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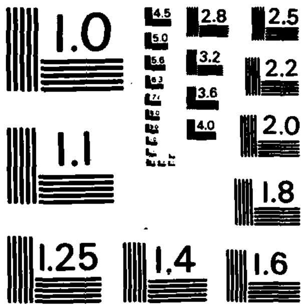

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RANGE ENHANCEMENT EQUATIONS FOR VARIOUS REFUELING OPTIONS

R. D. Shaver

N-1872-AF

June 1982

Prepared for The United States Air Force

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This Note presents a series of simplified equations and numerical calculations indicating the magnitude of range augmentation (or, equivalently, payload enhancement for a fixed range) that refueling by tankers can provide long range combat aircraft (LRCA). The treatment is theoretical and should apply to LRCAs of various design. T~~he Note examines~~ several refueling conditions, including two pre-fueling tactics for the tanker and two post-fueling tactics for the tanker. In addition, the author considers both pre- and post-strike refueling for the LRCA, and indicates the potential advantages in range enchancement derived from dropping the payload along the LRCA's flight path. Finally, for analytic convenience, most of the derivations and calculations contained in the Note assume that the LRCA and tanker are equal-size, being derived from the same basic airframe and engine.

A RAND NOTE

RANGE ENHANCEMENT EQUATIONS FOR VARIOUS REFUELING OPTIONS

R. D. Shaver

N-1872-AF

June 1982

Prepared for The United States Air Force

Rand
SANTA MONICA, CA. 90406

PREFACE

This Note is an extension of work performed in support of the 1980 Air Force Scientific Advisory Board (SAB) study of the Long Range Combat Aircraft (LRCA). It reproduces and expands upon equations useful for calculating the range extension obtainable from tanker support. The issue of tanker support for the LRCA was dealt with by the SAB only cursorily. The Board argued essentially that dependence on tankers was inherently bad, but it failed to specify the price (in terms of LRCA gross weight and, ultimately, life cycle cost) that the United States should be willing to pay to avoid that dependence. A companion Rand Note, N-1861-AF (forthcoming), attempts to deal with aspects of the tanker issue. (The present document contains the background mathematics for that Note.)

Rand's involvement in the SAB LRCA study was supported by Project AIR FORCE under the study effort "Assessment of Mixed Strategic Force Concepts for Flexible Requirements and Scenarios." Air Force planners interested in simple expressions for estimating range extension for one or more aircraft, under a variety of assumptions about tanker employment, may find this work useful.

SUMMARY

The Scientific Advisory Board of the USAF recommended in 1980 that the Air Force develop a new strategic bomber for use in the 1990s and beyond. The President's October 1981 decision specified that the new bomber would be a derivative of the B-1 (known as the B-1B) with an IOC of 1986. As a hedge against future growth in the threat, he also recommended developing an advanced technology bomber for deployment in the 1990s.

One issue not extensively treated by the SAB and only partially covered by the President's announcement was the proper role of tankers. Based on operational constraints, the SAB recommended that future bombers should be capable of performing their missions without tanker support. Cost and other factors may argue to the contrary.[1] This Note presents a series of equations that can be used to determine either the range augmentation tankers can provide or the corresponding increase in payload. For convenience of presentation, most equations assume equal-sized aircraft; extensions to different bomber/tanker sizes are straightforward.

Several tanker refueling options are discussed in this Note. They include: (1) where the tanker lands after completion of refueling, (2) how tankers and LRCA reach their rendezvous point, (3) the LRCA's status at refueling, and (4) how many LRCA each tanker services. Two options were included for where the tanker lands; i.e.,

- o Tankers land at the base from which they departed; we label this <u>radius missions</u>, or
- o Tankers land at forward bases; we assume throughout that these bases are at a constant distance from the refueling point, and label them <u>constant recovery distance missions</u>.

---

(1) See Reference 1.

For both the radius and constant recovery distance missions, we consider two ways for the tanker/LRCA pair to rendezvous:

- o The LRCA and tanker depart from the same base and fly in close proximity until the refueling point is reached; we call this the buddy-buddy approach, or
- o The tanker flies to a predetermined refueling point, meeting the LRCA there; we call this the filling station approach.

At the time of refueling, the bomber can be either outward- or inward-bound. We define these conditions to be the following:

- o Outward-bound LRCA are moving further away from the tanker's home base; thus a delay in refueling would require the tanker to expend more fuel to reach the refueling point,
- o Inward-bound LRCA are approaching the tanker's home base; thus delaying refueling increases the available tanker fuel for offloading.

In addition, the LRCA can either drop its payload somewhere along its flight path or carry it to the end. The former stretches the LRCA's total flying range. Finally, several options were considered regarding the number of tankers and LRCA involved:

- o One tanker services one LRCA; most of this Note treats this case.
- o Multiple tankers service one LRCA, using the buddy-buddy approach, i.e., all the tankers fly in formation with the LRCA, with refueling occurring sequentially as the fuel available for offloading from the i^{th} tanker equals the available fuel capacity of the LRCA.
- o One tanker services two LRCA, using the buddy-buddy

approach where several refueling conditions are examined.

- One tanker services n LRCA, using the filling station approach.

Not all combinations of the above options have been examined, but the basic mathematics to permit such examination are provided.

The results of this Note are two-fold. First, specific equations have been derived for the options mentioned above. The equation number for each option is listed in Table S-1. Second, optimum refueling conditions have been derived. For outward-bound bombers, refueling should occur at that distance where the fuel available onboard the tanker for offloading onto the bomber exactly matches the fuel required to completely refill the bomber. Refueling can occur earlier, without harm, so long as the last refueling occurs when this condition arises. For inward-bound bombers, refueling should occur at the latest possible time, usually when the bomber is at fuel exhaustion. The optimum refueling conditions either permit maximum payload weights over a fixed distance or maximum distance given a fixed payload.

Table S-1

LOCATION OF EQUATIONS DESCRIBING FINAL RESULTS

LRCA	Tankers	Tanker Departs From	Tanker Returns To	LRCA Headed	Other Comments	Equation Number
1	1	H	H	O		4.14
		H	F	O		4.20
		F	F	O		4.25
1	n	H	H	O		5.14
		H	F	O		5.12
2	1	H	H	O	Equal enhancement	6.14
		H	F	O	Equal enhancement	6.12
n	1	H	H	O	Filling station	7.24
		H	F	O	Filling station	7.26
		F	F	O	Filling station	7.27
1	1	H	H	I		9.9
		H	F	I		9.10
1	n	H	H	I		9.24
		H	F	I		9.25

NOTES: H = home base (same as LRCA);
F = forward base, assumed constant distance;
O = outward bound;
I = inward bound.

CONTENTS

CONTENTS - Continued

TABLES

FIGURES

I. INTRODUCTION

This Note presents a series of simplified equations and numerical calculations indicating the magnitude of range augmentation (or, equivalently, payload enhancement for a fixed range) that refueling by tankers can provide long range combat aircraft (LRCA). The treatment is theoretical and should apply to LRCAs of various designs.

This Note examines several refueling conditions. Two pre-refueling tactics are considered for the tanker. One assumes that the tanker leaves the home base with the LRCA it is to refuel, flying with the LRCA until the refueling point; we label this the buddy system. The second tactic assumes that the tanker leaves its home base and flies to a rendezvous point where it refuels one or more LRCAs; we label this the filling station approach. The buddy system is usually assumed to apply to LRCA's flying strategic missions requiring long range flights. The filling station approach appears to be more applicable to missions that require shorter flight distances and heavier payloads consistent with force projection missions.

Two post-refueling tactics are considered for the tanker. One assumes that, after all refuelings are accomplished, the tanker returns to the base from which it left; we label this the radius mission. The second assumes that the tanker recovers at a forward recovery base; for simplicity we shall specify that the distance from the last refueling point to the recovery base is a constant. We label this the constant recovery distance profile.

In addition, we consider both pre- and post-strike refueling for the LRCA, and indicate the potential advantages in range enhancement derived from dropping the payload along the LRCA's flight path. Longer range missions will require pre-strike refueling, but many force projection missions can be performed with post-strike refueling, at a considerable savings in total fuel required.

Finally, for analytic convenience, most of the derivations and calculations contained in this Note assume that the LRCA and tanker are equal-size, being derived from the same basic airframe and engine.

A section at the rear will relax this assumption, indicating how the equations can be modified to accommodate different LRCA/tanker designs.

This Note is organized in the following manner. Section II will derive the well-known Breguet equations expressing the relationship between flight range, fuel expended, and aircraft characteristics. These equations are used extensively in the following sections dealing with range augmentation and inflight refueling. Section III provides the reader a simple example of a derivation of a particular refueling equation. The assumptions employed in that section will be treated in detail in later sections. Sections IV through VII cover various tanker pre-refueling and post-refueling tactics in substantial detail, proving where necessary some of the conditions for selecting the best refueling point. Section VIII discusses the range augmentation obtained by payload delivery at mid-range. Section IX discusses the post-strike refueling equations. And Section X covers the equation derivations where the LRCA and the tankers are not similar. After reading Sections II and III, the reader should be able to skip sections that are not of particular interest; most sections are written to be self-sufficient.

II. BREGUET RANGE EQUATIONS

Many years ago Breguet developed a set of equations appropriate for estimating the range-payload tradeoffs for aircraft. While not quite appropriate for jet aircraft, these equations are still widely used. They are based on the following assumptions: (1) lift equals weight, (2) engine thrust equals drag, (3) the aircraft lift-to-drag ratio is constant, and (4) engine thrust equals a constant times the fuel mass flow rate. These are sensible assumptions for gas turbine powered aircraft flying under typical high altitude cruise conditions. They lead to the following differential equation:

$$\begin{aligned} \frac{dW}{dt} &= -cT && \text{(by 4)} \\ &= -cD && \text{(by 2)} \\ &= -\frac{cW}{(L/D)} && \text{(by 1 and 3)} \end{aligned} \tag{2.1}$$

where W is the vehicle's weight, T is the thrust of the engine, c is the engine specific fuel consumption, L is the vehicle's lift and D is the vehicle drag. For high altitude, maximum range flights, the (L/D) ratio is essentially constant. Therefore, we can integrate Eq. (2.1) and obtain:

$$W(t) = W(t_o)\exp\left\{-\frac{ct}{(L/D)}\right\} \tag{2.2}$$

Noting that $r = vt$, where v is the vehicle's velocity and r the range flown during time interval t,

$$W(r) = W(r_o)\exp\left\{-\frac{cr}{v(L/D)}\right\} \tag{2.3}$$

Since v is constant (thrust is assumed equal to drag), we can define a new constant $K = \frac{v(L/D)}{c}$. K is the well-known Breguet range factor, and is constant given our assumptions. Substituting K into Eq. (2.3), we obtain Breguet's fundamental relationship

$$W(r) = W(r_o)\exp\left\{-\frac{r}{K}\right\} \tag{2.4}$$

W(r) is the vehicle's gross weight after it has flown a distance r, and $W(r_o)$ is the aircraft's gross takeoff weight. Thus, the aircraft weight is simply an exponential function of the range flown.

The maximum range-payload equation follows quickly from Eq. (2.4). By definition

$$W(r_o) \stackrel{\text{def}}{=\!=} W_e + W_a + W_p + F_o \tag{2.5}$$

where W_e, W_a, and W_p are the aircraft dry weight (less avionics), avionics weight, and payload weight, respectively, and F_o is the maximum weight of fuel at aircraft takeoff. If R is the range where the fuel is completely expended,

$$W(R) \stackrel{\text{def}}{=\!=} W_e + W_a + W_p \tag{2.6}$$

Substituting Eqs. (2.5) and (2.6) into (2.4), we obtain

$$W_e + W_a + W_p = W_o\exp\left\{-\frac{R}{K}\right\}^*$$

or

$$R = K \log_e\left[\frac{W_o}{W_e + W_a + W_p}\right] \tag{2.7}$$

Equation (2.7) is the well-known Breguet range equation. Note that it

*We shall frequently use the shorter notation W_o for $W(r_o)$, if no confusion is likely.

applies to the no-refueling case. Also note that W_o can represent the aircraft's gross weight at any point along its flight path and R then becomes the remaining distance that the aircraft can fly until its fuel tanks are empty.

III. THE RANGE AUGMENTATION EQUATION: A SIMPLE EXAMPLE

The Breguet equations are well suited for deriving the potential range augmentation that can be obtained from a single tanker refueling. As a start, consider the following assumptions: (1) bombers and tankers have the same characteristics (e.g., the same gross takeoff weight, the same dry airframe weight, the same Breguet range factor), (2) both aircraft depart from the same base and fly in formation until the refueling point is reached, (3) there is a single refueling and it fills the bomber until its inflight weight equals its gross takeoff weight, (4) the tanker, after refueling, flies a fixed distance, s, to a forward recovery base, and (5) given the above, the bomber's unrefueled range exceeds the to-be-determined optimum refueling point. Later sections will alter some of these assumptions.

We will show later that the above assumptions imply that the maximum range augmentation is obtained if the refueling point occurs when the total fuel available to be offloaded from the tanker (i.e., still permitting the tanker to fly s miles to its recovery base) exactly equals the fuel needed to fully refuel the bomber (i.e., raise its gross weight to equal the takeoff weight) (see Section IV). The resultant range augmentation, Δr, is given by the following equation:

$$\Delta r = K \log_e \left[\frac{2}{1+\left(\frac{W_e}{W_o}\right)\exp\left\{\frac{s}{K}\right\}} \right] \tag{3.1}$$

where the variables have already been defined. The remainder of this section will be devoted to the derivation of this equation.

