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NAVAL POSTGRADUATE SCHOOL
Monterey, California

THESIS

A HORIZONTAL RANGE VS. DEPTH SOLUTION OF SOUND SOURCE POSITION UNDER GENERAL SOUND VELOCITY CONDITIONS USING THE LLOYD'S MIRROR INTERFERENCE PATTERN

by

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September 1983

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A Horizontal Range vs. Depth Solution of Sound Source Position Under General Sound Velocity Conditions Using the Lloyd's Mirror Interference Pattern

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I. INTRODUCTION

Sound propagation between a source and a receiver in the ocean may occur by many different ray paths, each dependent on existing physical and acoustical conditions. Crucial to the determination of which sound paths are of importance in any given problem are factors such as the sound velocity profile, the depth of the water, the bottom contour and composition, the sea surface conditions and the intensity and frequency of the sound.

This thesis deals with two specific paths of sound propagation from a source to a receiver. The first, called the direct path, does not touch the surface or bottom of the sea as it travels from the source to the receiver. The second path reflects off the sea surface before reaching the receiver and is called the reflected path.

The following explanation of the reception of sound from a single source via these two distinct paths is taken directly from Urick: [Ref. 1: pp. 120-123]

> "If the sea surface were perfectly smooth, it would form an almost perfect reflector of sound. The intensity of sound reflected from the smooth sea surface would be very nearly equal to that incident upon it."

He continues:

> "A criterion for the roughness or smoothness of a surface is given by the Rayleigh parameter, defined as $R = kH \sin\theta$; where k is the wave number, $(2\pi)/\lambda$ (λ is the wave length of the sound incident on the surface), H is

> the rms (sea) wave height (crest to trough), and θ is the grazing angle".

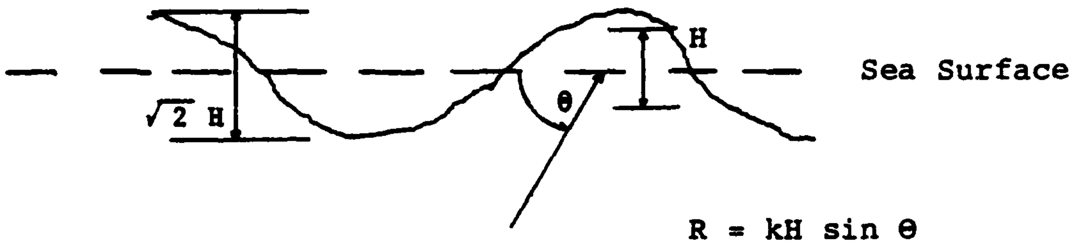


Figure I-1. Rayleigh Parameter (R) for Describing Sea Surface Roughness.

> "When R<<1, the surface is primarily a reflector and produces a coherent reflection at the specular angle equal to the angle of incidence. When R>>1, the surface acts as a scatterer, sending incoherent energy in all directions. When certain theoretical assumptions are made, (See: P. Beckmann, and A. Spizzichino: Scattering of Electromagnetic Waves From Rough Surfaces, p. 93, The Macmillan Company, N.Y., 1963.) the (pressure) amplitude reflection coefficient μ of an irregular surface (defined as the ratio of the reflected or coherent pressure amplitude of the return to the incoherent pressure amplitude) can be shown to be given by the simple expression $\mu = \exp(-R)$."

Urick goes on to explain the image interference.

> "When the sea surface is not too rough, it creates an interference pattern in the underwater sound field. This pattern is caused by constructive and destructive interference between the direct and surface reflected sound and is called the Lloyd mirror, or image interference, effect."

In Urick's sound propagation in the sea [Ref. 2: p. 9-5], he continues the discussion of surface reflection interference:

> "This sound field may be divided into three parts, 1) a NEAR FIELD close to the source in which the image source is too far away, and the reflected sound is too weak to produce appreciable interference, 2) an INTERFERENCE FIELD in which there are strong loops and nulls in the signal received by a receiver moving outward in range, and 3) a

FAR FIELD in which there is an increasingly out-of-phase condition between source and image and the intensity falls off as the inverse fourth power of the range (transmission loss increases as 40 log r)."

The interference effects felt at the receiver is expressable as the ratio of the combined direct and reflected sound intensities to the direct sound intensity alone.

Stated mathematically:

$$\text{Intensity Ratio} = \frac{I_{total}}{I_{direct}} = \frac{I_{(direct + reflected)}}{I_{direct}}$$

or more precisely

$$\text{Intensity Ratio} = 1 + \mu^2 \left(\frac{R_D}{R_R}\right)^2 - 2\mu \left(\frac{R_D}{R_R}\right) \cos(\omega\tau)$$

Where R_D is the direct ray path length, R_R is the reflected path length, μ is the surface pressure reflection coefficient, ω (where $\omega = 2\pi f$) is the angular frequency of the sound, and τ is the time delay caused by the difference in the two path lengths. A formal derivation of the intensity ratio is given in Appendix A. A more comprehensive treatment of the effects of surface roughness on acoustic waves is given by Clay and Medwin [Ref. 3].

II. APPLICATION of LLOYD MIRROR INTERFERENCE PHENOMENON to LOCATE the HORIZONTAL RANGE and DEPTH of the SOUND SOURCE FROM the RECEIVER

Tactical tracking information from the Lloyd's mirror phenomenon has been used operationally by the Navy for many years. In an excellent report on the analysis of Lloyd's mirror phenomenon under constant velocity gradient conditions, Shudde [Ref. 4: p. 3], discusses the difficulty of solving the surface reflection interference problem. Accurate error estimation for any acoustical condition other than the isovelocity case is not practical unless a large computer is used. Since Shudde's 1977 report, the advancements in memory size, and speed of operation of small portable desk top computers now allow an accurate solution of source position for all sound velocity conditions. The following mathematical solution is based on the lengths of the sound paths determined from Snell's Law. As explained by Urick, [Ref. 1: p. 116],

> "One of the most important practical results of ray theory is Snell's Law, which describes the refraction of sound rays in a medium of variable velocity."

The solution is not dependent on any isovelocity or constant gradient conditions other than the reasonable assumption that no horizontal sound speed gradients exist throughout the operating area. The mathematical solution presented in Section III makes no assumption as to the mathematical characteristics of the sound velocity profile nor does it use far field approximations. Past solutions depended on specific sound velocity profiles, such as an isovelocity or "straight line" profile which resulted in significant positional errors for some tactical applications when the actual sound velocity profile did not match the predicted model.

In applying the solution to the Lloyd's mirror interference effect one must understand the relationship of the propagating sound to the interference pattern. Coppens, Sanders and Dahl [Ref. 5 : p. 55] explained,

> "...the noise radiated by underwater vehicles consists of two contributions: broadband noise (similar to the hissing found between FM radio stations) and tonals (single frequency tones, like whistles). Thus, the radiated noise consists of all frequencies between certain broad limits with certain specific frequencies occurring with high intensity. Each of these frequencies, whether of broadband noise or tonal, has its own interference pattern.
>
> Because each frequency has its own period and wavelength, each will have a different phase delay for propagation over the direct and surface reflected paths. As a result, the ranges between source and receiver for which the surface reflected signal tends to cancel the direct signal will be different for each different frequency. Thus, the ranges at which different frequencies will have nulls at the receiver will be different. This means that as the range between source and receiver changes, the frequencies which have nulls at the receiver will also change."

Figure II-1 shows a schematic time vs. frequency plot of the interference pattern as the distance between the source and receiver is decreased and then increased. The distinct tonals will most readily show the effects of Doppler shift (apparent frequency change) as minimum separation range, is passed (see t = 0 in Figure II-1). The light (unshaded) hyperbolic shaped regions centered about t = 0 are known as the Lloyd's mirror interference pattern for the broad band noise. The frequency values F_1, F_2, F_3, F_4 represent the minimum frequency of each hyperbola. The actual frequency from the sound source that is being nullified by the surface reflection interface varies with time as the hyperbolas are traced out. The measured separation in hertz between any two successive nulls at any time t, is termed the difference frequency, Δf, and is the frequency value required in the problem solution discussed in the following Section. (See Section III.D.2)

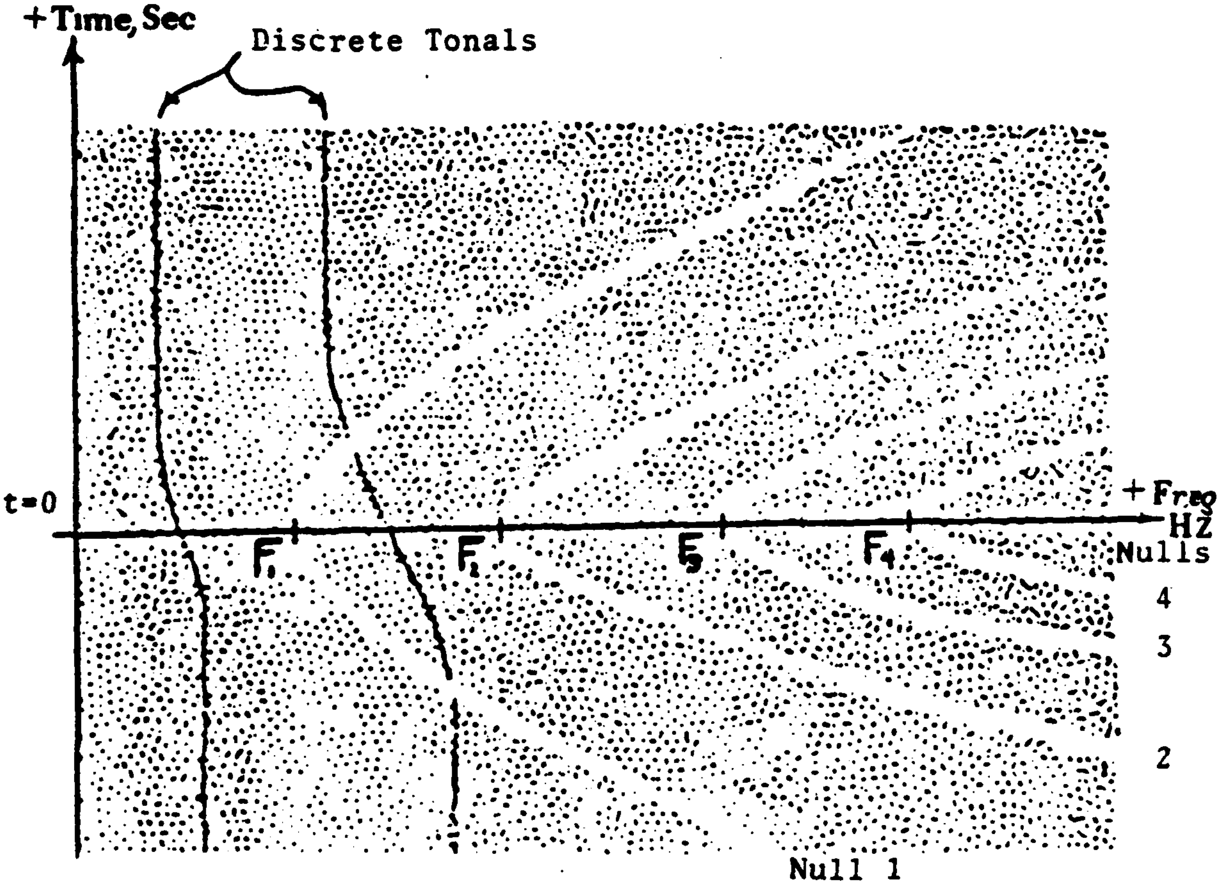


Figure II-1. Frequency vs. Time Display of Lloyd's Mirror Interference Pattern

III. RANGE vs. DEPTH SOLUTION USING the LLOYD'S MIRROR INTERFERENCE PHENOMENON

A. THEORETICAL BACKGROUND

1. Snell's Law

Snell's Law, [Ref. 1: p. 116, Ref. 6: p. 120], is a result of Fermat's minimum time principle which states that each ray will travel the path requiring minimum time of travel from source to receiver.

Mathematically, Snell's Law is expressed as:

$$\text{Snell Constant} = \frac{\sin \phi}{c(z)} \quad ;$$ ①

Where ϕ is the angle that the ray makes with the vertical and $c(z)$ is the sound velocity for a given depth z. (See Figure III-1)

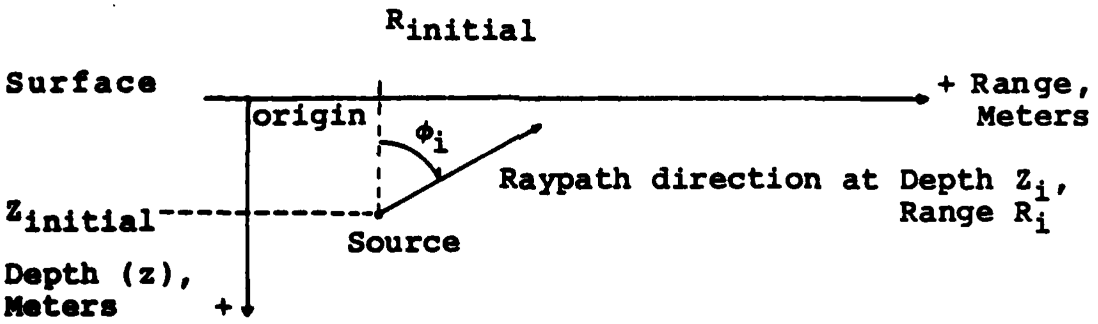


Figure III-1. Snell's Law Applied to Initial Sound Source Ray Angle.

