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ADAPTIVE ARRAYS FOR MULTIPLE SIMULTANEOUS DESIRED SIGNALS

The Ohio State University

Inder J. Gupta

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I. INTRODUCTION

Adaptive arrays for multiple simultaneous desired signals for conferencing, for example, are the subject of current research. Martin [1] has described an adaptive array algorithm for multiple simultaneous desired signals. The algorithm is a modification of the well known LMS adaptive array [2] and requires a priori information about the signal or the modulation method. In this report, steered beam adaptive arrays for multiple desired signals which do not require such a priori information are discussed. To facilitate the reception of these signals, the quiescent pattern of the array (array pattern in the absence of all signals) is assumed to be a multiple beam pattern with independent beams in each signal direction. It is shown that one should weight the different beams in accordance with their signal strengths. If one is interested in improving the signal-to-noise ratio (SNR) of weak signals at the expense of the strong ones, all beams should be given equal weights. The same is true when the range of the signal strengths is small. On the other hand, if one is interested in improving the SNR of strong signals and the range of desired signal strengths is large, the different beams should be weighted according to their signal strengths. If the range of desired signal strengths is large and one is interested in improving the SNR of all signals (the weak as well as the strong), appropriate weighting coefficients are given which require some information regarding the signal strengths. It is further shown that

all choices of the weighting coefficients lead to the rejection of strong jammers as long as the jammer is not in the immediate angular vicinity of desired signals.

In Section II, the problem of multiple simultaneous desired signals is formulated and expressions for the adapted pattern of the array and the improvement in the SNR of desired signals are given. It is shown that the two quantities depend upon the choice of the weighting coefficients for different beams. In Section III, various choices of weighting coefficients and their effect on the adaptive array performance are discussed. Interference rejection capability of steered beam adaptive arrays for multiple simultaneous desired signals is addressed in Section IV. Section V contains conclusions.

II. FORMULATION OF THE PROBLEM

The steady state weight vector that maximizes the output signalto-interference-plus-noise ratio (SINR) of an adaptive array in the presence of a single CW desired signal and multiple narrowband jammers is given by [3]

$$W = K \phi^{-1} U_{d}^{\star}$$
 (1)

where K is a constant, Φ is the covariance matrix of the signals incident on the array elements (including thermal noise), U_d is the desired signal vector and superscript * denotes complex conjugate. A feedback loop which leads to the same steady weights as given by Equation (1) is shown in Figure 1. This kind of feedback loop was first





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proposed by Applebaum [4]. For the maximum output SINR, the steering weight U_{si} should be equal to U_{di}^{*} . With this choice of steering weights, the quiescent pattern of the array (pattern in the absence of all signals) will have its main beam in the desired signal direction. In general, the steady state weight vector of the array will be given by

 $W = \phi^{-1} U_{e}$ (2)

where \boldsymbol{U}_{e} is the steering vector.

If the array is to be used in a multiple desired signal environment, the steering vector should be chosen such that the quiescent pattern of the array has major lobes in all the desired signal directions. Therefore, for multiple desired signals, let

$$U_{s} = \sum_{\ell=1}^{m} a_{\ell} U_{d\ell}^{*}$$
(3)

where $U_{d\ell}$ is the ℓ^{th} desired signal vector, a_{ℓ} is a constant to be defined and m is the total number of incident desired signals. Note that for $a_1 = a_2 = \dots = a_m$, the quiescent pattern of the array is a multiple beam pattern with an independent beam in each desired signal direction. Using Equation (3), the steady state weights for the feedback loop shown in Figure 1 will be

$$W = \phi^{-1} \sum_{\ell=1}^{m} a_{\ell} U_{d\ell}^{\star} .$$
 (4)

In the presence of m narrowband desired signals and in the absence of jammers, the covariance matrix, Φ , is given by

$$\Phi = \sigma^2 \left(I + \sum_{\ell=1}^{m} \varepsilon_{d\ell} U_{d\ell}^* U_{d\ell}^T \right)$$
(5)

where σ^2 is the thermal noise power added to each antenna element, I is an identity matrix, ξ_{dl} is the ratio of the l^{th} desired signal power to the thermal noise power and superscript T denotes transpose. In Equation (5), the thermal noise present in the array elements is assumed to be uncorrelated with each other and with desired signals. The desired signals are assumed to be narrowband and uncorrelated with each other. Let us further assume that the desired signals are incident from orthogonal directions to the array, i.e.,

$$U_{d\ell}^{T} U_{dk}^{*} = 0$$
 (6)
 $\ell \neq k$.