Consider Fig. 1. ΔF_r is the fuel required for the tanker to reach its recovery base, ΔF_a is the fuel available at range r to be loaded into the bomber, and ΔF_b is the fuel used in flying the distance r. ΔF_b applies to both the tanker and the bomber, since their characteristics are identical. Note also that we are assuming that the tanker reaches its recovery base empty. W_s is the tanker's gross weight immediately after refueling, and W_r is its weight at the start

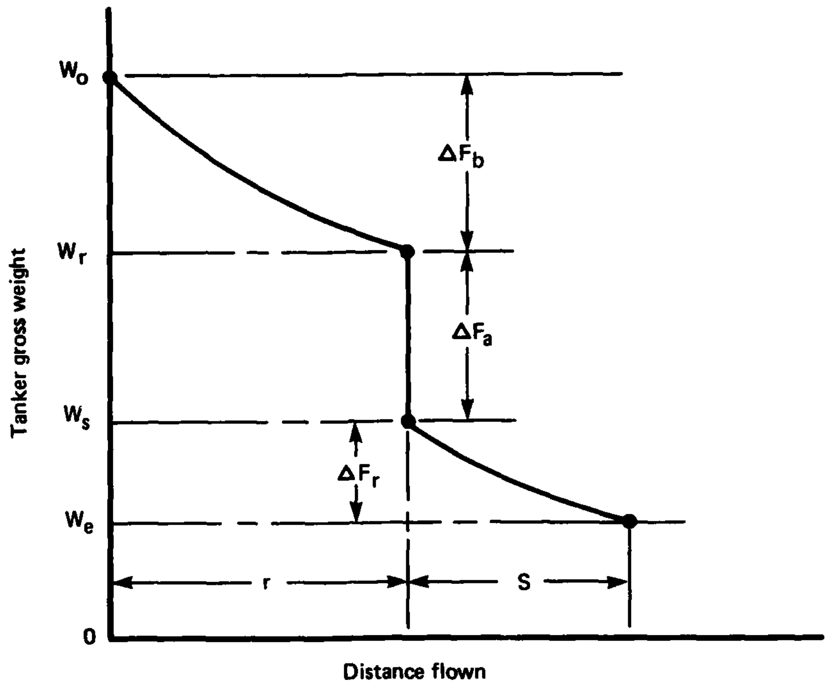

Fig. 1 — Tanker gross weight versus distance flown

of refueling. Assuming instantaneous refueling,

$$\Delta F_a = W_r - W_s \tag{3.2}$$

From Breguet's equation

$$W_e = W_s \exp\left\{-\frac{s}{K}\right\} \tag{3.3}$$

$$W_r = W_o \exp\left\{-\frac{r}{K}\right\} \tag{3.4}$$

Substituting (3.3) and (3.4) into (3.2)

$$\Delta F_a = W_o \left[\exp\left\{-\frac{r}{K}\right\} - \left(\frac{W_e}{W_o}\right) \exp\left\{\frac{s}{K}\right\}\right] \tag{3.5}$$

As we shall prove in Section IV but only assert here, optimum refueling occurs when ΔF_a equals the fuel weight needed to completely fill the bomber. ΔF_b is that weight. Therefore, to maximize the range augmentation,

$$\Delta F_a = \Delta F_b \tag{3.6}$$

From Fig. 1,

$$\begin{aligned}\Delta F_b &= W_o - W_r \\ &= W_o\left[1 - \exp\left\{-\frac{r}{K}\right\}\right]\end{aligned} \tag{3.7}$$

Substituting Eqs. (3.5) and (3.7) into (3.6)

$$W_o\left[1 - \exp\left\{-\frac{r}{K}\right\}\right] = W_o\left[\exp\left\{-\frac{r}{K}\right\} - \left(\frac{W_e}{W_o}\right)\exp\left\{\frac{s}{K}\right\}\right] \tag{3.8}$$

Solving for r, we obtain Eq. (3.1), i.e.,

$$r = K\log_e\left[\frac{2}{1+\left(\frac{W_e}{W_o}\right)\exp\left\{\frac{s}{K}\right\}}\right] \tag{3.9}$$

Since r is the distance from the bomber's home base, and the bomber is fully fueled at that distance (and can still fly its entire unrefueled range, R).* r corresponds to the added range a single refueling provides. Thus

$$\Delta r = r \qquad \text{Q.E.D.}$$

*We will use R_0 to represent the maximum unrefueled range when the bomber is fully fueled, and R to be the maximum remaining unrefueled range.

IV. RANGE ENHANCEMENT FROM A SINGLE (OUTBOUND) REFUELING

Section III treated a simple example of determining the maximum range enhancement achievable in a single refueling of an outbound LRCA. This section generalizes on that discussion. We start with some definitions:

- Base L: The LRCA's home base and the base from which it departs.
- Base T: The tanker's departure base.
- Base R: The tanker's recovery base.
- P: The point along the LRCA's flight path where refueling occurs.
- d_t: The minimum distance from base T to the LRCA flight path.
- d_R: The minimum distance from base R to the LRCA flight path.
- r_t: The distance from L along the LRCA flight path where the distance to T is minimum.
- r_R: The distance from L along the LRCA flight path where the distance to R is minimum.

Figure 2 depicts these definitions from arbitrary locations of L, T, and R.

The problem this section treats is to find P such that the maximum range augmentation occurs. Several configurations for L, T, and R will be considered and the appropriate rules for selecting P derived.

We start by recalling that ΔF_a is the fuel onboard the tanker available to be loading into the LRCA. ΔF_a is clearly a function of the tanker's flight distance up to that point. Similarly, recall that W(r) is the gross weight of the LRCA after it has flown a distance r. The total fuel that the LRCA could accept at that distance is obviously $W_o - W(r)$. We have labeled this ΔF_b.

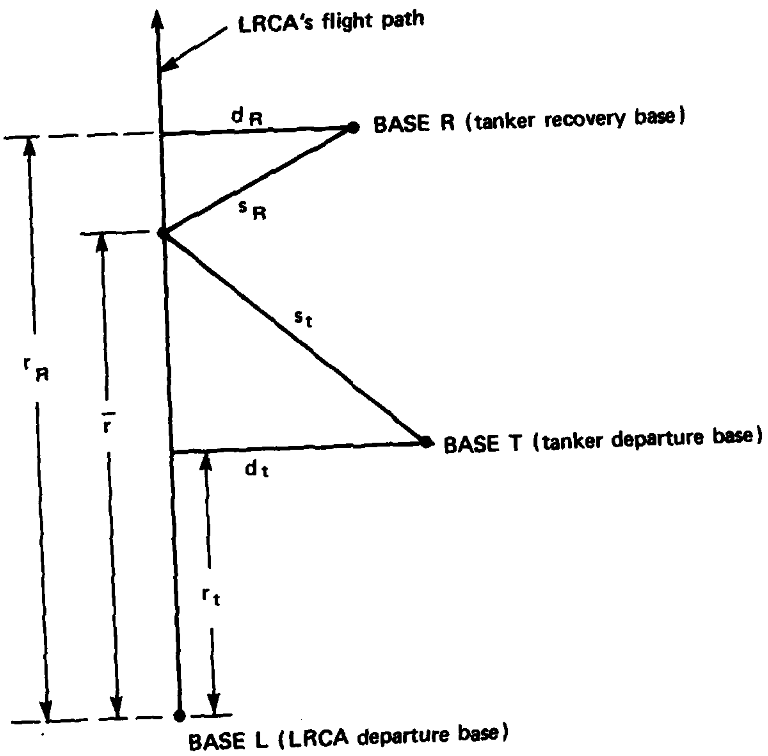

Fig. 2 — Geometry for LRCA/tanker basing

Assume for the moment that $\Delta F_a \leq \Delta F_b$. Then the entire available tanker fuel load, ΔF_a, could be transferred into the LRCA, increasing its weight to $W(r) + \Delta F_a$. Recalling Eq. (2.6), the new range to empty fuel tanks is

$$R_1 = K \log_e \left[\frac{W(r)+\Delta F_a}{W_e+W_a+W_p} \right] \tag{4.1}$$

and the total range obtained is simply the sum of the range to the refueling point and R_1, i.e., $r + R_1$. Recalling that R_o denotes the maximum <u>unrefueled</u> range, then the range augmentation, Δr_1, obtained by adding ΔF_a pounds of fuel at <u>P</u> is simply

$$\Delta r = r + R_1 - R_o \tag{4.2}$$

Clearly, we wish to find <u>P</u> such that Δr is a maximum.

Before we turn to maximizing Δr, however, consider the case where $\Delta F_a > \Delta F_b$. In this circumstance, only ΔF_b pounds of fuel can be loaded into the LRCA and its new gross weight would become its initial weight, W_o. Under these conditions, $R_1 = R_o$, i.e., the LRCA, fully fueled, is capable of flying its original unrefueled range, and the range augmentation, $\Delta r = r$. It is immediately evident that Δr grows with r. To maximize Δr, we should make r as large as possible. Therefore, we have our first refueling rule: If $\Delta F_a > \Delta F_b$, delay refueling, subject to the condition that the LRCA has sufficient fuel to keep flying.*

Now we return to Eq. (4.2). R_1 can be rewritten in the following form:

$$R_1 = K \log_e \left[\left(\frac{W_o}{W_e+W_a+W_p} \right) \left(\frac{W(r)+\Delta F_a}{W_o} \right) \right] \tag{4.3}$$

---

*We will discuss the consequence of this condition later.

or

$$R_1 = R_o + K \log_e\left(\frac{W(r)+\Delta F_a}{W_o}\right)$$

Therefore, Δr becomes simply

$$\Delta r = r + K \log_e\left(\frac{W(r)+\Delta F_a}{W_o}\right) \tag{4.4}$$

To determine the optimum refueling point, consider the derivative of Δr with respect to r. Three possibilities exist:

(1) $\frac{d}{dr}(\Delta r) < o$, for $o < r \leq R_o$, or

(2) $\frac{d}{dr}(\Delta r) > o$, for $o < r \leq R_o$, or

(3) $\frac{d}{dr}(\Delta r) = o$ at $r = \bar{r}$, $o < \bar{r} \leq R_o$*

If the first condition applies, then delaying refueling is bad and refueling should occur at the <u>earliest</u> feasible time. If the second condition applies, refueling should occur at the <u>latest</u> feasible time. If the third condition applies, refueling should occur at $\bar{r}$. To these three possibilities we add the two obvious constraints

$$\Delta F_b \geq \Delta F_a$$

$$\Delta F_b \leq W_o - \left(W_e + W_a + W_p\right)$$

The first constraint simply says that the fuel added cannot exceed the LRCA's capacity; the second that refueling occurs before the LRCA's fuel tanks run dry.

---

*It is possible that $\frac{d}{dr}(\Delta r) = o$ for all r. In this case, it does not matter where refueling occurs.

Consider Eq. (4.4). Then

$$\frac{d}{dr}(\Delta r) = 1 + \frac{K}{W(r)+\Delta F_a}\,\frac{d}{dr}\left(W(r)+\Delta F_a\right) \tag{4.5}$$

Recalling that $W(r) = W_o \exp\left\{-\frac{r}{K}\right\}$

$$\begin{aligned}\frac{d}{dr}(\Delta r) &= \frac{W(r) + \Delta F_a + K\left(-\frac{W_o}{K}\exp\left\{-\frac{r}{K}\right\} + \frac{d}{dr}\left(\Delta F_a\right)\right)}{W(r) + \Delta F_a} \\ &= \frac{\Delta F_a + K\,\frac{d}{dr}\left(\Delta F_a\right)}{W(r) + \Delta F_a}\end{aligned} \tag{4.6}$$

Since $W(r) + \Delta F_a$ is always positive, the sign of $\frac{d}{dr}(\Delta r)$ is simply the sign of $\Delta F_a + K\,\frac{d}{dr}(\Delta F_a)$.

At this point we note that the problem of deriving refueling rules has been reduced to an evaluation of the behavior of ΔF_a as a function of r. To go further, we need to specify ΔF_a. Consider Fig. 2. Given that P is the refueling point and recalling that ΔF_a is simply the total fuel available at P minus the fuel needed to return to base B, then

$$\begin{aligned}\Delta F_a &= W_o \exp\left\{-\frac{S_t}{K}\right\} - W_e \exp\left\{\frac{S_R}{K}\right\} \\ S_t &= \sqrt{d_t^2 + (r-r_t)^2} \\ S_R &= \sqrt{d_R^2 + (r-r_R)^2}\end{aligned} \tag{4.7}$$

Substituting Eqs. (4.7) into $\Delta F_a + K\,\frac{d}{dr}(\Delta F_a)$, and simplifying, we obtain

$$\Delta F_a + K\frac{d}{dr}(\Delta F_a) = W_o \exp\left\{-\frac{S_t}{K}\right\}\left[1 - \frac{(r-r_t)}{S_t}\right] - W_e \exp\left\{\frac{S_r}{K}\right\}\left[1 + \frac{(r-r_R)}{S_R}\right] \quad (4.8)$$

This transcendal equation is best evaluated by computer. Without proof, we state that all of the previously stated possibilities for $\frac{d}{dr}(\Delta r)$ can occur, depending on the locations of the various bases (L, A, and B) and specifying that the two constraints are met.

Some Specific LRCA/Tanker Basing Geometries

Certain specific basing geometries are more likely to be of concern than others. Therefore, we consider the following special cases:

Case Number	LRCA Departure Base	Tanker	
		Departure Base	Recovery Base
1	L	L	L
2	L	L	R ≠ L
3	L	L	Variable
4	L	T	T

In all cases, the LRCA departs from base L. In the first three of these special cases, the tanker is also assumed to depart from base L, flying with the LRCA in what we will term the buddy system. These cases differ by where the tankers recover. Case one assumes recovery at base L; tankers that depart from and return to the LRCA's home base are said to fly radius missions. Case two assumes that the recovery base R is not L. R is assumed to be a fixed site. Case three also assumes that the recovery base is not L. But this case assumes that the tanker can select from a number of potential bases such that the distance from the refueling point to the closest base is a constant s. Case three is clearly a simplification of case two. Case four assumes that the tanker leaves from and returns to the same base. In this case, however, that base is not the LRCA home base.

