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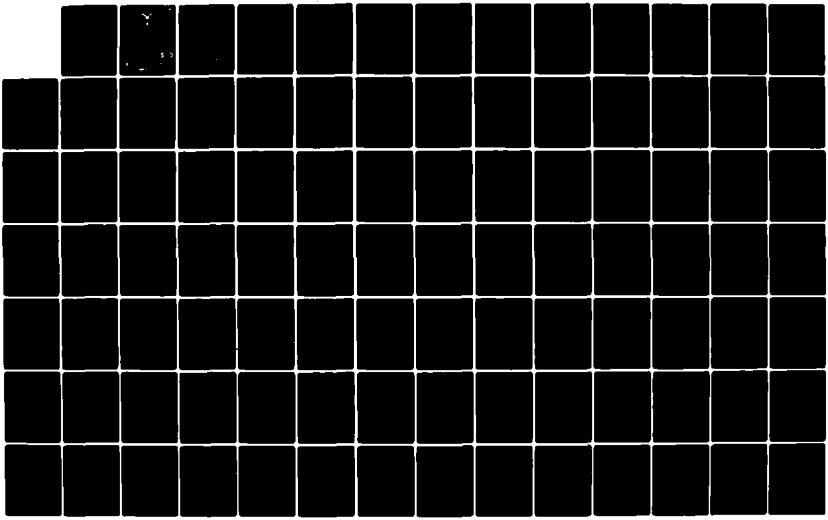

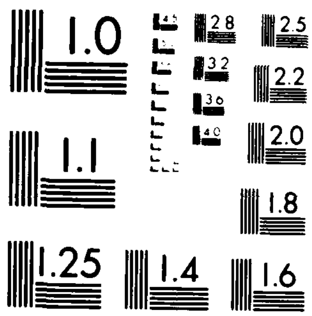

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OPTIMAL MAINTENANCE POLICIES:
A GRAPHICAL ANALYSIS

Patrick F. Doumit, Captain, USAF
Barbara A. Pearce, Captain, USAF

LSSR 28-83

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Probability maintenance models can be categorized according to three types of uncertainty: 1) uncertainty regarding when the next failure will occur is present for all stochastically failing components; 2) uncertainty regarding the component's present condition, good or failed, is present for some components; and 3) uncertainty regarding the component's underlying failure distribution is present in most real-world applications. Unfortunately, application of most models requires exact knowledge of the underlying failure distribution; however, graphical techniques, such as Total Time on Test (TTT), estimate optimal maintenance intervals based on empirical data; thus, they eliminate error resulting from type three uncertainty. The authors apply a TTT model to estimate optimal motor oil replacement intervals under conditions of three types of uncertainty. Their conclusions are: 1) There is no significant difference between synthetic (Stauffer and CONOCO) and petroleum motor oil lifetimes; 2) determining optimal oil replacement intervals requires application of a model more complex than the one applied; and 3) models that optimize an objective function without constraint are often not realistic. Thus, the authors develop and propose models that address three types of uncertainty and allow constraints (cost, availability, or failure risk) to be imposed on the model objective.

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OPTIMAL MAINTENANCE POLICIES: A GRAPHICAL ANALYSIS

A Thesis

Presented to the Faculty of the School of Systems and Logistics

of the Air Force Institute of Technology

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In Partial Fulfillment of the Requirements for the

Degree of Master of Science in Logistics Management

By

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This thesis, written by

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and

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has been accepted by the undersigned on behalf of the faculty of the School of Systems and Logistics in partial fulfillment of the requirements for the degree of

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CHAPTER I

INTRODUCTION

> In recent years delivering systems that perform adequately for a specified period of time in a given environment has become an important goal for both industry and government. In the space program, higher system reliability meant the difference between life and death. In general, the cost of maintaining and/or repairing electronic equipment during the first year of operation often exceeds the purchase cost, giving impetus to the study and development of reliability techniques [4:609].

Sound maintenance policies have their roots in reliability theory. From reliability theory, probability models have been developed for selecting optimum maintenance actions. In this context, maintenance actions include replacement, repair, and inspection. The purpose of this research project is to apply a theoretical preventive maintenance model using graphical solution techniques to an empirical data set to determine optimum replacement intervals for synthetic and petroleum motor oils.

Issue

Preventive maintenance is defined as follows:

> Equipment maintenance actions performed on a periodic basis, according to a specific set of instructions and a predetermined time schedule. The objective is to protect equipment capability and investment by removing the causes of failure and making adjustments to compensate for normal wear before failure occcurs [12:535].

In other words, preventive maintenance is considered as those actions designed to preclude failure and enhance system, subsystem, or component performance. On the other hand, corrective maintenance can be considered as those actions designed to restore a system, subsystem, or component and to enhance performance after failure has occurred. Further, the authors consider an optimum maintenance strategy as one that maximizes or minimizes the objective function of the maintenance policy, such as availability or cost. Thus, an optimum maintenance strategy could be either preventive or corrective.

Preventive maintenance policies can be categorized according to uncertainty about failure. This project addresses three types of uncertainty. The first two types are defined here, and the third type will be discussed as part of the problem statement.

1. Type one uncertainty exists when there is doubt regarding the component's life (i.e., when the next failure will occur). This type of uncertainty is present for all components demonstrating stochastic failure patterns.

2. Type two uncertainty exists when there is doubt regarding the present condition of the component (i.e., whether the component is good or failed). The presence of this type uncertainty depends on the nature of the component.

In cases where only type one uncertainty is present, and the component's condition, good or failed, is known at any point in time, either a planned replacement or repair policy may be optimal. In cases where both type one and type two uncertainty are present, and the component's condition is not known with certainty, either a planned inspection or what Talbott (11) has termed the "blind replacement" policy may be the optimum preventive maintenance strategy. However, depending upon the component's failure pattern and the pay-offs associated with maintenance actions versus failure, preventive maintenance may make no sense. The optimum policy may be to take no action until failure and then perform corrective maintenance.

Problem Statement

While theoretical probability models exist for selecting optimum maintenance policies, these techniques are often not adaptable to real-world preventive maintenance policies; thus, they receive little recognition in Air Force maintenance policies. A significant shortcoming of most probability models is that they require rather explicit knowledge of the component's underlying failure distribution.

All statistical procedures require an unbiased sample of a population. To the extent that the sample does not exactly model the population, a potential for error exists when drawing conclusions based on sample data. When this sample is component failure data, the error is a direct result of the type one uncertainty present in all stochastic processes. Additionally, if in order to apply a particular methodology, one must be able to describe the sample data in terms of some theoretical distribution, then further potential for error exists when drawing conclusions based on the theoretical distribution. This second source of error is a direct result of the extent to which the theoretical distribution does not exactly describe the sample data. Even when ample failure data is available, if failure patterns do not fit theoretical failure distributions, most probability models cannot reasonably be applied. Hence, the problem becomes one of development and application of sound maintenance strategies that addresses a third type of uncertainty: uncertainty about the underlying failure distribution of the component. Thus, type three uncertainty exists when there is doubt regarding the failure distribution of the component.

Scope

The authors believe an appropriate solution to this problem is to apply graphical solution techniques to preventive maintenance models. If a particular methodology does not require knowledge of the theoretical distribution, error resulting from type three uncertainty is eliminated. Graphical techniques do not require fitting a theoretical distribution to the component's failure distribution; hence, they eliminate error associated with type three uncertainty.

The particular graphical technique we will use is the total time on test (TTT) transform which Bergman (3:467-469) has already adapted for a preventive maintenance policy known as the age replacement model.

According to Talbott (11), techniques for adapting the TTT transform to other preventive maintenance policies have yet to appear in the literature but are currently under investigation. Additionally, application of these solutions to real-world situations using an empirical data set remains to be demonstrated. One particular maintenance model, termed the blind replacement model, proposes replacement of components at planned intervals without knowledge of component condition (6:190-192). Talbott (10:16) is developing a graphical solution technique for this maintenance model. We believe application of this technique to an empirical data set will demonstrate the validity of using graphical solutions for developing real-world maintenance strategies.

A disadvantage of TTT analysis is that it probably will not identify the truly optimal solution. The nature of this technique is to select from the observed failure data, the data point that is most optimal among all the data points for the preventive maintenance task. Thus, unless there is an observed failure at the optimal preventive maintenance interval, the model will not select the optimal interval, but rather, the data point closest to it. In other words, the model considers the empirical data set as a finite set of potential replacement intervals from which the optimal interval can be selected. We will elaborate on this disadvantage as we discuss the TTT methodology. However, in the interest of simplification, we will continue to address the interval selected by TTT analysis as optimal even though it may not be truly optimal.

Our empirical data base for this investigation is synthetic and petroleum lubricant-lifetime data made available by the Technical Support Center, Pensacola Naval Air Station, FL. This data base is part of a joint-service test of DOD vehicles to determine the merits of both types of lubricants. Application of the blind replacement model using graphical solutions should yield optimal oil replacement intervals for synthetic versus petroleum motor oils.

Research Question

Can the blind replacement model be applied to an empirical data set using TTT transform graphical solution techniques to determine optimal replacement intervals for synthetic and petroleum motor oils?

CHAPTER II

LITERATURE REVIEW

This chapter presents a brief look at the development of optimal maintenance policies based on reliability theory. Barlow and Proschan (2:84) categorize maintenance policies as replacement policies and inspection policies. The topics covered in this chapter are reliability theory, maintenance policies in general, replacement models, inspection models, and the blind replacement model. Discussion of the TTT transform graphical technique and its application to maintenance policies is reserved for Chapter Three, Methodology.

Reliability Theory

Barlow and Proschan (2:6) define reliability as "the probability of a device performing its purpose adequately for the period of time intended under the operating conditions encountered." Similarly, Hillier and Lieberman (4:594) state that

> reliability, $R(t)$, is the probability that a device performs adequately over the interval $(0,t)$. In general, it is assumed that unless repair or replacement occurs adequate performance at time t implies adequate performance during the interval $(0,t)$. The device under consideration may be an entire system, a subsystem, or a component.

Some probability models of reliability consider devices to be in one of two conditions: good or failed. For an individual component, this condition can be denoted by a random variable X_i, such that $X_i = 1$ if the i^{th} component is good and $X_i = 0$ if the i^{th} component is failed. Thus, X_i is a binary random variable defined as;

$$X_i(t) = \begin{cases} 1, & \text{if device i does not fail during } (0,t) \\ 0, & \text{if device i does fail during } (0,t) \end{cases}$$

Therefore, the probability of a device not failing during the interval (0,t) can be expressed as $P(X_i(t)=1)$ or $P(X=1)$ (4:595).

The common probability functions used in reliability theory treat system condition as a function of time. The function f(t) represents the failure probability density function (pdf)--that is, the probability of failure at time t; and F(t) represents the cumulative failure pdf--that is, the probability of failure within the interval (0,t). The function R(t) represents the survival distribution or reliability distribution and is the probability of survival throughout the interval (0,t); hence,

$$R(t) = P(X=1) = 1-F(t) \qquad (2.1)$$

The hazard rate, r(t), sometimes referred to as the instantaneous failure rate, is the conditional probability that a device surviving to time t will fail at time t.

Therefore, it is the ratio of the probability of failure at time t to the probability of survival throughout the interval (0,t); hence,

$$r(t) = f(t)/R(t) \tag{2.2}$$

For many devices, the hazard rate tends to increase as the device gets older due to deterioration associated with age. Devices having hazard rates that remain constant or increase with age are said to have an increasing failure rate (IFR). For some devices, the hazard rate tends to decrease as the device gets older due to a possible "work hardening" phenomenon that occurs during early life. Devices having a hazard rate that remains constant or decreases with age are said to have a decreasing failure rate (DFR). Devices having an exponential failure distribution have a random or poisson failure rate that is constant throughout time; hence, the exponential distribution is both IFR and DFR and represents a natural boundary on the survival probabilities of the IFR and DFR distributions (4:605-606).

Since R(t) is a continuous function which cannot assume negative values, the mean value of X (expected condition of the device) is represented by the following expression (5:57):

$$E(x) = \int_0^{\infty} R(x)dx \tag{2.3}$$

Furthermore, failure distributions represent probability models of the length of life of a component or system. Typical failure density functions popular in the literature are the exponential, the gamma, the Weibull, the normal, and the lognormal. The exponential distribution has limited utility (albeit wide application) because of its underlying property that previous use does not impact future life. In other words, a new device has the same probability of failing within the next time interval as an old device. For devices having exponential failure distributions, the future life distribution of the devi e remains unchanged as long as the device has not failed yet. Therefore, devices having exponential failure distributions essentially do not age (2:9-15).

Maintenance Policies

For some devices, failure during operation is undesirable because it is too costly or dangerous. In these situations, it may be wise to employ a maintenance policy that incorporates planned replacement/inspection of good devices. Barlow and Proschan (2:84) define an optimum maintenance policy as one "that minimizes total cost, maximizes availability, or in general attains the best value of the prescribed objective function."

Central to the development of maintenance policies is renewal theory. A renewal process may be thought of as any process that returns a device to a good-as-new condition; this includes both replacement and repair to a good-as-new condition. The renewal reward theorem states that the average long-term reward is equal to the expected reward in a renewal cycle divided by the expected cycle length. Mathematically, this is expressed as:

$$E(x) = \frac{E(x/\text{cycle})}{E(\text{cycle length})} \tag{2.4}$$

Here X represents some form of reward or payoff which can be positive, such as availability, or negative, such as down-time or cost. The objective function then becomes maximization of a positive long-term reward or minimization of a negative long-term reward (7:53).

In this research project, we will use the term <u>cost</u> to signify any negative reward, and <u>profit</u> for any positive reward. Also, the terms <u>renewal</u> and <u>replacement</u> are used interchangeably, such that replacement refers to either physical exchange of a device or repair to a good-as-new condition.

<u>Replacement Models</u>

Preventive maintenance policies are categorized by Barlow and Proschan (2:84) as either replacement type or inspection type policies. Replacement policies attempt to balance the cost of failure occurring during operation of the device against the cost of planned replacement. The following assumptions make application of a replacement policy reasonable: 1) cost of replacing a good device is less than cost of the device's failing; 2) detection and replacement of failed devices is instantaneous; and 3) the probability of failure is increasing with age. In the Barlow and Proschan models, $C(t)$ represents the expected long-term cost incurred during the interval $(0,t)$; C_1 represents the cost of device failure to include cost of replacing the failed device; and C_2 denotes the cost of replacing a good device. $N_1(t)$ represents the expected number of failures occurring during the interval $(0,t)$, while $N_2(t)$ represents the expected number of replacements of good devices. The basic replacement model can then be expressed as:

$$C(t) = C_1 N_1(t) + C_2 N_2(t) \tag{2.5}$$

Here, the objective is to minimize $C(t)$ for a finite time span or minimize $\lim_{t \to \infty} C(t)/t$ for an infinite time span (2:84-85).

