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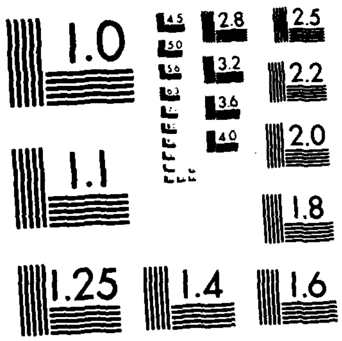

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External Injection Into a High Current Modified Betatron Accelerator

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By axially tapering the current density at each end of an axial pinch a scheme is developed for injecting electrons across magnetic field lines into a modified betatron accelerator. This scheme produces only a minimal perturbation to the circulating electron beam. The axial pinch provides beam equilibrium in the transverse field and the tapering reduces the non-adiabatic growth of the perpendicular particle velocity from 0.5 c to .05 c for the parameters of the NRL modified betatron.

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CONTENTS

EXTERNAL INJECTION INTO A HIGH CURRENT MODIFIED BETATRON ACCELERATOR

Introduction

Presently, there is substantial interest in increasing the current limits of the conventional betatron.[1-2] Several of these new approaches require at least a large toroidal magnetic field[3-6] in addition to the conventional betatron field. The added toroidal field, however, makes the injection into the accelerator considerably more involved, because the injected beam must be transported across field lines before entering the torus.

In the modified betatron, i.e., an accelerator that combines a betatron and toroidal field, this difficulty was avoided by locating the injector inside the torus.[6] Such an injection scheme has several advantages but also three short comings. First, it requires a large opening on the torus, which introduces large field perturbations. Second, the debris of the diode can have an adverse effect on the quality of the vacuum system and third, it requires an injector with a short fall time to avoid perturbing the beam after the first revolution.

To avoid the difficulties of internal injection, we have developed a scheme to propagate the beam across magnetic field lines and thus to locate the injector outside the torus. In the proposed scheme, an axial pinch provides an equilibrium for the beam to cross field lines. Tapering of the current density at both ends of the axial-pinch is necessary to minimize the increase of the perpendicular particle velocity which results from the axial-pinch.

I. Electron Beam Motion

The proposed external injection scheme for the high current modified betatron is shown in Fig. 1. The anode of the injector is located at $y = o$

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and the intense beam propagates in a combined transverse and axial external magnetic field until it reaches the torus. The components of the magnetic field for a uniform current density channel are given by,

$$B_x(z) = B_\psi \frac{z}{a} + B_{xo}, \tag{1}$$

$$B_z(x) = -B_\psi \frac{x}{a}, \tag{2}$$

$$B_y = B_{yo} \; . \tag{3}$$

where B_ψ is the magnitude of the plasma current magnetic field at the plasma radius a and B_{xo}, B_{yo} are the cartesian components of the toroidal magnetic field (B_θ) at the beam trajectory. The fields (B_ψ, B_{xo}, B_{yo}) are taken to be constants and the vertical magnetic field (B_{zo}) is ignored since its effect is the same as that of B_{xo}. In addition, the self fields are ignored, because the axial pinch plasma density is 5–6 orders of magnitude greater than a typical intense electron beam (~ $1KA/cm^2$) and thus charge and current neutralization is assumed.

After a change of variables (from time t to axial position y) using the transformation $\frac{d}{dt} = \upsilon_y \frac{d}{dy}$ and making the paraxial ray approximation [$(x'^2 + z'^2 << 1)$, where the prime is a derivative with respect to y] for the transverse motion, the equations of motion become

$$x'' + \frac{\Omega_\psi}{a\upsilon_o} x = - z' \frac{\Omega_y}{\upsilon_o}, \tag{4}$$

$$z'' + \frac{\Omega_\psi}{a\upsilon_o} \left(z + \frac{\Omega_x}{\Omega_\psi} a\right) = x' \frac{\Omega_y}{\upsilon_o}, \tag{5}$$

