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PHASE-ONLY NULLING AT SYMMETRIC PATTERN LOCATIONS

Robert A. Shore

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Nulls cannot be imposed in ideal linear array antenna patterns with phase-only weight control at locations symmetric with respect to the main beam if the phase perturbations are restricted to be small. If, however, no restriction is placed on the magnitude of the phase perturbations, then phase-only nulling at symmetric pattern locations is possible. Several examples are presented to illustrate symmetric nulling with phase-only control. The phase perturbations are calculated by a nonlinear programming computer algorithm. The cancellation patterns tend to be interferometric in shape and result in significant

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distortion over much of the pattern.

Preface

The author wishes to express his thanks to Randy Haupt and Dr. Hans Steyskal for their helpful comments and suggestions.

Contents

Illustrations

Illustrations

Phase-Only Nulling at Symmetric Pattern Locations

1. INTRODUCTION

Computational difficulties associated with the nonlinear problem of synthesizing nulls in linear array patterns with phase-only weight control are generally avoided by assuming that the phase perturbations are small. The small phase perturbation assumption enables the nulling equations to be linearized. A consequence of this assumption is[1,2,3] that the imposing of a null in the pattern of an ideal linear array is accompanied by the reinforcement of the pattern at the location symmetric with respect to the main beam, thus making it impossible to impose nulls at symmetric locations with small phase perturbations. It is the purpose of this report to draw attention to the fact that if the phase perturbations are not restricted in size, then it is indeed possible to impose nulls at symmetric locations with phase-only weight perturbations. Considerable pattern distortion, however, results from the fact that some of the phase perturbations are large.

(Received for publication 11 April 1983)

1. Baird, C., and Rassweiler, G. (1976) Adaptive sidelobe nulling using digitally controlled phase-shifters, IEEE Trans. Antennas Propag. AP-24:638-649.
2. Ananasso, F. (1981) Null-steering uses digital weighting, Microwave Systems News 11:78-94.
3. Steyskal, H. (1983) Simple method for pattern nulling by phase perturbation, IEEE Trans. Antennas Propag. AP-31:163-166.

2. ANALYSIS

We consider a linear array of N equispaced isotropic elements with inter-element spacing d and phase reference at the center of the array [illegible] a_n, n = 1, 2, ..., N, be the amplitude of the nth element, [illegible] amplitudes with respect to the reference center,

$$a_{N-n+1} = a_n, \quad n = 1, 2, \ldots, N$$

Then the array field pattern, $p_o(u)$, is

$$p_o(u) = \sum_{n=1}^{N} a_n e^{j d_n u}$$

where

$$d_n = \frac{N-1}{2} - (n-1), \quad n = 1, 2, \ldots, N$$

and

$$u = kd \sin \theta$$

with

$$k = 2\pi / \lambda$$

and θ the angle measured from broadside to the array. The $\{d_n\}$ are odd-symmetric with respect to the phase reference,

$$d_{N-n+1} = -d_n$$

and the pattern is real because of the symmetry of the $\{a_n\}$ and the $\{d_n\}$.

The equations for imposing nulls at a set of M locations, $u = u_m$, m = 1, 2, ..., M, with phase-only weight perturbations, $\{\exp(j\phi_n)\}$, are

$$\sum_{n=1}^{N} a_n e^{j\phi_n} e^{j d_n u_m} = 0, \quad m = 1, 2, \ldots, M. \qquad (1)$$

The set of Eqs. (1) is underdetermined if $M < N/2$, N even, or $M < (N-1)/2$, N odd, and has an infinity of solutions. However, a unique solution can be defined by imposing the requirement that the sum of the squares of the absolute values of the weight perturbations be a minimum; that is,

$$\sum_{n=1}^{N} \left[a_n \left| e^{j\phi_n} - 1 \right| \right]^2 = 4 \sum_{n=1}^{N} \left[a_n \sin\left(\frac{\phi_n}{2}\right) \right]^2 = \text{minimum} , \qquad (2)$$

a requirement that is useful to make in null synthesis to ensure that the perturbed pattern closely resemble the original pattern. The minimized weight perturbation, phase-only null synthesis problem is then to find the set of phases $\{\phi_n\}$ satisfying Eqs. (1) and (2). The problem is nonlinear in general (that is, if the phase perturbations are not assumed to be small) and cannot be solved analytically. Numerical solutions can be obtained, however, using nonlinear programming numerical techniques.[4] If nulls are required to be imposed at a pair of symmetric locations $u = \pm u_1$, the null equations are then

$$\sum_{n=1}^{N} a_n e^{j\phi_n} e^{\pm j d_n u_1} = 0 . \qquad (3)$$

It is simple to show (see Appendix A) that if the assumption of small phase perturbations is made and Eqs. (3) linearized via the approximation

$$\exp(j\phi_n) \sim 1 + j\phi_n \qquad (4)$$

then there is no solution possible to the resulting pair of equations. If the phase perturbations are not restricted to be small, however, then there is an infinity of possible solutions to Eqs. (3), and nonlinear programming methods can be used to obtain the solution that satisfies Eq. (2). Some examples are given in the following section.

For half wavelength element spacing, Eq. (2) is equivalent to minimizing the mean square difference between the original pattern and the perturbed pattern.

4. Shore, R. (1983) Phase-Only Nulling as a Nonlinear Programming Problem, RADC-TR-83-37.

3. RESULTS AND DISCUSSION

In this section we present a few examples of solutions to the problem of phase-only null synthesis at locations symmetric with respect to the main beam. All solutions were obtained using the nonlinear programming computer code LPNLP.[5] Initial values of 0.0001 radians were used for the unknown phases for all symmetric nulling examples.* The fact that the minimum phase perturbations are odd-symmetric with respect to the phase reference center[6] was used to reduce the number of unknown phases by a factor of two. All computations were performed for an array of 41 elements with half wavelength interelement spacing.

As the first example of imposing symmetrically located nulls, in Figures 1a-1c we show the original, perturbed, and cancellation pattern (that is, the perturbed field pattern minus the original field pattern) for the case of nulls imposed at ± 9.74°, the locations of the peaks of the third sidelobes, for a uniform amplitude distribution of the array. From the interferometric shape of the cancellation pattern in Figure 1c it might be guessed that only two elements of the array have significant phase perturbations, and this is indeed the case. The 15th and 27th element of the array (with a spacing of 6λ) have phase shifts of 154° while all other elements are shifted less than 1° in phase. The distortion of the pattern is considerable, exceeding 10 dB in many regions.

It can be clearly seen from Figures 1a and 1b that the perturbed pattern is not symmetric with respect to the main beam. This is a consequence of the odd-symmetry of the phase perturbations which, coupled with the even-symmetry of the element amplitudes, gives a cancellation pattern and a perturbed pattern that are not symmetric with respect to the location of the unperturbed main beam at $\theta = 0$, even though the nulling problem itself is completely symmetrical; that is, imposing nulls at symmetric locations in a symmetric pattern. For comparison with Figure 1, in Figures 2a-2c we show the original, perturbed, and cancellation pattern for the same array but with only one null imposed at +9.74°. All phase shifts for this case are less than 10.5° in magnitude, with 26 of the phase perturbations between 5.8° and 10.5°. Note the reinforcement of the pattern (~ 6 dB) at the location -9.74°. The cancellation pattern, Figure 2c, can be represented as the sum of two beams,[7] one directed toward the null location, and the other of opposite sign directed toward the symmetric location -9.74° adding in phase to the original pattern there. Note also how much more closely in general the perturbed pattern follows the original pattern in this example as compared with the first case.

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*Unlike for null synthesis at non-symmetric locations, LPNLP could not be started with zero initial values for the unknown phases.

(Due to the number of references cited above, they will not be listed here. See References, page 39.)

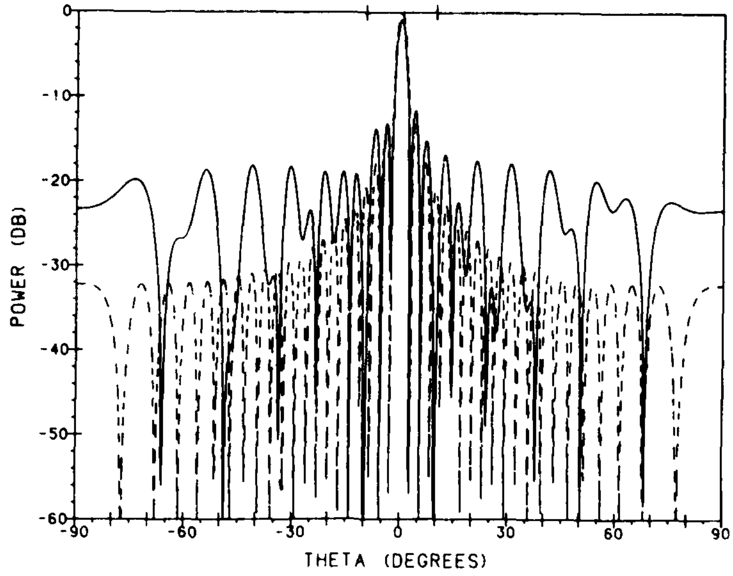

Figure 1a. Original Uniform Amplitude Pattern (-----) and Perturbed Pattern (———) With Nulls Imposed at ± 9.74°. θ = -90° to +90°

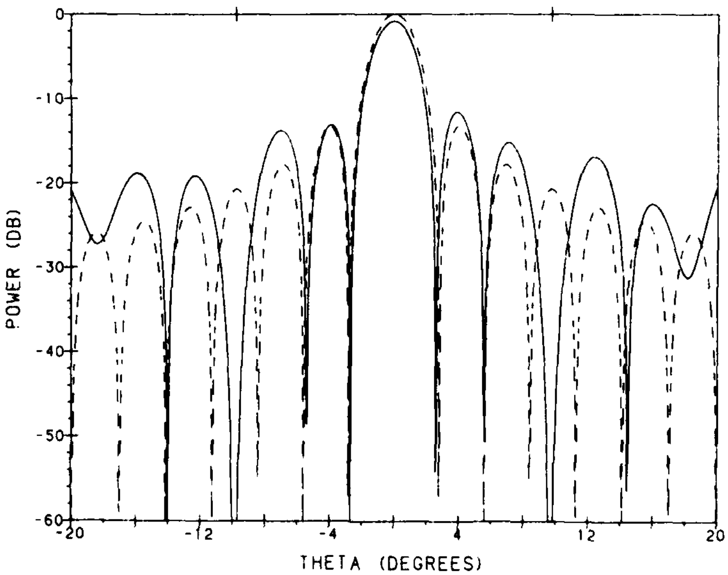

Figure 1b. Original Uniform Amplitude Pattern (-----) and Perturbed Pattern (———) With Nulls Imposed at ± 9.74°. θ = -20° to +20°