The most common replacement model is the age replacement model which proposes replacing devices at failure or at age T, whichever occurs first (where T is assumed to be a constant). Barlow and Proschan show that if the underlying failure distribution is continuous, then determining an optimum age of replacement requires exact knowledge of the failure distribution. Unfortunately, in actual practice, the failure distribution, F, may not be known. However, if the failure distribution, F, is known exactly, the optimum age of replacement for an infinite time span is that value of T which yields the minimum value for C(T) in the following equation (2:85-86).

$$C(T) = \frac{C_1F(T) + C_2R(T)}{\int_0^T R(x)dx} \tag{2.6}$$

This equation can be derived mathematically from equation 2.5 and is equivalent to the mathematical expression of the renewal reward theorem (equation 2.4) where:

$C(T)$ = expected long-term cost at time T

$C_1F(T)$ = failure cost multiplied by the probability of failure within the interval (0,T)

$C_2R(T)$ = planned replacement cost multiplied by the probability of survival throughout the interval (0,T)

$\int_0^T R(x)dx$ = the expected value of the cumulative survival pdf where R(x) is bounded by 0 and T (equation 2.3)

Hence, $C_1F(T) + C_2R(T)$ = E(cost per cycle), and $\int_0^T R(x)dx$ = E(cycle length) (2:85-86).

Regarding finite time spans, Barlow and Proschan (2:93) show that if F is known and continuous, then there exists an age of replacement for any finite time span (0,T);

however, no general formula exists for determining an optimum age replacement policy.

A second type of replacement policy is block replacement, which proposes replacing devices at calendar times kT (k = 1,2...), independent of component failure history; devices are also replaced at failure. Block replacement policies have an advantage over age replacement in that they eliminate the administrative burden of tracking device age. However, the disadvantage of a block replacement policy is that it throws away useful life. In comparing block and age replacement policies, Barlow and Proschan show that with block replacement, more unfailed devices are removed, more useful life is wasted, and the total number of both failed and good device removals is greater than with age replacement; however, the expected number of failures is less for block replacement policies (2:67,95).

Inspection Models

As stated earlier, a basic assumption of replacement policies is that detection and replacement of a failed device are instantaneous. Thus, replacement models assume that the condition of the device (failed or good) is known at any point in time. When type two uncertainty exists (i.e., the condition of the device cannot be determined instantaneously), replacement policies cannot logically be applied; and inspection policies, which make up the second major category of maintenance policies, are more suited for these items (11).

Inspection policies attempt to balance the cost of failure occurring during operation of the device against the cost of planned inspections. The following assumptions make application of an inspection policy reasonable: 1) the condition of the device can be determined only through inspection; 2) inspection is non-destructive; 3) the device cannot fail during inspection; 4) inspection involves a fixed cost, C_1; and 5) the time elapsed between system failure and discovery of failure at the next inspection costs C_2 per unit of time. As with the age replacement model, Barlow and Proschan apply a mathematical model to describe the relationship between the expected long-term cost C(t) and C_1 and C_2. They define an optimum inspection procedure as a "specification of specific checks $X_1 \leq X_2 \leq X_3 \ldots$" for which C(T) is minimized; and they show that if the failure distribution, F, is known and continuous with a finite mean, then an optimum inspection schedule must exist (2:107-108).

Blind Replacement Model

Radner and Jorgenson (6:184) categorize maintenance models according to sources of uncertainty. One source of uncertainty occurring with all maintenance policies developed for stochastically failing devices is predicting when a failure will occur (type one uncertainty). A second source of uncertainty is knowing the condition of the device (good or failed) at any point in time without inspection (type two uncertainty). Barlow and Proschan apply replacement models to cases where only type one uncertainty is present and apply inspection models to cases where both type one and type two uncertainty are present. Similarly, Radner and Jorgenson propose a category of preparedness models for situations having both types of uncertainty. Preparedness models include but are not restricted to the typical inspection type models addressed by Barlow and Proschan. For example, one assumption Barlow and Proschan make regarding inspection models is that inspection is nondestructive. However, in some situations, inspection may cause the device to fail. This is true of any device that must be expended in order to determine if the device is still good, such as munitions which must be fired to determine if they still work but clearly will not work again after they have been fired. For devices falling in this category, planned inspection makes no sense, and the optimum policy would be to replace at a planned interval. However, these devices do not meet the requisites of Barlow and Proschan's replacement policies since both type one and type two uncertainty are present.

Radner and Jorgenson developed a preparedness model for devices having an arbitrary distribution of times to failure, which Talbott (11) has termed the "blind replacement" model. Blind replacement proposes renewing a device at age or time T without knowledge of the condition. Assumptions for the blind replacement model are 1) the cost of replacement is less than cost of inspection or inspection is an unreasonable option, and 2) the probability of failure is increasing with age. If A(T) represents the average long-term profit (availability, in this case); T, the time between replacements; and K, the time to replace, then A(T) can be expressed in terms of reward renewal theorem (equation 2.4) as:

$$A(T) = \frac{\int_0^T R(x)dx}{T + K} \tag{2.7}$$

Here, the numerator of the above equation represents the mean life in a renewal cycle; and the denomonator represents the time between replacements plus time to replace, which is cycle length. The objective function then is to maximize A(T) (6:190-192).

Summary

Two important requisites exist for all the maintenance policies presented thus far. First, the failure distribution must be known; and second, it must be IFR, but not exponential. Although Barlow and Proschan provide several mathematical proofs of the requisite for an IFR failure distribution, it is intuitive that devices having purely exponential or DFR distributions are not reasonable candidates for preventive maintenance. Since devices having purely exponential failure distributions essentially do not age and have a constant hazard function throughout time, then clearly, replacing a good device will not impact the probability of failure within the next time interval; in other words, the new device would have the same probability of failing as the old. For devices having non-exponential, DFR failure distributions, planned replacement of a good device actually increases the probability of failure within the next time interval since the new device would have a greater probability of failure than the old. Therefore, an underlying assumption of all preventive maintenance policies discussed in this project is that their application is reasonable only if the device has a non-exponential, IFR failure distribution.

A major drawback of all the maintenance models presented thus far is that determining an optimum replacement/inspection strategy requires an exact description of the underlying failure distribution, F, in terms of some theoretical failure distribution such as a gamma or Weibull. As stated earlier, in actual practice, the failure distribution may not be known or may not reasonably fit a theoretical distribution. This fact alone makes application of theoretical maintenance policies difficult for most real-world preventive maintenance situations. Thus, these models could be made more useful if they could be adapted to data analysis techniques employing empirical distributions. One such technique is a graphical technique using total time on test transform, which is the subject of the next chapter.

CHAPTER III

METHODOLOGY

This chapter presents the total time on test methodology for analyzing observational data to determine optimal maintenance intervals. This technique was described by Barlow and Campo (1:451) as an effective means of analyzing failure data. In 1977, Bergman (3:467-469) adapted Barlow and Proschan's age replacement model to TTT analysis techniques. Also, Talbott (10:16) is currently adapting Radner and Jorgenson's blind replacement model for TTT analysis. In describing the development of the TTT transform technique, this chapter will cover the TTT process in general, the Bergman model, the Talbott model, and data application.

Total Time on Test Process

The total time on test (TTT) process was presented by Barlow and Campo in 1975 as a method of analyzing data that is useful for describing failure probability. An explanation of this process follows:

1. $t_{(i)}$ is a value from an ordered set of life observations from distribution F such that $t_{(1)} \leq t_{(2)} \leq t_{(3)} \leq \dots \leq t_{(n)}$. Since $t_{(1)}$ represents the life of the first device that failed, and $t_{(2)}$ the life of the second,

etc., then it follows that n devices all lived $t_{(1)}$ units of time, n-1 devices lived $t_{(2)}$ units of time, and only one device lived $t_{(n)}$ units of time. Thus $t_{(i)}$ represents the life of the i^{th} device in the ordered set of n lifetimes, where $t_{(0)}=0$.

2. T_i is the total time on test statistic and represents the life generated by the n devices in the interval $(t_{(0)}, t_{(i)})$, where a portion of the n devices survived throughout the interval and the remainder failed during the interval. Where $T_0=0$, then T_i is defined mathematically as,

$$T_i = \sum_{j=1}^{i} (n - j + 1)\ (t_{(j)} - t_{(j-1)}) \qquad (3.1)$$

3. U_i is the scaled total time on test statistic and is equal to the ratio of T_i to T_n. Thus, U_i represents the proportion of the total life generated by the n devices during the interval $(t_{(0)}, t_{(i)})$.

4. i/n is the ratio of i to n and represents the proportion of the devices that have failed by point in time $t_{(i)}$. Thus, it is approximately equal to the probability that an item fails within the interval $(t_{(0)}, t_{(i)})$.

5. The TTT plot is the graphical representation of the TTT process where U_i is plotted against i/n. Therefore,

each point on the TTT plot relates the proportion of the total life generated by time $t_{(i)}$ to the probability that a device fails by that time. Necessarily then, at the final point where i=n, U_i = i/n = 1 (1:452-457; 11).

Barlow and Campo (1:455) further show that if the data points on the TTT plot are joined by line segments, when n is sufficiently large, this function will approximate a curve. The curve will be concave for IFR distributions, a straight line for purely exponential, and convex for DFR distributions. Our examples of these functions are given in appendices A through C of this report.

The Bergman Model

Bergman made a major breakthrough in the application of probability based maintenance strategies to real-world scenarios by adapting Barlow and Proschan's age replacement model to TTT transform analysis. This method provides a useful means of estimating the optimum age replacement interval when only observational data is available, and the device's underlying failure distribution is not known with certainty or cannot be fitted to an empirical distribution. The model assumes a cost C is associated with component replacement and an additional cost C+K is associated with failure. If we let K=1, then from equation 2.6, the optimum age of replacement is that value of T which yields the minimum value for C(T) in the following equation:

$$C(T) = \frac{(C+1)(F(T)) + C(R(T))}{\int_0^T R(x)dx} \tag{3.2}$$

Since $R(T) = 1-F(T)$, the numerator of the above equation can be reduced mathematically to $C+F(T)$. Also since $F_n(T)$ is an empirical distribution of n observations that approaches the theoretical $F(T)$ as n approaches infinity, then $F(T) = \lim_{n \to \infty} F_n(T)$. Further, since $t_{(i)}$ represents one observation from that empirical distribution, then (using "$\doteq$" to represent "is approximately equal to") $F(T) \doteq F_n(T) \doteq F_n(t_{(i)})$ when n is sufficiently large. And necessarily, $C(T) \doteq C_n(T) \doteq C_n(t_{(i)})$. Hence,

$$C(T) = \frac{C + F(T)}{\int_0^T R(x)dx} \doteq \frac{C + F_n(t_{(i)})}{\int_0^{t_{(i)}} R_n(x)dx} = C_n(t_{(i)}) \tag{3.3}$$

Since T_i represents the cumulative life generated by the n components over the interval $(t_{(0)},t_{(i)})$ and since $\int_0^{t_{(i)}} R_n(x)dx$ represents the expected value of the cumulative survival pdf over that same interval, then for the empirical data set, $\int_0^{t_{(i)}} R_n(x)dx = T_i/n$. Also, since $t_{(i)}$ represents one data point in the empirical cumulative failure pdf, $F_n(T)$, then necessarily, $F_n(t_{(i)})$ is the probability of failure within the interval $(t_{(0)},t_{(i)})$ and is therefore approximately equal to i/n. Thus from equation 3.3, it can be shown that,

$$C_n(t_{(i)}) = \frac{C + F_n(t_{(i)})}{T_i/n} \doteq \frac{C + i/n}{(1/n)\ (T_n)\ (T_i/T_n)} \quad (3.4)$$

And finally, since $T_i/T_n = U_i$, then from equation 3.4,

$$C_n(t_{(i)}) \doteq \frac{C + i/n}{1/n\ (T_n)\ U_i} = \frac{1}{T_n/n} \times \frac{C + i/n}{U_i} \quad (3.5)$$

Thus,

$$C(T) \doteq C_n(T) \doteq C_n(t_{(i)}) \doteq \frac{1}{T_n/n} \times \frac{C + i/n}{U_i} \quad (3.6)$$

Note that the first part of the expression on the right hand side is a constant fixed by the sample, and the second part is the only part that is variable. Since the objective is to minimize long-term costs, this is equivalent to maximizing the reciprocal of $C_n(t_{(i)})$. Therefore the objective becomes maximizing the reciprocal of the variable portion of the right hand side of equation 3.6. Hence,

$$\min C(T) \implies \max \frac{U_i}{C + i/n} \quad (3.7)$$

For a particular value of i, plotting -C on the X-axis of the TTT plot yields a distance from -C to i/n that equals C + i/n. If a line is drawn from -C on the X-axis to the TTT data point corresponding to that value of i, that line is defined by its slope which is equal to $U_i/(C + i/n)$. For each line that can be drawn in this fashion, at least one line will be tangent to the function and will have a slope that is greater than or equal to the slope of any other line that can be drawn. Thus, that line defines the value of i (call it i*) that maximizes $U_i/(C + i/n)$ for a given value of C. The optimal replacement interval then is the value of $t_{(i)}$ defined by i* (3:467-469; 8:9-10).

As stated earlier, the disadvantage of TTT analysis is that it probably will not identify the truly optimal solution. Since the technique selects the value of $t_{(i)}$ defined by the point of tangency, it can consider only the observed lifetimes as candidates for the replacement interval. If the truly optimal solution is not a value of $t_{(i)}$, the point of tangency will be the $t_{(i)}$ closest to the truly optimal interval.

Additionally, if the distribution is purely exponential, the TTT function will be a straight line, and if it is DFR, the function will be convex. In either of these two cases, the point of tangency will be the last data point on the TTT plot. Hence, the model should be interpreted for purely exponential and DFR distributions as having an optimal strategy of replacing only at failure. Our examples of IFR, purely exponential and DFR distributions are given in Attachments A through C of this report.

The Talbott Model

From Bergman's insight, Talbott (10:16) is adapting Radner and Jorgenson's blind replacement model to TTT analysis techniques. Using a process similar to Bergman's, Talbott shows that as n approaches infinity, the empirical distribution approaches the theoretical distribution. Thus, from equation 2.7, he shows that

$$A(T) = \frac{\int_0^T R(x)dx}{T + K} \doteq \frac{\int_0^{t_{(i)}} R_n(x)dx}{t_{(i)} + K} \tag{3.8}$$

Since $\int_0^{t_{(i)}} R_n(x)dx$ represents the expected value of the cumulative survival pdf over the interval $(t_{(0)}, t_{(i)})$, then for the empirical data set, $\int_0^{t_{(i)}} R_n(x)dx = T_i/n$. Also since $U_i = T_i/T_n$, then from equation 3.8 we have,

$$A(T) \doteq \frac{T_i/n}{t_{(i)} + K} = \frac{(T_n/n)U_i}{t_{(i)} + K} = \tag{3.9}$$

$$\frac{T_n/n}{t_{(n)}} \times \frac{U_i}{t_{(i)}/t_{(n)} + K/t_{(n)}}$$

Therefore,

$$A(T) \qquad A_n(t_{(i)}) = \frac{T_n/n}{t_{(n)}} \times \frac{U_i}{t_{(i)}/t_{(n)} + K/t_{(n)}} \qquad (3.10)$$

Since the first part of the right-hand expression is fixed by the sample and is therefore constant, the second part is the only part that is variable. Hence, if the objective is to maximize the long-term reward, then,

$$\max A(T) \implies \max \frac{U_i}{t_{(i)}/t_{(n)} + K/t_{(n)}} \qquad (3.11)$$

By plotting $-K/t_{(n)}$ on the X-axis of the TTT plot, the value for i* can be found using the same techniques as in Bergman's model. Thus the optimum blind replacement interval is the value of $t_{(i)}$ defined by *i**. However, one problem is that there is some question as to the interpretation when i* = n (i.e., do not replace or replace at $t_{(n)}$) (11).

