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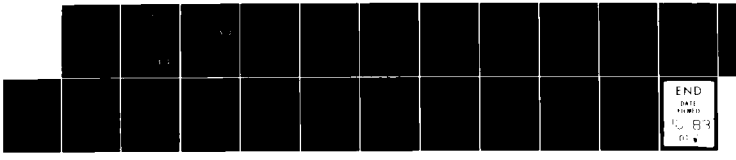

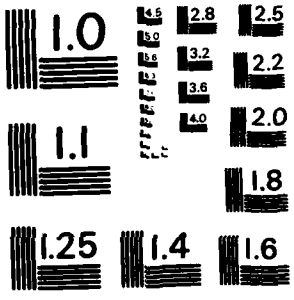

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AD-A133029

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MELBOURNE, VICTORIA

REPORT

MRL-R-861

DELAYED ARMING WITH A VELOCITY-SQUARE

RETARDED MECHANICAL INTEGRATING ACCELEROMETER

Y.-C. Thio and F.W. Shier

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Analytical details are given for three exemplary classes of motion of the bomb: constant deceleration, exponentially decaying deceleration and velocity-squared deceleration of the bomb. The relationship between the distance travelled by the moving part of the MIA and the separation between munition and aircraft is found to depend on the form of the deceleration of the bomb.

However, bounds for the relationship exist and are obtained explicitly. From these bounds, it is inferred that the relationship is almost independent of the initial motion of the bomb.

Further, the relationship is shown to be invariant with respect to a special but relatively large class of motions of the bomb. This includes the practical case of the munition experiencing a drag obeying the usual velocity-square law of aerodynamics.

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POSTAL ADDRESS: Director, Materials Research Laboratories
P.O. Box 50, Ascot Vale, Victoria 3032, Australia

SECURITY CLASSIFICATION OF THIS PAGE UNCLASSIFIED

DOCUMENT CONTROL DATA SHEET

REPORT NO.	AR NO.	REPORT SECURITY CLASSIFICATION
MRL-R-861	AR-003-044	UNCLASSIFIED

TITLE

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AUTHOR(S)
THIO, Y.-C. and
SHIER, F.W.

CORPORATE AUTHOR
Materials Research Laboratories
P.O. Box 50,
Ascot Vale, Victoria 3032

REPORT DATE	TASK NO.	SPONSOR
February, 1983	AIR 94/114	Air Force

CLASSIFICATION/LIMITATION REVIEW DATE

CLASSIFICATION/RELEASE AUTHORITY
Superintendent, MRL Physical Chemistry Division

SECONDARY DISTRIBUTION

Approved for Public Release

ANNOUNCEMENT
Announcement of this report is unlimited.

KEYWORDS
Accelerometers
Arming Devices
Mechanical integrating accelerometer
MIA
Delayed arming
Fuzes
Bombs
Rolamites
Deceleration

COSATI GROUPS
1901

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DELAYED ARMING WITH A VELOCITY-SQUARE RETARDED MECHANICAL INTEGRATING ACCELEROMETER

1. INTRODUCTION

In the design of fuzing systems for missiles and air-dropped weapons it is necessary for the fuze to remain in safe condition for some time after release to prevent inadvertant explosion close to the launch vehicle. Arming of the weapon can be suitably delayed by placing in the fuze a device which detects the acceleration of the weapon and by integration gives the distance travelled by the weapon. Mechanical devices which are used to perform this operation are known as mechanical integrating accelerometers (MIAs). Examples of such devices include runaway (or verge) escapements and flywheel integrators.

The rolamite mechanism (Figure 1) has the potential to detect and integrate acceleration. It consists of a pair of rollers (collectively called a cluster) designed to move freely in one dimension. One way of reducing the size of the device is to provide the cluster with some form of retardation, for example, a retardation proportional to the square of the cluster's velocity. However introduction of such a retardation complicates the motion of the cluster so that its use as a distance indicator involves more than a simple double integration step. In this report, the effects of retardation on the motion of a rolamite cluster and the implications on the way the distance should be measured are examined analytically.

2. THE MATHEMATICAL MODEL AND THE EQUATIONS OF MOTION

The mathematical model is based on high-drag aerial bombs but the analysis can be adapted to other situations. Typically, such bombs are subjected to an almost uni-directional deceleration which is ten to a hundred times greater than the acceleration due to gravity during the relevant phase of motion. The effects of gravity can therefore be neglected. It is appropriate then to adopt a one-dimensional geometry to study the dynamics of

the rolamite MIA provided the rolamite is free to move in the same direction as that of the initial motion of the bomb.

The coordinate systems used are shown in Figure 2. The bomb is released at time t = 0 directly above a fixed point 0 on the ground and initially continues to move in a straight line. The distance it travels in time t is denoted by y(t). The distance the rolamite cluster travels in the same time is denoted by x(t) when referred to the bomb and is denoted as s(t) when referred to the ground.