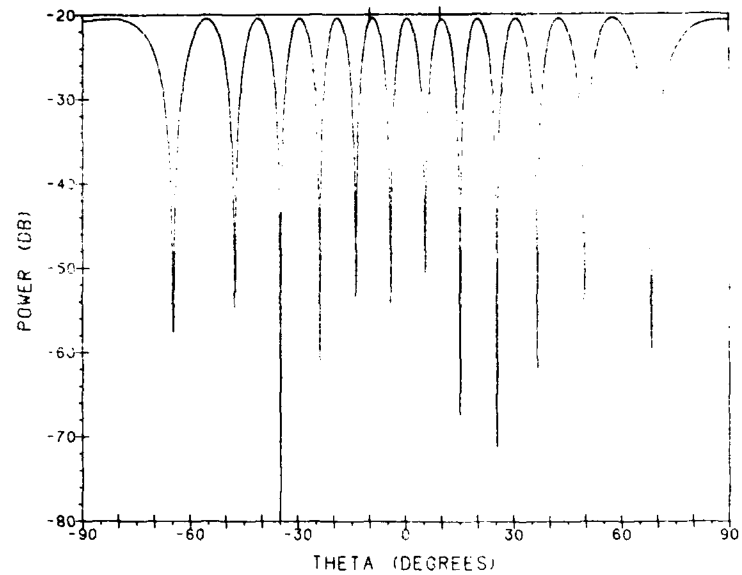

Figure 1c. Cancellation Pattern to Impose Nulls at ± 9.74° in Uniform Amplitude Pattern. θ = -90° to +90°

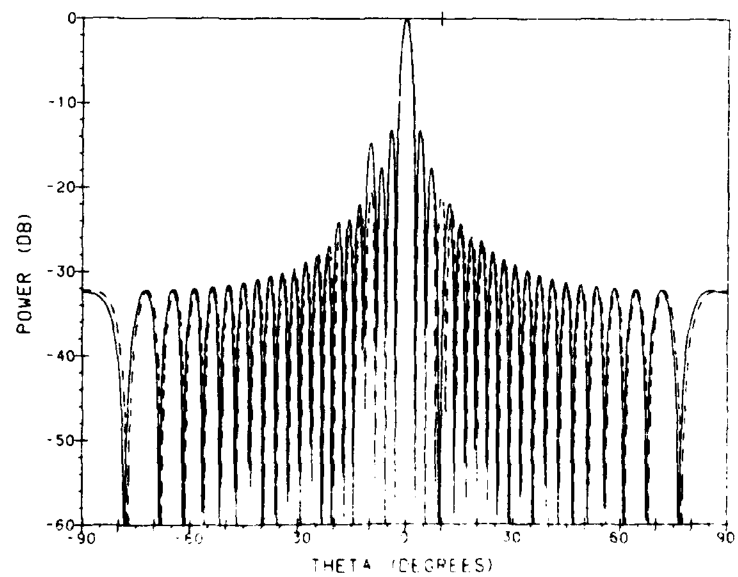

Figure 2a. Original Uniform Amplitude Pattern (-----) and Perturbed Pattern (———) With One Null Imposed at +9.74°. θ = -90° to +90°

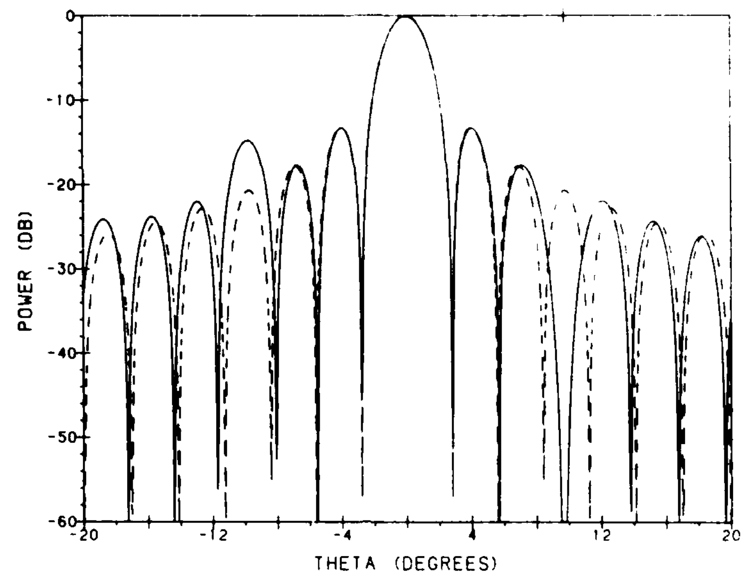

Figure 2b. Original Uniform Amplitude Pattern (-----) and Perturbed Pattern (———) With One Null Imposed at +9.74°. θ = -20° to +20°

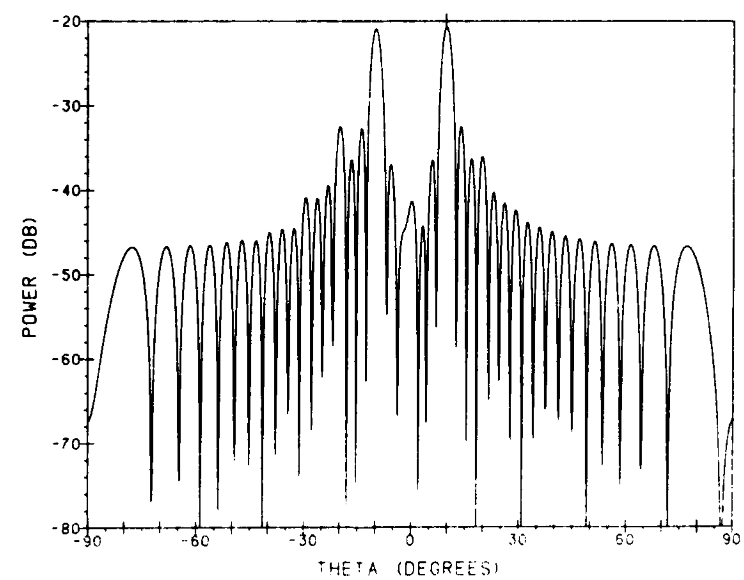

Figure 2c. Cancellation Pattern to Impose One Null at +9.74° in Uniform Amplitude Pattern. θ = -90° to +90°

The cancellation pattern resulting from equal and opposite shifts of the phases of two symmetrically placed elements is

$$2a_n \cos(\phi_n + d_n u) - 2a_n \cos(d_n u) = -4a_n \sin(\phi_n/2) \sin(\phi_n/2 + d_n u) .$$

This is an oscillatory function with amplitude $4a_n|\sin(\phi_n/2)|$ and period $2\pi/d_n$. For a cancellation pattern of this form to be able to produce nulls at symmetric pattern locations $u = \pm u_1$, it must be possible to find a value of n and a phase shift for which

$$-4a_n \sin(\phi_n/2) \sin(\phi_n/2 \pm d_n u_1) \approx -p_o(u_1) . \qquad (5)$$

This implies that there must be some element amplitude(s) of the order of $|p_o(u_1)|/4$, actually somewhat larger to allow for the decrease in magnitude from the product of the two sine factors. In the above example of nulls imposed at the peaks of the third sidelobes of a 41 element array with uniform amplitude and $\lambda/2$ spacing,

$$a_n = 1/41 = 0.02439$$

for all elements, and

$$p_o(9.74°) = -0.09241 .$$

As stated above, all phase shifts for this example were small except for shifts of 153.7° (2.68211 radians) in the 15th and 27th elements. Substituting

$$u_1 = 2\pi/\lambda \cdot \lambda/2 \sin(9.74°) = 0.53149$$

$$d_{15} = (41-1)/2 - (15-1) = 6$$

and

$$\phi_{15} = 2.68211$$

in Eq. (5) we obtain

$$(-0.09756)(0.97373)\begin{pmatrix}-0.98341\\ \\ -0.96186\end{pmatrix} = \left\{\begin{matrix}0.09342\\ \\ 0.09137\end{matrix}\right. \approx -p_o(9.74°) .$$

Suppose now that instead of imposing nulls at the peaks of the third sidelobes, we require nulls to be imposed at the locations of the peaks of the second sidelobes ($\pm$ 6.9°) of the same 41 element uniform amplitude array pattern. The value of the unperturbed pattern at $\pm$ 6.9° is 0.12913. In view of Eq. (5) it is clear that the cancellation pattern cannot be a simple interferometer pattern since

$$p_o(\pm u_1) = 0.12913 > 4a_n = 0.09756 .$$

In Figures 3a-3c we show patterns for this case. The cancellation pattern shown in Figure 3c is the superposition of two interferometric patterns, one from phase shifts of 176.4° in the 4th and 38th elements, and the other from shifts of 71.4° in the phases of the 5th and 37th elements. All other phase shifts are less than 6°. The relative amplitudes of the two interferometric patterns are

$$\sin(176.4/2)/\sin(71.4/2) = 1.7/1 .$$

The form of the cancellation pattern for symmetric, phase-only nulling can be more complicated than the first two examples have indicated. This can be seen in the next example in which we impose nulls at the outer 3 dB points of the third sidelobes ($\pm$ 10.48°) of the 41 element, uniform amplitude array pattern. Since the magnitude of the unperturbed pattern is only 0.0651 as compared with 0.0924 at the peak of the sidelobe, and since a simple interferometer pattern suffices to impose nulls at the locations of the peaks of the third sidelobes, it might be expected that the cancellation pattern for this case would also be a simple interferometric pattern. This, however, is not true as can be seen by referring to Figures 4a-4c where the patterns are shown. In this example, four pairs of elements undergo significant phase shifts: 103.7° for the 5th and 37th elements, 26.4° for the 16th and 26th elements, 14.9° for the 15th and 27th elements, and 10.3° for the 4th and 38th elements. The remainder of the phase shifts are less than 0.2°. The ratios of the amplitudes of the four interferometric patterns are

$$\sin(103.7/2) : \sin(26.4/2) : \sin(14.9/2) : \sin(10.3/2)$$

$$= 8.8 : 2.5 : 1.4 : 1 .$$

Even though four pairs of elements are shifted significantly in phase, the overall form of the cancellation pattern is quite strongly interferometric in character, dominated by the pattern corresponding to the phase shift of the 5th and 37th elements.