Data Application

Applications of Talbott's adaptation of the blind replacement model requires observational failure data. Our empirical data set for this investigation is synthetic and petroleum lubricant-lifetime data which has been made available by MEEP Project H79-1C. This data base is part of a joint service test of DOD vehicles to determine the merits of both types of lubricants. Application of the blind replacement model using graphical solutions should yield optimal oil replacement intervals for synthetic versus petroleum motor oils.

CHAPTER IV

DATA ANALYSIS

This chapter examines the procedures used to collect and analyze the oils comprising the data base for this research. It also discusses the method for determining K, the time required to replace failed oil.

The original motivation for the data collection effort was to determine if there were any advantages to using synthetic lubricants in DoD vehicles. Air Force Systems Command (AFSC) was tasked to perform a field evaluation to determine if using synthetic engine oils provided improved performance, greater reliability, and lower vehicle operating costs (9:1).

Data Collection

Oil sample data for the field evaluation was collected over a three-year period (1979-1981) through the Management and Equipment Evaluation Program (MEEP) Project H79-1C. The test included approximately 450 general purpose vehicles from twelve Air Force bases throughout the country. The large number of vehicles and the wide cross-section of bases should provide a relatively unbiased sample of Air Force vehicles. The lubricants selected for the program were MIL-L-46152 Qualified Products List (QPL) products.

Three different types of lubricants were evaluated: Stauffer's synthetic oil, a standard stock listed mineral based oil, and CONOCO's synthetic oil. To insure consistency in determining oil failure, oil samples were taken at regular intervals and sent to the Joint Oil Analysis Program (JOAP) Laboratory at the Naval Air Station, Pensacola, Florida for testing. Oil filters were changed each 6,000 miles or 12 months of operation, whichever came first. The personnel at the JOAP Laboratory made recommendations based on their findings whether or not to change oil/filter at more frequent intervals or to continue with routine sampling (9:1-6).

Oil was considered failed when certain established baseline factors were exceeded. Factors included various levels of metal content, changes in viscosity, and failure of a blotter test. Baseline data provided by the personnel at the JOAP Laboratory for determining failure of each type of oil is in Appendix D.

The entire set of raw data was provided by the JOAP Laboratory for this research. To maintain an unbiased sample, we collected only one instance of oil failure per vehicle in the test, although many of the vehicles experienced more than one oil change recommendation. To have collected more than one lifetime per vehicle would have biased our analysis in favor of those vehicles with short oil lifetimes. Data was compiled in terms of several factors as shown at Appendix E.

Although we received the raw data for approximately 450 Air Force vehicles, a thorough examination of the data revealed several problems which significantly reduced the total number in our research sample. For example, in many cases, an accurate number of miles between oil changes could not be determined. Some oil was not changed when failure was apparent and a recommendation was made. In some cases, the wrong type of oil was added to the engine crankcase, invalidating the test. Also, two bases were removed from the project due to engine problems and the use of improper oil. After discarding the samples we felt were inaccurate, the total number of cases remaining was 116. While this number is certainly not as large as originally anticipated, we consider it a fair cross-section of oil types and bases.

Statistical Analysis

Prior to performing the TTT analysis, we ran several Wilcoxon-Rank Sum (Mann-Whitney) tests on the data set. This test is a nonparametric technique for testing the null hypothesis that the probability distributions associated with two populations are identical. We made comparisons for three factors: oil type, vehicle make, and utilization and mileage. Based on an alpha value of .05, in only two cases could we conclude that the populations were significantly different. The failure distribution for vehicles having low utilization and low mileage was significantly different from both the distribution for vehicles having high utilization and high mileage and the distribution for vehicles having low utilization and high mileage. These two tests resulted in two-tailed probability values of .0097 and .0105, respectively. However, there was no significant difference between the failure distributions based on oil type, vehicle make, or the other utilization and mileage categories. Based on these results, we eliminated the low utilization/low mileage vehicles from the data set and retested for significant differences between oil types and vehicle makes. Again, we were unable to find significant differences among any of the failure distributions. Finally, we tested only the vehicles having low utilization/low mileage for significant differences based on oil type and vehicle make. Again, we found no significant differences. Thus, based on our sample data, we have no evidence that either oil type or vehicle make significantly affects the propensity of motor oil to fail. Results of these tests are shown at Appendix F.

Total Time on Test

To demonstrate the Blind Replacement model using graphical analysis, a value for replacement time K must be established. We estimated one day as the approximate time required to turn a vehicle into the motor pool, change the oil and filter, and return it to service. Since the life cycle length in the blind replacement model is expressed as replacement interval plus replacement time, (T+K), T and K must have the same unit of measure. Also, since it is desirable to express the replacement interval in miles, which is the common unit of measure for oil life, in order to maintain consistent units of measure, K should also be expressed in miles. Therefore, one day is translated into miles by dividing the total miles driven in the test by the number of days in the test to obtain an estimate of miles per day. Thus, replacement time is being measured as the expected number of miles that cannot be driven when the oil is being replaced. Using this technique, the following values for K were obtained:

All Oils Combined	= 24.83
Stauffer Synthetic Oil	= 20.92
Mineral Oil	= 28.30
CONOCO Synthetic Oil	= 25.46

We developed the FORTRAN computer program shown at Appendix G to perform the TTT analysis. Using the computed values for K, we applied our data set to this program and found the following replacement intervals provide maximum availability of good oil:

All oils combined (Appendix H)	= 1971 miles
Stauffer (Appendix I)	= 1139 miles
Mineral oil (Appendix J)	= 2039 miles
CONOCO (Appendix K)	= 2624 miles

Unfortunately, in all four cases, the value of $-K/t_{(n)}$ is so small, that when graphed, it can hardly be distinguished from the origin. Thus, the TTT plots in Appendices H through K do not provide very good illustrations of the TTT process.

Summary

Based on the tests performed, we conclude that the failure probability distributions are not significantly different for the three types of oils tested. However, since the TTT transform can select the optimal interval from only the observed data points, it would not select the same replacement interval for all three type oils unless each happened to have an oil change point of the same mileage and the value for K in each sample allowed that interval to be selected. For this reason, any interpretation of the differences in the intervals should be approached with caution. It should also be noted that the optimal interval selected is for the prescribed objective only--specifically, to maximize the availability of good oil. If the decision maker has other objectives, these intervals would no longer be optimal since use of the blind replacement model would be inappropriate.

CHAPTER V

MODEL ANALYSIS

While analyzing the results of this project, we noted major difficulties in our application of the Talbott model and weaknesses in the original Radner-Jorgenson formulation in terms of its real-world applicability. In this chapter, we will describe the weaknesses of the Blind Replacement model, explain how the Talbott model was misapplied for our data set and then present a proposed methodology for overcoming some of these problems.

Blind Replacement Model

The weaknesses of the blind replacement model can best be described in terms of Markov states. As stated earlier, type two uncertainty exists when there is doubt regarding the present condition of the device. Thus, as a device operates under conditions of type two uncertainty, it can be in any one of four possible states:

1. Good time - That time during which the device is in operation in an unfailed condition.

2. Bad time - That time during which the device is in operation in a failed condition.

3. Corrective maintenance - That time during which a failed device is being replaced.

4. Preventive maintenance - That time during which an unfailed device is being replaced.

In terms of the life cycle, at any point in time during the replacement interval T, the device is in either state one or state two; throughout the replacement time K, the device is in either state three or state four. Since a device can be available only when it is in state one, when T equals T*, the probability of being in state one versus any other state is maximized. Hence, blind replacement emphasizes state one at the expense of all the other states combined. Therefore, the probability of being in states two, three, and four is minimized unconstrained with respect to each other as if they were a single state. Since there cannot be a direct transition from state one to state three, only the conditions for states two or four can take a device out of state one. Thus, blind replacement assumes an equal preference for entering states two and four and further assumes that they both have the same intrinsic value in the mind of the decision maker. In other words, one must be able to say, "I would like the device to be available; but if it must stop being available, it makes no difference whether this is because it is staying in operation in a failed condition or because it is undergoing preventive replacement." Thus, where most models assume a cost of failure that is greater than the cost of replacement, the Radner-Jorgenson model implicitly assumes these costs are equal or of no concern.

Radner and Jorgenson specifically developed their model for devices (such as munitions) being kept in storage under conditions of type two uncertainty for use during an emergency. Here, the objective is to maintain the device in a state of operational readiness (state one) as much of the time as possible. Hence, states one and two constitute times that the device is in storage, not in operation. For this scenario, there may be little difference between failure and replacement costs. When the emergency hits, the cost of not having the device operationally ready is the cost of not being in state one. This cost is incurred regardless of which of the other three states the device is in; hence, it is not a failure cost. The only failure cost is the cost of the type two uncertainty, which is inherent in the problem to begin with. When the device is in state three or four, the device is known to be not available to respond to the emergency. Thus, the cost of failure is the cost of not having known the device to be failed.

Where this information has appreciable value, the blind replacement model should not be applied. Such a situation exists in the case of a munition device being "stored" on another piece of operational equipment, such as an aircraft ejection seat cartridge. Here, the cost of failure is possible loss of life since, knowing the device to be failed, one would not fly the aircraft. The situation also exists for munitions on alert if, for planning purposes, it is considered advantageous to reduce type two uncertainty at the expense of availability.

Another problem with application of the blind replacement model is most evident when there is a very small value for K relative to $t_{(n)}$. Necessarily, as K approaches 0, T approaches $t_{(1)}$. In fact, if K = 0, any value for T, where $0 \leq T \leq t_{(1)}$ will provide 100% availability. Therefore, with a very small value for K, maximum availability is realized when $T = t_{(1)}$. In this situation, all but one of the devices will be replaced while they still have life, and this could be costly. Thus, there is another cost involved: the cost of discarded life.

In a minimum cost model, not accounting for discarded life biases the model toward an earlier replacement interval. The greater the difference between failure and replacement costs, the relatively less expensive will be the discarded life, but the more discarded life will be incurred. However, if failure and replacement costs are relatively close, discarded life becomes relatively expensive but rarely occurs. Thus, when minimizing costs, even though the bias exists, the very nature of the model will constrain the significance of discarded life costs. On the other hand, when the objective of the model is to maximize availability, the model will discard life unconstrained. If the device is relatively inexpensive and K is relatively large, discarded life costs will be minimal.

However, if the reverse is true, discarded life could be quite significant, and the costs should not be ignored.

In conclusion, the Radner-Jorgenson and the Talbott models appear to be valid when applied strictly to the scenario for which they were developed; however, they appear to be weak in terms of real-world applications. The inherent problem with blind replacement is not in the Radner-Jorgenson formulation but, rather, in the objective function. Any model that maximizes availability without constraint is not a very useful model in our opinions. To the extent that there is a preference for entering state four versus state two or that discarded life is a concern, maximizing availability without constraint is not realistic. In other words, costs must truly be irrelevant for this objective to make sense, and we doubt this is often the case.

Application of the Talbott Model

The application of the Talbott model in this research project for finding an optimal oil replacement interval was inappropriate since engine oil does not constitute a device in storage waiting to respond to an emergency, and there are real costs associated with failure and discarded life which probably should not be ignored. Hence, the optimal replacement interval is only valid if the objective is to maximize availiability of the good oil and there is no concern for the cost of operating an engine on "bad" oil or for the cost of discarding good oil due to early replacement.

What would be an appropriate model for oil replacement is a question the authors are unable to answer. There are several problems associated with engine oil that make application of an appropriate replacement model complex. First, as oil transitions from state one to state two, failure costs are not incurred all at once; they are accumulated as the engine continues to operate on failed oil. Second, this accumulation of failure costs is probably not linear since operating an engine on failed oil will probably aggrevate engine damage in a nonlinear fashion. Thus, the cost of operating a vehicle for a mile on oil that just failed is probably less than the cost of operating a vehicle for a mile on oil that has been failed for a long time. Finally, this process of how failed oil damages an engine over time is probably stochastic and is, therefore, represented by some theoretical probability distribution. So, even if one were to obtain an unbiased sample of this process, the issue of how to address the type three uncertainty associated with this theoretical distribution would still be a problem. Thus, considering the intractability of accounting for failure cost, proposing an appropriate model for determining an optimal oil replacement interval presents a dilemna the authors have been unable to resolve.

Proposed Methodology

As stated earlier, type three uncertainty exists if the underlying failure distribution is not known with certainty. Both models that have been developed to address this type uncertainty, Bergman's and Talbott's, are somewhat limited in terms of real-world applications since neither considers the cost of discarded life and both maximize an objective function without constraint. The Bergman model minimizes cost without regard for availability, and the Talbott model maximizes availability without regard for cost. Therefore, the authors propose two models termed model one and model two which allow the decision maker to constrain the objective function. Both replacement models one and two address three concerns which the authors consider appropriate for most real-world scenarios. These concerns are cost, availability, and risk of failure. When the decision maker selects one of these concerns as the objective, the other two may remain in the model as constraints. We will address replacement model two first.

Replacement model two is appropriate for situations having all three types of uncertainty. To illustrate application of this model, we recall the example of the aircraft ejection seat cartridge. An appropriate application in this example might be to minimize cost, constrained by an acceptable level of risk for being in operation in a failed state (state two). Furthermore, in the example of munitions on alert, it might be more realistic to address availability as a constraint, not an objective. Thus, an appropriate application might be to minimize cost, constrained by a required level of readiness to meet mission requirements. Here, where availability operates as a constraint, the cost of not having known the device to be failed is minimized over the long run. By knowing the availability constraint, the decision maker always knows the expected number of devices in the readiness state (state one) and can plan accordingly.

