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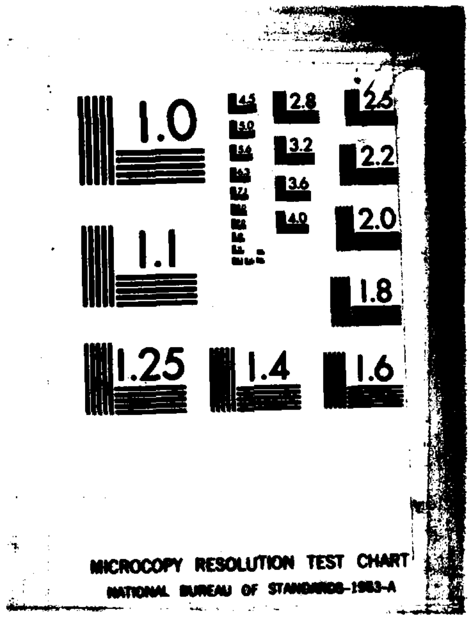
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MICROCOPY RESOLUTION TEST CHART
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AFWAL-TR-83- 4095

COMPOSITE LAMINATE WEIGHT OPTIMIZATION ON THE SHARP PC-1500 POCKET COMPUTER

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August 1983

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Approved for public release; distribution unlimited.

MATERIALS LABORATORY
AIR FORCE WRIGHT AERONAUTICAL LABORATORIES
AIR FORCE SYSTEMS COMMAND
WRIGHT-PATTERSON AIR FORCE BASE, OHIO 45433

NOTICE

This technical report has been reviewed and is approved for publication.

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REPORT DOCUMENTATION PAGE

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Microcomputer

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

An automated composite laminate sizing technique is presented, which optimizes for minimum weight. The technique can be coded for a microcomputer and a listing is given for the Sharp PC-1500. The program is interactive and easy to use. Ply ratios are optimized for point stress under multiple independent loads.

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FOREWORD

This report was prepared in the Mechanics and Surface Interactions Branch (AFWAL/MLBM), Nonmetallic Materials Division, Materials Laboratory, Air Force Wright Aeronautical Laboratories, Wright-Patterson Air Force Base, Ohio. The work was performed under the support of Project Number 2307, "Nonmetallic Structrual Materials", Task Number 2307P2, "Composite Materials and Mechanics Technology," and under Contract F33615-83-C-5001.

In this report, an automated composite laminate sizing technique is presented, which optimizes for minimun weight. The technique can be coded for a microcomputer and a listing is given for the Sharp PC-1500. The program is interactive and easy to use. Ply ratios are optimized for point stress under multiple independent loads.

This program is available on cassette tape and can be obtained by sending a blank 15 or 30-minute tape to AFWAL/MLBM, Wright-Patterson AFB, Ohio 45433 and referencing this report.

TABLE OF CONTENTS

SECTION I

PROGRAM DESCRIPTION

The program "OPTIM" is an optimization program designed to run on a small microcomputer. The listing presented here is for a Sharp PC-1500 with an 8K memory expansion. The version of Basic is standard enough that translations to other microcomputers is possible.

The program will find a minimum thickness laminate which will not fail under any of the load conditions entered. Ply orientations are chosen by the user. The program's capability in handling multiple, independent, loads could be useful for loads which change with time or for situations where there is uncertainty in calculating the loads. As the program is currently dimensioned, four independent load combinations and 6-ply orientations can be entered.

Only point stresses are considered, thus the program optimizes the laminate only at one point in the structure. Furthermore, the program assumes in-plane loads only and no out of plane deflections. This implies a symmetric laminate, but stacking sequence is not a factor in the program. The layer thicknesses generated by the program are the total and must be divided by 2 to get the halves of a symmetrical laminate.

No knowledge of optimization techniques is needed to run the program and very little knowledge of laminate plate theory is needed. In addition, material properities for three common advanced composites are stored in the program and new composite materials can be added by inputting their material constants into the program directly.

SECTION II

GENERAL INSTRUCTIONS

The Sharp PC-1500 manual includes tape loading instructions. Because "OPTIM" takes so long to read (approximately 5 minutes), it's a good idea to test tape player volume level with a short one or two line program to see if all is well. Load the tape using "OPTIM" as the name. An example of the display and appropriate responses are given in this report.

The program is started in RUN mode by instruction RUN (press the keys [R] [U] [N] and [ENTER]). User is guided through the program by simple questions. The user types the chosen answer and presses the [ENTER] key. At the completion of the routine, after all results have been displayed, the program will restart itself.

Run times can be quite long. They range from a minute for a 2-layer laminate, to 10 minutes for a 6-layer laminate subject to multiple loads.

The material constants are stored in the program. When "MATERIAL NO.=" is asked, input the material number;

1....T300/5208

2....BORON/5508

3....AS/3501

To add other materials (up to 6 more), user should type (in PRO mode) for a material with the material number N,

16N∅: DATA "name of material", E_x, E_y, ν_x, E_s, h_o, X, X', Y, Y', S : RETURN.

An example of adding a new material (Aluminum....Material number 4) is as follows:

Set PRO Mode and type,

```
1640 DATA "ALUMINUM", 69, 69, .3, 26.538, 125E-6, 400, 400, 400, 230 :
     RETURN
```

and press ENTER key.

The unit of engineering constants are in GPa and thickness of single ply h_0 is in meters.

SECTION III

METHOD

The goal is to minimize the total thickness of a composite laminate subject to failure constraints under static loads. Specifically

$$\sum_{k=1}^{L} h_k = \text{min.} \qquad \text{where } L = \text{number of layers}$$

subject to $h > 0$ and G $G_{ij}^{(\Theta_k)}\varepsilon_i^{(N)}\varepsilon_j^{(N)} + G_i^{(\Theta_k)}\varepsilon_i^{(N)} - 1 \leq 0$ where h_k is the total thickness of all the plies at the k'th orientation (which will be referred to as a "layer" in this report). The failure criteria is a first ply failure based on the Tsai-Wu tensor criteria in strain space. The G's are transformed to the laminate axis from the k'th layers orientation. The strains are associated with the N'th loading combination. This distinction is made since more than one independent loading may be considered. For the definition of the G's in terms of experimental strength data, see reference 1.