$$\frac{dy}{dt} = \upsilon_o \left(1 + x'^2 + z'^2\right)^{-1/2}, \tag{6}$$

where $\Omega_\psi = \frac{|e| B_\psi}{m\gamma}$, $\Omega_y = \frac{|e| B_{yo}}{m\gamma}$, $\Omega_x = \frac{|e| B_{xo}}{m\gamma}$, and $|e|$, m, γ and υ_o are the electronic charge, mass, relativistic factor and the magnitude of the initial velocity vector respectively. Note also, the paraxial ray approximation implies; $\Omega_x \ll \Omega_\psi < \Omega_y$ and small displacements.

According to Eq. (5) the B_x field displaces the particle orbits along the z-axis by, $-\frac{\Omega_x}{\Omega_\psi} a$. This is a general result and not a consequence of the paraxial ray approximation. Beam containment requires therefore that, $\Omega_x \ll \Omega_\psi$, that is, the peak field of the plasma current must be large compared to the crossed magnetic field.

The solution of Eqs. (4) and (5) is given by

$$z\ (y) = -\frac{\Omega_x}{\Omega_\psi} a + \left[x_o \sin k_y y + (z_o + \frac{\Omega_x}{\Omega_\psi} a) \cos k_y y \right] \cos ky$$

$$+ \left[(z_o' - x_o k_y) \cos k_y y + (x_o' + z_o k_y + k_y \frac{\Omega_x}{\Omega_\psi} a) \sin k_y y \right] \frac{\sin ky}{k}, \quad (7)$$

$$x\ (y) = \left[x_o \cos k_y y - (z_o + \frac{\Omega_x}{\Omega_\psi} a) \sin k_y y \right] \cos ky$$

$$+ \left[- (z_o' - x_o\ k_y) \sin k_y y + (k_y\ z_o + x_o' + k_y \frac{\Omega_x}{\Omega_\psi} a) \cos k_y y \right] \frac{\sin ky}{k}, \quad (8)$$

and indicate oscillatory motion around the displaced axis. In Eqs. (7) and (8) $k_y = \frac{\Omega_y}{2\upsilon_o}$, $k = \left(k_y^2 + \frac{\Omega_\psi}{a\upsilon_o}\right)^{1/2}$, and x_o, z_o, x_o', and z_o' are the initial positions and velocities respectively. Particle motion is characterized by two frequencies, a fast cyclotron motion at Ω_y and a slow rotation about the y-axis with a frequency of $\Omega_D = \upsilon_{yo}\ \Omega_\psi / a\ \Omega_y$, where υ_{yo} is the magnitude of the initial axial velocity.

Although the axial pinch can provide the means for crossed field transport it also is the source for the growth of perpendicular energy. From Eqs. (7)-(8), with $x_o' = z_o' = o$, the perpendicular energy is found to be,

$$\varepsilon_{\perp}(y) = 2m\,\upsilon_{yo}^{2}\left[\left(\frac{x_o\Omega_\psi}{a\,\Omega_y}\right)^2 + \left(\frac{z_o\Omega_\psi}{a\,\Omega_y} + \frac{\Omega_x}{\Omega_y}\right)\right]^2\left[1 + 4\,\frac{\Omega_D}{\Omega_y}\right]^{-1}\sin^2 ky.$$

Since $\Omega_\psi >> \Omega_x$ most of the perpendicular energy comes from the pinch, except for particles near the axis. However, in practice, most electron beams have more particles at a large radius.

