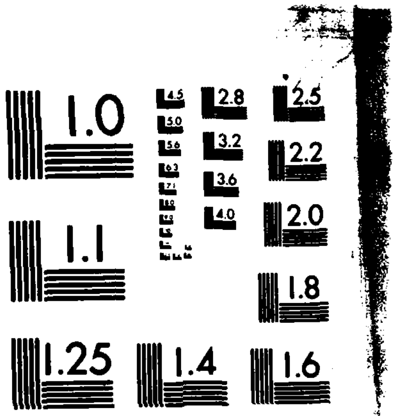
AD-A133 208 CONFIDENCE REGION FOR THE EVALUATION OF HF DF SINGLE SITE LOCATION SYSTEMS(U) NAVAL RESEARCH LAB WASHINGTON DC M H REILLY ET AL. 02 SEP 83 NRL-MR-5164 1/1

UNCLASSIFIED SBI-AD-E000 546 F/G 17/3 NL



MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS-1963-A

REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER: NRL Memorandum Report 5164	2. GOVT ACCESSION NO.: AD-A133 208
3. RECIPIENT'S CATALOG NUMBER	
4. TITLE (and Subtitle): CONFIDENCE REGION FOR THE EVALUATION OF HF DF SINGLE-SITE LOCATION SYSTEMS	5. TYPE OF REPORT & PERIOD COVERED: Interim report on a continuing NRL problem.
	6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s): M.H. Reilly and J. Coran	8. CONTRACT OR GRANT NUMBER(s)
9. PERFORMING ORGANIZATION NAME AND ADDRESS: Naval Research Laboratory Washington, DC 20375	10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS: 41-1661-0-0
11. CONTROLLING OFFICE NAME AND ADDRESS: Signal Warfare Laboratory Vint Hill Farms Warrenton, VA	12. REPORT DATE: September 2, 1983
	13. NUMBER OF PAGES: 22
14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office)	15. SECURITY CLASS. (of this report): UNCLASSIFIED
	15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

HF Direction-finding
Confidence ellipse
Single-site location
Distribution of fixes
Error statistics

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

A set of HF DF measurements in a single-site location (SSL) system will yield after processing, a scatter of latitude-longitude fixes of the position of a remote transmitter. A confidence ellipse associated with this distribution of fixes is defined and discussed for its applicability to SSL system evaluation.

CONTENTS

CONTENTS

CONFIDENCE REGION FOR THE EVALUATION OF HF DF SINGLE-SITE LOCATION SYSTEMS

I INTRODUCTION

It is desired to evaluate a system which processes measurements on a radio signal at a single site, along with information about the propagation medium, and finally yields the location of a remote ground HF transmitter. The radio signal is assumed to be propagated via the ionosphere. One basic element of the single site location (SSL) system is a radio direction-finder (DF) device, which measures the frequency, azimuth, and elevation angle of the incoming wavefront. The other basic element of the system is a propagation algorithm, which includes an ionosphere specified according to a statistical model, sounder measurements, or some combination of the two. The propagation algorithm converts measurements made at closely spaced times to a set of latitude-longitude fixes of the transmitter position. Equivalently, the fixes can be determined as a set of range and azimuth coordinates relative to the DF receiver position.

The SSL system may be tested on a fixed transmitter whose position is unknown beforehand. Fluctuations in the ionosphere, and in other parts of the system, affect the measurements. These fluctuations are thus transformed by the propagation algorithm into a statistical distribution of fixes. This distribution would surround the true position of the transmitter, except for the presence of systematic error. One important source of this error is expected to be inaccuracy of the ionospheric model. The effect of this type of error can be judged by the separation of the true position of the transmitter from, say, a 90% confidence region associated with the scatter of fixes. Specification of this confidence region is the next consideration.

Manuscript approved July 1, 1983.

The simplest assumption is that the range and azimuth cross-distance variables are statistically independent, and that the deviations from the mean of these variables are distributed according to the familiar bivariate normal distribution. Then it would be expected that, for the case of several fixes, these fixes would appear to be distributed approximately elliptically about a mean range and azimuth point, with major and minor axes in the range and cross-range directions. It is, however, often observed that, although the fixes are distributed somewhat elliptically, the axes deviate significantly from the range and cross-range directions. Variable ionospheric tilts can be a major cause of this behavior. For example, the higher elevation angle rays associated with some of the fixes may experience greater lateral ionospheric tilts, which would affect the measured azimuth angles for these rays. The range and cross-distance variables become correlated, thus affecting the size, orientation, and shape of the confidence region (e.g., see discussion of Gething [1978], pp 278-279).

A confidence region is characteristic of the distribution of a great many fixes under identical conditions, and should reflect the shape and appearance of the distribution. It is defined in Section 2, with this in mind. Section 3 describes the computer program, written in BASIC for a microcomputer, which calculates and draws the confidence ellipse associated with a set of latitude-longitude fixes. Some examples of program output are shown in Section 4, followed by a discussion of results and their application to SSL system tests in Section 5.

2. DETERMINATION OF THE CONFIDENCE REGION

The starting point is a set of N fixes (L_i, l_i), where L_i and l_i are, respectively, the latitude and longitude of the i^{th} fix. Each such fix is associated with a set of Cartesian coordinates (x_i, y_i, z_i) in the geocentric-equatorial coordinate system. The origin of these coordinates is at the center of the earth, the z axis passes through the North pole, the x axis passes through the equator in the Greenwich meridian, and the y axis also passes through the equator at right angles. With respect to this earth-fixed system,

$$(x_i, y_i, z_i) = (R \cos L_i \cos l_i,\ R \cos L_i \sin l_i,\ R \sin L_i), \tag{1}$$

where R is the earth's radius. The mean position of the fixes $(\bar{x},\bar{y},\bar{z})$ is defined by

$$(\bar{x},\bar{y},\bar{z}) = (\Sigma_i\, x_i,\ \Sigma_i\, y_i,\ \Sigma_i\, z_i)/N \quad , \tag{2}$$

and will lie some distance inside the earth's surface. This distance will be small, to the extent that the earth can be regarded as flat in the vicinity of the fixes. The latitude $\bar{L}$ and longitude $\bar{l}$ of the mean point are

$$(\bar{L},\bar{l}) = (\sin^{-1} [\bar{z}/\bar{r}],\ \sin^{-1} \left[\bar{y}/(\bar{x}^{\,2}+\bar{y}^{\,2})^{1/2}\right])$$
$$\text{where} \quad \bar{r} = (\bar{x}^2+\bar{y}^2+\bar{z}^2)^{1/2} \tag{3}$$