In the model two formulation, T represents the replacement interval; C(T) is the expected long term cost; C_1 is the cost of device failure to include cost of replacing the failed device; and C_2 is the cost of replacing a good device. Since discarded life is the cumulative life within the interval (T,∞), the expectation for the percent of the total life discarded can be expressed as:

$$E(x) = \frac{\int_T^{\infty} R(x)dx}{\int_0^{\infty} R(x)dx} \qquad (5.1)$$

Where cost of discarded life is measured as the cost of replacement multiplied by the expected percent of the total life discarded, then cost of discarded life equals $C_2[\int_T^\infty R(x)dx / \int_0^\infty R(x)dx]$. Further, in accordance with renewal reward theorem, the objective of a minimum cost model is to minimize expected costs per cycle divided by the expected cycle length (i.e., minimize cost per unit of life such as dollars per operating hour). Hence, similar to Barlow and Proschan's age replacement model, the optimum replacement interval for minimizing cost is that value of T which yields the minimum value for C(T) in the following equation:

$$C(T) = \frac{C_1F(T) + C_2R(T)[\int_T^\infty R(x)dx / \int_0^\infty R(x)dx]}{\int_0^T R(x)dx} \qquad (5.2)$$

For maximizing availability, the optimum replacement interval is found using the Radner-Jorgenson formulation, which maximizes the expected life over the expected cycle time of T + K; specifically,

$$A(T) = \frac{\int_0^T R(x)dx}{T + K} \qquad (5.3)$$

For minimizing risk of failure, we let Z(T) represent expected long term risk of the device being in state two. Since the objective is to minimize the expected time in a failed condition over the expected cycle time, the optimum replacement interval is that value of T which minimizes Z(T) in the following equation,

$$Z(T) = \frac{\int_0^T F(x)dx}{T + K} \tag{5.4}$$

Minimizing Z(T) unconstrained would not make sense since it would necessarily drive a replacement interval of $T = 0$, where the expected risk would also be 0.

In order for the model to address type three uncertainty (i.e., uncertainty regarding the underlying failure distribution), we apply some lessons learned from total time on test methodology. As stated earlier, $t_{(i)}$ represents a data point in the ordered set of life observations; T_i represents the accumulated life at point in time $t_{(i)}$; and n represents the number of observations. Then, we let $\underline{c}$ represent a cost per unit of life constraint; $\underline{z}$ represent an acceptable level of risk for the device being in state two; $\underline{a}$ represent an availability constraint in terms of the probability of the device being in state one; B_i represent the accumulated time in state two; and D_i represent the accumulated life discarded by preventive replacement. Then for any value of $T = t_{(i)}$,

$$B_i = \sum_{j=1}^{i-1} (t_{(i)} - t_{(j)}) \qquad (5.5)$$

$$D_i = \sum_{j=i+1}^{n} (t_{(j)} - t_{(i)}) \qquad (5.6)$$

$$T+K = t_{(i)}+K \qquad (5.7)$$

Since T_i/n equals the accumulated life for the n components divided by the number of components, it necessarily equals the expected life for the empirical data set and will approximate the mean of the theoretical probability distribution when n is sufficiently large. Thus, for any value of $T = t_{(i)}$,

$$\int_0^T R(x)dx = \lim_{n \to \infty} T_i/n \qquad (5.8)$$

And similarly,

$$\int_0^\infty R(x)dx = \lim_{n \to \infty} T_n/n \qquad (5.9)$$

$$\int_T^\infty R(x)dx = \lim_{n \to \infty} D_i/n \qquad (5.10)$$

$$\int_0^T F(x)dx = \lim_{n \to \infty} B_i/n \qquad (5.11)$$

Also, since $F(T) = \lim_{n \to \infty} F_n(T)$, then when n is sufficiently large, $F(T) \doteq F_n(T) \doteq F_n(t_{(i)})$. And similarly, $R(T) \doteq R_n(t_{(i)})$; $A(T) \doteq A_n(t_{(i)})$; $C(T) \doteq C_n(t_{(i)})$; and $Z(T) \doteq Z_n(t_{(i)})$. Further, since $t_{(i)}$ represents one data point in the empirical cumulative failure distribution, $F_n(T)$, then necessarily $F_n(t_{(i)})$ is the probability of failure within the interval $(t_{(0)}, t_{(i)})$ and is therefore approximately equal to i/n. Similarly, $R_n(t_{(i)}) \doteq 1-(i/n)$. Thus from equation 5.2,

$$C_n(t_{(i)}) \doteq \frac{C_1(i/n) + C_2[1-(i/n)](D_i/T_n)}{T_i/n} = \frac{C_1(i) + C_2(n-i)(D_i/T_n)}{T_i} \quad (5.12)$$

And from equation 5.3,

$$A_n(t_{(i)}) \doteq \frac{T_i/n}{t_{(i)} + K} = \frac{T_i}{n(t_{(i)}+K)} \quad (5.13)$$

And from equation 5.4,

$$Z_n(t_{(i)}) \doteq \frac{B_i/n}{t_{(i)} + K} = \frac{B_i}{n(t_{(i)}+K)} \quad (5.14)$$

Therefore, we can conclude that:

$$\min C(T) \implies \min \frac{C_1(i) + C_2(n-i)(D_i/T_n)}{T_i} \tag{5.15}$$

$$\max A(T) \implies \max \frac{T_i}{n(t_{(i)}+K)} \tag{5.16}$$

$$\min Z(T) \implies \min \frac{B_i}{n(t_{(i)}+K)} \tag{5.17}$$

Thus, the value of $t_{(i)}$ that minimizes C(T) where $B_i/[n(t_{(i)}+K)] \leq \underline{z}$ and $T_i/[n(t_{(i)}+K)] \geq \underline{a}$ is the replacement interval that minimizes long term costs subject to the probability of the device being in state two not exceeding $\underline{z}$ and probability of the device being in state one being greater than or equal to $\underline{a}$. Likewise the value of $t_{(i)}$ that maximizes A(T) where $Z_n(t_{(i)}) \leq \underline{z}$ and $C_n(t_{(i)}) \leq \underline{c}$ is the replacement interval that maximizes availability subject to probability of the device being in state two not exceeding $\underline{z}$ and cost per unit of life not exceeding $\underline{c}$. Either of these solutions can be found using a simple iterative computer program. Also, note that finding the value of $t_{(i)}$ that maximizes A(T) where there are no constraints established for cost and risk of failure is equivalant to finding the optimal replacement interval using the Talbott model. Thus, we now formulate replacement model one.

Replacement model one is appropriate for situations having only type one and type three uncertainty. The major distinction in this scenerio is that since there is no type two uncertainty (i.e., uncertainty regarding the present condition of the device), a device would not stay in operation in a failed condition; therefore, there is no state two. Hence, the optimum replacement inteval for maximizing availability is that value of T which maximizes the expected life over the expected cycle time in the following equation:

$$A(T) = \frac{\int_0^T R(x)dx}{\int_0^T R(x)dx + K} \qquad (5.18)$$

Here, maximizing A(T) unconstrained would not make sense since it would necessarily drive a replacement interval of T equal to infinity. For minimizing risk of failure, the optimum replacement interval is simply that value of T that minimizes F(T). The minimum cost formulation is identical to the formulation for replacement model two (Equation 5.2). Therefore, we can conclude:

$$\min C(T) \implies \min \frac{C_1(i) + C_2(n-i)(D_i/T_n)}{T_i} \qquad (5.19)$$

$$\max A(T) \implies \max \frac{T_i}{T_i + nK} \qquad (5.20)$$

$$\min F(T) \Longrightarrow \min i/n \qquad (5.21)$$

Thus, the value of $t_{(i)}$ that minimizes C(T) where $T_i/(T_i+nK) \geq \underline{a}$ and $F(T) \leq \underline{z}$ is the replacement interval that minimizes long term costs subject to the probability of the device being in state one not being less than $\underline{a}$ and the risk of failure not exceeding $\underline{z}$. Similarly, maximum availability and minimum risk solutions can be found as well. Furthermore, finding the value of $t_{(i)}$ that minimizes C(T) where $\underline{z} = 1$ and $\underline{a} = 0$ is similar to finding the optimal replacement interval using the Bergman model. The only difference is that the Bergman model does not account for discarded life.

Summary

Achieving the best value for a prescribed objective function, unconstrained by any other factors, may not be a realistic goal for many real-world preventive maintenance scenarios. This is particularly true when availability (readiness) or risk of failure are viable concerns, as they often are in the Air Force. Under conditions of type two uncertainty (i.e., uncertainty regarding the present condition of the device), maximizing availability without constraint can lead to several problems, as discussed earlier; and minimizing risk of failure without constraint may not be prudent. In the absence of type two uncertainty, neither maximizing availability nor minimizing risk of failure makes any sense as an unconstrained objective function. Hence, we believe models that allow the decision maker to impose constraints on the objective function are far more realistic.

Identifying an appropriate model for selecting an optimal oil replacement interval remains a dilemna. Due to the complex nature of motor oil failures, none of the models addressed in this project appear to be appropriate. We believe selecting an optimal preventive maintenance interval for motor oil requires development and application of a sophisticated maintenance model, far more complex than the ones addressed in this project.

CHAPTER VI

CONCLUSIONS AND RECOMMENDATIONS

The purpose of this chapter is to draw conclusions from this research project and make some recommendations for further research and study.

Research Question and Conclusions

The research question for this project was, "Can the blind replacement model be applied to an empirical data set using TTT transform graphical solution techiques to determine optimal replacement intervals for synthetic and petroleum motor oils?" Our answer is no. The major reason is that motor oils do not meet the assumptions of the blind replacement model since they are not in storage waiting to respond to an emergency. Also, we believe there are viable cost concerns associated with replacing motor oils. Specifically, there are real costs resulting from operating an engine on failed oil and discarding life of good oil. Moreover, these costs probably should not be ignored when selecting oil replacement intervals. Since blind replacement maximizes availability without regard for cost, we consider it an inappropriate model for determining optimal replacement intervals for motor oils. Therefore, we cannot recommend the replacement intervals determined in this research project for real-world applications.

In addition to the above, the authors draw the following conclusions based on the data and analysis of this research project.

1. When comparing synthetic (CONOCO and Stauffer) and petroleum (mineral) motor oils, the type of oil used in an Air Force general purpose vehicles does not significantly affect the propensity of the oil to fail. In other words, there is no significant difference between these three types of motor oils in terms of probability of oil failure.

2. In our opinions, due to the complex nature of oil failures, application of an appropriate model for determining optimal replacement intervals requires selection of a fairly complex preventive maintenance model.

3. Both the Radner-Jorgenson blind replacement model and the Talbott adaptation of this model to TTT transform techniques appear to be valid when applied strictly within the scenario for which they were developed. However, we believe their objective of maximizing availability without constraint makes them inappropriate for many real-world applications.

4. The TTT transform technique appears to be a viable approach for determining optimal replacement intervals when the underlying failure distribution is not known with certainty since it eliminates error resulting from type three uncertainty.

5. Preventive maintenance models that achieve the best value of a prescribed objective function, unconstrained by any other factors, may not be appropriate for many real-world preventive maintenance scenarios.

<u>Recommendations for Further Research</u>

The authors recommend the following areas for future research and study:

1. Development and demonstration of preventive maintenance models for motor oils and devices having similar complex failure patterns.

2. Further adaptations of mathematical models to TTT transform techniques or other methodologies that eliminate error resulting from type three uncertainty.

3. Further development and demonstration of preventive maintenance models that allow constraints to be imposed on the objective function.

4. Greater focus in the arena of reliability theory toward demonstration of preventive maintenance models in terms of their real-world tractability and applications.

APPENDIX A

TTT PROCESS FOR AN IFR DISTRIBUTION

TTT PROCESS FOR AN IFR DISTRIBUTION

EXAMPLE: Consider a set of five components where one fails after 5 hours of operation, one after 9 hours, one after 12 hours, one after 14 hours, and one after 15 hours. The ordered set of life distributions is then expressed as (5, 9, 12, 14, 15). From equation 3.1, T_i is computed for each component where n = 5 as follows:

$$T_i = \sum_{j=1}^{i} (5 - j + 1)\,(t_{(j)} - t_{(j-1)})$$

$$T_1 = (5 - 1 + 1)\,(5 - 0) = 25$$
$$T_2 = (5 - 2 + 1)\,(9 - 5) + 25 = 41$$
$$T_3 = (5 - 3 + 1)\,(12 - 9) + 41 = 50$$
$$T_4 = (5 - 4 + 1)\,(14 - 12) + 50 = 54$$
$$T_5 = (5 - 5 + 1)\,(15 - 14) + 54 = 55$$

And U_i can be computed as T_i/T_n; hence,

i	$t_{(i)}$	T_i		U_i	i/n
1	5	25	(5 components live 5 hours)	.45	.2
2	9	41	(4 components live another 4 hours)	.75	.4
3	12	50	(3 components live another 3 hours)	.91	.6
4	14	54	(2 components live another 2 hours)	.98	.8
5	15	55	(1 component lives another hour)	1.00	1.0

TTT plot for this distribution is given at Attachment C.

APPENDIX B

EXAMPLE TTT PROCESSES FOR EXPONENTIAL AND DFR DISTRIBUTIONS

EXAMPLE TTT PROCESSES FOR EXPONENTIAL AND DFR DISTRIBUTIONS

Example Exponential distribution:

i	$t_{(i)}$	T_i	U_i	i/n
1	3	15	.2	.2
2	6.75	30	.4	.4
3	11.75	45	.6	.6
4	19.25	60	.8	.8
5	34.25	75	1.0	1.0

Example DFR Distribution:

i	$t_{(i)}$	T_i	U_i	i/n
1	1	5	.05	.2
2	3.5	15	.15	.4
3	8.5	30	.30	.6
4	23.5	60	.60	.8
5	63.5	100	1.00	1.0

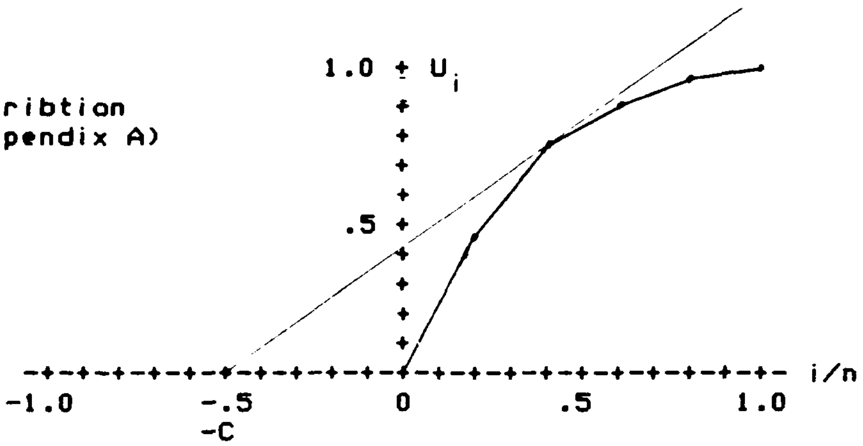
TTT plots for these distributions are given at Attachment C.