The above assumption is made to simplify the mathematics given below but it does not affect the results and conclusions.

One needs the inverse of the covariance matrix [Equation (5)] to compute the steady state weights [Equation (4)]. Using Equation (6), the inverse of the covariance matrix is given by

$$\Phi^{-1} = \frac{1}{\sigma^2} \begin{bmatrix} I - \sum_{\ell=1}^{m} & \frac{U_{d\ell}^{\dagger} U_{d\ell}^{\dagger}}{\xi_{d\ell}^{-1} + U_{d\ell}^{\dagger} U_{d\ell}^{\dagger}} \end{bmatrix} .$$
(7)

Substituting Equation (7) into Equation (4) and making use of the orthogonality [Equation (6)] one gets,

$$W = \frac{1}{\sigma^2} \sum_{\ell=1}^{m} a_{\ell} (1 + \xi_{d\ell} U_{d\ell}^{\mathsf{T}} U_{d\ell}^{\star})^{-1} U_{d\ell}^{\star} . \qquad (8)$$

Using the weights given by Equation (8), one can compute the adapted pattern and the output SNR for different signals. The adapted pattern of the array is given by

$$F(\theta,\phi,p) = U^{\mathsf{T}}W = \frac{1}{\sigma^2} \sum_{\ell=1}^{m} a_{\ell}(1 + \xi_{d\ell} U^{\mathsf{T}}_{d\ell} U^{\star}_{d\ell})^{-1} U^{\mathsf{T}} U^{\star}_{d\ell}$$
(9)

where U is the signal vector of the array for a signal incident from direction (θ, ϕ) and of polarization p. The output SNR of the array for the kth desired signal is

$$(SNR)_{k} = \frac{P_{dk}}{P_{n}}$$
(10)

where P_{dk} is the output power due to the kth desired signal,

$$P_{dk} = \frac{A_{dk}^2}{2} |W^T U_{dk}^{\star}|^2 , \qquad (11)$$

 P_n is the output thermal noise

$$P_n = \frac{\sigma^2}{2} |W^T W^*|$$
 (12)

and A_{dk}^2 is the kth desired signal power at the input, such that

$$\xi_{\rm dk} = A_{\rm dk}^2 / \sigma^2 \qquad (13)$$

Using Equation (8) in Equations (11) and (12)

$$P_{dk} = |a_{k}|^{2} \frac{A_{dk}^{2}}{2\sigma^{4}} \frac{|U_{dk}^{T} U_{dk}^{*}|^{2}}{(1 + \xi_{dk} U_{dk}^{T} U_{dk}^{*})^{2}}$$
(14)

and

$$P_{n} = \frac{1}{2\sigma^{2}} \sum_{\ell=1}^{m} |a_{\ell}|^{2} \frac{|U_{d\ell}^{T} U_{d\ell}^{*}|}{(1 + \xi_{d\ell} U_{d\ell}^{T} U_{d\ell}^{*})^{2}}$$
(15)

Thus, the output SNR of the kth desired signal [Equation (10)] is

$$(SNR)_{k} = \frac{\xi_{dk} |a_{k}|^{2} |U_{dk}^{T} U_{dk}^{*}|^{2} / (1 + \xi_{dk} U_{dk}^{T} U_{dk}^{*})^{2}}{\sum_{\ell=1}^{m} |a_{\ell}|^{2} |U_{d\ell}^{T} U_{d\ell}^{*}| / (1 + \xi_{d\ell} U_{d\ell}^{T} U_{d\ell}^{*})^{2}}$$
(16)

Or, the improvement $I_{\rm k}$ (in dB) of the SNR of the $k^{\rm th}$ desired signal by virtue of the array is

$$I_{k} = 10 \ \log_{10} \frac{(SNR)_{k}}{\xi_{dk}}$$

= 10 \log_{10} \frac{|a_{k}|^{2} |U_{dk}^{T} U_{dk}^{*}|^{2} / (1 + \xi_{dk} U_{dk}^{T} U_{dk}^{*})^{2}}{\sum_{\ell=1}^{m} |a_{\ell}|^{2} |U_{d\ell}^{T} U_{d\ell}^{*}| / (1 + \xi_{d\ell} U_{d\ell}^{T} U_{d\ell}^{*})^{2}} . (17)

From Equations (9) and (16), it is clear that the adapted pattern and the output SNR of the array depend on the choice of a_{ℓ} . In the next section, various choices of a_{ℓ} and their effect on the performance of adaptive arrays will be discussed.

