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## SIZE EFFECT AND STRENGTH VARIABILITY OF UNIDIRECTIONAL COMPOSITES

S. B. BATDORF and REZA GHAFFARIAN

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Size Effect and Strength Variability of Unidirectional Composites by S. B. Batdorf<sup>\*</sup> and Reza Ghaffarian<sup>\*\*</sup> Materials Science and Engineering Department University of California, Los Angeles 90024

### ABSTRACT

Statistical theories have been proposed for damage accumulation and tensile failure of unidirectionally reinforced fibrous composites. These theories involve the mechanical properties of the constituent fibers, the geometry of the composite, and some inadequately known functions such as stress distributions in the vicinity of single and multiple fiber breaks. As a result in trying to correlate theory and experiment some quantities have to be treated as unknown parameters.

It was found that use of Hedgepeth and Van Dyke's stress concentration factors led to good agreement between theory and Bullock's data on graphite epoxy only when the overloaded length of fiber at crack tips was assumed to have an unrealistically large value. A possible explanation is that the Hedgepeth and Van Dyke stress concentration factors were calculated for composites with a geometrically perfect fiber array. In real composites the fiber spacing is quite irregular. A theory is developed for strength of irregularly constructed composites and compared with experiment. Taking the effect into account improves agreement in the case of one experiment and impairs agreement in the case of another.

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#### Introduction

Griffith, who is generally considered to be the father of fracture mechanics, postulated failure of isotropic materials by crack instability. In his first paper on this subject [1], he gave as the criterion for failure that the strain energy released during crack extension should be equal to or greater than the energy required to create the resulting new crack surfaces. In a second paper on fracture [2], he suggested that a crack is unstable when the material at the crack tip is stressed beyond its intrinsic strength.

By an interesting coincidence the same two criteria are in use today in the study of the strength of unidirectionally reinforced composites. One school of thought, popular in England [3], and USSR [4], uses energy balance as the failure criterion. The other, pioneered by Rosen and Zweben in the USA [5] assumes that crack extension occurs when the stress at the crack tip exceeds the strength of the adjoining unbroken fiber. A feature of the second approach is that stable crack growth will generally precede the final failure.

As a result of complexity of their analysis, Zweben and Rosen were unable to arrive at a general failure criterion for 3-D composite. They did, however, analyze the 2-D case (tapes one fiber thick) in some detail and proposed the occurrence of the first double fiber break as a conservative approximation to the failure load. By a new approach, Harlow and Phoenix were able to find a virtually exact solution for the failure stress of highly idealized 2-D composites [6-9]. Among other things, they showed for the first time that the effective Weibull modulus of a composite is variable, and increases with increasing volume.

Recently Batdorf [10] proposed a general solution to 2-D and 3-D problems which is both simple and sufficiently accurate for most practical purposes. In comparing the latter theory with experimental data on the strength of graphite epoxy [11], Batdorf and Ghaffarian found good agreement was obtained only when the ineffective length was assumed to be nearly an order of magnitude larger than what is usually observed. Here, the ineffective length is used to describe the overloaded portion of undamaged fibers immediately adjacent to a break. This is the sum of the transfer length and the length of broken fiber that is debonded.

Up to this point all theories assumed that the fibers form a perfect array, i.e. the distances from a crack (multiple break) to its closest neighbors are all identical. In practice, construction is not perfect and normally there is a variation in these distances. As a result there must be a distribution in the stress concentration factors affecting the fibers adjacent to a crack. It has been shown [12] that such a variation increases the effective stress concentration factor, and this may make it unnecessary to assume an excessively large ineffective length in order to get good agreement with experiment.

One purpose of this paper is to reanalyze the data given in [12] together with additional data kindly supplied by Graham Dorey of the Royal Aircraft Establishment, Farnborough, UK, to learn to what extent the theory predicts the strength, effective modulus and ineffective length for these two sets of data. A second objective is to determine whether adequate characterization of a composite will in the future require determination of the variability in stress concentration factors in addition to the more familiar parameters such as basic structure, mechanical properties of fiber and matrix etc.

Theory

We first summarize the approximate theory reported in [10]. For simplicity, it was assumed that single fiber failure obeys the Weibull's 2-parameter distribution, i.e., on the application of uniform stress  $\sigma$ over the fiber length L, the cumulative probability of failure is given by

$$P_{f} = 1 - \exp[-L(\frac{\sigma}{\sigma_{o}})]$$
 (1)

where  $\sigma_0$  and m are the scale parameter and shape parameter (Weibull modulus), respectively.