Stacking sequence is not included in this formulation, and the laminate is assumed not to bend or warp. Therefore, strains and loads are related by

$$\vec{N} = |A| \ \vec{\varepsilon}$$

The optimization method applied is a modification of the method of feasible directions. The method can be demonstrated graphically with 2-dimensions, i.e. two layers. In figure 1 the two equalities

$$G_{ij}^{(0)}\varepsilon_i\varepsilon_j + G_i^{(0)}\varepsilon_i - 1 = 0$$

$$G_{ij}^{(90)}\varepsilon_i\varepsilon_j + G_i^{(90)}\ \varepsilon_i - 1 = 0$$

have been plotted as functions of $h^{[0]}$ and $h^{[90]}$ for the single loading condition shown. Any point above and to the right of these two curves is feasible, that is, failure will not occur. Points to the left and below the curves are infeasible. Because our objective function (the sum of the layer thicknesses) is linear, the optimum point will lie on one of these curves or the intersection of multiple curves.

The program starts by finding an initial feasible point (A) which lies on a constraint curve farthest from the origin on the line $h^{[90]}=h^{[0]}$. The distance from the origin is calculated using a strain ratio method. Along any vector which passes through the origin

$$h_k^{i+1} = h_k^{i} \cdot S/S_o$$

where S is a scalar distance, and $S_o = [\sum_{k=1}^{L} (h_k^i)^2]^{1/2}$

along this vector, strain can be found using

$$\varepsilon_i = \frac{\varepsilon_i^o \; S_o}{S}$$

where ε_i^o is a component of laminate strain evaluated at S_o.

Substituting into the failure criteria we have

$$\frac{G^{(\Theta_k)} \varepsilon_i^o \varepsilon_j^o S_o^2}{S^2} + \frac{G_i^{(\Theta_k)} \varepsilon_i^o S_o}{S} - 1 = 0$$

To ensure the calculated point lies slighty in the feasible region despite any numerical error, the program sets this function equal to the negative of a small number 1E(-6) rather than zero. Solving this equation for positive S we have

$$S = \frac{-B + \sqrt{(B^2-4AC)}}{2A}$$

where $A = 1 - 1E{-}6$ $\quad B = \sum_{i=1}^{3} - G^{(\Theta_k)} \varepsilon_i^{\circ} S_o$

$$C = \sum_{i=1}^{3} \sum_{j=1}^{3} -G^{(\Theta_k)} \varepsilon_i^{\circ} \varepsilon_j^{\circ} S_o^2$$

If S_o lies in the feasible region we solve the above equation for each layer and each load combination then take the smallest resulting S as the one that defines the boundary of the feasible region.

The next step in the optimization procedure is to establish a direction vector which will point away from the constraint A lies on and is parallel to the plane defined by $\Sigma\, h_k$=constant. In figure 1, this direction is shown as Z. Finding Z first requires calculation of the gradient of the active constraint evaluated at A. Let

$$C_{k,N} = G_{ij}^{(\Theta_k)} \varepsilon_i^{(N)} \varepsilon_j^{(N)} + G^{(\theta_k)} \varepsilon_i^{(N)} - 1$$

where k and N correspond to the layer and load combination of the active constraint. A constraint is considered active if $C_{k,N} > -E_2$ where E_2=1E-5 is a small number. Note that more than one constraint may be active. The gradient is then given by

$$\vec{\nabla} C_{k,N} = \sum_{L=1}^{L} [G_{ij}^{(\Theta_k)} (\frac{\partial \varepsilon_i^{(N)}}{\partial h_L} \varepsilon_j^{(N)} + \varepsilon_i^{(N)} \frac{\partial \varepsilon_j^{(N)}}{\partial h_L}) + G^{(\Theta_k)} \frac{\partial \varepsilon_i^{(N)}}{\partial h_L}] \; \hat{h}_L$$

where h_L is a unit vector. To find the partials of strain we start with the basic equation

$$\vec{N} = |A| \, \vec{\varepsilon}$$

$$0 = \frac{\partial}{\partial h_i} |A| \, \vec{\varepsilon} + A \frac{\partial}{\partial h_i} \vec{\varepsilon} \, , \quad \frac{\partial \vec{\varepsilon}}{\partial h_i} = -|A^{-1}| \frac{\partial}{\partial h_i} |A| \, \vec{\varepsilon}$$

and

$$\frac{\partial}{\partial h_i}|A| = \begin{bmatrix} Q_{11}^{(\Theta_i)} & Q_{12}^{(\Theta_i)} & Q_{13}^{(\Theta_i)} \\ Q_{21}^{(\Theta_i)} & Q_{22}^{(\Theta_i)} & Q_{23}^{(\Theta_i)} \\ Q_{13}^{(\Theta_i)} & Q_{23}^{(\Theta_i)} & Q_{33}^{(\Theta_i)} \end{bmatrix} = [Q^{(\Theta_i)}]$$

The gradient vector is normalized to unit length. If more than one constraint is active, the normalized gradients are summed together and the sum is then normalized to one. The negative of the gradient will point away from the constraint, into the feasible region. This vector is now projected onto the plane defined by the unit normal $\hat{n}$, where

$$\hat{n} = \frac{1}{\sqrt{L}} \sum_{i=1}^{L} \hat{h}_i \quad .$$

The projection can be made with a double cross product

$$\vec{Z} = \hat{n} \times (-\bar{\nabla}c \times \hat{n}) \quad .$$

With a vector identity, this can be rewritten as

$$\vec{Z} = (\bar{\nabla}c \cdot \hat{n})\hat{n} - \bar{\nabla}c \, .$$

Finally, $\vec{Z}$ is also normalized to unit length.