An accurate calculation of the perpendicular velocity cannot be based on the paraxial equations. However, the more general equations are not easily tractable analytically. For this reason, it is assumed that $\Omega_x = 0$. Under this assumption, it can be shown from the conservation of axial P_y and canonical angular P_ψ momentum that,

$$2t \cong \int_{\zeta_o}^{\zeta} \frac{d\zeta}{\left\{-\left[\frac{\Omega_y^2}{4} + \frac{P_y\,\Omega_\psi}{\gamma ma}\right]\zeta^2 + \left[\upsilon_o^2 - \frac{P_\psi\,\Omega_y}{\gamma m} - \left(\frac{P_y}{\gamma m}\right)^2\right]\zeta - \left(\frac{P_\psi}{\gamma m}\right)^2\right\}^{1/2}}, \quad (9)$$

where $\zeta = \rho^2$, and $\dfrac{\Omega_\psi\,\rho^2/2a}{P_y/\gamma m}$ is the small expansion parameter.

From (9) the maximum radial velocity is,

$$\upsilon_{\rho max} = \frac{\Omega_y\rho_o}{\sqrt{2}}\left\{1 + 2\,\frac{\Omega_D}{\Omega_y} - \left[1 + 4\,\frac{\Omega_D}{\Omega_y}\left(1 - 1/2\,\frac{\Omega_\psi a}{\upsilon_{yo}}\left(\frac{\rho_o}{a}\right)^2\right)\right]^{1/2}\right\}^{1/2},$$

where ρ_o is the initial value of the radius ρ and $\upsilon_{xo} = \upsilon_{zo} = 0$. For, B_y = 2KG, B_ψ = 2KG, $\rho_o = a = 2$ cm, and $\gamma = 7$, the ratio $\dfrac{\upsilon_{\rho max}}{\upsilon_{yo}} = 0.45$, which is too large to be acceptable.

By numerically integrating the exact relativistic particle equations we obtain $\upsilon_{\rho max}/c = 0.45$ (where c = speed of light). The numerically calculated radial velocity is shown in Fig. 2a. The results of Fig. 2a were obtained for the general case, i.e., when a transverse field is present, in addition to the axial and plasma current magnetic fields. The values of the various parameters are listed in the figure captions. Shown also in Fig. 2a are the axial profiles of the fields.

A subtantial reduction of the perpendicular velocity can be achieved by axially tapering the current density at both ends of the pinch. Although the pinch field is the primary source of $\upsilon_{\perp}$, nonadiabatic effects contribute significantly to $\upsilon_{\perp}$ but can be removed by an adiabatic taper. Results are shown in Fig. 2b. For a large taper length a perturbation calculation predicts a $\upsilon_{\perp}$ that is within 20% of the numerical results. To substantially reduce the perpendicular velocity the taper length must be several times the betatron wavelength. An expression for the betatron wavelength can be calculated from Eqs. (6), (7), (8) and is given by,

$$\lambda = 2\pi \frac{\upsilon_{yo}}{\Omega_y} [1 + 4 \frac{\Omega_D}{\Omega_y}]^{-1/2}$$

Figure 3 shows the reduction of the perpendicular velocity as a function of the ratio of taper length to betatron wavelength, this is shown with and without transverse fields. For $B_y = 2KG$, $B_\psi = 2KG$, $a = 2$ cm, $\gamma = 7$ the betatron wavelength is approximately, $\lambda = 10.4$ cm. To reduce $\upsilon_{\perp}/c$ substantially would require a taper length of 20-30 cm.

The feasibility of tapering the current density has been tested experimentally. Presently, a taper length of 25 cm has been achieved experimentally. The measured azimuthal magnetic field B_ψ of the pinch with and without tapering is shown in Fig. 4. This current density profile was achieved by geometrically tapering the vacuum chamber. A cylindrical resistor taper is presently being examined for making a radially compact taper.

