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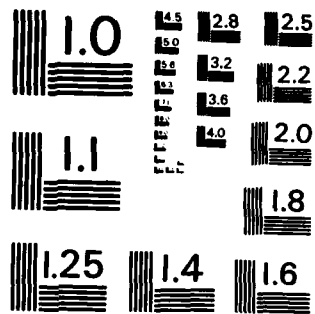
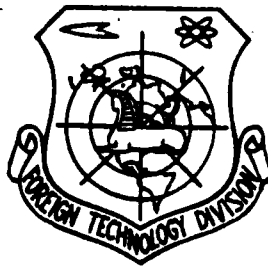MICROCOPY RESOLUTION TEST CHART

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CALCULATING THE MAGNETIC FIELD FROM A SADDLE-SHAPED
COIL WITHOUT A FERROMAGNETIC CIRCUIT

by

L. P. Gnedin, G. G. Borzov

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EDITED TRANSLATION

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Б б	*Б б*	B, b	С с	*С с*	S, s
В в	*В в*	V, v	Т т	*Т т*	T, t
Г г	*Г г*	G, g	У у	*У у*	U, u
Д д	*Д д*	D, d	Ф ф	*Ф ф*	F, f
Е е	*Е е*	Ye, ye; E, e*	Х х	*Х х*	Kh, kh
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Н н	*Н н*	N, n	Э э	*Э э*	E, e
О о	*О о*	O, o	Ю ю	*Ю ю*	Yu, yu
П п	*П п*	P, p	Я я	*Я я*	Ya, ya

*ye initially, after vowels, and after ъ, ь; e elsewhere.
When written as ё in Russian, transliterate as yё or ё.

RUSSIAN AND ENGLISH TRIGONOMETRIC FUNCTIONS

Russian	English	Russian	English	Russian	English
sin	sin	sn	sinh	arc sh	$\sinh^{-1}$
cos	cos	ch	cosh	arc ch	$\cosh^{-1}$
tg	tan	th	tanh	arc th	$\tanh^{-1}$
ctg	cot	cth	coth	arc cth	$\coth^{-1}$
sec	sec	sch	sech	arc sch	sech^{-1}
cosec	csc	csch	csch	arc csch	csch^{-1}

Russian	English
rot	curl
lg	log

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CALCULATING THE MAGNETIC FIELD FROM A SADDLE-SHAPED COIL WITHOUT A FERROMAGNETIC CIRCUIT

L. P. Gnedin, Dr. Tech. Sci.; G. G. Borzov, Engr.

Mathematical formulae for the components of induction
of the magnetic field and the magnetic flux from a coil
with saddle-shaped side sections in a system without a
magnetic circuit are given.

The use of superconductors to generate high-strength magnetic
fields in the excitation systems of MHD converters and electrical
machines makes it possible to eliminate the magnetic circuit in these
devices [1].

The most popular configuration of the excitation coil of an elec-
trical machine can be composed of two elements: a rectilinear side
part, and an arc-shaped front section with radius R (Fig. 1) - the
so-called C type [2]. The problem of calculating the induction of the
magnetic field from the arc-shaped front section was solved using the
Biot-Savart law in [3], in which the approximate formulae proposed in
[3] are only satisfied for small distances from the Z-axis.

The effect of the front sections was not considered in [4, 5].
In short coils, e.g., in the excitation coils of a DC collector or in
MHD converters with a short channel, the component of induction from
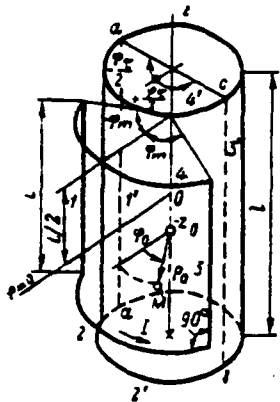the front section can exceed the induction from the side part.

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In order to calculate the magnetic field of an electrical machine, it is necessary to know both the induction at any point in space, e.g., for determining the losses in the armature circuits, and in the main working flux through the polar surface of the armature.

