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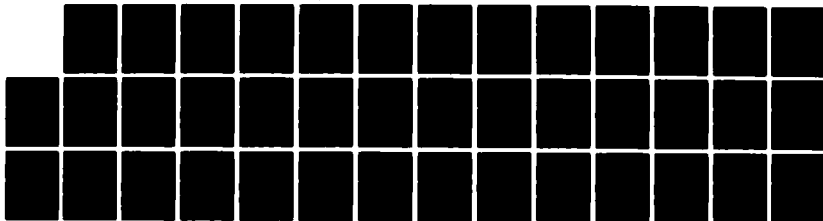
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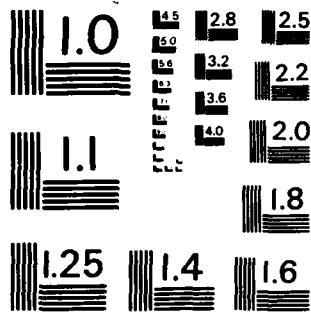
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PAFT F77

PROGRAM FOR THE ANALYSIS OF FAULT TREES

[September 1982, Fortran 77 Version]

Operations Research Center Research Report No. 83-14

Thomas A. Feo

October 1983

U. S. Army Research Office - Research Triangle Park

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Operations Research Center
University of California, Berkeley

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ABSTRACT

Document describes

PAFT F77, a program for the analysis of fault trees coded in Fortran 77, is presented. Given the structure of a fault tree and the probability or failure rates of its basic events, PAFT F77 calculates the probabilities of the top and all intermediate events, as well as the marginal importances of all basic events. When the input is in terms of failure rates instead of probabilities, the marginal occurrence rates of all basic events and the occurrence rate of the top event are also calculated. This additional information though, is invalid for fault trees comprising NOT gates or those gate types that must be modeled with NOT gates such as exclusive OR gates. The program is designed to handle simple fault trees and those with replicated basic events.

The introduction to this text describes the program's ability and limitations together with a brief theoretical review of fault tree analysis. The second section is a users guide to PAFT F77 on the UNIX operating system at the University of California, Berkeley. The final section gives an in depth description of the program's Fortran 77 code.

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INTRODUCTION TO PAFT F77

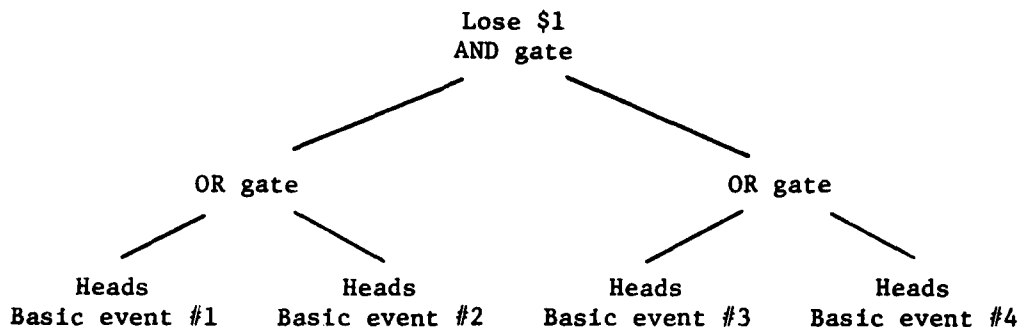
1.1 What is a Fault Tree?

The safety of nuclear reactors is one of the many engineering risk considerations in the forefront of today's political, environmental and social dialogue. Answering these questions of safety from an emotional or political point of view falls short of the scientific objectivity which our society expects and demands. Fault tree analysis is a method available to the engineer for determining reliability of complex systems. A fault tree is a directed acyclic graph representing the relationship between a system event and subsystem events. In turn, each subsystem event can be represented by more detailed subsystem events until fundamental or basic events are reached. The state of each fundamental component or basic event will determine the success or failure of the entire system, through the logic structure of the fault tree.

Constructing a fault tree for a complex system can be a formidable task. The millions of components of the system must be identified and mathematically modeled. Then this data must be used to construct a fault tree which correctly represents the logical dependence between the system's component events. For the fault tree of a nuclear reactor, the top event could be the catastrophic occurrence of core meltdown, while a basic event could be as insignificant as the outside humidity. Fault tree analysis is not limited to nuclear reactor reliability but is a commonly used statistical and engineering tool. The following example should give some idea of the variety of applications of this method.

You are offered a game of chance involving a fair coin. If either of the first two flips is a head and either of the second two flips is a head

you lose \$1, otherwise you win \$1. In this example, the top event is not a catastrophe, but still the basic principles of probability behind this game of chance can be applied to more serious and consequential cases. The fault tree (in this case it would more likely be called a game tree) is:



The probability of each occurrence of a head is $\frac{1}{2}$. The probability of each OR gate is $\frac{3}{4}$ and the probability of the AND gate or top event is $\frac{9}{16}$. Therefore, with the opportunity quoted, one would gracefully decline this game since your expected winnings are $-\frac{1}{8}$ of a dollar.

1.2 The Program's Ability

The input to PAFT F77 is the structure of a fault tree together with the probabilities or failure rates of its basic events. The program returns the probabilities of all gate events including the top event and the marginal importances of all basic events. The marginal importance of a basic event is the partial derivative of the probability of the top event with respect to the probability of the basic event. The marginal importance can also be interpreted as the probability of the top event given that the basic event has occurred, minus the probability of the top event given the basic event has not occurred. When analysing a system, it is useful to know which basic

events most influence the occurrence of the top event. The marginal importance is one measure of such influence.

When the input is in terms of failure rates instead of probabilities, the marginal occurrence rates of all basic events and the occurrence rate of the top event are also calculated. The marginal occurrence rate of a basic event, like its marginal importance, is a measure of the influence a basic event has on the top event. The occurrence rate of the top event is the sum of all of the marginal occurrence rates of the basic events. When a fault tree contains NOT gates or those gate types that must be modeled with NOT gates such as exclusive OR gates, the program, which assumes monotonicity of the top event with respect to basic events, calculates the marginal occurrence rates of the basic events under these gates incorrectly. Furthermore, the sum of these quantities produces an incorrect occurrence rate for the top event. Hence, marginal occurrence rates may be of little value for fault trees with NOT gates and exclusive OR gates.

1.3 Theoretical Background

The following functions are used to calculate the probabilities of AND, OR, EXOR, and NOT gates.

$$P_{\text{AND}} = \prod p_i$$

$$P_{\text{OR}} = 1 - \prod (1 - p_i)$$

$$P_{\text{EXOR}} = p_1 + p_2 - 2p_1p_2$$

$$P_{\text{NOT}} = 1 - p .$$

The remaining gate types (such as two out of three (2/3) or exactly 5 out of 7 (x5/7)) use the following generating function.

$$\prod_{i=1}^n (q_i + p_i z) .$$

For the two out of three case, the sum of the coefficients of z^2 and z^3 is the correct probability with $n = 3$; for the event exactly 5 out of 7, the coefficient of z^5 is the correct probability with $n = 7$.