The sound ray maintains the same Snell Constant value throughout its transmission path. Sound velocity changes result in an increasing or decreasing angle ϕ.

2. Ray Path Integration

Expanding the diagram of the sound ray, the differential raypath length, ds, can be described as a function of the differential change in depth, dz, and the differential change in horizontal range, dr. (Figure III-2)

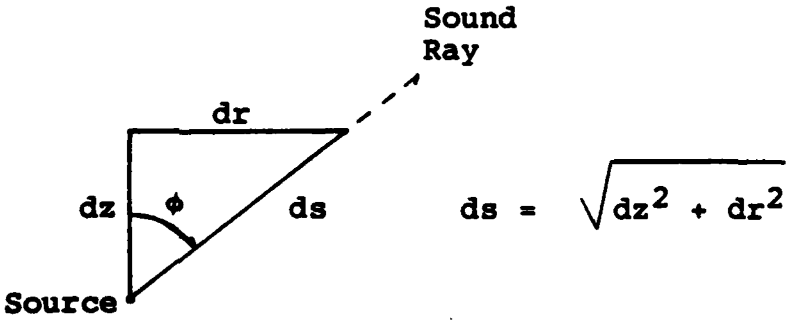


Figure III-2. Pythagorean Theorem Applied to Ray Path Length.

To calculate the actual path length of the sound ray, smaller and smaller segments of path length, Δs, horizontal range, Δr, and depth, Δz are used until the differential segments ds, dr, and dz are approached. This tends to smooth the curve of the ray into a sequence of straight line segments. The path length traveled by a particular ray can then be determined by integrating between the source depth and receiver depth. The possibility of the direct ray crossing the same depth more than once is not considered a problem in this solution due to the short ranges involved in Lloyd's mirror observations and the long ranges required for a sound ray to curve back through the same depth before reflecting off the surface. To handle the fact that the surface reflected ray does cross the same depth twice in the ranges pertinent to this discussion and to avoid errors in integration, the concept of a geometric equivalent image [Ref. 1: p. 123], of the source is used. The image reflected path and the actual reflected path are the same length. (Figure III-3)

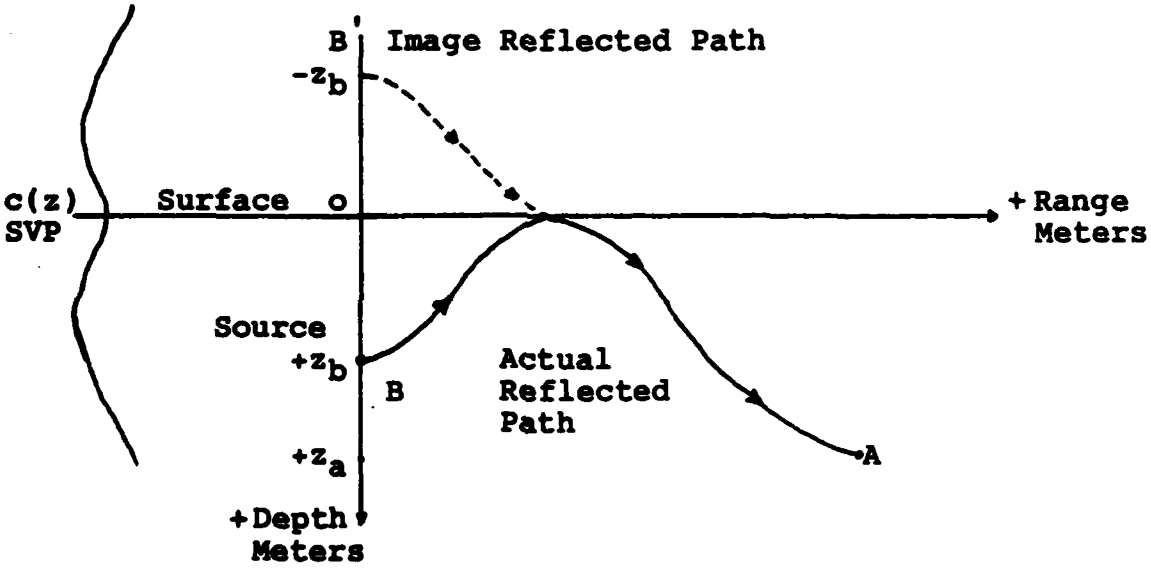


Figure III-3. Actual and Image Sound Paths From Source to Receiver for a Given Sound Velocity Profile (SVP).

The curved sound path shown in Figures III-3 and III-4 is due to the change (in general) in the sound velocity as the depth changes. It is possible to show that the path from A to B is the same length as the path from A to B'. Also, by inspection of the sketch, the horizontal range from A to B or A to B' is the same. Therefore, for the reflected path calculations of path length and horizontal range, the image reflection depth of $-z_b$ will be used.

B. HORIZONTAL RANGE SOLUTION

1. Direct Path

The horizontal range traversed by the direct path sound ray can be calculated from Snell's Law and the Pythagorean Theorem for right triangles applied to Figure III-2.

From ①, the Snell Constant, K_1 is:

$$K_1 = \frac{\sin \phi_i}{c(z_i)} \quad . \qquad \text{(1A)}$$

This is the Snell constant for the particular direct path ray that ultimately travels from the source to the receiver, but initially starts at depth z_i and moves in direction ϕ_i.

Using the Pythagorean Theorem on Figure III-2

$$\sin \phi = \frac{dr}{\sqrt{(dr)^2 + (dz)^2}} \qquad ②$$

Solving (1A) for $\sin \phi$, where ϕ = variable for the particular direct path ray tagged with Snell constant K_1,

$$\sin \phi = K_1 \, c(z) \qquad ③$$

setting ② and ③ equal to each other yields:

$$K_1\ c(z) = \frac{dr}{\sqrt{(dr)^2 + (dz)^2}} \quad ④$$

The right side of ④ can be simplified by dividing numerator and denominator by dr.

$$K_1\ c(z) = \frac{1}{\sqrt{1 + \left(\frac{dz}{dr}\right)^2}}$$

Solving for dz/dr follows in steps:

$$\sqrt{1 + \left(\frac{dz}{dr}\right)^2} = \frac{1}{K_1\ c(z)}$$

$$1 + \left(\frac{dz}{dr}\right)^2 = \frac{1}{[K_1\ c(z)]^2}$$

$$\left(\frac{dz}{dr}\right)^2 = \frac{1 - [K_1\ c(z)]^2}{[K_1\ c(z)]^2}$$

$$\frac{dz}{dr} = \pm\sqrt{\frac{1 - [K_1\ c(z)]^2}{[K_1\ c(z)]^2}}$$

$$\frac{dz}{dr} = \pm \frac{\sqrt{1 - [K_1\ c(z)]^2}}{K_1\ c(z)} \qquad \text{⑤}$$

Now, solving for dr, the incremental horizontal range yields:

$$dr = \pm \frac{K_1\ c(z)}{\sqrt{1 - [K_1\ c(z)]^2}}\,dz \qquad \text{⑥}$$

Integrating ⑥ with respect to depth, with the limits of integration being the depth of the source and the depth of the receiver, yields:

$$R\ (\text{direct path}) = \pm \int_{\substack{\text{Actual}\\ \text{Receiver}\\ \text{Depth}\\ \text{(along path of ray}\\ \text{"tagged" } K_1\text{)}}}^{\substack{\text{Source}\\ \text{Depth}}} \frac{K_1\ c(z)}{\sqrt{1 - [K_1\ c(z)]^2}}\,dz \qquad \text{⑦}$$

The "Range" is always positive and thus the appropriate + or - sign is used to make this integral always positive regardless of the integration limits. For ease of discussion in the rest of this solution, the receiver will be assumed to be at a shallower depth.

2. Reflected Path

Recalling the image depth discussion and noting a different angle, θ_i, for the reflected ray is required (Figure III-4), which results in a different Snell constant, K_2, where

$$K_2 = \frac{\sin(\theta_i)}{c(z_i)} \quad . \qquad ⑧$$

It follows directly that ⑦ can be rewritten for the reflected horizontal range as:

$$R\ (\text{reflected path}) = \int_{\substack{\text{Image}\\ \text{Receiver}\\ \text{Depth}}}^{\substack{\text{Source}\\ \text{Depth}}} \frac{K_2\, c(z)}{\sqrt{1 - [K_2\, c(z)]^2}}\, dz \qquad ⑨$$

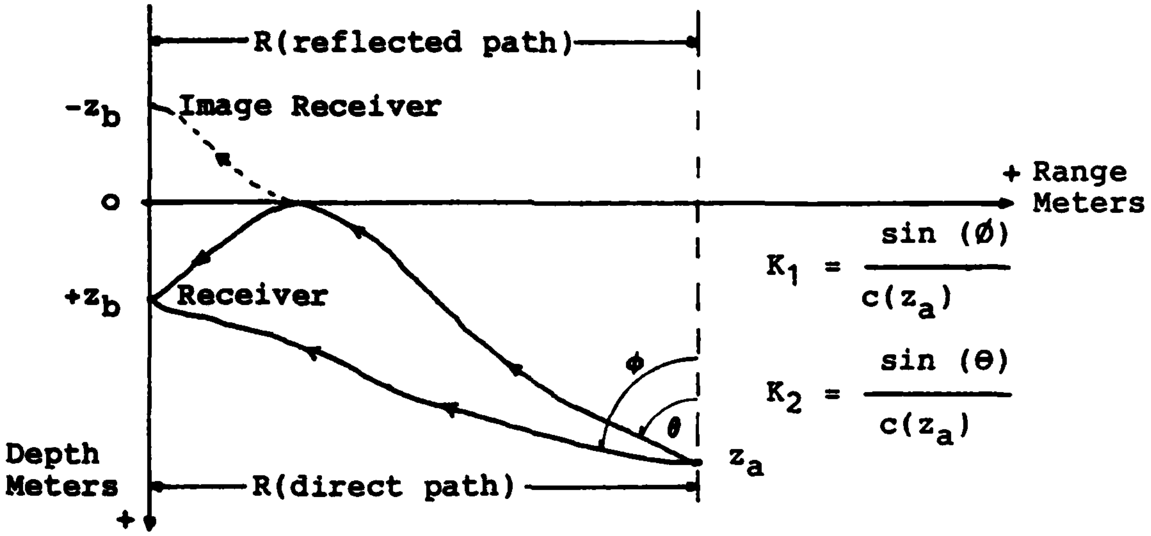


Figure III-4. Common Direct and Reflected Sound Paths From Source to Receiver.

C. SOUND RAY PATH LENGTH SOLUTION

1. Direct Path

The direct path ray length can be calculated by returning to Figure III-2, applying the Pythagorean Theorem and noting:

$$ds = \sqrt{(dr)^2 + (dz)^2}$$ ⑩

realizing the path length S from receiver to source is given by:

$$S = \int_{\text{Receiver Depth}}^{\text{Source Depth}} ds = \int_{\text{Receiver Depth}}^{\text{Source Depth}} \sqrt{(dr)^2 + (dz)^2}$$ ⑪

Which can be simplified by multiplying by $\dfrac{dz}{\sqrt{dz^2}}$

as follows:

$$S\ (\text{Direct Path}) = \int_{\substack{\text{Receiver}\\ \text{Depth}}}^{\substack{\text{Source}\\ \text{Depth}}} \sqrt{1 + \left(\frac{dr}{dz}\right)^2}\ dz$$

Substituting the equivalent of $\left(\dfrac{dr}{dz}\right)$ from the reciprocal of ⑤,

$$S\ (\text{Direct Path}) = \int_{\substack{\text{Receiver}\\ \text{Depth}}}^{\substack{\text{Source}\\ \text{Depth}}} \sqrt{1 + \frac{K_1^2\ c(z)^2}{1 - K_1^2\ c(z)^2}}\ dz$$

$$S\ (\text{Direct Path}) = \int_{\substack{\text{Receiver}\\ \text{Depth}}}^{\substack{\text{Source}\\ \text{Depth}}} \sqrt{\frac{1 - K_1^2\ c(z)^2 + K_1^2\ c(z)^2}{1 - [K_1\ c(z)]^2}}\ dz$$

$$S\ (\text{Direct Path}) = \int_{\substack{\text{Actual}\\ \text{Receiver}\\ \text{Depth}}}^{\substack{\text{Source}\\ \text{Depth}}} \frac{1}{\sqrt{1 - [K_1\ c(z)]^2}}\ dz \qquad \text{⑫}$$

2. Reflected Path

Using the same arguments about the reflected path as presented for horizontal range and substituting K_2 for K_1 and image receiver depth into ⑪ yields:

$$S\ (\text{Reflected Path}) = \int_{\substack{\text{Image}\\ \text{Receiver}\\ \text{Depth}}}^{\substack{\text{Source}\\ \text{Depth}}} \frac{1}{\sqrt{1 - [K_2\ c(z)]^2}}\ dz\ . \qquad \text{⑬}$$

D. EXPLANATION OF SOLUTION

1. Some Observations

Equations ⑦ and ⑫ are path integrals for the direct ray, using the same Snell constant and integration limits. Further, equations ⑨ and ⑬ are path integrals of the reflected ray and thus use a different Snell constant from the direct path integrals, but use the same integration limits respectively for the horizontal range and ray path lengths.

2. Solution

The key to the eventual numerical solution is the equality between the direct path horizontal range and the reflected path horizontal range. Although the actual path length of the reflected ray is longer than the direct path ray between common source and receiver, the horizontal distance traveled by both is the same.