On the assumption that the fibers carry the entire tensile load, and matrix serves only the function of transferring tensile stresses from one fiber to another through its shear stiffness, a simple relationship between the number of fiber breaks of various multiplets as a function of applied stress can be developed. If there are N fibers of length L in composite, then the number of isolated fracture (singlets) that will have been created by the time stress rises to  $\sigma$  is given by:

$$Q_{1} = N P_{f} \simeq NL \left(\frac{\sigma}{\sigma_{o}}\right)^{m}$$
(2)

A singlet becomes a doublet when one of the neighboring fibers breaks in its overstressed region. The number of doublets created in loading to stress  $\sigma$  is approximately

$$Q_2 = Q_1 n_1 \lambda_1 \left( \frac{c_1 \sigma}{\sigma_0} \right)^m$$
(3)

where  $n_1$  is the number of fibers immediately adjacent to a given singlet,  $c_1$  is the stress concentration factor,  $\lambda_1$  is the effective length of the overloaded region, that is the length which would have the same  $P_f$  when subjected to the uniform stress  $c_1 \sigma$  as the actual non-uniformly loaded fiber segment has.

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Generalizing these results

$$Q_{i+1} = Q_i n_i \lambda_i \left(\frac{c_i \sigma}{\sigma_o}\right)^m$$
(4)

Thus

$$Q_{j} = NL \left(\frac{\sigma}{\sigma_{0}}\right) \qquad \prod_{j=1}^{m} c_{j} n_{j} \lambda_{j} \qquad (5)$$

From (5) we see that a plot of  $\ln Q_i$  vs  $\ln \sigma$  is a straight line of slope im. The failure line is the envelope of the lines for the various Q's, and the failure stress  $\sigma_f$  is given by the intersection of the failure line with the horizontal line  $y = \ln Q_i = 0$  corresponding to  $Q_i = 1$ .

From (5) it is evident that  $Q_i \propto NL$ . Thus when the volume of the composite NL is changed, the failure stress  $\sigma_f$  also will be changed. A plot of  $\ln \sigma_f$  vs  $\ln NL$  is a broken line similar to the failure envelope. For a Weibull material a plot of  $\ln \sigma_f$  vs  $\ln V$  is a straight line of slope -1/m. For the composite a plot of  $\ln \sigma_f$  vs  $\ln V$ (where V=NL) is a broken line in which the segments have the slopes -1/m, -1/2m, -1/3m, etc., as illustrated in Fig 1. The segment of slope (-1/m) is a portion of the single fiber failure line. This segment covers the stress range over which an isolated break (singlet) is unstable. In general the segment of slope (-1/im) covers the stress range over which i-plets are responsible for failure.  $\sigma_s$ ,  $\sigma_d$ ,  $\sigma_t$  etc. are the stresses at which singlets, doublets, triplets etc. first occur for a composite of the size indicated.

The stress  $\sigma_i$  at the intersection of segments having the slopes -l/im and -l/(i+l)m is found by equating  $Q_{i+1}$  and  $Q_i$  in equation (4), that is

$$\frac{q_{i+1}}{q_i} = n_i \lambda_i \left(\frac{c_i \sigma_i}{\sigma_o}^m\right)^m = 1$$
 (6)

$$\sigma_{i} = \frac{\sigma_{o}}{c_{i}} \left(\frac{1}{n_{i}\lambda_{i}}\right)$$
(7)

When both the slopes of the line segments and the locations of the transition from one segment to another are known, the failure line can be readily constructed. Unfortunately, usually some of the quantities in (7) are not known and the problem therefore has to be attacked parametrically.

Every past theoretical treatment of composite strength, including ours, assumed the fibers in a composite form a perfect geometric array. However, micro-photographs of the real composites ( see for instance Fig. 2 ) normally show a variation in the distance between neighboring fibers. Thus one would expect that the stress concentration factors must vary correspondingly. In addition even in the regularly spaced fibers, there is generally some variation in  $c_i$  because of differences in the number of broken fibers in the vicinity of an overloaded unbroken fiber. To account for a distribution in  $c_i$ , we rewrite equation (6) in the form

$$\lambda_{i} \left( \begin{array}{c} \sigma_{i} & m & n_{i} \\ \sigma_{0} & f & f \\ \sigma_{0} & \rho & 0 \end{array} \right) \qquad f \qquad dn(c_{i}) c_{i} = 1.$$
 (8)

Here, dn denotes the number of neighboring fibers having a stress concentration factor in the range  $c_i$  to  $c_i + dc_i$ .