Case One: Tankers Fly Buddy-Buddy, Radius Missions. The equation for ΔF_a in this case is straightforward.

$$\Delta F_a = W_o \exp\left\{-\frac{r}{K}\right\} - W_e \exp\left\{\frac{r}{K}\right\} \tag{4.9}$$

Similarly

$$\Delta F_b = W_o \left[1 - \exp\left\{-\frac{r}{K}\right\}\right] \tag{4.10}$$

We now need to derive the correct refueling rules.

Recalling Eq. (4.4) and substituting for ΔF_a

$$\Delta r = r + K \log_e \left[2\exp\left\{-\frac{r}{K}\right\} - \left(\frac{W_e}{W_o}\right) \exp\left\{\frac{r}{K}\right\}\right]$$

Then

$$\frac{d}{dr}(\Delta r) = 1 + \frac{-2\exp\left\{-\frac{r}{K}\right\} - \left(\frac{W_e}{W_o}\right) \exp\left\{\frac{r}{K}\right\}}{2\exp\left\{-\frac{r}{K}\right\} - \left(\frac{W_e}{W_o}\right) \exp\left\{\frac{r}{K}\right\}}$$

$$= -\frac{2\left(\frac{W_e}{W_o}\right)\exp\left\{\frac{r}{K}\right\}}{2\exp\left\{-\frac{r}{K}\right\} - \left(\frac{W_e}{W_o}\right) \exp\left\{\frac{r}{K}\right\}}$$

Thus, for all r,

$$\frac{d}{dr}(\Delta r) \leq o$$

This inequality calls for refueling at the earliest time. However, at $r = o$, $\Delta F_a > \Delta F_b$. Therefore, until $\Delta F_a \leq \Delta F_b$, refueling should be delayed. Joining these two conditions, we obtain the proper rule for refueling, i.e.,

$$\Delta F_a = \Delta F_b \tag{4.11}$$

Substituting Eqs. (4.9) and (4.10) into (4.11),

$$W_o\left[1 - \exp\left\{-\frac{\bar{r}}{K}\right\}\right] = W_o \exp\left\{-\frac{\bar{r}}{K}\right\} - W_e \exp\left\{\frac{\bar{r}}{K}\right\} \qquad 4.12)$$

where $\bar{r}$ is the distance from base L where this equation holds. To solve this equation, define the variable z

$$z = \exp\left\{\frac{\bar{r}}{K}\right\}$$

Substituting z into Eq. (4.12)

$$1 - z^{-1} = z^{-1} - \left(\frac{W_e}{W_o}\right) z$$

or (4.13)

$$\left(\frac{W_e}{W_o}\right) z^2 + z - 2 = 0$$

Solving for z

$$z = \frac{-1+\sqrt{1+8\left(\frac{W_e}{W_o}\right)}}{2\left(\frac{W_e}{W_o}\right)} \tag{4.14}$$

which gives us $\bar{r}$, i.e., $\bar{r} = K \log_e(z)$. $\bar{r}$ is obviously equal to Δr (the maximum range enhancement) since the combat aircraft is fully fuelled at $\bar{r}$.

<u>Case Two: Tankers Fly Buddy Mission, Recover at Forward Base.</u>
For this case

$$\Delta F_a = W_o \exp\left\{-\frac{r}{K}\right\} - W_e \exp\left\{\frac{S_R}{K}\right\} \tag{4.15}$$

where S_R is given in Eq. (4.7). Substituting Eq. (4.15) into Eq. (4.4)

$$\Delta r = r + K \log_e \left[2\exp\left\{-\frac{r}{K}\right\} - \left(\frac{W_e}{W_o}\right) \exp\left\{\frac{S_R}{K}\right\}\right]$$

Thus

$$\frac{d}{dr}(\Delta r) = 1 + \frac{-2\exp\left\{-\frac{r}{K}\right\} - \left(\frac{W_e}{W_o}\right) \exp\left\{\frac{S_R}{K}\right\}\left(\frac{dS_R}{dr}\right)}{2\exp\left\{-\frac{r}{K}\right\} - \left(\frac{W_e}{W_o}\right) \exp\left\{\frac{S_R}{K}\right\}}$$

$$= \frac{-\left(\frac{W_e}{W_o}\right) \exp\left\{\frac{S_R}{K}\right\}\left[1 + \frac{dS_R}{dr}\right]}{2\exp\left\{-\frac{r}{K}\right\} - \left(\frac{W_e}{W_o}\right) \exp\left\{\frac{S_R}{K}\right\}}$$

Recalling Eq. (4.7) for S_R

$$\frac{dS_R}{dr} = \frac{(r-r_R)}{S_R}$$

Assuming that $d_R \neq 0$, $\frac{dS_R}{dr} > -1$. Therefore

$$\frac{d}{dr}(\Delta r) < 0$$

This inequality, coupled with the fact that for small r $\Delta F_a > \Delta F_b$ leads to the same refueling rules, i.e., refuel when

$$\Delta F_a = \Delta F_b \tag{4.16}$$

Recalling ΔF_b and substituting Eq. (4.15) into (4.16), we can solve for $\bar{r}$, the optimum distance at which refueling occurs.

$$W_o\left[1 - \exp\left\{-\frac{\bar{r}}{K}\right\}\right] = W_o\exp\left\{-\frac{\bar{r}}{K}\right\} - W_e\exp\left\{\frac{S_R}{K}\right\} \tag{4.17}$$

Substituting for S_R

$$2\exp\left\{-\frac{\bar{r}}{K}\right\} - 1 = \left(\frac{W_e}{W_o}\right)\exp\left\{\frac{\sqrt{d_R^2 + (\bar{r}-r_R)^2}}{K}\right\} \tag{4.18}$$

Equation (4.18) cannot be easily simplified. Solutions for $\bar{r}$ are best done numerically.

<u>Case Three: Tankers Fly Buddy Mission, Recover at an Unspecified Forward Base s Miles from the Refueling Point</u>. We assume s = constant. Therefore, the refueling conditions of Case Two apply. Equation (4.18) simplifies to

$$2\exp\left\{-\frac{\bar{r}}{K}\right\} - 1 = \left(\frac{W_e}{W_o}\right)\exp\left\{\frac{s}{K}\right\} \tag{4.19}$$

The solution for $\bar{r}$ is straightforward

$$\bar{r} = K\log_e\left[\frac{2}{1 + \left(\frac{W_e}{W_o}\right)\exp\left\{\frac{s}{K}\right\}}\right] \tag{4.20}$$

<u>Case Four: Tankers Depart from and Return to a Forward Base</u>. This case is a slight simplification of the general case. If we set $S_R = S_t$ in Eq. (4.7), then

$$\Delta F_a = W_o \exp\left\{-\frac{S_t}{K}\right\} - W_e \exp\left\{\frac{S_t}{K}\right\} \tag{4.21}$$

and

$$\Delta r = r + K \log_e\left[\exp\left\{-\frac{r}{K}\right\} + \exp\left\{-\frac{S_t}{K}\right\} - \left(\frac{W_e}{W_o}\right)\exp\left\{\frac{S_t}{K}\right\}\right] \tag{4.22}$$

Thus

$$\frac{d}{dr}(\Delta r) = 1 - \frac{\exp\left\{-\frac{r}{K}\right\} + \left[\exp\left\{-\frac{S_t}{K}\right\} + \left(\frac{W_e}{W_o}\right)\exp\left\{\frac{S_t}{K}\right\}\right]\frac{dS_t}{dr}}{\exp\left\{-\frac{r}{K}\right\} + \exp\left\{-\frac{S_t}{K}\right\} - \left(\frac{W_e}{W_o}\right)\exp\left\{\frac{S_t}{K}\right\}}$$

$$= \frac{\exp\left\{-\frac{S_t}{K}\right\}\left[1 - \frac{dS_t}{dr}\right] - \left(\frac{W_e}{W_o}\right)\exp\left\{\frac{S_t}{K}\right\}\left[1 + \frac{dS_t}{dr}\right]}{\exp\left\{-\frac{r}{K}\right\} + \exp\left\{-\frac{S_t}{K}\right\} - \left(\frac{W_e}{W_o}\right)\exp\left\{\frac{S_t}{K}\right\}}$$

Since the denominator is positive for all admissible r ($o < r \leq R_o$), the sign of $\frac{d}{dr}(\Delta r)$ depends only on the numerator. If we substitute for S_t,

$$\frac{d}{dr}(\Delta r) = \frac{\exp\left\{-\frac{S_t}{K}\right\}\left[1 - \frac{(r-r_t)}{S_t}\right] - \left(\frac{W_e}{W_o}\right)\exp\left\{\frac{S_t}{K}\right\}\left[1 + \frac{(r-r_t)}{S_t}\right]}{\exp\left\{-\frac{r}{K}\right\} + \exp\left\{-\frac{S_t}{K}\right\} - \left(\frac{W_e}{W_o}\right)\exp\left\{\frac{S_t}{K}\right\}} \tag{4.23}$$

Unfortunately no obvious simplifications of Eq. (4.23) are possible. If we set Eq. (4.23) equal to zero, we obtain the expression

$$\left[\frac{S_t - (\bar{r}-r_t)}{S_t + (\bar{r}-r_t)}\right] \exp\left\{-\frac{2S_t}{K}\right\} = \left(\frac{W_e}{W_o}\right) \qquad (4.24)$$

where $\bar{r}$ is the solution. In general, there is no reason to believe that a solution exists. If it does, we have the following refueling rules

(1) If $\Delta F_a(\bar{r}) > \Delta F_b(\bar{r})$, refuel when $\Delta F_a = \Delta F_b$

(2) If $\Delta F_b(\bar{r}) > W_o - (W_e+W_a+W_p)$, refuel at R_o

(3) Otherwise, refuel when $r = \bar{r}$

If no solution exists, then either $\frac{d}{dr}(\Delta r)$ is always positive or always negative, and the following refueling conditions apply

(4) If $\frac{d}{dr}(\Delta r) < o$ for all r, refuel when $\Delta F_a = \Delta F_b$

(5) If $\frac{d}{dr}(\Delta r) > o$ for all r, refuel at R_o

The last of these conditions is self evident; refuel at the last possible refueling point. That obviously occurs when the LRCA's fuel tanks are empty, i.e., when the LRCA reaches its maximum unrefueled range, R_o. Rule (4) is simply a condition that arose in our earlier cases.

One simplification of this case can be made by assuming that S_t is independent of r, i.e., equals a constant. In this case, $\frac{d}{dr}(\Delta F_a) = o$ and $\frac{d}{dr}(\Delta r)$ is always positive. Thus refuel at R_o.

To obtain Δr, recall Eq. (4.22) and substitute s (= constant) for S_t. Thus

$$\Delta r = R_o + K \log_e\left[\exp\left\{-\frac{R_o}{K}\right\} + \exp\left\{-\frac{s}{K}\right\} - \left(\frac{W_e}{W_o}\right)\exp\left\{\frac{s}{K}\right\}\right]$$

Recall also that

$$R_o = K \log_e \left[\frac{W_o}{W_e+W_a+W_p}\right]$$

Thus

$$\Delta r = K \log_e \left[1 + \frac{\left(W_e+W_a+W_p\right)}{W_o}\left(\exp\left\{-\frac{s}{K}\right\} - \left(\frac{W_e}{W_o}\right)\exp\left\{\frac{s}{K}\right\}\right)\right] \qquad (4.25)$$

Figure 3 shows the relative range enhancement $\left(\frac{\Delta r}{K}\right)$ as a function of $\left(\frac{W_e}{W_o}\right)$ for cases 1 and 3. Realistic values for $\left(\frac{W_e}{W_o}\right)$ probably exceed 0.4 for most normal aircraft designs. Therefore the range enhancement gains relative to the Breguet range factor are about 0.3.

Multiple Refuelings for a Single LRCA/Tanker Pair

Although the constraint that refueling must occur before the LRCA runs out of fuel was stated, so far it has not entered into our derivations. In general, if $\frac{d}{dr}(\Delta r) > o$ for all r and $\Delta F_a(R_o) \leq \Delta F_b(R_o)$, then a single refueling at R_o is optimum. However, what should be done if the $\Delta F_a > \Delta F_b$ at R_o. From above we know that refueling should occur at R_o. But in contrast to the above, the tanker will have excess fuel available after that refueling. Therefore a second refueling is possible. To be determined are (1) how much fuel should be passed on the first refueling, (2) when should the second refueling occur, and (3) how much range augmentation is possible.

We shall restrict our attention to those cases where the tanker and the LRCA fly buddy missions, i.e., both depart from base L. For the time being, we also assume that at R_o the LRCA is fully refueled (we will test later whether this assumption leads to optimum range enhancements).