By calculating all of the initial ray angles that are possible for each depth of the source of interest, two horizontal range arrays, one for the direct path and one for the reflected path, are formed. Each of these arrays is linked respectively to a direct and reflected ray path length array by the corresponding Snell Constant used to determine both the horizontal range and sound ray length for each path, (See Figure III-4). For example, suppose the initial direct path ray angle of 15 degrees from the vertical covered a horizontal range of 20 meters in traveling from the source to the receiver. Similarly, the initial reflected path ray angle of 8 degrees from the vertical covered the same horizontal range of 20 meters from the source to the receiver. Then the difference between the reflected ray path length for 8 degrees and the direct ray path length for 15 degrees could be computed (Figure III-5).

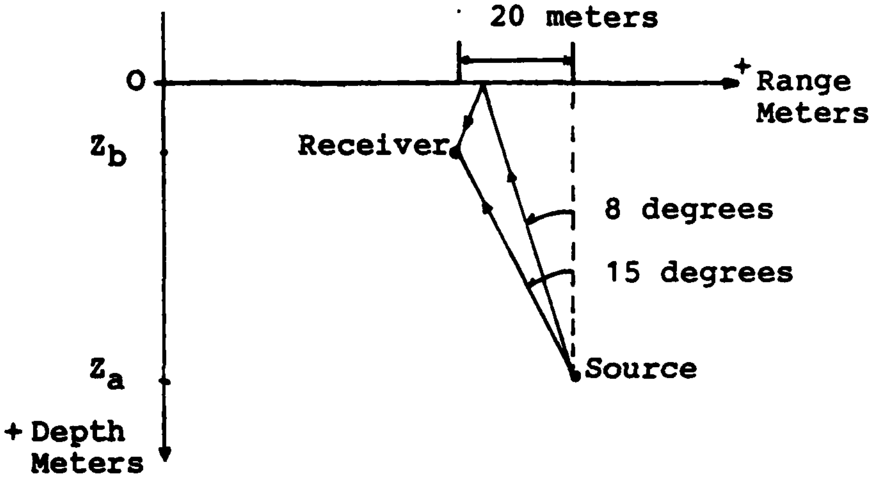


Figure III-5. Example of Reflected Path Ray Interfering With Direct Path Ray at the Receiver.

It is this difference in ray path length, $\Delta S = S_R - S_D$, for a wide band of frequencies that causes the time delayed phase shift sensed at the receiver and known as the Lloyd's mirror interference phenomenon. Since both the direct and reflected rays are traveling at the same sound velocity <u>at the receiver</u>, the ray path length difference can be directly converted to the frequency difference, Δf <u>as observed at the receiver</u> and seen in the Lloyd's mirror interference pattern.

The intensity ratio at the receiver is given by

$$\text{Intensity ratio} = 1 + \mu^2 \left(\frac{R_D}{R_R}\right)^2 - 2\mu\left(\frac{R_D}{R_R}\right) \cos(\omega \tau)$$

The ratio reaches a minimum value corresponding to maximum interference when $\cos(\omega\tau) = 1$. This occurs when $\omega\tau = 2\pi$ or any integer multiple of 2π.

Note that $\lambda = c/f$ where λ = wavelength (m)
c = phase speed (m/s)
f = frequency in (hz)

k = wave number (m^{-1})

and $k = 2\pi/\lambda$

Now let $\Delta\phi$ equal the phase difference seen at the receiver due to the two path length differences and defined as,

$$\Delta\phi = k(S_R - S_D), \text{ where}$$

S_R = Path length of the reflected ray and

S_D = Path length of the direct ray.

Let $\Delta S = (S_R - S_D)$

So that the first null occurs when

$\Delta\phi = k(\Delta S) = 2\pi = \omega\tau$

So $k(\Delta S) = \omega\tau = 2\pi = 2\pi(\Delta S)/\lambda$ or

$\Delta S = 2\pi\lambda/2\pi = \lambda$

therefore $\Delta S = c/f_1$

so the first null occurs at frequency

$f_1 = c/\Delta S$

The second null occurs at

$\omega\tau = 4\pi$ so that

$\Delta S = 4\pi\lambda/2\pi = 2\lambda = 2c/f_2$

therefore second null occurs at frequency

$$f_2 = 2c/\Delta S = 2(f_1)$$

which shows that $f_2 - f_1 = f_1$

and so on. So that for the n^{th} null

$$f_n = nc/\Delta S = nf_1$$

and $f_n - f_{n-1} = f_1$ or more generally

$$\frac{f_m - f_n}{m - n} = f_1 = \frac{c}{\Delta S}$$

where m >> n for best accuracy so that the measured difference in frequency between any two successive nulls can be related to the difference in path length required to produce that null by the equation

$$f = \frac{C(\text{receiver})}{\Delta S}$$

where $C_{(\text{receiver})}$ is the actual sound velocity at the receiver at the time of reception.

IV. COMPARISON of the ANALYTICAL and COMPUTER SOLUTIONS for the ISOVELOCITY SOUND PROPAGATION CASE

This analysis demonstrates the accuracy of the computer solution by using an isovelocity sound propagation model. The constant sound velocity condition results in straight line transmission paths as the sound travels from the source to the receiver. By solving the geometric model analytically and comparing the results to the computer output one can see the accuracy of the Romberg integration technique coupled with the four integrals derived in section III . The isovelocity case is the simplest propagation model yet it exercises all phases of the computer program's main calculation algorithm. The only section of the overall program not tested by the isovelocity case are the sound velocity calculations performed as the user inputs a varying temperature profile. Table IV-1 compares the difference in path lengths between the direct and reflected sound paths as calculated analytically and by the computer. Of the 20 data points compared, the computer missed the exact ray path length for a horizontal range of 750 meters by only 19.16 centimeters in the worst case and averaged 5.02 centimeters error over all 20 cases. Table IV-2 shows the resulting horizontal range error due to the error of the computer's calculation presented in Table IV-1. The average range error for the 20 points is only 1.52 meters.

The computer program steps through sea depths of 0 to 200 meters in 10 meter increments where each depth represents a possible target depth. To calculate a complete set of Snell constants, initial sound ray angles of 3 thru 177 degrees in increments of 3 degrees (measured from the vertical) are used. Ultimately the integration section derives a path length difference between the surface reflected ray and the direct path ray for each of these selected starting ray directions.The 59 initial rays times the 20 depth increments yields a total of 1,180 data points. These point solutions are used to interpolate values of path length difference for points every 10 meters of horizontal range out to 1000 meters which results in a two dimensional matrix containing 2000 (20 depth increments times 100 range increments) horizontal range vs depth elements.

To compare the computer solution to actual geometric results, five ranges for each of four depths were chosen at random. In all comparisons the receiver is set at a depth of 27 meters (approximately 90 feet).

TABLE IV-1

ISOVELOCITY CASE
COMPARISON BETWEEN
GEOMETRIC AND COMPUTER SOLUTIONS

PATH LENGTH DIFFERENCE

$$\frac{\text{ANALYTICAL}}{\text{COMPUTER}}$$

SOURCE DEPTHS METERS	HORIZONTAL RANGES, METERS 20	100	380	750
10	15.8107	5.1908	1.4170	0.7195
	15.8228	5.1939	1.4418	.6693
50	49.0755	23.5992	7.0274	3.5897
	49.0196	23.6220	7.0389	3.5714
90	52.5987	35.7218	12.4171	6.4289
	52.6316	35.7143	12.4688	6.2267
130	53.3450	42.5840	17.4439	9.2169
	53.3808	42.6136	17.5644	9.0253
170	53.6208	46.4312	22.0132	11.9302
	53.5714	46.4396	22.0264	11.7739

TABLE IV-2

COMPARISON of PROGRAM RANGES vs. ACTUAL RANGES

SOURCE DEPTHS METERS	ACTUAL HORIZONTAL RANGES, METERS			
	20	100	380	750
10	19.954	99.935	373.42	806.3
50	20.134	99.875	379.37	753.87
90	19.745	100.038	378.33	774.84
130	19.42	99.815	377.075	766.41
170	21.26	99.93	379.725	760.475

COMPUTER

HORIZONTAL RANGES, METERS

V. COMPUTER PROGRAM FLOW CHART

A. FLOW CHART and PROGRAM BACKGROUND

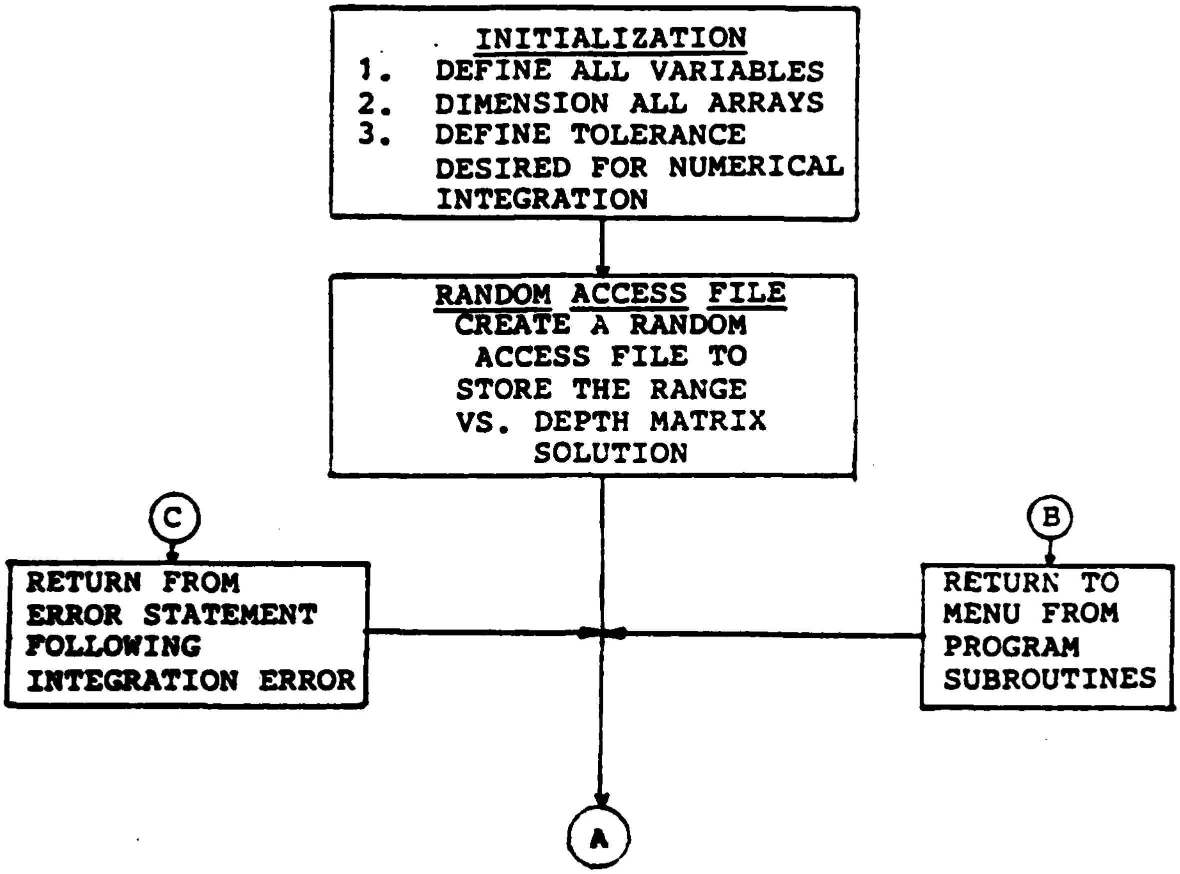
The following block diagram flow chart demonstrates the specific steps required to obtain an accurate horizontal range vs. depth solution using the Lloyd mirror interference phenomenon. Specific values or data used in the included program will be given at the bottom of each appropriate block.

