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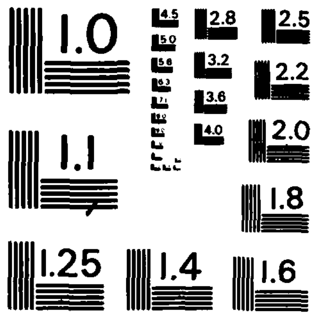

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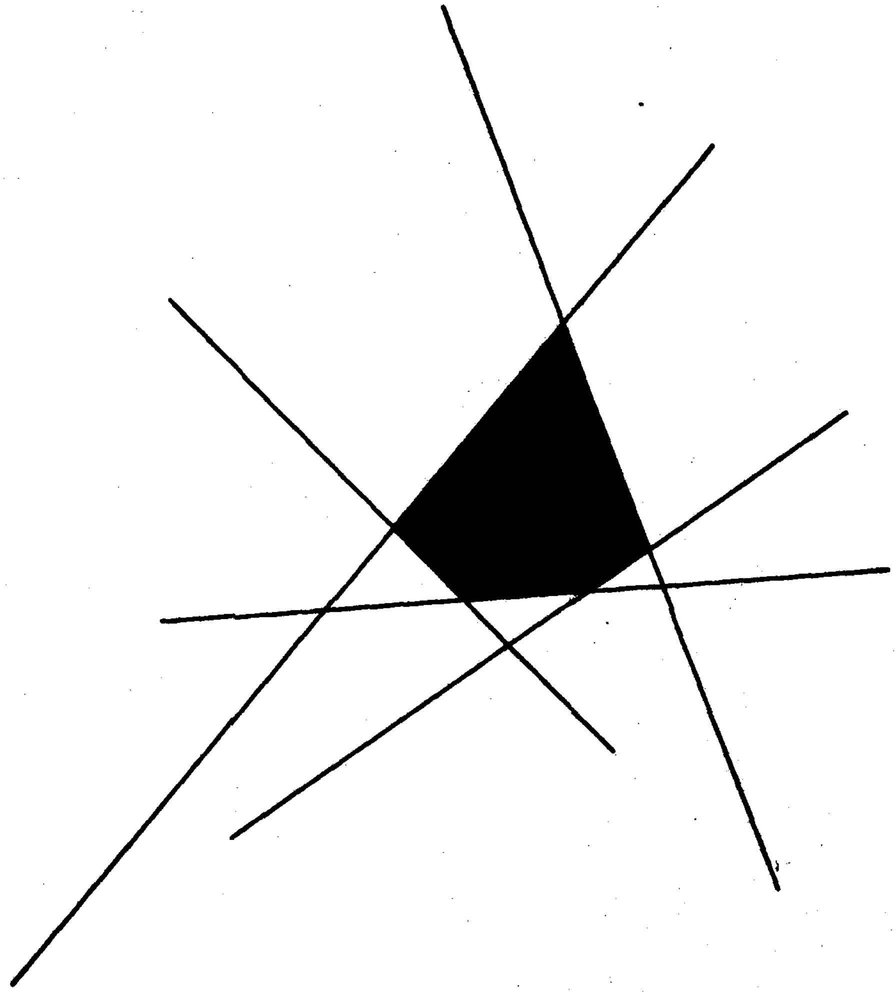
by
RICHARD E. BARLOW



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Richard E. Barlow

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This apparent paradox was first brought to my attention by Dr. William Vesely, many years ago.

ABSTRACT

For the exponential life distribution model and any prior distribution for the failure rate parameter, the predictive distribution has a decreasing failure rate. We give a Bayesian explanation of why this is logically reasonable.

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For many devices, wear-out does not seem likely--at least not within the time frame of interest. For such devices, an exponential life distribution model may be used; i.e., for random lifetime X,

$$P[X > x \mid \lambda] = e^{-\lambda x} .$$

However, information about λ must come from data and/or engineering judgment. In the exponential case and relative to λ, the data can be summarized by k, the observed number of failures and T, the observed total time on test. Given (k,T), suppose a posterior density is calculated based on a prior $\pi(\lambda)$. The predictive survival probability to age x is then

$$\bar{F}(x \mid k,T) = \int_0^\infty P[X > x \mid \lambda]\pi(\lambda \mid k,T)d\lambda . \qquad (1)$$

It is well known that for all priors, π, the predictive distribution has a decreasing failure rate function in x [cf. Theorem 4.7, Page 103, Barlow and Proschan (1981)] when it exists. However, at first glance, this result seems highly unreasonable. If the device does not wear out, why should we predict its future life by a model [the predictive distribution] which actually has a decreasing failure rate?