It is convenient to transform to the SEZ coordinate system with the origin at the mean position, so that fix coordinates are $(u_i,\ v_i,\ w_i)$, where the u axis points south, the v axis points east, and the w axis points toward the zenith. The uv plane is thus perpendicular to the radius vector from the center of the earth through the mean position, and is parallel to the plane tangent to the earth at $(\bar{L},\bar{l})$. The transformation to (u_i, v_i, w_i) coordinates is given in matrix notation as

$$\begin{bmatrix} u_i \\ v_i \\ w_i \end{bmatrix} = \begin{bmatrix} \sin\bar{L}\cos\bar{l} & \sin\bar{L}\sin\bar{l} & -\cos\bar{L} \\ -\sin\bar{l} & \cos\bar{l} & 0 \\ \cos\bar{L}\cos\bar{l} & \cos\bar{L}\sin\bar{l} & \sin\bar{L} \end{bmatrix} \begin{bmatrix} x_i-\bar{x} \\ y_i-\bar{y} \\ z_i-\bar{z} \end{bmatrix} \tag{4}$$

This transformation is the result of a rotation by angle $\bar{l}$ about the z axis, followed by a rotation by angle $\bar{L}$ about the new y axis. Evidently, the (u_i, v_i, w_i) values have zero means, and are suitable for consideration as random variables. It is desired to define a confidence ellipse in two dimensions. For this purpose the fix coordinate w_i will be set to zero, which amounts to a projection of the fixes onto the uv plane. In this plane the fixes may appear as illustrated in Figure 1. The fixes are the points, which are distributed more or less elliptically about the XY axes, which are rotated by amount Θ from the uv axes. Also sketched is a dashed line estimate of, say, a 90% confidence ellipse, which reflects the shape of the distribution of fixes. The fix coordinates (u_i, v_i) in the uv plane become fix coordinates (X_i, Y_i) in the XY plane through the transformation

$$\begin{bmatrix} X_i \\ Y_i \end{bmatrix} = \begin{bmatrix} \cos\Theta & \sin\Theta \\ -\sin\Theta & \cos\Theta \end{bmatrix} \begin{bmatrix} u_i \\ v_i \end{bmatrix} \tag{5}$$

The angle θ is determined from the condition

$$\Sigma_i X_i Y_i = 0, \tag{6}$$

so that the distribution of fixes among the quadrants formed by X and Y axes is about equal (cf. Fig. 1). Hence, Θ is given by

$$\Theta = \frac{1}{2} \tan^{-1} \left[\frac{2\,\Sigma_i u_i v_i}{\Sigma_i (u_i^2 - v_i^2)} \right] \tag{7}$$

In order to define the confidence region it will be assumed that fixes are distributed according to a bivariate normal distribution, so that the probability per unit area for a fix at (X,Y) is

$$P(X,Y) = N(X,\sigma_x)\, N(Y,\sigma_y) \quad , \tag{8a}$$

where

$$N(X,\sigma_x) = (2\pi\sigma_x^2)^{-1/2} \exp(-X^2/2\sigma_x^2) \tag{8b}$$

Here, the standard deviation parameters are estimated from the data as

$$\sigma_x^2 = \Sigma_{i=1}^{N} X_i^2 \;/\; (N-1) \qquad \sigma_y^2 = \Sigma_{i=1}^{N} Y_i^2 \;/\; (N-1) \tag{9}$$

The use of (8) implies that X and Y are independent statistical variables. This is consistent with the condition (6), which would give a calculated correlation coefficient of zero between these variables. It is worth noting, however, that while (6) is a necessary condition for variable independence, it is not a sufficient condition. Nevertheless, the use of (6) - (9) in conjunction with the data seems to give a logical basis for defining a confidence region. Contours of constant probability constitute the family of ellipses

$$X^2/\sigma_x^2 + Y^2/\sigma_y^2 = A^2 \tag{10}$$

Confidence C can be associated with each ellipse in (10) as the integrated probability within its area ($0 < C < 1$), and the associated confidence region has the least possible area. It is well known, and is demonstrated in Appendix A, that A in (10) is determined from a given confidence C according to

$$A = \sqrt{-2 \ln (1-C)} \tag{11}$$

Hence, for example, for a 90% confidence ellipse (C = 0.9), A = 2.146.

The above confidence ellipse in the XY plane maps back into latitude-longitude coordinates by inverting the above series of transformations. A point on the ellipse in (10) has (u,v) coordinates obtained from (5) through the inverse of the matrix there, which is simply its transpose. The value of w which projects this point onto the sphere is found from

$$w = (R^2 - u^2 - v^2)^{1/2} - \bar{r} \quad , \tag{12}$$

where $\bar{r}$ is defined in (3).

Then (x,y,z) on the sphere is given by the inverse of the relation in (4), which again involves the transpose of the matrix there. Finally, the latitude-longitude coordinates (L,l) of the confidence ellipse points are given by (3) without the overhead bars and with $\bar{r}$ replaced by R.

3. COMPUTER PROGRAM FOR THE CONFIDENCE ELLIPSE

The program was written in BASIC for use on the Tektronix 4050-4054 microcomputers. A listing is found in Appendix B. The listing shows several starred REMARK statements which partition the program into its sections. The "INPUTS" section prompts the user to specify the distribution of fixes and the ellipse confidence factor. The "FIND GEO. EQ. COORDINATES" section finds geocentric - equatorial fix coordinates from (1) and the mean position from (2), and locates latitude and longitude extrema P1-P4 of the fixes for later use in the graphical display. The latitude and longitude of the mean point are found from (3) in the "CENTER OF GRAVITY" section. The "FIND S.E.Z. COORDINATES" section computes (u,v,w) coordinates of the fixes from (4), and saves $u^2 + v^2$ for each fix. The "TRIMMING SECTION" section eliminates fixes one at a time, based on distance from the mean point, until the number of fixes specified in the "INPUTS" section have been trimmed out of the set. After each fix is eliminated, the mean position and (u,v,w) coordinates of the remaining fixes are recalculated. In the "X-Y NORMALIZED COORDINATES" section the orientation of the XY axes in the uv plane is found from (7), the (X,Y) coordinates of the fixes are found from (5), and the variance parameter summations in (9) are calculated. The "DRAWING GRAPH BACKGROUND" section draws and labels the graph grid lines, using the information that

$$P1 \leq \text{lon.} \leq P2 \quad \text{and} \quad P3 \leq \text{lat.} \leq P4.$$

It plots the latitude-longitude fixes as crosses by calling the subroutine in the "DRAWING POINTS ON GRAPH" section, beginning at Statement 2440, and encircles the points that were eliminated from the set of fixes. The "SET ELLIPSE IN X-Y COORDINATE SYSTEM" section calculates and stores the (X,Y) coordinates of 121 points on the confidence ellipse according to (10) and (11). In the "ELLIPSE IN S.E.Z" section, the (u,v,w,) coordinates of each ellipse point are found from (12) and the inverse of (5). The geocentric-equatorial coordinates of the ellipse point are found from the inverse of (4) in the "GEO. EQ. COORDINATES" section. The "LAT. AND LON." section evaluates lat.-lon. coordinates of the ellipse point from an equation analagous to (3), and draws a straight line to it from the preceding ellipse point, thus tracing out a portion of the confidence region. The "FINDING AXIS POINTS OF ELLIPSE" section is branched to from the preceding section. It stores the points where the ellipse axes intersect the ellipse. The "OUTPUTS" section prints out this information along with other relevant data next to the graph.