II. Plasma Interaction

Calculations indicate that the axial pinch does not substantially interact with the electron beam nor with the external fields. However, there are limits imposed on the plasma density. To avoid an unacceptably large displacement of the plasma column from the $J_y \times B_{xo}$ force and enhancement of the beam emittance, the plasma density should satisfy the inequalities (in CGS)

$$\frac{\varepsilon_N^2 \ \beta^2}{8\pi \ DZ \ (Z+1) \ \ell \ r_e^2 \ \rho_o^2 \ell n \ [a_o \gamma \beta^2 / 2Z^{4/3} r_e]} > n > \frac{B_\psi \ B_{xo} \tau^2}{4\pi \ \Delta z A m_p a} \ ,$$

where ε_N is the maximum beam emittance, ℓ is the length of the plasma column, Z, A are the atomic number and weight of the gas, Δz is the maximum plasma channel displacement, m_p is the proton mass, $r_e = \frac{e^2}{mc^2}$, $a_o = \frac{\hbar^2}{me^2}$, D = 1 for monotomic and D = 2 for diatomic gas. Also, τ is the time that has elapsed from the beginning of the pinch to when the electron beam is injected. Instabilities will be considered elsewhere.

Summary

Using a tapered axial pinch of suitable plasma density and current appears so far to provide a low perturbation scheme for beam injection into a modified betatron. Extensive analytical and computational work indicates that the transverse particle velocity at the exit of the plasma column is substantially smaller than in an untapered pinch. Preliminary experimental results indicates that a pinch with tailored axial current density is achievable.