APPENDIX C

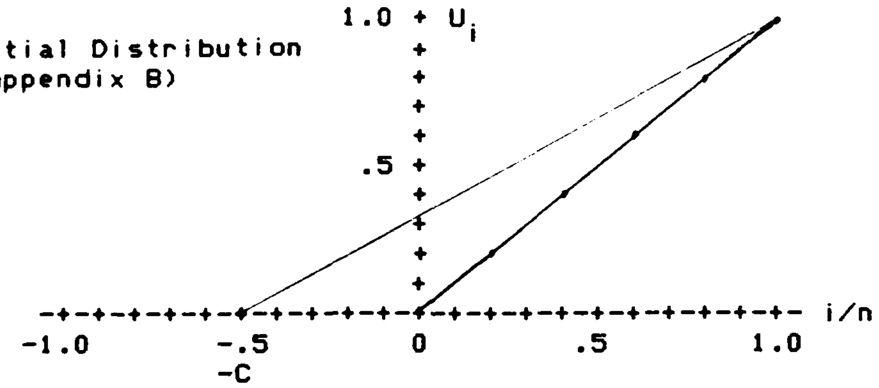
EXAMPLE TTT PLOTS

EXAMPLE TTT PLOTS

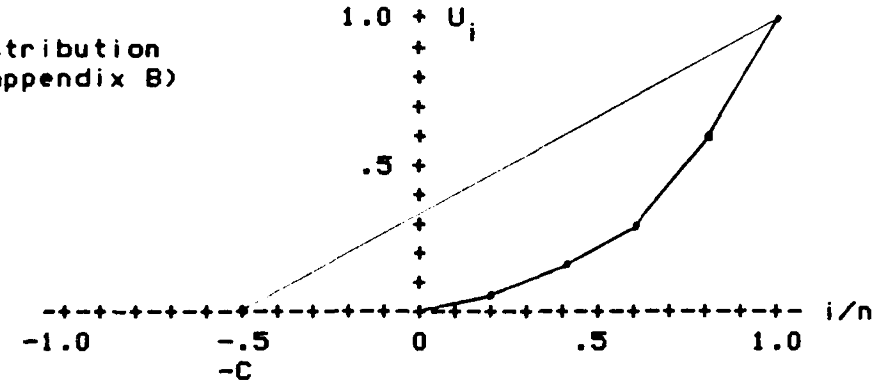
IFR Distribtion
(From Appendix A)



Exponential Distribution
(From Appendix B)



DFR Distribution
(From Appendix B)



APPENDIX D

CRITERIA FOR FAILED OIL

Baseline Data for Synthetic Oils

OIL ANALYSIS RECORD | Equipment Model All | Equipment Serial Number | Type Equip Code (EIC) | C u s t | Cust. Ident. | Lab | End Model/Hull Number | End Item Serial Number | Type Oil | REMARKS

Sample	Hours Miles Since OVHL	Hours Miles Since OIL CHG	Reason for Sample	FE	AG	AL	BE	CR	CU	MG	NA	NI	PB	SI	SN	TI	B	BA	CD	MN	MO	V	ZN	Validation	Lab Recommend	Action Taken	Discrepancy
Conoco (Green)				2						663	22		4	7			2	998					722				
Stauffers (Yellow)				4						15	32		2	22			11	118					998				

BASELINE DATA FOR CONOCO & STAUFFER SYNTHETIC OILS

CUSTOMER SAMPLE NUMBER	DATA INDEX	JULIAN DATE SAMPLE ANAL.	DAYS IN TRANS.	LAST LAB ADV.	HOURS MILES SINCE OVHL	HOURS MILES SINCE OIL CHG	REASON FOR SMPL.	Fe	Ag	Al	Be	Cr	Cu	Mg	Na	Ni	Pb	Si	Sn	Ti	B	Ba	Cd	Mn	Mo	V	Zn	VALIDATION	LAB ADVICE	MAINT ACTION	DISCREP. ITEM	HOW MAL	HOW FOUND
STAUFFERS						$\bar{x}$		4.8	0	0	0	0	0	15.4	32.3	0	2.2	21.6	0	0	11.2	118	0	0	0	0	>998						
						σ		[illegible]	–	–	–	–	–	0.63	1.40	–	0.56	0.83	–	–	0.56	5.10	–	–	–	–	–						
						n		15	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–						
CONOCO						$\bar{x}$		1.8	0	0	0	0	0	663	22.2	0	4.0	6.7	0	0	2.4	>998	0	0	0	0	722						
						σ		0.44	–	–	–	–	–	46	1.93	–	0.53	0.47	–	–	0.51	–	–	–	–	–	55						
						n		15	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–						
ppm		STAUFFERS						4						15	32		2	22			11	118					>998						
		CONOCO						2						663	22		4	7			2	>998					722						
Run @ 0-1000 rpm																																	

RSL: 13 Nov 79

OIL ANALYSIS RECORD

Equipment Model APL	Equipment Serial Number	Type Equip Code (GIC)	Cust	Cust. Ident.	Lab	End Model/Hull Number	End Item Serial Number	Type Oil	REMARKS

SAMPLE NUMBER	DATA INDEX	SAMPLE DATE / SAMPLE ANALYZED	RESPONSE TIME	LAST LAB RECOMMENDATION	Hours Miles Since: OVHL	Hours Miles Since: OIL CHG	REASONFORSMPL	VALIDATION	LAB RECOMMEND	ACTION TAKEN

SPECTROMETRIC ANALYSIS

FE	[illegible] > 200 / 200-500 High / > 500 Very High
AG	/ / / / / / / / /
AL	[illegible] > 30 / 30-50 High / > 50 Very High
BE	/ / / / / / / / /
CR	[illegible] > 30 / 30-50 High / > 50 Very High
CU	[illegible] > 20 / 20-40 High / > 40 Very High
MG	Same as Boron (Sealer) ~ ~ / Mineral oil only K-99F
NA	Same as Boron (Sealer) ~ ~ / Mineral oil only
NI	/ / / / / / / / /
PB	
SI	[illegible] > 30 / 30-50 High / > 50 Very High
SN	Object[illegible]
TI	/ / / / / / / / /
B	[illegible] > 30 High / 30-50 High / > 50 Very High
BA	Same as Boron (Sealer) [illegible]
CD	/ / / / / / / / /
MN	
MO	[illegible] > 30 / 30-50 High / > 50 Very High
V	/ / / / / / / / /
ZN	

NOTE: when mileage exceeds 6000 and day [illegible], fill into "Very High" on exceed guidelines for physical tests, recommend oil change for mineral oils and on synthetics are [illegible].

Physical Analysis	
SAMPLE NUMBER	
OIL ADDITION GALLONS	
TEMPERATURE	
VIS	Refer to Vis. Spec. Sheets; < SPEC Require Fuel Dilution by G.C.; > SPEC Require TAN/TBN; ≥ Spec ±25% Require [illegible]
WATER	Change Filter & Oil if > 0.5 % by IR & Recm Inspect Coolant Sys.
FILTER SPOT	Spots indicating H_2O/Coolant Require IR Analysis. Spots indicating Excessive Solids Require Insolubles/TAN/TBN
SOLIDS	Solids > 0.3 ml Recm: Oil & Filter Change. In Cases with high solids Consider Si and if both or High Recm: OC/F/AIR
FUEL DIL.	Fuel Dilution > 5.0 % Recm: Inspect Fuel System or if Cause is excessive idling or Cold Starts recm. run out
ACID	Refer to TAN/TBN Spec Sheet. When Specs. are exceeded Recm: Oil & Filter Change.
BASE	See "Acid"
FLASH POINT	N/A
IR	USE if Water or Glycol is Suspected, H_2O > 0.5% or any Glycol is excessive. Recm. Oil & Filter Change after Inspect [illegible]
LAB RECOMMENDATIONS	
REMARKS	

NOTE: Refer to Baseline Data Card for Average Spec Values

STAUFFER SL-2100

TEMPERATURE °F	VISCOSITY* TRUE VALUE	LOW LIMIT (25% TV)	HIGH LIMIT (25% TV)	
68	125.75	94.22	157.19	
69	122.40	91.80	153.00	
70	119.18	89.39	148.99	
71	116.10	87.08	145.13	
72	113.34	85.01	141.68	
73	110.29	82.72	137.86	
74	107.55	80.66	134.44	
75	104.91	78.68	131.14	
76	102.37	76.78	127.96	
77	99.93	74.95	124.91	
78	97.57	73.18	121.96	
79	95.30	71.48	119.13	
80	93.11	69.83	116.39	
81	91.00	68.25	113.75	
82	88.95	66.71	111.19	
83	86.98	65.24	108.73	
84	85.08	63.81	106.35	
85	83.24	62.43	104.05	

*Use 10^3 scale only with Stauffer SL-2100 oils.

17 Jan 1980/Griffin L. J

AF FORM SEP 77 3126 GENERAL PURPOSE

STAUFFER SL-2100

	Batch 9751 H-1-1	Original Values	Military Specification Limits
Gravity, $^{\circ}$API	26.6	27.1	± 1
Flash Point ($^{\circ}$F)	425	450	± 50
Viscosity @ 210°F (cs)	14.32	14.02	± 0.5
Viscosity @ 100°F	88.15	84.95	± 15.0
Apparent Viscosity @ 0°F (cp)	1640	1520	≮ 2400
Viscosity @ 0°F (cs)	2537	2300	Report
Viscosity Index	180	182	"
Pour Point ($^{\circ}$F)	-45	-45	"
Stable Pour Point ($^{\circ}$F)	-25	-25	"
Total Base Number-D664, mgKOH/g	6.08	6.02	"
Total Base Number-D2896, mgKOH/g	6.48	6.32	"
Total Acid Number-D664, mgKOH/g	2.75	2.81	"
Sulfated Ash, %	1.02	0.99	0.90-1.20

CRC L-38 Crankcase Oil Oxidation Test

Bearing weight loss (mg)			
Top half	13.5	16.2	-
Bottom half	17.4	15.9	-
Total	30.9	32.1	≮ 40.0
Varnish Deposits	59.3	59.4	-
Piston Skirt Varnish	9.6	9.6	≯ 9.0

CONOCO DN-600

TEMPERATURE °F	VISCOSITY* TRUE VALUE	LOW LIMIT (25% TV)	HIGH LIMIT (25% TV)	
68	102.47	76.85	128.09	
69	99.60	74.70	124.50	
70	96.85	72.64	121.06	
71	94.21	70.66	117.76	
72	91.68	68.76	114.60	
73	89.25	67.24	111.56	
74	86.92	65.19	108.65	
75	84.67	63.50	105.84	
76	82.52	61.89	103.15	
77	80.44	60.33	100.55	
78	78.45	58.84	98.06	
79	76.52	56.64	94.40	
80	74.67	56.00	93.34	
81	72.89	54.67	91.11	
82	71.17	53.38	88.96	
83	69.51	52.13	86.89	
84	67.90	50.93	84.88	
85	66.36	49.77	82.95	

17 Jan 1980/Griffin L. Jones

*Use 10^2 scale only with Conoco DN-600 oils.

AF FORM SEP 77 3126 GENERAL PURPOSE

APPENDIX E

LUBRICANT LIFETIME DATA

SYMBOL TABLE

Vehicle make:

AMC = American Motors Corporation
CHEV = Chevrolet
DODG = Dodge
FORD = Ford
IHC = International Harvester Corporation
OLDS = Oldsmobile
PLYM = Plymouth

Vehicle Type:

AMB = Ambulance
C/A = Carry-all
CJ7 = Jeep
M/S = Multi-step
P/U = Pick-up
Pan = Panel truck
S/P = Special purpose
S/V = Step van
SDN = Sedan
SW = Station wagon
TRAC = Tractor
1 Ton = One ton truck
1.5TN = 1-1/2 ton truck
4X4 = Four-wheel drive truck

Usage Code:

A = High utilization/low mileage
B = High utilization/high mileage
C = Low utilization/low mileage
D = Low utilization/high mileage

LUBRICANT LIFETIME DATA

STAUFFER

MAKE	MODEL	USE	VEHICLE	ENG	START MILES	END MILES	OIL LIFE	TEST DAYS	AVG MPD
GRAND FORKS AFB									
CHEV	C/A	B	791737	350	44516	48512	3996	85	47.01
OFFUTT AFB									
MINOT AFB									
LACKLAND AFB									
AMC	SDN	D	762826	232	28114	34081	5967	220	27.12
DODG	P/U	C	764048	225	23286	30259	6973	402	17.35
AMC	SW	A	785344	200	50426	56651	6225	193	32.25
AMC	SDN	D	788302	232	10535	25535	15000	442	33.94
RANDOLPH AFB									
IHC	TRAC	B	724309	392	83212	89272	6060	310	19.55
IHC	M/S	A	732296	345	54835	60467	5632	204	27.61
FORD	P/U	A	761781	1900	25220	33194	7974	422	18.90
AMC	SW	B	762448	232	39469	48384	8915	305	29.23
DODG	P/U	C	770149	318	4544	5251	707	330	2.14
CHEV	P/U	B	781244	250	39335	42231	2896	294	9.85
CHEV	P/U	C	781247	250	13779	19063	5284	440	12.01
FORD	SW	B	785346	200	39725	48238	8513	224	38.00
AMC	SDN	B	788305	232	16493	26925	10432	361	28.90
PLYM	SDN	B	789172	225	48537	54752	6215	84	73.99
GEORGE AFB									
CHEV	S/V	C	761615	350	27175	32709	5534	339	16.32
IHC	1.5TN	A	771097	345	10003	15554	5551	244	22.75
DODG	P/U	A	780446	318	23216	26309	3093	187	16.54
DODG	P/U	A	780452	318	14853	18137	3284	245	13.40
CHEV	PAN	A	783521	250	29246	34569	5323	289	18.42
FORD	1.5TN	A	785850	300	17445	21467	4022	200	20.11
PLYM	SDN	B	789080	225	33106	38055	4949	93	53.22
DODG	1 TON	A	789357	360	11639	18660	7021	336	20.90
DODG	PAN	A	792534	318	11396	13367	1971	134	14.71

MYRTLE BEACH AFB									
AMC	AMB	C	760278	360	2445	3584	1139	527	2.16
DODG	PAN	A	792473	225	2604	11413	8809	586	15.03
HANCOCK FIELD									
AMC	SDN	A	733165	304	55928	59672	3744	375	9.98
CHEV	P/U	C	740738	350	58785	62747	3962	385	10.29
FORD	P/U	A	751429	300	35068	43936	8868	497	17.84
FORD	P/U	A	751430	300	29419	33403	3984	381	10.46
PETERSON FIELD									
CHEV	P/U	A	742105	350	46519	52898	6379	320	19.93
DODG	P/U	B	780139	225	23147	28117	4970	209	23.78
DODG	P/U	C	780141	225	7639	11715	4076	383	10.64
CHEV	M/S	B	784570	292	10645	13244	2599	267	9.73
PLYM	SW	B	785110	225	49242	56395	7153	62	115.37
USAF ACADEMY									
PLYM	SDN	A	788159	225	15345	28276	12931	233	55.50
FORD	SDN	B	795660	200	12535	29364	16829	241	69.83