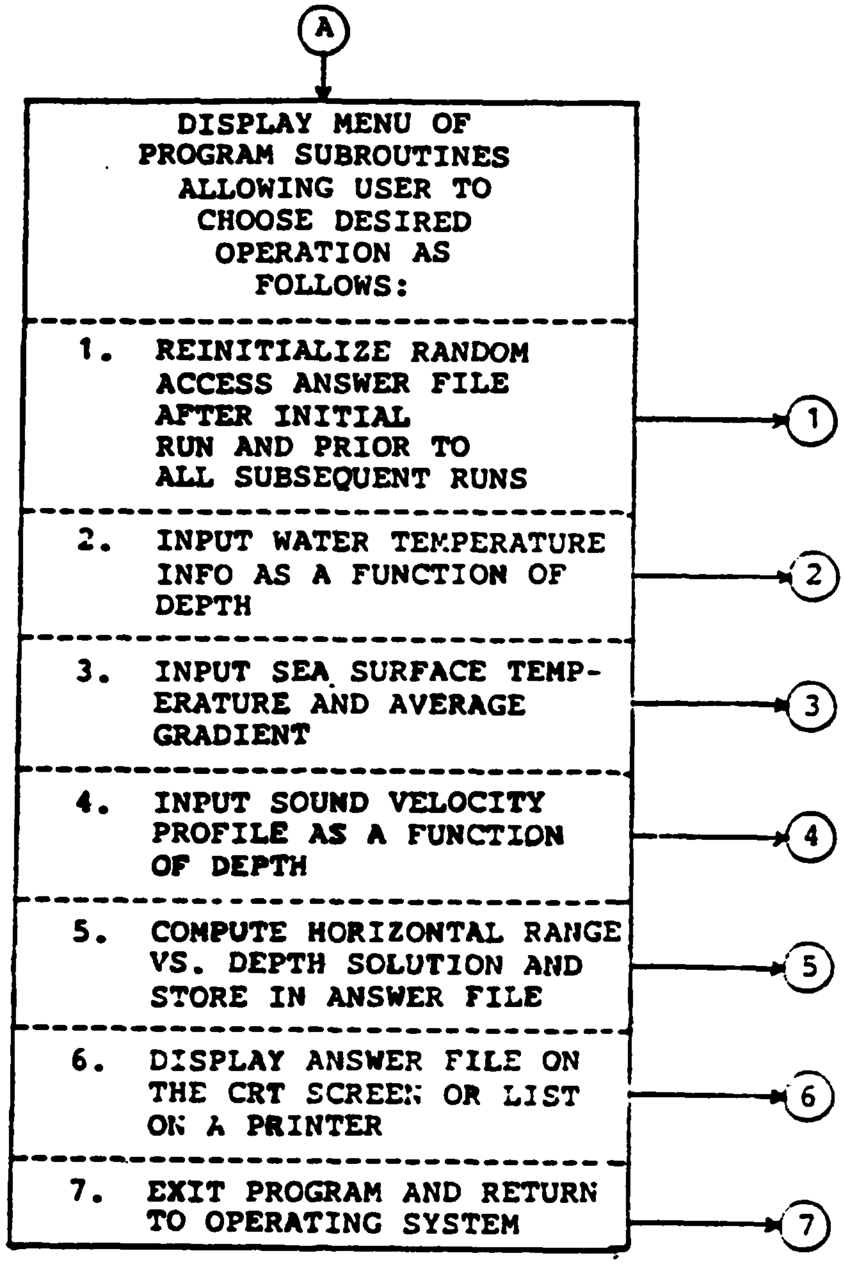
The actual numerical integration section of the program is taken directly from the Romberg integration program provided by Miller [Ref. 7: pp. 215-218].

The sound velocity equation used in the program was developed by Coppens [Refs. 6 and 8] and selected after several equations of varying complexity were considered. Specifically, equations derived by Wilson and Leroy [Ref. 1: p. 105], plus Mackenzie [Ref. 9] were compared with Coppens. All results were within 1 to 2 centimeters per second of each other. Coppen's equation was chosen because of the ability to correct for not only temperature, pressure and salinity, but also, latitude variations.

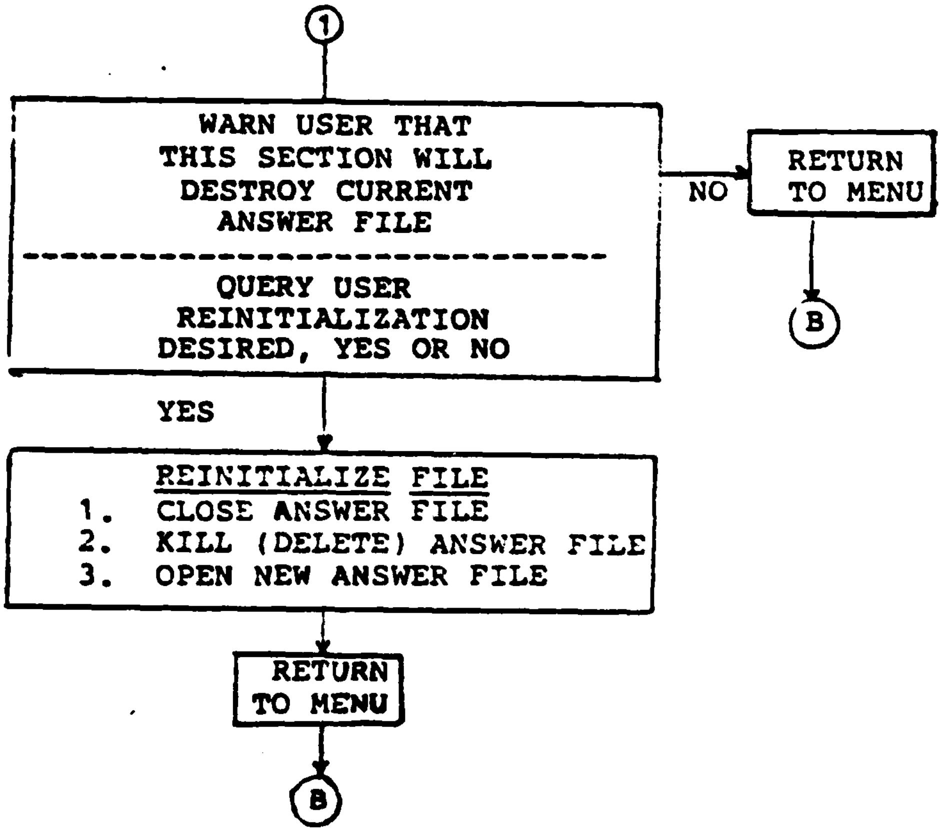
B. FLOW CHART

1. Initialization and Main Menu

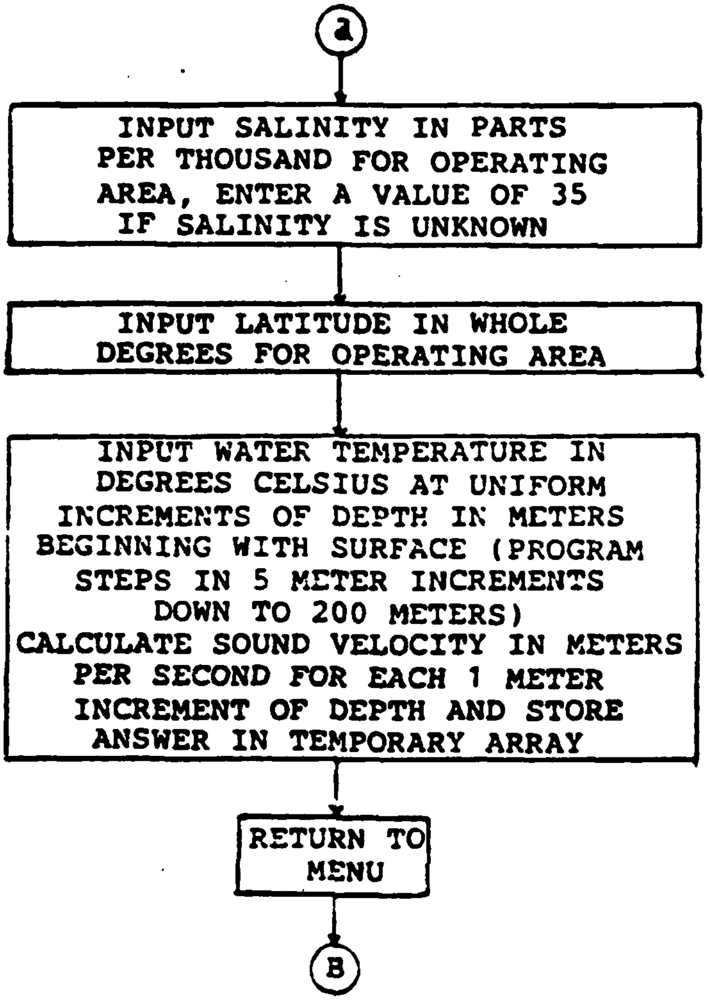



A
DISPLAY MENU OF PROGRAM SUBROUTINES ALLOWING USER TO CHOOSE DESIRED OPERATION AS FOLLOWS:
1. REINITIALIZE RANDOM ACCESS ANSWER FILE AFTER INITIAL RUN AND PRIOR TO ALL SUBSEQUENT RUNS
1
2. INPUT WATER TEMPERATURE INFO AS A FUNCTION OF DEPTH
2
3. INPUT SEA SURFACE TEMPERATURE AND AVERAGE GRADIENT
3
4. INPUT SOUND VELOCITY PROFILE AS A FUNCTION OF DEPTH
4
5. COMPUTE HORIZONTAL RANGE VS. DEPTH SOLUTION AND STORE IN ANSWER FILE
5
6. DISPLAY ANSWER FILE ON THE CRT SCREEN OR LIST ON A PRINTER
6
7. EXIT PROGRAM AND RETURN TO OPERATING SYSTEM
7

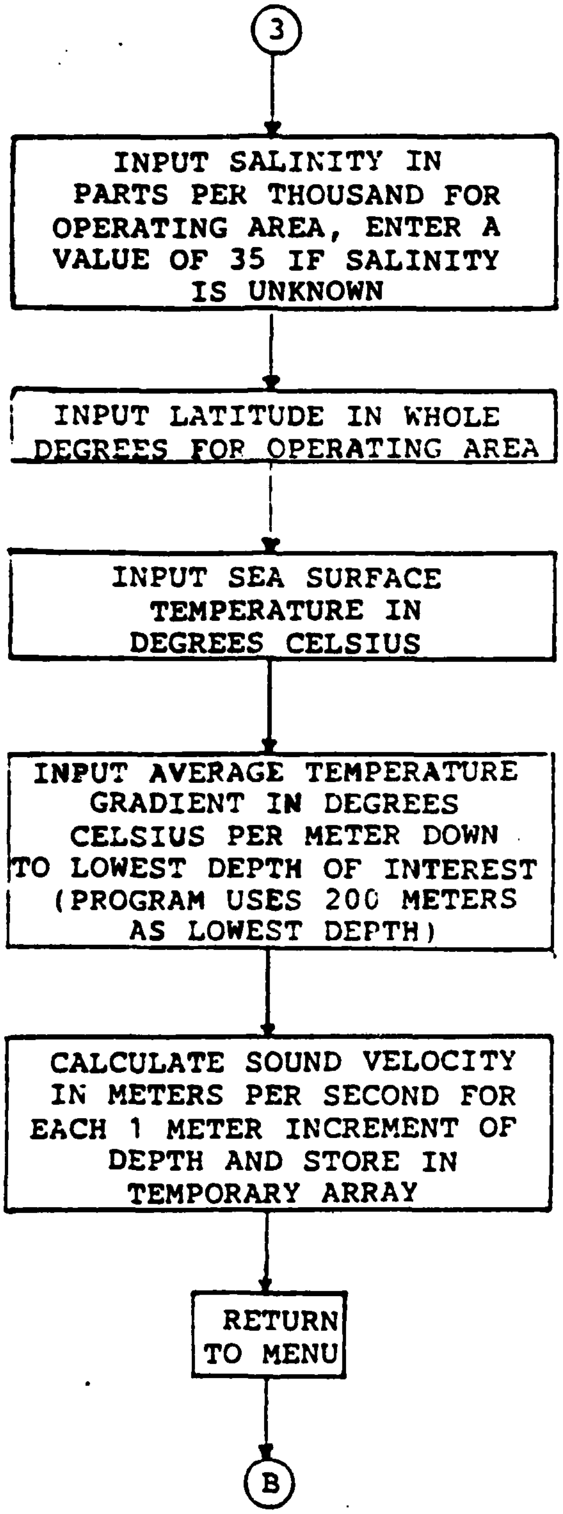
2. Reinitialize Random Access Answer File