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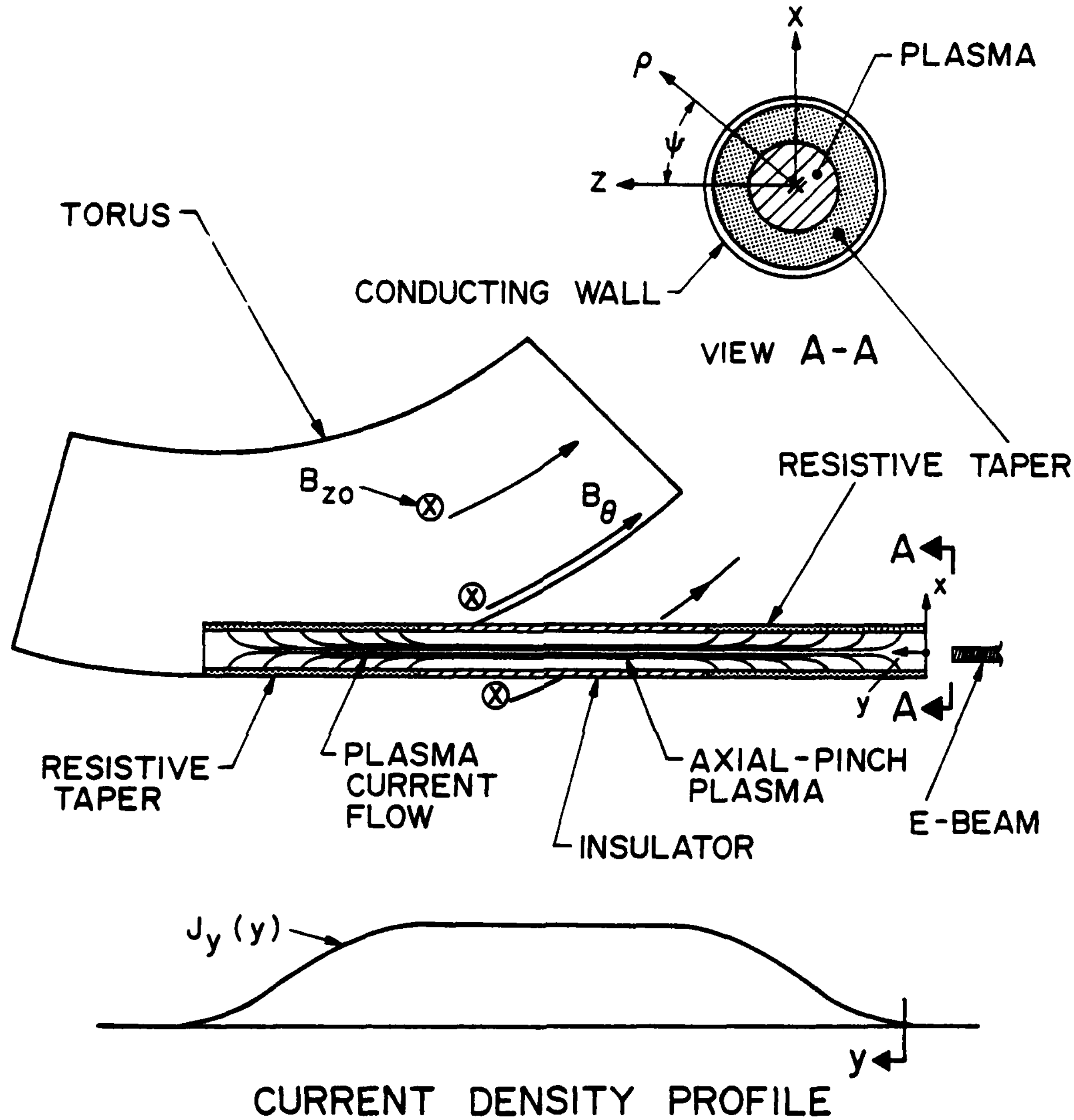
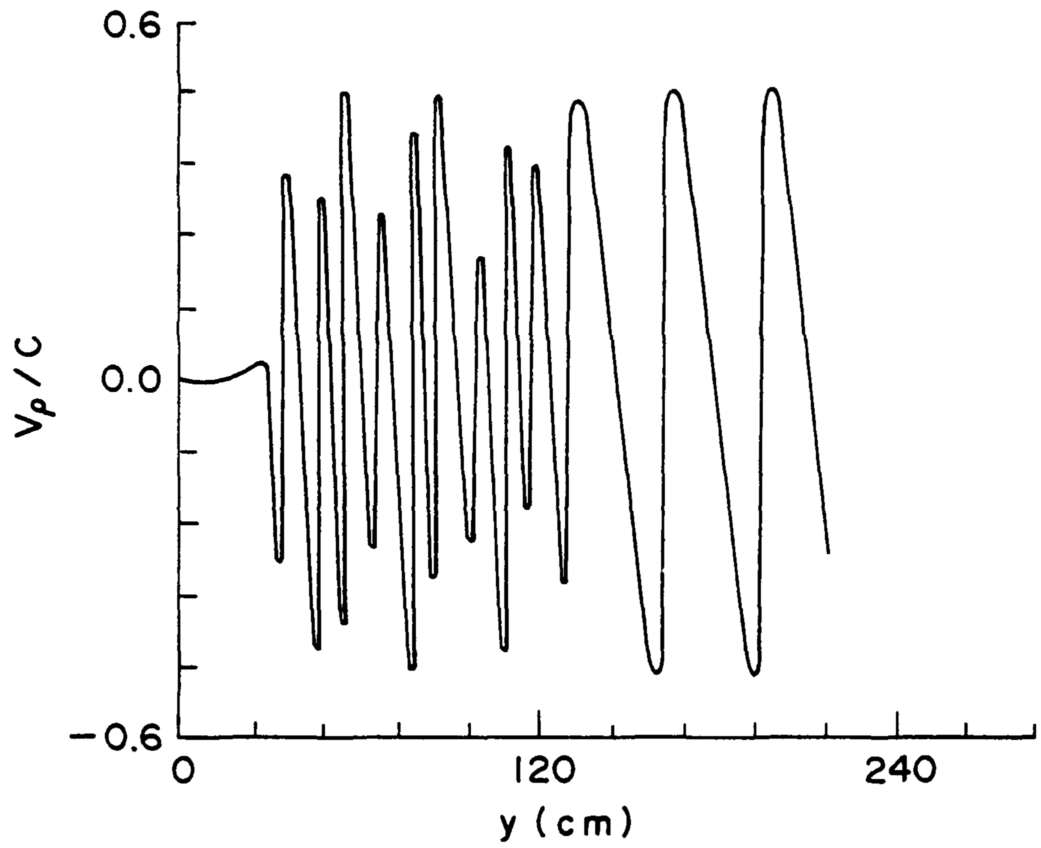


Figure 1 Schematic of tapered axial-pinch injector.