The total kinetic energy of the rollers is

$$T = \tfrac{1}{2} m\dot{s}^2 + \tfrac{1}{2} I_1 w_1^2 + \tfrac{1}{2} I_2 w_2^2$$

where m is the combined mass of the rollers, I_1 and I_2 the moment of inertia of the primary and secondary rollers respectively, and w_1 and w_2 their respective angular velocities. If r_1 and r_2 are the respective radii of the rollers, then $w_1 = \dot{x}/r_1$ and $w_2 = \dot{x}/r_2$. By using these relationships the above expression for the kinetic energy can be cast in terms of the generalised coordinates x and y, thus:

$$T = \tfrac{1}{2} m\dot{s}^2 + \tfrac{1}{2} I_1 r_1^{-2} \dot{n}^2 + \tfrac{1}{2} I_2 r_2^{-2} \dot{x}^2$$

$$= \tfrac{1}{2} m(\dot{x} + \dot{y})^2 + \tfrac{1}{2} m_r \dot{x}^2$$

where $m_r = I_1 r_1^{-2} + I_2 r_2^{-2}$ is an effective mass associated with the rotation of the rollers.

The Lagrange's equation of motion applied to the rollers which are associated with the coordinate x is,

$$\frac{d}{dt}\left(\frac{\delta T}{\delta \dot{x}}\right) - \frac{\delta T}{\delta x} = Q \qquad (2.2)$$

Here, Q represents the applied forces acting on the rollers. For the forces intended to be applied to the rollers,

$$Q = - k|\dot{x}|\dot{x}$$

where k is a constant. The equation of motion (2.2) on expanding now becomes

$$m_e x''(t) + k|x'(t)|x'(t) = - my''(t) \qquad (2.3)$$

where $m_e = m + m_r = m + I_1 r_1^{-2} + I_2 r_2^{-2}$.

In the case of aerial bombs, the bomb is continuously decelerated with the result that the rolamite cluster is continuously accelerated in the same direction and hence x'(t) is always positive for t > o. Equation (2.3) may then be written as

$$x''(t) + \kappa[x'(t)]^2 = - \alpha y''(t) \tag{2.4}$$

where $\kappa = k/m_e$, $\alpha = m/m_e = m/(m + I_1 r_1^{-2} + I_2 r_2^{-2})$. This is the primary equation of motion governing the distances travelled by the rollers.

3. OBJECT DISTANCE, IMAGE DISTANCE AND THEIR RELATIONSHIP : PRELIMINARIES

For mathematical convenience, we introduce here an auxiliary function defined thus,

$$z(t) = V_o t - y(t) \tag{3.1}$$

where $V_o = y'(o)$ is the initial velocity of the bomb. The function z(t) gives the separation between the aircraft and the bomb if the aircraft continues to move with the same uniform velocity V_o after the bomb is released. Note that, z''(t) = -y''(t),

$$z(o) = z'(0) = 0, \tag{3.2}$$

and the equation of motion (2.4) can be re-written in terms of the function z(t) as

$$x''(t) + \kappa[x'(t)]^2 = \alpha z''(t) \tag{3.3}$$

The distance x(t) travelled by the rolamite cluster in time t will be referred to as the image distance at time t; the function z(t) the object distance at time t. Given some form of deceleration z''(t), equation (3.3) may be solved for x(t) subject to the initial conditions:

$$x(o) = x'(0) = 0 \tag{3.4}$$

By comparing the image distance x(t) and the object distance z(t) over a range of values of t, a relationship between the image and the object distance can be obtained. This relationship will be referred to as a distance relationship for the accelerometer.

For an ideal distance measurer, the distance relationship should be invariant with respect to all forms of deceleration z''(t) for the bomb. For the velocity-square-retarded MIA, however, the distance relationship is in general not invariant and depends on the deceleration z''(t). This dependence may be indicated explicity by writing

$$z = W(x; z'') \tag{3.5}$$

The governing equation (3.3) is a general Riccati equation for the function $v(t) = x'(t)$. By using a standard change of variable (e.g. Goldstein and Braun 1973).

$$x(t) = \frac{1}{\kappa} \ln \theta(t) \tag{3.6}$$

the non-linear equation (3.3) can be put in a linear form thus,

$$\theta''(t) - \alpha\kappa z''(t)\,\theta(t) = 0 \tag{3.7}$$

The corresponding initial conditions for $\theta(t)$ follows from (3.4) and (3.6) and are given by

$$\theta(0) = 1, \quad \theta'(0) = 0 \tag{3.8}$$

Accounts on the solution of equation (3.7) can be found readily in the literature (e.g. Goldstein and Braun, 1973; Rainville, 1964; Murphy, 1960; Kamke, 1967), but none is available which is directly relevant to the distance relation $W(x; z'')$.