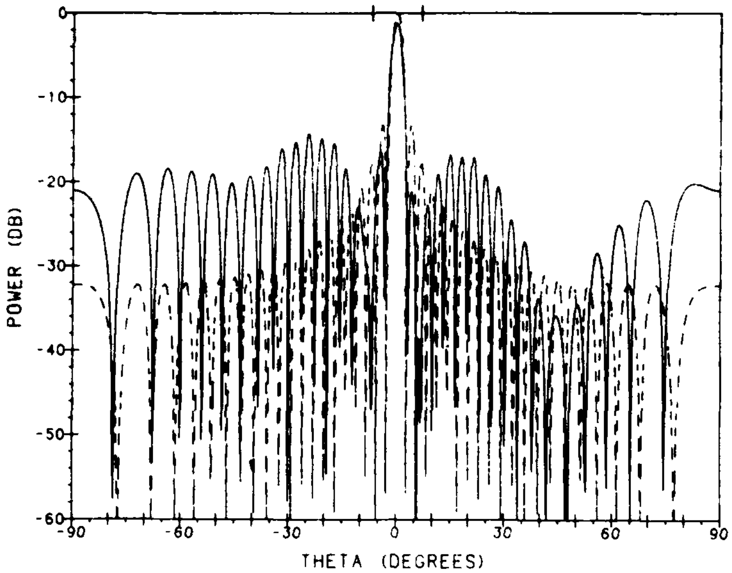

Figure 3a. Original Uniform Amplitude Pattern (-----) and Perturbed Pattern (———) With Nulls Imposed at ± 6.90° θ = -90° to +90°

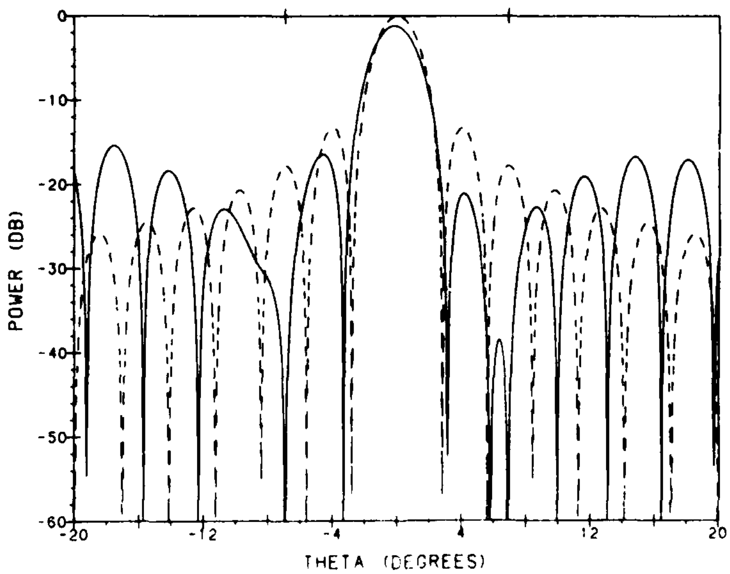

Figure 3b. Original Uniform Amplitude Pattern (-----) and Perturbed Pattern (———) With Nulls Imposed at ± 6.90°, θ = -20° to +20°

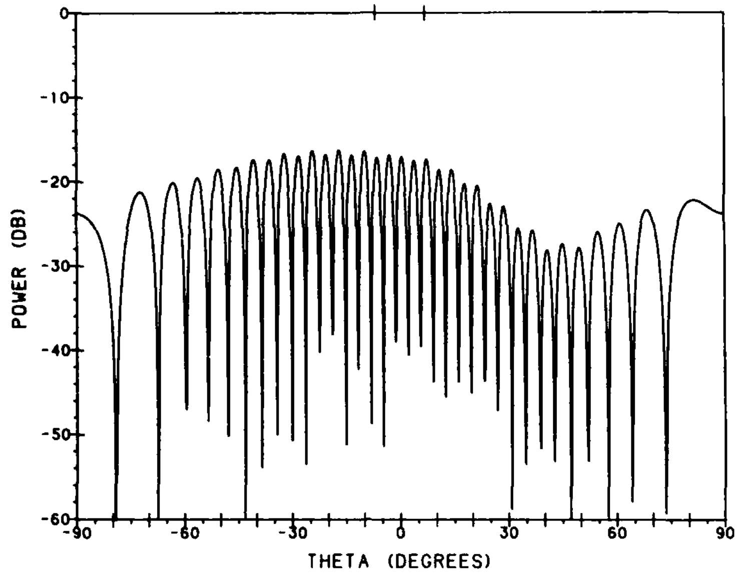

Figure 3c. Cancellation Pattern to Impose Nulls at ± 6.90 in Uniform Amplitude Pattern. θ = -90° to +90°

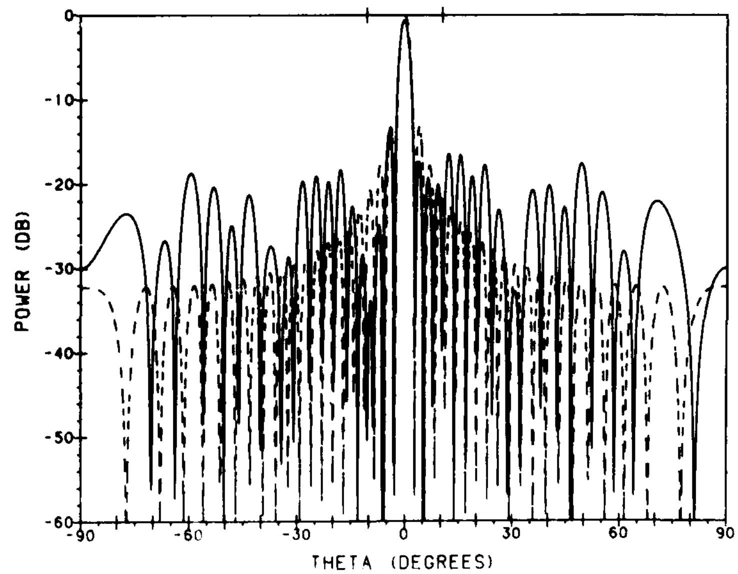

Figure 4a. Original Uniform Amplitude Pattern (-----) and Perturbed Pattern (———) With Nulls Imposed at ± 10.48°. θ = -90° to +90°

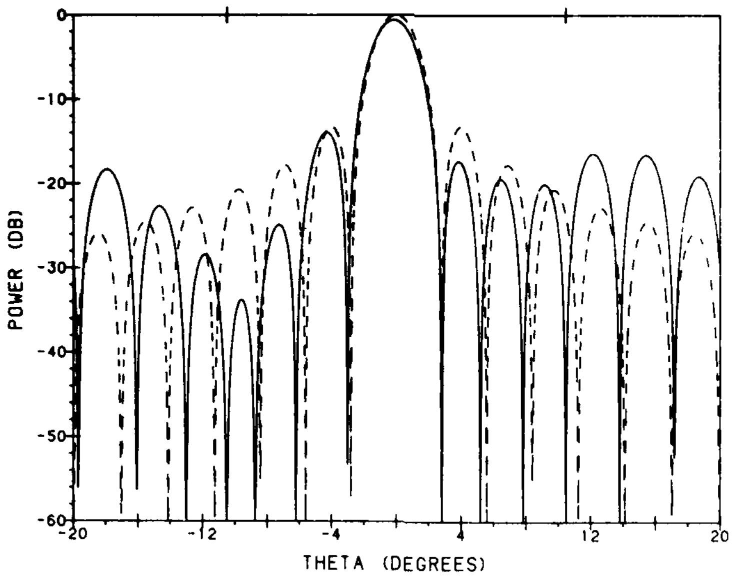

Figure 4b. Original Uniform Amplitude Pattern (-----) and Perturbed Pattern (———) With Nulls Imposed at ± 10.48. θ = -20° to +20°

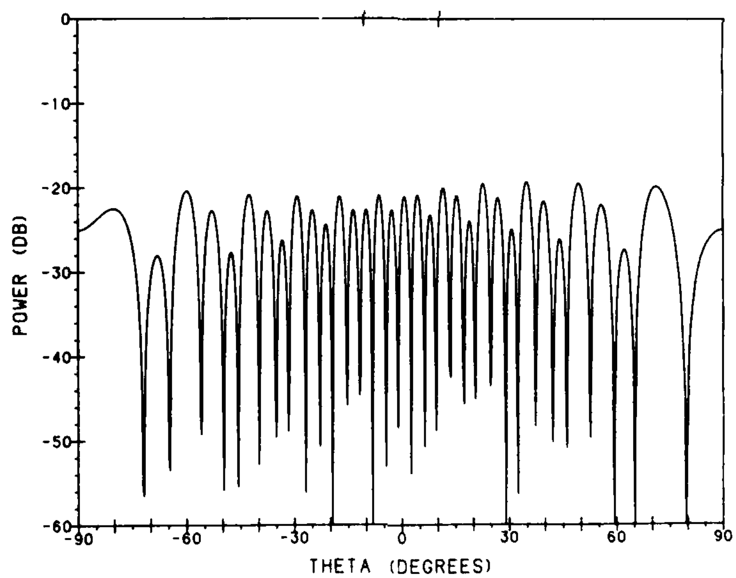

Figure 4c. Cancellation Pattern to Impose Nulls at ± 10.48° in Uniform Amplitude Pattern. θ = -90° to +90°

For comparison with Figure 4, in Figure 5 we show patterns for the case of nulls imposed at the non-symmetric pair of locations +10.48° and -7.62°, the outer 3 dB point of the second sidelobe on the left. The cancellation pattern of Figure 5c is characterized by two beams directed closely at the null locations. Pattern distortion is quite small beyond the fourth sidelobes. Phase shifts for this example are all less than 19° with 24 phase shifts in the range between 6° and 19°.*

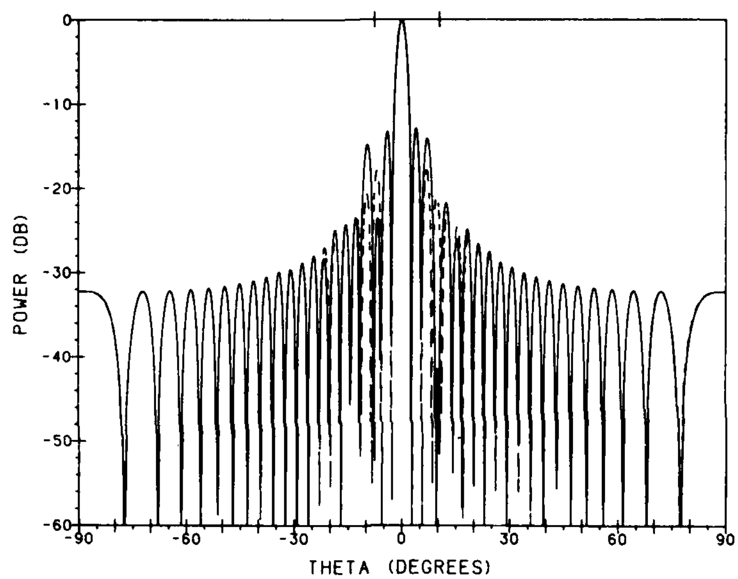

Figure 5a. Original Uniform Amplitude Pattern (-----) and Perturbed Pattern (———) With Nulls Imposed at + 10.48° and -7.62°. θ = -90° to +90°

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*Since two non-symmetrically located nulls are imposed in this example, in view of the discussion of Figure 2c one might expect the cancellation pattern in Figure 5c to be composed of two pairs of beams, one pair directed at ± 10.48° and the other at ±7.62°. The reason that this structure is not apparent is the relatively small displacement of the two beam pairs. For an example of the cancellation pattern for a pair of non-symmetrically located nulls with a greater separation between the beams see Figure 7c. See also the discussion of Figure 8 on page 26.