During an engineering calculation of an electrical machine without using an electronic computer or for checking a program which has been written, it is desirable to express the induction and flow using elementary functions. This problem is considered in the proposed article.

Fig. 1. Calculating the magnetic field of a saddle-shaped coil.

We will define the magnetic induction as the vector potential $\bar{A}$ in cylindrical coordinates:

$$
\left.
\begin{aligned}
B_\rho &= \frac{1}{\rho_0} \frac{\partial A_z}{\partial \varphi_0} - \frac{\partial A_\varphi}{\partial z_0}, \\
B_z &= \frac{1}{\rho_0} \frac{\partial (\rho_0 A_\varphi)}{\partial \rho_0} - \frac{1}{\rho_0} \frac{\partial A_\rho}{\partial \varphi_0}, \\
B_\varphi &= \frac{\partial A_\rho}{\partial z_0} - \frac{\partial A_z}{\partial \rho_0},
\end{aligned}
\right\}
\tag{1}
$$

where $\rho_0,\ \varphi_0,\ z_0$ are the coordinates of the current point.

.The origin of coordinates coincides with the rotational axis of the armature, and the counting axis of the angles passes through the center of the polar surface of the armature.

We will make the following assumptions.

1. We will replace the cross section of the coil with an infinitely thin conductor at the geometric center of the cross section with current i equivalent to the current of the entire cross section of the coil.

2. We will consider the plane in which the front part is located to be perpendicular to the side parts of the coil.

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INDUCTION COMPONENTS FROM FRONT PART OF EXCITATION COIL
(from circuit 2 in Fig. 1)

a) The radial induction component is $B_{\rho 2}$. The component $A_Z = 0$
in expression (1) for $B_{\rho 2}$ (assumption 2). For the direction of the
currents used in Fig. 1, the component

$$A_\varphi = \frac{\mu_0 i R}{4\pi} \int_{-\varphi_m}^{\varphi_m} \frac{\cos(\varphi_0 - \varphi)\, d\varphi}{\sqrt{\rho_0^2 + R^2 - 2R\rho_0 \cos(\varphi_0 - \varphi) + (L/2 + z_0)^2}}, \qquad (2)$$

where φ_m is the angle of placement of the side part of the excitation
coil; φ is the angular position of the current element; L is the
length of the excitation coil.

Substituting (2) in expression B_ρ and setting $\varphi_0 - \varphi = \pi - 2\gamma$,
we obtain:

$$B_{\rho 2} = \frac{\mu_0 i k^3 (L/2 + z_0)}{16\pi\rho_0^2 \sqrt{q}} \int_\alpha^\beta \frac{(2\sin^2\gamma - 1)\, d\gamma}{(\sqrt{1 - k^2 \sin^2 \gamma})^3}, \qquad (3)$$

where

$$\alpha = \frac{\pi - \varphi_m - \varphi_0}{2}, \quad \beta = \frac{\pi + \varphi_m - \varphi_0}{2},$$

$$k^3 = \frac{4\rho_0 R}{(R + \rho_0)^2 + (L/2 + z_0)^2}, \quad q = \frac{R}{\rho_0}.$$

The integral in expression (3) forms the sum of the integrals,
one of which $\int \frac{\cos^2\gamma\, d\gamma}{(\sqrt{1 - k^2\sin^2\gamma})^3}$ gives us $V = \int \frac{\cos\gamma\, d\gamma}{(\sqrt{1 - k^2 \sin^2 \gamma})^3}$ during the
solution by parts.

The integral V is solved by the replacement of $\sin\gamma = t$, with the
subsequent substitution of $\sqrt{(1 - kt)(1 + kt)} = (1 + kt)x$.

