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A TIME SERIES ANALYSIS OF SOME INTERRELATED
LOGISTICS PERFORMANCE VARIABLES*

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ABSTRACT

This paper is a case study. We show how the powerful methods of time
series analysis can be used to investigate the interrelationships between Alert
Availability, a logistics performance variable, and Flying Hours, an operational
requirement, in the presence of a major change in operating procedures and us-
ing contaminated data. The system considered is the fleet of C-141 aircraft of
the U.S. Air Force. The major change in operating procedures was brought
about by what is known as Reliability Centered Maintenance, and the contam-
inated data were due to anomalies in reporting procedures. The technique used
is a combination of transfer function modeling and intervention analysis.

1. INTRODUCTION AND SUMMARY

In January 1976, the U.S. Air Force began some experimental modifications to the exist-
ing maintenance policies for the fleet of
C-141 aircraft. These modifications were a part of a Department of Defense project known as
"Reliability Centered Maintenance," henceforth denoted by RCM. The modifications involved
an *extension* of the maintenance intervals and a *reduction* in the amount of scheduled mainte-
nance. The experimental phase of the project ended in June 1977, and the modified policies
were officially and permanently instiued at that time. The anticipated benefit from RCM was a
decrease in scheduled maintenance activity, with a consequent increase in "alert availability."
Alert availability, henceforth denoted by AA, is the instantaneous probability that a typical air-
craft is available to react to an execution order. In practice, its average value, over say a
month, is computed by dividing the monthly total number of "fleet operational hours" by the
total number of "fleet available hours."† A plot of AA from October 1973 through November
1979 is shown in Figure 1.

†*Fleet available hours* is the cumulative time of possession across a fleet of aircraft, whereas *fleet operational hours* is the
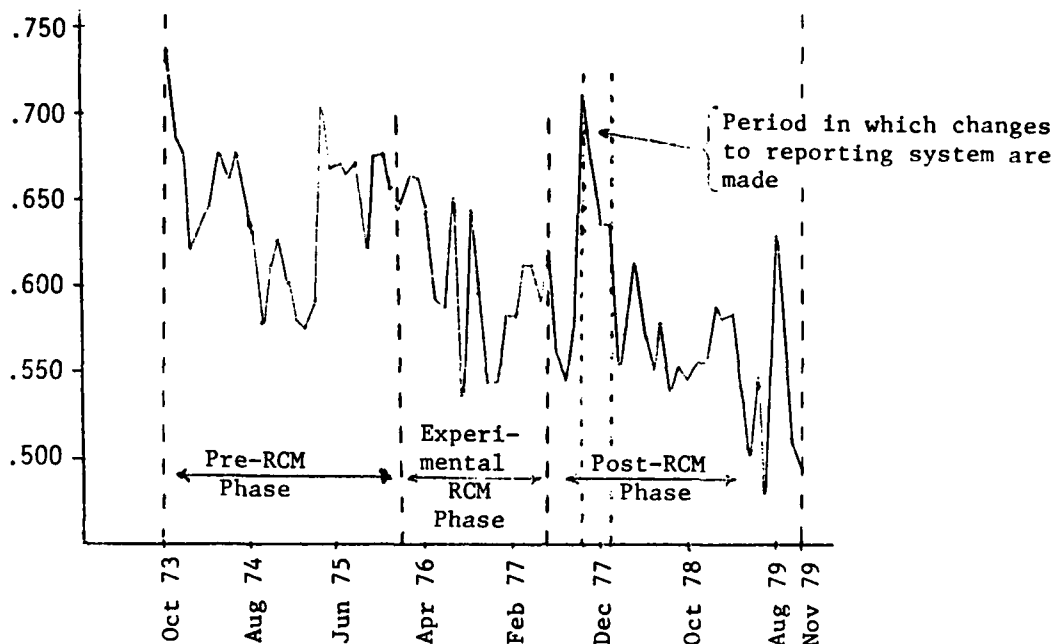cumulative amount of operational time (i.e., up-time) for the fleet.