To evaluate the integral in equation (8) we must choose a specific distribution function. It turns out that use of the Weibull distribution

or

facilitates a relatively simple analysis, so we choose it. Such a distribution for Weibull modulus m'=4 is shown in Fig. 3. In general dn is given by

dn = 
$$n_i \frac{d}{dc_i} \{1 - \exp[-(\frac{c_i}{c_{oi}})^m]\} dc_i$$
 (9)

which satisfies the necessary relation

$$\int_{0}^{n_{i}} dn = \int_{0}^{\infty} \frac{dn}{dc_{i}} dc_{i} = n_{i}$$
(10)

Substituting equation (9) into the integral of (8) and integrating by parts we obtain

$$\int_{0}^{\infty} \frac{dg(c_i)}{dc_i} c_i^m dc_i = -g(c_i) c_i^m | + m \int_{0}^{\infty} g(c_i) c_i^{m-1} dc_i (11)$$

where

$$g(c_i) = \exp \left[ - \left( \frac{c_i}{c_{oi}} \right)^{m'} \right]$$
 (12)

The integrated term vanishes. The remaining term can be rewritten:

$$m \int_{0}^{\infty} g(c_{i}) c_{i}^{m-1} dc_{i} = \frac{m}{m^{T}} c_{0i}^{m} \int_{0}^{\infty} e^{t} dt$$

$$= c_{0i}^{m} \Gamma(1 + \frac{m}{m^{T}}) \qquad (13)$$

Here

$$t = \left(\frac{c_{i}}{\epsilon_{oi}}\right)^{m'}$$
(14)

Putting all this together, we obtain for the revised form of the expression for the stress at the ith vertex in the failure curve:

$$\sigma_{i} = \frac{\sigma_{o}}{(c_{i})_{eff}} \left[ \frac{1}{n_{i} \lambda_{i}} \right]$$
(15)

where

$$(c_i) = c_{oi} [\Gamma(1 + \frac{m}{m'})]$$
 (16)  
eff

The effective value of  $c_i$  exceeds the mean value by a factor A that can be derived from the probability distribution. By definition, the average stress concentration factor is given by

$$\overline{c}_{i} = \int_{0}^{\infty} c_{i} \frac{d}{dc_{i}} \exp\left[-\left(\frac{c_{i}}{c_{oi}}\right)^{m'}\right] dc_{i} = c_{oi} \Gamma\left(1 + \frac{1}{m'}\right) (17)$$

Then, the ratio of effective stress concentration to average stress concentration can be found from

$$\frac{(c_i)_{eff}}{\overline{c_i}} = A = \frac{[\Gamma(1 + \frac{m}{m'})]}{\Gamma(1 + \frac{1}{m'})}$$
(18)

A plot of this factor is shown in Fig. 4. for several values of m, as a function of the stress concentration shape factor (m').

## Experimental Data

Bullock [13]reported statistical data on the strengths of impregnated tows of T300 fibers using Narmco 5208 as the matrix material and compared the results with the tensile and bending strengths of laminated coupons composed of such tows. His results are summarized in Table 1. In this

table  $\sigma_f$  is the mean failure stress and m is the Weibull modulus of an individual fiber. Here we consider only fibers and tows

In [11]  $n_i$  was determined by assuming that multiplets are as nearly penny-shaped as the geometry permits and the stress concentration factors were

determined by passing a smooth curve through the values for c<sub>i</sub> given by Hedgepeth and Van Dyke [14] for fibers forming a square array. The results are shown in Table 2.

Since m and  $\lambda$  are both unknown, they were treated as disposible parameters. Failure lines were constructed for various assumed values of m and  $\lambda$ . Most of the failure lines missed the data point for 25mm tow rather badly, but some combinations of m and  $\lambda$  result in failure lines that pass through tow data point. Such combinations include (m=2,  $\lambda$ =1.75 mm), (m=3,  $\lambda$ = 1.25mm), (m=4,  $\lambda$ = 1 mm). (m=7,  $\lambda$ =.625 mm), leading to effective Weibull moduli of 24, 30, 36, and 49 respectively. The best choice, taking into account the observed effective shape parameter for the tow was obtained with m=3 and  $\lambda$ = 1.25 mm. Now m= 3 is an entirely possible value but  $\lambda$  is an order of magnitude larger than normally observed. If a distribution in c<sub>i</sub> is chosen such that  $\Gamma(1 + \frac{m}{m^{-1}})$ becomes considerably larger than one, then it is possible to obtain good agreement with observed failure stress while assuming a reasonable value for  $\lambda$ . It will be seen later that choice of m probably should not be influenced by the observed Weibull modulus for a tow.