Along $\vec{Z}$, another constraint will eventually be reached (point B in Figure 1). The point is found iteratively by a bisection technique. Since the bisection method is very time consuming, the constraint line is only found within a relatively large error band. What we are really interested in is a point approximately midway between A and B, which is C in the figure. From point C, the strain ratio technique is used to analytically calculate D. Starting at D, the entire procedure repeats.

The program terminates when the distance $\overline{AB}$ or $\overline{CD}$ is small (say 1/10 of a ply thickness) or the magnitude of $\vec{Z}$ before normalization is very small (implying $\hat{n}$ and $\vec{\nabla}c$ are almost parallel).

In some cases, $h_k \geq 0$ constraint may be reached. When this happens, that orientation is completely dropped from further calculations. Thus, the constraints associated with a zero thickness layer cannot affect the results. Once an orientation reaches zero thickness, it is never reinstated in later iterations.

Figure 1 shows a case where the program reaches the intersection of two constraints. However, simultaneous failure should not be considered a criteria for optimization. The constraint line for the +45° layer is completely in the infeasible region. The line $h^{[45]} + h^{[-45]} = $ const. has been included to show that point D is the minimum thickness (See Figure 2).

References

1. S.W. Tsai, H.T. Hahn, Introduction to Composite Materials, Technomic Publishing Company, Westport, Connecticut, 1980.

2. D.M Himmelblan, Applied Nonlinear Programming, McGraw-Hill, New York, 1972.

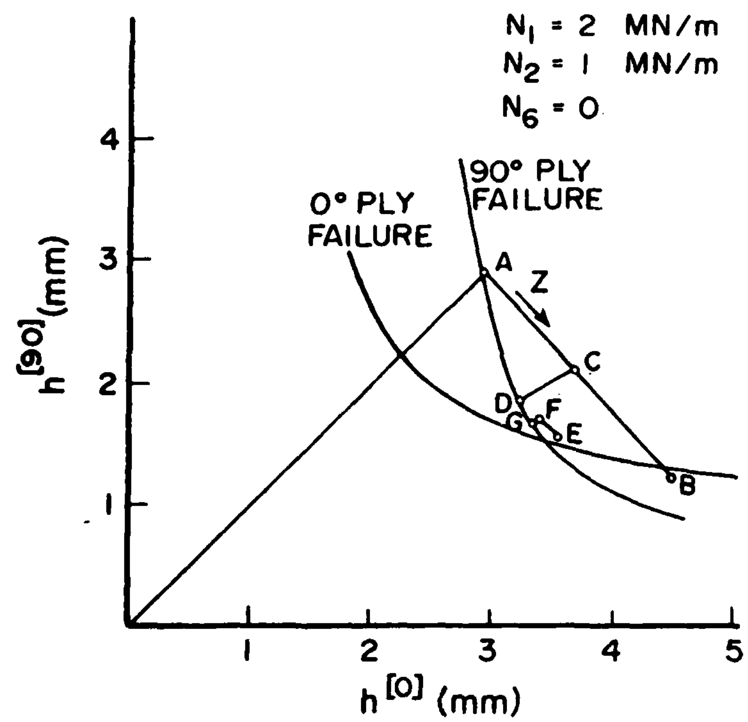


Figure 1. Constraint Surfaces and Optimization Trajectory for 0/90 under Biaxial Load.

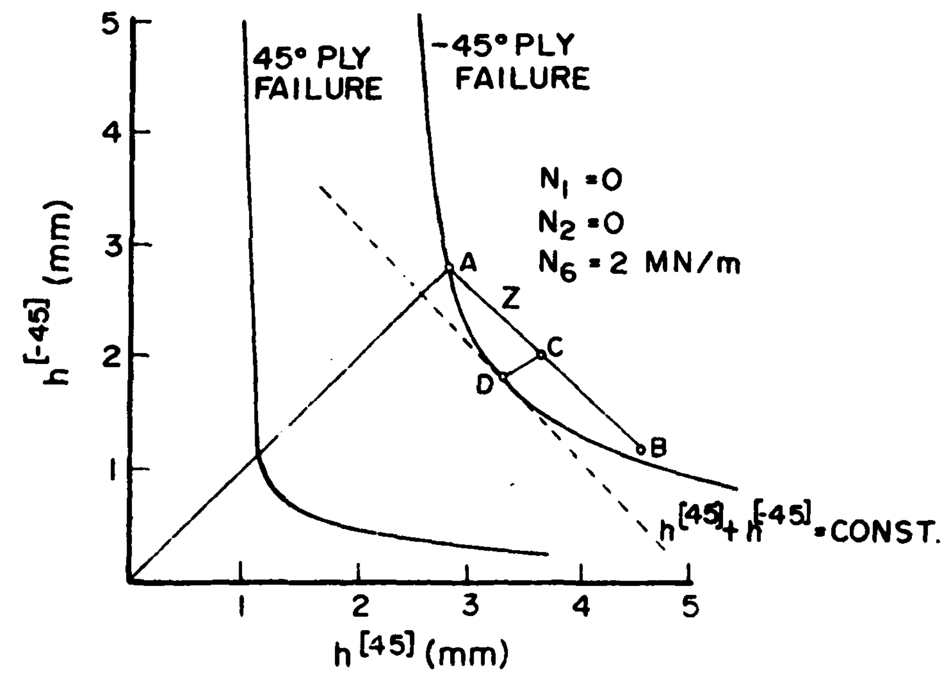
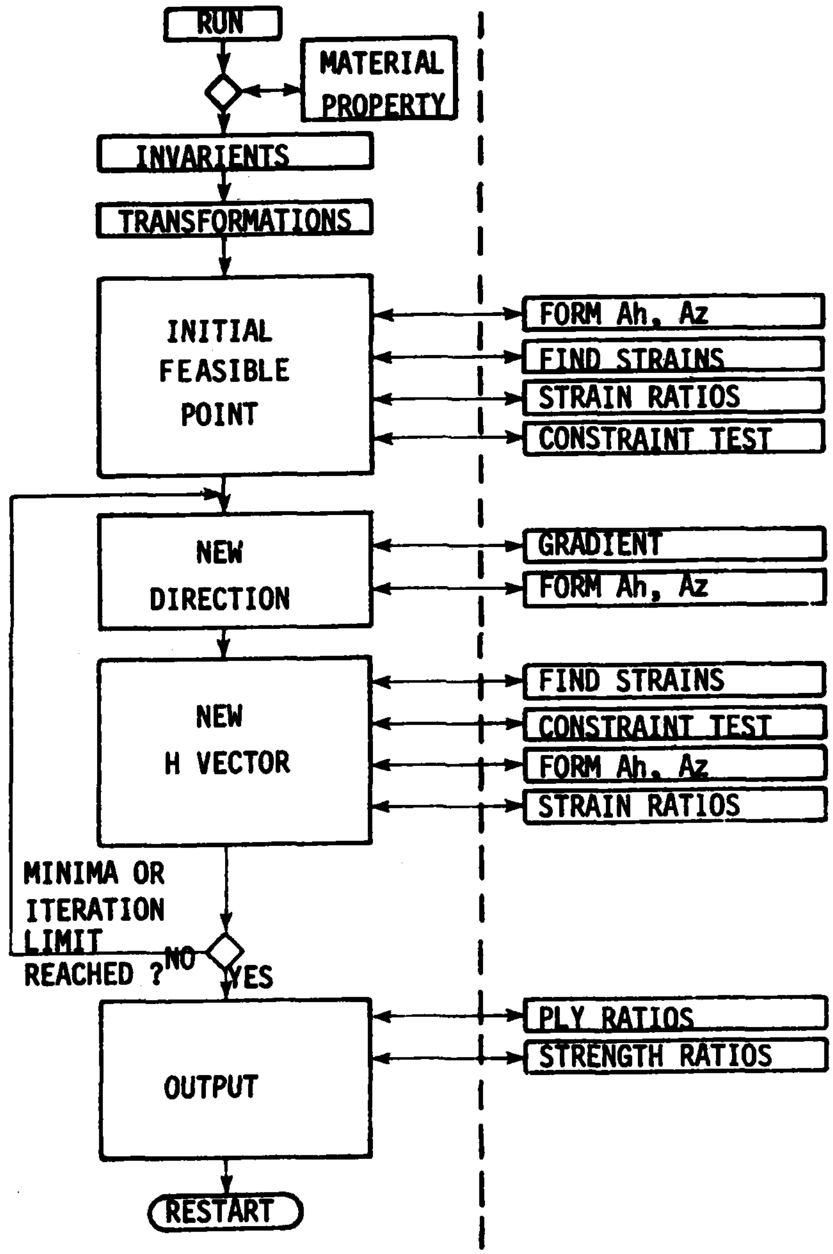