4. THREE TYPICAL CLASSES OF MOTION

We begin by presenting specific results for the distance relationship for three typical classes of motion: (a) constant deceleration of the bomb; (b) deceleration of the bomb decaying exponentially; (c) deceleration of the bomb proportional to the square of its velocity. Analysis is made in each case giving the behaviour of the distance relationship in the late stages of motion. The behaviour in the early phase of the motion is treated in a general way later (Section 6).

Class (a) : Constant deceleration, i.e. $z''(t) = c$ where c is a constant. Equation (3.7) in this case is $\theta''(t) - \alpha\kappa c\theta(t) = 0$. This gives $\theta(t) = \cosh([\alpha\kappa x]^{1/2} t)$ which satisfies also the initial conditions (3.8). Consequently, by (3.6), the image distance is given by

$$x(t) = \frac{1}{\kappa} \ln \cosh([\alpha\kappa c]^{1/2} t) \tag{4.1}$$

The object distance is obtained by integrating $z''(t)$ twice subject to the initial conditions (3.2). This gives

$$z(t) = \tfrac{1}{2} c t^2 \tag{4.2}$$

Upon eliminating t between (4.1) and (4.2), the distance relationship can be obtained as

$$x = \frac{1}{\kappa} \ln \cosh([2\alpha\kappa z]^{1/2}) \tag{4.3a}$$

or

$$z = [\cosh^{-1} e^{\kappa x}]^2 / 2\alpha\kappa \qquad (4.3b)$$

Note that the constant c giving the magnitude of the bomb's deceleration does not enter into the distance relation. The distance relation is therefore invariant with respect to the class of motion generated by different values of the deceleration. A graph of the distance relation (4.3) is shown as curve (a) in Figure 3.

For large values of κx, the right member of the expression (4.3b) may be expanded by using a formula of Abramowitz and Stegun (1965, p. 88), giving

$$z = \frac{1}{2\alpha\kappa} [\ln 2 + \xi - \frac{1}{4} e^{-2\xi} - \frac{3}{3z} e^{-4\xi} + \ldots.]^2$$

$$= \frac{1}{2\alpha\kappa} [(\ln 2)^2 + \xi^2 + 2\xi \ln 2 - \tfrac{1}{2} \xi e^{-2\xi} + O(\xi e^{-4\xi})]$$

$$- \frac{1}{2\alpha\kappa} \xi^2 [1 + O(\xi^{-1})] \qquad (4.4)$$

where $\xi = \alpha\kappa x$. The last expression gives the principal term asymptotically for large values of κx. Provided $x >> 1/\kappa$, the necessary increment in the image distance per unit increment in the object distance (i.e. the differential coefficient dx/dz) is inversely proportional to the image distance x and a significant increment in the object distance z can be indicated by smaller and smaller changes in the image distance x as x increases.

Class (b) : Deceleration of the bomb decaying exponentially with time. Here $z''(t) = a^2 e^{-bt}$ where a and b are two parameters characterising such a decay. Equation (3.7) in this case is $\theta''(t) - \alpha\kappa a^2 e^{-bt} \theta(t) = 0$. Upon solving this equation with the initial conditions (3.8), the image distance can be obtained as

$$x(t) = \frac{1}{\kappa} \ln p\{K_1(p) I_o(pe^{-bt/2}) + I_1(p) K_o(pe^{-bt/2})\} \qquad (4.5)$$

where $p = 2(\alpha\kappa)^{1/2}(a/b)$, I_ν and K_ν are modified Bessel functions of the first and second kind respectively (the normalization for these functions follows those of Abramowitz and Stegun (1965)). By integrating the retardation z''(t) twice, the object distance for this case is given by

$$z(t) = (\frac{a}{b})^2 [bt + e^{-bt} - 1] \qquad (4.6)$$

The product bt may be eliminated between (4.5) and (4.6), giving a single relationship between the object distance and the image distance for a fixed ratio a/b and fixed values of α and κ. A graph of the distance relationship for the ratio ab = 1 is shown as curve (b) in Figure 3.

The behaviour of the distance relationship at late stages may be seen by making the following expansions for the functions $I_o(pe^{-bt/2})$ and

$K_o(pe^{-bt/2})$ which occurs in the expression for the image distance, (4.5):

$$I_o(pe^{-bt/2}) = 1 + \frac{1}{4}p^2e^{-bt} + ... = 1 + O(p^2e^{-bt})$$

$$K_o(pe^{-bt/2}) = -\{\ln(1/2\,pe^{-bt/2}) + \gamma\}I_o(pe^{-bt/2}) + \frac{1}{4}p^2e^{-bt} + ...$$

$$= \frac{bt}{2} - \gamma - \ln\left(\frac{p}{2}\right) + O(btp^2e^{-bt}) \qquad (4.7)$$

where $\gamma(= .57721 ...)$ is the Euler's constant (Abramowitz and Stegun 1965, p. 375). For large values of the time variable t, the object distance (4.6) may be approximated for