The marginal importance of a basic event is

$$P(\text{top} \mid \text{basic event}) - P(\text{top} \mid \text{not basic event}) .$$

The marginal occurrence rate is (at a specified time t)

$$\frac{(\text{failure rate of basic event})(\text{marginal importance of basic event})}{(1 - P(\text{basic event})) / (1 - P(\text{top}))} .$$

If $r_i(t)$ is the occurrence rate of basic event i at time t , $F_i(t)$ is the probability event i occurs (for the first time) by time t , $I_i(t)$ is the marginal importance of basic event i at time t and $P(T > t)$ is the probability the top event has *not* occurred by time t , then the basic event occurrence rate at time t is

$$r_i(t) [1 - F_i(t)] I_i(t) / P(T > t) .$$

If input failure rates are constant in time, then the distribution of time to first occurrence of a basic event is exponential.

The occurrence rate of the top event is

$$\sum_{\text{all basic events}} \text{marginal occurrence rate} .$$

Again the formula for marginal occurrence rates holds only for trees not comprising NOT, EXOR, or exclusive x out of y gates.

1.4 The Program's Limitations

PAFT F77 is an exponential time process in terms of the number of replicated basic events contained in the inputted fault tree. Thus, the program is limited to trees with a few replicated basic events, approximately ten or less. The maximum dimension of an inputted tree is ninety-nine gates, including all basic events. Of these ninety-nine, a maximum of 20 replicated basic events can be specified, though a tree with this number will take days to compute. Replicated gates are not allowed.

A second limitation to the program is its inability to handle NO EXOR, or exclusive x out of y gates properly when calculating marginal occurrence rates. An enhancement to solve this problem can be readily coded into the state enumeration framework of the existing program.

An inadequate user interface is a third limitation to PAFT F77. An interface enhancement should prompt input either from an existing file or manual CRT entry. It should have limited editing capabilities in order to facilitate changes to existing input. And, idiot-proofing should be done to check for the correct data types and sizes of descriptive labels, probabilities between one and zero, and complete, connected, and acyclical fault trees.

USER'S GUIDE

2.1 Using PAFT F77 on UNIX

After having logged on successfully on the UNIX system, the PAFT F77 program can be run by giving the command,

```
ftree/paft/code < {input file} > {output file}
```

ftree/paft/code specifies the location of the PAFT F77 machine code command file. The first parameter, preceded by '<' is the file name containing the data that defines the fault tree to be analysed. The second parameter preceded by '>' is optional. If given, output from the program is written into a file with the given name. The default is to have output directed back to one's terminal.

2.2 Input File Structure

The input file structure to PAFT F77 is a specific sequence. The following list indicates this order, with each number representing a line in the input file.

- ① Fault tree name - any alpha-numeric entry up to 60 positions in length with no intermediate blanks.
- ② Probabilities or failure rates - the character 'p' or 'f' is entered to specify the input as either in terms of probabilities or failure rates, respectively.

If failure rates are indicated, then lines ③ - ⑤ must be entered.

- ③ Number of time intervals - a digit is entered from 1 to a maximum of 9.

- ④ Time intervals - real nonnegative values are entered separated by blanks.
- ⑤ Time interval units - alpha-numeric entry up to 8 positions in length with no intermediate blanks.
- ⑥ Total number of nodes and number of replicated basic events - an integer from 1 to a maximum of 99, and an integer from 0 to a maximum of 20, respectively, separated by a blank.
- ⑦ Replicated node numbers (if any) - integer values separated by blanks.

Data ⑧ - ⑩ is entered for each node in the fault tree, starting with the top node as number 1.

- ⑧ Node description, node type, number of nodes above, number of nodes below - description is an alpha-numeric entry up to 16 positions in length with no intermediate blanks, type is an integer (see Table 1 for the index), number of nodes above and number of nodes below are integer entries from 0 to a maximum of 20. All entries on this line are separated by blanks.

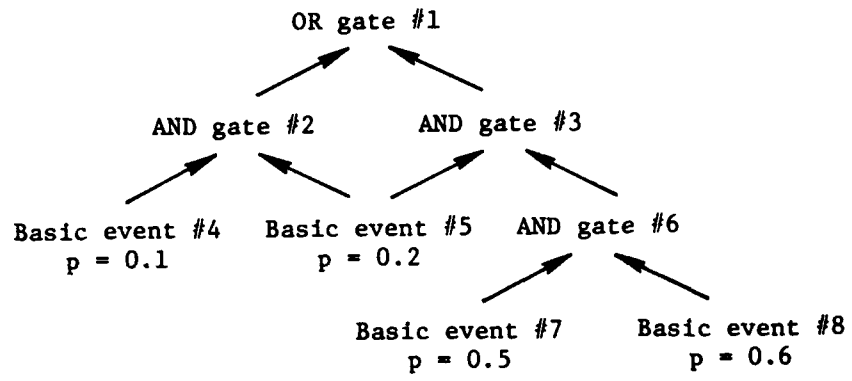
If the node is a basic event, then line ⑨ must be entered.

- ⑨ Probability or failure rate - real nonnegative value.
- ⑩ Node number adjacencies - integer entries of node numbers adjacent from above then adjacent from below, separated by blanks.

Repeat ⑧ - ⑩ until all nodes have been defined.