Figure 2. Constraint Surfaces and Optimization Trajectory for ± 45 Laminate Under Pure Shear.

APPENDIX A

FLOW DIAGRAM



APPENDIX B

KEY OPERATION PROCEDURE

DISPLAY	INPUT	PRINT OUT & REMARKS
MATERIAL NUMBER =	RUN [ENTER] 1 [ENTER]	T300/5208
NUMBER OF PLY ANGLES ? PLY ANGLE-1 ? PLY ANGLE-2 ?	2 [ENTER] 0 [ENTER] 90 [ENTER]	NO.OF PLY ANGLE= 2 ANGLE= 0.0 ANGLE= 90.0
NUMBER OF INDEPENDENT LOADINGS LOADING-1 N_1=? N_2=? N_6=? LOADING-2 N_1=? N_2=? N_6=?	2 [ENTER] 4 [ENTER] 1 [ENTER] 0 [ENTER] 1.25 [ENTER] 3.25 [ENTER] 1.299 [ENTER]	NO.OF LOADINGS= 2 LOADING UNIT is in MN/m LOADING-1 N1= 4.000 N2= 1.000 N6= 0.000 LOADING-2 N1= 1.250 N2= 3.250 N6= 1.299
		AFTER 4 ITERATIONS TOTAL THICKNESS = 2.282E 00 cm 182.57 PLIES

DISPLAY	INPUT	PRINT OUT & REMARKS
		ANGLE= 0.0 RATIO= 0.304 # PLIES= 55.63 ANGLE= 90.0 RATIO= 0.695 # PLIES= 126.94 STRENGTH RATIOS 1=ULTIMATE STRAIN AND >1=SAFE LOADING-1 PLY RATIO 0.0 2.301 90.0 1.371 LOADING-2 PLY RATIO 0.0 1.000 90.0 1.023
MATERIAL NUMBER	[ON] [ENTER]	--BREAK--

APPENDIX C

MEMORY CONTENTS

MEMORY	DESCRIPTION	MEMORY	DESCRIPTION
A(3,3)	Value of Thickness	Z(6)	Direction Vector
B(6,9)	G_{ij} (Strength Parameters in Strain Space)	A$(6)	Subscript Indicators
C(6,9)	Q_{ij} (Modulus Components)	U1--U5	Modulus invariants
D(3,3)	$\Sigma Q^{k}_{ij} Z_k$	V1--V7	Strength invariants (Strain Space)
E(4,3)	Strains	C2	COS 2Θ
G(3,3)	Strength Parameter Matrix	C4	COS 4Θ
N(4,3)	Loads	S2	sin 2Θ
P(3,3)	Inverse of A Matrix	S4	sin 4Θ
Q(3,3)	Modulus Matrix	S	Final scalar distance to be moved
C$(10,2)	List of active constraints	S1--S2	Points in feasible region
H(6)	Thickness for each ply	SR	Distance to origin
R(3)	Intermediate results	SM	Distance to first h=0 constraint
S(3)	Strength parameter components	NP	Number of ply orientations
T(6)	Angle of each ply	NL	Number of independent loadings

MEMORY CONTENTS (CONTINUED)