$$z(t) = \left(\frac{a}{b}\right)^2[bt - 1 + O(e^{-bt})] \qquad (4.8)$$

Combining the expansions (4.7), (4.8) with the expression for the image distance (4.5) we obtain, after some algebra, the following relationship between object and image distances valid for the late stages of the motion:

$$z = (\alpha\kappa)^{-1/2}(a/b)^2e^{\kappa x}$$

$$- (a/b)^2[1 + 2K_1(\alpha\kappa)^{1/2}(a/b) + \ln\left(\frac{\alpha\kappa a^2}{b^2}\right) + 2\gamma] \qquad (4.9)$$

$$+ O(\alpha\kappa(a/b)^4bt\,e^{-bt})$$

As can be seen from the expression, the asymptotic behaviour is by no means simple with respect to the various design parameters (α, κ, a and b); the expression is useful as a means of gaining insight into the distance relationship and providing a guide to more elaborate and exact numerical studies in specific cases.

Class (c) : Deceleration of the bomb in proportion to the square of its velocity. In this case the distance travelled by the bomb in time t is y(t) where y(t) satisfies the equation

$$y''(t) = -\mu[y'(t)]^2 \qquad (4.10)$$

for some constant μ. Solving the equation with the initial conditions $y(0) = 0$, $y'(0) = V_o$ gives

$$y(t) = \frac{1}{\mu}\log(1 + \tau) \qquad (4.11)$$

where $\tau = \mu V_o t$. The object distance function is then

$$z(t) = V_o t - y(t) = \frac{1}{\mu}\,[\tau - \log\,(1 + \tau)] \tag{4.12}$$

The deceleration of the bomb $z''(t)$ is given by

$$z''(t) = \frac{1}{\mu}\left(\frac{\mu V_o}{1 + \mu V_o t}\right)^2 \tag{4.13}$$

and the governing equation (3.7) for this case is

$$\theta''(t) - \beta\left(\frac{\mu V_o}{1 + \mu V_o t}\right)^2 \theta(t) = 0 \tag{4.14}$$

where $\beta = \alpha\kappa/\mu$. The solution of this equation which satisfies also the initial conditions (3.8) can be shown to be

$$\theta(t) = (1 + 4\beta)^{-1/2}\{V_2(1 + \tau)^{-v_1} + v_1(1 + \tau)^{v_2}\} \tag{4.15}$$

where $v_1 = \frac{1}{2}(\sqrt{1 + 4\beta} - 1)$ and $v_2 = \frac{1}{2}(\sqrt{1 + 4\beta} - 1)$. Accordingly, the image distance is given by

$$x(t) = \frac{1}{\kappa}\ln(1 + 4p)^{-1/2}\{v_2(1 + \tau)^{-v_1} + v_1(1 + \tau)^{v_2}\} \tag{4.16}$$

Upon eliminating the variable τ between (4.16) and (4.12), a relationship between the object distance and the image distance can be obtained which is independent of the initial velocity V_o, and which is unique for fixed values of the design parameters α, κ and μ .

For large values of τ, $\mu z \sim \tau$, while

$$e^{\kappa x} = (1 + 4\beta)^{-1/2} v_1 (1 + \tau)^{v_2}\{1 + \left(\frac{v_2}{v_1}\right)(1 + \tau)^{-(v_1 + v_2)}\}$$

$$\sim (1 + 4\beta)^{-1/2} v_1 \tau^{v_2}\{1 + O(\tau^{-(v_1 + v_2)})\}$$

$$\sim (1 + 4\beta)^{-1/2} v_1 (\mu z)^{v_2}\{1 + O(\tau^{-(v_1 + v_2)})\} \tag{4.17}$$

or

$$z \sim \frac{1}{\mu}\{v_1(1 + 4\beta)^{-1/2}\}^{1/2} e^{\kappa x/v_2}\{1 + O)\tau^{-v_1 + v_2)}\} \tag{4.18}$$

A plot of $z(t)$ against $x(t)$ which is derived from the expressions (4.12) and (4.16) is shown in Figure 3 for the case $\beta = \alpha\kappa/\mu = 0.25$. Expression (4.17) or (4.18) may be used to study the characteristics of the distance relationship at the late stages of the motion. They may also be used conveniently to extent the graph of Figure 3 indefinitely.

5. BOUNDS ON THE OBJECT DISTANCE

On integrating both sides of the equation (3.3) twice with respect to time and using the initial conditions (3.2) and (3.4), we obtain,

$$z(t) = \frac{1}{\alpha} x(t) + \left(\frac{\kappa}{\alpha}\right) \int_o^t dt_2 \int_o^{t_2} dt_1 \{x'(t_1)\}^2 \tag{5.1}$$

The integrand in the double integral is always positive. The integral then must be positive. Hence,

$$z(t) \geq \frac{1}{\alpha} x(t) \tag{5.2}$$

This gives a lower bound for the object distance z_L for a given image distance x as

$$z_L = x/\alpha \tag{5.3}$$

The expression (5.3) is the distance relationship in the absence of any retardation of the MIA cluster if z_L is interpreted as the object distance. The lower bound indicates that if retardation is present, the object distance is increased. This in turn permits miniaturization of the MIA.