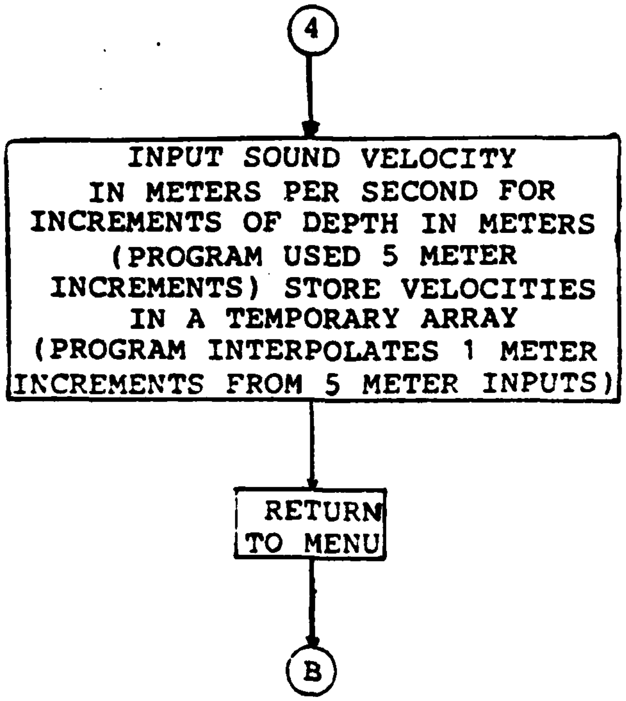
3. Input Water Temperature Profile



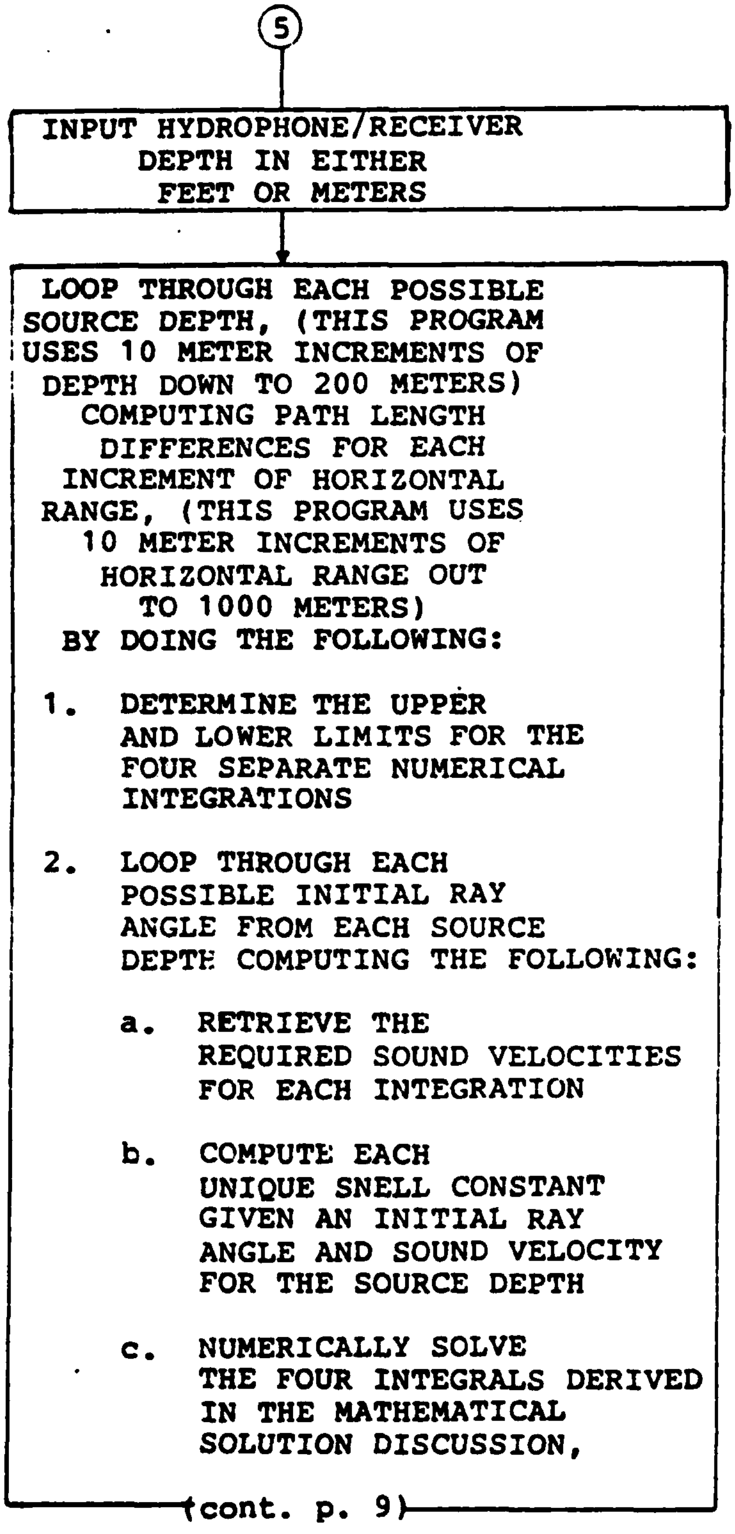
4. Input Sea Surface Temperature And Average Gradient



5. Input Sound Velocity Profile



6. Compute Range vs. Depth Solution And Store In Random Access File



INPUT HYDROPHONE/RECEIVER DEPTH IN EITHER FEET OR METERS

LOOP THROUGH EACH POSSIBLE SOURCE DEPTH, (THIS PROGRAM USES 10 METER INCREMENTS OF DEPTH DOWN TO 200 METERS) COMPUTING PATH LENGTH DIFFERENCES FOR EACH INCREMENT OF HORIZONTAL RANGE, (THIS PROGRAM USES 10 METER INCREMENTS OF HORIZONTAL RANGE OUT TO 1000 METERS) BY DOING THE FOLLOWING:

1. DETERMINE THE UPPER AND LOWER LIMITS FOR THE FOUR SEPARATE NUMERICAL INTEGRATIONS

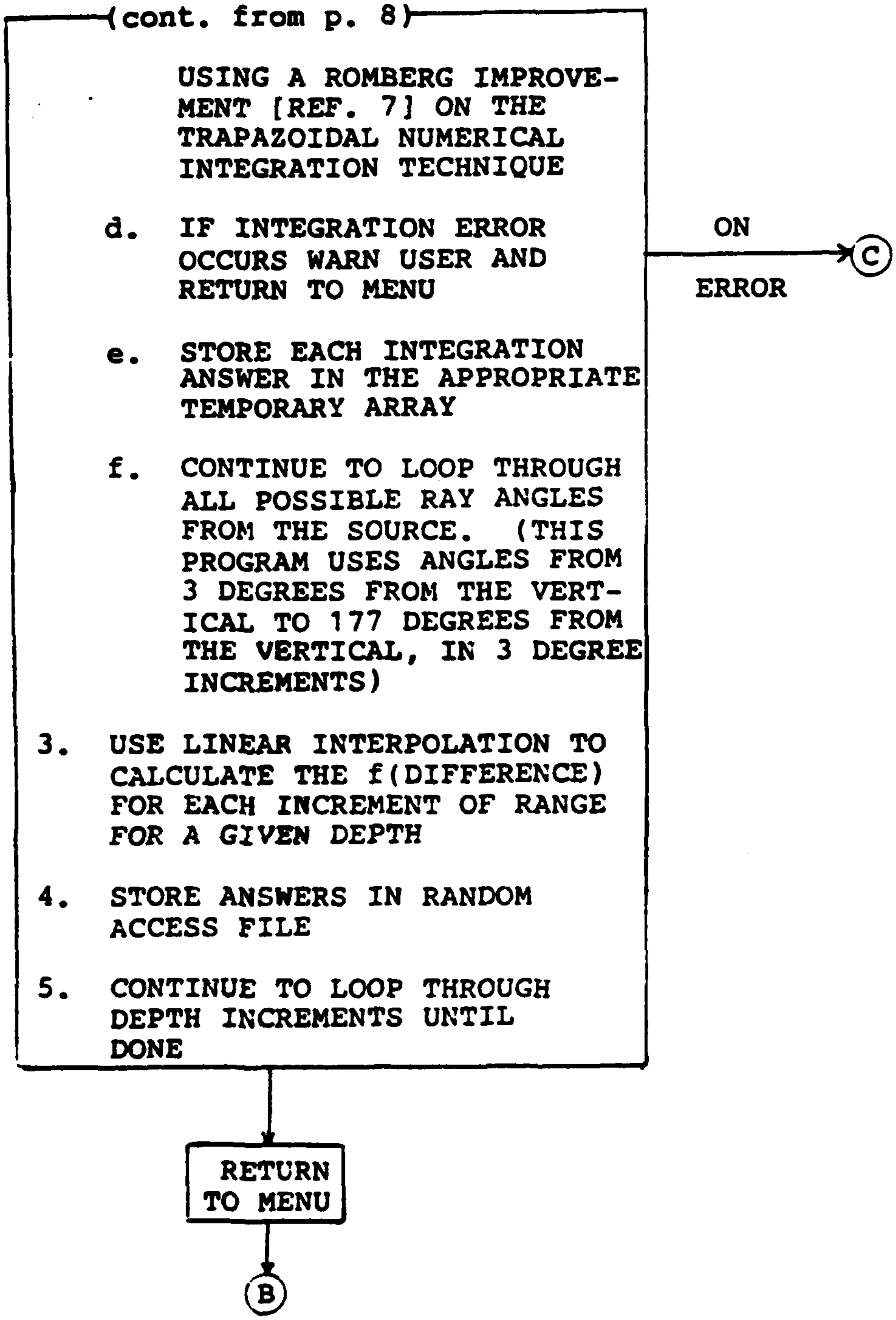
2. LOOP THROUGH EACH POSSIBLE INITIAL RAY ANGLE FROM EACH SOURCE DEPTH COMPUTING THE FOLLOWING:

   a. RETRIEVE THE REQUIRED SOUND VELOCITIES FOR EACH INTEGRATION

   b. COMPUTE EACH UNIQUE SNELL CONSTANT GIVEN AN INITIAL RAY ANGLE AND SOUND VELOCITY FOR THE SOURCE DEPTH

   c. NUMERICALLY SOLVE THE FOUR INTEGRALS DERIVED IN THE MATHEMATICAL SOLUTION DISCUSSION,

(cont. p. 9)



USING A ROMBERG IMPROVEMENT [REF. 7] ON THE TRAPAZOIDAL NUMERICAL INTEGRATION TECHNIQUE

d. IF INTEGRATION ERROR OCCURS WARN USER AND RETURN TO MENU

e. STORE EACH INTEGRATION ANSWER IN THE APPROPRIATE TEMPORARY ARRAY

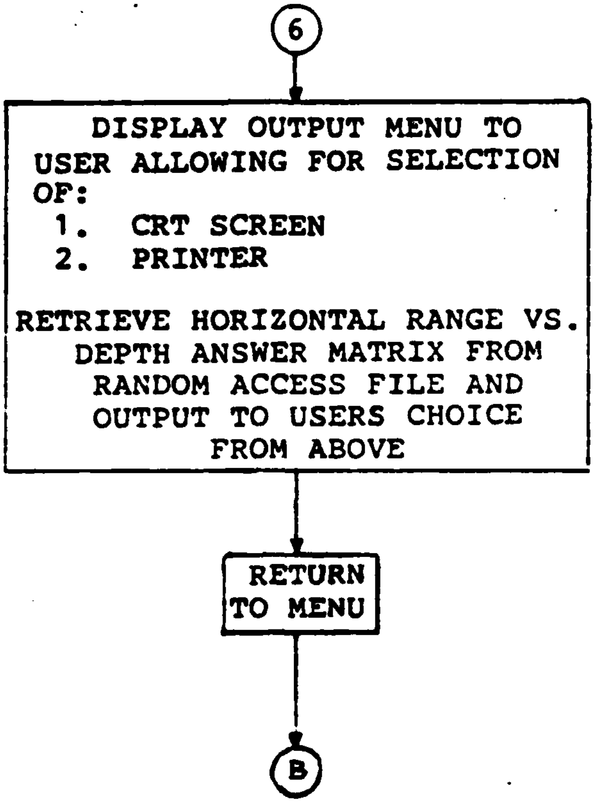
f. CONTINUE TO LOOP THROUGH ALL POSSIBLE RAY ANGLES FROM THE SOURCE. (THIS PROGRAM USES ANGLES FROM 3 DEGREES FROM THE VERTICAL TO 177 DEGREES FROM THE VERTICAL, IN 3 DEGREE INCREMENTS)

3. USE LINEAR INTERPOLATION TO CALCULATE THE f(DIFFERENCE) FOR EACH INCREMENT OF RANGE FOR A GIVEN DEPTH

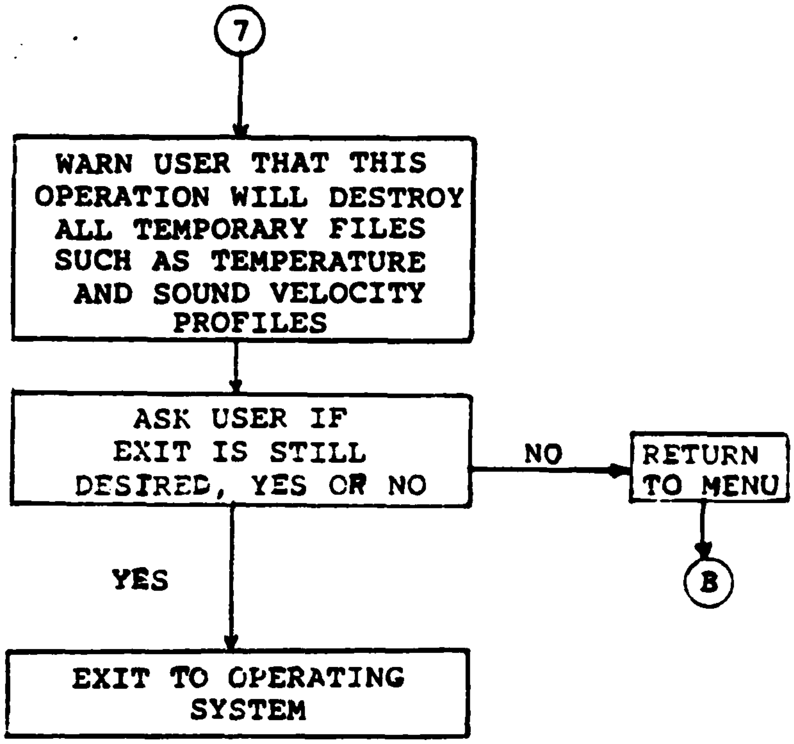
4. STORE ANSWERS IN RANDOM ACCESS FILE

5. CONTINUE TO LOOP THROUGH DEPTH INCREMENTS UNTIL DONE

7. Display Answer File On CRT Or List On Printer



8. Exit Program And Return To Operating System



VI. DEFINITION of COMPUTER PROGRAM VARIABLES

ADD	Equation used to compute raypath length in meters for both the direct and reflected paths.
AFLAG	Branching flag used in solving for answer matrix elements
ANS	Answer obtained from numeric integration
AOUT	Answer matrix element output to CRT screen or printer
BFLAG	Branching flag used in solving for answer matrix elements
BNDLOW	Lower limit for evaluating numeric integration II
BOUNDHI	Upper limit for evaluating numeric integration
BOUNDLO	Direct path lower limit
CLAT	Latitude variation used in sound velocity equation
DDEEP	First half of depth effect used in sound velocity equation
DEEP	Second half of depth effect used in sound velocity equation
DEG.	Average whole degree latitude of operating area used to calculate CLAT
DELTA	Line integral uniform divisions used in Romberg numeric integration routine
DHI	Direct path horizontal range upper bound used in interpolating direct path length for desired range increment
DIRFRAC	Direct path interpolation fraction
DIRRAY	Direct path interpolated path length