For the LRCA: $W_L(R_o) = \frac{W_e+W_a+W_p}{W_o}$ (before refueling)

$W_L(R_o) = W_o$ (after refueling)

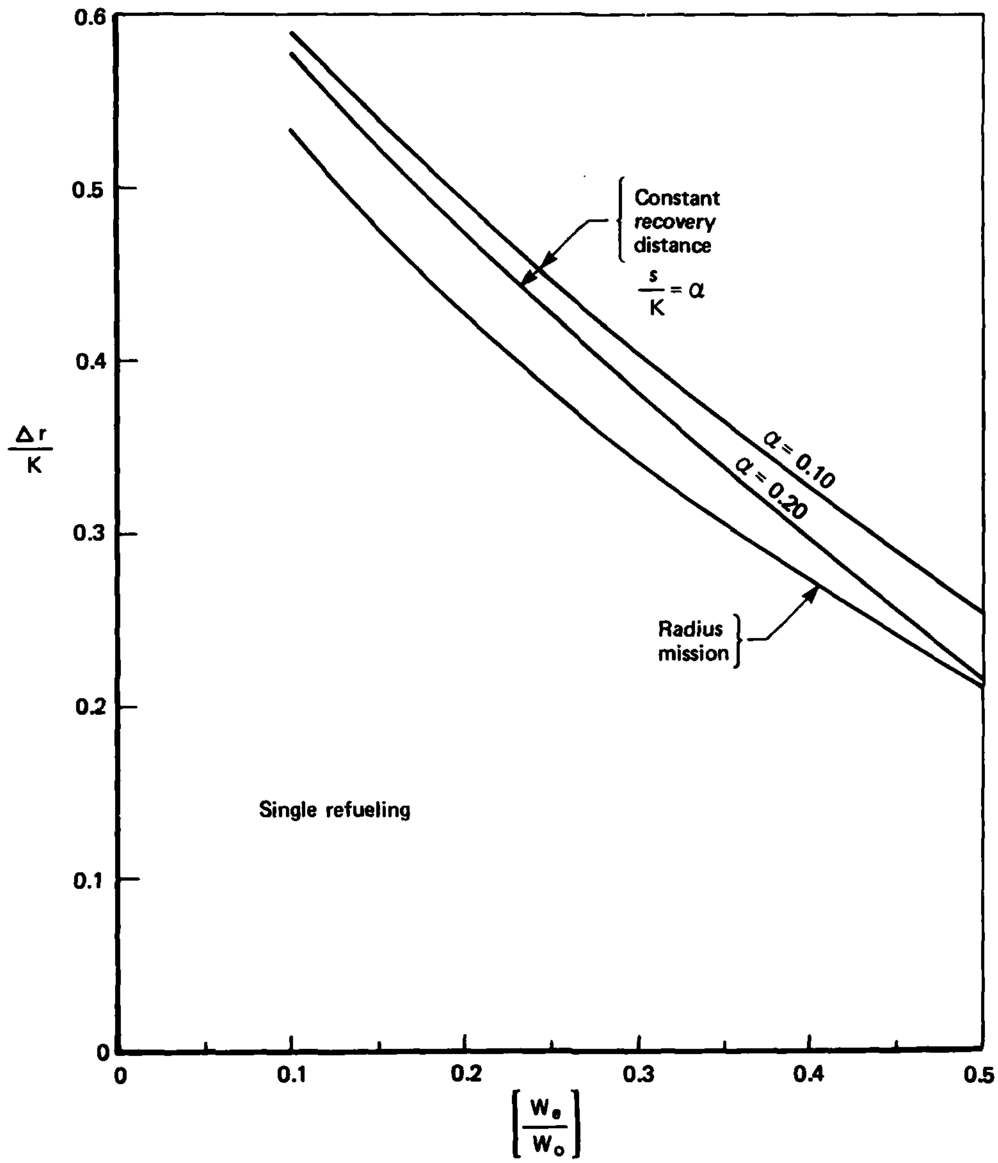

Fig. 3 — Range augmentation from a single refueling

<u>For the tanker</u>: $W_T(R_o) = W_o \exp\left\{-\frac{R_o}{K}\right\}$ (before refueling)

$$W_T(R_o) = W_o \exp\left\{-\frac{R_o}{K}\right\} - F_b \tag{4.26}$$

$$= W_o\left[2\exp\left\{-\frac{R_o}{K}\right\} - 1\right] \quad \text{(after refueling)}$$

We have used the subscripts L and T to differentiate the LRCA and tanker equations.

By assumption, the LRCA, fully fueled, continues along its outbound path. We assume that the tanker does likewise, still flying in formation with the LRCA. At some point a second refueling should occur. To determine that point, we need to be specific about which case we are considering.

<u>Case One: Tanker Radius Missions</u>. The equation for tanker fuel availability is as before

$$\Delta F_a = W_T(r) - W_e \exp\left\{\frac{r}{K}\right\} \qquad r > R_o \tag{4.27}$$

where $W_T(r)$ is the tanker gross weight at total range r. From Breguet's formula

$$W_T(r) = W_T(R_o)\exp\left\{-\frac{r-R_o}{K}\right\} \tag{4.28}$$

where the first fuel transfer has occurred at R_o. Therefore,

$$\Delta F_a = W_o\left[2\exp\left\{-\frac{R_o}{K}\right\} - 1\right]\exp\left\{-\frac{r-R_o}{K}\right\} - W_e \exp\left\{\frac{r}{K}\right\} \tag{4.29}$$

Similarly

$$\Delta F_b = W(o)\left[1 - \exp\left\{- \frac{r-R_o}{K}\right\}\right] \qquad (4.30)$$

Because the refueling conditions are identical with the earlier Case One, the optimum <u>second</u> refueling point corresponds to $\Delta F_b = \Delta F_a$. Thus

$$\left[2 \exp\left\{- \frac{R_o}{K}\right\} - 1\right]\exp\left\{- \frac{\bar{r}-R_o}{K}\right\} - \left(\frac{W_e}{W_o}\right)\exp\left\{\frac{\bar{r}}{K}\right\} = 1 - \exp\left\{- \frac{\bar{r}-R_o}{K}\right\} \qquad (4.31)$$

Setting $z = \exp\left\{\frac{\bar{r}}{K}\right\}$

$$\left[2 \exp\left\{- \frac{R_o}{K}\right\} - 1\right]\exp\left\{\frac{R_o}{K}\right\}z^{-1} - \left(\frac{W_e}{W_o}\right)z = 1 - \exp\left\{\frac{R_o}{K}\right\}z^{-1}$$

or

$$\left[2 = \exp\left\{\frac{R_o}{K}\right\}\right]z^{-1} - \left(\frac{W_e}{W_o}\right)z - 1 + \exp\left\{\frac{R_o}{K}\right\}z^{-1} = 0$$

or

$$\left(\frac{W_e}{W_o}\right)z^2 + z - 2 = 0 \qquad (4.32)$$

which leads to Eq. (4.14). Therefore, for this case, the intermediate refueling at R did not alter the total range obtained by optimally using a single dedicated tanker.

<u>Case 3: Constant Recovery Distance</u>. As before

$$\Delta F_a = W_T(r) - W_e\exp\left\{\frac{s}{K}\right\} \qquad r > R_o \qquad (4.33)$$

The remaining equations shown in Case 1 are unaltered. Thus $\Delta F_a = \Delta F_b$ yields

$$\left[2 \exp\left\{-\frac{R_o}{K}\right\}\right] - 1 \exp\left\{-\frac{\bar{r}-R_o}{K}\right\} - \left(\frac{W_e}{W_o}\right)\exp\left\{\frac{s}{K}\right\} = 1 - \exp\left\{-\frac{r-R_o}{K}\right\} \qquad (4.34)$$

or

$$2 \exp\left\{-\frac{\bar{r}}{K}\right\} - \left(\frac{W_e}{W_o}\right)\exp\left\{\frac{s}{K}\right\} - 1 = 0 \qquad (4.35)$$

This is identical to Eq. (4.19). As above, the intermediate refueling at R_o did not alter the total range obtained by optimally using a dedicated tanker.

By implication, multiple mid-course refuelings would also not change these results. Furthermore, since we have not used the conditions describing the combat aircraft being empty of fuel (we just stated it as occurring at some range R_o), this observation applies to any mid-course refueling, so long as that refueling fills the combat aircraft.

Finally, we wish to test the assumption that fully fueling the LRCA at R_o was the correct rule. Therefore, consider a lesser refueling Δf, i.e.,

$$\Delta f < \Delta F_b$$

Then after refueling

$$\begin{aligned} W_L(R_o) &= W_o \exp\left\{-\frac{R_o}{K}\right\} + \Delta f \\ W_T(R_o) &= W_o \exp\left\{-\frac{R_o}{K}\right\} - \Delta f \end{aligned} \qquad (4.36)$$

Assuming that Δf was sufficiently large to permit $\Delta F_a = \Delta F_b$ before the LRCA runs out of fuel for the second time,

$$\begin{aligned} \Delta F_a &= W_T(R_o)\exp\left\{-\frac{r-R_o}{K}\right\} - W_e\exp\left\{\frac{r}{K}\right\} && r > R_o \\ \Delta F_b &= \left[W_o - W_L(R_o)\right] + W_L(R_o)\left[1 - \exp\left\{-\frac{r-R_o}{K}\right\}\right] && r > R_o \end{aligned} \tag{4.37}$$

Equating ΔF_a and ΔF_b, and substituting Eq. (4.36)

$$\begin{aligned} &\left[W_o\exp\left\{-\frac{R_o}{K}\right\} - \Delta f\right]\exp\left\{-\frac{\bar{r}-R_o}{K}\right\} - W_e\exp\left\{\frac{\bar{r}}{K}\right\} \\ &\qquad = W_o - \left[W_o\exp\left\{-\frac{R_o}{K}\right\} + \Delta f\right]\exp\left\{-\frac{\bar{r}-R_o}{K}\right\} \end{aligned} \tag{4.38}$$

This equation simplifies to

$$\begin{aligned} &\exp\left\{-\frac{\bar{r}}{K}\right\} - \Delta f\exp\left\{-\frac{r-R_o}{K}\right\} - \left(\frac{W_e}{W_o}\right)\exp\left\{\frac{\bar{r}}{K}\right\} = \\ &\qquad 1 - \exp\left\{-\frac{\bar{r}}{K}\right\} - \Delta f\exp\left\{-\frac{r-R_o}{K}\right\} \end{aligned} \tag{4.39}$$

These terms involving Δf cancel and we are left with

$$\left(\frac{W_e}{W_o}\right)z^2 + z - 2 = 0$$

This equation is identical with Eq. (4.15). Therefore we conclude that it doesn't matter how much fuel is transferred prior to $\bar{r}$. Therefore all previous equations and rules pertain.

V. RANGE ENHANCEMENT FROM TWO OR MORE TANKERS PER COMBAT AIRCRAFT

The derivation for two or more tankers per combat aircraft follows the previous derivation for a single aircraft. We assume the buddy refueling approach, i.e., both the LRCA and the tankers depart from the same base. For simplicity we will derive equations only for the constant recovery range case, but will state the equations for the radius missions as well. We assert without proof that optimal refueling points still correspond to $\Delta F_b = \Delta F_a$, where ΔF_a relates to a specific tanker. Therefore each successive refueling fills the combat aircraft and empties the tanker except for the fuel needed to recover to base.

Let $\bar{r}_i$ be the <u>total</u> LRCA range enhancement obtained from i tankers. At $\bar{r}_{i-1}$

$$W_L(\bar{r}_{i-1}) = W(o) \qquad \text{(LRCA)} \tag{5.1}$$

$$W_T(\bar{r}_{i-1}) = W(o)\exp\left\{-\frac{\bar{r}_{i-1}}{K}\right\} \qquad (\underline{\text{next}}\text{ tanker weight}) \tag{5.2}$$

Thus, for the next tanker,

$$\Delta F_a = W(o)\exp\left\{-\frac{r}{K}\right\} - W_e\exp\left\{\frac{s}{K}\right\} \tag{5.3}$$

and for the LRCA,

$$\Delta F_b = W(o)\left[1 - \exp\left\{-\frac{r-\bar{r}_{i-1}}{K}\right\}\right] \qquad r \geq \bar{r}_{i-1} \tag{5.4}$$

where we impose the assumption that refueling times are negligible.* Equating ΔF_a and ΔF_b

*If refueling times are <u>not</u> negligible (and in reality they would not be), there exists a minimum distance that the LRCA flies from the start of one refueling to the next. Thus $r_i - r_{i-1}$ must be greater than or equal to that minimum distance.

$$\exp\left\{-\frac{\overline{r}_i}{K}\right\} - \left(\frac{W_e}{W_o}\right)\exp\left\{\frac{s}{K}\right\} = 1 - \exp\left\{\frac{\overline{r}_{i-1}}{K}\right\}\exp\left\{-\frac{\overline{r}_i}{K}\right\} \tag{5.5}$$

Solving for $\overline{r}_i$,

$$\overline{r}_i = K \log_e \left[\frac{1 + \exp\left\{\frac{\overline{r}_{i-1}}{K}\right\}}{1 + \left(\frac{W_e}{W_o}\right)\exp\left\{\frac{s}{K}\right\}}\right] \tag{5.6}$$

Eq. (5.6) is obviously a more general version of Eq. (4.20).

It is possible to solve for $\overline{r}_{i-1}$ and state $\overline{r}_i$ solely as a function of the aircraft design characteristics. We do this by induction. Define the dominator in Eq. (5.6) by D, i.e.,

$$D = 1 + \left(\frac{W_e}{W_o}\right)\exp\left\{\frac{s}{K}\right\} \tag{5.7}$$

Then, substituting into Eq. (5.6),

$$\exp\left\{\frac{r_1}{K}\right\} = \frac{2}{D} \qquad (r_o = o)$$

$$\exp\left\{\frac{r_2}{K}\right\} = \frac{1 + \exp\left\{\frac{r_1}{K}\right\}}{D} = \frac{1 + \frac{2}{D}}{D} = \frac{D + 2}{D^2}$$

$$\exp\left\{\frac{r_3}{K}\right\} = \frac{1 + \exp\left\{\frac{r_2}{K}\right\}}{D} = \frac{1 + \frac{D+2}{D^2}}{D} = \frac{D^2 + D + 2}{D^3}$$

Observing the pattern, assume that

$$\exp\left\{\frac{r_n}{K}\right\} = \frac{D^{n-1} + D^{n-2} + \cdots + D + 2}{D^n} \tag{5.8}$$

To prove that this is the correct expression, we only need to show that if it holds for r_n, then it is also true for r_{n+1}. Thus

$$\exp\left\{\frac{r_{n+1}}{K}\right\} = \frac{1 + \exp\left\{\frac{r_n}{K}\right\}}{D} = \frac{D^n + D^{n-1} + \cdots + D + 2}{D^{n+1}} \quad \text{Q.E.D.} \quad (5.9)$$

and the proof is complete.