Substituting $\theta(t) = 1 + g(t)$ in equations (3.7) gives

$$z''(t) = \frac{1}{\alpha\kappa} g''(t) - g(t)z''(t) \tag{5.4}$$

A double integration with respect to time gives

$$z(t) = \frac{1}{\alpha\kappa} g(t) - \int_o^t dt_2 \int_o^{t_2} dt_1 \; g(t_1)z''(t_1) \tag{5.5}$$

From the definition of d(t), (3.6), and the definition of g(t), we have

$$g(t) = e^{\kappa x(t)} - 1 \tag{5.6}$$

Since $\kappa > 0$ and $x(t) > o$, g(t) is positive. By assumption, z"(t) is also positive for $t > o$. The double integral in (5.5) is thus positive, making

$$z(t) \leq \frac{1}{\alpha\kappa} g(t) \tag{5.7}$$

This provides an upper bound for the object distance attainable with a given image distance x as

$$z_u = \frac{1}{\alpha\kappa} (e^{\kappa x} - 1) \tag{5.8}$$

Conversely, for a given object distance, the above expression sets a limit to the degree of miniaturization of the accelerometer (minimising x).

6. THE EARLY PHASE OF THE MOTION

The expression for the upper bound of the object distance (5.8) may be expanded as

$$z_u = \frac{1}{\alpha\kappa} \{\kappa x + \frac{1}{2!} (\kappa\tau)^2 + \frac{1}{3!} \kappa x)^3 + \ldots\}$$

During the sufficiently early phase of the motion when $\kappa x << 1$, the upper bound may be approximated by the leading term in the above series, giving

$$z_u \sim x/\alpha \tag{6.1}$$

up to second order in the product κx . Comparing this with the expression for the lower bound (5.3), it is seen that the upper and the lower bounds for the object distance coincide during this phase of the motion up to second order in κx . It follows that the relationship between the object and the image distance is simply

$$z = x/\alpha + O((\kappa\tau)^2/\alpha) \tag{6.2}$$

and is independent of any particular nature of bomb's deceleration (i.e. any function for z"(t)).

The result may be understood physically as follows. When the distance x travelled by the cluster is less than a characteristic length ($x << 1/\kappa$), the effects of the retardation are negligible. Consequently, the distance relationship corresponds to the case in which there is no retardation.

This result (6.2) can be illustrated by using the case where the bomb experiences constant deceleration. For this case the distance relationship is given by expression (4.3a). For small values of $\alpha\kappa z$, the relationship (4.3a) can be expanded in a series of ascending powers of $\alpha\kappa z$ thus,

$$x = \frac{1}{\kappa} \ln \cosh \left(\{2\alpha\kappa z\}^2\right)$$

$$= \frac{1}{\kappa} \ln \{1 + a\kappa z + 0(\{\alpha\kappa z\}^2)\}$$

$$= \alpha z + 0(\kappa(\alpha z)^2)$$

On inverting the relationship, we obtain

$$z = x/\alpha + 0\ ((\kappa x)^2/\alpha)$$

in agreement with (6.2).

7. A GENERAL INVARIANCE PROPERTY

The question of invariance of the distance relationship W(x;z") with repsect to the motions of the bomb is now considered. We propose the following theorem, the proof of which is given below.

Consider the class of motions of the bomb in which any two object-distance functions $z_1(t)$ and $z_2(t)$ can be related as $z_2(t) = z_1(vt)$ where v is a suitable constant. With respect to such a class of motions the distance relationship is invariant for fixed values of the design parameters κ and α .

Proof

We note that for two object-distance functions related as $z_2(t) = z_1(vt)$, the corresponding decelerations of the bomb are related as

$$z_2"(t) = v^2\ z_1"(vt) \tag{7.1}$$

and vice versa.

Let $x_1(t)$ and $x_2(t)$ be the image-distance functions corresponding to the decelerations $z_1"(t)$ and $z_2"(t)$ respectively. That is, $x_1(t)$ and $x_2(t)$ each satisfies the governing equation (3.3):

$$x_1"(t) + \kappa\ \{x_1'(t)\}^2 = \alpha\, z_1"(t) \tag{7.3}$$

$$x_2''(t) + \kappa \{x_2'(t)\}^2 = \alpha z_2''(t) \tag{7.4}$$

Define a function f as follows:

$$f(t) = x_1(vt) \tag{7.5}$$

Then $f'(t) = vx_1'(vt)$, $f''(t) = v^2x_1''(vt)$ so that

$$f''(t) + \kappa \{f'(t)\}^2 = v^2x_1''(vt) + \kappa \{vx_1'(vt)\}^2$$

$$= v^2(x_1''(vt) + \kappa \{x_1'(vt)\}^2)$$

By using (7.3), the last expression can be simplified, giving

$$f''(t) + \kappa \{f'(t)\}^2 = a\, v^2 z_1''(vt) \quad .$$

By (7.1), it follows that

$$f''(t) + \kappa \{f'(t)\}^2 = \alpha z_2''(t) \quad . \tag{7.6}$$

From its definition (7.5), the function f has the initial properties

$$f(0) = 0, \; f'(0) = 0$$

By comparing (7.6) with (7.4), it can be seen that the function f(t) satisfies the same equation as the function $x_2(t)$ and the same initial conditions. This implies equality between the two functions $x_2(t)$ and f(t), or

$$x_2(t) = x_1(vt)$$

On eliminating t between the above relationship for the image distances and the assumed relationship for the object distances

$$z_2(t) = z_1(vt)$$

it is seen that z_2 is related to x_2 as z_1 is related to x_1. The above argument is general for any two motions represented by z_1 and z_2 in the class and thus invariance of the distance relationship with respect to the whole class of motions follows.

Application

The case of constant decleration of the bomb (class (a) of section 4) and the case of exponentially decaying deceleration of the bomb (class (b) of Section 4) for a fixed ratio of a/b are trivial examples of Theorem 7.1. The case of the bomb undergoing velocity-squared deceleration may be used to illustrate the application of the theorem to obtain invariance of the distance relationship, without actually solving the equation of motion for the bomb.

The equation of motion for the last case is given by equation (4.10). Let

$$z(t) = V_o t - y(t) = \mu^{-1} \phi(t) \tag{7.7}$$

where $\tau = \mu V_o t$. Then equation (4.10) transforms into

$$\phi''(\tau) - \phi'(t)^2 + 2\,\phi'(t) = 1 \tag{7.8}$$

with the initial conditions $\phi(0) = \phi'(0) = 0$. The solution of the above equation with these initial conditions is unique. Thus all object distance functions z(t) which arise out of solutions of equation of motion (4.10) can be put in the form of (7.8), or

$$z(t) = \mu^{-1}\, \phi(\, \mu V_o t) \tag{7.9}$$

It can be checked by straight forward algebra that if $z_a(t)$ and $z_b(t)$ are two solutions given by (7.9) corresponding to two values of V_o, namely V_a and V_b respectively, then

$$z_b(t) = z_a(\{V_b/V_a\}t)$$

provided μ is fixed. The conditions of Theorem 7.1 are therefore satisfied and invariance of the distance relationship with respect to the class of motion follows.

8. REMARKS

In the case of an aerial bomb which is decelerated by a parachute, the whole assembly consisting of the bomb and the fully opened parachute experiences an aerodynamic drag which is proportional to the square of its velocity. According to Section 7, the distance relationship of the MIA for this case is unique, since the aerodynamic drag coefficient for the bomb-parachute assembly and its presented area would be almost constant (thus μ is constant) and the coefficients α and κ are constant for a particular design of the the MIA. The opening of the parachute introduces some complications. However, if the opening time of the parachute and the retardation coefficient κ on the rolamite are suitably chosen so that the opening of the parachute occurs within the 'early' phase of the motion of the rolamite ($\kappa z \ll 1$), then according to Section 6, the motion of the bomb during this period would have negligible effects on the distance relationship of the resultant MIA.

9. SUMMARY

The distance relationship for a mechanical integrating accelerometer using a velocity-squared retardation on its moving parts is examined. The principal results are as follows.

(i) The distance relationship is found to depend on the particular nature of deceleration of the bomb.

(ii) Bounds for the object distance corresponding to a given image distance exist. Expressions for the lower bound and the upper bound are obtained explicitly, (5.3) and (5.8).

(iii) From these bounds, it is inferred that during the early phase of the motion, the distance relationship is almost independent of the particular form of the motion of the bomb. This allows design effort to be concentrated on the late phase of the motion.

(iv) Further, the distance relationship is found to be invariant with respect to a large class of motion of the bomb whose decelerations admit transformation of the form (7.1).

(v) This invariance property of the distance relationship applies in particular to the practical case of a bomb undergoing deceleration which is proportional to the square of its velocity.

(vi) The object distance, the image distance and their relationships are analysed in detail for three exemplary classes of motion of the bomb, namely, constant deceleration, exponentially decaying deceleration, velocity-square deceleration. Quantitative results in graphical form (Figure 3) are presented. In view of (iii), expressions are given from which the asymptotic behaviour of the distance relationship at late stages of the motion can be studied in detail.

10. ACKNOWLEDGEMENTS

The present report is based on unpublished work of one of us (Y.C.T.) who greatly benefited from the interaction with Mr I. Griffiths of the Explosives Development Unit.

11. REFERENCES

1. Abramowitz, M. and Stegun, A.I. 1965. Handbook of Mathematical Functions, Dover, New York.

2. Goldstein, M.E., and Braun, W.H. 1973. Advanced Methods for the Solution of Differential Equations, NASA SP-316, Scientific and Technical Information Office, NASA, Washington, D.C.