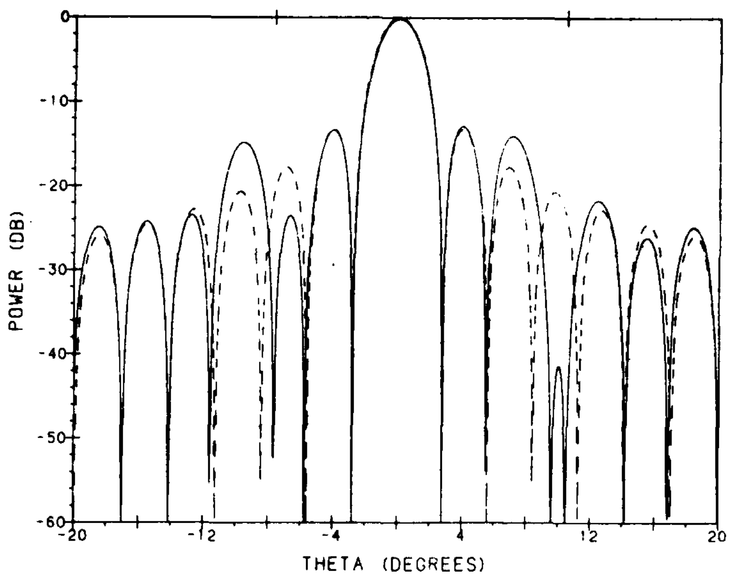

Figure 5b. Original Uniform Amplitude Pattern (-----) and Perturbed Pattern (———) With Nulls Imposed at + 10.48° and -7.62°. θ = -20° to +20°

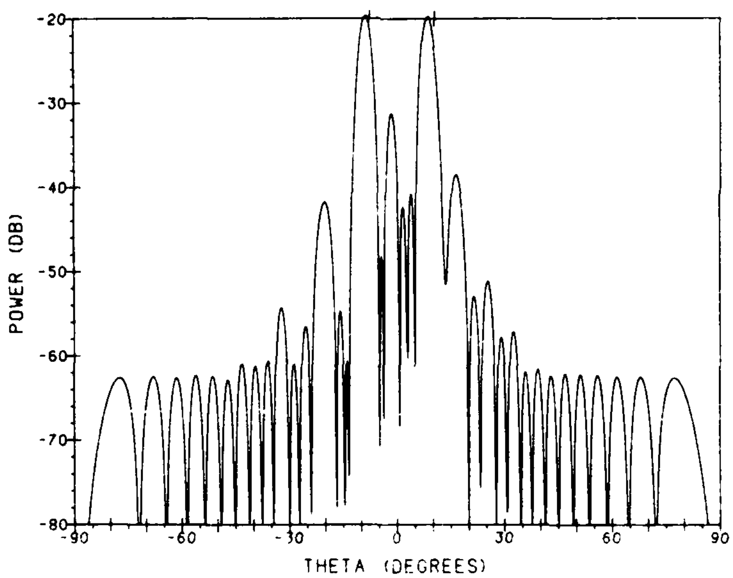

Figure 5c. Cancellation Pattern to Impose Nulls at +10.48° and -7.62° in Uniform Amplitude Pattern. θ = -90° to +90°

In Figures 6a-6c, patterns are shown for the example of nulls imposed at the symmetric pair of locations ± 15.23°, the locations of the peaks of the fifth sidelobes, for a 40 dB Chebyshev taper of the array. The interferometric cancellation pattern of Figure 6c is attributable to shifts of 92° in the phases of the 2nd and 40th elements of the array (spacing 19λ), all other elements being shifted less than 0.2° in phase. For comparison, in Figures 7a-7c we show patterns when a pair of non-symmetric nulls are imposed in the same pattern, one as before at +15.23° and the other at -27.15°, the location of the peak of the ninth sidelobe on the left. All phase shifts are less than 5.5° for this case. The cancellation pattern shown in Figure 7c is the superposition of two paired-beam patterns of the form of Figure 2c, one pair of beams directed at the locations ± 15.23° with the beam directed at -15.23° adding in phase with the original pattern there, and the other pair of beams directed at the locations ± 27.15° with the beam directed at +27.15° adding in phase with the original pattern at that location, thus resulting in the two 6 dB increases in the perturbed pattern apparent in Figures 7a and 7b.

Since the contrast between the interferometric cancellation pattern of Figure 6c and the two paired-beam cancellation pattern of Figure 7c is so striking, it is interesting to see the transition between the two patterns as the left imposed null location is moved towards symmetry with the right null location.

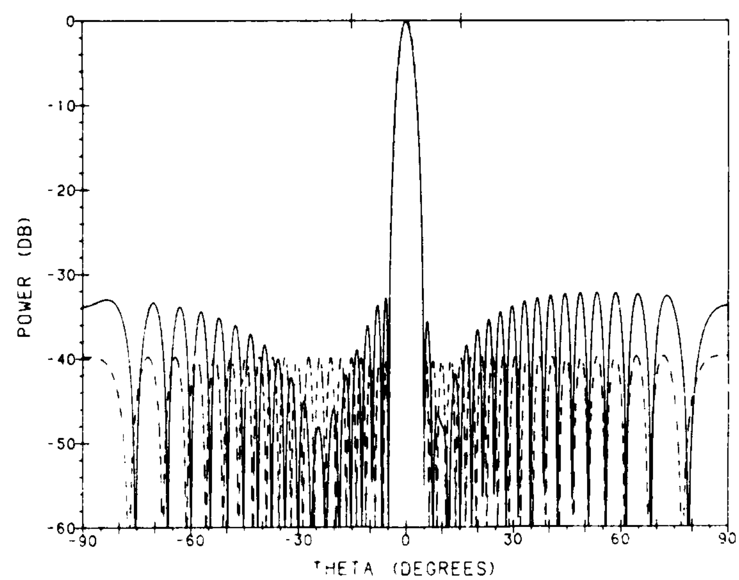

Figure 6a. Original 40 dB Chebyshev Pattern (-----) and Perturbed Pattern (———) With Nulls Imposed at ± 15.23°, θ = -90° to +90°

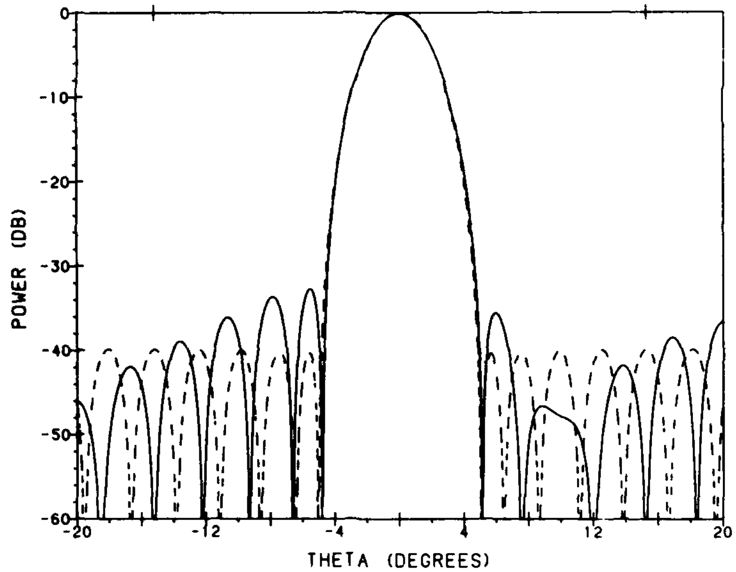

Figure 6b. Original 40 dB Chebyshev Pattern (-----) and Perturbed Pattern (———) With Nulls Imposed at ± 15.23°. θ = -20° to +20°

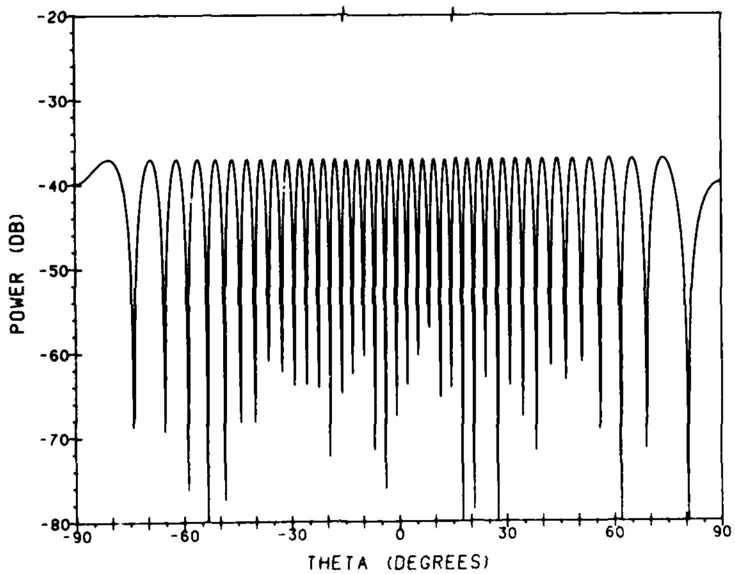

Figure 6c. Cancellation Pattern to Impose Nulls at ± 15.23° in 40 dB Chebyshev Pattern. θ = -90° to +90°