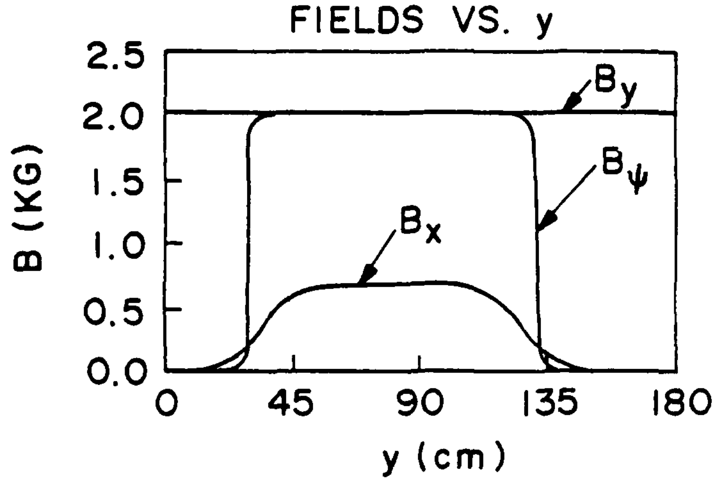
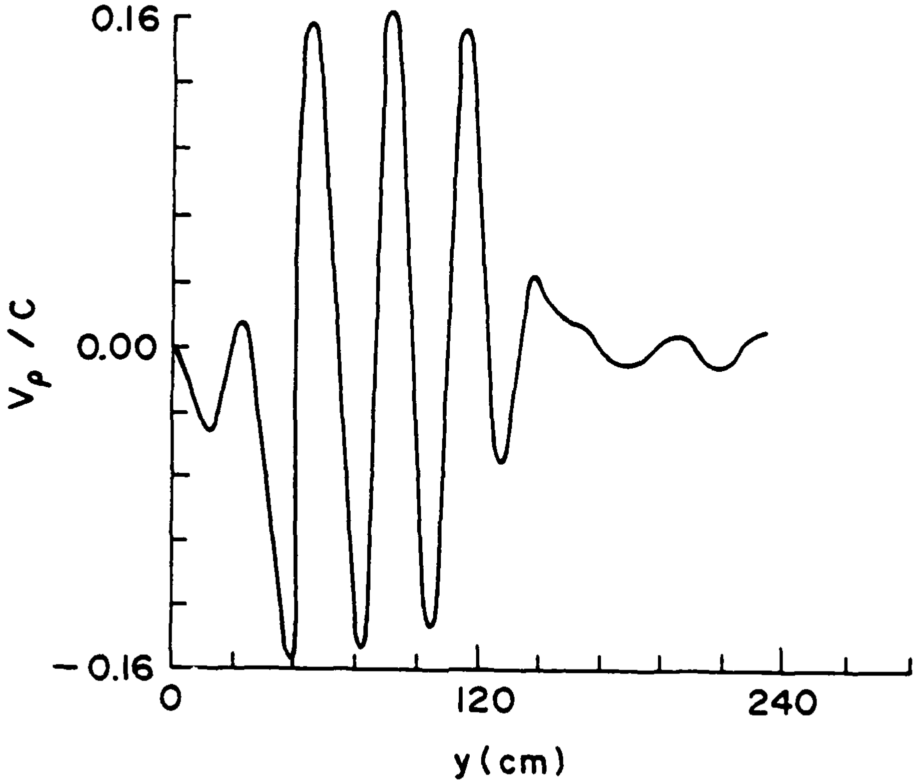


Figure 2a Transverse particle velocity vs axial position. The particle parameters are: γ_y=7, $\gamma_\perp$ = 0 and ρ_0 = a = 2 cm. Taper length from 10-90% amplitude is 2.2 cm. C= speed of light.