FIGURE 1. A plot of C-141 alert availablity from October 1973 to November 1979

In a previous study (Singpurwalla and Talbott [5]), we investigated the effects of RCM on several variables which describe what is known as the "logistics performance" of the fleet. Our conclusion was that there was no evidence of an improvement in the logistics performance of the fleet due to RCM; on the contrary, in some cases there was a clear indication of deterioration in performance. These conclusions were particularly true of AA, which is considered to be an important logistics performance variable. One criticism of this previous study, and a valid one, is that it did not take into consideration the influence of other "operational variables," which in addition to RCM may affect the logistics performance variables. An important operational variable, which is suspected of being strongly related to the AA, is "flying hours." *Flying hours* is the total number of hours flown by the fleet of the C-141's over a certain period of time, say one month. A plot of the monthly flying hours, from October 1973 through November 1979, is shown in Figure 2. Another criticism of our previous study, and again a valid one, is that it did not adequately account for the fact that some of the AA data were "messy." Specifically, there were some revisions to the information system for reporting the operational status of the aircraft that resulted in some possible anomalies in the reported values of the AA.

The analysis that is described here was initiated with a view towards rectifying the limitations of our previous study. Here, using the AA anu the flying hours as examples, we demonstrate how a procedure for investigating the interrelationships between the two, using messy data and RCM as an intervention, can be developed and used. The approach we take is a combination of those described in Box and Jenkins [1], pp. 335-420) for the analysis of multiple time series, and in Box and Tiao [2] for intervention analysis. Our conclusion is that the AA is
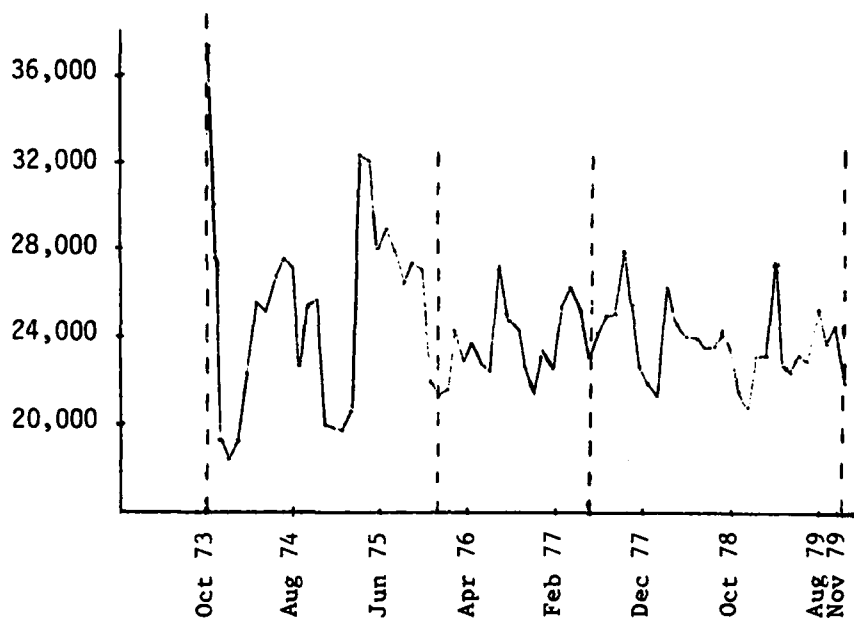
FIGURE 2. A plot of C-141 flying hours from October 1973 to November 1979

indeed related to the flying hours, as has been conjectured, but that even after taking this relationship into consideration, and in the presence of messy data, our previous conclusion still holds, namely, that there is no clear evidence of improvement in the AA after the initiation of RCM.

Before going into the details of our analysis, we want to emphasize that the above conclusion and its practical implications are not intended to be the main theme of this paper. Rather, our aim is to suggest and to demonstrate how the powerful methods of time series analysis can be used to analyze the messy and interrelated data that often arise in a study of the reliability and the logistics performance of large military systems.

In what follows, we presume that the reader has a knowledge of autoregressive integrated moving average (ARIMA) processes and is familiar with the notation, terminology, and methodology described in Box and Jenkins [1].

2. NOTATION, PRELIMINARIES, AND AN OUTLINE OF THE PROCEDURE

Let X and Y be two variables of interest, and let X_t and Y_t be their values at time t. In our case, we let X_t denote the total flying hours for the fleet of C-141's during the tth month, and Y_t the alert availability during that month; t varies from October 1973 through November 1979. A sequence of values $X_t, X_{t-1}, \ldots$ will be denoted by $\{X_t\}$.

To discern the relationship between X_t and Y_t, we strive to obtain a *linear transfer function model* (Box and Jenkins [1], p. 379) of the form

$$(2.1) \qquad (1 - \delta_1 B - \ldots - \delta_r B^r) Y_t = (w_0 - w_1 B - \ldots - w_s B^s) X_{t-b} + N_t,$$

where $B^m X_t = X_{t-m}$, $m = 0, 1, \ldots$, and $\delta_1, \ldots, \delta_r, w_0, \ldots, w_s$ are unknown constants to be estimated; b represents the lag of Y_t with respect to X_t; and N_t denotes a noise component described by a suitable ARIMA process.

