a small distance Δ ahead of the crack tip.

Numerical results have been obtained for a material with the following mechanical properties, which are comparable to those of CrMnSiNi Steel:

Young's modulus: $E = 2.06 \times 10^{11}$ N/m^2

Poisson's ratio: $\nu = 0.3$

Yield stress in shear: $k = 8.13 \times 10^8$ N/m^2

Plane stress fracture toughness: $K_C = 16.7 \times 10^7$ N/m^2

The geometry considered was an edge crack of initial length

$a_o = 50$mm

in a half-plane. The half-plane was subjected to distant tensile stresses of magnitude $\sigma(t)$. The relevant stress intensity factor was taken as

$$K_I = 1.1215\ [\pi a(t)]^{\frac{1}{2}} \sigma(t) \tag{65}$$

The first example attempts to consider a case where the stress intensity factor varies in such a manner that a steady-state situation can be established. We choose

$$a(t) = a_o \qquad t \leq t_s \tag{66a}$$

$$= a_o + a_1(t-t_s) + a_1\{\exp[-(t-t_s)] - 1]\} \qquad t \geq t_s \tag{66b}$$

and

$$\sigma(t) = (t/t_s)\sigma_o \qquad t \leq t_s \tag{67a}$$

$$= \left[\frac{a_o}{a(t)}\{1 + (\frac{a_c}{a_o} - 1)\frac{t-t_s}{t_c-t_s}\}\right]^{\frac{1}{2}} \sigma_o \qquad t_s \leq t \leq t_c \tag{67b}$$

$$= \left[\frac{a_c}{a(t)}\right]^{\frac{1}{2}} \sigma_o \qquad t_c \leq t \tag{67c}$$

Here σ_o was taken as the time that K_I reaches the value of the fracture toughness

$$\sigma_o = K_C/(1.1215\sqrt{\pi a_o}) , \tag{68}$$

while t_c is the time that the crack tip arrives at a distance Δ from the position of the elastic-plastic boundary at time $t = t_s$. Thus t_c can be computed from

$$a(t_c) + \Delta = a_o + x_p(t_s) \tag{69}$$

The length a_c was taken as a_c = 63.9mm, and the time t_s was taken as t_s = 30s. It follows from (67c) and (65) that K_I remains constant for $t > t_c$. The crack-tip speed $\dot{a}(t)$ and the distant tensile stress $\sigma(t)$ have been plotted in Fig. 2a and Fig. 2b, respectively.

The strain ε_y has been plotted in Fig. 3. The upper curve represents the strain at a fixed position Δ = 1mm ahead of the crack tip. For $t < t_s$ = 30s, this is a fixed position. For $t > t_s$ the position moves with the crack tip. The curves numbered 2-10 represent the strains for specific fixed material points. Points 2, 3 and 4 were located inside the plastic zone at time $t = t_s$, and the corresponding strains were computed by Eq.(60 with upper limit $t \leq t_e$. Points 5-10 were outside the plastic zone at time $t = t_s$, and for these points the strains were computed by Eq.(55). Curves 2-10 all end at a time t_e at which the crack tip is a distance Λ from the material point. Time t_e is computed from Eq.(61). It is noted that the strain quickly approaches an apparent steady-state value, which is just equal to the steady-state value that can be computed from Eq.(64). Thus, for the distant stress given by Eq.(67), stable crack propagation at a constant strain ε_y is established quickly.

For the second example we choose

$$\sigma(t) = \left(\frac{t}{t_s}\right)\sigma_o \quad \text{for} \quad t \leq t_s \tag{70a}$$

$$= \sigma_o \quad \text{for} \quad t \geq t_s \, , \tag{70b}$$

where σ_o is defined by Eq.(68). The crack starts to propagate with constant velocity c_F at time $t = t_s$:

$$a(t) = a_o \qquad \text{for } t \leq t_s \tag{71a}$$

$$= a_o + c_F(t-t_s) \qquad \text{for } t \geq t_s \tag{71b}$$

For three crack-tip speeds, the strains at a location which remains at a fixed distance of $\Delta = 3\text{mm}$ ahead of the crack tip are shown in Fig. 4. Thus, the location where the strain is computed is stationary for $t \leq t_s$, and it moves with the crack-tip speed c_F for $t > t_s$. The strains increase with time, and the rate of change depends significantly on the crack tip speed.