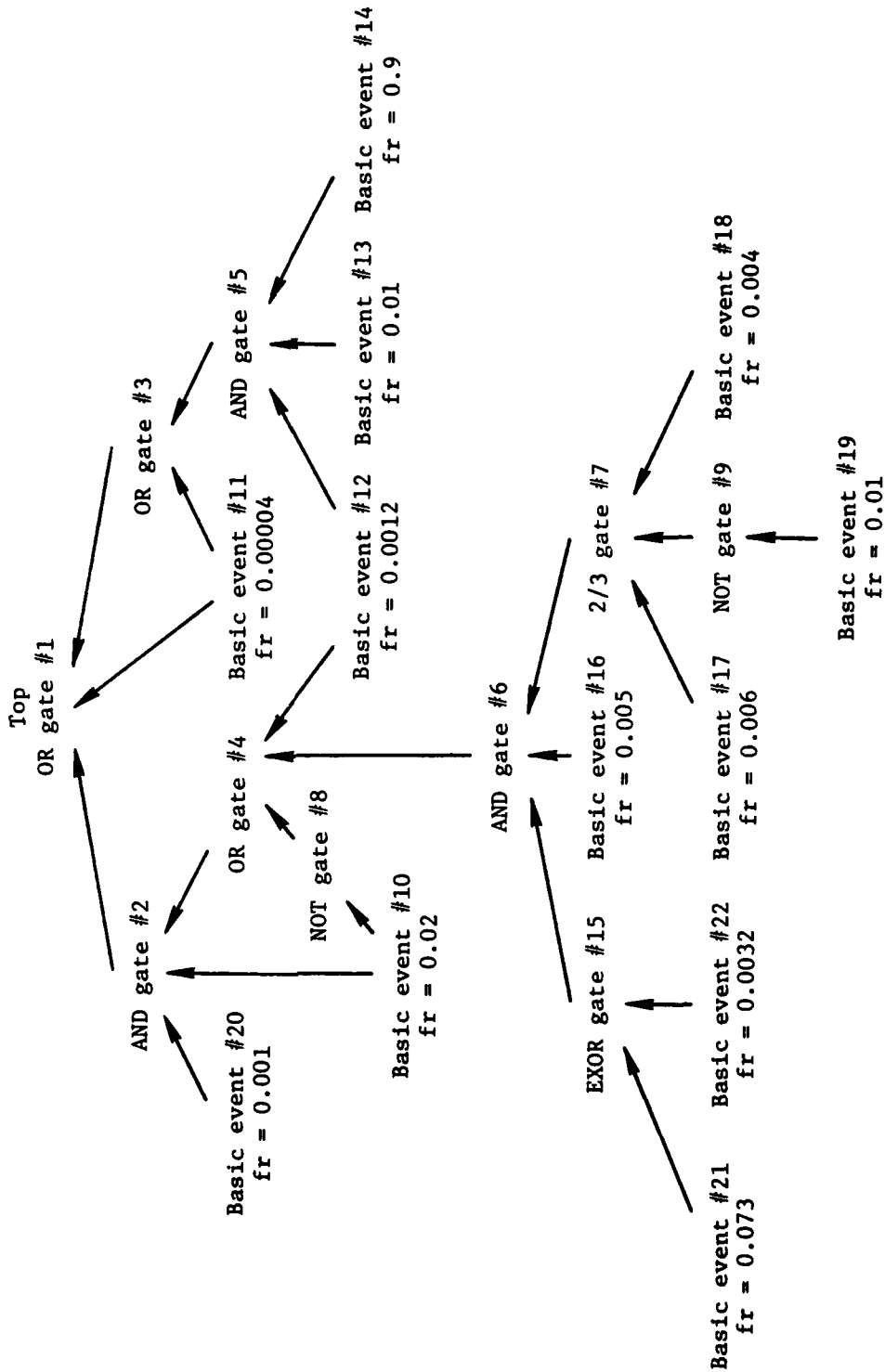
The following two examples of input file structures for the two fault trees below should prove helpful.

TEST.PAFTCODE



A.COMPLEX.FAULT.TREE.EXAMPLE

Time intervals: 2.0, 3.5, 5.0 hours



- ① Test.paftcode
- ② p
- ⑥ 8 1
- ⑦ 5
- ⑧ top#or 3 0 2
- ⑩ 2 3
- ⑧ gate#2and 2 1 2
- ⑩ 1 4 5
- ⑧ gate#3and 2 1 2
- ⑩ 1 5 6
- ⑧ be#4 1 1 0
- ⑨ 0.1
- ⑩ 2
- ⑧ be#5 1 2 0
- ⑨ 0.2
- ⑩ 2 3
- ⑧ gate#6and 2 1 2
- ⑩ 3 7 8
- ⑧ be#7 1 1 0
- ⑨ 0.5
- ⑩ 6
- ⑧ be#8 1 1 0
- ⑨ 0.6
- ⑩ 6

INPUT FILE FOR: Test.paftcode

① A.compex.fault.tree.example
② f
③ 3
④ 2.0 3.5 5.0
⑤ hours
⑥ 22 3
⑦ 10 11 12
⑧ Top 3 0 3
⑩ 2 3 11
⑧ Gate2 2 1 3
⑩ 1 4 10 20
⑧ Gate3 3 1 2
⑩ 1 5 11
⑧ Gate4 3 1 3
⑩ 2 6 8 12
⑧ Gate5 2 1 3
⑩ 3 12 13 14
⑧ Gate6 2 1 3
⑩ 4 7 15 16
⑧ Gate7 6 1 3
⑩ 6 9 17 18
⑧ Gate8 5 1 1
⑩ 4 10
⑧ Gate9 5 1 1
⑩ 7 19
⑧ RBE10 1 2 0
⑨ 0.02
⑩ 2 8
⑧ RBE11 1 2 0
⑨ 0.00004
⑩ 1 3
⑧ RBE12 1 2 0
⑨ 0.0012

INPUT FILE FOR: A.compex.fault.tree.example

(continued)

⑩ 4 5
⑧ BE13 1 1 0
⑨ 0.01
⑩ 5
⑧ BE14 1 1 0
⑨ 0.9
⑩ 5
⑧ Gate15 4 1 2
⑩ 6 21 22
⑧ BE16 1 1 0
⑨ 0.005
⑩ 6
⑧ BE17 1 1 0
⑨ 0.006
⑩ 7
⑧ BE18 1 1 0
⑨ 0.004
⑩ 7
⑧ BE19 1 1 0
⑨ 0.01
⑩ 9
⑧ BE20 1 1 0
⑨ 0.001
⑩ 2
⑧ BE21 1 1 0
⑨ 0.073
⑩ 15
⑧ BE22 1 1 0
⑨ 0.0032
⑩ 15

INPUT FILE FOR: A.complex.fault.tree.example

TABLE 1
NODE TYPE INDEX

<u>Type index #</u>	<u>Gate type</u>
1	Basic event
2	AND
3	OR
4	EXOR
5	NOT
6	2/3
7	x2/3
8	2/4
9	x2/4
10	3/4
11	x3/4
12	2/5
.	.
.	.
t	a/b
.	.
.	.
346	19/20
347	x19/20

$$t = b^2 - 5b + 2a + 8 \text{ (adding 1 for exclusive cases)}$$

2.3 Output File Examples

The following two outputs are the results of running Test.paftcode and A.complex.fault.tree.example.

Note that in the second example the marginal occurrence rates of the basic events and the occurrence rates for the top event are invalid due to the presence of NOT gate #8, NOT gate #9 and EXOR gate #15.

