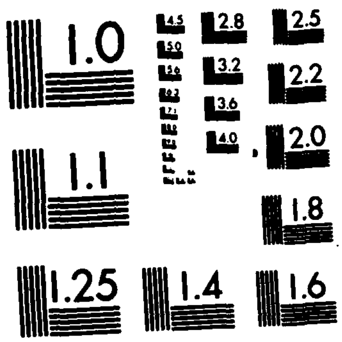
AD-A134 673   NUMERICAL SOLUTION OF SINGULAR INTEGRAL EQUATIONS ARISING IN MIXED BOUNDA..(U) STATE UNIV OF NEW YORK AT STONY BROOK R P SRIVASTAV OCT 83 ARO-16807.12-MA   1/1

UNCLASSIFIED   DAAG29-80-C-0030   F/G 12/1   NL

END



MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS-1963-A

(12)

Unclassified

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

AD-A134 673

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER 16807.12-MA	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) Numerical Solution of Singular Integral Equations Arising in Mixed Boundary Value Problems of Elasticity		5. TYPE OF REPORT & PERIOD COVERED Final: 1 Nov 79 - 30 Jun 83
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) Ram P. Srivastav		8. CONTRACT OR GRANT NUMBER(s) DAAG29 80 C 0030
9. PERFORMING ORGANIZATION NAME AND ADDRESS State University of New York Stony Brook, NY 11794		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
11. CONTROLLING OFFICE NAME AND ADDRESS U. S. Army Research Office Post Office Box 12211 Research Triangle Park, NC 27709		12. REPORT DATE Oct 83
		13. NUMBER OF PAGES 4
14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office)		15. SECURITY CLASS. (of this report) Unclassified
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

The view, opinions, and/or findings contained in this report are those of the author(s) and should not be construed as an official Department of the Army position, policy, or decision, unless so designated by other documentation

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

numerical analysis
boundary value problems
elasticity

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

The central theme of this project was the development and analysis of direct methods based on collocation for the solution of singular integral equations with a principal value integral. These equations arise in such diverse fields as linear elastic fracture mechanics, neutron transport, long water waves, image reconstruction and radiative transfer. In the classical approach, the singular integral equation is regularized to yield a Fredholm integral equation of the second kind. The numerical implementation of the regularization is usually quite cumbersome. While

DD FORM 1473, 1 JAN 73 EDITION OF 1 NOV 65 IS OBSOLETE

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

83 11 07 039

ABSTRACT (cont.)

direct methods proposed by Erdogan (1969), Erdogan and Gupta (1972), and Theocaris and Ioakimidis (1977) have been used in Fracture Mechanics, their convergence and stability has received scant attention. The work at Stony Brook has made important strides in both directions. We have analysed the behavior of approximate solutions enhancing our understanding of the convergence and stability of the methods based on Gaussian quadrature. We have also proposed methods which can be employed in the situations where the Gaussian quadrature-collocation schemes fail.

FINAL REPORT

I. ARO Proposal Number : 16807 - M

II. Contract Number : DAAG 29 - 80-C-0030

III. Title of the proposal : Numerical Solution of Singular Integral Equations arising in Mixed Boundary Value Problems of Elasticity

IV. Contract Period : November 1, 1979 - June 30, 1983

V. Name of the Institution : State University of New York at Stony Brook

VI. Scientific Personnel supported by the contract:

1. Ram P. Srivastav : Principal Investigator and Project Director
2. Chen, Kim-Joan : Graduate Research Assistant (awarded Ph.D.)
3. Don, Eugene : Graduate Research Assistant
4. Driscoll, Michael : Graduate Research Assistant (awarded M.S.)
5. Jen, Erica : Graduate Research Assistant (awarded Ph.D.)
6. Ong, Michael K. : Graduate Research Assistant
7. Osorio, Luis : Graduate Research Assistant (awarded M.S.)
8. Suresh, Ambadi : Graduate Research Assistant
9. Svirsky, Vladimir: Graduate Research Assistant (awarded M.S.)
10. Venturino, Ezio : Graduate Research Assistant (awarded M.S.)

VII. A statement of the problem studied and a summary of the most important resul (The numbers refer to the list of publications in Section VIII of this report. Other references are included in the last part of this section.)

The central theme of this project was the development and analysis of direct methods based on collocation for the solution of singular integral equations with a principal value integral. These equations arise in such diverse fields as linear elastic fracture mechanics, neutron transport, long water waves, image reconstruction and radiative transfer. In the classical approach, the singular integral equation is regularized to yield a Fredholm integral equation of the second kind. The numerical implementation of the regularization is usually quite cumbersome. While direct methods proposed by Erdogan(1969), Erdogan and Gupta(1972) , and Theocaris and Ioakimidis(1977) have been used in Fracture Mechanics, their convergence and stability has received scant attention. The work at Stony Brook has made important strides in both directions.

We have analysed the behavior of approximate solutions enhancing our understanding of the convergence and stability of the methods based on Gaussian quadrature. We have also proposed methods which can be employed in the situations where the Gaussian quadrature-collocation schemes fail.