Acknowledgement

This work was carried out in the course of research sponsored by the U.S. Office of Naval Research (Contract No. N00014-76-C-0063).

References

[1] Rice, J.R., Elastic Plastic Crack Growth, in Mechanics of Solids (H.G. Hopkins and M.J. Sewell, eds.), Pergamon Press, Oxford and New York, 1982, pp. 539-562.

[2] Rice, J.R., Mathematical Analysis in the Mechanics of Fracture, in Fracture: An Advanced Treatise, Vol. II, (H. Liebowitz, ed.), Academic Press, New York and London, 1968, pp. 192-314.

[3] Achenbach, J.D. and Dunayevsky, V., Crack Growth under Plane Stress Conditions in an Elastic Perfectly Plastic Material, Journal of the Mechanics and Physics of Solids, to appear.

[4] *Hutchinson*, J.W., Journal of the Mechanics and Physics of Solids, Vol.16, 1968, pp. 337-347.

[5] Creager, M., and Paris, Paul C., International Journal of Fracture Mechanics, Vol 3, 1967, pp. 247-252.

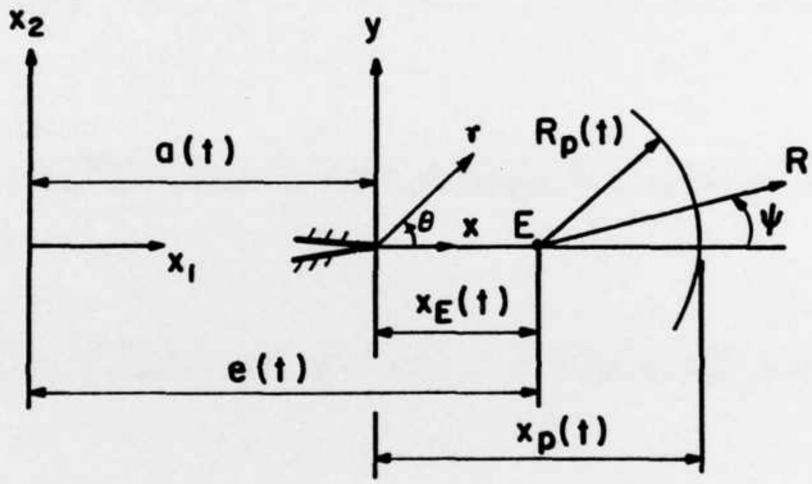


Fig. 1: Center of elastic field E, for slender notch of tip radius ρ

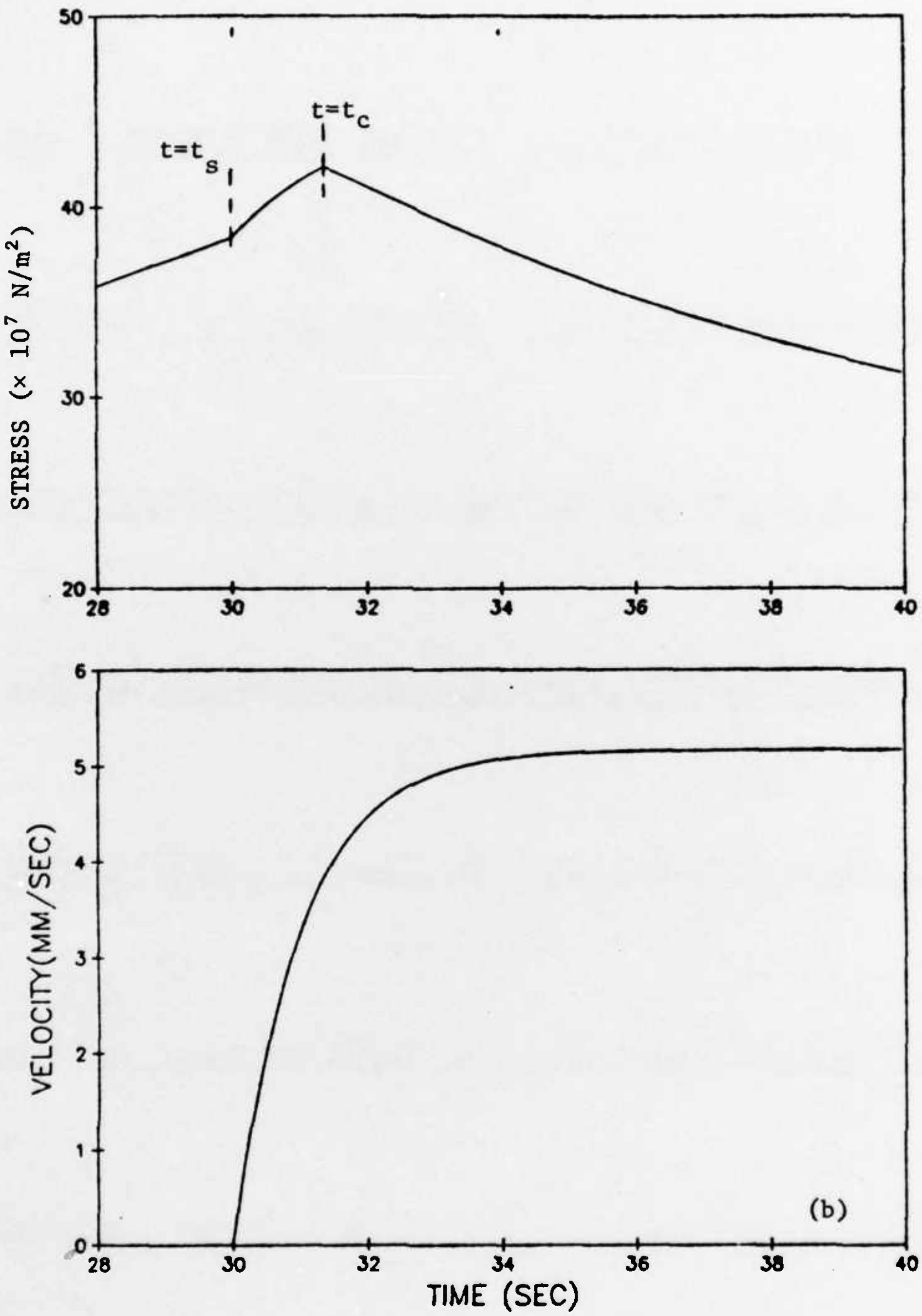


Fig. 2: (a) Stress field $\sigma(t)$ according to (67 a-c), and (b) Crack-tip speed from Eq.(66b).