MEMORY	DESCRIPTION	MEMORY	DESCRIPTION
W(6)	Normalized gradient of a constraint	NC	Number of active constraints
X(6)	Normalized sum of gradients	IT	Iteration counter
Y(3)	Intermediate results	TP	Individual thickness
IM	Maximum number of iterations	GR,GS GP,GQ	Strength parameter in strain space
NN	$(1-\nu_x \nu_y)^{-1}$	GX,GY	
A, B, C, D, G, H, CN, NM, TS, SV	Intermediate calculations	F$	Flag to halt program when "F"
		G$	Flag "F" if a constraint is violated
I, J, K, L	Loop counters	M$	Material Numbers
P, II	Ply orientation pointers	W$	Material Names
N	Load pointers	X$	Subscript indicators
Z	$\sqrt{NP}$		
O, Q, R, T	Engineering constants		
U, V, W, X, Y	Strengths		
Q1-Q4	Modulus		

(D) SAMPLE PROBLEMS

```
T300/5208

NO.OF PLY ANGLE= 2
  ANGLE=-10.0
  ANGLE= 70.0

NO.OF LOADINGS= 2
   LOADING UNIT is
     in  MN/m

  LOADING-1
    N1=        4.000
    N2=        1.000
    N6=        0.000

  LOADING-2
    N1=        1.250
    N2=        3.250
    N6=        1.299

AFTER 5 ITERATIONS

TOTAL THICKNESS =
  1.133E 00  cm
   90.68     PLIES

ANGLE=-10.0
RATIO=  0.500
# PLIES=  45.34

ANGLE= 70.0
RATIO=  0.500
# PLIES=  45.34

  STRENGTH RATIOS

  1=ULTIMATE STRAIN
    AND  >1=SAFE

    LOADING-1
  PLY           RATIO
 -10.0          1.388
  70.0          1.000

    LOADING-2
  PLY           RATIO
 -10.0          1.068
  70.0          [illegible]
```

```
T300/5208

NO.OF PLY ANGLE= 4
  ANGLE=  0.0
  ANGLE= 45.0
  ANGLE=-45.0
  ANGLE= 90.0

NO.OF LOADINGS= 4
   LOADING UNIT is
     in  MN/m

  LOADING-1
    N1=        1.000
    N2=        1.000
    N6=        1.000

  LOADING-2
    N1=        2.000
    N2=        1.000
    N6=        0.000

  LOADING-3
    N1=        2.000
    N2=       -1.000
    N6=        0.000

  LOADING-4
    N1=        0.000
    N2=        0.000
    N6=       -1.500

AFTER 4 ITERATIONS

TOTAL THICKNESS =
  7.727E-01  cm
   61.82     PLIES

ANGLE=  0.0
RATIO=  0.301
# PLIES=  18.62

ANGLE= 45.0
RATIO=  0.254
# PLIES=  15.72

ANGLE=-45.0
RATIO=  0.232
# PLIES=  14.35

ANGLE= 90.0
RATIO=  0.212
# PLIES=  13.12
```

```
STRENGTH RATIOS

1=ULTIMATE STRAIN
  AND  >1=SAFE

   LOADING-1
 PLY           RATIO
  0.0          1.320
 45.0          2.271
-45.0          1.074
 90.0          1.384

   LOADING-2
 PLY           RATIO
  0.0          1.839
 45.0          1.476
-45.0          1.513
 90.0          1.298

   LOADING-3
 PLY           RATIO
  0.0          2.396
 45.0          1.260
-45.0          1.264
 90.0          1.000

   LOADING-4
 PLY           RATIO
  0.0          1.269
 45.0          1.010
-45.0          2.453
 90.0          1.271
```

```
T300/5208

NO.OF PLY ANGLE= 4
  ANGLE=  0.0
  ANGLE= 90.0
  ANGLE= 45.0
  ANGLE=-45.0

NO.OF LOADINGS= 2
   LOADING UNIT is
     in  MN/m

  LOADING-1
    N1=          3.000
    N2=          2.000
    N6=          0.500

  LOADING-2
    N1=          1.000
    N2=          4.000
    N6=          0.000

AFTER 3 ITERATIONS

TOTAL THICKNESS =
  1.154E 00  cm
   92.32     PLIES

ANGLE=  0.0
RATIO=  0.203
# PLIES=  18.75

ANGLE= 90.0
RATIO=  0.335
# PLIES=  30.97

ANGLE= 45.0
RATIO=  0.230
# PLIES=  21.30

ANGLE=-45.0
RATIO=  0.230
# PLIES=  21.30

 STRENGTH RATIOS

 1=ULTIMATE STRAIN
   AND  >1=SAFE

   LOADING-1
  PLY         RATIO
   0.0        1.426
  90.0        1.004
  45.0        1.324
 -45.0        1.043

   LOADING-2
  PLY         RATIO
   0.0        1.000
  90.0        1.769
  45.0        1.220
 -45.0        1.220
```

```
BORON/5505

NO.OF PLY ANGLE= 4
  ANGLE=  0.0
  ANGLE= 90.0
  ANGLE= 45.0
  ANGLE=-45.0

NO.OF LOADINGS= 2
   LOADING UNIT is
     in  MN/m

  LOADING-1
    N1=          3.000
    N2=          2.000
    N6=          0.500

  LOADING-2
    N1=          1.000
    N2=          4.000
    N6=          0.000

AFTER 5 ITERATIONS

TOTAL THICKNESS =
  1.194E 00  cm
   95.56     PLIES

ANGLE=  0.0
RATIO=  0.203
# PLIES=  19.41

ANGLE= 90.0
RATIO=  0.337
# PLIES=  32.21

ANGLE= 45.0
RATIO=  0.229
# PLIES=  21.97

ANGLE=-45.0
RATIO=  0.229
# PLIES=  21.97

 STRENGTH RATIOS

 1=ULTIMATE STRAIN
   AND  >1=SAFE

   LOADING-1
  PLY         RATIO
   0.0        1.379
  90.0        1.002
  45.0        1.321
 -45.0        1.046

   LOADING-2
  PLY         RATIO
   0.0        1.000
  90.0        1.654
  45.0        1.300
 -45.0        1.300
```

```
AS/3501

NO.OF PLY ANGLE= 4
  ANGLE=  0.0
  ANGLE= 90.0
  ANGLE= 45.0
  ANGLE=-45.0

NO.OF LOADINGS= 2
   LOADING UNIT is
     in  MN/m

  LOADING-1
    N1=          3.000
    N2=          2.000
    N6=          0.500

  LOADING-2
    N1=          1.000
    N2=          4.000
    N6=          0.000

AFTER 7 ITERATIONS

TOTAL THICKNESS =
  1.000E 00  cm
   80.04     PLIES

ANGLE=  0.0
RATIO=  0.203
# PLIES=  16.26

ANGLE= 90.0
RATIO=  0.333
# PLIES=  26.67

ANGLE= 45.0
RATIO=  0.231
# PLIES=  18.56

ANGLE=-45.0
RATIO=  0.231
# PLIES=  18.56

 STRENGTH RATIOS

 1=ULTIMATE STRAIN
   AND  >1=SAFE

   LOADING-1
  PLY         RATIO
   0.0        1.362
  90.0        1.019
  45.0        1.287
 -45.0        1.054

   LOADING-2
  PLY         RATIO
   0.0        1.000
  90.0        1.591
  45.0        1.195
 -45.0        1.195
```