Substituting for D, we therefore obtain

$$\overline{r}_n = K \log_e \left[\frac{\displaystyle\sum_{i=0}^{n-1} \left(1 + \left(\frac{W_e}{W_o}\right) \exp\left\{\frac{s}{K}\right\}\right)^i + 1}{\left(1 + \left(\frac{W_e}{W_o}\right) \exp\left\{\frac{s}{K}\right\}\right)^n} \right] \quad (5.10)$$

If we note that the right hand side of Eq. (5.8) satisfies the relationship

$$\frac{D^{n-1} + D^{n-2} + \cdots + D + 2}{D^n} = \left(\frac{1}{D}\right) + \left(\frac{1}{D}\right)^2 + \cdots + \left(\frac{1}{D}\right)^{n-1} + 2\left(\frac{1}{D}\right)^n$$

and recall that

$$1 + x + x^2 + \cdots + x^{n-1} = \frac{1-x^n}{1-x} \qquad x < 1$$

then

$$\left(\frac{1}{D}\right) + \left(\frac{1}{D}\right)^2 + \cdots + \left(\frac{1}{D}\right)^{n-1} + 2\left(\frac{1}{D}\right)^n$$

$$= \left(\frac{1}{D}\right)\left[\frac{1 - \left(\frac{1}{D}\right)^n}{1 - \left(\frac{1}{D}\right)}\right] + \left(\frac{1}{D}\right)^n = \frac{\left(\frac{1}{D}\right) + \left(\frac{1}{D}\right)^n - 2\left(\frac{1}{D}\right)^{n+1}}{1 - \left(\frac{1}{D}\right)}$$

Thus

$$\exp\left\{\frac{\bar{r}_n}{K}\right\} = \frac{\left(\frac{1}{D}\right) + \left(\frac{1}{D}\right)^n - 2\left(\frac{1}{D}\right)^{n+1}}{1 - \left(\frac{1}{D}\right)} = \frac{D^n + D - 2}{D^n(D-1)} \tag{5.11}$$

or, substituting D,

$$r_n = K \log_e \left\{ \frac{\left[1 + \left(\frac{W_e}{W_o}\right)\exp\left\{\frac{s}{K}\right\}\right]^n + \left(\frac{W_e}{W_o}\right)\exp\left\{\frac{s}{K}\right\} - 1}{\left[1 + \left(\frac{W_e}{W_o}\right)\exp\left\{\frac{s}{K}\right\}\right]^n \left(\frac{W_e}{W_o}\right)\exp\left\{\frac{s}{K}\right\}} \right\} \tag{5.12}$$

In the limit as $n \to \infty$, $\left(\frac{1}{D}\right)^{n-1} \to 0$, and

$$\exp\left\{\frac{r_\infty}{K}\right\} = \frac{1}{D-1} = \left(\frac{W_e}{W_o}\right)\exp\left\{-\frac{s}{K}\right\} \tag{5.13}$$

r_∞ represents the absolute maximum LRCA range augmentation obtainable for tankers on radius missions. Not surprisingly, this also corresponds to the range for the tanker where ΔF_a equals zero.

The expression for case 1 (tanker radius missions) can be derived in a similar fashion. If we define

$$z_i = \exp\left\{\frac{\bar{r}_i}{K}\right\}$$

then

$$z_i = \frac{-1 + \sqrt{1 + 4\left(\frac{W_e}{W_o}\right)\left[1 + z_{i-1}\right]}}{2\left(\frac{W_e}{W_o}\right)} \tag{5.14}$$

No simple expression has been found that permits the reduction of this recursive relationship to an expression that is solely a function of the aircraft's design characteristics.

Figure 4 shows the total range enhancements achieved with multiple tankers for the two cases above. By seven refuelings the added range enhancement becomes quite small. These figures also indicate the maximum range extension obtainable with an unlimited number of tankers, assuming that they fly either a radius mission (out and back to the same base) or fly a constant distance (1000 n mi) after refueling. The advantages of being able to recover at a forward base (the constant distance case) over returning to home base are obvious.

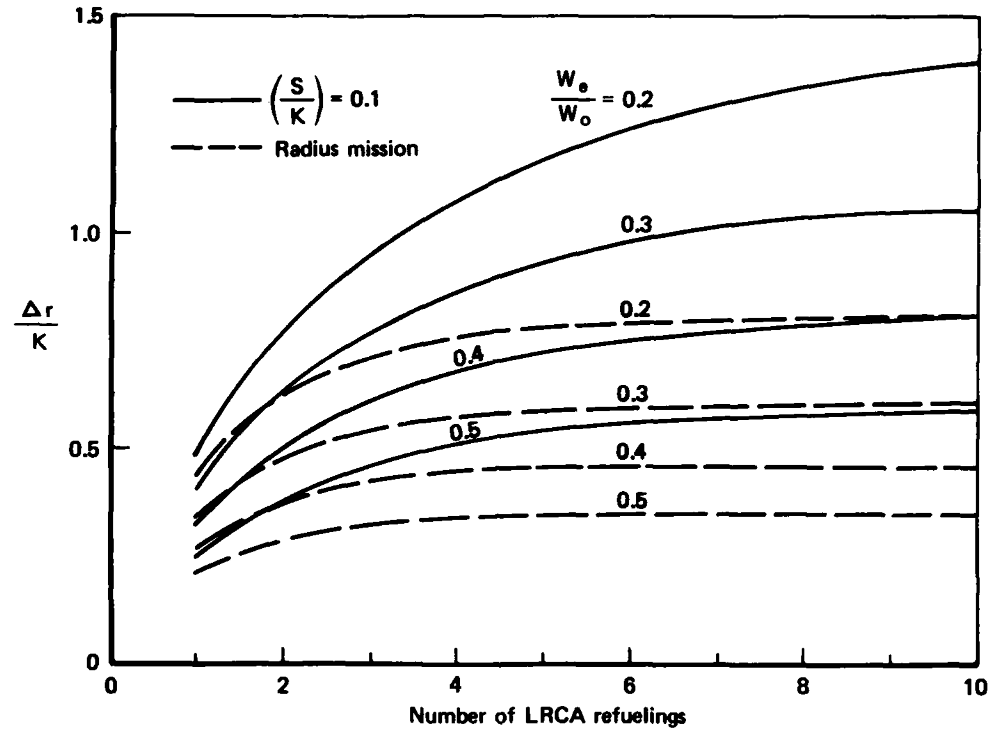

Fig. 4 — Range augmentation from n refuelings, using the buddy-buddy tactic

VI. RANGE ENHANCEMENTS WHERE ONE TANKER SUPPORTS TWO COMBAT AIRCRAFT FLYING THE BUDDY TACTIC

One tanker may have enough fuel to provide two or more combat aircraft with sufficient range extension to carry out the prescribed mission. This section will discuss range enhancements for two LRCA per tanker and will assume a variant of the buddy tactic. The following section will discuss multiple LRCAs per tanker using a different refueling tactic (the filling station approach).

We assume that the tanker and the two combat aircraft leave their common base simultaneously and fly in formation until both combat aircraft are refueled. The recovery conditions are Cases One and Three discussed in Section IV. As was true in the prior section, we will derive the equations for the constant recovery distance case, and simply state the equations for the other case. The question to be answered is when should each combat aircraft be refueled? We shall treat three cases: (1) the range augmentation is equal for both combat aircraft, (2) both LRCA have their tanks completely filled, and (3) the tanker offloads equal fuel loads into both aircraft. These cases will produce different range enhancements for each aircraft, different average range enhancements for both, and different refueling locations.

We start with some necessary notation. Let r_1 and r_2 be the two refueling distances from the base. In all cases the first refueling will fully refuel the first combat airplane; therefore the range augmentation provided that airplane, Δr_1, is simply equal to r_1. It is not true that the second airplane will in all cases be fully refueled. Therefore, Δr_2, its range augmentation, is not necessarily equal to r_2. Consider Fig. 5.

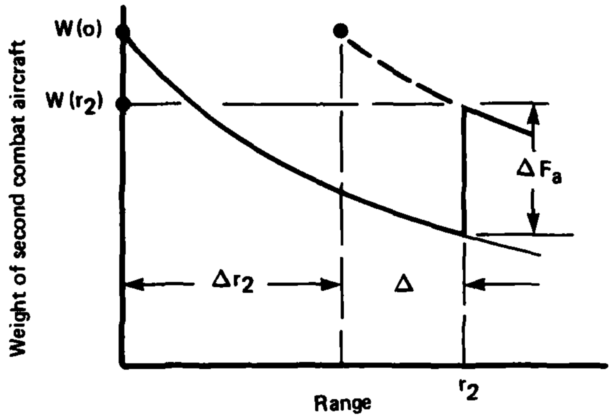

Fig. 5 — Aircraft gross weight versus range with refueling

The range augmentation, Δr_2, satisfies the relation

$$\Delta r_2 = r_2 - \Delta \tag{6.1}$$

where Δ is the range lost because the LRCA was not fully fueled and satisfies the equation

$$W(r_2) = W(o)\exp\left\{-\frac{\Delta}{K}\right\} \qquad \text{(Breguet's Equation)} \tag{6.2}$$

Therefore

$$\Delta r_2 = r_2 - K \log_e\left|\frac{W(o)}{W(r_2)}\right| \tag{6.3}$$

Assume that the distance between the first and second refuelings is x.* Thus $r_2 = r_1 + x$. Substituting into Eq. (6.3)

---

*We will show later that x should be as small as possible for maximum performance. Clearly the minimum distance between completion of refuelings is fixed by the mechanics of disengaging from one aircraft, coupling with the second and transferring the fuel load.

$$\Delta r_2 = r_1 + x - K \log_e \left| \frac{W(o)}{W(r_2)} \right|$$

The average range enhancement, $\overline{\Delta r}$, is

$$\overline{\Delta r} = 1/2(\Delta r_1 + \Delta r_2) = \Delta r_1 + \frac{x}{2} - \frac{K}{2} \log_e \left| \frac{W(o)}{W(r_2)} \right| \tag{6.4}$$

and, as noted, the refueling distances are r_1 and r_2 (= $r_1 + x$).*

<u>Assumption 1: Range augmentation is equal for both combat aircraft.</u>

In this case, $\Delta r_1 = \Delta r_2$. Therefore

$$\Delta = x \tag{6.5}$$

or

$$W(r_2) = W(o)\exp\left\{- \frac{x}{K}\right\} \tag{6.6}$$

Recall that

$$W(r_2) = W(o)\exp\left\{- \frac{r_2}{K}\right\} + \Delta F_a(r_2) \tag{6.7}$$

To determine r_2 we must first determine $\Delta F_a(r_2)$.

The tanker gross weight immediately after the first refueling is

$$W_T(r_1) = W(o)\exp\left\{- \frac{r_1}{K}\right\} - \Delta F_a(r_1)$$

As noted $\Delta F_a(r_1) = \Delta F_b(r_1)$. Also, as before, $\Delta F_b = W(o)\left|1 - \exp\left\{- \frac{r}{K}\right\}\right|$. Therefore

---

*Because not all SIOP mission lengths are the same, the average range enhancement may be a reasonable measure of performance gain.

$$W_T(r_1) = W(o)\left[2\ \exp\left\{-\frac{r_1}{K}\right\} - 1\right] \tag{6.8}$$

The tanker fuel available at r_2 is simply

$$\begin{aligned} \Delta F_a(r_2) &= W_T(r_1)\exp\left\{-\frac{r_2-r_1}{K}\right\} - W_e\exp\left\{\frac{s}{K}\right\} \\ &= W(o)\left[2\ \exp\left\{-\frac{r_1}{K}\right\} - 1\right]\exp\left\{-\frac{x}{K}\right\} - W_e\exp\left\{\frac{s}{K}\right\} \end{aligned} \tag{6.9}$$

Combining Eqs. (6.6), (6.7), and (6.9), we obtain

$$\begin{aligned} \exp\left\{-\frac{x}{K}\right\} = \exp\left\{-\frac{r_1}{K}\right\}\exp\left\{-\frac{x}{K}\right\} + \left[2\ \exp\left\{-\frac{r_1}{K}\right\} - 1\right]\exp\left\{-\frac{x}{K}\right\} \\ - \left(\frac{W_e}{W_o}\right)\exp\left\{\frac{s}{K}\right\} \end{aligned} \tag{6.10}$$

Solving for r_1,

$$3\ \exp\left\{-\frac{r_1}{K}\right\} = 2 + \left(\frac{W_e}{W_o}\right)\exp\left\{\frac{x+s}{K}\right\} \tag{6.11}$$

which leads to

$$r_1 = \Delta r_1 = \overline{\Delta r} = K\ \log_e\left[\frac{3}{2 + \left(\frac{W_e}{W_o}\right)\exp\left\{\frac{x+s}{K}\right\}}\right] \tag{6.12}$$

Equation (6.12) gives the average range enhancement which, in this case, also represents the range enhancement for both aircraft.

The equations for tankers that must recover to their home base (i.e., radius missions, or Case 1) are obtained by substituting r_2 for s in Eq. (6.10). Thus

$$3\ \exp\left\{-\frac{r_1}{K}\right\} = 2 + \left(\frac{W_e}{W_o}\right)\exp\left\{\frac{2x}{K}\right\}\exp\left\{\frac{r_1}{K}\right\} \tag{6.13}$$

Solving for r_1,

$$r_1 = \Delta r_1 = K \log_e \left[\frac{-1 + \sqrt{1 + 3\left(\frac{W_e}{W_o}\right)\exp\left\{\frac{2x}{K}\right\}}}{\left(\frac{W_e}{W_o}\right)\exp\left\{\frac{2x}{K}\right\}} \right] \tag{6.14}$$

Careful examination of Eqs. (6.12) and (6.14) would show that the average range augmentation decreases as x increases. Table 1 shows the sensitivity of $\overline{\Delta r}$ for several values of $\frac{x}{K}$ and $\left(\frac{W_e}{W_o}\right)$.