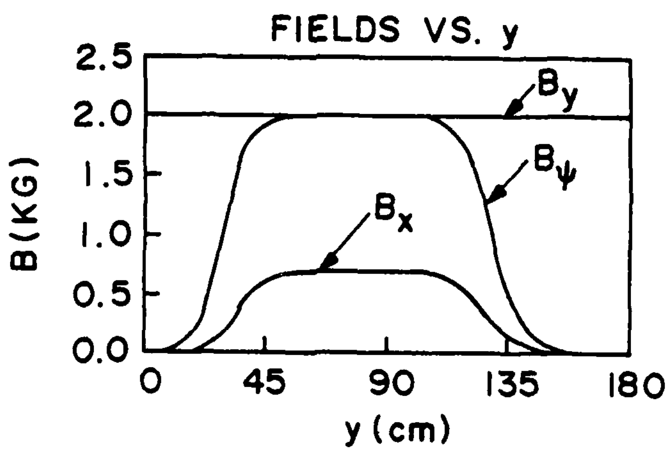


Figure 2b Transverse particle velocity vs axial position. The same parameters as in Fig. 2a but the taper length is 22 cm.

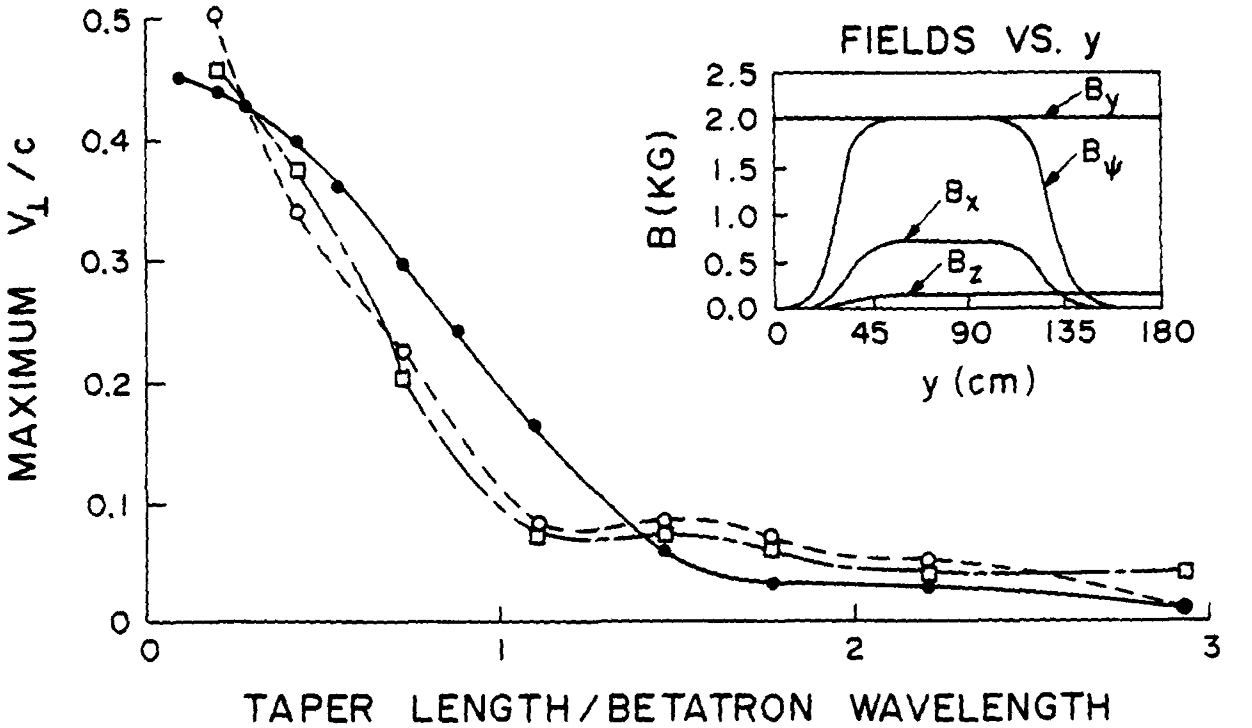


Figure 3 Maximum transverse particle velocity vs. taper length. The same parameters as in Fig. 2a but with variable