Equation (2.1) describes the alert availability at time t in terms of the previous values of the alert availability and the present and previous values of the flying hours.

Following the procedures suggested by Box and Jenkins [1] and by Haugh and Box [3], we first consider the sequence (series) $\{X_t\}$ and transform it to a "white noise series," $\{a_t\}$, by way of an appropriate univariate time series model. Similarly, we also reduce the sequence $\{Y_t\}$ to a white noise series, $\{b_t\}$, using an appropriate "intervention analysis model" (see Box and Tiao [2]). Such a model is necessary here in order to account for RCM as well as for the presence of some anomalous observations in $\{Y_t\}$. This procedure of reducing $\{X_t\}$ and $\{Y_t\}$ to $\{a_t\}$ and $\{b_t\}$, respectively, is known as *prewhitening* the respective series. We then cross correlate the two prewhitened sequences $\{a_t\}$ and $\{b_t\}$ in order to obtain an indication of the relationship between $\{X_t\}$ and $\{Y_t\}$. The cross correlation function is used to suggest values for r,s, and b in (2.1). Finally, we use the prewhitened sequences $\{a_t\}$ and $\{b_t\}$ to estimate the constants $\delta_1, \ldots, \delta_r, w_0, \ldots, w_s$ for some selected values for r, s, and b.

Regarding the prewhitening of $\{Y_t\}$ using a model for intervention analysis, we remark that intervention due to RCM can be described by a sequence of indicator variables, say $\{Z_t\}$, where Z_t takes a value of 0 for all t representing the months prior to January 1976, and a value 1 for all t thereafter. Recall that January 1976 is the date of intervention—the date at which experimental RCM was initiated. The response of the sequence $\{Y_t\}$, the *output sequence* denoting AA, to the *input sequence* $\{Z_t\}$ can take various functional forms. These are depicted in Figure 3; they have been taken from Box and Tiao [2]. We strive to use the most appropriate form of the response function for the situations at hand.

3. PREWHITENING THE FLYING HOURS SERIES $\{X_t\}$

An inspection of Figure 2 finds that the fluctuations of the series about its mean changes over time. In such situations it is common to take the natural logarithms of the data. Accordingly, our analysis of flying hours involves a logarithmic transformation of the original data.

We find that flying hours can best be described by an ARIMA $(2,0,0) * (2,0,0)_4$ process. However, the residuals from this model reveal significant autocorrelations at lags 5, 10, 15, $\ldots$, etc. Consequently, we fit an ARIMA $(0,0,0) * (0,0,1)_5$ model to these residuals. Based upon these considerations, an appropriate model for prewhitening can be written as

$$(3.1) \qquad (1 - .54B + .11B^2)(1 + .38B^4 + .37B^8)(X_t - 10.09)$$
$$= (1 + .18B^5)a_t.$$

This model produces residuals whose autocorrelation function (ACF) and log spectral density are shown in Figure 4, in (a) and (b), respectively. These residuals are listed in Table 1.