<u>Assumption 2: The second refueling exactly fills the second combat aircraft.</u>

Thus $\Delta = 0$ and $\Delta r_2 = \Delta r_1 + x$. Also

$$\Delta F_a(r_2) = \Delta F_b(r_2) = W(o)\left[1 - \exp\left\{-\frac{r_2}{K}\right\}\right] \tag{6.15}$$

since the refueling exactly fills the second bomber. From Eq. (6.9)

$$\Delta F_a(r_2) = W(o)\left[2\exp\left\{-\frac{r_1}{K}\right\} - 1\right]\exp\left\{-\frac{x}{K}\right\} - W_e\exp\left\{\frac{s}{K}\right\}$$

Therefore

$$1 - \exp\left\{-\frac{r_1}{K}\right\}\exp\left\{-\frac{x}{K}\right\} = \left[2\exp\left\{-\frac{r_1}{K}\right\} - 1\right]\exp\left\{-\frac{x}{K}\right\} - \left(\frac{W_e}{W_o}\right)\exp\left\{\frac{s}{K}\right\} \tag{6.16}$$

Solving for r_1,

$$3\exp\left\{-\frac{r_1}{K}\right\}\exp\left\{-\frac{x}{K}\right\} = 1 + \exp\left\{-\frac{x}{K}\right\} + \left(\frac{W_e}{W_o}\right)\exp\left\{\frac{s}{K}\right\} \tag{6.17}$$

Table 1

RANGE AUGMENTATION: TWO LRCAs AND ONE TANKER
(Both Aircraft Achieve Same Total Range)

Spacing Between Refuelings (x/k)	Aircraft Dry Weight Fraction				Tanker Recovery Condition
	0.2	0.3	0.4	0.5	
0	.301	.252	.206	.162	Constant Distance = 0.1 x k
.05	.296	.245	.197	.150	
.10	.290	.237	.187	.139	
.15	.285	.229	.177	.127	
.20	.279	.221	.166	.115	
0	.281	.232	.189	.150	Radius Mission
.05	.276	.225	.181	.141	
.10	.270	.218	.172	.132	
.15	.265	.211	.164	.122	
.20	.259	.203	.155	.112	

or

$$r_1 = \Delta r_1 = K \log_e \left[\frac{3 \exp\left\{- \frac{x}{K}\right\}}{1 + \exp\left\{- \frac{x}{K}\right\} + \left(\frac{W_e}{W_o}\right) \exp\left\{\frac{s}{K}\right\}} \right] \tag{6.18}$$

Also

$$\Delta r_2 = r_1 + x \tag{6.19}$$

$$\overline{\Delta r} = r_1 + \frac{x}{2}$$

The equations for tankers flying radius missions are obtained by substituting r_2 (= r_1 + x) for s in Eq. (6.17). Careful algebra yields

$$\Delta r_1 = K \log_e \left[\frac{- \left(1 + \exp\left\{+ \frac{x}{K}\right\}\right) + \sqrt{\left(1 + \exp\left\{+ \frac{x}{K}\right\}\right)^2 + 12\left(\frac{W_e}{W_o}\right) \exp\left\{+ \frac{2x}{K}\right\}}}{2\left(\frac{W_e}{W_o}\right) \exp\left\{+ \frac{2x}{K}\right\}} \right] \tag{6.20}$$

Table 2 shows Δr_1 and Δr_2 for several values of $\frac{x}{K}$ and aircraft gross takeoff weight. Unlike the prior case, here $\Delta r_1 \neq \Delta r_2$ unless x = 0.

Assumption 3: Both combat aircraft receive equal fuel offloads.

Thus, since the first refueling fills the first combat aircraft,

$$\Delta F_a(r_2) = \Delta F_b(r_1) \tag{6.21}$$

From Eq. (6.9)

$$\Delta F_a(r_2) = W_o \left[2 \exp\left\{- \frac{r_1}{K}\right\} - 1 \right] \exp\left\{- \frac{x}{K}\right\} - W_e \exp\left\{\frac{s}{K}\right\} \tag{6.22}$$

Table 2

RANGE AUGMENTATION: TWO LRCAs AND ONE TANKER[a]
(Second LRCA Fully Refueled)

Spacing Between Refuelings (x/k)	Aircraft Dry Weight Fraction			
	0.2	0.3	0.4	0.5
CONSTANT RECOVERY DISTANCE s = 0.1 x k				
0	.301	.252	.206	.162
.05	.273/.323	.223/.273	.176/.226	.131/.181
.10	.244/.344	.194/.294	.146/.246	.099/.199
.15	.215/.365	.164/.314	.114/.264	.068/.218
.20	.186/.386	.133/.333	.083/.283	.035/.235
RADIUS MISSION				
0	.281	.232	.189	.150
.05	.251/.301	.200/.250	.156/.206	.116/.166
.10	.219/.319	.167/.267	.121/.221	.081/.181
.15	.187/.337	.134/.284	.087/.237	.045/.195
.20	.155/.355	.100/.300	.051/.251	.009/.209

[a]First number is $\frac{\Delta r_1}{K}$, the second is $\frac{\Delta r_2}{K}$.

$$\Delta F_b(r_1) = W(o)\left[1 - \exp\left\{-\frac{r_1}{K}\right\}\right] \tag{6.23}$$

Substituting into Eq. (6.21) and solving for r_1

$$\left[2\exp\left\{-\frac{r_1}{K}\right\} - 1\right]\exp\left\{-\frac{x}{K}\right\} - \left(\frac{W_e}{W_o}\right)\exp\left\{\frac{s}{K}\right\} = 1 - \exp\left\{-\frac{r_1}{K}\right\} \tag{6.24}$$

or

$$\exp\left\{-\frac{r_1}{K}\right\}\left[1 + 2\exp\left\{-\frac{x}{K}\right\}\right] = 1 + \exp\left\{-\frac{x}{K}\right\} + \left(\frac{W_e}{W_o}\right)\exp\left\{\frac{s}{K}\right\}$$

Thus

$$r_1 = \Delta r_1 = K \log_e\left[\frac{1 + 2\exp\left\{-\frac{x}{K}\right\}}{1 + \exp\left\{-\frac{x}{K}\right\} + \left(\frac{W_e}{W_o}\right)\exp\left\{\frac{s}{K}\right\}}\right] \tag{6.25}$$

To determine $\overline{\Delta r}$, recall $\Delta r_2 = \Delta r_1 + x - \Delta$. Also

$$\Delta = K \log_e\left[\frac{W(o)}{W(r_2)}\right]$$

and

$$W(r_2) = W(o)\exp\left\{-\frac{r_2}{K}\right\} + \Delta F_a(r_2)$$

Therefore

$$W(r_2) = W(o)\left[\exp\left\{-\frac{r_1}{K}\right\}\left(\exp\left\{-\frac{x}{K}\right\} - 1\right) + 1\right]$$

Solving for Δ

$$\Delta = K \log \left[\frac{1}{1 - \exp\left\{-\frac{r_1}{K}\right\}\left|1 - \exp\left\{-\frac{x}{K}\right\}\right|} \right] \qquad (6.26)$$

This equation and Δr_1 in Eq. (6.25) can be solved to yield $\overline{\Delta r}$, i.e.,

$$\overline{\Delta r} = \Delta r_1 + \frac{x-\Delta}{2} \qquad (6.27)$$

The equations for the tanker radius missions can be obtained by substituting $r_2 = r_1 + x$ for s in Eq. (6.24). Thus

$$\exp\left\{-\frac{r_1}{K}\right\}\left|2 \exp\left\{-\frac{x}{K}\right\} + 1\right| = 1 + \exp\left\{-\frac{x}{K}\right\} + \left(\frac{W_e}{W_o}\right)\exp\left\{\frac{r_1}{K}\right\}\exp\left\{\frac{x}{K}\right\}$$

Solving for r_1,

$$r_1 = K \log_e \left[\frac{-\left(1 + \exp\left\{\frac{x}{K}\right\}\right) + \sqrt{\left(1 + \exp\left\{\frac{x}{K}\right\}\right)^2 + 4\left(\frac{W_e}{W_o}\right)\left(2 + \exp\left\{\frac{x}{K}\right\}\right)\exp\left\{\frac{2x}{K}\right\}}}{2\left(\frac{W_e}{W_o}\right)\exp\left\{\frac{2x}{K}\right\}} \right] \qquad (6.28)$$

The value of Δ is obtained from Eq. (6.26), i.e.,

$$\Delta = -K \log_e \left|1 + \exp\left\{-\frac{r_1}{K}\right\}\left(\exp\left\{-\frac{x}{K}\right\} - 1\right)\right| \qquad (6.29)$$

Table 3 shows Δr_1 and Δr_2 for several values of x and $\left(\frac{W_e}{W_o}\right)$.

Table 3

RANGE AUGMENTATION: TWO LRCAs AND ONE TANKER
(Equal Fuel Weight Offloads)

Spacing Between Refueling (x/k)	Aircraft Dry Weight Fraction			
	0.2	0.3	0.4	0.5
CONSTANT RECOVERY DISTANCE s = 0.1 x k				
0	.301/.301	.252/.252	.206/.206	.162/.162
.05	.290/.303	.240/.251	.193/.202	.148/.155
.10	.279/.304	.228/.249	.180/.197	.134/.147
.15	.268/.305	.216/.247	.167/.192	.120/.138
.20	.257/.306	.204/.244	.154/.185	.106/.128
RADIUS MISSION				
0	.281/.281	.232/.232	.189/.189	.150/.150
.05	.266/.277	.215/.224	.170/.178	.130/.136
.10	.250/.273	.197/.215	.150/.164	.109/.119
.15	.234/.267	.178/.204	.130/.149	.087/.100
.20	.217/.260	.160/.192	.110/.132	.065/.079

VII. RANGE ENHANCEMENT: ONE TANKER SUPPORTING ONE OR MORE COMBAT AIRCRAFT (THE FILLING STATION APPROACH)

Under some circumstances it is possible to phase the departure of combat aircraft from their base so that they arrive at a given range spaced in time, thereby permitting a tanker to refuel the aircraft sequentially without flying even further from its home base. We call this approach the filling station concept. For simplicity we will assume that all refuelings occur at a fixed range from the LRCA's home base, and that the range penalty imposed on the tanker for the time needed for refueling one LRCA and loitering while awaiting for the next is also fixed, i.e., equals x. As a consequence, every refueling is the same, offloading the same amount of fuel and augmenting the range of every LRCA by the same distance. In addition, we assume that the tanker flies to the refueling location with the first aircraft and returns to its original base after the last refueling, i.e., the tanker radius mission. The equations for tanker basing (departure and recovery), assuming constant range to and from the refueling point will be included for completeness.

As before,

$$\Delta F_b = W(o)\left|1 - \exp\left\{-\frac{r}{K}\right\}\right|$$

By assumption, this holds for all LRCAs and the fuel offloaded to each LRCA by the tanker equals this amount. For a single refueling the fuel available in the tanker is simply

$$\Delta F_a^{(1)} = W(o)\exp\left\{-\frac{r}{K}\right\} - W_e\exp\left\{\frac{r}{K}\right\} \tag{7.1}$$

Setting $\Delta F_b = \Delta F_a^{(1)}$, we obtain

$$z^2\left(\frac{W_e}{W_o}\right) + z - 2 = 0 \tag{7.2}$$

where for convenience we have adopted the notation $z = \exp\left\{\frac{r}{K}\right\}$. Solving for r,

$$r = K \log_e \left[\frac{-1 + \sqrt{1 + 8\left(\frac{W_e}{W_o}\right)}}{2\left(\frac{W_e}{W_o}\right)} \right] \tag{7.3}$$

which is the same as Eq. (4.14). For a single refueling the filling station approach is the same as the buddy tactic.

<u>One Tanker, Two LRCA</u>. At the end of the first refueling, the weight of the tanker is $W_T(1)$, where

$$W_T(1) = W(o)\exp\left\{-\frac{r}{K}\right\} - \Delta F_b = W(o)\left[2\ \exp\left\{-\frac{r}{K}\right\} - 1\right] \tag{7.4}$$

The tanker weight at the start of the second refueling is $W_T(1)\exp\left\{-\frac{x}{K}\right\}$. Therefore the available fuel for the second refueling is

$$\Delta F_a^{(2)} = W_T(1)\exp\left\{-\frac{x}{K}\right\} - W_e\exp\left\{\frac{r}{K}\right\} \tag{7.5}$$

where the second term is the minimum tanker weight needed to be able to fly r miles back to the home base. Setting $\Delta F_b = \Delta F_a$ and using Eqs. (7.4) and (7.5)

$$1 - \exp\left\{-\frac{r}{K}\right\} = \left[2\ \exp\left\{-\frac{r}{K}\right\} - 1\right]\exp\left\{-\frac{x}{K}\right\} - \left(\frac{W_e}{W_o}\right)\exp\left\{\frac{r}{K}\right\} \tag{7.6}$$

Substituting z, and simplifying, we obtain

$$z^2\left(\frac{W_e}{W_o}\right) + z\left[1 + \exp\left\{-\frac{x}{K}\right\}\right] - \left[1 + 2\ \exp\left\{-\frac{x}{K}\right\}\right] = 0 \tag{7.7}$$

To further shorten the notation, let $y = \exp\left\{-\frac{x}{K}\right\}$. Then

$$z^2\left(\frac{W_e}{W_o}\right) + z(1 + y) - (1 + 2y) = 0 \tag{7.8}$$

Solving for z,

$$z = \frac{-(1 + y) + \sqrt{(1 + y)^2 + 4\left(\frac{W_e}{W_o}\right)(1 + 2y)}}{2\left(\frac{W_e}{W_o}\right)} \tag{7.9}$$

Note that this equation is similar to, but not identical with Eq. (7.3).

One Tanker, Three LRCA. Our intent is to derive a general formula applicable for any number of LRCAs to be refueled. Toward that end we now consider three LRCAs and will compare the derived equations with those for two and one LRCAs. This comparison will suggest a general formula.