Fault tree name: Test.paftcode

Total number of events: 8 Number of replicated events: 1

Node#	Description	Type	Probability	Above#	/	Below#
1	top#or	OR	0.74000e-01		/	2 3
2	gate#2and	AND	0.20000e-01		1/	4 5
3	gate#3and	AND	0.60000e-01		1/	5 6
4	be#4	BE	0.10000e+00		2/	
R- 5	be#5	BE	0.20000e+00	2	3/	
6	gate#6and	AND	0.30000e+00		3/	7 8
7	be#7	BE	0.50000e+00		6/	
8	be#8	BE	0.60000e+00		6/	

Node#	Description	Type	Probability	Marginal Importance
1	top#or	OR	0.74000e-01	0.10000e+01
R- 5	be#5	BE	0.20000e+00	0.37000e+00
4	be#4	BE	0.10000e+00	0.14000e+00
7	be#7	BE	0.50000e+00	0.10800e+00
8	be#8	BE	0.60000e+00	0.90000e-01
2	gate#2and	AND	0.20000e-01	
3	gate#3and	AND	0.60000e-01	
6	gate#6and	AND	0.30000e+00	

OUTPUT FILE FOR: Test.paftcode

Fault tree name: A.complex.fault.tree.example
 Total number of events: 22 Number of replicated events: 3
 Time intervals: hours 2.00000 3.50000 5.00000

Node#	Description	Type	Fail Rate	Above#	/	Below#
1	Top	OR			/	2 3 11
2	Gate2	AND		1/		4 10 20
3	Gate3	OR		1/		5 11
4	Gate4	OR		2/		6 8 12
5	Gate5	AND		3/		12 13 14
6	Gate6	AND		4/		7 15 16
7	Gate7	2/3		6/		9 17 18
8	Gate8	NOT		4/		10
9	Gate9	NOT		7/		19
R- 10	RBE10	BE	0.20000e-01	2	8/	
R- 11	RBE11	BE	0.40000e-04	1	3/	
R- 12	RBE12	BE	0.12000e-02	4	5/	
13	BE13	BE	0.10000e-01		5/	
14	BE14	BE	0.90000e+00		5/	
15	Gate15	EXOR		6/		21 22
16	BE16	BE	0.50000e-02		6/	
17	BE17	BE	0.60000e-02		7/	
18	BE18	BE	0.40000e-02		7/	
19	BE19	BE	0.10000e-01		9/	
20	BE20	BE	0.10000e-02		2/	
21	BE21	BE	0.73000e-01		15/	
22	BE22	BE	0.32000e-02		15/	

Time interval: 2.00000 hours
 Occurrence rate of top event: 0.86746e-04

Node#	Description	Type	Probability	Marginal Importance	Marginal Occur Rate
1	Top	OR	0.11980e-03	0.10000e+01	0.86746e-04
R- 11	RBE11	BE	0.79997e-04	0.99996e+00	0.40000e-04
R- 12	RBE12	BE	0.23971e-02	0.16604e-01	0.19879e-04
13	BE13	BE	0.19801e-01	0.20006e-02	0.19612e-04
20	BE20	BE	0.19980e-02	0.93493e-04	0.93317e-07
14	BE14	BE	0.83470e+00	0.47459e-04	0.70612e-05
R- 10	RBE10	BE	0.39211e-01	0.47640e-05	0.91555e-07
17	BE17	BE	0.11928e-01	0.74371e-06	0.44095e-08
18	BE18	BE	0.79681e-02	0.74080e-06	0.29399e-08
16	BE16	BE	0.99502e-02	0.21311e-06	0.10551e-08
21	BE21	BE	0.13584e+00	0.14901e-07	0.94014e-09
22	BE22	BE	0.63796e-02	0.10994e-07	0.34961e-10
19	BE19	BE	0.19801e-01	-0.15069e-07	-0.14773e-09
2	Gate2	AND	0.18992e-06		
3	Gate3	OR	0.11961e-03		
4	Gate4	OR	0.96088e+00		
5	Gate5	AND	0.39620e-04		
6	Gate6	AND	0.27135e-04		
7	Gate7	2/3	0.19411e-01		
8	Gate8	NOT	0.96079e+00		
9	Gate9	NOT	0.98020e+00		
15	Gate15	EXOR	0.14049e+00		

OUTPUT FILE FOR: A.complex.fault.tree.example

Time interval: 3.50000 hours
 Occurrence rate of top event: 0.12454e-03

Node#	Description	Type	Probability	Marginal Importance	Marginal Occur Rate
1	Top	OR	0.27894e-03	0.10000e+01	0.12454e-03
R- 11	RBE11	BE	0.13999e-03	0.99986e+00	0.40000e-04
R- 12	RBE12	BE	0.41912e-02	0.33144e-01	0.39618e-04
13	BE13	BE	0.34395e-01	0.40101e-02	0.38732e-04
20	BE20	BE	0.34939e-02	0.28297e-03	0.28206e-06
14	BE14	BE	0.95715e+00	0.14410e-03	0.55590e-05
R- 10	RBE10	BE	0.67606e-01	0.14624e-04	0.27278e-06
17	BE17	BE	0.20781e-01	0.63009e-05	0.37030e-07
18	BE18	BE	0.13902e-01	0.62585e-05	0.24693e-07
16	BE16	BE	0.17348e-01	0.18094e-05	0.88926e-08
21	BE21	BE	0.22547e+00	0.13248e-06	0.74924e-08
22	BE22	BE	0.11138e-01	0.74386e-07	0.23545e-09
19	BE19	BE	0.34395e-01	-0.22557e-06	-0.21788e-08
2	Gate2	AND	0.10214e-05		
3	Gate3	OR	0.27795e-03		
4	Gate4	OR	0.93269e+00		
5	Gate5	AND	0.13798e-03		
6	Gate6	AND	0.13347e-03		
7	Gate7	2/3	0.33222e-01		
8	Gate8	NOT	0.93239e+00		
9	Gate9	NOT	0.96561e+00		
15	Gate15	EXOR	0.23159e+00		

Time interval: 5.00000 hours
 Occurrence rate of top event: 0.15866e-03

Node#	Description	Type	Probability	Marginal Importance	Marginal Occur Rate
1	Top	OR	0.49130e-03	0.10000e-01	0.15866e-03
R- 11	RBE11	BE	0.19998e-03	0.99971e+00	0.40000e-04
R- 12	RBE12	BE	0.59820e-02	0.48671e-01	0.58084e-04
13	BE13	BE	0.48771e-01	0.59116e-02	0.56260e-04
20	BE20	BE	0.49875e-02	0.57552e-03	0.57293e-06
14	BE14	BE	0.98889e+00	0.29155e-03	0.29164e-05
R- 10	RBE10	BE	0.95163e-01	0.30163e-04	0.54612e-06
17	BE17	BE	0.29554e-01	0.23738e-04	0.13828e-06
18	BE18	BE	0.19801e-01	0.23514e-04	0.92238e-07
16	BE16	BE	0.24690e-01	0.68309e-05	0.33328e-07
21	BE21	BE	0.30580e+00	0.52343e-06	0.26538e-07
22	BE22	BE	0.15873e-01	0.20998e-06	0.66159e-09
19	BE19	BE	0.48771e-01	-0.12255e-05	-0.11663e-07
2	Gate2	AND	0.30079e-05		
3	Gate3	OR	0.48843e-03		
4	Gate4	OR	0.90544e+00		
5	Gate5	AND	0.28851e-03		
6	Gate6	AND	0.35756e-03		
7	Gate7	2/3	0.46421e-01		
8	Gate8	NOT	0.90484e+00		
9	Gate9	NOT	0.95123e+00		
15	Gate15	EXOR	0.31197e+00		

OUTPUT FILE FOR: A.complex.fault.tree.example

PAFT F77 LISTING DESCRIPTION

3.1 Overview of Program Structure

PAFT F77 is comprised of fifteen Fortran F77 modules, in alphabetical order these are:

Subroutine bin(i) - adds 1 to the binary array bin(i) through a recursive procedure.