The key to successful analysis of the aforesaid direct methods was the discovery that the coefficient matrix of both Gauss-Chebyshev and Lobatto-Chebyshev techniques have a closed form inverse, which is the transpose of the coefficient matrix multiplied by a diagonal matrix [1], [2]. Using Jackson's theorem on "best approximation", the convergence of interpolatory polynomials can be easily established. By refining the techniques we are able to prove [3] the convergence under weaker condition. [4] contains an extension of the analysis of [1] to the complete singular integral equation. The nodes of quadrature and collocation now are the zeros of Jacobi polynomials. To cover the general case of a Fredholm kernel added to a Cauchy kernel, the theory of collectively compact operators is used. [5] is addressed to the problem of computing bounded solutions. In this case a compatibility condition has to be satisfied. Often the discrete system representing the continuous is overdetermined and has one equation more than the number of unknowns. We find that the solutions vary with the equation ignored (the usual engineering practice) and the error peaks in the vicinity of the collocation point which is discarded to obtain a compatible system. Use of general collocation points in conjunction with Gaussian quadrature is examined in [6]. The question of an optimal selection of nodes is still open.

While both Gauss-Chebyshev and Lobatto-Chebyshev methods are easy to code, they are not suitable for functions which are highly oscillatory in some small neighborhood. Since the collocation points are the zeros of Chebyshev polynomials, the only way their concentration can be increased is by increasing the degree of the polynomial. Since the largest distance between the zeros of Chebyshev polynomials is approximately π/n, for a resolution of the order .001 a polynomial of order 3000 has to be considered.

In [7] we propose a method based on the use of piecewise linear functions. [8] develops the procedure further employing cubic splines. The order of singularity is explicitly built into the solution. Nemet-Nasser and Horii (1982) found that the results obtained through piecewise polynomial approximation indistinguishable from the corresponding results obtained by the Erdogan-Gupta method for the problem of kinked crack extension under tension. [9] contains a review of the Gaussian integration methods for the numerical solution of singular integral equations.

Some of the results of our investigation are yet to be published. To reduce an integral equation to an algebraic system whose solution leads to a polynomial approximation to the solution usually one of the two approaches is adopted:1. The weighted norm of the difference between the left hand side and the right hand side is minimized by choosing the coefficients of the polynomial (The Galerkin Method) 2. The equation is satisfied only over a finite set of points and the integral is replaced by a Quadrature formula (The collocation method). We have been able to show that the two solutions are related by means of a Gaussian quadrature formula [10] . [11] contains a comparison of the accuracy of stress intensity factors for a Griffith crack problem using Gauss-Chebyshev and Lobatto-Chebyshev quadrature formulas. [12] contains a proof that the coefficient matrix obtained through the "tanh rule" and collocation is nonsingular. Finally, [13] treats the problem of a cruciform crack opened by internal pressure varying with time. For time dependent problems it brings into focus the need for methods which are both fast and accurate.

We are in the process of writing the final versions of the papers [10,11,12] and U.S. Army Research Office will be furnished copies when they are ready.

References

Erdogan, F. and Gupta, G.D. "On the numerical solution of singular integral equations", Q.Appl.Math. (1972) Vol. 30, pp. 525-534

Theocaris, P.S. and Ioakimidis,N.I. "Numerical integration methods for the solution of singular integral equations", Q.Appl.Math.(1977) vol. 35, pp. 173-183

Nemet-Nasser,S. and Horii, H. " Compression-induced Nonplanar Crack extension with application to Splitting, Exfoliation, and Rockburst" J. Geophysical Res., Vol. 87, pp. 6805-6821.

VIII. Research Publications

1. Numerical Solution of Singular Integral Equations Using Gauss-Type Formulae - I: Quadrature and Collocation on Chebyshev nodes, IMA Journal on Numerical Analysis (to appear soon).

2. Numerical Solution of Singular Integral Equations Using Gauss-Type Formulae - II: Lobatto-Chebyshev Quadrature and Collocation on Chebyshev Nodes (with Erica Jen), IMA Journal on Numerical Analysis (to appear soon).

3. On the Polynomials Interpolating Approximate Solutions of Singular Integral Equations (with Erica Jen), Applicable Analysis 14 (1983), 275-285.

4. On the Solvability of Singular Integral Equations in a Gauss-Jacobi Quadrature (with A. Gerasoulis), International J. Computer Math. 12 (1982), 59-75.

5. Solving Singular Integral Equations Using Gaussian Quadrature and Overdetermined Systems (with Erica Jen), Computers and Mathematics with Applications 9 (1983), 625-632.

6. On Solving Singular Integral Equations of Cauchy Type Using Gaussian Quadrature and General Collocation Scheme. To be published in the Proceedings of the Silver Jubilee Symposium on Mechanics and Approximation, Indian Institute of Technology, Bombay.

7. A Method for the Numerical Solution of Singular Integral Equations with a Principal Value Integral (with A. Gerasoulis), Int. J. Engng. Sci. 19 (1981), 1293-1298.

8. Cubic Splines and Approximate Solution of Singular Integral Equations (with Erica Jen), Math. Comp. 37 (1981), 417-423.

9. On the Gaussian Integration Method for the Numerical Solution of Singular Integral Equations - Advances in Computer Methods for Partial Differential Equations, Proc. 4th IMACS Conference (1981), pp. 332-334.

10. On the Approximate Solutions of Singular Integral Equations Using Galerkin-Type Methods and Collocation (with E. Venturino) (to be published).

11. A Comparison of Stress Intensity Factors for a Griffith Crack Problem Using Gauss-Chebyshev and Lobatto-Chebyshev Quadrature Formulae (to be published).

12. On Solving Singular Integral Equations of Cauchy Type Using "tanh rule" (with E. Venturino) (to be published).

13. A Cruciform Crack Opened by Internal Pressure Varying with Time (with M. K. Ong), submitted to International J. Engng. Sci.