4. PREWHITENING THE ALERT AVAILABILITY SERIES $\{Y_t\}$

In our previous study we considered alert availability as two separate series, one ending prior to January 1976 and the other beginning with January 1976. We found that the loga-
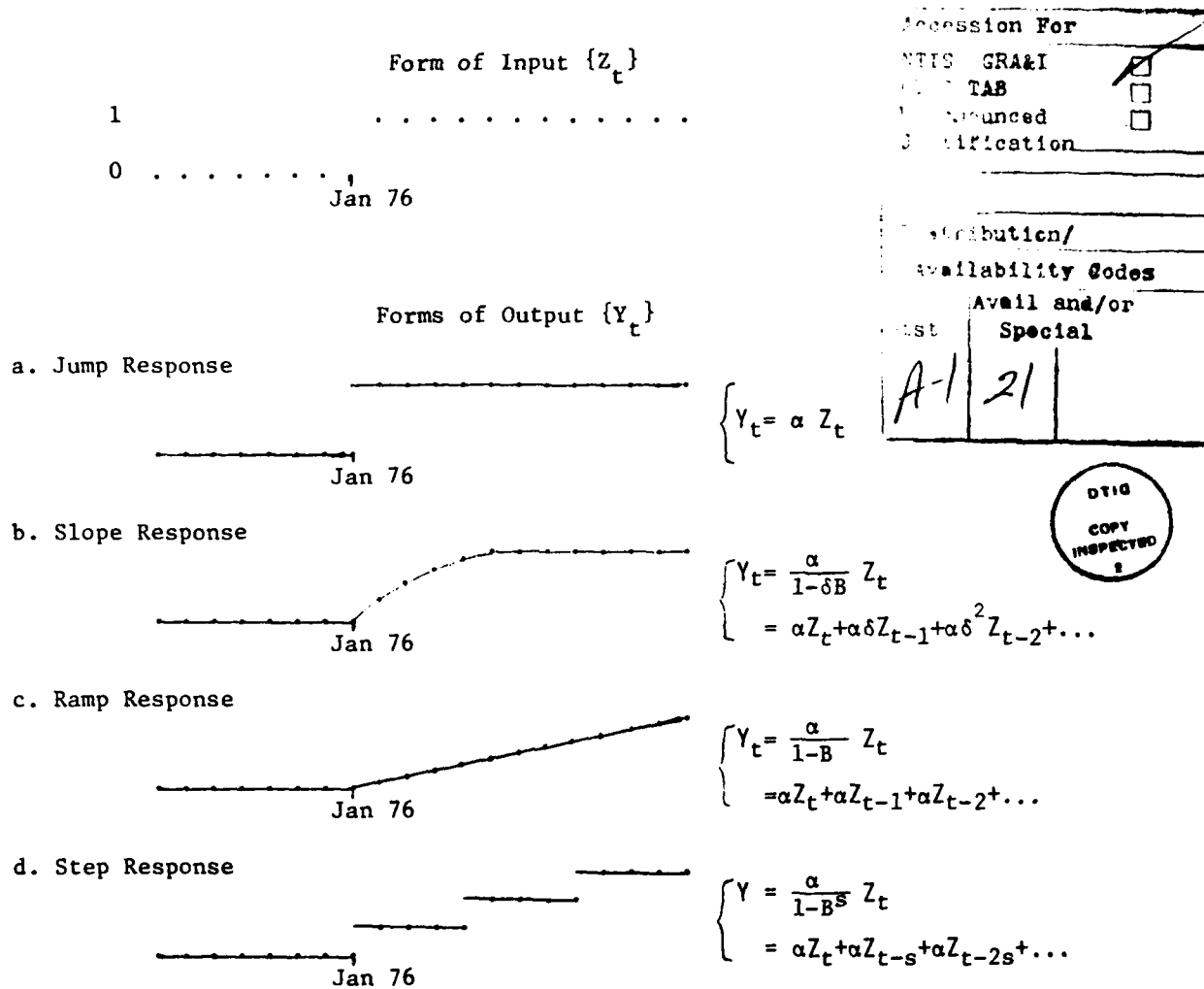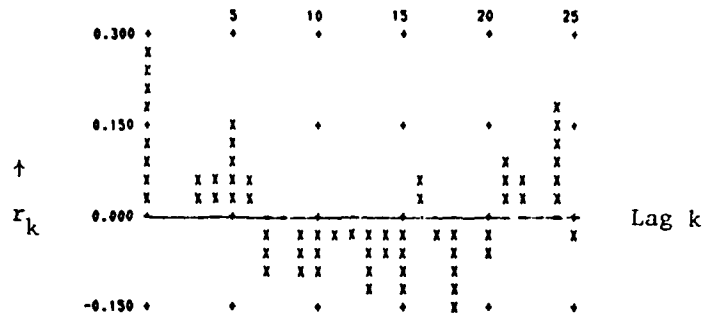
Form of Input $\{Z_t\}$

FIGURE 3. Typical forms of the response $\{Y_t\}$ to the input $\{Z_t\}$

Forms of Output $\{Y_t\}$

a. Jump Response

$$\left\{ Y_t = \alpha \, Z_t \right.$$

b. Slope Response

$$\left\{ \begin{array}{l} Y_t = \dfrac{\alpha}{1-\delta B} \, Z_t \\ = \alpha Z_t + \alpha \delta Z_{t-1} + \alpha \delta^2 Z_{t-2} + \ldots \end{array} \right.$$

c. Ramp Response

$$\left\{ \begin{array}{l} Y_t = \dfrac{\alpha}{1-B} \, Z_t \\ = \alpha Z_t + \alpha Z_{t-1} + \alpha Z_{t-2} + \ldots \end{array} \right.$$

d. Step Response

$$\left\{ \begin{array}{l} Y = \dfrac{\alpha}{1-B^s} \, Z_t \\ = \alpha Z_t + \alpha Z_{t-s} + \alpha Z_{t-2s} + \ldots \end{array} \right.$$