The final "OUTPUTS" section listings will be discussed with reference to specific examples below. In terms of methodology, however, the lengths of the semi-major and semi-minor axes are determined from the law of cosines in spherical trigonometry, and printed out in statements (St's.) 3010-3150. The computed angle T3 in St. 3180 is the negative of the angle T9 found from equation (7). Recall that Θ measured the angle of the X axis from south to east. The angle T3 measures the angle of the -X axis from north to east. Whether this is the major or the minor axis of the ellipse depends on which one is more closely aligned with the north-south direction.

In the first "INPUTS" section, there are two types of inputs, keyboard and data file. The data file is specified in St. 160 by keyboard, and in St's. 230-290 the confidence factor in percent T7 and trimming percentage E4 are specified. The binary data file M contains in sequence: (1) a character string containing the date, (2) a transmitter I.D. code, (3) the number of fixes N, and (4) an NX2 array of lat.-lon. fixes ($-90 \leq L1 \leq 90$, $-180 \leq L0 \leq 180$). This data file is read using the "READ @ 33:" commands.

4. EXAMPLES OF COMPUTER PROGRAM OUTPUT

mple ou' its of the preceding computer program are shown in Figures

2-5. In Fig. 2 a 90% confidence ellipse (C = 0.9) is shown for a distribution of fixes shown by crosses. The principal reason for the shape distortion of the ellipse is the nature of the lat-lon grid. Distances associated with longitude changes at high latitude are substantially smaller than corresponding changes at low latitudes.

Data listed to the side of the graph in Fig.2 consist of: (1) the date and transmitter identification no., (2) C listed as a percent and the associated value of A from (11), (3) the percent trim desired, (4) the total number of fix points and this number minus those trimmed in the evaluation of the confidence ellipse (none were trimmed in this example), (5) the lat-lon coordinates of the X axis intercepts with the ellipse, and the same for the Y axis intercepts, (6) the lat.-lon. coordinates of the mean point, which is the center of the ellipse in the uv plane, (7) the lengths of the semi-minor and semi-major (measured from the center of the ellipse) axes on the surface of the earth, with the first number associated with the Y axis and the second number with the X axis, and (8) the orientation angle (between 0 and 180) of the major axis of the ellipse, as measured from north to east. Latitude is measured along the vertical axis in the plot; longitude is measured along the horizontal axis.

Figure 3 shows the 90% ellipse for the same set of fixes, but with two fixes trimmed out of the set. The trimmed values are encircled on the graph by the program. Note that the number of points used in the calculation is 43 out of the initial 45. As expected, the confidence ellipse is slightly smaller. Evidently, the trim option has been used arbitrarily and only for the sake of illustration. Ross's [1975] discussion shows that caution and restraint are in order for the use of this option.

In Figure 4 an 80% confidence ellipse has been calculated for the same set of 45 fixes. Naturally, it is somewhat smaller.

In Figure 5 a 90% confidence ellipse has been calculated for a different distribution of fixes. The elliptical shape is less distorted in appearance. This is apparently because of the more southerly location of this example, as compared to Figure 2. Because of this, there is less deviation in distance per degree of longitude as the latitude changes. This causes less distortion in the mapping from the uv plane to the linear latitude-longitude grid used here.

5. DISCUSSION

A statistical confidence region has been defined with respect to a distribution of fixes obtained by a SSL system. The size and shape of this region is dictated by the characteristics of the fix distribution and the degree of confidence that another fix would be inside the region. Fluctuations in the system environment are responsible for the amount of scatter of the fix distribution. A confidence region thus represents the limit of the ability of an SSL system to locate the true position.

On the other hand, there may be systematic errors associated with SSL system algorithms for converting measurements to fixes. The effect of these, barring a fortuitous cancellation of systematic errors, is to displace a confidence region with respect to the true transmitter position. The amount of this displacement relative to a relevant dimension of the confidence region is thus one way to evaluate the importance of systematic error relative to inherent random error. Hence, one procedure for judging this is as follows: (1) Start with the latitude-longitude position (L,l) of the true transmitter position, known after the fact, (2) find its coordinates X,Y in the plane perpendicular to and including the radius vector to the mean point, as described in Section 2, (3) use equation (10) of that section to determine the parameter A, and (4) let A/2.146 be the parameter which characterizes the systematic error displacement relative to the appropriate dimension of the 90% confidence ellipse. Systematic errors in an SSL system cannot be considered to be eliminated until this parameter is calculated to be consistently on the order of unity or less for several tests on check transmitters.

It is possible that two SSL systems which operate differently will have significantly different confidence regions. The relative sizes of these confidence regions is not necessarily the criterion for selecting one system over the other. The system with the smaller confidence region, assuming that both systems have a logical rationale for converting measurements to fixes, may hold the greater promise of the two. In terms of present system performance, however, it would seem that both systems have to be judged on their relative success in obtaining mean positions close to the true positions of check transmitters which are known only after the fact. These results would reflect the relative amounts of residual systematic and random error in the two systems. The rms average of these displacements is, of course, one meaningful summary parameter for such tests.

ACKNOWLEDGMENTS

The authors would like to thank D.R. Uffelman, A.J. Martin, and J.M. Goodman for helpful discussions on this problem.