DLONG	Dirct path length upper bound used in interpolating path length for desired range increment
DLOW	Direct path hoirzontal range lower bound used in interpolating direct path length for desired range increment
DPH	Depth increment used for outputing answer matrix
DSHORT	Direct path length lower bound used in interpolating path length for desired range increment
FANSWER.FL1	Random access file used to store the horizontal range vs. depth answer matrix
FINAL ( )	Answer array used to temporarily hold the interpolated values before storing them in random access file
GRAD	Average temperature gradilent input by user
HYDEEP	Hydrophone/receiver depth in feet
HYDPTH	Hydrophone/receiver depth in meters
HYSPD	Sound velocity at hydrophone/receiver depth
I	Loop increment
IFLAG	Loop increment used to branch through one of four integrated equations
II	Loop increment
IN	Loop increment
INCR	Loop increment
INN	Loop increment
J	Counter
JMP	Array element pointer
K	Counter
M	Counter
METER	Array element pointer

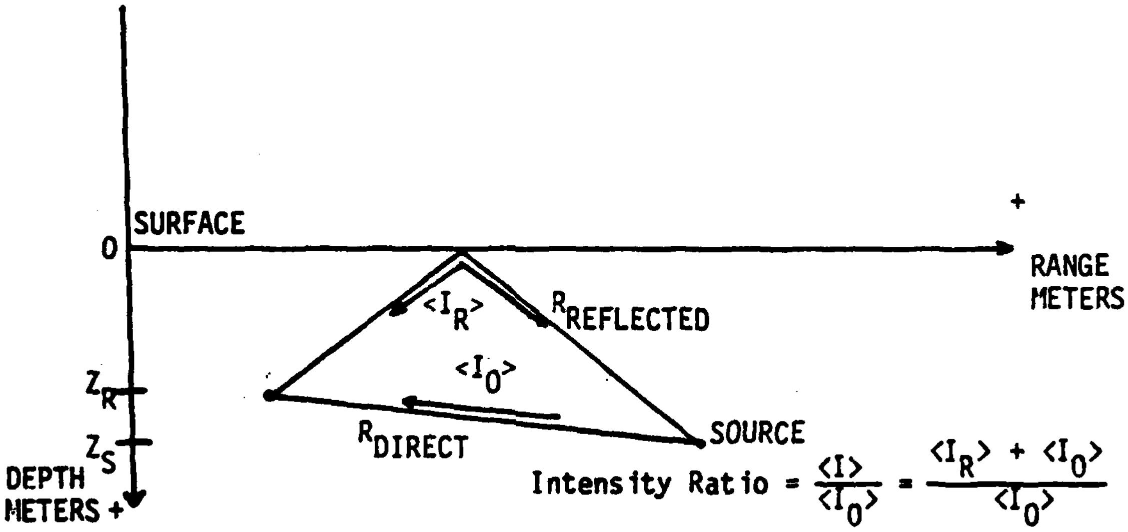
MSTEP	Loop increment
MSV	Array element pointer
MTRHI	Array element pointer
MTRLO	Array element pointer
MTRS	Array element pointer
MTRSTEP	Array element pointer
MULT	Random access file pointer
N	Array element pointer
NANGL	Ray angle from source
NMTR	Horizontal range increment
NN	Counter
NSTEP	Ray angle from source
NT	Array element pointer
NTRA	Array element pointer
NUM	*Main menu index selection*
NUMB	Output menu index selection
NUPLMT	Incremented source depth
NX ( )	Array of counters used during integration accuracy testing loop
PART	Partitioning factor used in numerical integration routine
PIECES	Incremented divisor used to partition depth integration limits
RAD	Conversion of ray angle from degrees to radians
RAYDIR ( )	Direct ray path length calculated from equation #12 page 25
RAYREF	Reflected path length calculated from equation #13 page 25

REFFRAC	Reflected path interpolation fraction
REFLO	Reflected path lower limit. Negative value of boundlo
REFRAY	Reflected path interpolated path length
RHI	Reflected path horizontal range upper bound used in interpolating reflected path length for desired range
RLONG	Reflected path length upper bound used in interpolating path length for desired range increment
RLOW	Reflected path horizontal range lower bound used in interpolating reflected path length for desired range
RMTR	Depth increment for interpolation routine
RNGDIR ( )	Direct path horizontal range array calculated from equation #7 page 21
RNGREF ( )	Reflected path horizontal range array calculated from equation #9 page 22
RSHORT	Reflected path length lower bound used in interpolating path length
SALIN	Salinity value for operating area input by user
SNDHI	Sound velocity at numeric integral upper bound
SNDLO	Sound velocity at numeric integral lower bound
SNDSPS ( )	Sound velocity profile array input directly by user or calculated from temperature, salinity, and latitude inputs
SNELL	Snell constant equation result calculated from initial ray angle and depth of source
SOUND	Sound velocity at incremented source depth
SST	Sea surface temperature
START	Initial estimate of integral answer
SUM	Equation used to compute horizontal range in meters for both the direct and reflected paths

SVP	Input sound velocity profile by user
TEMP	Degrees celsius temperature divided by 10
TEST ( )	Successive numeric integration answer array used to check accuracy of answer
TGTDPTH	Incremented source depth
TOL	Accuracy desired for numerical integration routine answer
TOT	Summation of partitioned numerical ilntegration values
TTEMP	Input temperature used in calculating sound velocity profile
WALK	Integration routine smoothing factor
X	Incremented depth value for each partition

APPENDIX A

Urick [Ref. 1: p. 124] briefly discusses the intensity of sound reaching a receiver via a direct path and a surface reflected path. The following derives the ratio of the total intensity of the signal received at a point when multipath Lloyd's mirror interference is present, to the total intensity resulting only from the direct path.



$$\text{Intensity Ratio} = \frac{\langle I \rangle}{\langle I_0 \rangle} = \frac{\langle I_R \rangle + \langle I_0 \rangle}{\langle I_0 \rangle}$$

MUST SHOW:

$$\frac{\langle I \rangle}{\langle I_0 \rangle} = 1 + \mu^2 \left(\frac{R_D}{R_R}\right)^2 - 2\mu\left(\frac{R_D}{R_R}\right) \cos(\omega\tau)$$

WHERE: $\langle I \rangle$, the time averaged intensity and I, the instantaneous intensity are defined as follows:

(1) $\langle I \rangle \ (1/T) \int_0^T I \, dt$ $\quad T = 1$ period $\quad T = \frac{1}{f}$

(2) $I = \frac{(P_0(r) + P_R(r))^2}{2\rho c}$; $P_0(r)$ and $P_R(r)$ are both functions of time

GIVEN:

(1) $P_{RECEIVED} = P_{DIRECT} + P_{REFLECTED}$

$$= \frac{P_0}{R_D} \cos \omega t - \frac{\mu P_0}{R_R} \cos [\omega(t-\tau)]$$

where: μ = surface pressure reflection coefficient

τ = time delay caused by the difference in the two path lengths

(2) $I_0 = \frac{P^2}{\rho c} = \frac{P_0^2}{2\rho c R_D^2}$

where: P = RMS pressure amplitude of the direct path ray

NOTE: It is assumed that the transmission loss is due only to spherical spreading and that absorption in the medium is negligible.

SOLUTION:

$$P_{DIR} = \frac{P_0}{R_D} \cos (\omega t) \qquad P_{REF} = \frac{-\mu P_0}{R_R} \cos[\omega(t-\tau)]$$

$$P_{RECEIVED} = P_{DIRECT} + P_{REFLECTED}$$

$$P_{RECEIVED} = P_0\left[\frac{1}{R_D} \cos(\omega t) - \frac{\mu}{R_R} \cos (\omega(t-\tau))\right]$$

$$I = \frac{P^2}{\rho c} = \frac{P_0^2}{2\rho c}\left[\frac{\cos^2(\omega t)}{R_D^2} + \frac{\mu^2}{R_R^2} \cos^2(\omega(t-\tau)) - \frac{2\mu}{R_R R_D} \cos (\omega t) \cos [\omega (t-\tau)]\right]$$

SINCE: $\langle I\rangle = \frac{1}{T}\int_0^T I \; dt$

AND $\omega = 2\pi f = \frac{2\pi}{T}$ $\qquad T = \frac{2\pi}{\omega}$

$$\langle I\rangle = \frac{\omega}{2\pi}\int_0^{2\pi/\omega} I \; dt$$

$$\langle I\rangle = \frac{\omega P_0^2}{2\pi\rho c}\left[\frac{1}{R_D^2}\int_0^{2\pi/\omega}\cos^2(\omega t)\,dt + \frac{\mu^2}{R_R^2}\int_0^{2\pi/\omega}\cos^2(\omega(t-\tau))\,dt\right.$$

$$\left. - \frac{2\mu}{R_D R_R}\int_0^{2\pi/\omega}\cos(\omega t)\cos(\omega(t-\tau))\,dt\right]$$

Solving the three integrals separately

$$\frac{1}{R_D^2}\int_0^{2\pi/\omega}\cos^2(\omega t)\,dt = \frac{1}{R_D^2}\left[\frac{t}{2} + \frac{1}{4\omega}\sin(2\,\omega t)\right]\Bigg|_0^{2\pi/\omega}$$

$$= \frac{1}{R_D^2}\left[\frac{\pi}{\omega} + \frac{1}{4\omega}\sin(4\pi)\right]$$

$$= \boxed{\frac{\pi}{\omega R_D^2}}$$

$$\frac{\mu^2}{R_R^2}\int_0^{2\pi/\omega}\cos^2(\omega(t-\tau))\ dt = \frac{\mu^2}{R_R^2}\left[\frac{t}{2}+\frac{1}{4\omega}\sin(2\omega(t-\tau))\right]\Bigg|_0^{2\pi/\omega}$$

$$= \frac{\mu^2}{R_R^2}\left[\frac{\pi}{\omega}+\frac{1}{4\omega}\left(\sin(4\pi-2\omega\tau)-\sin(-2\omega\tau)\right)\right]$$

$$= \boxed{\frac{\mu^2\pi}{\omega R_R^2}}$$

$$\frac{2\mu}{R_RR_D}\int_0^{2\pi/\omega}\cos(\omega t)\ \cos(\omega(t-\tau))\ dt$$

$$= \frac{2\mu}{R_RR_D}\int_0^{2\pi/\omega}\cos^2(\omega t)\ \cos(\omega\tau)+\sin(\omega t)\ \cos(\omega t)\ \sin(\omega\tau)\ dt$$

$$= \frac{2\mu\cos(\omega\tau)}{R_RR_D}\int_0^{2\pi/\omega}\cos^2(\omega t)\ dt + \frac{2\mu\sin(\omega\tau)}{R_RR_D}\int_0^{2\pi/\omega}\sin(\omega t)\ \cos(\omega t)\ dt$$

$$= \frac{2\mu\cos(\omega\tau)}{R_RR_D}\ \left(\frac{t}{2}+\frac{1}{4\omega}\sin(2\omega t)\right|_0^{2\pi/\omega} + \frac{2\mu\sin(\omega\tau)}{R_RR_D\,2\omega}\left(\sin^2(\omega t)\right)\Bigg|_0^{2\pi/\omega}$$

$$= \frac{2\mu\cos(\omega\tau)}{R_RR_D}\left(\frac{\pi}{\omega}\right) + \frac{2\mu\sin(\omega\tau)}{R_RR_D}(0)$$

$$= \boxed{\frac{2\pi\mu}{\omega R_RR_D}\cos(\omega\tau)}$$

Assembling the answers to the three integrals yields:

$$\langle I\rangle = \frac{\omega P_0^2}{2\pi\rho c}\left[\frac{\pi}{\omega R_D^2} + \frac{\mu^2\pi}{\omega R_R^2} - \frac{2\pi\mu}{\omega R_R R_D}\cos(\omega\tau)\right]$$

$$\langle I\rangle = \frac{P_0^2}{2\rho c}\left[\frac{1}{R_D^2} + \frac{\mu^2}{R_R^2} - \frac{2\mu}{R_R R_D}\cos(\omega\tau)\right]$$

$$\langle I\rangle = \frac{P_0^2}{2\rho c R_D^2}\left[1 + \frac{\mu^2 R_D^2}{R_R^2} - \frac{2\mu R_D}{R_R}\cos(\omega\tau)\right]$$

NOTE:

$$\langle I_0\rangle = \frac{P_0^2}{2\rho c R_D^2}$$

$$\langle I\rangle = \langle I_0\rangle\left[1 + \frac{\mu^2 R_D^2}{R_R^2} - \frac{2\mu R_D}{R_R}\cos(\omega\tau)\right]$$

Finally

$$\boxed{\frac{\langle I\rangle}{\langle I_0\rangle} = 1 + \mu^2\left(\frac{R_D}{R_R}\right)^2 - 2\mu\left(\frac{R_D}{R_R}\right)\cos(\omega\tau)}$$

This result shows the dependence of the intensity at the receiver on the difference in path lengths R_{DIRECT} and $R_{REFLECTED}$ and the time delay, τ, caused by this difference. This intensity ratio is a minimum (corresponding to a null on the Lloyd's mirror interference pattern) when $\cos\omega\tau = 1$ or $\omega\tau$ = integer multiples of 2π.

COMPUTER OUTPUT

A sample of the actual computer output is listed below. Data values are frequency measurements Δf (Hertz) between successive nulls on the Lloyd's mirror pattern (see Figure II-1).