Dorey has obtained sets of data on the statistical strength properties of several different carbon fibers as well as tows and minitows constructed of fibers [15]. These data are summarized in Table 3. Because of the similarity of the different fiber types, only type A results are shown here. The data as supplied to us did not include the Weibull modulus. In the table m was estimated using the approximate relation

$$m = 1.2 / c.v.$$
 (19)

where c.v. is the coefficient of variation. As in the case of Bullock's data, stress concentration factors were assumed to be those calculated by Hegepeth and Van Dyke. The fibers were assumed to form a square array, and m was taken to be the measured value, 7. Thus the only unknown parameter was the effective length  $\lambda$ .

The failure lines calculated for composites constructed of type A fibers with various assumed values of  $\lambda$  are shown in Fig. 5. The best fit was obtained with = 0.1 mm. This is within the general range of observations reported by Manders and Bader [16].

These numbers were obtained assuming the fibers in the composite formed a regular square array. If irregularities are assumed such that m'= 4 (the value obtained for Bullock's data), A= 1.17 and the value of  $\lambda$  leading to best agreement with Dorey's data turns out to be 0.033 mm., which seems a bit low.

Up to this point our concern has been to reconcile theory and experiment with respect to composite strength. They can also be compared on the basis of strength variability of fibers and tows (which are small composites). Here there is a marked discrepancy. The theory implies that m approaches infinity as composite size increases without limit, whereas experiments indicate that m levels off. There are a number of possible reasons for this discrepancy. Among them: the theory is defective in this area; the experiments are faulty; there are sources of variability that are not considered in the theoretical analysis.

The theory outlined here admittedly contains many approximations. However, it shares the feature under consideration (unlimited increase of the effective modulus with increasing composite size) with a virtually exact theory for a single ply tape under local load sharing and approximate theories related thereto [6-9]. It therefore seems unlikely that this is the cause of the discrepancy.

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The experiments may be at fault. Whitney and Knight attributed anomalies in experimentally determined composite moduli to the difficulty of avoiding a small amount of bending during tensile testing, [17] and random amounts of such inadvertent bending would result in the observed effect.

A third possible explanation is the presence of other sources of variability. It is well known that there is some variation in fiber diameter within a tow, and some investigators have reported a decrease in strength with increasing diameter. Somewhat better established is a dependence of Young's modulus with diameter [18]. Since in the absence of residual stress all fibers within a bundle under load are subjected to the same strain, there will be a variation in fiber stress. While effects such as these will increase the variability in composite strength, they will not prevent the variability from approaching zero as composite size increases. This is because they affect the individual fibers in a random manner. Their influence is essentially to decrease the Weibull modulus of fibers in situ compared to the modulus of the same fibers in isolation.

Systematic variability is another matter. Consider, for instance, the case of residual fiber stress caused by matrix shrinkage that varies

somewhat from one sample of the composite to another because of inadvertent small differences in composition or the processing variables. No matter how large the composite is, the variability in apparent strength will never be less than the variability in residual stress. Thus one must expect an upper limit to effective Weibull modulus of composites, in accord with experimental findings.

## Summary

Attempts to correlate statistical theories of composite strength with experiment are handicapped by insufficient information regarding some of the important parameters. Often the mechanical properties of the constituent fibers and matrix materials are unknown. Fragmentary theoretical information is available on the stress concentration factor affecting a fiber adjacent to a crack (multiple fiber break), but the length of the overloaded region is generally unknown. Treating the unknowns parametrically, good agreement between theory and Bullock's data was only obtained when improbably large overload regions were assumed [11]. A possible explanation for this is that theories generally assume that the fibers in the matrix form a geometrically perfect array, which is far from the case in real composites.

If the distances between a fiber and its nearest neighbors vary, the stress concentration factors will also vary. It can be inferred from this that by proper choice of the variance in stress concentration factor, the theory will give the observed value of composite strength with reasonable values of the size of overloaded region.