MINERAL

MAKE	MODEL	USE	VEHICLE	ENG	START MILES	END MILES	OIL LIFE	TEST DAYS	AVG MPD
GRAND FORKS AFB									
CHEV	C/A	B	791753	350	43211	49814	6603	62	106.50
OFFUTT AFB									
CHEV	M/S	A	722544	350	42094	44133	2039	128	15.93
CHEV	M/S	A	753606	350	41759	49110	7351	195	37.70
MINOT AFB									
CHEV	C/A	C	740088	454	164995	170078	5083	403	12.61
CHEV	C/A	C	744124	454	163183	168057	4874	252	19.34
CHEV	C/A	B	760519	350	117567	133079	15512	214	72.49
CHEV	C/A	B	791755	350	37750	43697	5947	86	69.15
CHEV	C/A	B	791764	350	49380	53237	3857	20	192.85
CHEV	C/A	B	792709	350	16038	24616	8578	184	46.62
CHEV	C/A	B	793380	350	17765	28147	10382	149	69.68
CHEV	C/A	B	793381	350	27407	39101	11694	158	74.01
CHEV	C/A	B	793383	350	30919	39775	8856	116	76.34
LACKLAND AFB									
DODG	C/A	C	755338	318	64995	68712	3717	126	29.50
CHEV	AMB	A	760113	350	49704	55002	5298	405	13.08
AMC	SDN	B	762919	232	45238	53343	8105	267	30.36
CHEV	P/U	A	781259	250	22498	30926	8428	269	31.33
DODG	P/U	C	782261	225	17169	22578	5409	247	21.90
AMC	SDN	C	788418	232	5603	11205	5602	470	11.92
CHEV	P/U	B	794279	2300	9171	17362	8191	298	27.49
RANDOLPH AFB									
CHEV	M/S	A	734641	350	52768	56410	3642	140	26.01
CHEV	C/A	B	770785	350	43730	47060	3330	113	29.47
CHEV	P/U	B	781245	250	19425	35145	15720	506	31.07
CHEV	P/U	B	781246	250	28093	35474	7381	371	19.89
FORD	SW	B	787738	200	37805	43587	5782	89	64.97
AMC	SDN	A	788657	232	15792	28178	12386	431	28.74

GEORGE AFB

CHEV	S/V	B	761619	350	36339	39661	3322	170	19.54
CHEV	S/V	B	761622	350	41330	46108	4778	147	32.50
IHC	1.5TN	A	771098	345	29456	35872	6416	206	31.15
DODG	P/U	A	780447	318	23776	29663	5887	222	26.52
PLYM	SDN	B	789081	225	34206	37540	3334	133	25.07
DODG	1 TON	A	789358	318	7151	14730	7579	337	22.49

MYRTLE BEACH AFB

OLDS	AMB	C	730453	455	59300	66539	7239	258	28.06
CHEV	M/S	A	742721	250	41526	46168	4642	511	9.08
DODG	P/U	C	754213	318	14264	17984	3720	276	13.48

HANCOCK FIELD

PETERSON FIELD

CHEV	M/S	B	722632	350	53882	59038	5156	235	21.94
FORD	P/U	C	761807	1900	31590	41858	10268	387	26.53
DODG	4X4	B	770077	225	4708	10049	5341	357	14.96
PLYM	SW	B	785109	225	40787	55673	14886	558	26.68
FORD	SDN	B	783768	400	40832	46272	5440	91	59.78

USAF ACADEMY

CHEV	S/P	A	683520	250	62503	66641	4138	299	13.84
DODG	P/U	A	780131	225	19908	25351	5443	240	22.68
DODG	P/U	A	790861	225	6773	18544	11771	231	50.96

CONOCO

MAKE	MODEL	USE	VEHICLE	ENG	START MILES	END MILES	OIL LIFE	TEST DAYS	AVG MPD
GRAND FORKS AFB									
OFFUTT AFB									
MINOT AFB									
CHEV	C/A	C	744121	454	159156	164482	5326	293	18.18
CHEV	C/A	B	760516	350	133095	139086	5991	106	56.52
CHEV	C/A	B	791757	350	56977	61031	4054	54	75.07
CHEV	C/A	B	791762	350	36924	40912	3988	82	48.63
CHEV	C/A	B	793387	350	14252	20337	6085	308	19.76
CHEV	C/A	B	793388	350	31466	37194	5728	88	65.09
LACKLAND AFB									
DODG	C/A	A	750870	318	51827	69226	17399	602	28.90
DODG	P/U	D	764049	225	37162	47983	10821	267	40.53
CHEV	P/U	A	781235	250	28602	38397	9795	351	27.91
CHEV	P/U	A	781261	250	12671	23916	11245	428	26.27
AMC	SW	B	785432	200	56456	65300	8844	138	64.09
RANDOLPH AFB									
FORD	P/U	B	761774	1900	46601	56756	10155	216	47.01
CHEV	C/A	B	770428	350	65402	70377	4975	116	42.89
CHEV	P/U	B	781248	250	17146	25590	8444	318	26.55
FORD	SW	B	785347	200	52692	61004	8312	270	30.79
PLYM	SDN	B	789173	225	63092	75892	12800	147	87.07
GEORGE AFB									
DODG	P/U	B	780442	318	30767	33391	2624	188	13.96
DODG	P/U	B	780460	318	27207	30688	3481	122	28.53
FORD	SW	B	787944	302	26953	32057	5104	164	31.12
PLYM	SDN	B	789079	225	33043	39500	6457	139	46.45
MYRTLE BEACH AFB									
DODG	P/U	C	754212	318	14202	18118	3916	472	8.30
AMC	CJ7	C	789616	258	16862	21165	4303	322	13.36
HANCOCK FIELD									
FORD	SDN	B	714579	302	94743	98742	3999	112	35.71
AMC	SDN	A	733260	304	49598	57033	7435	633	11.75
FORD	P/U	A	751427	300	33875	37462	3587	220	16.30
DODG	P/U	A	754106	318	26777	33360	6583	577	11.41
CHEV	P/U	A	771056	300	12532	18636	6104	545	11.20
PLYM	SW	B	785038	225	19386	27894	8508	152	55.97

PETERSON FIELD									
CHEV	M/S	B	722548	350	56406	63370	6964	296	23.53
AMC	SDN	B	733766	304	47214	55909	8695	495	17.57
FORD	P/U	A	753204	300	39135	44267	5132	552	9.30
CHEV	M/S	B	784568	292	19114	27352	8238	278	29.63
CHEV	M/S	A	784569	292	18250	24779	6529	231	28.26
USAF ACADEMY									
IHC	M/S	A	731969	345	64510	71333	6823	323	21.12
FORD	S/P	A	750245	318	43688	47660	3972	184	21.59
DODG	P/U	A	790858	225	6934	14413	7479	177	42.25
FORD	SDN	B	795659	200	12161	18434	6273	95	66.03

TOTAL OIL LIFE: 776275
TOTAL TEST DAYS: 31267
AVERAGE MILES PER DAY: 24.83

APPENDIX F

WILCOXON-RANK SUM STATISTICAL ANALYSIS

WILCOXON-RANK SUM STATISTICAL ANALYSIS

The first test compared populations using the entire data set. Note: The two-tailed probability (P) values are compared to alpha = .05/2 = .025.

TEST ONE

Factor: Oil Type

OIL TYPE	NUMBER	2-TAILED P
STAUFFER	37	.2977
MINERAL	42	
STAUFFER	37	.1583
CONOCO	37	
MINERAL	42	.7162
CONOCO	37	

Factor: Make of Vehicle

MAKE	NUMBER	2-TAILED P
AMC	14	.1240
CHEVROLET	46	
AMC	14	.0622
DODGE	24	
AMC	14	.3989
FORD	17	
AMC	14	.3426
IHC	5	
AMC	14	.9333
OLDSMOBILE	1	
AMC	14	.7813
PLYMOUTH	9	
CHEVROLET	46	.3790
DODGE	24	

CHEVROLET	46	.5986
FORD	17	
CHEVROLET	46	.6115
IHC	5	
CHEVROLET	46	.6383
OLDSMOBILE	1	
CHEVROLET	46	.1014
PLYMOUTH	9	
DODGE	24	.2040
FORD	17	
DODGE	24	.4476
IHC	5	
DODGE	24	.5600
OLDSMOBILE	1	
DODGE	24	.0785
PLYMOUTH	9	
FORD	17	.8795
IHC	5	
FORD	17	.8889
OLDSMOBILE	1	
FORD	17	.3666
PLYMOUTH	9	
IHC	5	.3333
OLDSMOBILE	1	
IHC	5	.2398
PLYMOUTH	9	
OLDSMOBILE	1	1.0000
PLYMOUTH	9	

Factor: Utilization of Vehicles

USAGE CODE	NUMBER	2-TAILED P
HI UTE,LO MI	41	.5279
HI UTE,HI MI	54	
HI UTE,LO MI	41	.0286
LO UTE,LO MI	18	

HI UTE,LO MI	41	.1306
LO UTE,HI MI	3	
HI UTE,HI MI	54	.0097 *
LO UTE,LO MI	18	
HI UTE,HI MI	54	.1560
LO UTE,HI MI	3	
LO UTE,LO MI	18	.0105 *
LO UTE,HI MI	3	

TEST TWO

Test two excludes those observations with Usage code C (low utilization/low mileage).

Factor: Oil type

OIL TYPE	NUMBER	2-TAILED P
STAUFFER	30	.4926
MINERAL	34	
STAUFFER	30	.2879
CONOCO	34	
MINERAL	34	.7779
CONOCO	34	

Factor: Make of Vehicle

MAKE	NUMBER	2-TAILED P
AMC	11	.0296
CHEVROLET	40	
AMC	11	.0590
DODGE	17	
AMC	11	.0712
FORD	16	
AMC	11	.0517
IHC	5	
AMC	11	.7103
PLYMOUTH	9	
CHEVROLET	40	.8753
DODGE	17	
CHEVROLET	40	.9855
FORD	16	
CHEVROLET	40	.9581
IHC	5	

CHEVROLET	40	.1620
PLYMOUTH	9	
DODGE	17	.7898
FORD	16	
DODGE	17	.9396
IHC	5	
DODGE	17	.2409
PLYMOUTH	9	
FORD	16	.7190
IHC	5	
FORD	16	.3014
PLYMOUTH	9	
IHC	5	.2398
PLYMOUTH	9	

TEST THREE

Test three includes only those observations with Usage code C (low utilization/low mileage).

Factor: Oil Type

OIL TYPE	NUMBER	2-TAILED P
STAUFFER	7	.3357
MINERAL	8	
STAUFFER	7	1.0000
CONOCO	3	
MINERAL	8	.4970
CONOCO	3	

Factor: Make of Vehicle

MAKE	NUMBER	2-TAILED P
AMC	3	.7143
CHEVROLET	6	
AMC	3	.8333
DODGE	7	
AMC	3	.5000
FORD	1	
AMC	3	.5000
OLDSMOBILE	1	
CHEVROLET	6	.2343
DODGE	7	
CHEVROLET	6	.2857
FORD	1	
CHEVROLET	6	.2857
OLDSMOBILE	1	
DODGE	7	.2500
FORD	1	

DODGE	7	.2500
OLDSMOBILE	1	
FORD	1	1.0000
OLDSMOBILE	1	

APPENDIX G

FORTRAN PROGRAM

```
      PROGRAM BLIND
C
C     APPLICATION OF THE TALBOTT MODEL
C
C     PEARCE AND DOUMIT; SEP 1983
C
C
C         SYMBOL TABLE
C
C    REAL:
C     FAIL - OBSERVED LIFETIMES (UNORDERED)
C     LIFE(I) - OBSERVED LIFETIMES (ORDERED)
C     T(I) - CUMULATIVE LIFE AT POINT IN TIME LIFE(I)
C     TAN - VALUE OF LIFE(I) DEFINED BY POINT OF TANGENCY
C           (THE OPTIMAL INTERVAL)
C     U(I) - TTT SCALED STATISTIC; T(I)/T(N); Y-AXIS
C            VARIABLE
C     X(I) - LIFE(I)/LIFE(N); X-AXIS VARIABLE
C     SL(I) - SLOPE OF THE LINE CONNECTING K/LIFE(N) TO
C             THE TTT DATA POINT
C     MSL - MAXIMUM VALUE OF SL(I)
C     K - TIME TO REPLACE
C     C - CRITICAL TTT POINT: K/LIFE(N)
C     B,E,O,V,W,Y,Z,WW,ZZ - RANDOM REAL VALUES
C
C    INTEGER
C     I - OBSERVATION NUMBER
C     N,J - INDICES
C
C    FILES
C     DATA - INPUT DATA FILE CONTAINING OBSERVED OIL LIFES
C            AND OTHER DESCRIPTIVE DATA
C     TRACK - OUTPUT REPORT FILE
C
C
C         INITIALIZE PROGRAM
C
      REAL FAIL,LIFE(150),T(150),TAN,U(150),
     $ X(150),SL(150),MSL,K,C,W,WW,Y,
     $ ZZ,Z,B,E,O,V
      INTEGER I,N,J
      OPEN (10,FILE='DATA')
      OPEN (11,FILE='TRACK')
      REWIND 10
      REWIND 11
C
C
C    READ IN VALUES FOR N AND K
```

```
C
         READ(10,*)N,K
         WRITE (11,800)
         WRITE (11,820)N,K
         WRITE (11,810)
C
C
C     READ IN AND ORDER LIFETIMES
C
        READ(10,*)O,B,Z,W,Y,V,E,WW,ZZ,LIFE(1)
         DO 20 I=2,N
           READ(10,*)O,B,Z,W,Y,V,E,ZZ,YY,FAIL
           LIFE(I)=FAIL
           DO 10 J=1,I-1
             IF (LIFE(I-J) .GT. FAIL) THEN
               LIFE(I-J+1)=LIFE(I-J)
               LIFE(I-J)=FAIL
             ELSE
               GO TO 20
             END IF
 10        CONTINUE
 20      CONTINUE
         C=K/LIFE(N)
C
C
C     COMPUTE TTT STATISTIC
C
         T(1)=LIFE(1)*N
         DO 30 I=2,N
           T(I)=((LIFE(I)-LIFE(I-1))*(N-I+1.))+T(I-1)
 30      CONTINUE
C
C
C     COMPUTE AXIS VARIABLES AND FIND MAX SLOPE
C
         MSL=0.
         DO 40 I=1,N
           U(I)=T(I)/T(N)
           X(I)=LIFE(I)/LIFE(N)
           SL(I)=U(I)/(C+X(I))
           WRITE(11,920)I,LIFE(I),T(I),U(I),X(I),SL(I)
           IF (SL(I) .GT. MSL) THEN
             MSL=SL(I)
             TAN=LIFE(I)
           END IF
 40      CONTINUE
         WRITE(11,930)TAN
C
C
C     FORMAT STATEMENTS
C
```

AD-A134 470 OPTIMAL MAINTENANCE POLICIES: A GRAPHICAL ANALYSIS(U) AIR FORCE INST OF TECH WRIGHT-PATTERSON AFB OH SCHOOL OF SYSTEMS AND LOGISTICS P F DOUMIT ET AL. SEP 83 2/2