(a) Graph of observed series ACF
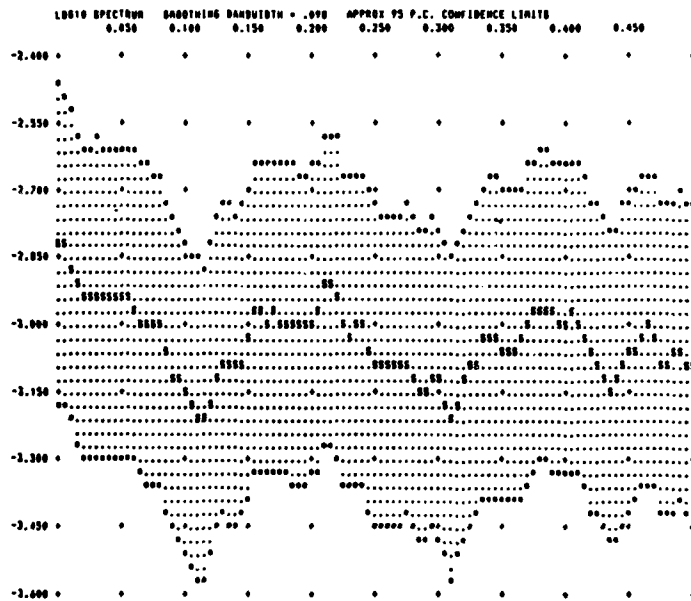
(b) Graph of log spectral density

FIGURE 4. Behavior of the ACF and the log of the spectral density of the residuals from an ARIMA $(0,0,0) \bullet 0,0,1)_5$ model fit to residuals from an ARIMA $(2,0,0) \bullet 2,0,0)_4$ model

TABLE 1 — *Residuals from Prewhitening Models*
for Flying Hours and Alert Availability

TIME T	A (T)	B (T)	TIME T	A (T)	B (T)
JAN 1975	−.131303		JUL 1977	.037152	−.053020
FEB 1975	−.026086	−.074950	AUG 1977	.022717	.012580
MAR 1975	−.034914	−.003150	SEP 1977	.021623	−.045700
APR 1975	.283323	.108000	OCT 1977	.080163	.053860
MAY 1975	.060148	.026820	NOV 1977	.037052	.039220
JUN 1975	−.032649	.056160	DEC 1977	−.048706	−.063430
JUL 1975	.135207	.085220	JAN 1978	−.047455	−.011080
AUG 1975	.124353	.105500	FEB 1978	−.058318	.016660
SEP 1975	−.007629	.039270	MAR 1978	.130172	−.010630
OCT 1975	.059647	−.003746	APR 1978	−.053118	.013570
NOV 1975	.046232	.035070	MAY 1978	−.008318	.001221
DEC 1975	.020893	.007938	JUN 1978	.020092	−.023900
JAN 1976	−.025669	.033680	JUL 1978	.036141	.067210
FEB 1976	−.021861	.054060	AUG 1978	−.073053	−.013030
MAR 1976	.109077	.095080	SEP 1978	−.002367	−.015090
APR 1976	−.101662	−.033110	OCT 1978	−.050561	−.099890
MAY 1976	.001382	−.081130	NOV 1978	−.054985	.001532
JUN 1976	−.044392	−.057940	DEC 1978	−.110742	.036330
JUL 1976	−.012095	.068650	JAN 1979	.041226	.051630
AUG 1976	.068336	−.175700	FEB 1979	−.043154	−.043154
SEP 1976	−.047820	.075940	MAR 1979	.106237	.047260
OCT 1976	−.035721	−.047270	APR 1979	−.173297	−.027790
NOV 1976	−.066278	−.066570	MAY 1979	.007886	−.090300
DEC 1976	−.044366	−.042450	JUN 1979	−.031905	.030150
JAN 1977	.003072	−.040260	JUL 1979	−.014306	−.183300
FEB 1977	−.073529	.076220	AUG 1979	−.027671	.163600
MAR 1977	.045873	−.014810	SEP 1979	−.030462	.029880
APR 1977	.097139	.062520	OCT 1979	.026290	−.062220
MAY 1977	.008354	.098570	NOV 1979	−.070995	−.015390
JUN 1977	−.089749	.137600			

rithms of the observations in both these series could best be described by an ARIMA $(1,0,0)$ *
$(1,1,0)_6$ model. Here, we prewhiten the *entire* series by considering the onset of RCM as an
intervention and incorporating this effect into the ARIMA $(1,0,0)$ * $(1,1,0)_6$ model.