REFERENCES

1. Gething P.J.D. (1978), Radio Direction Finding, Peter Peregrinus Ltd., on behalf of the IEEE, England.

2. Ross W., (1975), "Wild Bearings in High-Frequency Direction-Finding", Proc. IEE, 122, pp 337-339.

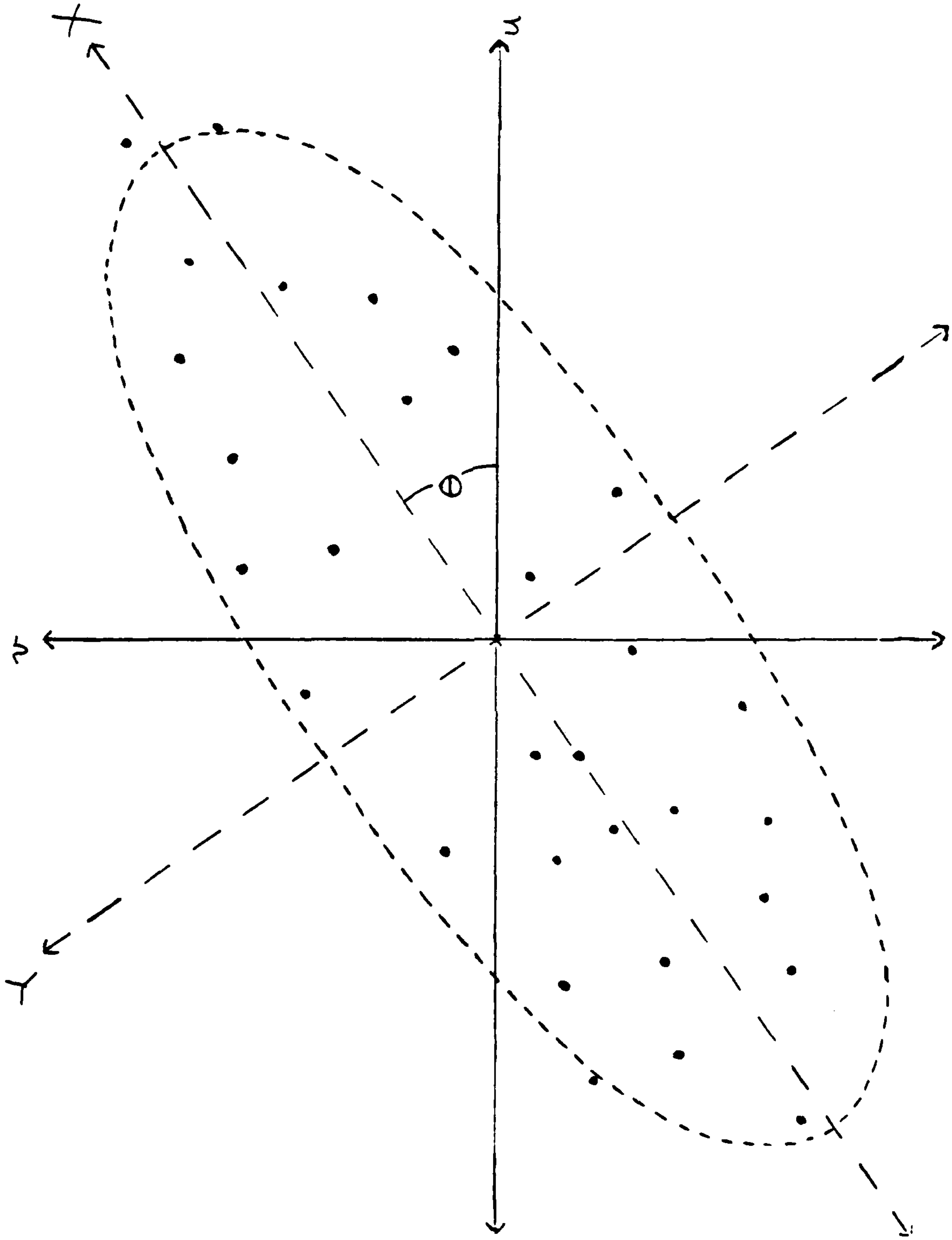


Fig. 1: Illustration of transformation from uv to XY axes. the set of fixes are sketched as points.

```
==================
DATE: 82/12/15
XMTR I.D. 20
==================
 90% CONFIDENCE
DEV. MULT. 2.146
==================
 0% TRIM
==================
POINTS DRAWN
 45
POINTS USED
 45
==================
HORIZ.(64.72 69.93)
      (34.56 38.46)

VERT. (52.96 41.6)
      (47.83 55.81)
==================
CENTER AT
      (50.61 49.09)
==================
SEMI-AXIS LENGTHS
 577.35 KM
 1980.4 KM
==================
ORIENTATION ANGLE
 29.77 DEGREES
==================
```

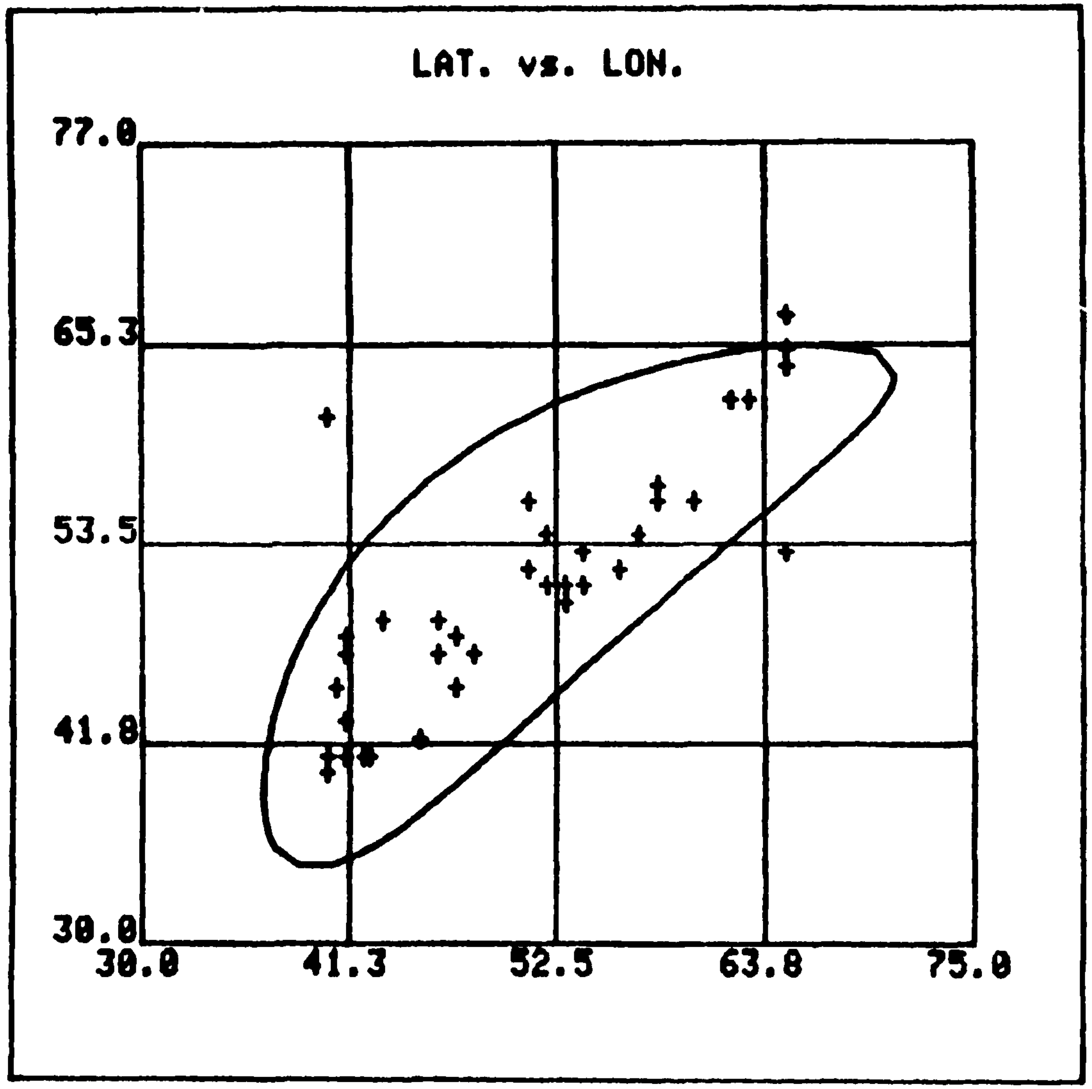


Fig. 2: Computed 90% confidence ellipse - no trim.

```
===================
DATE: 82/12/15
XMTR I.D. 20
===================
 90% CONFIDENCE
DEV. MULT. 2.146
===================
 5% TRIM
===================
POINTS DRAWN
 45
POINTS USED
 43
===================
HORIZ.(62.51 67.18)
      (35.52 38.23)

VERT. (52.36 41.45)
      (46.97 55.05)
===================
CENTER AT
      (49.86 48.63)
===================
SEMI-AXIS LENGTHS
 572.5 KM
 1802.5 KM
===================
ORIENTATION ANGLE
 31.74 DEGREES
===================
```