III. DETERMINATION OF THE COEFFICIENTS ag

First consider the case where all the coefficients are the same, i.e.,

$$a_1 = a_2 = \cdots = a_m \qquad (18)$$

For this choice of the a_{ℓ} , the adapted pattern [Equation (9)] and the improvement in the SNR of the kth desired signal [Equation (17)] are, respectively,

$$F(\theta,\phi,p) = \frac{1}{\sigma^2} \sum_{\ell=1}^{m} \frac{U^{\mathsf{T}} U^{\mathsf{T}}_{d\ell}}{(1 + \xi_{d\ell} U^{\mathsf{T}}_{d\ell} U^{\mathsf{T}}_{d\ell})}$$
(19)

and

The second

$$I_{k} = 10 \log_{10} \frac{|U_{dk}^{T} U_{dk}^{*}|^{2}/(1 + \xi_{dk} U_{dk}^{T} U_{dk}^{*})^{2}}{\sum_{\ell=1}^{m} |U_{d\ell}^{T} U_{d\ell}^{*}|/(1 + \xi_{d\ell} U_{d\ell}^{T} U_{d\ell}^{*})^{2}}$$
(20)

Or,

$$I_{k} = 10 \log_{10} \frac{|U_{dk}^{T} U_{dk}^{*}|^{2}}{D(1 + \xi_{dk} U_{dk}^{T} U_{dk}^{*})^{2}}$$
(21)

where

$$D = \sum_{\ell=1}^{m} \frac{|U_{d\ell}^{T} U_{d\ell}^{*}|}{(1 + \xi_{d\ell} U_{d\ell}^{T} U_{d\ell}^{*})^{2}}$$
(22)

First, let us consider the adapted pattern of the array [Equation (19)]. A factor $(1 + \xi_{dl} U_{dl}^{\dagger} U_{dl}^{\star})$ appears in the denominator of the adapted pattern expression. The stronger the desired signal, the larger the factor will be. This in turn will result in more suppression of the beam in the direction of the desired signal. Thus, the choice of identical a_{l} will suppress the lobes in strong signal directions more than in weak signal directions. Figure 2 shows the adapted pattern of a linear array of ten isotropic elements (Figure 3) for a signal environment consisting of two desired signals. One of the signals is incident from $\theta = 60^{\circ}$ and has an input SNR (ξ_d) of 10 dB. The other signal is incident from $\theta = 120^{\circ}$ and has an input SNR of 0 dB. The two signals are assumed to be of the same frequency. There is no jammer







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present and the spacing between the array elements is half a wavelength. Note that the lobe in the stronger signal direction, i.e., $\theta = 60^{\circ}$, has been suppressed while the array maintains the lobe along $\theta = 120^{\circ}$.

Next, consider the improvement in the SNR [Equation (21)]. For a given signal environment, the factor D [Equation (22)] is fixed and is the same for all signals. Another factor $(1 + \xi_{dk} U_{dk}^T U_{dk}^*)^2$ appears in the denominator of the expression. The stronger the signal, the larger will be this factor and thus, the improvement in the SNR of a strong signal will be less than that for a weak signal. In fact, the SNR of strong signals may be degraded. But, if all the signals are very weak, i.e.,

then

$$(1 + \xi_{d\ell} \cup_{d\ell}^{T} \cup_{d\ell}^{*}) = 1$$

 $\ell = 1, 2, ..., m$
(24)

and from Equation (21)

$$I_{k} = 10 \log_{10} \frac{|U_{dk}^{T} U_{dk}^{*}|^{2}}{D}$$
(25)

where

$$D = \sum_{\ell=1}^{m} |U_{d\ell}^{\mathsf{T}} U_{d\ell}^{\star}| \qquad (26)$$

Again, D is the same for all signals and thus the improvement in the output SNR will be the same, provided the array has similar radiation characteristics for all desired signals. Next, if all the signals are of the same strength, or they vary in signal strength over a limited range, i.e.,

$$\xi_{d1} U_{d1}^{\mathsf{T}} U_{d1}^{\star} = \xi_{d2} U_{d2}^{\mathsf{T}} U_{d2}^{\star} = \cdots = \xi_{dm} U_{dm}^{\mathsf{T}} U_{dm}^{\star}$$
(27)

then, from Equation (21)

$$I_{k} = 10 \log_{10} \frac{|U_{dk}^{T} U_{dk}^{*}|^{2}}{D}$$
(28)

where

$$D = \sum_{\ell=1}^{m} |U_{d\ell}^{\mathsf{T}} U_{d\ell}^{\star}| \qquad (29)$$

Comparing Equations (25) and (28), one notes that the two are the same. The improvement in the SNR of the various desired signals will, therefore, be independent of their input SNR's. Thus, the choice of a_{ℓ} as given by Equation (18) is appropriate when one is interested in weak signals, or the variation of the desired signal strengths is limited.