Figure 1 indicates clearly that the AA had been decreasing after January 1976, the onset
of RCM. However, the rate of decrease of the AA during the experimental phase of RCM is
greater than the rate of decrease during the post-RCM phase. This suggests that the effect of
RCM during the experimental phase may be different from the effect after the experimental
phase. One possible reason for this difference is that there was a piecemeal introduction of
RCM, air base by air base, during the experimental phase, along with several trial revisions in
the maintenance policies. We also notice some large spikes in value of the AA during October,
November, and December 1977 and January 1978. A cause for these large values may be that
the information system for reporting the operational status of aircraft was revised during this
period. The AA may have been artificially increased during this period due to anomalies of
reporting. Thus, it appears that we need three distinct components to our intervention analysis
model. We represent these components by three indicator variables, $J_t, K_t,$ and $L_t,$ where:

$$J_t = \begin{cases} 1 & \text{for Jan 76} \leqslant t \leqslant \text{May 77} \\ 0 & \text{otherwise} \end{cases}$$

$$K_t = \begin{cases} 1 & \text{for Jun 77} \leqslant t \leqslant \text{Sep 77} \\ 1 & \text{for Feb 78} \leqslant t \leqslant \text{Nov 79} \\ 0 & \text{otherwise} \end{cases}$$

$$L_t = \begin{cases} 1 & \text{for Oct 77} \leqslant t \leqslant \text{Jan 78} \\ 0 & \text{otherwise} \end{cases}$$

Note that the union of the three sequences $\{J_t\}, \{K_t\},$ and $\{L_t\}$ constitutes the series $\{Z_t\},$
defined earlier.

As for the functional form of the response (series $\{Y_t\}$) to these components (see Figure
3), we remark that ramp and step responses appear to be possible candidate forms for $\{J_t\}$ and
$\{K_t\}$, whereas a jump response is appropriate for $\{L_t\}$. We exclude a jump response for the first
two because it would not account for the gradual decline evident in the data, and we consider a
slope response to be inappropriate due to the absence of a sustained leveling of the data. We
conjecture that a step response might have a period of three months, which coincides with the
length of time between scheduled minor inspections. Our aim now is to compare the functional
forms of the ramp response versus the step response in order to determine which combination
might best represent the relationship between AA and the series $\{Z_t\}$.

We fit all reasonable combinations of response forms with the variables $J_t,$ $K_t,$ and $L_t,$
and find that step type responses, along with the original ARIMA model, i.e., a model of the
form

$$(4.1) \qquad Y_t = \frac{\alpha_1}{1 - B^3} J_t + \frac{\alpha_2}{1 - B^3} K_t + \alpha_3 L_t + N_t,$$

where N_t denotes an ARIMA $(1,0,0) * (1,1,0)_6$ process, produces residuals that are most satisfactory (see Figure 5). The model (4.1) describes the series $\{Y_t\}$ as a step function with three-month incremental decreases of size α_1 during the experimental RCM phase, as a step function with three-month incremental decreases of size α_2 after implementation of RCM, and as a jump function with height α_3 during the October 1977-January 1978 time frame. Thus we have the prewhitening model

$$(4.2) \qquad (1 - .18B)(1 + .35B^6)(1 - B^6)\, Y_t = \frac{-.02}{1 - B^3} J_t + \frac{-.01}{1 - B^3} K_t$$

$$+ .15 L_t + b_t.$$

We remark that the estimated parameters α_1 and α_2 are negative, as expected, with α_1 greater than α_2. Also, the estimated parameter α_3 is positive, which is intuitively apparent. We also remark that in the residual series (see Figure 5) there remain some significant auto-correlations at lag 12. We can account for this by fitting an ARIMA $(0,0,0)*(2,0,0)_{12}$ model to the residuals. However, this would provide us with only 34 observations with which to perform a cross correlation analysis. Since paucity of data can lead us to questionable results,* we elect to continue working with the residuals of the model (4.2). These residuals are given in Table 1 as values of b_t; also given there are values of a_t, the prewhitened flying hours.
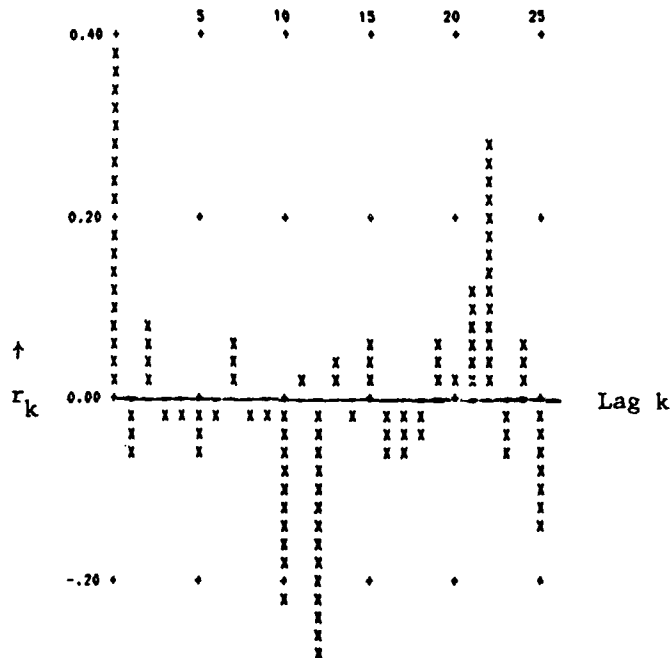