Function calpb(n) - calculates and returns the probability of node n .

Function calspb(n) - calculates and returns the probability of node n during state enumeration.

Subroutine import - calculates the marginal importance of all non-replicated basic events.

Subroutine input - reads the input file.

Program paft - the main module, performs subroutine calls and data initialization.

Subroutine match - sets flags for nodes dependent on replicated basic events.

Subroutine occur(ndone) - calculates the marginal occurrence rates of all basic events and the occurrence rate of the top event for the ndoneth time interval.

Subroutine output(ndone) - outputs data for a probability input, or for the ndoneth time interval of a failure rate input.

Subroutine qsort(m,n) - sorts the array S from subscript m to n ; S indexes the marginal importances of basic events in decreasing order. Qsort uses a recursive procedure.

Subroutine rduce(i) - reduces the fault tree by calculating all gate probabilities not dependent on any replicated basic events. Rduce uses a recursive procedure starting with node 1.

Subroutine remark(i) - temporarily resets the flags signifying nodal

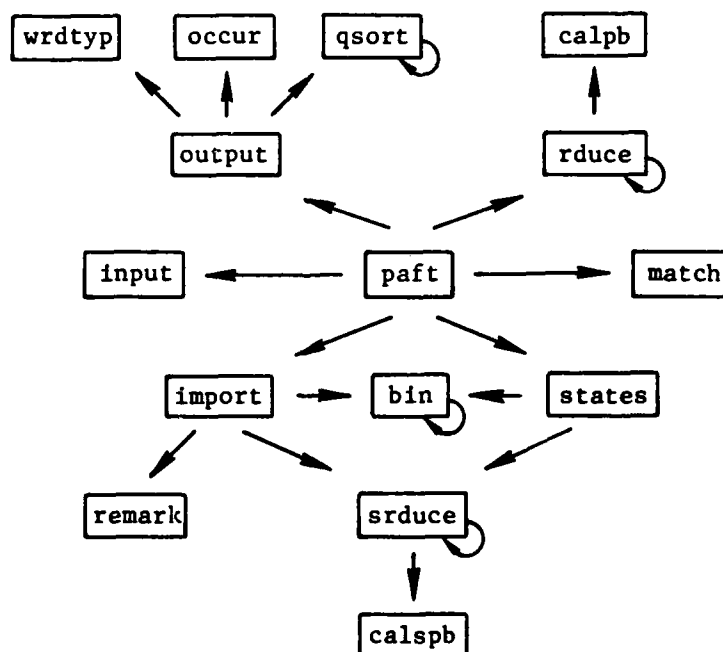
dependence on replicated basic events, for the nodes above non-replicated basic event i .

Subroutine srduce(i) - reduces the fault tree by calculating all gate probabilities during a specific state enumeration. Srduce uses a recursive procedure starting with node 1.

Subroutine states - calculates all gate probabilities dependent on replicated basic events through the state enumeration method. Also calculates the marginal importances of all replicated basic events.

Function wrdtyp(i) - a character function returning the alphanumeric equivalent of the type index i .

The diagram below shows the structure of the procedural dependence between the modules.



Two common data blocks are used to define global variables; `nn` , for numerical data, and `cc` , for character data.

3.2 Global Variable Definitions

The following list gives the global variable definitions for PAFT F77. They are ordered according to their position in the common blocks `cc` and `nn` .

Common block `cc` -

`name` - the fault tree name; up to 60 characters.

`descr(99)` - node descriptions; up to 16 characters each.

`units` - time units for failure rate input; up to 8 characters.

`pf` - probability or failure rate input indicator; 1 character.

Common block `nn` -

`type(99)` - node type indexes; integer.

`above(99)` - number of nodes above; integer.

`below(99)` - number of nodes below; integer.

`nodes(99,20)` - adjacent node numbers above and then below; integer.

`prb(99)` - probability of nodes; real.

`sprb(99)` - scratch probability of nodes during state enumeration; real.

`prt(99)` - marginal importance of nodes; real.

`fail(99)` - basic event failure rates; real.

`time(9)` - time intervals; real.

`sumocr` - occurrence rate of top event; real.

`ocr(99)` - marginal occurrence rates of basic events; real.

`total` - total number of nodes; integer.

nrep - number of replicated basic events; integer.

nbe - number of basic events; integer.

replc(20) - replicated basic event node numbers; integer.

numti - number of time intervals; integer.

mark(99) - flag signifying nodal dependence on a replicated basic event or an enumerated basic event; integer.

store(99) - scratch store for mark(99); integer

bn(20) - binary array signifying states of replicated basic events; integer.

depend(99) - flag signifying nodal dependence on a replicated basic event through two or more paths; integer.

s(99) - index for order of marginal importances; integer.

APPENDIX

```

subroutine bin(i)
integer bn(20)
common /nn/ type(99),above(99),below(99),nodes(99,20),prb(99),sprb(99),
&          prt(99),fail(99),time(9),sumocr,ocr(99),total,nrep,nbe,
&          replc(20),numti,mark(99),store(99),bn(20),dpend(99),s(99)
c change i th member of bn, 0 to 1 or 1 to 0
if (bn(i).eq.0) then
  bn(i)=1
c if 0 to 1 return
  return
  else
  bn(i)=0
c if 1 to 0 change i+1 th member of bn
  call bin(i+1)
end if
return
end

function calpb(n)
real p,pl
integer a,b,c,d,fin,start,num,bn(20),type(99),above(99),below(99),
&          nodes(99,20)
common /nn/ type(99),above(99),below(99),nodes(99,20),prb(99),sprb(99),
&          prt(99),fail(99),time(9),sumocr,ocr(99),total,nrep,nbe,
&          replc(20),numti,mark(99),store(99),bn(20),dpend(99),s(99)
a=above(n)
b=below(n)
c type of gate defines calpb operation
if (type(n),ge.6) go to 50
p=prb(nodes(n,a+1))
c=2
go to (10,20,30,40) type(n)-1
c AND gate
10 do 1 i=a+2, a+b
  p=p*prb(nodes(n,i))
1 continue
go to 70
c OR gate
20 c=1
c EXOR gate
30 do 2 j=a+2, a+b
  pl=prb(nodes(n,j))
  p=p+pl-c*p*pl
2 continue
go to 70
c NOT gate
40 p=1-p
go to 70
c all other gates
50 p=0.0