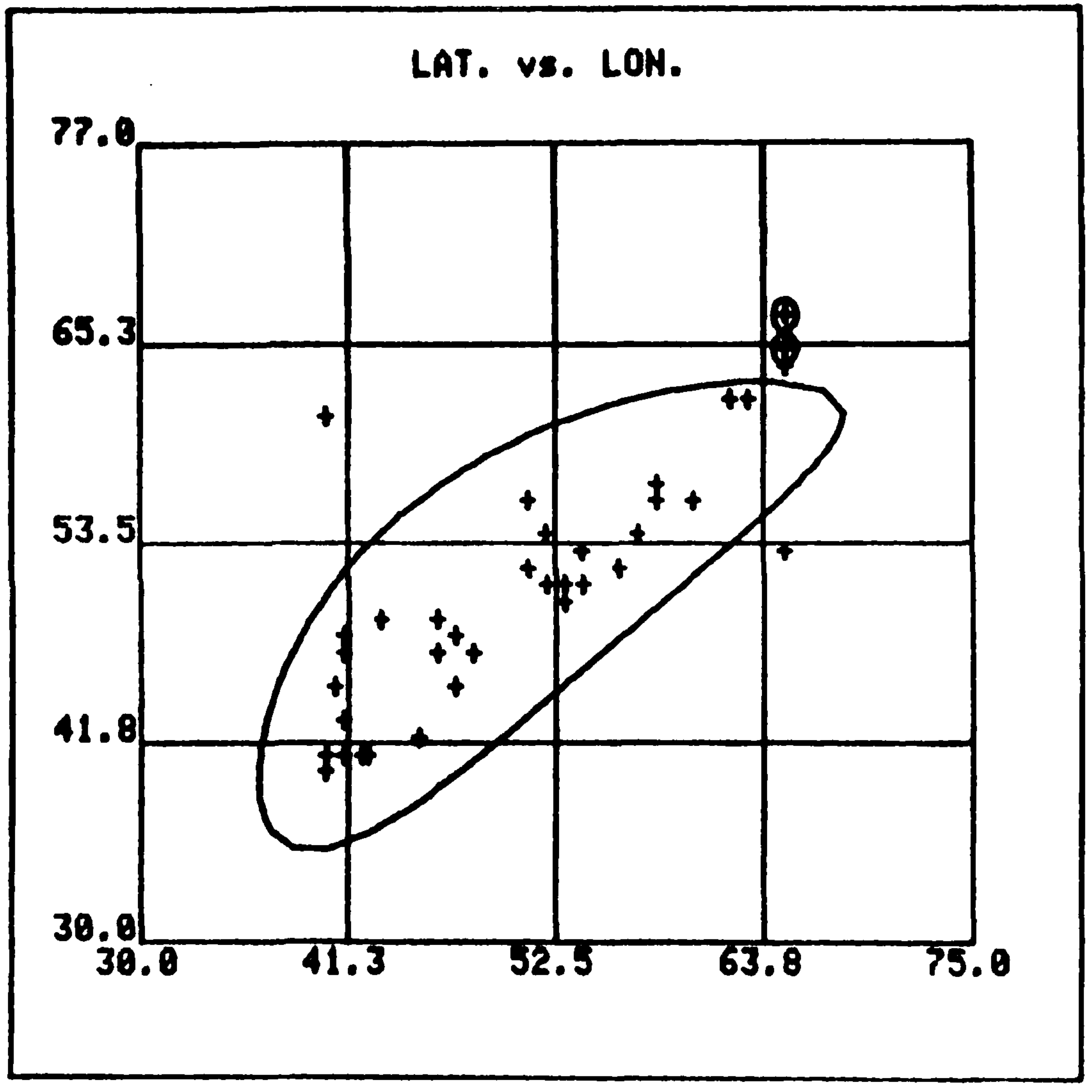


Fig. 3: Computed 90% confidence ellipse with trim.

```
==================
DATE: 82/12/15
XMTR I.D. 20
==================
 80% CONFIDENCE
DEV. MULT. 1.7941
==================
 0% TRIM
==================
POINTS DRAWN
 45
POINTS USED
 45
==================
HORIZ.(62.62 65.12)
      (37.32 39.9)

VERT. (52.6 42.88)
      (48.31 54.76)
==================
CENTER AT
      (50.61 49.09)
==================
SEMI-AXIS LENGTHS
 482.6 KM
 1647.5 KM
==================
ORIENTATION ANGLE
 29.77 DEGREES
==================
```