3. Kamke, E. 1967. Differentialgleichungen, Losungsmethoden und Losungen, Vol I, Academische Verlagsgesellschaft. Geest & Portig K.-G.

4. Rainville, E.D. 1964. Intermediate Differential Equations, Second Edition, MacMillan.

5. Murphy, G.M. 1960. Ordinary Differential Equations and Their Solutions, D. Van Nostrand.

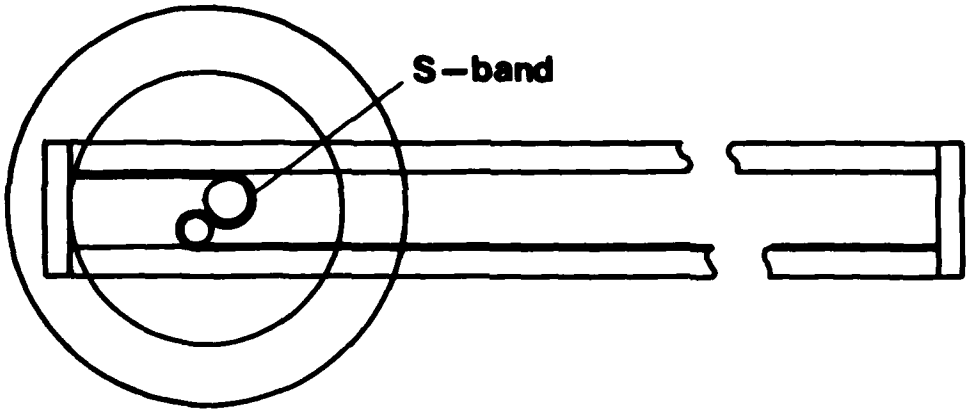

Figure 1. The rolamite mechanism. It consists of two rollers which are suspended by an S-band and are designed to move freely in one dimension. Flywheels are attached to the rollers to increase their moments of inertia.

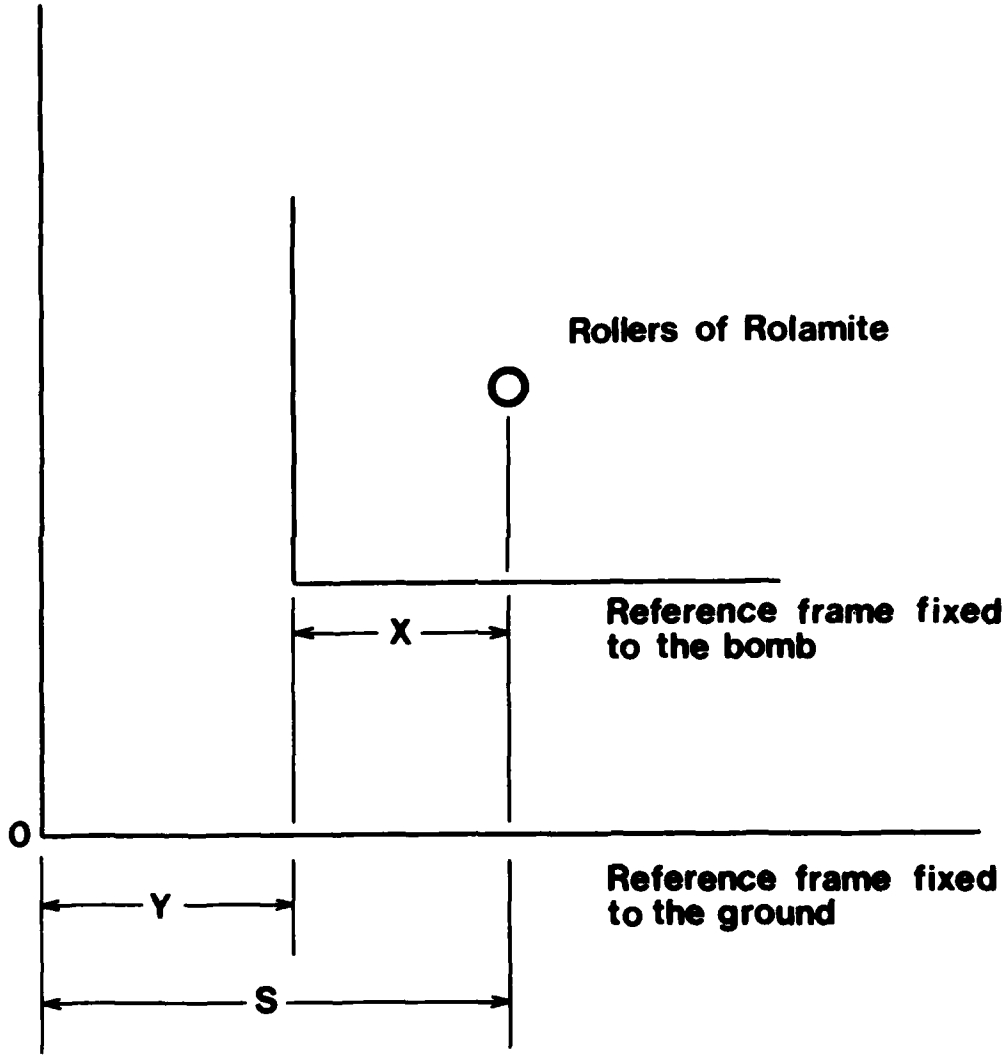

Figure 2. The reference frames used to develop the equations of notion for the system.