Denote this predictive failure rate function by $r(x \mid k,T)$ where, using (1),

$$r(x \mid k,T) \stackrel{\text{Def}}{=} \frac{\int_0^\infty \lambda e^{-\lambda x} \pi(\lambda \mid k,T) d\lambda}{\int_0^\infty e^{-\lambda x} \pi(\lambda \mid k,T) d\lambda} . \tag{2}$$

Figure 1 illustrates our apparent paradox.

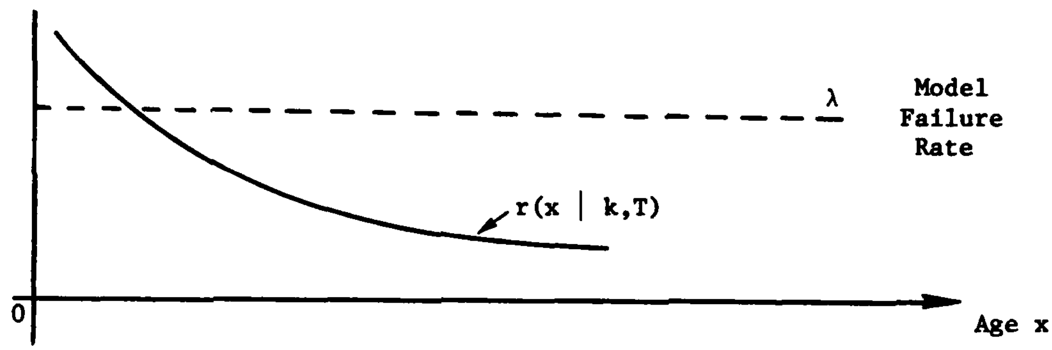

FIGURE 1

COMPARISON OF PREDICTIVE FAILURE
RATE AND MODEL FAILURE RATE, λ

A Bayesian explanation of Figure 1 may be instructive. Note that from (2),

$$r(0 \mid k,T) = \int_0^\infty \lambda \pi(\lambda \mid k,T) d\lambda = E[\lambda \mid k,T] \tag{3}$$

so the posterior mean for λ estimates $r(0 \mid k,T)$. Now suppose a new device (exchangeable with our sample devices) *were* to survive x hours, then from (2)

$$r(x \mid k,T) = \frac{\int_0^\infty \lambda e^{-\lambda x}\pi(\lambda \mid k,T)d\lambda}{\int_0^\infty e^{-\lambda x}\pi(\lambda \mid k,T)d\lambda} .$$

But, by Bayes theorem

$$\pi(\lambda \mid k,T,X > x) = \frac{e^{-\lambda x}\pi(\lambda \mid k,T)}{\int_0^\infty e^{-\lambda x}\pi(\lambda \mid k,T)d\lambda}$$

so that

$$\begin{aligned} r(x \mid k,T) &= E[\lambda \mid k,T,X > x] \\ &= E[\lambda \mid k,T + x] . \end{aligned} \tag{4}$$

Hence our estimate for $r(x \mid k,T)$ based on our updated information, namely that another device has survived time x, produces an estimate for λ different from $r(0 \mid k,T)$.

It is proved in Barlow and Proschan (1979) that for g nondecreasing, $\int_0^\infty g(\lambda)\pi(\lambda \mid k,T)d\lambda$ is decreasing in T. Expression (3) is the special case $g(\lambda) = \lambda$ so that $E[\lambda \mid k,T] \geq E[\lambda \mid k,T + x]$ for all priors.

<u>Example</u>:

If $\pi(\lambda \mid a,b) = \frac{b^a \lambda^{a-1} e^{-b\lambda}}{\Gamma(a)}$, the natural conjugate prior, then

$$\bar{F}(x \mid k,T) = \left(\frac{b + T}{b + T + x}\right)^a$$

and

$$r(x \mid k,T) = \frac{a}{b + T + x}$$

which is obviously decreasing in x .

Conclusion:

The source of the confusion concerns the difference between a *model* based on prior assumptions, such as constant failure rate, and a *predictive distribution* based on *current* information. We would *not* use the predictive distribution (1) as our model, since we believe in constant failure rate. However, were we asked to predict whether or not a *new* device (exchangeable with the sample devices) would survive to age x , we would use (1) based on current information.

On the other hand, were we to characterize a new set of say, m , devices, exchangeable with our sample, we would say that each has an exponential life distribution with expected failure rate $E[\lambda \mid k,T]$ and $\pi(\lambda \mid k,T)$ would fully measure our uncertainty about that failure rate. However, were we asked to predict how many will fail in time period $[0,t]$, we would use the predictive distribution and calculate

$$mE_{\lambda}[1 - e^{-\lambda t} \mid k,T] = mF(t \mid k,T) .$$

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