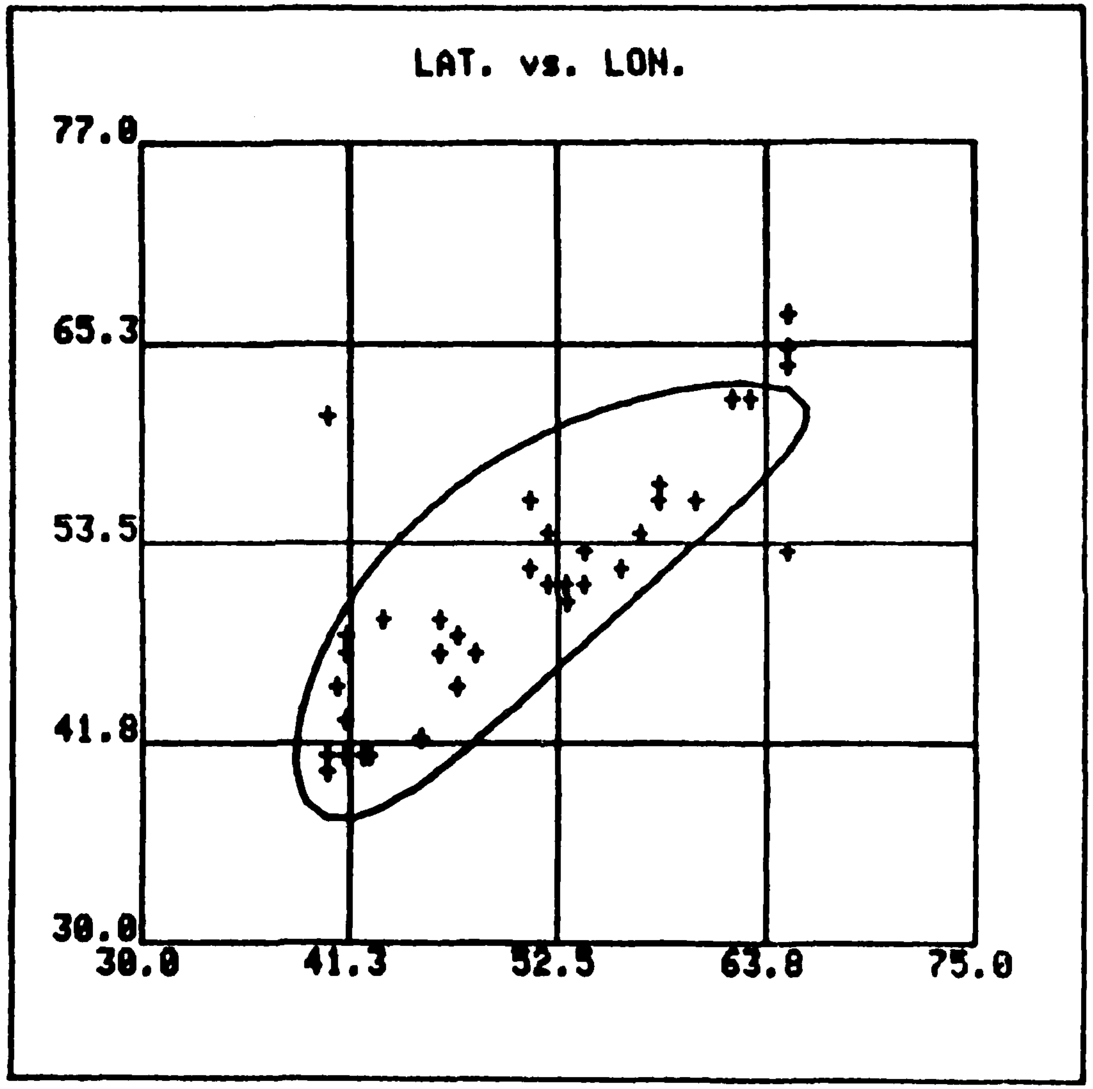


Fig. 4: Computed 80% confidence ellipse - no trim.

```
==================
DATE: 82/12/20
XMTR I.D. 40
==================
 90% CONFIDENCE
DEV. MULT. 2.146
==================
 0% TRIM
==================
POINTS DRAWN
 20
POINTS USED
 20
==================
HORIZ.(30.3 -128.9)
      (15.65 -119.13)

VERT. (20.45 -128.22)
      (25.53 -119.11)
==================
CENTER AT
      (23.05 -123.75)
==================
SEMI-AXIS LENGTHS
 544.85 KM
 954.65 KM
==================
ORIENTATION ANGLE
 148.71 DEGREES
==================
```

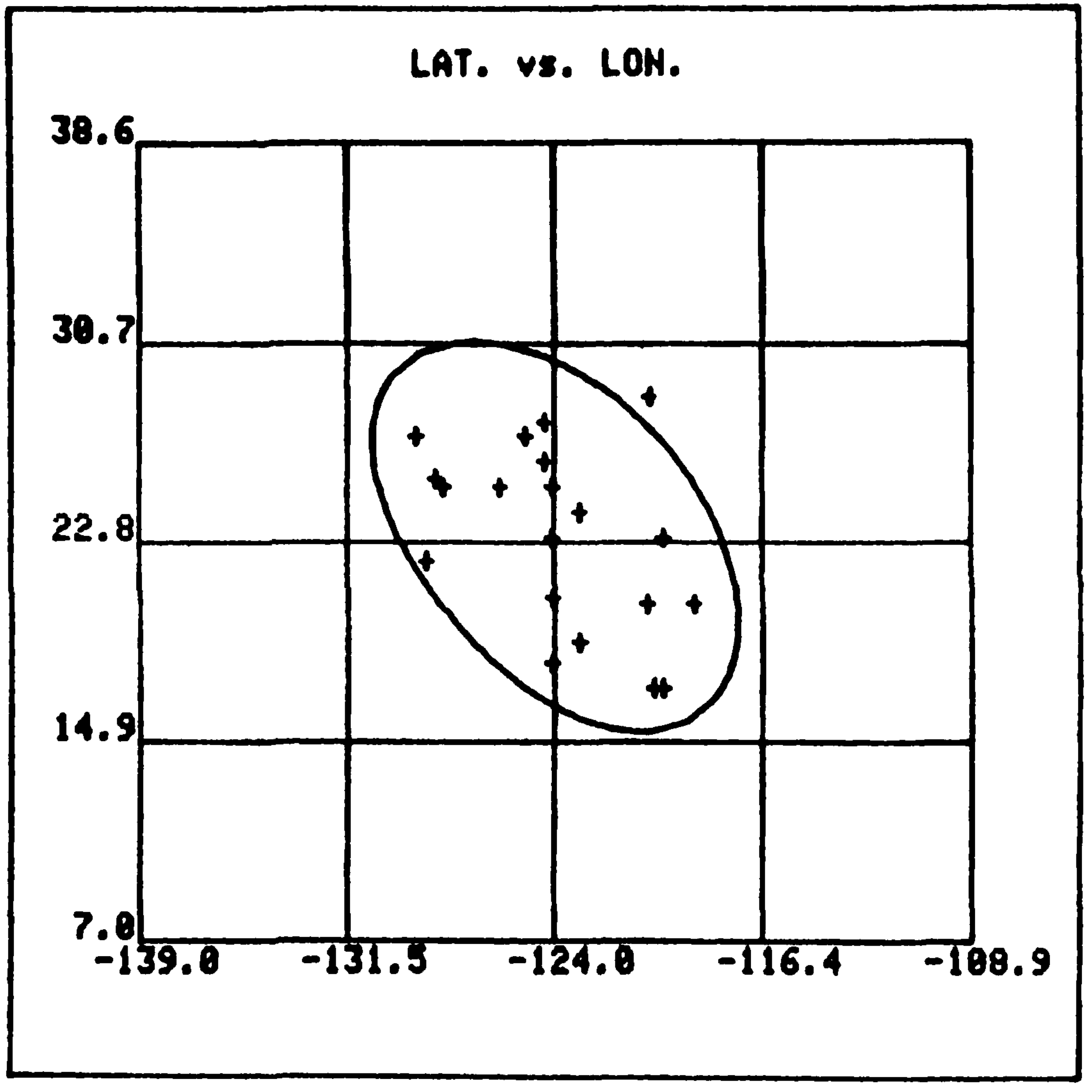


Fig. 5: Computed 90% confidence ellipse - different fixes - no trim.

APPENDIX A - CONFIDENCE ELLIPSE RELATION

The value of A in (10) of the text defines the boundary of the domain of integration D, such that the integration of P (X,Y) in (8) over D yields the prescribed confidence C. With the transformation to dimensionless variables

$$\xi = X / \sigma_x , \quad \eta = Y / \sigma_y \tag{A1}$$

the integral becomes

$$C = \iint_D P\,(X,Y)\, d\,X\, d\,Y = (2\pi)^{-1} \iint_{D'} \exp\,[-(\xi^2 + \eta^2)\,/2]\, d\,\xi\, d\eta, \tag{A2}$$

where the transformed domain D' is now the interior of a circle of radius A in the $\xi\eta$ plane. Hence, circular polar coordinates are advantageous, and the relation becomes

$$C = (2\,\pi)^{-1} \int^{2\pi} d\theta \int_0^A dr \cdot r \exp\,(-r^2/2) = 1 - \exp\,(-A^2\,/2) \tag{A3}$$

This is equivalent to (11) of the text.