Figure 4 shows the output SNR of the linear array of Figure 3 for an environment consisting of two desired signals. The input SNR of the signal incident at $\theta = 120^{\circ}$ is varied between -30 dB and 30 dB while the rest of the parameters are the same as in Figure 2. The output SNR of



Figure 4. Output SNR of the two signals vs ξ_{d2} . $\theta_{d1} = 60^{\circ}$, $\theta_{d2} = 120^{\circ}$, $\xi_{d1} = 10 \, dB$, $a_1 = a_2 = 1$. No jammer.

the two signals is plotted as a function of the input SNR of the signal incident from $\theta = 120^{\circ}$. Note that the SNR of the weaker signal has improved, while the SNR of the stronger signal has degraded except when the two have approximately the same input SNR's. The larger the difference between the two signal strengths, the more pronounced the suppression of the stronger signal. The choice of equal a_{ℓ} is, therefore, appropriate when one is interested in relatively weak signals or signal strengths vary little.

The next case we consider is the following:

$$a_{\underline{\ell}} = \xi_{\underline{d}\underline{\ell}} \quad (30)$$

Note that this choice of a_{ℓ} requires a prior knowledge of signal strengths. In practice, this information is obtainable since while estimating the angles of arrival of the desired signals one can estimate the signal strengths also [5]. Using Equation (30), the adapted pattern [Equation (9)] and the improvement in the SNR of the kth desired signal [Equation (17)] will be given by

$$F(\theta,\phi,p) = \frac{1}{\sigma^2} \sum_{\ell=1}^{m} \frac{\xi_{d\ell}}{(1+\xi_{d\ell} \cup_{d\ell}^{T} \cup_{d\ell}^{\star})} U^{T} \cup_{d\ell}^{\star}$$
(31)

and

$$I_{k} = 10 \log_{10} \frac{\xi_{dk}^{2} |U_{dk}^{T} |U_{dk}^{*}|^{2}}{(1 + \xi_{dk} |U_{dk}^{T} |U_{dk}^{*}|^{2} \cdot n)}$$
(32)

where

$$D = \sum_{\ell=1}^{m} \frac{\xi_{d\ell}^2 |U_{d\ell}^T U_{d\ell}^*|}{(1 + \xi_{d\ell}^2 U_{d\ell}^T U_{d\ell}^*)^2}$$
(33)

Consider the adapted pattern first. For a weak signal [Equation (23)], the denominator in the adapted pattern expression [Equation (31)] is unity and thus the lobe in the signal direction depends on the signal strength. The weaker the signal the smaller the lobe. But for strong signals, i.e.,

$$(1 + \xi_{d\ell} U_{d\ell}^{T} U_{d\ell}^{*}) > 10$$
, (34)

$$\frac{\xi_{d\ell}}{(1 + \xi_{d\ell} \cup_{d\ell}^{\dagger} \cup_{d\ell}^{\star})} \stackrel{\simeq}{=} \frac{1}{\bigcup_{d\ell}^{\dagger} \cup_{d\ell}^{\star}}$$
(35)

the lobe in the stronger signal direction will be independent of the signal strength. The array, therefore, will maintain its lobes in the stronger signal directions while the lobes in the weaker signal directions may be attenuated.

Next, consider the improvement in the SNR of the kth desired signal. For weak signals [Equation (23)], the improvement [Equation (32)] is given by

$$I_{k} = 10 \log_{10} \frac{\xi_{dk}^{2} |U_{dk}^{T} U_{dk}^{*}|^{2}}{D}$$
 (36)

Again, for a given signal environment, D [Equation (33)] is the same for all signals and thus the improvement in SNR of a weak signal depends upon its input SNR. The smaller the input SNR the smaller the improvement. In fact, the SNR of weak signals may be degraded. For strong signals [Equation (34)], Equation (32) yields

$$I_{k} \cong 10 \log_{10} \frac{1}{D}$$
 (37)

and the improvement in the SNR is independent of the input SNR. The SNR of all the strong signals will be improved by the same amount. This choice of a_{ℓ} is, therefore, suitable when one is interested only in strong signals or when all desired signals are relatively strong.