At the end of the second refueling the tanker weight, $W_T(2)$ is

$$\begin{aligned} W_T(2) &= W_T(1)\exp\left\{-\frac{x}{K}\right\} - \Delta F_b \\ &= W(o)\left|2\exp\left\{-\frac{r}{K}\right\} - 1\right|\exp\left\{-\frac{x}{K}\right\} - W(o)\left[1 - \exp\left\{-\frac{r}{K}\right\}\right] \end{aligned} \tag{7.10}$$

The tanker weight at the start of the third refueling is $W_T(2)\exp\left\{-\frac{x}{K}\right\}$. Therefore the available fuel for the third refueling is

$$\Delta F_a^{(3)} = W_T(2)\exp\left\{-\frac{x}{K}\right\} - W_e\exp\left\{\frac{r}{K}\right\} \tag{7.11}$$

Setting $\Delta F_a = \Delta F_b$ at the third refueling, and substituting y and z for convenience,

$$1 - z^{-1} = \left\{\left[2z^{-1} - 1\right]y - \left(1 - z^{-1}\right)\right\} y - \left(\frac{W_e}{W_o}\right) z \tag{7.12}$$

or

$$z^2\left(\frac{W_e}{W_o}\right) + z\left[1 + y + y^2\right] - \left[1 + y + 2y^2\right] \tag{7.13}$$

The solution for z is

$$z = \frac{-(1 + y + y^2) + \sqrt{(1 + y + y^2)^2 + 4\left(\frac{W_e}{W_o}\right)(1 + y + 2y^2)}}{2\left(\frac{W_e}{W_o}\right)} \tag{7.14}$$

<u>One Tanker, n LRCA</u>. A comparison of Eqs. (7.2), (7.8), and (7.13) strongly suggest a pattern. To derive this pattern consider the following. For n refuelings, $\Delta F_a(n) = \Delta F_b$. Also

$$\Delta F_a(n) = W_T(n-1)y - W_e z \tag{7.15}$$

where $W_T(n-1)$ is the weight of the tanker after n-1 refuelings. Now

$$W_T(n-1) = W_T(n-2)y - W_e z \tag{7.16}$$

i.e., the weight of the tanker after n-1 refuelings is just the weight of the tanker after n-2 refuelings times the percent weight reduction caused by the flying time between the refueling minus the fuel off-loaded during the n-1th refueling. Thus, by successive application of Eq. (7.16)

$$\begin{aligned}
W_T(n-1) &= W_T(n-2)y - W_o(1-z^{-1}) \\
&= \left|W_T(n-3)y - W_o(1-z^{-1})\right| y - W_o(1-z^{-1}) \\
&= W_T(n-3)y^2 - W_o(1-z^{-1})(1+y) \\
&= \left|W_T(n-4)y - W_o(1-z^{-1})\right| y^2 - W_o(1-z^{-1})(1+y) \\
&= W_T(n-4)y^3 - W_o(1-z^{-1})(1+y+y^2) \\
&= W_T(1)y^{n-2} - W_o(1-z^{-1})(1+y+y^2+\cdots+y^{n-3})
\end{aligned} \tag{7.17}$$

Now

$$W_T(1) = W_o z^{-1} - W_o(1-z^{-1}) \tag{7.18}$$

Thus

$$W_T(n-1) = W_o z^{-1} y^{n-2} - W_o(1-z^{-1})(1+y+y^2+\cdots+y^{n-2}) \tag{7.19}$$

Substituting $W_T(n-1)$ into Eq. (7.15),

$$\Delta F_a(n) = W_o z^{-1} y^{n-1} - W_o(1-z^{-1})(y+\cdots+y^{n-1}) - W_e z \tag{7.20}$$

Setting $\Delta F_a(n) = \Delta F_b$, we obtain

$$(1-z^{-1}) = z^{-1}y^{n-1} - (1-z^{-1})(y+\cdots+y^{n-1}) - \left(\frac{W_e}{W_o}\right)z \tag{7.21}$$

or

$$z^2\left(\frac{W_e}{W_o}\right) + (z-1)(1+y+y^2+\cdots+y^{n-1}) - y^{n-1} = 0 \tag{7.22}$$

If we define $S_n = \sum_{i=0}^{n-1} y^i$, then

$$z^2\left(\frac{W_e}{W_o}\right) + S_n z - \left(S_n + y^{n-1}\right) = 0 \tag{7.23}$$

Solving for z

$$z = \frac{-S_n + \sqrt{S_n{}^2 + 4\left(\frac{W_e}{W_o}\right)\left[S_n + y^{n-1}\right]}}{2\left(\frac{W_e}{W_o}\right)} \tag{7.24}$$

This equation matches Eqs. (7.9) and (7.14) for n = 2 and 3 respectively.

If we assume that the tanker still flies the buddy svstem to the first refueling point, but flies a constant distance, s, to a forward recovery base, Eq. (7.21) becomes

$$(1-z^{-1}) = z^{-1}y^{n-1} - (1-z^{-1})(y+y^2+\ldots+y^{n-1}) - \left(\frac{W_e}{W_o}\right)\exp\left\{\frac{s}{K}\right\} \tag{7.25}$$

Soving for z

$$z = \frac{y^{n-1} + S_n}{S_n + \left(\frac{W_e}{W_o}\right)\exp\left\{\frac{s}{K}\right\}} \tag{7.26}$$

If we further assume that the tanker leaves and returns a constant distance, s, to a forward base, Eq. (7.21) becomes

$$z = \frac{S_n}{y^{n-1}\exp\left\{-\frac{s}{K}\right\} - S_n - \left(\frac{W_e}{W_o}\right)\exp\left\{\frac{s}{K}\right\}} \tag{7.27}$$

Figure 6 is a plot of Eq. (7.24).

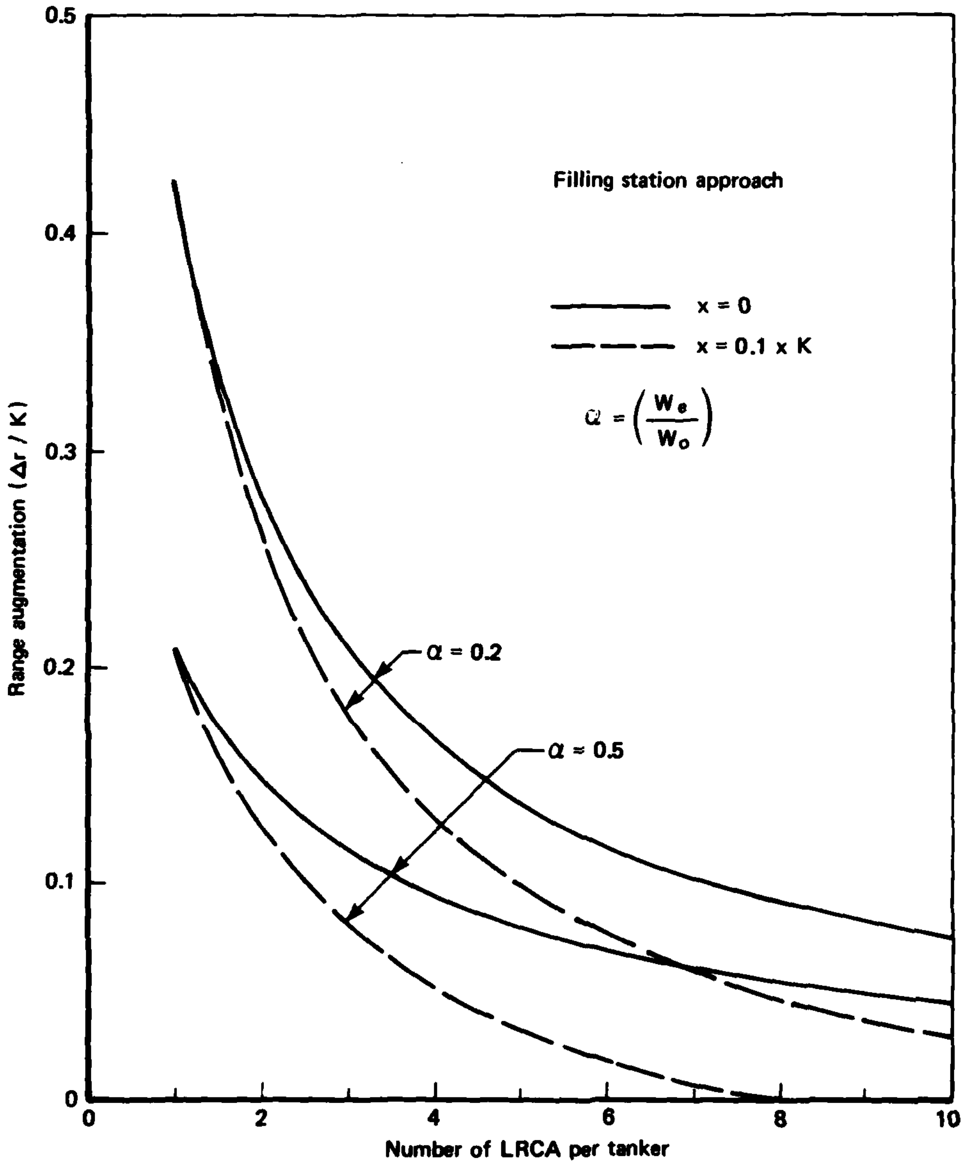

Fig. 6 — Range augmentation provided n LRCA by one tanker (filling station approach)

VIII. RANGE EXTENSION ARISING FROM PAYLOAD DELIVERY AT MID-RANGE

The delivery of payload during the LRCA's flight helps extend its range. In this section we will derive the equations pertinent to estimating this range extension, both with and without refueling.

Let R_o be the LRCA's maximum range without refueling as derived from Breguet's formula and let r_p ($r_p \leq R_o$) be the range where the payload is dropped. The true maximum range can be estimated by the techniques of refueling already derived. For example,

$$W_L(r_p) = W_o \exp\left\{-\frac{r_p}{K}\right\} - W_p \tag{8.1}$$

$$(W_a + W_e) = W_L(r_p) \exp\left\{-\frac{r_e}{K}\right\} \tag{8.2}$$

where W_L is the weight of the LRCA and r_e is the range beyond r_p that the LRCA can fly before running out of fuel. Thus

$$W_a + W_e = W_o \exp\left\{-\frac{r_p + r_e}{K}\right\} - W_p \exp\left\{-\frac{r_e}{K}\right\} \tag{8.3}$$

The maximum range is obviously $r_p + r_e$. Solving for r_e,

$$r_e = K \log_e \left[\frac{W_o \exp\left\{-\frac{r_p}{K}\right\} - W_p}{W_a + W_e}\right] \tag{8.4}$$

r_e and the range extension is $\Delta R = r_p + r_e - R_o$. Figure 7 is a graph of $\frac{\Delta R}{K}$ for several different parameter values.

Equation (8.4) applies if there has been no refueling. If we assume that refueling always occurs before payload delivery, then the above derivation applies if we adjust r_p to reflect the fact that the LRCA is fully fueled at the refueling range r_1 (we will only discuss the single refueling case). Therefore, it follows that

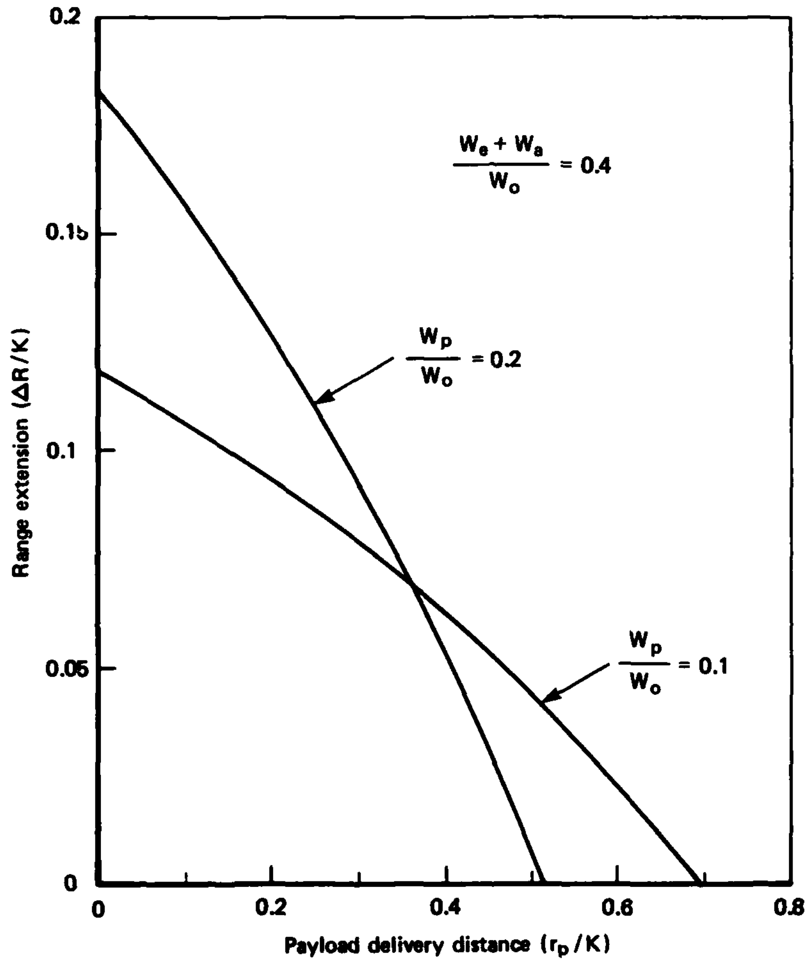

Fig. 7 — Unrefueled range extension arising from mid-range payload delivery

$$r_e = K \log_e \left[\frac{W_o \exp\left\{- \frac{r_p - r_1}{K}\right\} - W_p}{W_a + W_e} \right] \tag{8.5}$$

and the additional range extension is $\Delta R = r_p + r_e - (R+r_1)$.