taper length and crossed fields. -•- $B_z=B_x=0$, -o- $B_z=0$, B_x as shown, -□- B_z, B_x as shown.

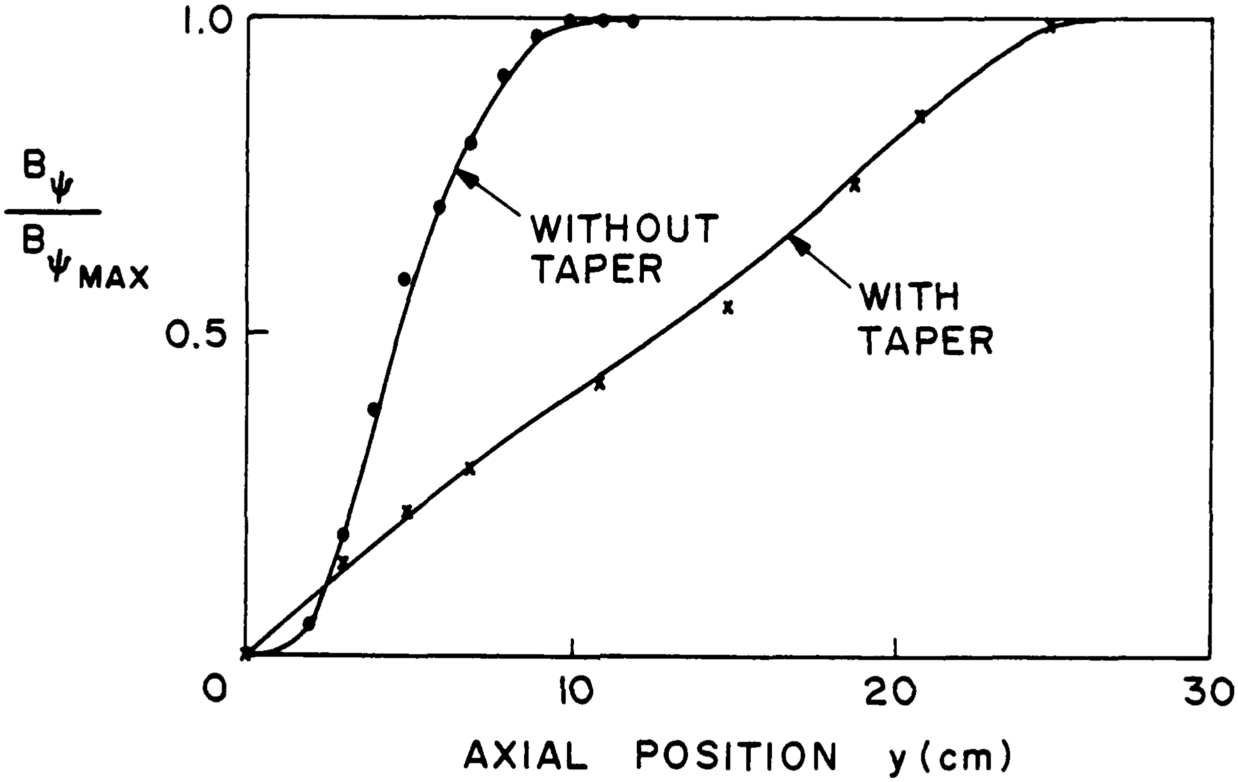


Figure 4 Axial pinch magnetic field vs axial position. Dots and crosses are experimental data and the curves indicate trend of data.

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