Figure 5 shows the output SNR of the linear array for the same environment as in Figure 2. The a_{1} are chosen according to Equation (30) and the output SNR's of the two signals are plotted as a function of the input SNR of the signal incident from $\theta = 120^{\circ}$. Note that the output SNR of the signal incident at $\theta = 60^{\circ}$ is almost independent of the input SNR of the signal incident at $\theta = 120^{\circ}$. For $\xi_{d2} > 0$ dB, the SNR of the signal incident at $\theta = 120^{\circ}$. For $\xi_{d2} > 0$ dB, the SNR of the signal incident at $\theta = 120^{\circ}$ has improved by 7 dR. For $\xi_{d2} < -10$ dB, there is hardly any improvement in the SNR of the signal. In fact, the SNR has degraded.

The last case involves a choice of a_{ℓ} in accordance with the following equation

$$a_{\ell} = (1 + \xi_{d\ell} \cup_{d\ell}^{T} \cup_{d\ell}^{\star})$$

$$\ell = 1, 2, \dots m$$
(38)



Figure 5. Output SNR of the two signals vs. ξ_{d2} . $\theta_{d1}=60^{\circ}$, $\theta_{d2}=120^{\circ}$, $\xi_{d1}=10$ dB, $a_1=10$, $a_2=\xi_{d2}$. No jammer.

Note that this choice of a_{ℓ} needs a prior knowledge of the signal strengths as well as angles of arrival. The same was true for the steering vector chosen in the previous case. Hence, one does not need anymore information to generate the steering vector. Using Equation (39), the adapted pattern [Equation (9)] and the improvement in the SNR of the kth desired signal [Equation (17)] will be given by

$$F(\theta,\phi,p) = \frac{1}{\sigma^2} \sum_{\ell=1}^{m} U^T U_{d\ell}^{\dagger}$$
(39)

and

$$I_{k} = 10 \log_{10} \frac{|U_{dk}^{T} U_{dk}^{*}|^{2}}{D}$$
(40)

where

$$D = \sum_{\ell=1}^{m} |U_{d\ell}^{\mathsf{T}} U_{d\ell}^{\star}| \qquad (41)$$

From Equations (39) and (40), it is clear that neither the lobe in the k^{th} desired signal direction nor the improvement in the SNR of the k^{th} desired signal depends upon its input SNR. The improvement in the SNR of all desired signals is the same. Thus this choice of a_{ℓ} is appropriate when one is dealing with signals whose strengths vary over a wide range and one is interested in improving the SNR of all signals.

Figure 6 shows the output SNR of the linear array for the same signal environment as in Figure 2. The output SNR of the two signals is



Figure 6. Output SNR of the two signals vs. ξ_{d2} . $\theta_{d1}=60^{\circ}$, $\theta_{d2}=120^{\circ}$, $\xi_{d1}=10$ dB, $a_1=101$, $a_2=(1+10 \xi_{d2})$. No jammer.

plotted as a function of the input SNR of the signal incident at $\theta = 120^{\circ}$. The a_{ℓ} are chosen according to Equation (38). Note that the SNR of the two signals have improved by 7 dB, irrespective of the input SNR of the signal incident at $\theta = 120^{\circ}$.

IV. PERFORMANCE IN THE PRESENCE OF JAMMERS

In the last section, we studied the performance of adaptive arrays for multiple simultaneous desired signals in the absence of all jammers. Nulling strong jammers while still retaining its desired mainbeam characteristics is the most attractive feature of an adaptive array. The work, therefore, will not be complete until the interference rejection capabilities of the adaptive array are discussed. In this section, some examples of the array performance in the presence of multiple jammers are given. It is shown that the steered beam adaptive arrays for multiple simultaneous desired signals null the jammers quite effectively as long as the jammers are not within the major lobes of the array. When a jammer falls inside a major lobe of the array, the output signal-to-interference-plus-noise ratio (SINR) of the desired signal incident from that major lobe direction is poor, as expected. Under these conditions, the output SINR of other desired signals will also be affected.