APPENDIX E

PROGRAM LISTING

```
10:"OPTIM":CLEAR
12:DIM H(6),R(3),
   S(3),T(6),W(6)
   ,X(6),Y(3),Z(6
   )
14:DIM A(3,3),B(6
   ,9),C(6,9),D(3
   ,3),E(4,3),G(3
   ,3),N(4,3),P(3
   ,3),Q(3,3)
16:DIM A$(6),C$(1
   0,2)
18:IT=1:IM=10
20:DATA "1","2","
   3","4","5","6"
21:RESTORE 20:FOR
   I=1TO 6:READ A
   $(I):NEXT I
30:GOSUB 1200
40:GOSUB 1700:
   GOSUB 1100:
   GOSUB 1000
42:GOSUB 800
45:IF F$="F"GOTO
   1300
50:GOSUB 700
60:IT=IT+1
70:IF F$="F"OR IT
   >IMTHEN GOTO 1
   300
75:GOTO 42
200:G$="P":NC=0
210:FOR P=1TO NP
215:IF H(P)=0GOTO
    280
220:II=P:GOSUB 650
225:FOR N=1TO NL
230:CN=-1
235:FOR K=1TO 3:
    FOR J=1TO 3
240:CN=CN+G(K,J)*E
    (N,J)*E(N,K):
    NEXT J
245:CN=CN+S(K)*E(N
    ,K):NEXT K
250:IF CN>0LET G$=
    "F":GOTO 295
253:IF CN<-.1GOTO
    275
255:NC=NC+1
260:C$(NC,1)=CHR$
    P:C$(NC,2)=
    CHR$ N
275:NEXT N
280:NEXT P
295:RETURN
300:NM=0,II=P:
    GOSUB 650
305:FOR L=1TO NP
310:IF H(L)=0GOTO
    370
315:II=L:GOSUB 600
320:FOR J=1TO 3:R(
    J)=0
325:FOR K=1TO 3
330:R(J)=R(J)-Q(J,
    K)*E(N,K)
335:NEXT K:NEXT J
340:FOR J=1TO 3:Y(
    J)=0
342:FOR K=1TO 3:Y(
    J)=Y(J)+P(J,K)
    *R(K)
345:NEXT K:NEXT J
350:W(L)=0
352:FOR J=1TO 3:
    FOR K=1TO 3
354:W(L)=W(L)+G(J,
    K)*(Y(J)*E(N,K
    )+E(N,J)*Y(K))
356:NEXT K
358:W(L)=W(L)+S(J)
    *Y(J):NEXT J
360:NM=NM+W(L)*W(L
    )
370:NEXT L
375:NM=√(NM)
380:FOR L=1TO NP
385:W(L)=W(L)/NM:
    NEXT L
390:RETURN
400:FOR I=1TO 3:
    FOR J=1TO 3
405:E(I,J)=A(I,J)+
    D(I,J)*S
410:NEXT J:NEXT I
415:T=E(1,1),U=E(1
    ,2),V=E(1,3),W
    =E(2,2),X=E(2,
    3),Y=E(3,3)
420:D=T*W*Y+2*U*X*
    V-W*V*V-T*X*X-
    Y*U*U
425:F=(W*Y-X*X)/D
    O=(T*Y-V*V)/D
    G=(V*X-U*Y)/D
430:R=(T*W-U*U)/D:
    H=(U*X-W*V)/D:
    Q=(U*V-T*X)/D
435:P(1,1)=F,P(1,2
    )=G,P(1,3)=H,P
    (2,1)=G,P(2,2)
    =O
440:P(2,3)=Q,P(3,1
    )=H,P(3,2)=Q,P
    (3,3)=R
450:FOR I=1TO NL:
    FOR J=1TO 3
455:E(I,J)=0
460:FOR K=1TO 3
470:E(I,J)=E(I,J)+
    P(J,K)*N(I,K)
480:NEXT K:NEXT J
490:NEXT I:RETURN
500:FOR I=1TO 3:
    FOR J=1TO 3
510:A(I,J)=0:D(I,J
    )=0
520:NEXT J:NEXT I
530:FOR I=1TO NP:I
    I=I
540:GOSUB 600
550:FOR J=1TO 3:
    FOR K=1TO 3
560:A(J,K)=A(J,K)+
    Q(J,K)*H(I):D(
    J,K)=D(J,K)+Q(
    J,K)*Z(I)
570:NEXT K:NEXT J:
    NEXT I
580:RETURN
600:FOR J=1TO 3:
    FOR K=1TO 3
610:Q(J,K)=C(II,J*
    K)
620:NEXT K:NEXT J
640:RETURN
650:FOR J=1TO 3
    FOR K=1TO 3
660:G(J,K)=B(II,J*
    K)
670:NEXT K:NEXT J
680:S(1)=B(II,7):S
    (2)=B(II,8):S(
    3)=B(II,5)
690:RETURN
700:SM=1E10
732:FOR I=1TO NP
734:IF Z(I)<>0LET
    S=-H(I)/Z(I)
```

```
736:IF S>0AND S<SM
    THEN LET SM=S
737:NEXT I
738:F$="P"
740:IF SM>10LET F$
    ="F":RETURN
744:S1=0:S2=SM:S=S
    M
746:IF NC=0GOTO 77
    5
748:GOSUB 400:
    GOSUB 200
750:IF G$="F"LET S
    2=S
752:IF G$="P"LET S
    1=S
754:IF S1=SMGOTO 7
    65
755:S=(S1+S2)/2
756:IF (S2-S1)<1E-
    5AND S1=0THEN
    LET F$="F"
758:IF F$="F"GOTO
    786
760:IF S1/(S2-S1)<
    4GOTO 748
762:S=S1/2
765:SR=0
767:FOR I=1TO NP:H
    (I)=H(I)+Z(I)*
    S
768:IF H(I)<1E-5
    LET H(I)=0
770:SR=SR+H(I)*H(I
    ):NEXT I