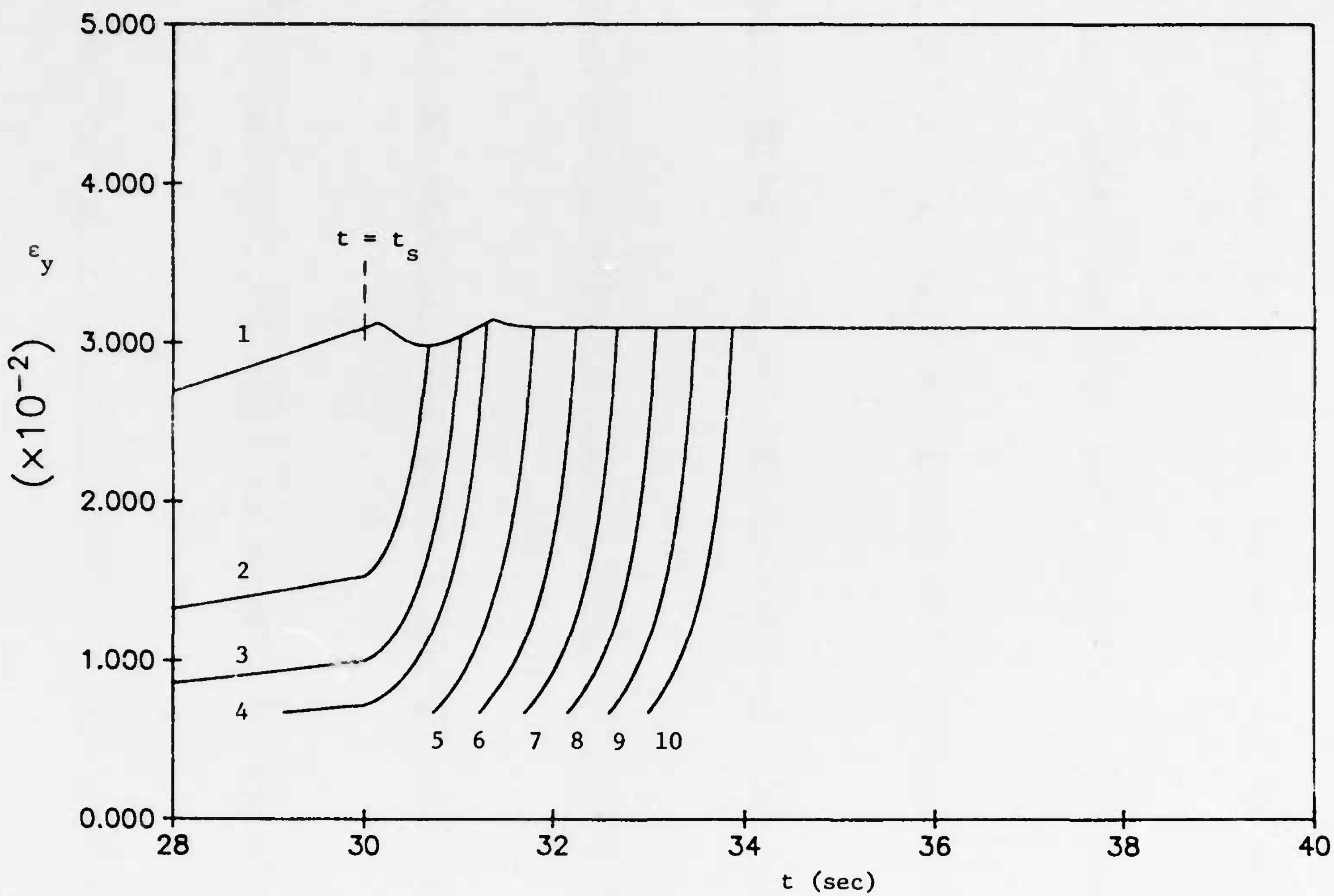


Fig. 3: Strain ε_y versus time; curve 1: at fixed distance Δ ahead of the stationary ($t < t_s$) and the moving crack tip ($t \geq t_s$); curves 2-10: at fixed material points; Δ = 1mm.