As a result, we obtain:

$$B_{\rho 2} = \frac{\mu_0 i (L/2 + z_0) k}{8\pi\rho_0^2 \sqrt{q}} \left\{ \frac{2 - k^2}{2(1 - k^2)} [E_{(\beta, k)} - E_{(\alpha, k)}] - \right.$$

$$- [F_{(\beta, k)} - [F_{(\alpha, k)}] - \frac{2 - k^2}{2(1 - k^2)} \frac{k^2}{2} \times \qquad (4)$$

$$\left. \times \left(\frac{\sin 2\beta}{\sqrt{1 - k^2 \sin^2 \beta}} - \frac{\sin 2\alpha}{\sqrt{1 - k^2 \sin^2 \alpha}} \right) \right\},$$

where E, F are the incomplete elliptical integrals of the second and
first types, respectively, taken from the table in report [6], where
$F - (\varphi) = -F(\varphi)$, $F(2\pi n \pm \varphi) = 2nK \pm F(\varphi)$; K is the total integral, The situation is
analogous for E.

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b) With A_φ determined from (2) with the replacement of $\cos(\varphi_0 - \varphi)$ by $-\sin(\varphi_0 - \varphi)$, the component of induction on the Z-axis will be:

$$B_{z2} = \frac{\mu_0 i k}{8\pi\rho_0 \sqrt{q}} \left\{ [F_{(\beta, k)} - F_{(\alpha, k)}] + \right.$$
$$+ \frac{k^2(q+1)-2}{2(1-k^2)} [E_{(\beta, k)} - E_{(\alpha, k)}] -$$
$$\left. - \frac{k^2(q+1)-2}{2(1-k^2)} \frac{k^2}{2} \left(\frac{\sin 2\beta}{\sqrt{1-k^2 \sin^2 \beta}} - \frac{\sin 2\alpha}{\sqrt{1-k^2 \sin^2 \alpha}} \right) \right. \quad (5)$$

c) The tangential component of induction:

$$B_{\varphi 2} = \frac{\mu_0 i (L/2 + z_0) k}{8\pi\rho_0^2 \sqrt{q}} \times \quad (6)$$
$$\times \left(\frac{1}{\sqrt{1-k^2 \sin^2 \beta}} - \frac{1}{\sqrt{1-k^2 \sin^2 \alpha}} \right).$$

For conductor 4, expressions (4)-(6) are valid with the replacement of $(z_0 + L/2)$ by $(z_0 - L/2)$ and the circuit -i (the value of k changes in this case).

The point $\rho_0 = 0$ is a singular point for expressions (4)-(6). The following expressions of the induction components were obtained for this point:

$$B_{\rho (\rho=0)} = \frac{\mu_0 i R (L/2 + z_0) \sin \varphi_m \cos \varphi_0}{2\pi [R^2 + (L/2 + z_0)^2]^{3/2}};$$
$$B_{z (\rho=0)} = \frac{\mu_0 i R^2 \varphi_m}{2\pi [R^2 + (L/2 + z_0)^2]^{3/2}}, \quad (7)$$
$$B_{\varphi (\rho=0)} = \frac{\mu_0 i R (L/2 + z_0) \sin \varphi_m \sin \varphi_0}{2\pi [R^2 + (L/2 + z_0)^2]^{3/2}}.$$

Components of Induction from the Side Part of the Excitation Coil (Circuit 3). Only

$$A_z = \frac{\mu_0 i}{4\pi} \int_{-L/2}^{+L/2} \frac{dz}{\sqrt{R^2 + \rho_0^2 - 2R\rho_0 \cos(\varphi_m - \varphi_0) + (z - z_0)^2}} \quad (8)$$

is present in (1).