```

```

c      change type index to (x)a/b form
      d=(type(n)-b*b+5*b-8)/2
      do 3 g=1, b
        bn(g)=0
3     continue
c     set up correct iteration bounds for generating function operations
      if (mod(type(n),2).eq.1) then
        start=d
        fin=d

        else

          if (d.le.b/2) then
            start=0
            fin=d-1

            else

              start=d
              fin=b
            end if
          end if
c     generating function operation
      do 4 k=1, b**2
        num=0
        do 5 l=1, b
          if (bn(l).eq.1) num=num+1
5     continue
        if ((num.lt.start).or.(num.gt.fin)) go to 60
        pl=1.0
c     probability calculation
        do 6 m=1, b
          if (bn(m).eq.1) then
            pl=pl*prb(nodes(n,a+m))
            else
              pl=pl*(1-prb(nodes(n,a+m)))
          end if
6     continue
        p=p+pl
60    if (k.lt.b**2) call bin(1)
4     continue
      if (d.le.b/2) p=1-p
70   calpb=p
      return
      end

      function calspb(n)
      real p,pl
      integer a,b,c,d,fin,start,num,bn(20),type(99),above(99),below(99),
&      nodes(99,20)
      common /nn/ type(99),above(99),below(99),nodes(99,20),prb(99),sprb(99),
&      prt(99),fail(99),time(9),sumocr,ocr(99),total,nrep,nbe,
&      replc(20),numti,mark(99),store(99),bn(20),dpend(99),s(99)
      a=above(n)
      b=below(n)

```

```

c   type of gate defines calspb operation
   if (type(n).ge.6) go to 50
   p=sprb(nodes(n,a+1))
   c=2
   go to (10,20,30,40) type(n)-1
c   AND gate
10  do 1 i=a+2, a+b
    p=p*sprb(nodes(n,i))
1   continue
    go to 70
c   OR gate
20  c=1
c   EXOR gate
30  do 2 j=a+2, a+b
    pl=sprb(nodes(n,j))
    p=p+pl-c*p*pl
2   continue
    go to 70
c   NOT gate
40  p=1-p
    go to 70
c   all other gates
50  p=0.0
c   change type index to (x)a/b form
    d=(type(n)-b*b+5*b-8)/2
    do 3 g=1, b
      bn(g)=0
3   continue
c   set up correct iteration bounds for generating function operations
    if (mod(type(n),2).eq.1) then
      start=d
      fin=d
      else
        if (d.le,b/2) then
          start=0
          fin=d-1
        else
          start=d
          fin=b
        end if
c   generating function operation
    end if
    do 4 k=1, b**2
      num=0
      do 5 l=1, b
        if (bn(l).eq.1) num=num+1
5     continue
        if ((num.lt.start).or.(num.gt.fin)) go to 60
        pl=1.0
c   sprob calculations
    do 6 m=1, b

```



```

        if (bn(m).eq.1) then
            pl=pl*sprb(nodes(n,a+m))
            else
                pl=pl*(1-sprb(nodes(n,a+m)))
            end if
6         continue
          p=p+pl
60        if (k.lt.b**2) call bin(1)
4         continue
          if (d.le.b/2) p=1-p
70        calspb=p
          return
        end

subroutine import
integer l,c,type(99),above(99),total,nrep,replc(20),bn(20),
&         store(99),mark(99)
real p,x
common /nn/ type(99),above(99),below(99),nodes(99,20),prb(99),sprb(99),
&         prt(99),fail(99),time(9),sumocr,ocr(99),total,nrep,nbe,
&         replc(20),numti,mark(99),store(99),bn(20),depend(99),s(99)
c         top event always has a marginal importance of 1.0
          prt(1)=1.0
c         reclaim original mark flags
          do 1 i=1, total
            store(i)=mark(i)
1         continue
c         select only nonreplicated basic events
          do 2 n=1, total
            if (type(n).gt.1) go to 2
            if (above(n).gt.1) go to 2
            call remark(n)
            sprb(n)=0.0
c         basic event first off then on
          do 3 m=1, 2
            if (m.eq.2) sprb(n)=1.0
            do 4 g=1, nrep
              bn(g)=0
4             continue
            c=2**nrep
c         state enumeration on nonreplicated basic event
          do 5 j=1,c
            p=1.0
            do 6 k=1, nrep
              l=replc(k)
              sprb(l)=bn(k)
c         probability calculation
            if (bn(k).eq.1) then
              p=p*prb(l)
              else
                p=p*(1-prb(l))
            end if

```

```

6         continue
          call sreduce(1)
          x=p*sprb(1)
c     marginal importance calculation
          if (m.eq.2) then
            prt(n)=prt(n)+x
          else
            prt(n)=prt(n)-x
          end if
          if (j.lt.c) call bin(1)
5         continue
3         continue
          do 7 h=1, total
            if (store(h).eq.1) sprb(h)=prb(h)
7         continue
2     continue
        return
        end

        subroutine input
        integer type(99),above(99),below(99),nodes(99,20),total,numti,
&         nrep,replc(20)
        character name*60,descrp(99)*16,units*8,pf
        common /nn/ type(99),above(99),below(99),nodes(99,20),prb(99),sprb(99),
&         prt(99),fail(99),time(9),sumocr,ocr(99),total,nrep,nbe,
&         replc(20),numti,mark(99),store(99),bn(20),depend(99),s(99)
        common /cc/ name,descrp(99),units,pf
        read(5,*) name
        read(5,*) pf
        if (pf.eq.'p') go to 10
        read(5,*) numti
        read(5,*) (time(k),k=1, numti)
        read(5,*) units
10       read(5,*) total,nrep
        if (nrep.gt.0) read(5,*) (replc(n),n=1, nrep)
        do 1 i=1, total
          read(5,*) desrcp(i),type(i),above(i),below(i)
          if (type(i).eq.1) then
            read(5,*) prb(i)
c     fail to be changed to a probability in main routine
            fail(i)=prb(i)
          end if
          read(5,*) (nodes(i,j),j=1, above(i)+below(i))
1         continue
        return
        end

        program paft
        integer j,n,type(99),total,nrep,above(99),below(99),replc(20),numti,
&         nbe,nodes(99,20),mark(99),s(99),bn(20),depend(99),store(99)
        real prb(99),sprb(99),prt(99),fail(99),time(9),sumocr,
&         ocr(99)