Figures 7 through 9 show the output SINR of the ten element array (Figure 3) for two desired signals in the presence of three CW jammers. The weighting coefficients and other parameters in Figures 7 through 9 are the same as in Figures 4 through 6, respectively. The three jammers



Figure 7. Output SINR of the two signals in the presence of three jammers vs. ξ_{d2} . $\theta_{d1}=60^{\circ}$, $\theta_{d2}=120^{\circ}$, $\xi_{d1}=10$ dR, $a_1=a_2=1$, $\theta_{i1}=0^{\circ}$, $\theta_{i2}=90^{\circ}$, $\theta_{i3}=140^{\circ}$, $\xi_{i1}=\xi_{i2}=\xi_{i3}=30$ dB.



Figure 8. Output SINR of the two signals in the presence of three jammers vs. ξ_{d2} . $\theta_{d1}=60^{\circ}$, $\theta_{d2}=120^{\circ}$, $\xi_{d1}=10^{\circ}$ dB, $a_1=10^{\circ}$, $a_2=\xi_{d2}$, $\theta_{i1}=0^{\circ}$, $\theta_{i2}=90^{\circ}$, $\theta_{i3}=140^{\circ}$, $\xi_{i1}=\xi_{i2}=\xi_{i3}=30^{\circ}$ dB.



Figure 9. Output SINR of the two signals in the presence of three jammers vs. ξ_{d2} . $\theta_{d1}=60^{\circ}$, $\theta_{d2}=120^{\circ}$, $\xi_{d1}=10^{\circ}$ dB, $a_1=101$, $a_2=(1+10 \xi_{d2})$, $\theta_{i1}=0^{\circ}$, $\theta_{i2}=90^{\circ}$, $\theta_{i3}=140^{\circ}$, $\xi_{i1}=\xi_{i2}=\xi_{i3}=30^{\circ}$ dB.

are incident from 0°, 90° and 160°, respectively, which corresponds to the sidelobe peaks of the array when its two beams are along 60° and 120° (the desired signal directions), respectively. Each jammer is 30 dB stronger than the thermal noise added to each element. Comparing the output SINR in Figures 7 through 9 with the output SNR in Figures 4 through 6, respectively, one can see that the two are about the same. The adaptive array is, therefore, completely rejecting the interfering signals without negative effects on the desired signals.

In the above example, all the jammers were outside the major lobes of the array. A jamming scenario with jammers falling inside the major lobes of the array will be considered next.

Figure 10 shows the output SINR of the ten element array for two desired signals in the presence of three jammers. The two signals are incident at 60° and 120° and are, respectively, 10 dB and 0 dB stronger than the thermal noise. $a_1 = 10$ and $a_2 = 1$ in the plot. Two of the jammers are incident from 0° and 90° while the third jammer is swept across the whole visible space. Each jammer is 30 dB stronger than the thermal noise and the output SINR is plotted as a function of the third jammer angle of arrival. Note that the output SINR of the two signals is approximately 7 dB higher than the input SNR as long as the third jammer is not in the angular vicinity of a desired signal. This is true even for small angular separations between the jammers. When the third jammer approaches one of the two desired signals, the output SINR for that desired signal drops, as expected. The output SINR of the other desired signal is also affected for this jamming scenario. The change



Figure 10. Output SINR of the two signals in the presence of three jammers vs. a jammer direction. $\theta_{d1}=60^{\circ}$, $\theta_{d2}=120^{\circ}$, $\xi_{d1}=10 \text{ dB}$, $\xi_{d2}=0 \text{ dB}$, $a_1=10$, $a_2=1$, $\theta_{i1}=0^{\circ}$, $\theta_{i2}=90^{\circ}$, $\xi_{i1}=\xi_{i2}=\xi_{i3}=30 \text{ dB}$.

in the output SINR of the other desired signal is due to the following reason. For small but non-zero angular separation between a desired signal and a jammer, the array tries to maintain the two beams while minimizing the total output power. In the process of maintaining the two beams, a residual portion of the interference power appears at the array output and thus the output SINR of the two desired signals drops. When the jammer direction coincides with one of the two desired signal directions, the array has no choice but to turn off the beam in that direction to minimize the output power. Turning off the beam will suppress the jammer and the array can maintain its mainbeam in the other desired signal direction. The output SINR of the other desired signal will, thus, again increase. This is true for all choices of the weighting coefficients. The weighting coefficients, therefore, should be chosen according to the range of the desired signal strengths of interest and irrespective of jammer scenario.

V. CONCLUSIONS

Steered beam adaptive arrays for multiple simultaneous desired signals were discussed. It was shown that a steered beam adaptive array can be used for multiple simultaneous desired signal environments. For optimum performance, the different beams should be weighted according to the expected range of signal strengths.

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