We obtain the following expressions:

$$B_{\rho 3} = - \frac{\mu_0 i R \sin 2\beta}{4\pi (R + \rho_0)^2 (1 - m^2 \sin^2 \beta)} \times \left[\frac{L/2 - z_0}{\sqrt{1 - m^2 \sin^2 \beta + \left(\frac{L/2 - z_0}{R + \rho_0}\right)^2}} + \frac{L/2 + z_0}{\sqrt{1 - m^2 \sin^2 \beta + \left(\frac{L/2 + z_0}{R + \rho_0}\right)^2}} \right], \quad (9)$$

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$$B_{\varphi\varphi} = \frac{\mu_0 i \, (\rho_0 + R \cos 2\beta)}{4\pi \, (R + \rho_0)^2 \, (1 - m^2 \sin^2\beta)} \times$$

$$\times \left[\frac{L/2 - z_0}{\sqrt{1 - m^2 \sin^2\beta + \left(\frac{L/2 - z_0}{R + \rho_0}\right)^2}} + \right.$$

$$\left. + \frac{L/2 + z_0}{\sqrt{1 - m^2 \sin^2\beta + \left(\frac{L/2 + z_0}{R + \rho_0}\right)^2}} \right] \qquad (9)$$

where $\qquad m^2 = 4\rho_0 R \, (R + \rho_0)^2.$

For circuit 1, formulae (9) are valid when β is replaced by α and with the current $-i$. The components of the t-th excitation coil are determined from the above expressions when φ_0 is replaced by $\varphi_{0t} = [\pi/p(t-1) + \varphi_0]$, where p is the number of pairs of poles in the excitation system (with the angle counted in the positive direction).

Calculating the Main Magnetic Flux. We will define the flow through the polar surface a2'bc4'da (Fig. 1) as $\Phi = \int \vec{A} \, dl$ [7]. We will express Φ as the sum of the fluxes from the current in the side parts of the coil as Φ_6 , and in the front parts Φ_n. Based on the sym-metry of the magnetic system, the sides are found under identical magnetic conditions; therefore, da and bc

$$\Phi_6 = 2 \int_{-l/2}^{l/2} A_{3,3'} \, dz_t - 2 \int_{-l/2}^{l/2} A_{1,3'} \, dz_t. \qquad (10)$$

where $A_{3,3'}$ is the vector potential from the current in circuit 3 on line 3' of the armature contour; $A_{1,3'}$ - from the current in circuit 1 on line 3'.

The vector potential from the current in circuit m (1 or 3) on line 3'

$$A_{m,3'} = -\frac{\mu_0 i}{4\pi} \ln \frac{(z_t - L/2) + \sqrt{(z_t - L/2)^2 + a_{m,3'}^2}}{(z_t + L/2) + \sqrt{(z_t + L/2)^2 + a_{m,3'}^2}}, \qquad (11)$$

where z_1 is the coordinate of the current point on line 3' on the Z-axis,

$$a_{1,3'}^2 = R^2 + \rho_0^2 - 2R\rho_0 \cos(\varphi_0 + \varphi_m);$$

$$a_{3,3'}^2 = R^2 + \rho_0^2 - 2R\rho_0 \cos(\varphi_0 - \varphi_m).$$

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Solving expression (9), we obtain:

$$\Phi_6 = + \frac{\mu_0 i}{2\pi} (f_{3,3'} - f_{1,3'}).$$

where

$$f_{m,3'} = 2\sqrt{\left(\frac{L+l}{2}\right)^2 + a_{m,3'}^2} - 2\sqrt{\left(\frac{l-L}{2}\right)^2 + a_{m,3'}^2} +$$

$$+ \frac{l-L}{2} \ln \frac{\frac{l-L}{2} + \sqrt{\left(\frac{l-L}{2}\right)^2 + a_{m,3'}^2}}{-\frac{l-L}{2} + \sqrt{\left(\frac{l-L}{2}\right)^2 + a_{m,3'}^2}} + \frac{l+L}{2} \ln \frac{-\frac{l+L}{2} + \sqrt{\left(\frac{l+L}{2}\right)^2 + a_{m,3'}^2}}{\frac{l+L}{2} + \sqrt{\left(\frac{l+L}{2}\right)^2 + a_{m,3'}^2}}. \quad (12)$$