To see whether variability of stress concentration factor is a major effect, in the present paper additional data are analyzed. These data

compare the strengths of isolated fibers, minitows(20 fibers) and tows (2000 fibers). It is found that the theory leads to the observed failure stress assuming an overload region that is close to the length of the debonded region. Assuming an imperfect array with the same variance in stress concentration factor used earlier to analyze Bullock's data leads to a smaller and less plausible overload region. Thus we do not have a definitive answer to the question whether it is important to take account of variability in stress concentration factor. The data are too few- more work is needed to resolve the issue.

The theory implies that the larger the composite the smaller will be the variance in failure stress, and that in the limit of large composites the variance approaches zero. Here there is a significant discrepancy between theory and experiment. In laboratory tests the variance reaches a non-zero lower limit. One possible explanation is experimental difficulties in producing a state of simple tension; usually some bending is also present. The presence of bending would account for the observed discrepancy, but the discrepancy might be limited to laboratory specimens and not apply to field situations. Another possible explanation is the presence of residual stress, which can vary from specimen to specimen due to small variations in composition and processing variables. Such variations can hardly be avoided in manufacturing. If this is the explanation, it would imply that, in practical applications the theory for strength variability can apply only up to a certain size; above that size, other sources of variability not included in the theory

become dominant. And this in turn means that as composite size increases, the effective Weibull modulus also increases until it reaches a limiting value.

Before closing, we note that most experimental arrangements for tensile testing tows will introduce non-uniform stresses over cross sections near the ends of the tow. These die out in accord with St. Venant's principle, but will have the effect of reducing the failure load. The effect will be much more pronounced for a 25mm tow (Bullock case) than a 150mm tow (Dorey case). This may be one reason that the experimental failure stress was appreciably lower than the theoretical (which neglects non-uniformity) in the 25mm case, but was not in the 150mm case.

#### Conclusions

- In practical composites the fibers do not form a perfect geometric array; instead nearest-neighbor distances vary considerably. This must lead to variations in the stress concentration factor in the various fibers ahead of a crack tip.
- 2. An analysis of the effect of variation in stress concentration factor shown that such variability leads to an increase in the effective stress concentration factor.
- 3. Taking variability into account improves the agreement of theory with observations on the strength-size relation of composites in one experiment and makes agreement poorer in the case of another experiment. Thus the need for including this complication in composite strength analyses is at this time an open question.
- 4. Theory indicates that as composite size increases, the coefficient of variation of the fracture stress decreases without limit. This is contradicted by experiment. The explanation for the discrepancy may be presence of undesired bending stresses during the experiments or to the presence of unaccounted-for sources of variability in the composite itself. One such source is presence of residual stresses.

#### ACKNOWLEDGMENT

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	σ <sub>f</sub> (Gpa)	m	Remarks
50mm single fiber	2.48	?	
25mm tow	2.48	29.1	2000 fibers
Coupon in tension	2.04	23.3	Contains 11.75 m of tow
Coupon in 3-point bending	2.76	24.6	Contains 11.75 m of tow

## Table II

i	ı	2	3	4	5	6	7	8	9	10	11	12	
° i	1.145	1.18	1.225	1.27	1.31	1.34	1.375	1.405	1.43	1.46	1.48	1.5	
n,	4	6	7	8	9	10	11	11	12	13	14	15	

# Table III

Fiber type	Single fiber (25 mm) Gpa (c.v.)	Mini-tow (20 fibers 25mm) Gpa (c.v.)	Tows (10,000 fibers 150mm) Gpa (c.v.)
A	2.79 (17%)	3.52 (7.6%)	2.98 (9.0%)
A etched	3.43 (17%)	3.66 (7.7%)	
XA	3.20 (18.1%)	4.01 (4.1%)	3.46 (5.7%)
XAS	3.09 (14.5%)	4.02	3.22 (8.7%)

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# Table I





Fig. 1 - Plot of failure stress vs. size for unidirectional composite (schematic). Space between top and bottom lines is damage accumulation region. GRAPHITE EPOXY SECTION

(CROSSMAN et al.)



Fig. 2. Kicrographs showing irregular fiber spacing

in typical composite





services ( constant backson assessment) and

Fig 4. Variation of Stress Concentration Factor with m and m'

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150

LOG(STRESS)