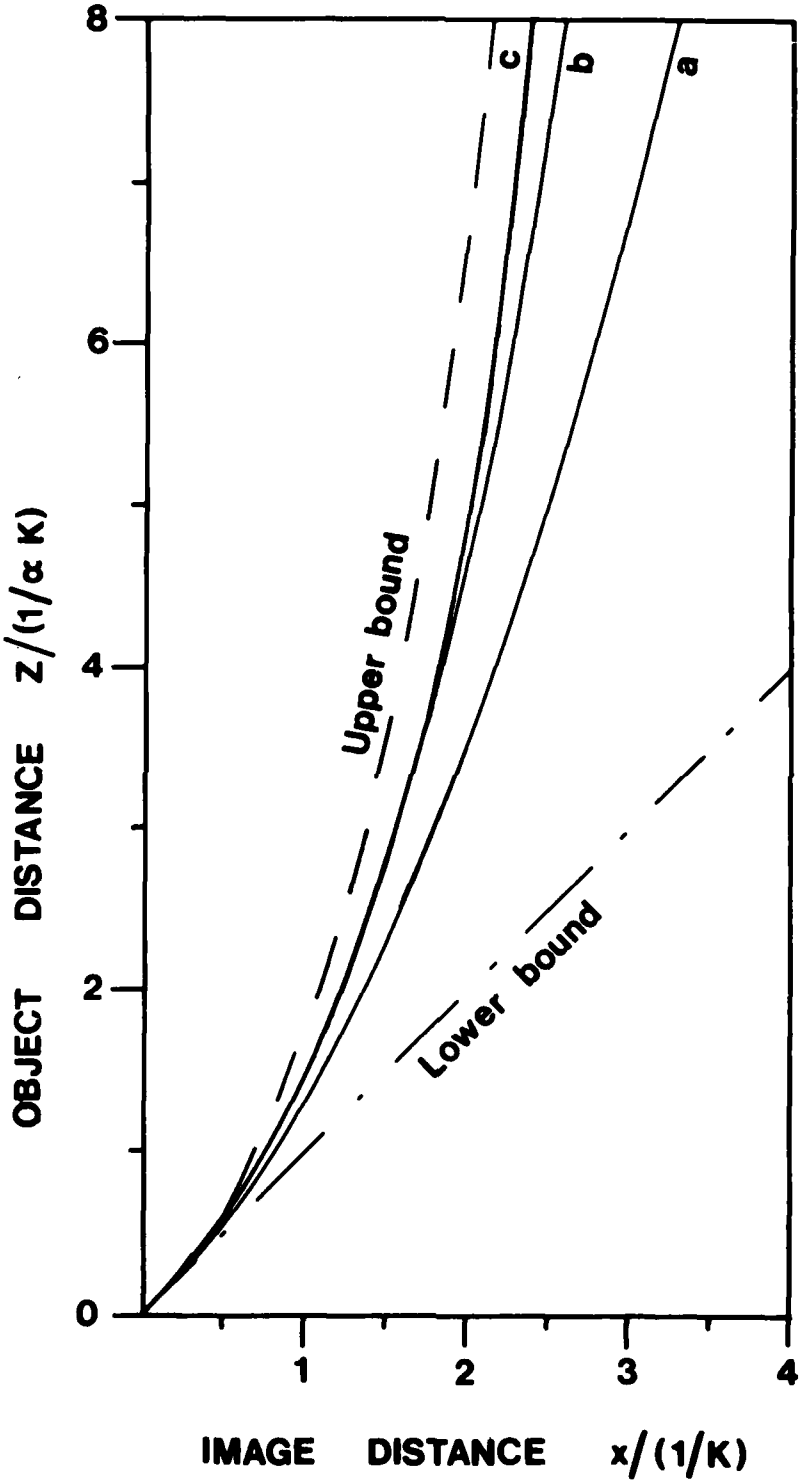

Figure 3. The relationship between the object distance (z) and the image distance (x). The dash curve gives the upper bound for the object distance whereas the dot-dash curve gives the lower bound. The solid curves apply to three typical classes of motions of the bomb: (a) constant deceleration of the bomb, (b) the bomb's deceleration being given by $a^2 e^{-at}$, (c) the bomb's deceleration being given by μ x (its velocity)^2.

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