```

```

character name*60,descr(99)*16,units*8,pf
common /nn/ type(99),above(99),below(99),nodes(99,20),prb(99),sprb(99),
&          prt(99),fail(99),time(9),sumocr,ocr(99),total,nrep,nbe,
&          replc(20),numti,mark(99),store(99),bn(20),depend(99),s(99)
common /cc/ name,descr(99),units,pf
call input
c  prt(top) is always equal to 1.0
  s(1)=1
  nbe=1
c  j keeps count of non-basic events
  j=total
c  set up s array for correct input to qsort and output routines
  do 1 k=1, total
    dpend(k)=0
    mark(k)=1
    if (k.eq.1) go to 1
    n=total+2-k
    if (type(n).eq.1) then
      nbe=nbe+1
      s(nbe)=n
    else
      s(j)=n
      j=j-1
    end if
1  continue
  call match
c  number of output loops
  if (pf.eq.'p') numti=1
  do 2 ndone-1, numti
c  initializations; fail into prb,prb into sprb,and prt
  do 3 i=1, total
    if (type(i).eq.1) then
      if (pf.eq.'f') prb(i)=1-exp(-fail(i)*time(ndone))
    else
      prb(i)=0.0
    end if
    sprb(i)=prb(i)
    prt(i)=0
3  continue
  call rduce(1)
  if (nrep.eq.0) go to 10
  call states
10  call import
  call output(ndone)
2  continue
  stop
  end

subroutine match
integer l,c,n,j,list(99),above(99),nodes(99,20),depend(99),nrep,
&          mark(99),replc(20)

```

```

common /nn/ type(99),above(99),below(99),nodes(99,20),prb(99),sprb(99),
& prt(99),fail(99),time(9),sumocr,ocr(99),total,nrep,nbe,
& replc(20),numti,mark(99),store(99),bn(20),depend(99),s(99)
do 1 i=1, nrep
c list stores replicated basic events already processed
  list(1)=replc(i)
  l=1
  c=1
20 do 2 k=1, above(list(c))
c process starts at replicated basic event and marks up the fault tree
  n=nodes(list(c),k)
  j=1
10 if(list(j).eq.n) dpend(n)=1
  if (dpend(n).eq.1) go to 2
  j=j+1
  if (l.le.1) go to 10
  l=l+1
  list(l)=n
  mark(n)=0
2 continue
c next replicated basic event
  c=c+1
  if (c.le.1) go to 20
1 continue
return
end

subroutine occur(ndone)
integer type(99),total
real q
common /nn/ type(99),above(99),below(99),nodes(99,20),prb(99),sprb(99),
& prt(99),fail(99),time(9),sumocr,ocr(99),total,nrep,nbe,
& replc(20),numti,mark(99),store(99),bn(20),depend(99),s(99)
sumocr=0.0
q=1-prb(1)
do 1 k=2, total
  if (type(k).eq.1) then
    ocr(k)=(1-prb(k))*fail(k)*prt(k)/q
    sumocr=sumocr+ocr(k)
  end if
1 continue
ocr(1)=sumocr
return
end

subroutine output(ndone)
integer a,b,type(99),above(99),below(99),nodes(99,20),total,numti,
& nrep,s(99),nbe
character name*60,descrp(99)*16,blanks*11,units*8,word*6,wrdtype*6,r*2,pf
common /nn/ type(99),above(99),below(99),nodes(99,20),prb(99),sprb(99),
& prt(99),fail(99),time(9),sumocr,ocr(99),total,nrep,nbe,
& replc(20),numti,mark(99),store(99),bn(20),depend(99),s(99)

```

```

common /cc/ name,descr(99),units,pf
blanks='
c   go to time interval output code if ndone is greater than 1
   if (ndone.gt.1) go to 1000
   open(6,form='print')
   write(6,10) name
10  format('Fault tree name: ',a60)
   write(6,20) total.nrep
20  format(' Total number of events: ',i2,'   Number of replicated',
&      ' events: ',i2)
c   go to failure rate output code if input is in terms of failure rates
   if (pf.eq.'f') go to 2000
c   probability's initial output
   write(6,30)
30  format('ONode#   Description           Type   Probability   ',
&      'Above# / Below#')
do 1 k=1, total
   a=above(k)
   b=below(k)
   r='
   if (a.gt.1) r='R-'
   word=wrdtype(type(k))
   write(6,40) r,k,descr(k),word,prb(k)
40  format(lx,a2,i3,3x,a16,lx,a4,2x,e12.5,$)
   if (a.eq.0) write(6,50) blanks
   if (a.eq.1) write(6,60) nodes(k,1)
   if (a.eq.2) write(6,70) nodes(k,1),nodes(k,2)
   if (a.gt.2) write(6,80) (nodes(k,i),i=1, a)
50  format(lx,a11,$)
60  format(9x,i3,$)
70  format(6x,2i3,$)
80  format(3x,20(i3,$))
   write(6,90) (nodes(k,j),j=a+1, a+b)
90  format(lx,'/',20i3)
1   continue
c   probability's secondary output
   write(6,100)
100 format('0                                     Marginal')
   write(6,110)
110 format(' Node#   Description           Type   Probability   Importance')
   call qsort(2,nbe)
do 2 n=1, total
   r='
   if (above(s(n)).gt.1) r='R-'
   word=wrdtype(type(s(n)))
   if ((n.eq.1).or.(type(s(n)).eq.1)) then
       write(6,120) r,s(n),descr(s(n)),word,prb(s(n)),prt(s(n))
           else
       write(6,120) r,s(n),descr(s(n)),word,prb(s(n))
   end if
120 format(lx,a2,i3,3x,a16,lx,a4,2x,e12.5,2x,e12.5,2x,e12.5)

```

```

2      continue
      return
c      failure rate's initial output
2000   write(6,130) units,(time(f),f=1, numti)
130    format(' Time intervals: ',a8,3f8.5)
      write(6,140)
140    format('ONode#   Description           Type   Fail Rate   Above#',
&      ' / below#')
      do 3 l=1, total
          a=above(l)
          b=below(l)
          r='
          if (a.gt.1) r='R-'
          word=wrctyp(type(l))
          if (type(l).eq.1) then
              write(6,150) r,l,descr(l),word,fail(l)
          else
              write(6,160) r,l,descr(l),word,'
          end if
150    format(1x,a2,i3,3x,a16,1x,a4,2x,e12,5,$)
160    format(1x,a2,i3,3x,a16,1x,a4,a14,$)
          if (a.eq.0) write(6,50) blanks
          if (a.eq.1) write(6,60) nodes(1,1)
          if (a.eq.2) write(6,70) nodes(1,1),nodes(1,2)
          if (a.gt.2) write(6,80) (nodes(1,m),m=1, a)
          write(6,90) (nodes(1,g),g=a+1, a+b)
3      continue
c      failure rate's secondary output
1000   write(6,170) time(ndone),units
170    format('OTime interval: ',f8.5,1x,a8)
      call occur(ndone)
      write(6,180) sumocr
180    format(' occurrence rate of top event: ',e12.5)
      write(6,190)
190    format('                               Marginal ',
&      ' Marginal')
      write(6,200)
200    format(' Node#   Description           Type   Probability   Importance',
&      ' Occur Rate')
      call qsort(2,nbe)
      do 4 h=1, total
          r='
          if (above(s(h)).gt.1) r='R-'
          word=wrctyp(type(s(h)))
          if ((h.eq.1).or.(type(s(h)).eq.1)) then
              write(6,120) r,s(h),descr(s(h)),word,prb(s(h)),prt(s(h)),ocr(s(h))
          else
              write(6,120) r,s(h),descr(s(h)),word,prb(s(h))
          end if
4      continue
      return
      end