Given a particular source depth and a Δf, the source range can be determined from the corresponding output column. The source depth can be determined from the appropriate output row if the horizontal range to the source from the receiver and the Δf are known.

```
            Group of Ranges From 10 to 70 Meters

    Depth of 10 Meters
      80.6    94.8   114.1   136.0   159.8   184.4   210.1

    Depth of 20 Meters
      41.9    50.2    60.1    71.0    82.7    94.8   107.3

    Depth of 30 Meters
      31.6    37.3    43.8    50.8    58.3    66.1    74.3

    Depth of 40 Meters
      29.2    32.5    36.8    41.7    46.9    52.5    58.3

    Depth of 50 Meters
      28.5    30.6    33.5    36.9    40.8    44.9    49.4
```

COMPUTER PROGRAM

```
10 DEFINT I,J,K,L,M,N
20 DIM NX(16),TEST(136),SNDSPD(201),RNGDIR(180)
30 DIM RNGREF(180),RAYDIR(180),RAYREF(180),FINAL(101)
40 TOL = .05
50 OPEN "R",#1,"FANSWER.FL1",4
60 FIELD #1, 4 AS F1$
100 PRINT CHR$(26): PRINT "SELECT OPERATION":PRINT
110 PRINT 1,"INITIALIZE ANSWER FILE,DO AT THE BEGINNING OF
EACH NEW BT INPUT"
120 PRINT 2,"INPUT BT INFO IN 5 METER INCREMENTS"
130 PRINT 3,"INPUT SEA SURFACE TEMP AND GRADIENT FOR FIRST
200 METERS"
135 PRINT 4,"INPUT SOUND VELOCITY PROFILE IN 5 METER
INCREMENTS"
140 PRINT 5,"CALCULATE LLOYD'S MIRROR HORIZONTAL RANGE .VS.
DEPTH CHART"
150 PRINT 6,"OUTPUT CHART TO PRINTER"
160 PRINT 7,"EXIT PROGRAM"
170 PRINT:PRINT:INPUT "ENTER THE NUMBER OF THE OPERATION
DESIRED   ",NUM
180 IF (NUM<1) OR (NUM>7) THEN PRINT "IMPROPER INPUT":GOTO
100
190 ON NUM GOSUB 1000,2000,5000,5500,6000,9600,10000
200 GOTO 100
1000 REM  INITIALIZE ANSWER FILE
1010 PRINT CHR$(26):PRINT "ARE YOU SURE YOU WANT TO ZERO OUT
THE ANSWER FILE"
1030 INPUT "ENTER Y FOR YES, N FOR NO   ",A$
1040 IF A$<>"Y" THEN RETURN
1050 CLOSE #1
1060 KILL "FANSWER.FL1"
1070 OPEN "R",#1,"FANSWER.FL1",4
1080 FIELD #1, 4 AS F1$
1090 RETURN
2000 REM INPUT BT INFO IN 5 METER INCREMENTS,THEN CALCULATE
SOUND VELOCITY
2010 REM  USING ALAN R. COPPENS EQUATION PRESENTED IN J.
ACOUS. SOC. OF AMER.
2015 REM  VOLUME 69, PAGE 862, 1981 AND PUBLISHED IN
FUNDAMENTALS OF ACOUSTICS
2016 REM   THIRD EDITION, BY KINSLER FREY COPPENS AND
SANDERS
2020 GOSUB 3010: GOSUB 4000:PRINT CHR$(26)
2030 PRINT "ENTER BT INFO IN 5 METER INCREMENTS STARTING
WITH THE SURFACE"
2040 FOR I = 0 TO 200 STEP 5
2050 PRINT:PRINT I "METERS":INPUT "TEMP IN CELSIUS =
",TTEMP:PRINT
2060 PRINT "YOU INPUT "TTEMP" DEGREES CELSIUS":
```

```
    INPUT "IS THIS CORRECT ? ENTER Y FOR YES  N FOR NO
";B$
2070 IF B$<>"Y" THEN PRINT "REENTER TEMP":GOTO 2050
2075 TEMP = TTEMP / 10!
2080 METER = I
2100 DEEP = (16.23 +.253*TEMP)* CLAT * CSNG(I)/1000! +(.213-
.1*TEMP) *
      (CLAT * (CSNG(I)/1000!))^2
2105 DDEEP = (.016 + .0002 * (SALIN-35)) * (SALIN-
35)*TEMP*CLAT*CSNG(I)/1000!
2110 SNDSPD(METER)=1449.05 + 45.7 * TEMP - 5.21 * (TEMP^2) +
.23 * (TEMP^3) +
       (1.333-.126*TEMP+.009*(TEMP^2))*(SALIN-
35)+(DEEP+DDEEP)
2111 IF I = 0 GOTO 2120
2112 SLOW = SNDSPD( I - 5 )
2113 SHI = SNDSPD( I )
2114 FOR J = 1 TO 4
2115 SNDSPD( I - 5 + J ) = SLOW + ( J / 5 ) * ( SHI - SLOW )
2116 NEXT J
2120 NEXT I
2130 RETURN
3000 REM  ENTRY OF OPAREA LATITUDE
3010 PRINT CHR$(26):PRINT "ENTER AVERAGE DEGREES LATITUDE OF
OPAREA"
3020 INPUT "IN WHOLE DEGREES    ",DEG
3030 PRINT:PRINT "YOU ENTERED "DEG" DEGREES, IS THIS CORRECT
?"
3040 INPUT "ENTER Y FOR YES  N FOR NO   ",CC$
3050 IF CC$<>"Y" THEN PRINT "REENTER LATITUDE ":GOTO 3010
3060 CLAT =  1 - .0026 * (COS( DEG * 3.14159 / 90!))
3070 RETURN
4000 REM  ENTRY OF SALINITY VALUE FOR SOUND SPEED
COMPUTATIONS
4010 PRINT CHR$(26)
4020 PRINT "ENTER THE SALINITY VALUE AS PARTS PER THOUSAND
FOR THE OPAREA"
4030 INPUT "IF UNKNOWN, ENTER A VALUE OF 35, SALINITY =
",SALIN
4040 PRINT:PRINT "THE SALINITY LEVEL IS "SALIN" PARTS PER
THOUSAND"
4050 INPUT "IS THIS CORRECT ? ENTER Y FOR YES  N FOR NO
",BB$
4060 IF BB$ <> "Y" GOTO 4010
4070 RETURN
5000 REM ENTER SEA SURFACE TEMP AND AVERAGE GRADIENT FOR
FIRST 200 METERS
5010 PRINT CHR$(26)
5020 INPUT "ENTER THE SEA SURFACE TEMPERATURE IN DEGREES
CELSIUS   ",SST
5030 PRINT:PRINT "YOU ENTERED "SST" DEGREES CELSIUS FOR THE
```

```
SEA SURFACE TEMP"
5040 INPUT "IS THIS CORRECT ?  ENTER Y FOR YES  N FOR NO
",C$
5050 IF C$ <> "Y" THEN PRINT "REENTER SEA SURFACE TEMP":GOTO
5020
5060 PRINT
5070 PRINT "ENTER THE TEMPERATURE GRADIENT (DEG
CELSIUS/METER)"
5075 INPUT "FOR THE FIRST 200 METERS   ",GRAD:PRINT
5080 PRINT "YOU ENTERED A TEMPERATURE GRADIENT OF "GRAD"
DEGREES CELSIUS/METER"
5090 INPUT "IS THIS CORRECT ?  ENTER Y FOR YES  N FOR NO
",D$
5100 IF D$ <> "Y" THEN PRINT "REENTER GRADIENT":GOTO 5070
5110 GOSUB 3010
5120 GOSUB 4000
5125 PRINT CHR$(26):PRINT "PLEASE WAIT, CALCULATING SOUND
VELOCITY VALUES"
5130 FOR I = 0 TO 200
5140 TEMP = (SST + (CSNG(I) * GRAD)) / 10!
5150 DEEP = (16.23 + .253*TEMP)* CLAT * CSNG(I) /1000! +
(.213-.1*TEMP)
* (CLAT * (CSNG(I) /1000! ))^2
5155 DDEEP = (.016 + .0002 * (SALIN - 35)) * (SALIN-35) *
TEMP * CLAT *
 CSNG(I) / 1000!
5170  SNDSPD(I) = 1449.05 + 45.7 * TEMP - 5.21 * (TEMP^2) +
.23 * (TEMP^3 )
     +(1.333  - .126  *  TEMP  +  .009*TEMP^2)*(SALIN  -
35)+DEEP+DDEEP
5180 NEXT I
5190 RETURN
5510 REM ENTRY OF SOUND VELOCITY PROFILE
5525 PRINT CHR$(26)
5530 PRINT "ENTER SOUND VELOCITY PROFILE IN 5 METER
INCREMENTS "
5540 PRINT "STARTING WITH THE SURFACE SOUND VELOCITY IN
METERS/SECOND"
5550 FOR II = 0 TO 200 STEP 5
5560 PRINT:PRINT II "METERS" :INPUT " SOUND
VELOCITY(METERS/SEC)=  ",SVP
5570 PRINT:PRINT "YOU INPUT "SVP" METERS/SEC"
5580 INPUT "IS THIS CORRECT ? ENTER Y FOR YES N FOR NO
";SS$
5590 IF SS$<>"Y" THEN PRINT "REENTER SOUND VELOCITY":GOTO
5560
5610 SNDSPD(II) = SVP
5611 IF II = 0 GOTO 5620
5612 SLOW = SNDSPD( II - 5 )
5613 SHI = SNDSPD( II )
5614 FOR JJ = 1 TO 4
```

```
5615 SNDSPD( II - 5 + JJ ) = SLOW + ( JJ / 5 ) * ( SHI -
SLOW )
5616 NEXT JJ
5620 NEXT II
5630 RETURN
6000 REM CALCULATE LLOYD'S MIRROR HORIZONTAL RANGE .VS.
DEPTH CHART
6005 PRINT:PRINT:PRINT "WILL HYDROPHONE DEPTH BE INPUT IN
FEET OR METERS ?"
6006 INPUT "ENTER F FOR FEET OR M FOR METERS    ", AHYD$
6007 IF AHYD$ = "F" GOTO 6010 ELSE IF AHYD$ = "M" GOTO 6052
6008 PRINT:PRINT:PRINT "INPUT ERROR" :GOTO 6005
6010 PRINT CHR$(26):INPUT "ENTER HYDROPHONE DEPTH IN FEET
",HYDEEP
6020 PRINT:PRINT "YOU ENTERED A HYDROPHONE DEPTH OF "HYDEEP"
FEET"
6030 INPUT "IS THIS CORRECT ? ENTER Y FOR YES  N FOR NO
",AA$
6040 IF AA$ <> "Y" GOTO 6010
6050 HYDPTH = (HYDEEP * .3048)
6051 GOTO 6060
6052 PRINT CHR$(26):INPUT "ENTER HYDROPHONE DEPTH IN METERS
",HYDPTH
6053 PRINT:PRINT "YOU ENTERED A HYDROPHONE DEPTH OF
"HYDPTH" METERS"
6054 INPUT " IS THIS CORRECT  ?  ENTER Y FOR YES N FOR NO
",BHY$
6055 IF BHY$ <> "Y" GOTO 6052
6060 HYSPD = SNDSPD( HYDPTH )
6170 FOR NUPLMT = 10 TO 200 STEP 10
6175 PRINT "NUPLMT=" NUPLMT
6180 TGTDPTH = CSNG( NUPLMT )
6185 IF HYDPTH = TGTDPTH THEN HYDPTH = HYDPTH - 3
6190 IF TGTDPTH > HYDPTH GOTO 6230
6200 BOUNDLO = TGTDPTH
6210 BOUNDHI = HYDPTH
6220 GOTO 6250
6230 BOUNDLO = HYDPTH
6240 BOUNDHI = TGTDPTH
6250 REFLO = -(BOUNDLO)
6360 FOR NSTEP = 3 TO 177 STEP 3
6365 PRINT "NSTEP=" NSTEP
6370 RAD = CSNG(NSTEP) * 3.14159 / 180!
6380 MTRSTEP = TGTDPTH
6390 SOUND = SNDSPD(MTRSTEP)
6400 MTRHI = BOUNDHI
6410 SNDHI = SNDSPD(MTRHI)
6420 MTRLO = BOUNDLO
6430 SNDLO = SNDSPD(MTRLO)
6440 SNELL = SIN( RAD ) / SOUND
6480 FOR IFLAG = 1 TO 4
```

```
7000 REM ROMBERG NUMERICAL INTEGRATION SUBROUTINE
7015 START = 0!
7020 PIECES = 1!
7030 NX(1) = 1
7040 IF (IFLAG=2) OR (IFLAG=4) THEN BNDLOW=REFLO ELSE
BNDLOW=BOUNDLO
7041 IF (IFLAG = 2) OR (IFLAG = 4) GOTO 7045
7042 IF (TGTDPTH > HYDPTH) AND (NSTEP > 90) GOTO 8105
7043 IF (TGTDPTH < HYDPTH) AND (NSTEP < 90) GOTO 8105
7044 GOTO 7050
7045 IF NSTEP > 90 GOTO 8105
7050 DELTA = (BOUNDHI - BNDLOW) / PIECES
7051 DENOM1 = (SNELL)^2 * (SNDLO)^2