The flux from the front parts

$$\Phi_z = 2\rho_0 \int_{-\frac{\varphi_1}{2}}^{\frac{\varphi_1}{2}} (A_{2,2'} - A_{4,2'}) \, d\varphi_0. \quad (13)$$

The value of the vector potential is found by solving expression (2), as a result of which we obtain the formula for calculating the flow

$$\Phi_z = \frac{\mu_0 i \rho_0 \sqrt{q}}{\pi} \int_{-\frac{\varphi_1}{2}}^{\frac{\varphi_1}{2}} f(\varphi_0) \, d\varphi_0. \quad (14)$$

where

$$f(\varphi_0) = \frac{2 - k_2^2}{2k_2} [F_{(\beta, k_2)} - F_{(\alpha, k_2)}] -$$

$$- \frac{1}{k_2} [E_{(\beta, k_2)} - E_{(\alpha, k_2)}] - \frac{2 - k_4^2}{2k_4} [F_{(\beta, k_4)} - F_{(\alpha, k_4)}] +$$

$$+ \frac{1}{k_4} [E_{(\beta, k_4)} - E_{(\alpha, k_4)}].$$

$$k_2^2 = \frac{4\rho_0 R}{\left(\frac{L-l}{2}\right)^2 + (R + \rho_0)^2}, \quad k_4^2 = \frac{4\rho_0 R}{\left(\frac{L+l}{2}\right)^2 + (R + \rho_0)^2},$$

$\varphi_1 = \pi/p$ is the angular value of the polar spacing.

The integral of expression (14) can be found by any numerical method (e.g., using the Simpson rule [8]).

The value of the flow from the t-th pole is determined from the formulae given above, with φ_0 replaced by $\varphi_{0t} = \pi/p(t-1) + \varphi_0$; the sign of expression (14) is equal to $(-1)^{t-1}$ in this case.

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The induction component $B_{\rho} = f(z)$ of the superconductive excitation system of a dipole machine was calculated using the above formulae at $\tau_0 = 0$ and $\rho_0 = 14.75 \cdot 10^{-2}$ m (for the surface of a cryostat). The results of the calculation are compared with the experimental data in Fig. 2.
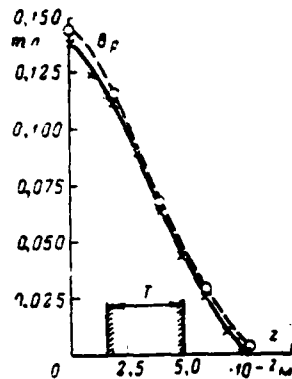
The calculated value of B_{φ} at point $\varphi_0 = \pi/2$, $\rho_0 = 3.9 \cdot 10^{-2}$ m, $z = 0$ was experimentally verified. The results agree, within the limits of the precision of measurement of induction with instrument type Ye-11.

Fig. 2. Change in induction along machine at $\rho_0 = 14.75$ cm, $\varphi_c = 0$.
T – zone of front part of excitation winding;
_____ – experimental curve;
– – – – calculated curve;
· · · · · – computer calculation.

The main flux of the machine on the calculated diameter of the armature was determined from (12) and (14). The arc spacing was 18°. The excitation coil was replaced by an infinitely thin turn. The divergence of the experimental value of the flux, determined from the machine's EMF, from the calcualted value is less than 5%.

CONCLUSIONS

1. The proposed formulae can be used in the electromagnetic calculation of an electrical machine with superconductive excitation without a magnetic circuit.

2. The magnetic flux of a machine with an armature diameter of 100–150 mm can be calculated by replacing the coil cross section with one infinitely thin conductor in the geometric center of the cross section.

3. During the precise calculation of induction in both the zone of the armature winding, and in the points close to the excitation winding, it is necessary to divide the coil cross section into several

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elementary cross sections, and to replace each by an infinitely thin conductor. The breakdown of the cross section into more than 16 sections virtually does not change the result of the calculation at points which are at a distance equal to the height of the coil.

4. The formulae can be used to calculate magnetic systems with saddle-shaped MHD converters.

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