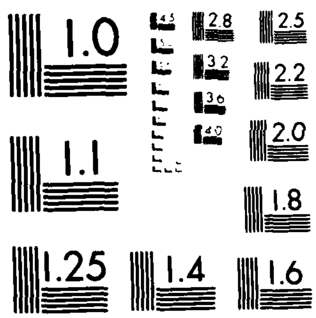
UNCLASSIFIED AFIT-LSSR-28-83 F/G 12/1 NL

END

DATE FILMED

11 83

DTIC



MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS 1963 A

```
800     FORMAT(///T20,'TOTAL TIME ON TEST'//)
810     FORMAT(//T3,'I',T8,'LIFE',T19,'TI',T28,'UI',
    $ T34,'X(I)',T43,'SL'/)
820     FORMAT(' N=',I3,'  K=',F6.2)
920     FORMAT(T2,I3,T7,F6.0,T17,F7.0,T26,F5.3,T31,F7.3,
    $ T40,F8.6)
930     FORMAT(//'OPTIMAL INTERVAL IS ',F6.0,' MILES')
        STOP
        END
```

APPENDIX H

TTT RESULTS: ALL OILS COMBINED

TOTAL TIME ON TEST
ALL OILS COMBINED

N=116 K= 24.83

I	LIFE(I)	T(I)	U(I)	X(I)	SLOPE	
1	707.	82012.	.106	.041	2.511747	
2	1139.	131692.	.170	.065	2.536171	
3	1971.	226540.	.292	.113	2.544076	*
4	2039.	234224.	.302	.117	2.543702	
5	2599.	296944.	.383	.149	2.536574	
6	2624.	299719.	.386	.151	2.536115	
7	2896.	329639.	.425	.166	2.529537	
8	3093.	351112.	.452	.178	2.524073	
9	3284.	371740.	.479	.189	2.518103	
10	3322.	375806.	.484	.191	2.516742	
11	3330.	376654.	.485	.191	2.516406	
12	3334.	377074.	.486	.192	2.516212	
13	3481.	392362.	.505	.200	2.508446	
14	3587.	403280.	.520	.206	2.502581	
15	3642.	408890.	.527	.209	2.499335	
16	3717.	416465.	.536	.214	2.494613	
17	3720.	416765.	.537	.214	2.494410	
18	3744.	419141.	.540	.215	2.492656	
19	3857.	430215.	.554	.222	2.484035	
20	3916.	435938.	.562	.225	2.479395	
21	3962.	440354.	.567	.228	2.475614	
22	3972.	441304.	.568	.228	2.474747	
23	3984.	442432.	.570	.229	2.473646	
24	3988.	442804.	.570	.229	2.473258	
25	3996.	443540.	.571	.230	2.472440	
26	3999.	443813.	.572	.230	2.472117	
27	4022.	445883.	.574	.231	2.469532	
28	4054.	448731.	.578	.233	2.465807	
29	4076.	450667.	.581	.234	2.463160	
30	4138.	456061.	.587	.238	2.455517	
31	4303.	470251.	.606	.247	2.435388	
32	4642.	499066.	.643	.267	2.396871	
33	4778.	510490.	.658	.275	2.382312	
34	4874.	518458.	.668	.280	2.372083	
35	4949.	524608.	.676	.284	2.364028	
36	4970.	526309.	.678	.286	2.361722	
37	4975.	526709.	.679	.286	2.361153	
38	5083.	535241.	.689	.292	2.348668	
39	5104.	536879.	.692	.293	2.346210	
40	5132.	539035.	.694	.295	2.342841	
41	5156.	540859.	.697	.296	2.339879	
42	5284.	550459.	.709	.304	2.323993	
43	5298.	551495.	.710	.305	2.322243	

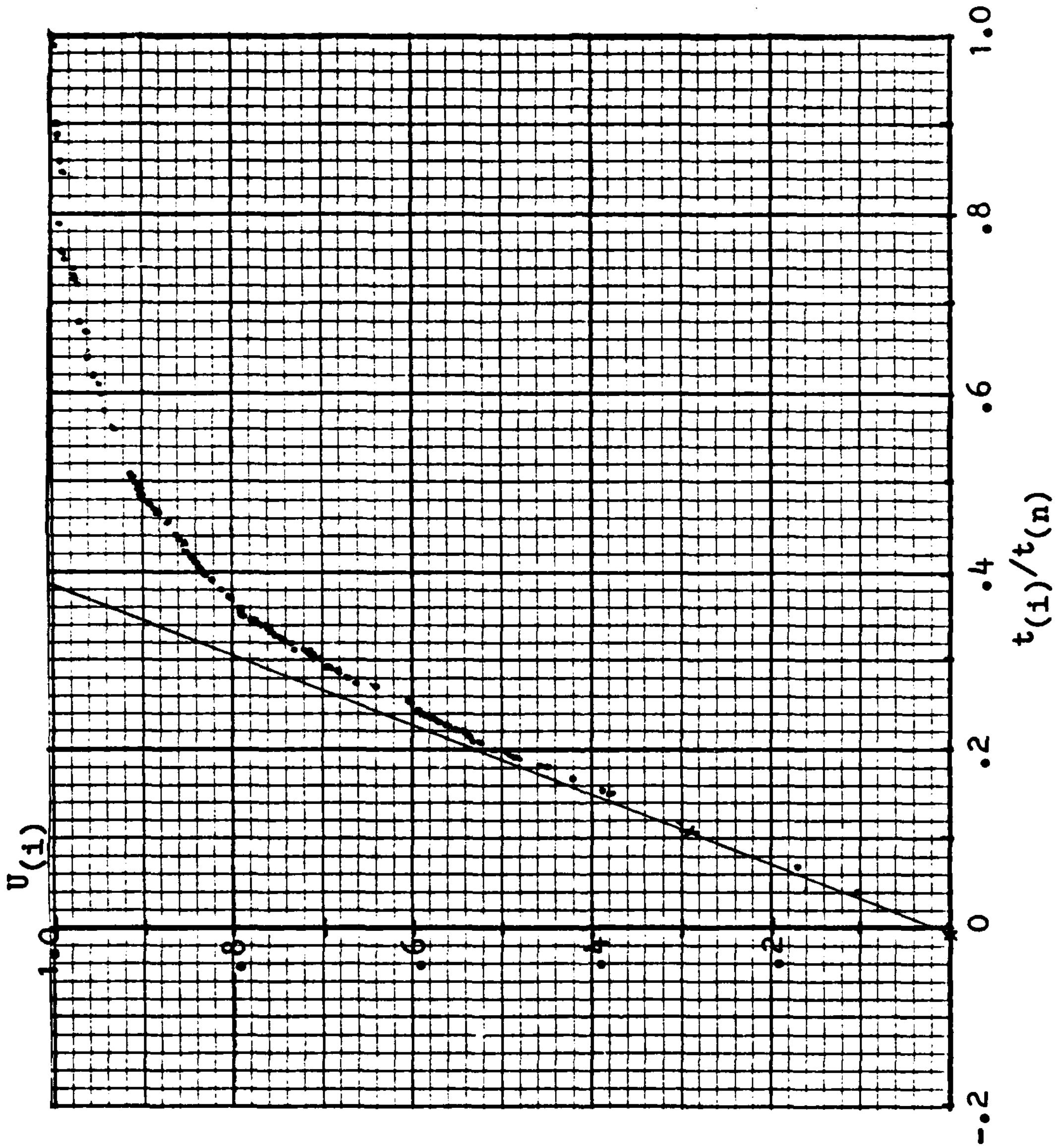
44	5323.	553320.	.713	.306	2.319036
45	5326.	553536.	.713	.306	2.318640
46	5341.	554601.	.714	.307	2.316607
47	5409.	559361.	.721	.311	2.307251
48	5440.	561500.	.723	.313	2.302936
49	5443.	561704.	.724	.313	2.302508
50	5534.	567801.	.731	.318	2.289399
51	5551.	568923.	.733	.319	2.286929
52	5602.	572238.	.737	.322	2.279405
53	5632.	574158.	.740	.324	2.274924
54	5728.	580206.	.747	.329	2.260525
55	5782.	583554.	.752	.332	2.252426
56	5887.	589959.	.760	.338	2.236704
57	5947.	593559.	.765	.342	2.227743
58	5967.	594739.	.766	.343	2.224721
59	5991.	596131.	.768	.344	2.221032
60	6060.	600064.	.773	.348	2.210334
61	6085.	601464.	.775	.350	2.206425
62	6104.	602509.	.776	.351	2.203407
63	6215.	608503.	.784	.357	2.185741
64	6225.	609033.	.785	.358	2.184144
65	6273.	611529.	.788	.361	2.176380
66	6379.	616935.	.795	.367	2.159277
67	6416.	618785.	.797	.369	2.153310
68	6457.	620794.	.800	.371	2.146637
69	6529.	624250.	.804	.375	2.134873
70	6583.	626788.	.807	.378	2.126035
71	6603.	627708.	.809	.380	2.122731
72	6823.	637608.	.821	.392	2.086938
73	6964.	643812.	.829	.400	2.064730
74	6973.	644199.	.830	.401	2.063314
75	7021.	646215.	.832	.404	2.055671
76	7153.	651627.	.839	.411	2.034767
77	7239.	655067.	.844	.416	2.021291
78	7351.	659435.	.849	.422	2.003871
79	7381.	660575.	.851	.424	1.999204
80	7435.	662573.	.854	.427	1.990735
81	7479.	664157.	.856	.430	1.983793
82	7579.	667657.	.860	.436	1.968021
83	7974.	681087.	.877	.458	1.908468
84	8105.	685410.	.883	.466	1.889634
85	8191.	688162.	.886	.471	1.877362
86	8238.	689619.	.888	.473	1.870635
87	8312.	691839.	.891	.478	1.859999
88	8428.	695203.	.896	.484	1.843394
89	8444.	695651.	.896	.485	1.841097
90	8508.	697379.	.898	.489	1.831827
91	8513.	697509.	.899	.489	1.831096
92	8578.	699134.	.901	.493	1.821494
93	8695.	701942.	.904	.500	1.804272
94	8809.	704564.	.908	.506	1.787640

95	8844.	705334.	.909	.508	1.782531
96	8856.	705586.	.909	.509	1.780759
97	8868.	705826.	.909	.510	1.778961
98	8915.	706719.	.910	.512	1.771847
99	9795.	722559.	.931	.563	1.649218
100	10155.	728679.	.939	.584	1.604370
101	10268.	730487.	.941	.590	1.590693
102	10382.	732197.	.943	.597	1.576951
103	10432.	732897.	.944	.600	1.570911
104	10821.	737954.	.951	.622	1.525019
105	11245.	743042.	.957	.646	1.477763
106	11694.	747981.	.964	.672	1.430589
107	11771.	748751.	.965	.677	1.422714
108	12386.	754286.	.972	.712	1.362210
109	12800.	757598.	.976	.736	1.324024
110	12931.	758515.	.977	.743	1.312223
111	14886.	770245.	.992	.856	1.157806
112	15000.	770815.	.993	.862	1.149871
113	15512.	772863.	.996	.892	1.114933
114	15720.	773487.	.996	.904	1.101092
115	16829.	775705.	.999	.967	1.031589
116	17399.	776275.	1.000	1.000	.998575

OPTIMAL INTERVAL IS 1971. MILES

TOTAL TIME ON TEST PLOTS

ALL OILS COMBINED



APPENDIX I

TTT RESULTS: STAUFFER SYNTHETIC OIL

TOTAL TIME ON TEST
STAUFFER SYNTHETIC OIL

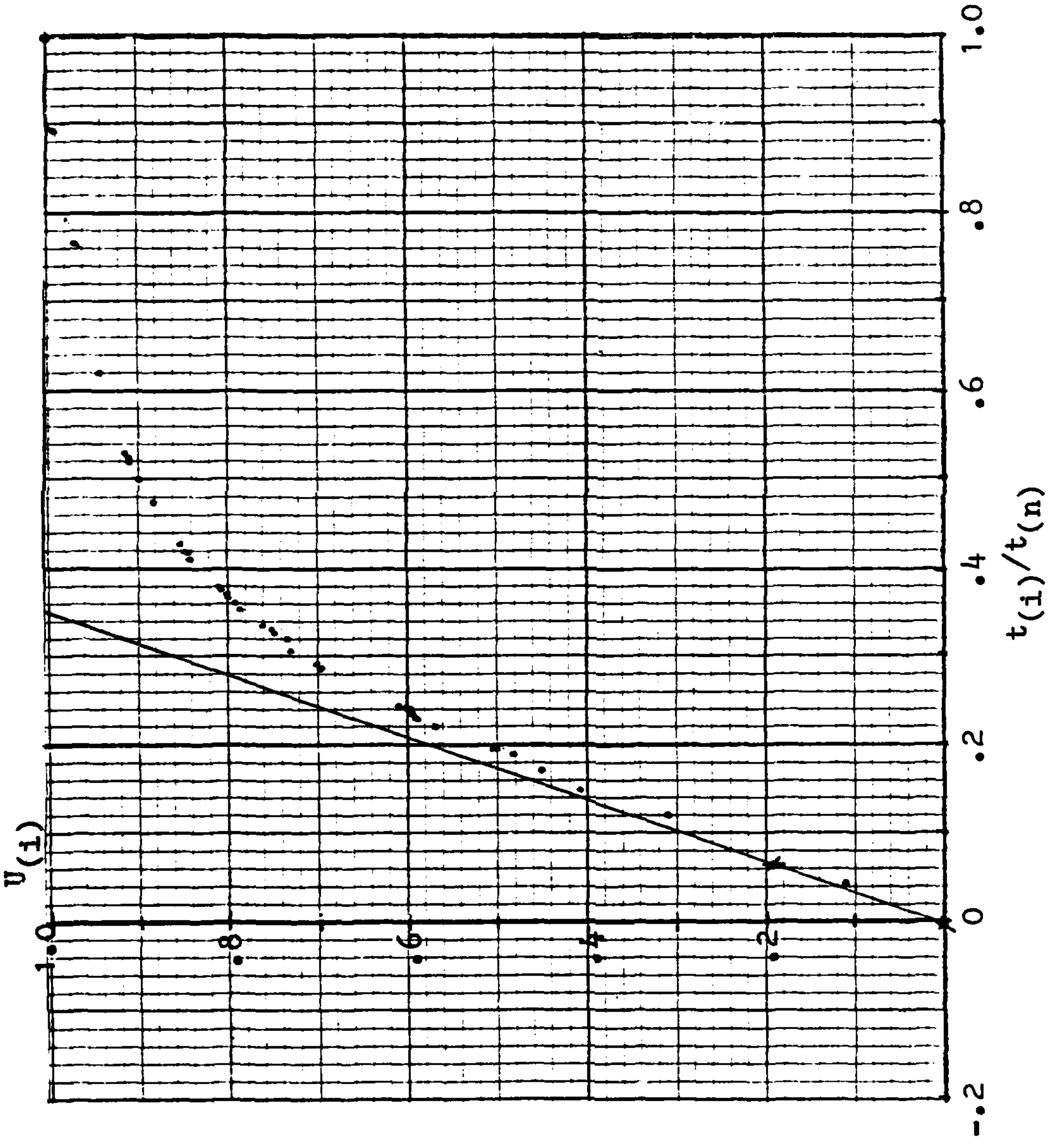
N= 37 K= 20.92

I	LIFE(I)	T(I)	U(I)	X(I)	SLOPE	
1	707.	26159.	.115	.042	2.664454	
2	1139.	41711.	.184	.068	2.666203	*
3	1971.	70831.	.312	.117	2.636466	
4	2599.	92183.	.406	.154	2.608756	
5	2896.	101984.	.449	.172	2.592258	
6	3093.	108288.	.477	.184	2.578360	
7	3284.	114209.	.503	.195	2.562183	
8	3744.	128009.	.564	.222	2.520899	
9	3962.	134331.	.592	.235	2.500607	
10	3984.	134947.	.595	.237	2.498274	
11	3996.	135271.	.596	.237	2.496791	
12	4022.	135947.	.599	.239	2.493132	
13	4076.	137297.	.605	.242	2.484702	
14	4949.	158249.	.697	.294	2.360817	
15	4970.	158732.	.699	.295	2.358059	
16	5284.	165640.	.730	.314	2.315033	
17	5323.	166459.	.733	.316	2.309500	
18	5534.	170679.	.752	.329	2.278101	
19	5551.	171002.	.753	.330	2.275449	
20	5632.	172460.	.760	.335	2.261967	
21	5967.	178155.	.785	.355	2.205935	
22	6060.	179643.	.791	.360	2.190341	
23	6215.	181968.	.802	.369	2.163541	
24	6225.	182108.	.802	.370	2.161739	
25	6379.	184110.	.811	.379	2.132915	
26	6973.	191238.	.843	.414	2.027329	
27	7021.	191766.	.845	.417	2.019069	
28	7153.	193086.	.851	.425	1.995561	
29	7974.	200475.	.883	.474	1.859160	
30	8513.	204787.	.902	.506	1.779199	
31	8809.	206859.	.911	.523	1.736954	
32	8868.	207213.	.913	.527	1.728378	
33	8915.	207448.	.914	.530	1.721237	
34	10432.	213516.	.941	.620	1.514480	
35	12931.	221013.	.974	.768	1.265186	
36	15000.	225151.	.992	.891	1.111343	
37	16829.	226980.	1.000	1.000	.998758	