APPENDIX B - PROGRAM LISTING

```
100 REM T3F8                  CONFIDENCE ELLIPSE FOR S.S.L. TEST
110 REM                       ==================================
120 REM
130 INIT
140 PAGE
150 REM *** INPUTS **************************************************
160 PRINT "WHICH DATA FILE ?"
170 INPUT M
180 FIND M
190 READ @33:D$
200 PRINT "DATE: ";D$
210 READ @33:D1
220 PRINT "XMTR I.D.  ";D1
230 PRINT "ELLIPSE CONFIDENCE FACTOR ?"
240 INPUT T7
250 PRINT "TRIM ?"
260 INPUT E$
270 IF E$="NO" THEN 310
280 PRINT "PERCENT TRIM ?"
290 INPUT E4
300 E3=100/E4
310 READ @33:N
320 N9=N
330 R=6371.2
340 DELETE L1,LO,X,Y,Z,U,V,W,X1,Y1,R3
350 DIM L1(N),LO(N),X(N),Y(N),Z(N),U(N),V(N),W(N)
360 DIM X1(N),Y1(N),R3(N),X3(121),Y3(121)
370 X9=0
380 Y9=0
390 Z9=0
400 S1=0
410 S2=0
420 C=0
430 Y8=0
440 X8=0
450 I9=0
460 FOR J9=1 TO N
470 READ @33:L1(J9)
480 READ @33:LO(J9)
490 L1(J9)=L1(J9)*(PI*2/360)
500 LO(J9)=LO(J9)*(PI*2/360)
510 NEXT J9
520 P1=180
530 P2=-180
540 P3=90
550 P4=-90
560 C9=-1
570 E=0
580 IF E$="NO" THEN 600
590 E=INT(N/E3)
600 C9=C9+1
610 X9=0
620 Y9=0
630 Z9=0
640 REM *** FIND GEO. EQ. COORDINATES ***********************************
650 FOR J8=1 TO N
660 X(J8)=R*COS(L1(J8))*COS(LO(J8))
```

```
670 Y(J8)=R*COS(L1(J8))*SIN(L0(J8))
680 Z(J8)=R*SIN(L1(J8))
690 X9=X(J8)+X9
700 Y9=Y(J8)+Y9
710 Z9=Z(J8)+Z9
720 IF L1(J8)>P3 THEN 740
730 P3=L1(J8)
740 IF L1(J8)<P4 THEN 760
750 P4=L1(J8)
760 IF L0(J8)>P1 THEN 780
770 P1=L0(J8)
780 IF L0(J8)<P2 THEN 800
790 P2=L0(J8)
800 NEXT J8
810 X5=X9/N
820 Y5=Y9/N
830 Z5=Z9/N
840 R8=SQR(X5^2+Y5^2+Z5^2)
850 REM *** CENTER OF GRAVITY ******************************************
860 L7=ASN(Z5/R8)
870 W3=1
880 L8=ASN(Y5/(X5^2+Y5^2)^0.5)
890 IF X5=>0 THEN 930
900 IF Y5=0 THEN 920
910 W3=SGN(Y5)
920 L8=-L8+PI*W3
930 REM *** FIND S.E.Z. COORDINATES ************************************
940 FOR J7=1 TO N
950 X2=X(J7)-X5
960 Y2=Y(J7)-Y5
970 Z2=Z(J7)-Z5
980 U9=Z2*COS(L7)
990 U(J7)=X2*SIN(L7)*COS(L8)+Y2*SIN(L7)*SIN(L8)-U9
1000 V(J7)=Y2*COS(L8)-X2*SIN(L8)
1010 W9=Z2*SIN(L7)
1020 W(J7)=X2*COS(L7)*COS(L8)+Y2*COS(L7)*SIN(L8)+W9
1030 R3(J7)=U(J7)^2+V(J7)^2
1040 NEXT J7
1050 REM *** TRIMMING SECTION ******************************************
1060 IF E=C9 THEN 1250
1070 R6=0
1080 FOR K7=1 TO N
1090 IF R6>R3(K7) THEN 1120
1100 R6=R3(K7)
1110 N0=K7
1120 NEXT K7
1130 L4=L0(N0)
1140 L5=L1(N0)
1150 N=N-1
1160 IF N0=N+1 THEN 1210
1170 FOR C6=N0 TO N
1180 L0(C6)=L0(C6+1)
1190 L1(C6)=L1(C6+1)
1200 NEXT C6
1210 L0(N+1)=L4
1220 L1(N+1)=L5
1230 GO TO 600
1240 REM *** X-Y NORMALIZED COORDINATES ********************************
1250 FOR J6=1 TO N
1260 S1=S1+U(J6)*V(J6)
```

```
1270 S2=S2+U(J6)^2-V(J6)^2
1280 NEXT J6
1290 T9=0.5*ATN(2*S1/S2)
1300 FOR K5=1 TO N
1310 X1(K5)=U(K5)*COS(T9)+V(K5)*SIN(T9)
1320 Y1(K5)=V(K5)*COS(T9)-U(K5)*SIN(T9)
1330 X8=X8+X1(K5)^2
1340 Y8=Y8+Y1(K5)^2
1350 NEXT K5
1360 FOR K4=1 TO N9
1370 L1(K4)=L1(K4)*360/(2*PI)
1380 L0(K4)=L0(K4)*360/(2*PI)
1390 NEXT K4
1400 REM *** DRAWING GRAPH BACKROUND ***********************************
1410 PAGE
1420 VIEWPORT 38,128,7,93
1430 WINDOW 0,100,0,100
1440 MOVE 0,0
1450 DRAW 0,100
1460 DRAW 100,100
1470 DRAW 100,0
1480 DRAW 0,0
1490 MOVE 50,100
1500 PRINT "JJHHHHHHLAT. vs. LON."
1510 VIEWPORT 49,119,18,82
1520 P1=P1*360/(2*PI)-10
1530 P2=P2*360/(2*PI)+10
1540 P3=P3*360/(2*PI)-10
1550 P4=P4*360/(2*PI)+10
1560 WINDOW P1,P2,P3,P4
1570 MOVE P1,P3
1580 DRAW P2,P3
1590 DRAW P2,P4
1600 DRAW P1,P4
1610 DRAW P1,P3
1620 H=(P2-P1)/4
1630 H2=(P4-P3)/4
1640 FOR I5=1 TO 3
1650 MOVE P1+I5*H,P3
1660 DRAW P1+I5*H,P4
1670 MOVE P1,I5*H2+P3
1680 DRAW P2,I5*H2+P3
1690 NEXT I5
1700 FOR I6=1 TO 5
1710 I7=I6-1
1720 MOVE I7*H+P1,P3
1730 S9=INT((I7*H+P1)*100+0.5)/100
1740 PRINT USING """JHH""FD.1D":S9
1750 MOVE P1,P3+I7*H2
1760 S9=INT((I7*H2+P3)*100+0.5)/100
1770 PRINT USING """HHHHHH""4D.1D":S9
1780 NEXT I6