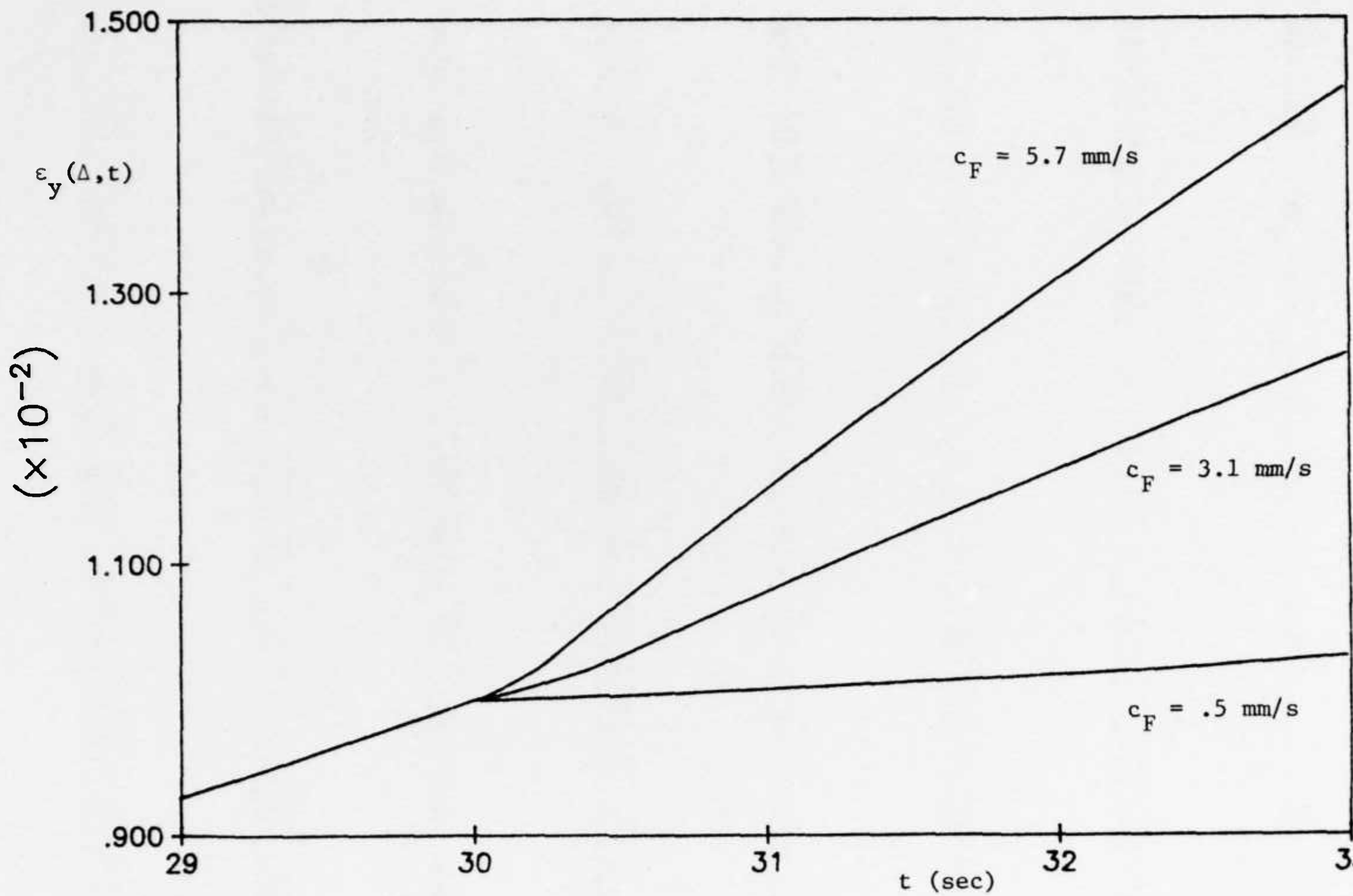


Fig. 4: Strain ε_y at fixed distance Δ ahead of crack tip, for ramp stress defined by (70a,b) and constant crack tip speed, see (71a,b); Δ = 3mm.

REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER: NU SML-TR-83-1	2. GOVT ACCESSION NO.: AD-A134868
3. RECIPIENT'S CATALOG NUMBER	
4. TITLE (and Subtitle): Plane Stress Crack-Line Fields for Crack Growth in an Elastic Perfectly-Plastic Material	5. TYPE OF REPORT & PERIOD COVERED: Interim
	6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s): J.D. Achenbach and Z. L. Li	8. CONTRACT OR GRANT NUMBER(s): N00014-76-C-0063
9. PERFORMING ORGANIZATION NAME AND ADDRESS: Northwestern University, Evanston, IL 60201	10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
11. CONTROLLING OFFICE NAME AND ADDRESS: Office of Naval Research, Mechanics Division, Code 432, Department of the Navy, Arlington, VA 22217	12. REPORT DATE: September 1, 1983
	13. NUMBER OF PAGES: 22
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)	15. SECURITY CLASS. (of this report): Unclassified
	15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

Presented at 16th ASTM National Symposium on Fracture Mechanics

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

crack propagation
Mode-I
elastic-perfectly-plastic behavior
strain on crack line

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Mode-I crack growth in an elastic pefectly-plastic material under conditions of generalized plane stress has been investigated. In the plastic loading zone, near the plane of the crack, the stresses and strains have been expanded in powers of the distance, y, to the crack line. Substitution of the expansions in the equilibrium equations, the yield condition and the constitutive equations yields a system of simple ordinary differential equations for the coefficients of the expansions. This system is solvable if it is assumed that the cleavage stress is uniform on the crack line. By matching the relevant stress components and

particle velocities to the dominant terms of appropriate elastic fields at the elastic-plastic boundary, a complete solution has been obtained for ε_y in the plane of the crack. The solution depends on crack-line position and time, and applies from the propagating crack tip up to the moving elastic-plastic boundary. Numerical results are presented for the edge crack geometry.

END

FILMED

12-83

DTIC