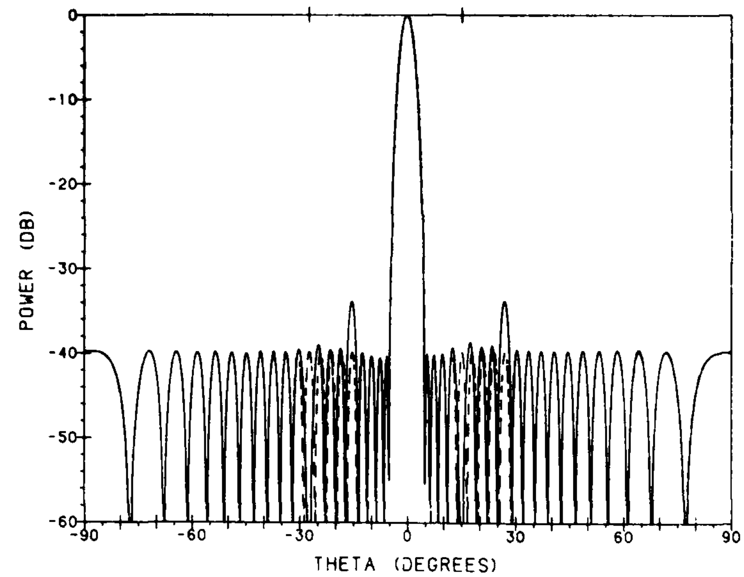

Figure 7a. Original 40 dB Chebyshev Pattern (-----) and Perturbed Pattern (———) With Nulls Imposed at +15.23° and -27.15°. θ = -90° to +90°

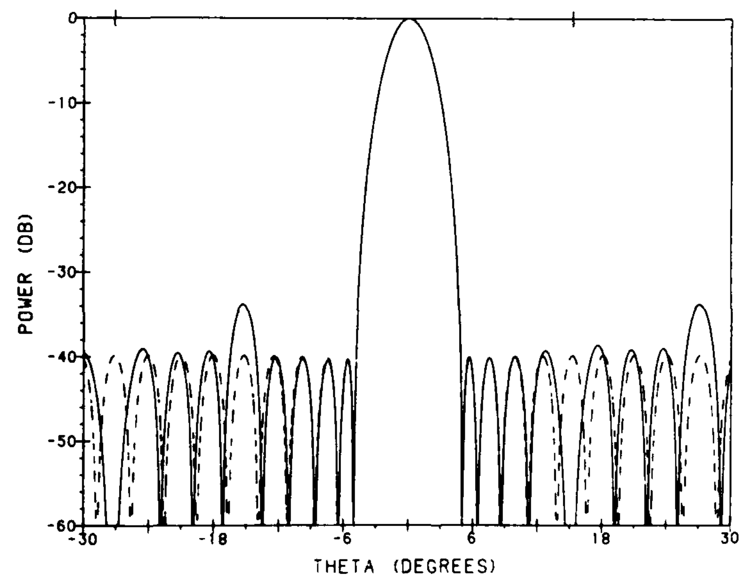

Figure 7b. Original 40 dB Chebyshev Pattern (-----) and Perturbed Pattern (———) With Nulls Imposed at +15.23° and -27.15°. θ = -30° to +30°

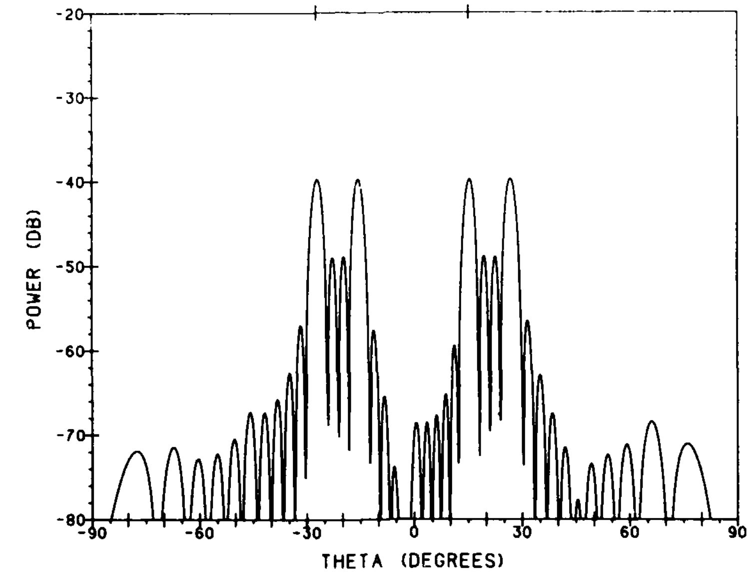

Figure 7c. Cancellation Pattern to Impose Nulls at +15.23° and -27.15° in 40 dB Chebyshev Pattern. θ = -90° to +90°

In Figures 8a-8j we show a series of original, perturbed, and cancellation patterns in which the right imposed null location is held fixed at +15.23°, as in Figures 6 and 7, while the left imposed null is moved successively to -21.02°, -18.09°, -17.12°, -16.17°, and -15.65°, the respective locations of the peak of the left seventh sidelobe, the peak and inner 6 dB point of the left sixth sidelobe, and the outer 6 dB and 1 dB points of the left fifth sidelobe. Adding the two endpoints of the series, Figures 7a, 7c, and 6a, 6c, a gradual change can be clearly seen between a paired-beam cancellation pattern, pattern reinforcement at the locations symmetric with the imposed null locations, and small pattern distortion in general, on the one hand; and an interferometric cancellation pattern and widespread significant pattern distortion on the other. Comparing the cancellation pattern of Figure 8b (with the left null imposed at -21.02°) with that of Figure 8d (with the left null imposed at -18.09°) it can be seen that the two beam-pair components of the cancellation pattern of Figure 8b (one directed towards the locations ± 15.23° and the other towards the locations ∓ 21.02° with the -15.23° and the +21.02° beams adding in phase with the original pattern) have been apparently replaced by the two single beams of Figure 8d. The reason for this difference in structure, as has been noted above in connection with Figure 5c, is that the +18.09° and -15.23° beams have been merged with the +15.23° and -18.09° beams respectively, causing a broadening as well as a raising of the levels of these latter beams. The -18.09° beam must not only cancel the original pattern there, but must also cancel a significant component of the -15.23° beam as well, and similarly for the +15.23° beam.

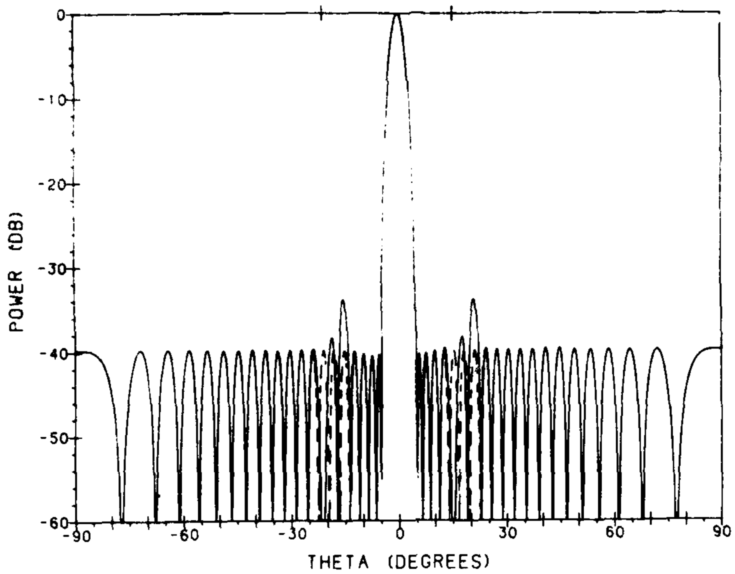

Figure 8a. Original 40 dB Chebyshev Pattern (-----) and Perturbed Pattern (———) With Nulls Imposed at +15.23° and -21.02°. θ = -90° to +90°

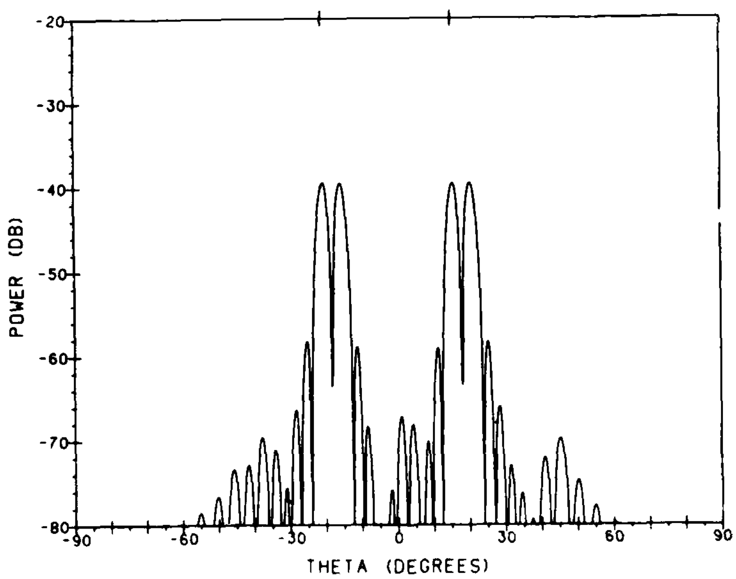

Figure 8b. Cancellation Pattern to Impose Nulls at +15.23° and -21.02° in 40 dB Chebyshev Pattern. θ = -90° to +90°

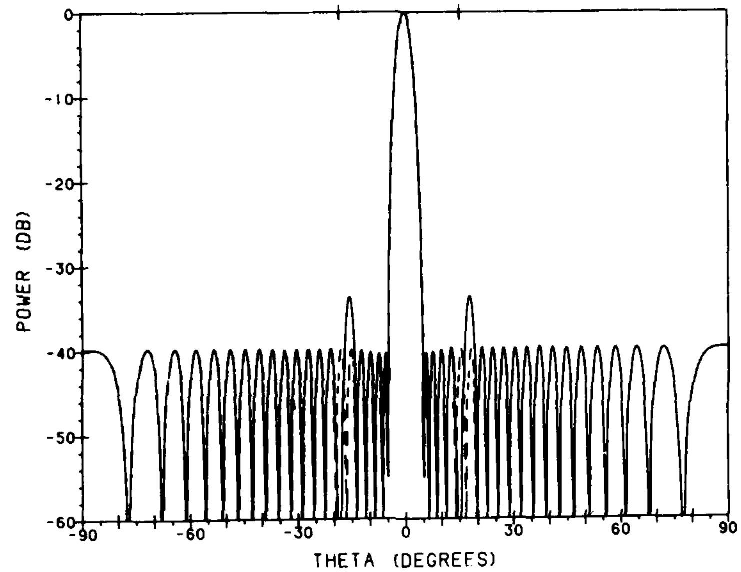

Figure 8c. Original 40 dB Chebyshev Pattern (-----) and Perturbed Pattern (———) With Nulls Imposed at +15.23° and -18.09°. θ = -90° to +90°