775:S=0:GOSUB 500:
    GOSUB 400
778:LET SR=√(SR)
780:GOSUB 900
782:IF (SR-S)<1E-5
    LET F$="F"
784:FOR I=1TO NP:H
    (I)=H(I)*S/SR:
    NEXT I
786:S=0:GOSUB 500:
    GOSUB 400
790:GOSUB 200:
    RETURN
800:NM=1,Z=0
802:FOR I=1TO NP:X
    (I)=0
804:Z=Z+SGN H(I):
    NEXT I
806:Z=1/√Z
807:IF NC=0GOTO 85
    0
808:FOR I=1TO NC
810:P=ASC C$(I,1):
    N=ASC C$(I,2):
    GOSUB 300
812:FOR J=1TO NP:X
    (J)=X(J)-W(J)
814:NEXT J:NEXT I
820:NM=0:FOR J=1TO
    NP
825:NM=NM+X(J)*X(J
    ):NEXT J
830:NM=√(NM):TS=0
850:FOR I=1TO NP:X
    (I)=X(I)/NM
855:TS=TS+X(I)*Z*
    SGN H(I):NEXT
    I
860:NM=0:FOR I=1TO
    NP
865:Z(I)=X(I)-TS*Z
    *SGN H(I)
870:NM=NM+Z(I)*Z(I
    ):NEXT I
872:F$="P"
874:IF NM<1E-6LET
    F$="F"
876:IF F$="F"THEN
    RETURN
878:NM=√(NM)
880:FOR I=1TO NP:Z
    (I)=Z(I)/NM:
    NEXT I
885:GOSUB 500
890:RETURN
900:FOR P=1TO NP
905:IF H(P)=0GOTO
    980
910:II=P:GOSUB 650
915:FOR N=1TO NL:B
    =0:C=0
920:FOR I=1TO 3:
    FOR J=1TO 3
925:C=C-SR*SR*G(I,
    J)*E(N,I)*E(N,
    J)
930:NEXT J
940:B=B-SR*S(I)*E(
    N,I):NEXT I
950:SU=(-B+√(B*B-4
    *C*(1-1E-6)))/
    (2*(1-1E-6))
960:IF SU>SLET S=S
    U
970:NEXT N
980:NEXT P:RETURN
1000:Z=1/√(NP)
1010:FOR I=1TO NP
     Z(I)=Z:H(I)
     =Z:NEXT I
1020:GOSUB 500
1030:S=0:SR=1:
     GOSUB 400
1040:S=0:GOSUB 90
     0
1050:FOR I=1TO NP
     :H(I)=H(I)*S
     :NEXT I
1060:S=0:GOSUB 50
     0
1070:GOSUB 400:
     GOSUB 200:
     RETURN
1100:FOR I=1TO NP
1110:C2=COS (2*T(
     I)):C4=COS (
     4*T(I))
1120:S2=SIN (2*T(
     I)):S4=SIN (
     4*T(I))
1130:B(I,1)=U1+C2
     *U2+C4*U3:B(
     I,4)=U1-C2*U
     2+C4*U3
1140:B(I,2)=U4-C4
     *U3:B(I,9)=U
     5-C4*U3
1150:B(I,3)=S2/2*
     U2+S4*U3:B(I
     ,6)=S2/2*U2-
     S4*U3
1160:B(I,7)=U6+C2
     *U7:B(I,8)=U
     6-C2*U7:B(I,
     5)=S2*U7
1170:C(I,1)=U1+C2
     *U2+C4*U3:C(
     I,4)=U1-C2*U
     2+C4*U3
1180:C(I,2)=U4-C4
     *U3:C(I,9)=U
     5-C4*U3
1190:C(I,3)=S2/2*
     U2+S4*U3:C(I
     ,6)=S2/2*U2-
     S4*U3
1195:NEXT I:
     RETURN
1200:GOSUB 1600:
     LF (1)
1210:GOSUB 1830
1220:INPUT "NO.OF
      PLY ANGLES
     ?";NP
1230:LPRINT "NO.O
     F PLY ANGLE=
     ";NP
1235:FOR I=1TO NP
```

```
1240:PAUSE "PLY A
     NGLE-"+STR$
     (I):INPUT T(
     I)
1241:GOSUB 1810
1242:LPRINT "  AN
     GLE=";T(I)
1245:NEXT I:LF (1
     )
1248:GOSUB 1830
1250:INPUT "NO.OF
      INDEP. LOAD
     INGS=";NL
1252:LPRINT "NO.O
     F LOADINGS="
     ;NL
1253:LPRINT "   L
     OADING UNIT
     :s     in  M
     N/m   ".LF (1
     )
1255.FOR I=1TO NL
     :LPRINT "  L
     OADING-"+A$(
     I)
1256:FOR J=1TO 3
1260:L=J:IF J=3
     LET L=6
1262:X$="     N"+A
     $(L)+"="
1265:GOSUB 1800
1267:LPRINT X$.
     INPUT N(I,J)
     :LF (-1)
1268:LPRINT N(I,J
     )
1269:N(I,J)=N(I,J
     )*1E6
1270:NEXT J:LF (1
     ):NEXT I.
     RETURN
1300:TS=0
1310:FOR I=1TO NP
1320:TS=TS+H(I):
     NEXT I
1322:LPRINT "AFTE
     R "+STR$ (IT
     )+" ITERATIO
     NS".LF (1)
1325:GOSUB 1840
1330.LPRINT "TOTA
     L THICKNESS
     =";TS*100.LF
     (-1)
1340:LPRINT "
               cm"
1345:B=INT ((TS/T