FIGURE 5. Intervention model for alert availability; graph of the ACF of the residual series

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*Actually, we did perform a cross correlation analysis with only 34 observations and obtained results that were counter-intuitive.

5. CROSS CORRELATION ANALYSIS

Using the prewhitened series $\{a_t\}$ and $\{b_t\}$, we now develop the transfer function model (2.1). We remark that the $\{a_t\}$ series with its 59 observations must be modified by omitting the January 1975 observation so that it will match the $\{b_t\}$ series, which has 58 observations.

Following procedures of Box and Jenkins and using the estimated "impulse response weights," V_k, shown in Figure 6, we remark that possible values of (r,s,b) in (2.1) are (1,3,0) or (2,3,0). From the autocorrelation functions of the generated noise series (see Singpurwalla and Talbott [4]), we remark that a possible model for the noise N_t is an ARIMA (1,0,1).

K	V(K)
0	0.224
1	0.260
2	0.035
3	0.126
4	0.026
5	0.000
6	0.078
7	0.127
8	-0.025
9	0.023
10	-0.016
11	0.149
12	-0.227
13	-0.213
14	-0.021
15	0.146

FIGURE 6. Estimated impulse response weights $V(K)$ from a cross correlation of prewhitened flying hours and alert availability

We fit both the (2,3,0) and the (1,3,0) transfer function models with an ARIMA (1,0,1) noise component to the data and find that the more parsimonious (1,3,0) model results in better residuals. This estimated model, whose residuals have an ACF as shown in Figure 7, is

$$(5.1) \qquad (1 + .53B)b_t = (.25 + .51B + .24B^2 + .1B^3)a_t + \frac{1 - .11B}{1 + .12B}e_t,$$

where e_t is random noise.

6. THE COMBINED INTERVENTION ANALYSIS TRANSFER FUNCTION MODEL

We can expand the transfer function model (5.1) for the prewhitened series $\{a_t\}$ and $\{b_t\}$ by using the following relationships from (3.1) and (4.2):

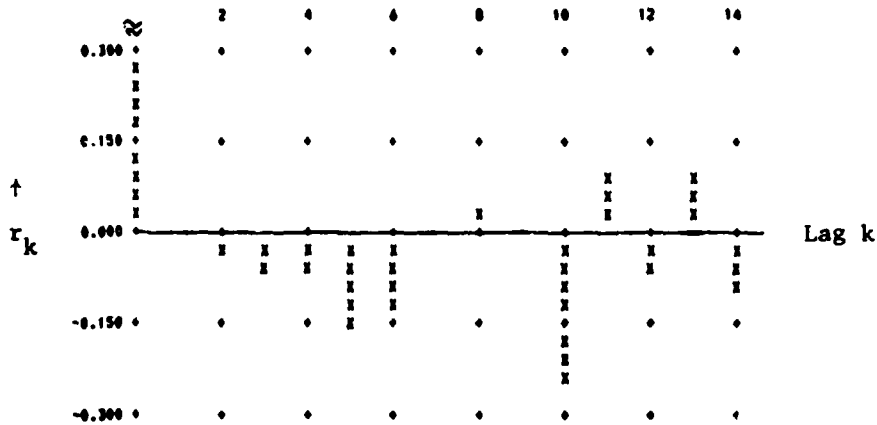$$(6.1) \qquad a_t = \frac{(1 - .54B + .11B^2)(1 + .38B^4 + .37B^8)(\tilde{X}_t)}{1 + .18B^5}$$

FIGURE 7. A (1,3,0) transfer function model between flying hours and alert
availability; graph of the ACF of the residual series

(6.2) $$b_t = (1 - .18B)(1 + .35B^6)(1 - B^6)Y_t + \frac{.02}{1 - B^3}J_t + \frac{.01}{1 - B^3}K_t$$
$$- .15L_t,$$

where $\bar{X}_t = (X_t - 10.09)$.