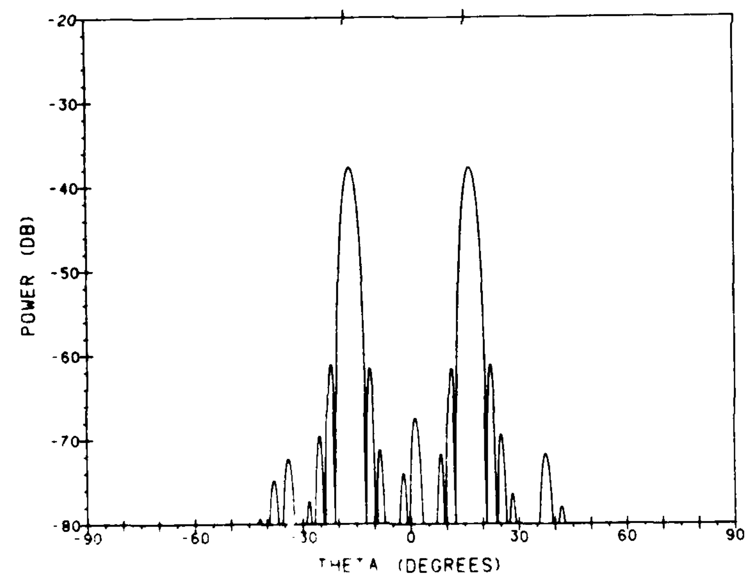

Figure 8d. Cancellation Pattern to Impose Nulls at +15.23° and -18.09° in 40 dB Chebyshev Pattern. θ = -90° to +90°

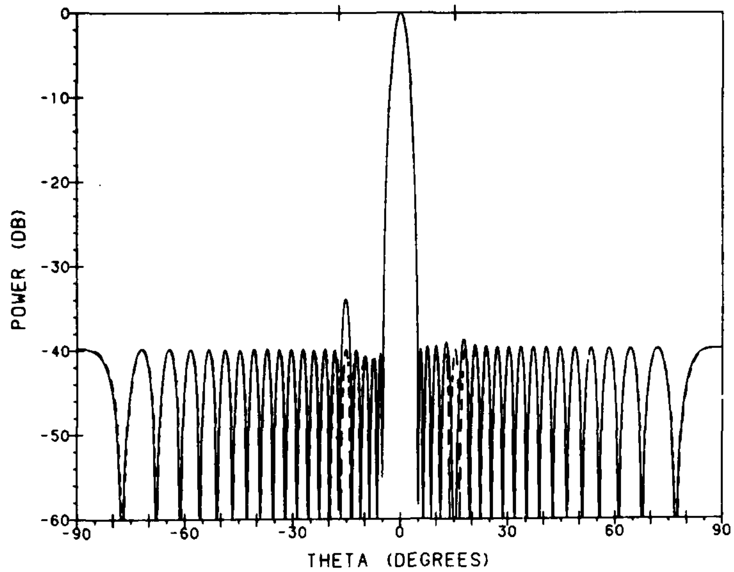

Figure 8e. Original 40 dB Chebyshev Pattern (-----) and Perturbed Pattern (———) With Nulls Imposed at +15.23° and -17.12°. θ = -90° to +90°

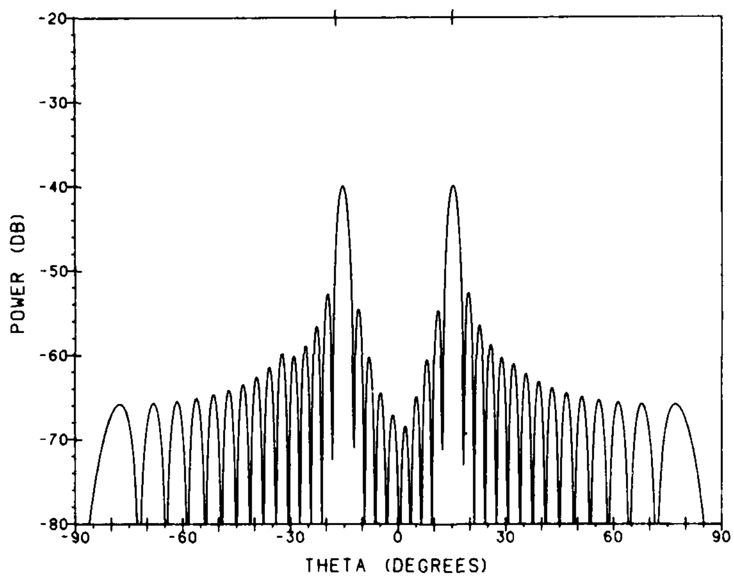

Figure 8f. Cancellation Pattern to Impose Nulls at +15.23° and -17.12° in 40 dB Chebyshev Pattern. θ = -90° to +90°

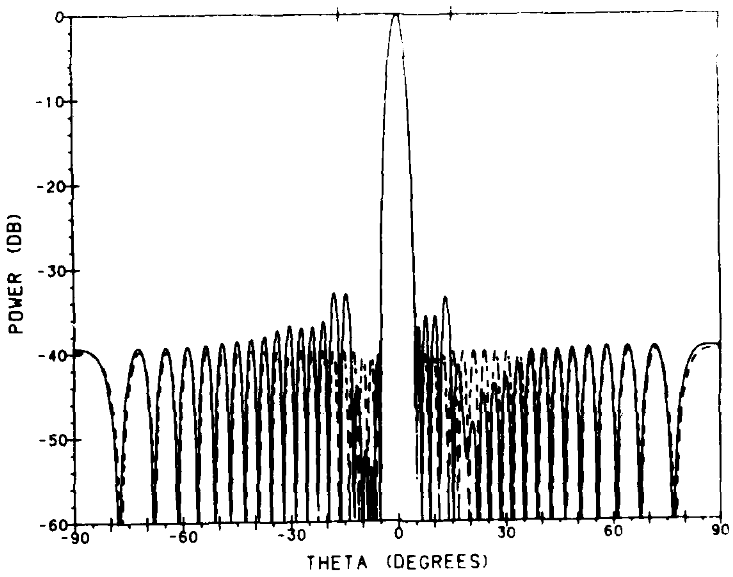

Figure 8g. Original 40 dB Chebyshev Pattern (-----) and Perturbed Pattern (———) With Nulls Imposed at +15.23° and -16.17°. θ = -90° to +90°

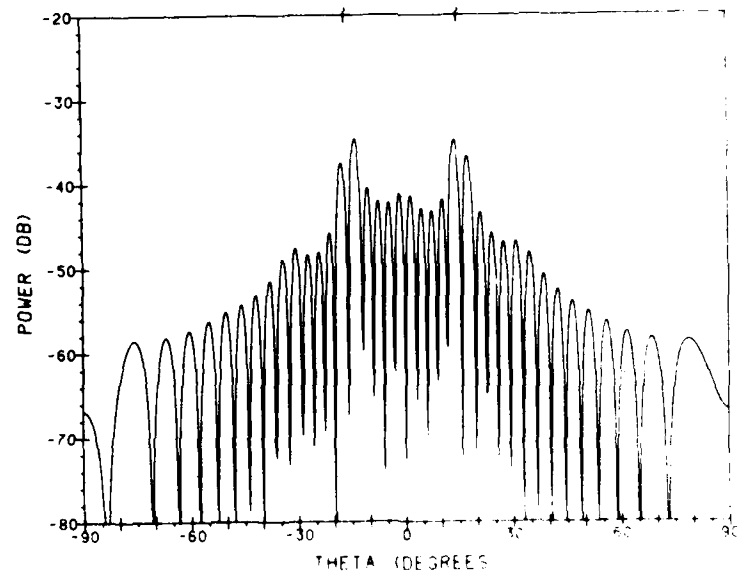

Figure 8h. Cancellation Pattern to Impose Nulls at +15.23° and -16.17° in 40 dB Chebyshev Pattern. θ = -90° to +90°

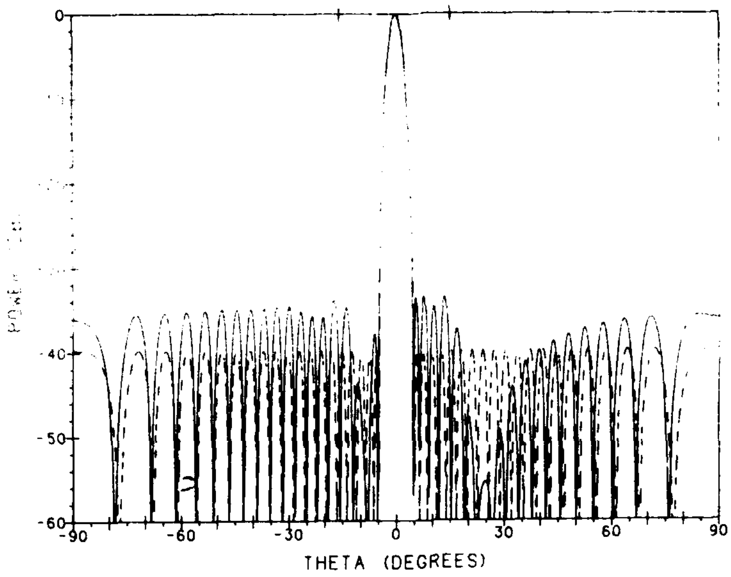

Figure 8i. Original 40 dB Chebyshev Pattern (-----) and Perturbed Pattern (———) With Nulls Imposed at +15.23° and -15.65°. θ = -90° to +90°