1790 FOR Z4=1 TO N9
1800 MOVE L0(Z4),L1(Z4)
1810 GOSUB 2440
1820 NEXT Z4
1830 SET DEGREES
1840 IF E=0 THEN 1960
1850 FOR H7=N+1 TO N9
1860 MOVE L0(H7),L1(H7)
```

```
1870 RMOVE (P2-P1)/70,0
1880 FOR L9=1 TO 10
1890 L6=L9*36
1900 X4=(P2-P1)/70*COS(L6)
1910 Y4=(P4-P3)/50*SIN(L6)
1920 DRAW L0(H7)+X4,L1(H7)+Y4
1930 NEXT L9
1940 NEXT H7
1950 REM *** SET ELLIPSE IN X-Y COORDINATE SYSTEM **********************
1960 SET RADIANS
1970 G9=SQR(-2*LOG(1-T7/100))
1980 A=G9*SQR(X8/(N-1))
1990 B=G9*SQR(Y8/(N-1))
2000 FOR J=1 TO 30
2010 C=J-1
2020 X3(J)=C*(A/30)
2030 Y3(J)=SQR(B^2*(1-X3(J)^2/A^2))
2040 X3(60+2-J)=X3(J)
2050 X3(60+J)=-X3(J)
2060 X3(122-J)=-X3(J)
2070 Y3(122-J)=Y3(J)
2080 Y3(62-J)=-Y3(J)
2090 Y3(60+J)=-Y3(J)
2100 NEXT J
2110 X3(31)=A
2120 X3(91)=-A
2130 Y3(31)=0
2140 Y3(91)=0
2150 REM *** ELLIPSE IN S.E.Z. ******************************************
2160 FOR J4=1 TO 121
2170 U3=X3(J4)*COS(T9)-Y3(J4)*SIN(T9)
2180 V3=X3(J4)*SIN(T9)+Y3(J4)*COS(T9)
2190 R9=SQR(U3^2+V3^2)
2200 W3=SQR(R^2-R9^2)-R8
2210 REM *** GEO. EQ. COORDINATES ***************************************
2220 X6=U3*SIN(L7)*COS(L8)-V3*SIN(L8)+W3*COS(L7)*COS(L8)+X5
2230 Y6=U3*SIN(L7)*SIN(L8)+V3*COS(L8)+W3*COS(L7)*SIN(L8)+Y5
2240 Z6=W3*SIN(L7)-U3*COS(L7)+Z5
2250 REM *** LAT. AND LON. **********************************************
2260 L3=ASN(Z6/R)
2270 L4=ASN(Y6/SQR(Y6^2+X6^2))
2280 IF X6>0 THEN 2320
2290 L4=PI-L4
2300 IF Y6>0 THEN 2320
2310 L4=L4-2*PI
2320 L4=L4*(360/(2*PI))
2330 L3=L3*(360/(2*PI))
2340 IF J4>1 THEN 2370
2350 MOVE L4,L3
2360 GO TO 2380
2370 DRAW L4,L3
2380 IF J4=1 THEN 2510
2390 IF J4=31 THEN 2540
2400 IF J4=61 THEN 2570
2410 IF J4=91 THEN 2600
2420 NEXT J4
2430 GO TO 2630
2440 REM *** DRAWING POINTS ON GRAPH ************************************
2450 RMOVE 0,(P4-P3)/100
2460 RDRAW 0,-(P4-P3)/50
```

```
2470 RMOVE (P2-P1)/140,(P4-P3)/100
2480 RDRAW -(P2-P1)/70,0
2490 RETURN
2500 REM *** FINDING AXIS POINTS OF ELLIPSE ****************************
2510 G1=INT(100*L3+0.5)/100
2520 K1=INT(L4*100+0.5)/100
2530 GO TO 2420
2540 G2=INT(100*L3+0.5)/100
2550 K2=INT(100*L4+0.5)/100
2560 GO TO 2420
2570 G3=INT(100*L3+0.5)/100
2580 K3=INT(100*L4+0.5)/100
2590 GO TO 2420
2600 G4=INT(100*L3+0.5)/100
2610 K4=INT(100*L4+0.5)/100
2620 GO TO 2420
2630 HOME
2640 REM *** OUTPUTS ***********************************************
2650 PRINT
2660 PRINT
2670 PRINT
2680 PRINT "==================== "
2690 PRINT "DATE: ";D$
2700 PRINT "XMTR I.D. ";D1
2710 PRINT "==================== "
2720 G9=INT(10000*G9+0.5)/10000
2730 PRINT T7;"% CONFIDENCE"
2740 PRINT "DEV. MULT. ";G9
2750 PRINT "==================== "
2760 IF E$="NO" THEN 2790
2770 P0=E4
2780 GO TO 2800
2790 P0=0
2800 PRINT P0;"% TRIM"
2810 PRINT "==================== "
2820 PRINT "POINTS DRAWN"
2830 PRINT N9
2840 PRINT "POINTS USED"
2850 PRINT N
2860 PRINT "==================== "
2870 PRINT "HORIZ.(";G4;K4;")"
2880 PRINT "       (";G2;K2;")"
2890 PRINT
2900 PRINT "VERT. (";G3;K3;")"
2910 PRINT "       (";G1;K1;")"
2920 PRINT "==================== "
2930 L8=L8*180/PI
2940 L7=L7*180/PI
2950 PRINT "CENTER AT"
2960 I8=INT(100*L8+0.5)/100
2970 I7=INT(100*L7+0.5)/100
2980 PRINT "       (";I7;I8;")"
2990 PRINT "==================== "
3000 PRINT "SEMI-AXIS LENGTHS"
3010 H9=1
3020 SET DEGREES
3030 Q1=K3
3040 Q2=K1
3050 Q3=G3
3060 Q4=G1
```

```
3070 GO TO 3130
3080 Q1=K4
3090 Q2=K2
3100 Q3=G4
3110 Q4=G2
3120 H9=2
3130 R7=R*ACS(SIN(Q4)*SIN(Q3)+COS(Q4)*COS(Q3)*COS(Q1-Q2))*PI/180
3140 PRINT 0.05*INT(R7*10);" KM"
3150 IF H9=1 THEN 3280
3160 PRINT "====================== "
3170 PRINT "ORIENTATION ANGLE"
3180 T3=-180*T9/PI
3190 T5=INT(T3*100+0.5)/100
3200 IF R7<V9 THEN 3240
3210 IF T5>0 THEN 3250
3220 T5=T5+180
3230 GO TO 3250
3240 T5=T5+90
3250 PRINT T5;" DEGREES"
3260 PRINT "====================== "
3270 END
3280 V9=R7
3290 GO TO 3080
```