OPTIMAL INTERVAL IS 1139. MILES

TOTAL TIME ON TEST PLOTS

STAUFFER SYNTHETIC OIL



APPENDIX J

TTT RESULTS: MINERAL OIL

TOTAL TIME ON TEST
MINERAL OIL

N= 42 K= 28.30

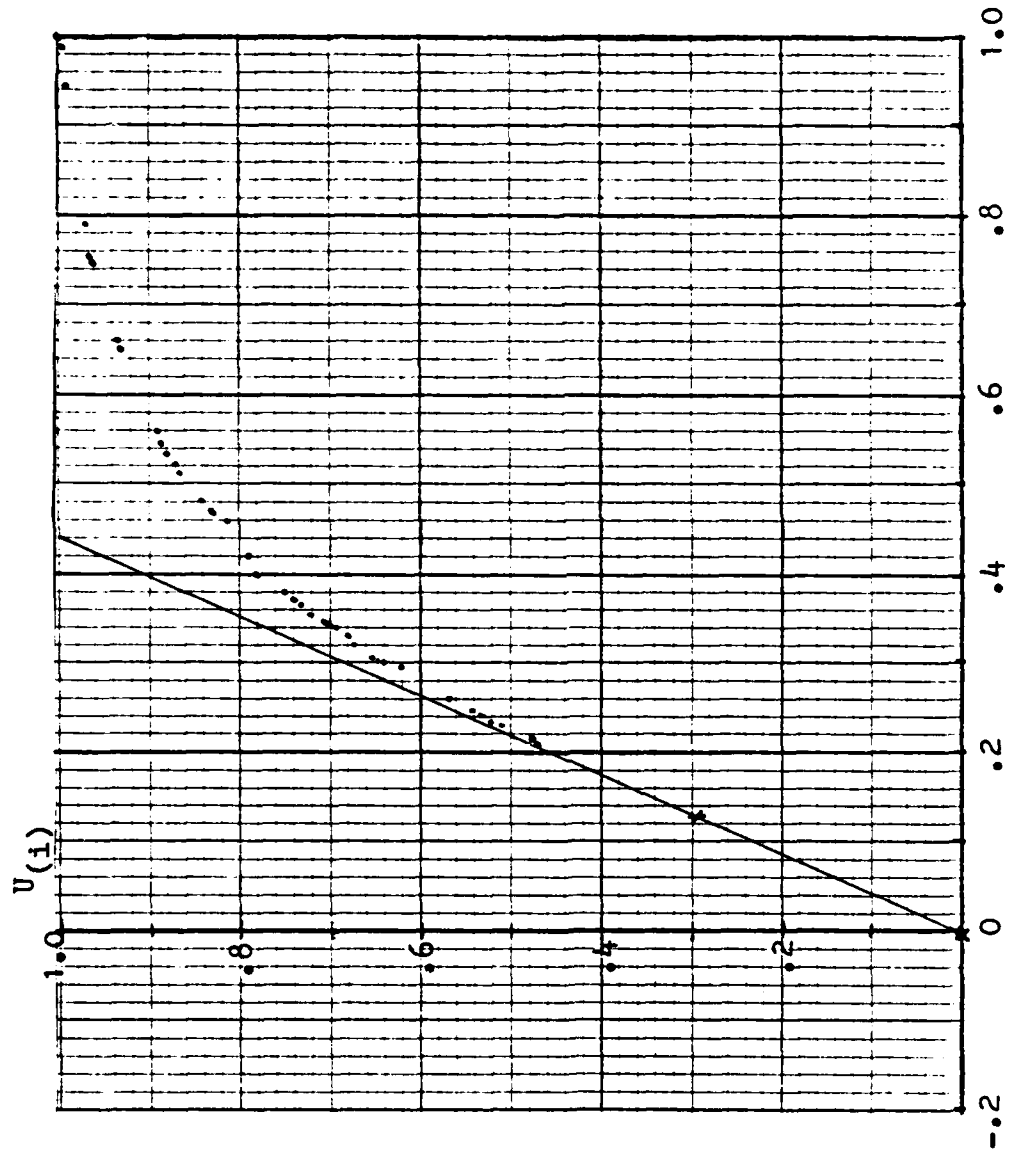
I	LIFE(I)	T(I)	U(I)	X(I)	SLOPE	
1	2039.	85638.	.292	.130	2.221569	*
2	3322.	138241.	.472	.211	2.212839	
3	3330.	138561.	.473	.212	2.212678	
4	3334.	138717.	.473	.212	2.212534	
5	3642.	150421.	.513	.232	2.197878	
6	3717.	153196.	.523	.236	2.193600	
7	3720.	153304.	.523	.237	2.193390	
8	3857.	158099.	.539	.245	2.182234	
9	4138.	167653.	.572	.263	2.158030	
10	4642.	184285.	.629	.295	2.116128	
11	4778.	188637.	.644	.304	2.104809	
12	4874.	191613.	.654	.310	2.096147	
13	5083.	197883.	.675	.323	2.076222	
14	5156.	200000.	.682	.328	2.068886	
15	5298.	203976.	.696	.337	2.053762	
16	5341.	205137.	.700	.340	2.048911	
17	5409.	206905.	.706	.344	2.040725	
18	5440.	207680.	.708	.346	2.036756	
19	5443.	207752.	.709	.346	2.036345	
20	5602.	211409.	.721	.356	2.013672	
21	5782.	215369.	.735	.368	1.987840	
22	5887.	217574.	.742	.374	1.972545	
23	5947.	218774.	.746	.378	1.963508	
24	6416.	227685.	.777	.408	1.894765	
25	6603.	231051.	.788	.420	1.868555	
26	7239.	241863.	.825	.460	1.784814	
27	7351.	243655.	.831	.468	1.770748	
28	7381.	244105.	.833	.470	1.766836	
29	7579.	246877.	.842	.482	1.740391	
30	8105.	253715.	.866	.516	1.672923	
31	8191.	254747.	.869	.521	1.662153	
32	8428.	257354.	.878	.536	1.632102	
33	8578.	258854.	.883	.546	1.613003	
34	8856.	261356.	.892	.563	1.577633	
35	10268.	272652.	.930	.653	1.420117	
36	10382.	273450.	.933	.660	1.408677	
37	11694.	281322.	.960	.744	1.287027	
38	11771.	281707.	.961	.749	1.280378	
39	12386.	284167.	.969	.788	1.227575	
40	14886.	291667.	.995	.947	1.048772	

41	15512.	292919.	.999	.987	1.010846
42	15720.	293127.	1.000	1.000	.998203

OPTIMAL INTERVAL IS 2039. MILES

TOTAL TIME ON TEST PLOTS

MINERAL OIL



APPENDIX K

TTT RESULTS: CONOCO SYNTHETIC OIL

TOTAL TIME ON TEST
CONOCO SYNTHETIC OIL

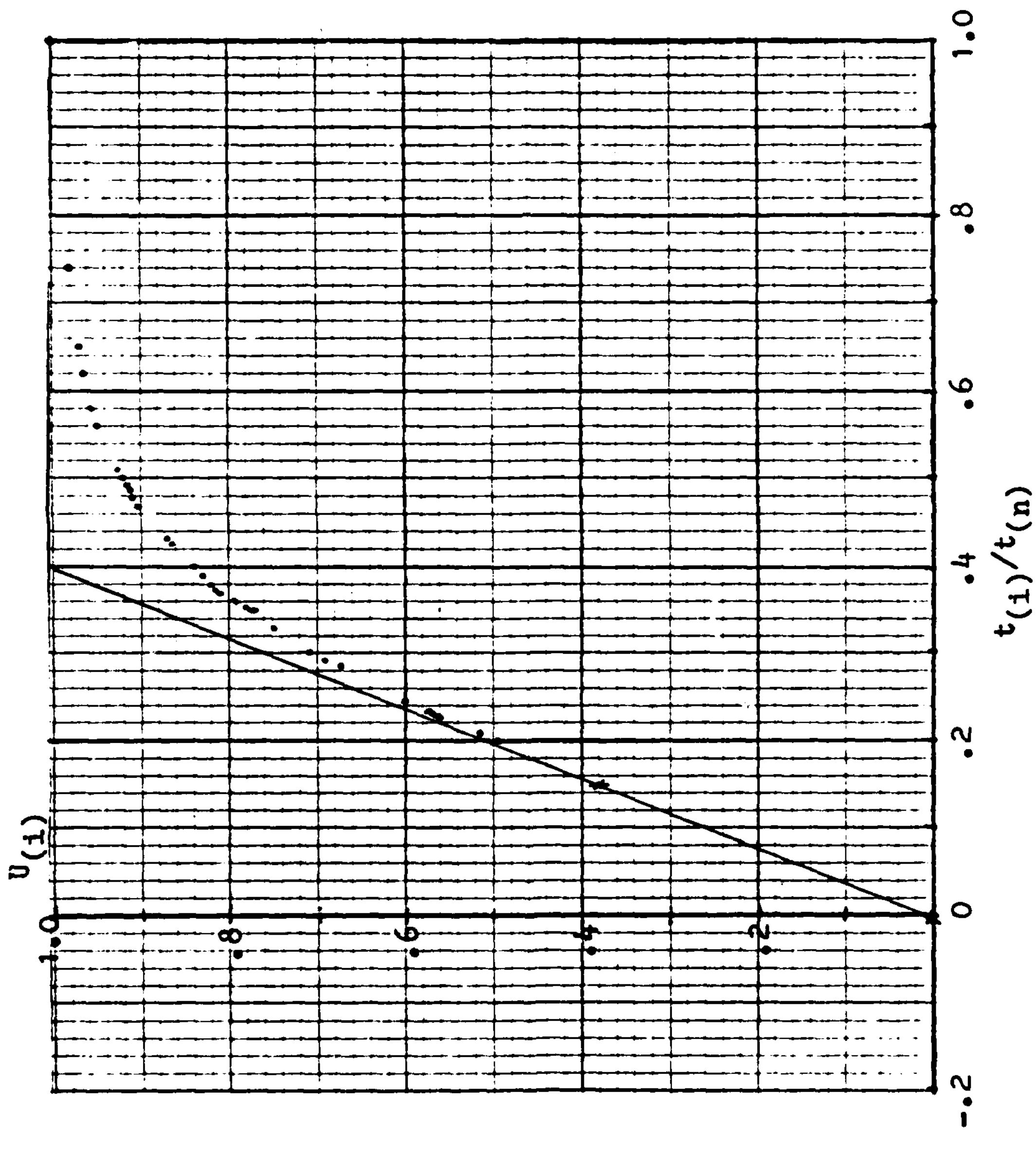
N= 37 K= 25.46

I	LIFE(I)	T(I)	U(I)	X(I)	SLOPE	
1	2624.	97088.	.379	.151	2.488901	*
2	3481.	127940.	.499	.200	2.478203	
3	3587.	131650.	.514	.206	2.475240	
4	3916.	142836.	.558	.225	2.461387	
5	3972.	144684.	.565	.228	2.458305	
6	3988.	145196.	.567	.229	2.457170	
7	3999.	145537.	.568	.230	2.456208	
8	4054.	147187.	.575	.233	2.450565	
9	4303.	154408.	.603	.247	2.422902	
10	4975.	173224.	.676	.286	2.352868	
11	5104.	176707.	.690	.293	2.339815	
12	5132.	177435.	.693	.295	2.336699	
13	5326.	182285.	.712	.306	2.313546	
14	5728.	191933.	.749	.329	2.265792	
15	5991.	197982.	.773	.344	2.235034	
16	6085.	200050.	.781	.350	2.223638	
17	6104.	200449.	.782	.351	2.221166	
18	6273.	203829.	.796	.361	2.198017	
19	6457.	207325.	.809	.371	2.172257	
20	6529.	208621.	.814	.375	2.161825	
21	6583.	209539.	.818	.378	2.153595	
22	6823.	213379.	.833	.392	2.116207	
23	6964.	215494.	.841	.400	2.094069	
24	7435.	222088.	.867	.427	2.021896	
25	7479.	222660.	.869	.430	2.015219	
26	8238.	231768.	.905	.473	1.904982	
27	8312.	232582.	.908	.478	1.894706	
28	8444.	233902.	.913	.485	1.875762	
29	8508.	234478.	.915	.489	1.866278	
30	8695.	235974.	.921	.500	1.837910	
31	8844.	237017.	.925	.508	1.815021	
32	9795.	242723.	.948	.563	1.678721	
33	10155.	244523.	.955	.584	1.631367	
34	10821.	247187.	.965	.622	1.547879	
35	11245.	248459.	.970	.646	1.497313	
36	12800.	251569.	.982	.736	1.332243	
37	17399.	256168.	1.000	1.000	.998539	

OPTIMAL INTERVAL IS 2624. MILES

TOTAL TIME ON TEST PLOTS

CONOCO SYNTHETIC OIL



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