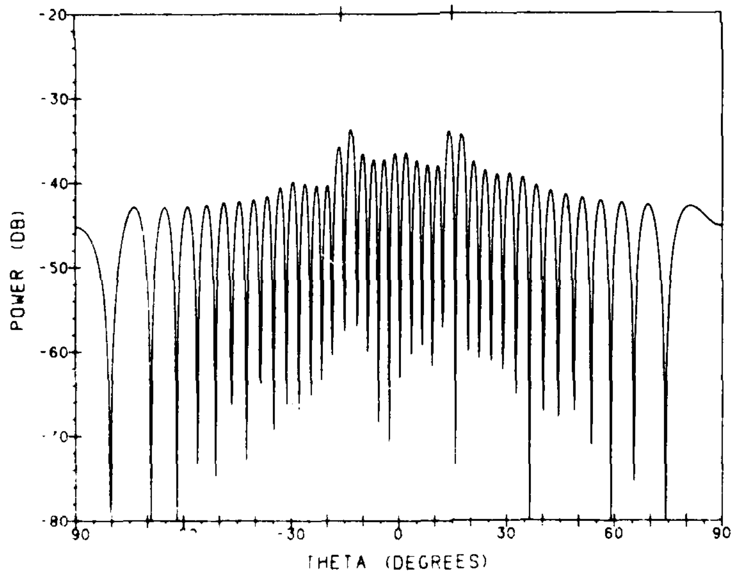

Figure 8j. Cancellation Pattern to Impose Nulls at +15.23° and -15.65° in 40 dB Chebyshev Pattern. θ = -90° to +90°

Returning to the main flow of this section, in Figures 9a-9c patterns are shown for the case of nulls imposed at ± 15.94°, the outer 3 dB points of the fifth sidelobes, for a 40 dB Chebyshev taper of the array. The cancellation pattern in Figure 9c is the superposition of two interferometric patterns, the principal one from shifts of 84.6° in the 2nd and the 40th elements of the array, and a secondary one from shifts of 11.7° in the 3rd and 39th elements. All other phase shifts are less than 3°. The ratio of the amplitudes of the two interferometer patterns is given by

$$\frac{a_2 \sin(\phi_2/2)}{a_3 \sin(\phi_3/2)} = \frac{0.00489 \sin(84.6/2)}{0.00647 \sin(11.7/2)} = \frac{5.0}{1}$$

so that the cancellation pattern is dominated by the shifts in the 2nd and 40th elements. Here as in the example of Figure 4, the cancellation pattern for nulls imposed at symmetric locations not directly at the peaks of sidelobes involves more array elements and is more complicated than when the nulls are imposed at the sidelobe maxima.

In Figures 10a-10c, we show patterns for the example of nulls imposed in the pattern of a 41 element array with a 20 dB Chebyshev taper at the symmetric locations ± 14.70°, the peaks of the fifth sidelobes. The cancellation pattern, Figure 10c, as for Figure 3c, is the superposition of two interferometric patterns, one from phase shifts of 178.9° in the 2nd and 40th elements, and the other from phase shifts of 111.7° in the 9th and 33rd elements. All other phase shifts are less than 1.4°. The ratio of the amplitudes of the two interferometer patterns is

$$\frac{a_2 \sin(\phi_2/2)}{a_9 \sin(\phi_9/2)} = \frac{0.0125 \sin(178.9/2)}{0.0214 \sin(111.7/2)} = \frac{0.7}{1} .$$

Here, despite the larger phase shifts in the 2nd and 40th elements, the larger amplitudes of the 9th and 33rd elements result in a somewhat stronger interferometer pattern than that from the 2nd and 40th elements. The large ripples in the cancellation pattern are a consequence of the fact that neither interferometric component is strongly dominant. It is also interesting to note that in this example there are no fewer than ten pairs of elements with amplitudes greater than 1/4 the magnitude of the unperturbed pattern at the imposed null locations, in contrast with the example of Figure 3 (nulls imposed at the peaks of the second sidelobes of a uniform array pattern) in which, as noted, no pair of elements had amplitudes exceeding 1/4 the magnitude of the original pattern at the imposed null locations. Thus it is not apparent why the cancellation pattern in this example is a superposition of two interferometer patterns rather than a simple interferometer pattern.

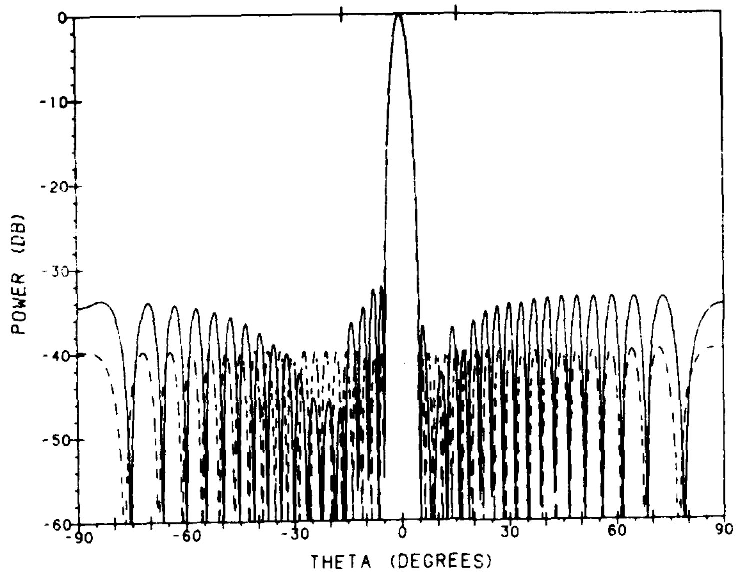

Figure 9a. Original 40 dB Pattern (-----) and Perturbed Pattern (———) With Imposed Nulls at ± 15.94°. θ = -90° to +90°

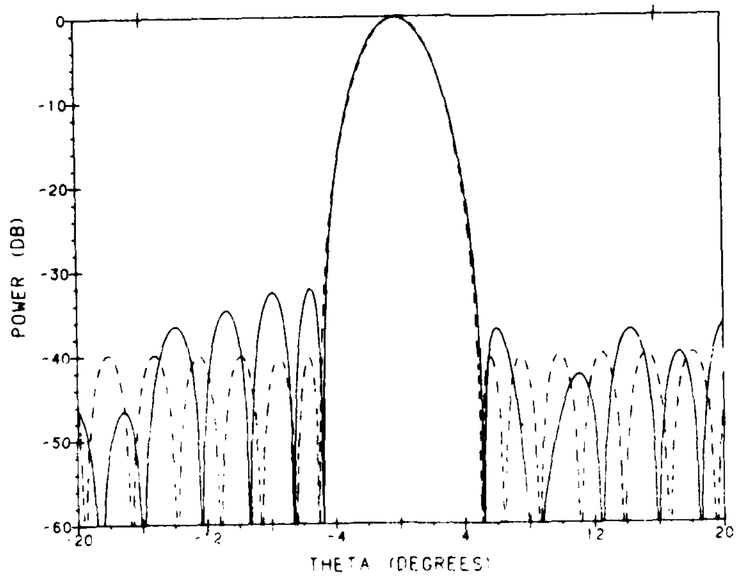

Figure 9b. Original 40 dB Chebyshev Pattern (-----) and Perturbed Pattern (———) With Nulls Imposed at ± 15.94°. θ = -20° to +20°

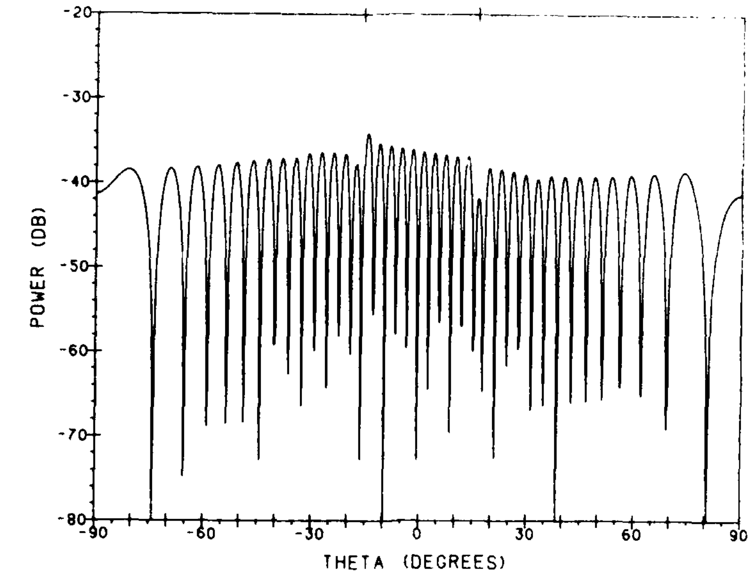

Figure 9c. Cancellation Pattern to Impose Nulls at ± 15.94° in 40 dB Chebyshev Pattern.
θ = -90° to +90°

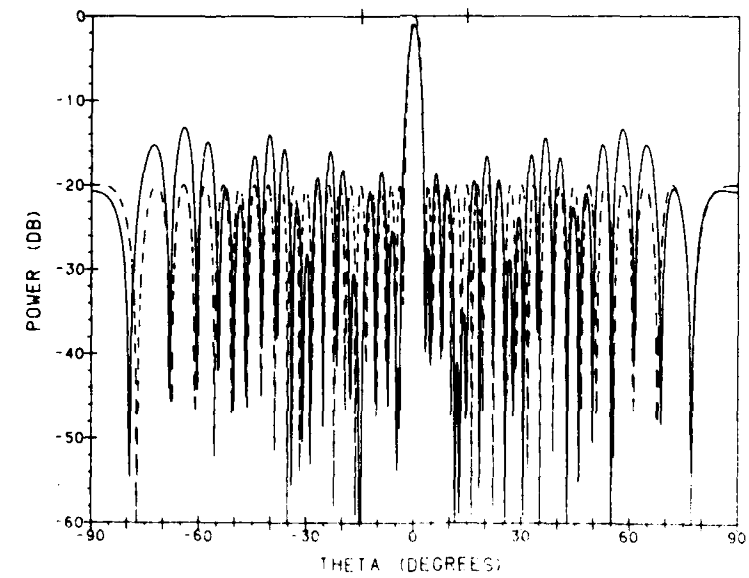

Figure 10a. Original 20 dB Chebyshev Pattern (-----) and Perturbed Pattern (———) With Nulls Imposed at ± 14.70°. θ = -90° to +90°