```

```

subroutine qsort(m,n)
integer i(99),ii(99),j(99),jj(99),l,k,s(99),total
real x
common /nn/ type(99),above(99),below(99),nodes(99,20),prb(99),sprb(99),
& prt(99),fail(99),time(9),sumocr,ocr(99),total,nrep,nbe,
& replc(20),numti,mark(99),store(99),bn(20),dpend(99),s(99)
if ((m.eq.2).and.(n.eq.total)) l=0
c l is recursive level indicator
l=l+1
ii(1)=m
jj(1)=n
i(1)=m
j(1)=n
x=prt(s(j(1)))
c standard recursive quicksort routine
10 if (prt(s(i(1))).le.x) go to 20
i(1)=i(1)+1
go to 10
20 if (prt(s(j(1))).ge.x) go to 30
j(1)=j(1)-1
go to 20
30 if (i(1).le.j(1)) then
k=s(i(1))
s(i(1))=s(j(1))
s(j(1))=k
i(1)=i(1)+1
j(1)=j(1)-1
end if
if (i(1).le.j(1)) go to 10
c resort all smaller than x
if (ii(1).lt.j(1)) then
call qsort(ii(1),j(1))
l=l-1
end if
c resort all larger than x
if (i(1).lt.jj(1)) then
call qsort(i(1),jj(1))
l=l-1
end if
return
end

subroutine rduce(i)
integer n(99),k(99),l,above(99),below(99),nodes(99,20),dpend(99)
common /nn/ type(99),above(99),below(99),nodes(99,20),prb(99),sprb(99),
& prt(99),fail(99),time(9),sumocr,ocr(99),total,nrep,nbe,
& replc(20),numti,mark(99),store(99),bn(20),dpend(99),s(99)
if (i.eq.1) then
l=0
n(1)=1
end if
c l is recursive level indicator
l=l+1

```

```

c      k is iterative breadth pointer
      k(1)=above(n(1))+1
10     n(1+1)=nodes(n(1),k(1))
c      recurse depth first
      if (below(n(1+1)).gt.0) call rduce(n(1+1))
      iterate breadth second
      if (k(1).lt.above(n(1))+below(n(1))) then
        k(1)=k(1)+1
        to to 10
      end if
c      probability calculations
      if (depend(n(1)).eq.0) then
        prb(n(1))=calpb(n(1))
        sprb(n(1))=prb(n(1))
      end if
      l=l-1
      return
      end

      subroutine remark(i)
      integer j,n,store(99),nodes(99,20),mark(99),total
      common /nn/ type(99),above(99),below(99),nodes(99,20),prb(99),sprb(99),
&      prt(99),fail(99),time(9),sumocr,ocr(99),total,nrep,nbe,
&      replc(20),numti,mark(99),store(99),bn(99),depend(99),s(99)
c      store original mark in store array
      do 1 k=1, total
        mark(k)=store(k)
1      continue
      j=i
10     n=nodes(j,1)
c      unmark all nodes above i
      mark(n)=0
      j=n
      if (j.gt.1) go to 10
      return
      end

      subroutine sreduce(i)
      integer n(99),k(99),l,above(99),below(99),nodes(99,20),mark(99)
      common /nn/ type(99),above(99),below(99),nodes(99,20),prb(99),sprb(99),
&      prt(99),fail(99),time(9),sumocr,ocr(99),total,nrep,nbe,
&      replc(20),numti,mark(99),store(99),bn(20),depend(99),s(99)
      if (i.eq.1) then
        l=0
        n(1)=1
      end if
c      l is recursive level indicator
      l=l+1
c      k is iterative breadth pointer
      k(1)=above(n(1))+1
10     n(1+1)=nodes(n(1),k(1))
c      recurse depth first
      if (mark(n(1+1)).eq.0) call sreduce(n(1+1))

```



```

c      iterate breadth second
      if (k(1).lt.above(n(1))+below(n(1))) then
        k(1)=k(1)+1
        go to 10
      end if
c      sprb calculations
      sprb(n(1))=calspb(n(1))
      l=l-1
      return
      end

subroutine states
integer l,c,type(99),total,nrep,replc(20),bn(20),depend(99)
real p,x
common /nn/ type(99),above(99),below(99),nodes(99,20),prb(99),sprb(99),
&      prt(99),fail(99),time(9),sumocr,ocr(99),total,nrep,nbe,
&      replc(20),numti,mark(99),store(99),bn(20),depend(99),s(99)
do 1 i=1, nrep
  bn(i)=0
  continue
state enumeration process
c=2**nrep
do 2 j=1, c
  p=1.0
  do 3 k=1, nrep
    l=replc(k)
    sprb(l)=bn(k)
c      probability calculations
    if (bn(k).eq.1) then
      p=p*prb(l)
    else
      p=p*(1-prb(l))
    end if
3      continue
    call srduce(l)
    do 4 n=1, total
      if (depend(n).eq.0) go to 4
      x=p*sprb(n)
      prb(n)-prb(n)+x
      if (n.gt.1) go to 4
c      replicated basic event marginal importances calculations
      do 5 m=1, nrep
        l=replc(m)
        if (bn(m).eq.1) then
          prt(l)=prt(l)+x/prb(l)
        else
          prt(l)=prt(l)-x/(1-prb(l))
        end if
5          continue
4          continue
        if (j.lt.c) call bin(l)
2      continue
      return
      end

```

```

character*6 function wrdtyp(i)
integer a,b,ind
character word(5)*6, cha*2, chb*2
data word/'BE','AND','OR','EXOR','NOT'/
c if type index is less than 6 return correct data word
if (i.lt.6) then
  wrdtyp=word(i)
  return
end if
c change type index to (a)a/b alpha-numeric form
b=(sqrt(4*i-23.))+5)/2
a=(i-b*b+5*b-8)/2
write(cha,(i2)) a
write(chb,(i2)) b
if (mod(i,2).ne.0) then
  wrdtyp='X'
  ind=2
else
  ind=1
end if
if (a.ge.10) then
  wrdtyp(ind:)=cha//''
  ind=ind+3
else
  wrdtyp(ind:)=cha(2:2)//''
  ind=ind+2
end if
if (b.ge.10) then
  wrdtyp(ind:)=chb
  ind=ind+2
else
  wrdtyp(ind:)=chb(2:2)
  ind=ind+1
end if
return
end

```

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