Substituting (6.1) and (6.2) for a_t and b_t in Equation (5.1), we obtain a multiplicative transfer function model relating the input series $\{X_t\}$ to the output series $\{Y_t\}$ as

$$(1 + .18B^5)(1 + .53B)(1 - .18B)(1 + .35B^6)(1 - B^6)(1 + .12B)Y_t$$

$$= (.25 + .51B + .24B^2 + .1B^3)(1 - .54B + .11B^2)(1 + .38B^4 + .37B^8)$$

$$\cdot (1 + .12B)\bar{X}_t - \frac{(1 + .12B)(1 + .53B)}{(1 + .18B^5)(1 - B^3)}[.02J_t]$$

(6.3) $$- \frac{(1 + .12B)(1 + .53B)}{(1 + .18B^5)(1 - B^3)}[.01K_t]$$

$$+ \frac{(1 + .12B)(1 + .53B)}{(1 + .18B^5)}[.15L_t] + (1 - .11B)e_t.$$

Note that X_t and Y_t in (6.3) are in terms of the logarithms of the original data.

Expanding the polynomials and rewriting Equation (6.3) in conventional notation (in the interest of parsimony, coefficients with a value less than .1 are arbitrarily deleted), we can reduce the transfer function model to

$$Y_t = -.47Y_{t-1} + .1Y_{t-2} - .18Y_{t-5} + .59Y_{t-6} + .31Y_{t-7} + .12Y_{t-11}$$
$$+ .39Y_{t-12} + .17Y_{t-13}$$

(6.4)
$$+ .25\tilde{X}_t + .68\tilde{X}_{t-1} + .11\tilde{X}_{t-3} + .26\tilde{X}_{t-5} + .25\tilde{X}_{t-9}$$
$$+ .42e_{t-1} + .18e_{t-5}$$
$$+ Z_t + .65Z_{t-1} - .18Z_{t-5} - .12Z_{t-6}$$

where, following the notation of Section 4,

$$Z_t = \{[-.02J_t - .02J_{t-3} - .02J_{t-6} - .02J_{t-9} - \ldots]$$
$$+ [-.01K_t - .01K_{t-3} - .01K_{t-6} - .01K_{t-9} - \ldots] + .15L_t\}.$$

7. CONCLUDING REMARKS

From Equation (6.4), we remark that the nonzero coefficients associated with the $\tilde{X}_t$'s imply that flying hours *do* have an effect on the alert availability in a manner specified by the functional form of the equation. Furthermore, an examination of Figure 2, together with the prewhitening transformation, Equation (3.1), reveals that there was no upward or downward trend in the flying hours during the period of study. However, there does appear to be a reduction in the variability of the flying hours as of the inception of RCM. In any case, it appears that Equation (6.4) supports the adage that "the more you fly, the less you fail," within limits.

The negative coefficients (albeit small) associated with the variables J_t and K_t in Equation (6.4) do, in the absence of any upward or downward trend in flying hours, support our premise that RCM has a tendency to reduce the alert availability.

The model, Equation (6.4), can be used not only for interpretative purposes as is done above, but within limits and with proper care, it can also be used to predict future availability given its previous values, and the present and previous values of the flying hours.

REFERENCES

[1] Box, G.E.P. and G.M. Jenkins, *Time Series Analysis: Forecasting and Control* (Holden-Day, San Francisco, 1976).
[2] Box, G.E.P. and G.C. Tiao, "Intervention Analysis with Applications in Economic and Environmental Problems," *Journal of the American Statistical Association*, 70-79 (March, 1975).
[3] Haugh, L.D. and G.E.P. Box, "Identification of Dynamic Regression (Distributed Lag) Models Connecting Two Time Series," *Journal of the American Statistical Association*, 121-130 (March, 1977)
[4] Singpurwalla, N.D. and C.M. Talbott, "A Combined Intervention Analysis/Transfer Function Model for Describing the Relationship Between C-141 Flying Hours and Alert Availability," AFLMC Report No. 790702-2, Air Force Logistics Management Center, Gunter AFB, Alabama (July, 1979)

[5] Singpurwalla, N.D. and C.M. Talbott, "An Assessment of the Benefits of Reliability Centered Maintenance on the C-141," Air Force Journal of Logistics, 11-15 (Winter, 1980)