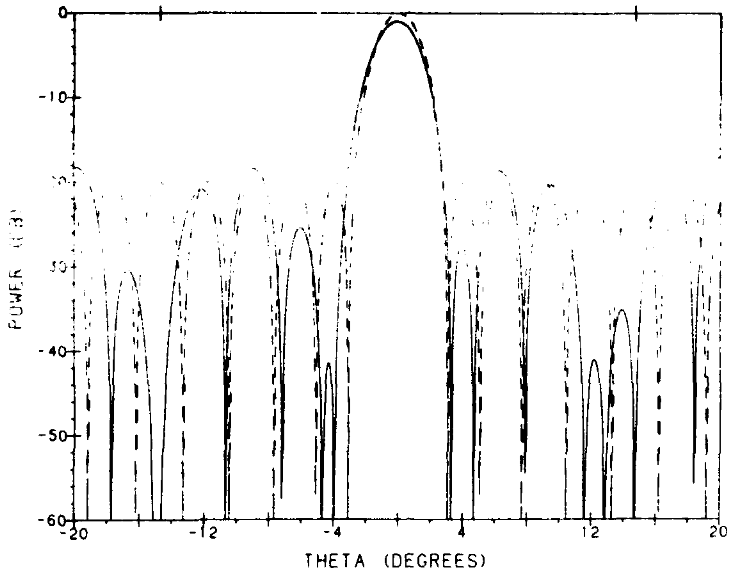

Figure 10b. Original 20 dB Chebyshev Pattern (-----) and Perturbed Pattern (———) With Nulls Imposed at ± 14.70°. θ = -20° to +20°

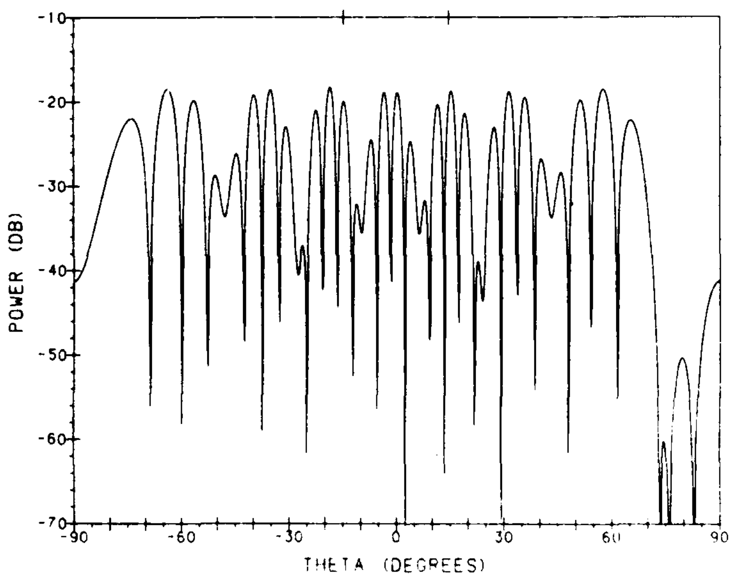

Figure 10c. Cancellation Pattern to Impose Nulls at ± 14.70° in 20 dB Chebyshev Pattern. θ = -90° to +90°

As our last example, in Figures 11a-11c we show patterns for the case of two pairs of symmetrically located imposed nulls, one pair at ± 15.95° and the other pair at ± 17.36°, locations on either side of the null between the fifth and sixth side-lobes for a 40 dB Chebyshev taper of the array. The cancellation pattern of Figure 11c results principally from phase shifts of 86.7° for the 2nd and 40th elements, 33.8° for the 1st and 41st elements, and to a lesser extent from shifts of about 5° for the 3rd and 39th, and the 7th and 35th elements. The ratio of the amplitudes of the two principal interferometer patterns components is given by

$$\frac{a_2 \sin(\phi_2/2)}{a_1 \sin(\phi_1/2)} = \frac{0.0049 \sin(86.7/2)}{0.0072 \sin(33.8/2)} = \frac{1.6}{1} .$$

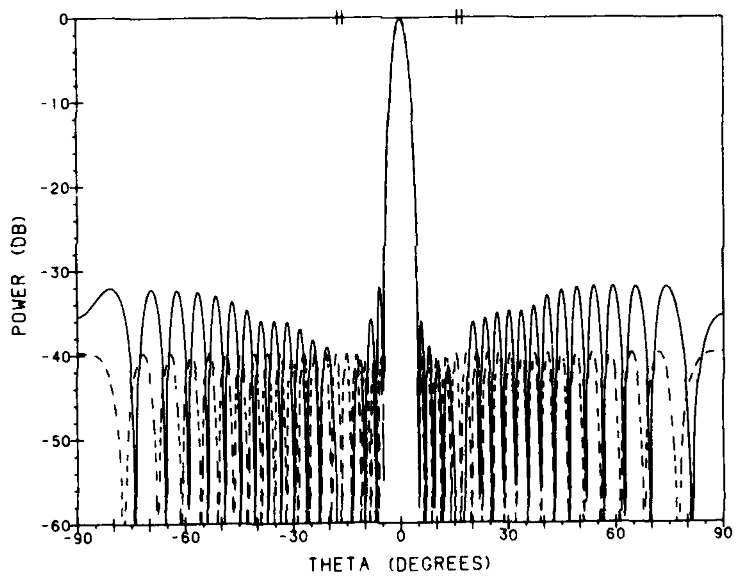

Figure 11a. Original 40 dB Chebyshev Pattern (-----) and Perturbed Pattern (———) With Nulls Imposed at ± 15.95° and ± 17.36°. θ = -90° to 90°

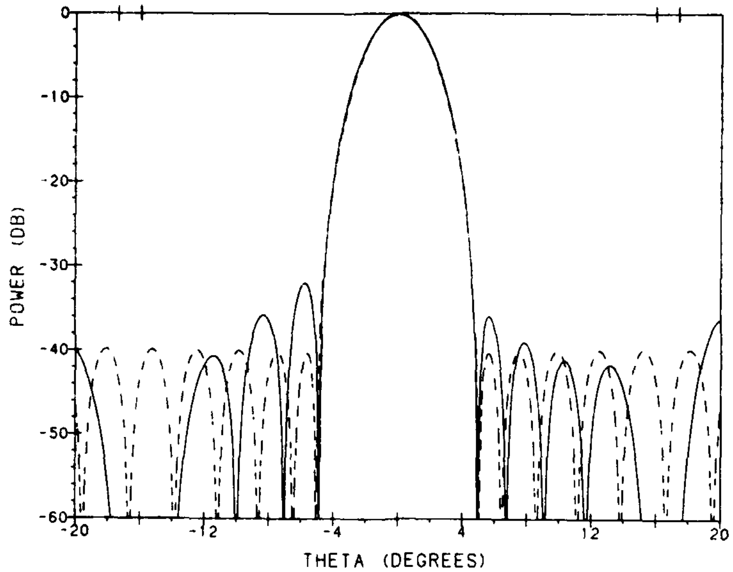

Figure 11b. Original 40 dB Chebyshev Pattern (-----) and Perturbed Pattern (———) With Nulls Imposed at ± 15.95° and ± 17.36°. θ = -20° to +20°

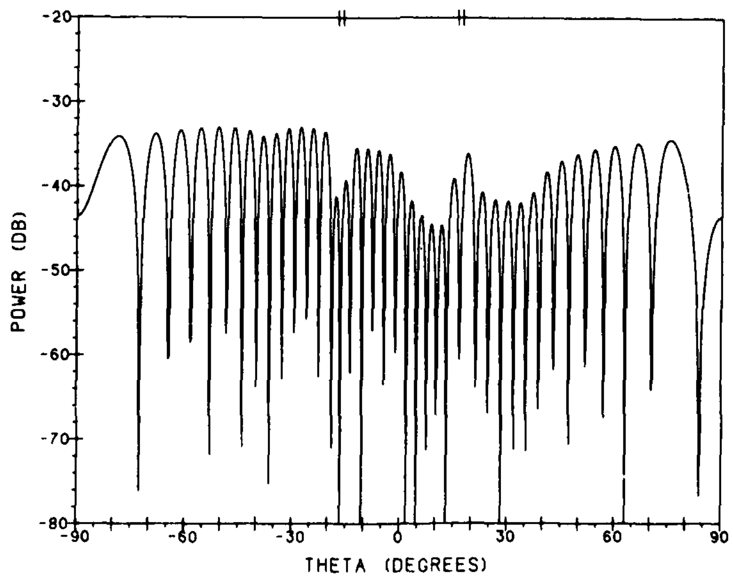

Figure 11c. Cancellation Pattern to Impose Nulls at ± 15.95° and ± 17.36° in 40 dB Chebyshev Pattern. θ = -90° to +90°

4. CONCLUSIONS

The imposing of nulls with phase-only weight control at locations in a linear array antenna pattern that are symmetric with respect to the main beam, is feasible provided that the phase perturbations are not restricted to be small. The cancellation patterns for phase-only nulls at symmetric pattern locations tend to be interferometric, and result in significant distortion over much of the pattern. In contrast, the paired-beam shape that characterizes the cancellation patterns of phase-only nulling at non-symmetric pattern locations, results in considerably less pattern distortion away from the imposed null locations. Because of the odd-symmetry of the phases in phase-only nulling, the cancellation and perturbed patterns for imposing phase-only nulls at symmetric pattern locations are not symmetrical, even though the original pattern and imposed null locations are completely even-symmetric with respect to the main beam.

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Appendix A

Proof of the Impossibility of Nulling at Symmetric Pattern Locations With Small Phase Perturbations

To prove the impossibility of imposing symmetrically located nulls with small phase-only weight perturbations, substitute the small phase approximation, Eq. (4), in Eqs. (3) for nulls at a pair of symmetric locations. The resulting equations are

$$\sum_{n=1}^{N} a_n \,(1 + j\phi_n)\, \exp[\pm\, j\, d_n u_1] = 0 \, .$$

Rearranging then gives

$$\sum_{n=1}^{N} a_n \,\phi_n \exp[j\, d_n u_1] = -j \sum_{n=1}^{N} a_n \exp[j\, d_n u_1] = -j\, p_o\,(u_1) \tag{A1a}$$

$$\sum_{n=1}^{N} a_n \,\phi_n \exp[-j\, d_n u_1) = -j \sum_{n=1}^{N} a_n \exp[-j\, d_n u_1) = -j\, p_o\,(u_1) \tag{A1b}$$

where $p_o(u_1)$ is the value of the unperturbed pattern at $\pm u_1$. But the left hand sides of Eqs. (A1a, A1b) are complex conjugates and so cannot both be equal to the same imaginary quantity. Hence the assumptions of symmetric imposed null locations and small phase perturbations are incompatible.

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