7052 DENOM2 = (SNELL)^2 * (SNDHI)^2
7053 IF DENOM1 = 1! THEN DENOM1 = .999 :IF DENOM2 = 1! THEN
DENOM2 = .999
7054 IF ( DENOM1 > 1! ) OR ( DENOM2 > 1! ) GOTO 8105
7060 IF (IFLAG=1) OR (IFLAG=2) GOTO 7085
7065 FOR IN = 1 TO 2
7068 IF IN = 1 THEN DENOM = DENOM1 ELSE DENOM = DENOM2
7070 START = START + ((1!)/ SQR(1! - DENOM))
7075 NEXT IN
7080 GOTO 7100
7085 FOR INN = 1 TO 2
7088 IF INN = 1 THEN DENOM = DENOM1 ELSE DENOM = DENOM2
7089 IF INN = 1 THEN SPEED = SNDLO ELSE SPEED = SNDHI
7090 START = START + ((SNELL * SPEED) / SQR(1! - DENOM))
7095 NEXT INN
7100 TOT = START / 2!
7110 TEST(1) = DELTA * START / 2!
7120 N = 1
7130 NN = 2
7140 N = N + 1
7150 WALK = 4!
7160 NX(N) = NN
7170 MSTEP = CINT( PIECES )
7180 PIECES = PIECES * 2!
7190 DELTA = (BOUNDHI - BNDLOW) / PIECES
7500 REM    COMPUTE TRAPEZOIDAL SUM
7510 FOR INCR = 1 TO MSTEP
7520 PART = CSNG(INCR) * 2! - 1!
7530 X = BNDLOW + (DELTA * PART)
7540 MTRS = CINT ( ABS (X) )
7545 IF MTRS > 200 THEN MTRS = 200
7550 SPEED = SNDSPD( MTRS )
7552 DENOM3 = (SNELL)^2 * (SPEED)^2
7554 IF DENOM3 = 1! THEN DENOM3 = .999
7555 IF DENOM3 > 1! GOTO 8105
7570 IF (IFLAG=1) OR (IFLAG=2) GOTO 7600
7580 TOT = TOT + ( (1!) / SQR(1! - DENOM3))
7590 GOTO 7610
```

```
7600 TOT = TOT + ( (SNELL * SPEED) / SQR(1! - DENOM3))
7610 NEXT INCR
7620 TEST( NN ) = TOT * DELTA
7630 K = N - 1
7640 NTRA = NX( K )
7650 FOR M = 1 TO K
7660 J = NN + M
7670 NT = NTRA + M - 1
7680 TEST(J) = ( TEST(J-1) * WALK - TEST(NT)) / (WALK-1!)
7690 WALK = WALK * 4!
7700 NEXT M
7705 IF (IFLAG = 1 OR IFLAG = 2) AND (TEST(J) > 3000!) GOTO
8105
7710 IF N < 5 GOTO 7760
7720 IF TEST( NN + 1 ) = 0! GOTO 7750
7730 IF ABS(TEST(NTRA+1)-TEST(NN + 1)) <= (ABS(TEST(NN +
1)*TOL)) GOTO 8000
7740 IF ABS( TEST(NN-1) - TEST(J)) <= (ABS( TEST(J) * TOL))
GOTO 8000
7750 IF N > 15 GOTO 9500
7760 NN = J + 1
7770 GOTO 7140
8000 ANS = TEST(J)
8030 ON IFLAG GOTO 8040,8060,8080,8100
8040 RNGDIR(NSTEP) = ANS
8050 GOTO 8105
8060 RNGREF(NSTEP) = ANS
8070 GOTO 8105
8080 RAYDIR(NSTEP) = ANS
8090 GOTO 8105
8100 RAYREF(NSTEP) = ANS
8105 NEXT IFLAG
8110 NEXT NSTEP
8115 REM  USE LINEAR INTERPOLATION TO CALCULATE ANSWER CHART
VALUES IN RANGE
8117 REM       AND DEPTH INCREMENTS OF 10 METERS
8120 FOR NMTR = 1 TO 100
8130 RMTR = CSNG( NMTR ) * 10!
8135 DIRRAY = 0! : REFRAY = 0!
8140 AFLAG = 0!
8150 BFLAG = 0!
8160 IF TGTDPTH < HYDPTH GOTO 8179
8161 FOR NANGL = 3 TO 87 STEP 3
8162 IF RNGDIR( NANGL ) <= RMTR GOTO 8171
8163 DLOW = RNGDIR( NANGL - 3 ):DHI = RNGDIR( NANGL )
8164 IF DLOW = DHI THEN DIRFRAC = 0 :GOTO 8166
8165 DIRFRAC = (RMTR - DLOW) / (DHI - DLOW)
8166 IF DLOW > 0 GOTO 8168
8167 DSHORT = BOUNDHI - BOUNDLO :GOTO 8169
8168 DSHORT = RAYDIR( NANGL - 3 )
8169 DLONG = RAYDIR( NANGL )
```

```
8170 DIRRAY = DSHORT + ( DLONG - DSHORT ) * DIRFRAC : GOTO
8210
8171 IF ( NANGL = 87 ) AND ( RNGDIR( 87 ) > 0 ) GOTO 8172
ELSE GOTO 8177
8172 DLOW = RNGDIR( 87 )
8173 IF DLOW = RMTR THEN DIRRAY = RAYDIR( 87 ) : GOTO 8210
8174 DIRFRAC  = ( RMTR - DLOW ) / ( 1000 - DLOW )
8175 DSHORT = RAYDIR( NANGL ) : CLONG = SQR( 1000^2 +
(BOUNDHI - BOUNDLO)^2 )
8176 DIRRAY = DSHORT + ( CLONG - DSHORT ) * DIRFRAC : GOTO
8210
8177 NEXT NANGL
8178 GOTO 8410
8179 FOR MANGL = 93 TO 177 STEP 3
8180 IF RMTR >= RNGDIR( MANGL ) GOTO 8181 ELSE GOTO 8195
8181 IF ( MANGL = 93 ) AND ( RNGDIR( 93 ) > 0 ) GOTO 8182
ELSE GOTO 8186
8182 DLOW = RNGDIR( 93 )
8183 IF DLOW = RMTR THEN DIRRAY = RAYDIR( 93 ) : GOTO 8210
8184 DIRFRAC = ( RMTR - DLOW ) / ( 1000 - DLOW ) : DSHORT =
RAYDIR( 93 ) :
     BLONG = SQR ( 1000^2 + ( BOUNDHI - BOUNDLO )^2 )
8185 DIRRAY= DSHORT + ( BLONG - DSHORT ) * DIRFRAC : GOTO
8210
8186 DLOW = RNGDIR( MANGL ) :DHI = RNGDIR( MANGL - 3 )
8187 IF ( DLOW <= RMTR ) AND ( DHI >= RMTR ) GOTO 8188 ELSE
GOTO 8195
8188 IF DLOW = DHI THEN DIRFRAC = 0 : GOTO 8190
8189 DIRFRAC = ( RMTR - DLOW ) / ( DHI - DLOW )
8190 IF DLOW > 0 GOTO 8192
8191 DSHORT = BOUNDHI - BOUNDLO : GOTO 8193
8192 DSHORT = RAYDIR( MANGL ): DLONG = RAYDIR( MANGL - 3)
8194 DIRRAY = DSHORT + ( DLONG - DSHORT ) * DIRFRAC : GOTO
8210
8195 IF ( MANGL <> 177 ) GOTO 8199
8196 DLOW = 0 : DHI = RNGDIR( 177 ) : DIRFRAC = RMTR / DHI
8197 DSHORT = BOUNDHI - BOUNDLO : DLONG = RAYDIR( 177 )
8198 DIRRAY = DSHORT + ( DLONG - DSHORT ) * DIRFRAC : GOTO
8210
8199 NEXT MANGL
8200 GOTO 8410
8205 IF DHI = DLOW THEN DIRFRAC = 0 : GOTO 8211
8210 FOR NREF = 3 TO 87 STEP 3
8211 IF RNGREF( NREF ) <= RMTR GOTO 8220
8212 RLOW = RNGREF( NREF - 3 ) : RHI = RNGREF( NREF )
8213 IF RLOW = RHI THEN REFFRAC = 0 : GOTO 8215
8214 REFFRAC = ( RMTR - RLOW ) / ( RHI - RLOW )
8215 IF RLOW > 0 GOTO 8217
8216 RSHORT = BOUNDHI + BOUNDLO : GOTO 8218
8217 RSHORT = RAYREF( NREF - 3 ): RLONG = RAYREF(NREF)
8219 REFRAY = RSHORT + ( RLONG - RSHORT ) * REFFRAC :GOTO
```

```
8400
8220 IF ( NREF = 87 ) AND (RNGREF(87) > 0 ) GOTO 8221 ELSE
GOTO 8226
8221 RLOW = RNGREF( 87 )
8222 IF RLOW = RMTR THEN REFRAY = RAYREF( 87 ): GOTO 8400
8223 REFFRAC = ( RMTR - RLOW ) / ( 1000 - RLOW )
8224 RSHORT = RAYREF( NREF ) : ALONG = SQR ( 1000^2 + (
BOUNDHI + BOUNDLO )^2 )
8225 REFRAY = RSHORT +  ( ALONG - RSHORT ) * REFFRAC : GOTO
8400
8226 NEXT NREF
8227 GOTO 8410
8370 REM  OUTPUT FINAL ANSWERS FOR EACH DEPTH AND RANGE TO
DISK FILE
8385 LPRINT "HYSPD=" HYSPD
8400 FINAL( NMTR ) = HYSPD / ( REFRAY - DIRRAY )
8410 NEXT NMTR
8420 MULT = (TGTDPTH ± 10) - 1
8430 FOR JMP = 1 TO 100
8440 LSET F1$ = MKS$(FINAL(JMP))
8450 RECNUM% = JMP + (MULT * 100)
8460 PUT #1, RECNUM%
8470 NEXT JMP
8480 FOR NUL = 0 TO 100
8490 FINAL(NUL) = 0!
8500 NEXT NUL
8510 FOR NUL = 0 TO 135
8520 TEST(NUL) = 0!
8530 NEXT NUL
8540 FOR NUL = 0 TO 15
8550 NX(15) = 0!
8560 NEXT NUL
8570 FOR NUL = 0 TO 179
8580RNGDIR(NUL)=0!:RNGREF(NUL)=0:RAYDIR(NUL)=0:
     RAYREF(NUL)=0
8620 NEXT NUL
9000 NEXT NUPLMT
9010 RETURN
9500 PRINT:PRINT:PRINT "INPUT DATA HAS CAUSED AN ERROR"
9510 GOTO 100
9600 REM PRINT OUT GRAPH OF HORIZONTAL RANGE .vs. DEPTH
9610 PRINT CHR$(26)
9620 PRINT:PRINT:PRINT " SELECT OUTPUT DEVICE ": PRINT
9630 PRINT 1," VIDIO SCREEN ": PRINT
9640 PRINT 2," PRINTER ": PRINT
9650 INPUT " ENTER THE NUMBER FOR OUTPUT DEVICE DESIRED
",NUMB
9660 IF ( NUMB < 1 OR NUMB > 2 ) THEN PRINT "IMPROPER INPUT
":GOTO 9620
9665 PRINT:PRINT
9670 PRINT "   THE OUTPUT MATRIX IS PRESENTED IN 10 METER
```

```
INCREMENTS FOR BOTH "
9680 PRINT " HORIZONTAL RANGES AND FOR DEPTHS OF 10 METERS
TO 200 METERS. "
9690 PRINT "  HORIZONTAL RANGES FROM THE RECEIVER TO THE
TARGET , IN "
9700 PRINT " GROUPS OF 100 METERS, INCREASE FROM LEFT TO
RIGHT, "
9710 PRINT " ie. 10 TO 100 METER RANGES WILL BE DISPLAYED
FIRST, FOLLOWED"
9720 PRINT " BY 110 TO 200 METERS AND SO ON UP TO 1000
METERS."
9730 PRINT "  DEPTHS OF 0 TO 200 METERS WILL BE DISPLAYED
FOR EACH GROUP OF "
9740 PRINT " RANGES, WITH THE FIRST ROW REPRESENTING 10
METERS, THE "
9750 PRINT " NEXT ROW 20 METERS, ON DOWN THE PAGE UNTIL ROW
20 IS "
9760 PRINT " PRINTED FOR A DEPTH OF 200 METERS.":PRINT:PRINT
9770 FOR I = 0 TO 9
9780 THIS = 10 + 100 * I
9790 THAT = 100 + 100 * I
9800 IF NUMB = 1 THEN PRINT:
     PRINT "GROUP OF RANGES FROM "THIS" TO
"THAT"METERS":GOTO 9820
9805 LPRINT:LPRINT:
9810 LPRINT:LPRINT "GROUP OF RANGES FROM "THIS" TO "THAT"
METERS"
9820 FOR J = 0 TO 19
9830 DPH = 10 + 10 * J
9840 IF NUMB = 1 THEN PRINT:PRINT "DEPTH OF "DPH"
METERS":GOTO 9860
9850 LPRINT:LPRINT "DEPTH OF "DPH" METERS"
9860 FOR K = 1 TO 10
9870 RECNUM% = K + 100 * J + 10 * I
9880 GET #1,RECNUM%
9890 AOUT = CVS( F1$ )
9900 IF NUMB = 2 GOTO 9930
9910 PRINT USING "####.#  ";AOUT;
9920 GOTO 9940
9930 LPRINT USING "####.#  ";AOUT;
9940 NEXT K
9950 NEXT J
9960 NEXT I
9965 LPRINT
9970 RETURN
10000 REM CLOSE ALL FILES AND EXIT PROGRAM
10010 PRINT CHR$(26):PRINT "DO YOU REALLY WANT TO QUIT"
10020 INPUT "ENTER A Y FOR YES, AN N FOR NO   ",Z$
10030 IF Z$ = "Y" THEN END ELSE RETURN
```

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