As a specific application of the above equations, consider the case where the total range is fixed at 5000 n mi and we wish to measure the increase in payload capacity if we assume that the payload is dropped at 2500 n mi. Define P_1 as the maximum payload that can be carried if we do not account for dropping the payload mid-way, and P_2 as the payload that can be carried if we did. From the Breguet equation

$$P_1 = W_o \exp\left\{- \frac{5000}{K}\right\} - (W_e + W_a) \tag{8.6}$$

To determine P_2

$$\begin{aligned} W_L(r_p) &= W_o \exp\left\{- \frac{r_p}{K}\right\} - P_2 \\ W_L(5000) &= W_L(r_p) \exp\left\{- \frac{5000 - r_p}{K}\right\} \\ &= \left[W_o \exp\left\{- \frac{r_p}{K}\right\} - P_2 \right] \exp\left\{- \frac{5000 - r_p}{K}\right\} \\ &= W_o \exp\left\{- \frac{5000}{K}\right\} - P_2 \exp\left\{- \frac{5000 - r_p}{K}\right\} \end{aligned} \tag{8.7}$$

Since $W_L(5000) = W_e + W_a$ by assumption, and $r_p = 2500$ n mi,

$$P_2 = \exp\left\{\frac{2500}{K}\right\} \left[W_o \exp\left\{- \frac{5000}{K}\right\} - (W_e + W_a) \right] \tag{8.8}$$

or

$$P_2 = P_1 \exp\left\{\frac{2500}{K}\right\} \tag{8.9}$$

For the range of K's of interest, this yields P_2's which are 25 percent greater than P_1's.

IX. RANGE EXTENSION BY POST-STRIKE REFUELING

The previous derivations have always assumed that refueling occurs when the combat aircraft is heading away from its home base. For the two tanker refueling tactics examined, refueling always occurred (if possible) when the amount of fuel available equaled the fuel needed. We shall now extend those results to include post-strike refueling where we shall assume that (1) the combat aircraft is headed toward its base, and (2) the tanker is based at the same airfield. Under these conditions it is easy to show that refueling should occur when the combat aircraft is just about to run out of fuel and that the amount of fuel it should receive is only enough to get it back to its base, if that is possible.

One Tanker/One LRCA. Based on the above,

$$W_E = (W_E + \Delta F)\exp\left\{-\frac{r}{K}\right\} \tag{9.1}$$

where $W_E = W_e + W_a + W_p$, ΔF is the fuel added to the empty LRCA and r is the resultant range extension.* Solving for ΔF

$$\Delta F = W_E\left[\exp\left\{\frac{r}{K}\right\} - 1\right] \tag{9.2}$$

To maximize the range extension, ΔF should be as large as possible. How large ΔF is depends on the tanker refueling tactic assumed. For simplicity we will treat tanker radius missions in detail and will only state the equations for a constant recovery distance. Assuming therefore that the tanker returns to the same recovery base as the LRCA,

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*We will assume throughout (unless explicitly stated otherwise) that the range extensions are based on the assumption that the payload is not delivered and is to be recovered.

$$\Delta F_a = W_o \exp\left\{-\frac{r}{K}\right\} - W_e \exp\left\{\frac{r}{K}\right\} \tag{9.3}$$

Setting ΔF_a equal to ΔF, and substituting z,

$$W_E(z-1) = W_o z^{-1} - W_e z \tag{9.4}$$

or

$$(W_E+W_e)z^2 - W_E z - W_o = 0 \tag{9.5}$$

To shorten the notation, define α and β accordingly

$$\beta = \left(\frac{W_E}{W_o}\right) \tag{9.6}$$

$$\alpha = \beta + \left(\frac{W_e}{W_o}\right) \tag{9.7}$$

Thus

$$\alpha z^2 - \beta z - 1 = 0 \tag{9.8}$$

and

$$r = K \log_e\left[\frac{\beta + \sqrt{\beta^2 + 4\alpha}}{2\alpha}\right] \tag{9.9}$$

Equation (9.9) applies for tanker radius missions. If we specified a <u>tanker recovery distance equal to s</u>, then

$$r = K \log_e \frac{\left[\beta - \left(\frac{W_e}{W_o}\right)\exp\left\{\frac{s}{K}\right\}\right] + \sqrt{\left[\beta - \left(\frac{W_e}{W_o}\right)\exp\left\{\frac{s}{K}\right\}\right]^2 + 4\beta}}{2\beta} \tag{9.10}$$

In general the post-strike refueling range augmentation values exceed those of the pre-strike refuelings. This is true because lighter aircraft having the same empty weight (but less fuel) use less fuel per mile.

Two Tankers/One LRCA. We maintain the assumptions above; therefore, the tankers fly radius missions, with the LRCA assumed to be empty of fuel at both refuelings. Thus Eq. (9.1) applies for the second refueling, i.e.,

$$W_E = (W_E + \Delta F_2)\exp\left\{-\frac{r_2}{K}\right\} \tag{9.11}$$

where the subscripts on ΔF and r indicate that this equation is for the second refueling. Similarly,

$$W_E = (W_E + \Delta F_1)\exp\left\{-\frac{r_1 - r_2}{K}\right\} \tag{9.12}$$

where r_1 is the distance from the base for the first post-strike refueling. The fuel available equation for both tankers is as before

$$\Delta F_a(r) = W_o \exp\left\{-\frac{r}{K}\right\} - W_e \exp\left\{\frac{r}{K}\right\} \tag{9.13}$$

At both refuelings, $\Delta F = \Delta F_a$. Thus we have two equations

$$W_E(z_2 - 1) = W_o z_2^{-1} - W_e z_2 \tag{9.14}$$

$$W_E(z_1 z_2^{-1} - 1) = W_o z_1^{-1} - W_e z_1 \tag{9.15}$$

The solution for r_2 is the same as for a single tanker and is given in Eq. (9.9). Rewriting Eq. (9.15)

$$z_1^2\left[\left(\frac{W_e}{W_o}\right)z_2^{-1} + \left(\frac{W_e}{W_o}\right)\right] - \left(\frac{W_E}{W_o}\right)z_1 - 1 = 0 \tag{9.16}$$

Substituting β

$$z_1^2\left[\beta z_2^{-1} + \left(\frac{W_e}{W_o}\right)\right] - \beta z_1 - 1 = 0 \tag{9.17}$$

where

$$z_2^{-1} = \frac{2\alpha}{\beta + \sqrt{\beta^2 + 4\alpha}} \tag{9.18}$$

and

$$z_1 = \frac{\beta + \sqrt{\beta^2 + 4\left[\beta z_2^{-1} + \left(\frac{W_e}{W_o}\right)\right]}}{2\left[\beta z_2^{-1} + \left(\frac{W_e}{W_o}\right)\right]} \tag{9.19}$$

If both tankers fly a <u>fixed recovery range</u>, then $r_2 = r$ in Eq. (9.9) and

$$z_1 = \frac{\left[\beta - \left(\frac{W_e}{W_o}\right)\exp\left\{\frac{t}{K}\right\}\right] + \sqrt{\left[\beta - \left(\frac{W_e}{W_o}\right)\exp\left\{\frac{t}{K}\right\}\right]^2 + 4\beta z_2^{-1}}}{2\beta z_2^{-1}} \tag{9.20}$$

<u>One Tanker/n LRCA</u>. Assuming the filling station approach, the tanker fuel availability equation is still Eq. (7.20), i.e.,

$$\Delta F_a(n) = W_o\left\{y^{n-1}z^{-1}(1-z^{-1})(y+y^2+\cdots+y^{n-1}) - \left(\frac{W_e}{W_o}\right)z\right\} \tag{9.21}$$

and the LRCA fuel needed equation is Eq. (9.2), i.e.,

$$\Delta F = W_o\beta(z-1) \tag{9.22}$$

Equating $\Delta F_a(n)$ and ΔF (i.e., assuming n refuelings and calculating the range augmentation possible)

$$\beta(z-1) = y^{n-1}z^{-1} - (1-z^{-1})(y+y^2+\cdots+y^{n-1}) - \left(\frac{W_e}{W_o}\right)z$$

or

$$\alpha z^2 + z\left|y+y^2+\cdots+y^{n-1}-\beta\right| - \left|y+y^2+\cdots+2y^{n-1}\right| = 0 \tag{9.23}$$

Defining $T_n = \sum_{i=1}^{n} y^i$, Eq. (A.97) becomes

$$\alpha z^2 + \left|T_{n-1}-\beta\right|z - \left|T_{n-1}+y^{n-1}\right| = 0$$

which leads to

$$r_n = K \log_e \frac{-(T_{n-1}-\beta) + \sqrt{(T_{n-1}-\beta)^2 + 4\alpha(T_{n-1}+y^{n-1})}}{2\alpha} \tag{9.24}$$

If we define $T_o = 0$, Eq. (9.24) applies for all $n \geq 1$.

The similar equations for the constant recovery distance assumption are obtained by replacing $\left(\frac{W_e}{W_o}\right)z^2$ in Eq. (9.23) by $\left(\frac{W_e}{W_o}\right)z \exp\left\{\frac{s}{K}\right\}$. Thus

$$r_n = K \log_e \frac{-\left[T_{n-1} + \left(\frac{W_e}{W_o}\right)\exp\left\{\frac{s}{K}\right\} - \beta\right] + \sqrt{\left[T_{n-1} + \left(\frac{W_e}{W_o}\right)\exp\left\{\frac{s}{K}\right\} - \beta\right]^2 + 4\beta(T_{n-1} + y^{n-1})}}{2\beta} \qquad (9.25)$$

Pre- and Post-Strike Refueling. The above post-strike range enhancements can obviously be used regardless of prior refuelings in the pre-strike portion of the LRCA's flight. The post-strike r_n will always be greater than the pre-strike r_n if a single refueling is sufficient to get the LRCA back to base. However, post-strike refuelings imply operational uncertainties that are clearly greater than those of pre-strike refueling. Therefore it is not self evident that post-strike refuelings should be preferred. In general, the advantages of pre-strike refueling would appear to offset the modest range enhancement gains associated with post-strike refueling. Of course, post strike refueling may be of interest when used in combination with pre-strike (double refueling).

X. RANGE ENHANCEMENT WHERE THE TANKER AND LRCA ARE DISSIMILAR

The above derivations assumed that the tanker and combat aircraft were similar, i.e., their gross weight, structural fraction, and Breguet range factor, are the same. As combat aircraft gross weight was scaled, so too was the tanker's. In this section we will display some of the equations pertinent to different tanker designs that do not scale with those of the LRCA.

One Tanker/One LRCA. Assume that the tankers fly radius missions. Then the weight of the tanker after refueling at r is

$$W_T(r) = W_T(o)\exp\left\{-\frac{r}{K_T}\right\} - \Delta F_a \tag{10.1}$$

and

$$W_{Te} = W_T(r)\exp\left\{-\frac{r}{K_T}\right\} \tag{10.2}$$

Therefore

$$\Delta F_a = W_T(o)\left[\exp\left\{-\frac{r}{K_T}\right\} - \left(\frac{W_{Te}}{W_{To}}\right)\exp\left\{\frac{r}{K_T}\right\}\right] \tag{10.3}$$

For the combat aircraft,

$$\Delta F_b = W_A(o)\left[1-\exp\left\{-\frac{r}{K_A}\right\}\right] \tag{10.4}$$

Equating $\Delta F_b = \Delta F_a$,*

*This is still the condition for optimum refueling.

$$W_A(o)\left[1-\exp\left\{-\frac{r}{K_A}\right\}\right] = W_T(o)\left[\exp\left\{-\frac{r}{K_T}\right\} - \left(\frac{W_{Te}}{W_{To}}\right)\exp\left\{\frac{r}{K_T}\right\}\right] \qquad (10.5)$$

This equation is best solved numerically. Figure 8 is an example of a graphical solution for Eq. (10.5).

One Tanker/n LRCA. From Eq. (7.20)

$$\Delta F_a(n) = W_T(o)\left\{z_T^{-1}y^{n-1} - (1-z_T^{-1})(y+y^2+\cdots+y^{n-1}) - \left(\frac{W_{Te}}{W_{To}}\right)z_T\right\} \qquad (10.6)$$

where

$$z_T = \exp\left\{\frac{r}{K_T}\right\}$$

Also

$$\Delta F_b = W_A(o)\left[1-z_A^{-1}\right] \qquad z_A = \exp\left\{\frac{r}{K_A}\right\}$$

Thus, for $\Delta F_a(n) = \Delta F_b$,

$$W_A(o)(1-z_A^{-1}) = W_T(o)\left\{z_T^{-1}y^{n-1} - (1-z_T^{-1})(y+y^2+\cdots+y^{n-1}) - \left(\frac{W_{Te}}{W_{To}}\right)z_T\right\} \qquad (10.7)$$

As above, this equation must be solved numerically (or graphically).

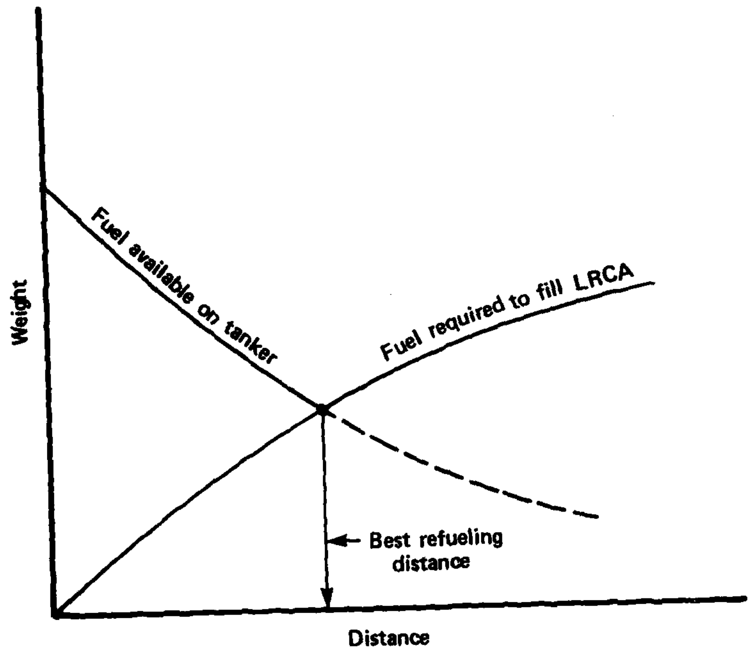

Fig. 8 — Graphical solution for single LRCA/tanker refueling point

REFERENCE

1. Shaver, R. D., H. G. Massey with A. A. Barbour and J. L. Birkler, "The Impact of Tanker Support on Long Range Combat Aircraft Size Selection," The Rand Corporation, N-1861-AF (forthcoming).