     P+.005)*100)
     /100
1347:GOSUB 1850
1350:LPRINT " ";B
     ;"     PLIES"
1355:GOSUB 1500
1360:GOSUB 1400:
     GOTO 20
1400:LF (1)
1405:LPRINT " STR
     ENGTH RATIOS
     ":LF (1)
1410:LPRINT " 1=U
     LTIMATE STRA
     IN   AND  >1
     =SAFE".LF (1
     )
1420:FOR I=1TO NL
1430:LPRINT "
     LOADING-"+
     STR$ (I)
1440:LPRINT "  PL
     Y":LF (-1)
1445:LPRINT "
               RAT
     IO"
1446:GOSUB 1450
1448:NEXT I:
     RETURN
1450:FOR P=1TO NP
1452:IF H(P)=0
     GOTO 1490
1455:II=P:GOSUB 6
     50
1460:A=0:B=0
1465:FOR J=1TO 3:
     FOR K=1TO 3
1470:A=A+G(J,K)*E
     (I,J)*E(I,K)
     :NEXT K
1475:B=B+S(J)*E(I
     ,J):NEXT J
1480:A=(-B+√(B*B+
     4*A))/2/A
1482:A=INT ((A+.5
     /1E3)*1E3)/1
     E3
1483:GOSUB 1810
1484:LPRINT " ";W
     (P):LF (-1)
1485:GOSUB 1800
1486:LPRINT A
1490:NEXT P
1495:LF (1):
     RETURN
1500:FOR I=1TO NP
1505:LF (1)
1510:W(I)=INT ((T
     (I)+.05)*10)
     /10
1520:A=INT ((H(I)
     /TS+.5/1E4)*
     1E4)/1E4
1530.B=INT ((H(I)
     /TP+.005)*10
     0)/100
1535.GOSUB 1810
1540:LPRINT "ANGL
     E=";W(I)
1545:GOSUB 1800
1550:LPRINT "RATI
     O=";A
1555:GOSUB 1850
1560:LPRINT "# PL
     IES=";B
1570:NEXT I:
     RETURN
1600:INPUT "MATER
     IAL NO.= ";M
     $
1602:I=VAL M$*10+
     1600:GOSUB I
     :RESTORE I:
     READ W$,O,Q,
     R,T,TP,U,V,W
     ,X,Y:LPRINT
     W$:GOTO 1697
1610:DATA "T300/5
     208",181.,10
     .3,.28,7.17,
     125E-6,1500,
     1500,40.,246
     ,68.:RETURN
1620:DATA "BORON/
     5505",204.,1
     8.5,.23,5.59
     ,125E-6,1260
     ,2500,61.,20
     2.,67.,
     RETURN
1630.DATA "AS/350
     1",138.,8.96
     ,.30,7.1,125
     E-6,1447,144
     7,51.7,206.,
     93.:RETURN
1640.DATA " ".
     RETURN
1650.DATA " "
     RETURN
1660.DATA " ".
     RETURN
1670.DATA " ".
     RETURN
1680:DATA " "
     RETURN
1690:DATA " "
     RETURN
1697.RETURN
```

```
1700:O=O*1E9, Q=Q*
     1E9, T=T*1E9,
     U=U*1E6, V=V*
     1E6, W=W*1E6,
     X=X*1E6, Y=Y*
     1E6
1705:NN=1/(1-R*R*
     Q/O):Q1=NN*O
     :Q2=NN*Q:Q3=
     NN*R*Q:Q4=T
1710:U1=(3*Q1+3*Q
     2+2*Q3+4*Q4)
     /8:U2=(Q1-Q2
     )/2:U3=(Q1+Q
     2-2*Q3-4*Q4)
     /8
1720:U4=(Q1+Q2+6*
     Q3-4*Q4)/8:U
     5=(Q1+Q2-2*Q
     3+4*Q4)/8
1730:O=1/(U*U):Q=
     1/(W*X):T=1/
     (Y*Y):G=1/U-
     1/V:H=1/W-1/
     X
1740:F=-√(O*Q)/2
1750:GP=O*Q1*Q1+2
     *F*Q1*Q3+Q*Q
     3*Q3:GQ=O*Q3
     *Q3+2*F*Q3*Q
     2+Q*Q2*Q2
1760:GR=O*Q1*Q3+F
     *(Q1*Q2+Q3*Q
     3)+Q*Q3*Q2:G
     S=T*Q4*Q4
1765:GX=G*Q1+H*Q3
     :GY=G*Q3+H*Q
     2
1770:V1=(3*GP+3*G
     Q+2*GR+4*GS)
     /8:V2=(GP-GQ
     )/2
1775:V3=(GP+GQ-2*
     GR-4*GS)/8:V
     4=(GP+GQ+6*G
     R-4*GS)/8
1780:V5=(GP+GQ-2*
     GR+4*GS)/8
1790:V6=(GX+GY)/2
     :V7=(GX-GY)/
     2:RETURN
1800:USING "###.#
     ##":RETURN
1810:USING "###.#
     ":RETURN
1830:USING "##":
     RETURN
1840:USING "####.
     ###^":RETURN
1850:USING "####.
     ##":RETURN
1890:END
```


*USGPO: 1983-659-062-1116

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END

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