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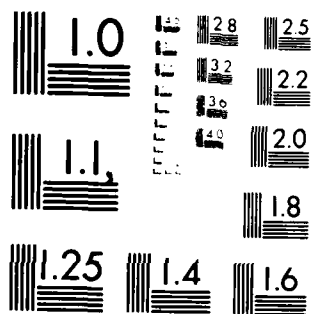
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BEAM PLASMA TURBULENCE STUDY

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CENTER FOR SPACE RESEARCH
MASSACHUSETTS INSTITUTE OF TECHNOLOGY
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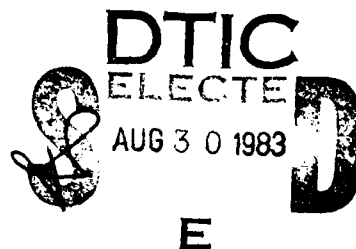
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AIR FORCE SYSTEMS COMMAND
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This technical report has been reviewed and is approved for publication

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number) We are concerned with the plasma wave radiations and turbulence, generated by natural or artificially produced charged beams and currents or plasma anisotropy, in the Earth's ionosphere and magnetosphere--particularly in the topside ionosphere and the suprathermal region. Specific problems studied during the three-year contract period include: ion conics and beams, lower hybrid waves and hybrid whistlers, nonlinear electrostatic ion cyclotron waves and cavitons, the beam-plasma-discharge phenomenon, strong plasma turbulence, electrostatic shocks across auroral field lines, particle-acceleration.		

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cont. > mechanisms, collisional effects, and anisotropy-driven magnetic reconnection processes. Major research contributions are described in detail in this report. Complete lists of the titles of scientific publications and technical presentations sponsored by this contract are also given, and important related professional activities are outlined.

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I. SYNOPSIS OF CONTRACT ACTIVITIES

It has been a rather fruitful and eventful three-year period at the MIT Center for Space Research with regard to our ionospheric and magnetospheric research, beam-plasma turbulence studies, and other related professional activities. We have made some definitive strides towards the basic understanding of the relevant plasma instabilities, weak and strong turbulence, and wave-particle interactions in the Earth's ionosphere and magnetosphere. The following is a summary of our contract activities:

1. Using the idea of electron-beam generated lower hybrid turbulence, we were able to demonstrate how the auroral F-region ions can be energized and transported to the supraauroral region of the magnetosphere. We were able to show that the resulting energetic ion conics and beams could excite the observed electrostatic ion cyclotron waves and ion cyclotron harmonic waves at higher altitudes. We have published four papers and presented several invited lectures and special topics seminars on this subject.

Our research has been aided by the useful interactions with the able plasma theorists, Dr. J. R. Jasperse at the Air Force Geophysics Laboratory, Drs. B. Basu and J. Retterer of the Space Data Analysis Laboratory, Boston College, and Drs. R. L. Lysak and M. K. Hudson of the University of California at Berkeley. Dialogues with the experimental groups (Drs. J. D. Winningham and J. Burch at the Southwest Research Institute, Dr. D. Klumpar of the University of Texas at Dallas, Dr. P. Kintner of the Cornell University, Dr. J. Sharber of the Florida Institute of Technology, and Dr. M. Temerin at the University of California at Berkeley) responsible for the particle and wave measurements of the DE, ISIS-2, and S3-3 satellites and other high altitude sounding rockets were instrumental in the development of our basic theoretical model.

2. We have made some relevant plasma wave calculations related to the interesting beam-plasma-discharge (BPD) phenomenon produced by an electron beam injected in a vacuum chamber under ambient plasma and magnetic field conditions simulating the ionospheric environment. Similar ideas on the plasma waves and the associated wave-particle interactions induced by artificially produced charged (ion and electron) particle beams injected in the ionosphere aboard sounding rockets have also been developed. We have enjoyed the discussions with Mr. H. Cohen and Dr. W. Denig at the Air Force Geophysics Laboratory, Dr. P. Kellogg at the University of Minnesota, Dr. P. Kintner at the Cornell University, and Drs. R. Arnoldy and T. Moore of the University of New Hampshire at Durham, in connection with the various experimental facets of these fascinating beam-generated plasma phenomena in space and in the laboratory.

3. We have made exact nonlinear plasma wave calculations capable of describing sawtooth and double-spike shaped finite amplitude electrostatic ion cyclotron waves similar to those observed in the supraauroral region. We have also demonstrated the existence of a class of nonlinear electrostatic ion cyclotron cavitons. The structures of these cavitons bear striking resemblance of the paired electrostatic shocks frequently detected near $1 R_E$ above the auroral zones. We have published two papers and presented several seminars on this subject.

4. Jointly with Dr. J. R. Jasperse at the Air Force Geophysics Laboratory, we have been studying the general properties of the collision operators that are relevant to the Boltzmann-Fokker-Planck equation for ionospheric applications. We have presented several papers on this subject at the AGU and APS-Division of Plasma Physics Meetings.

5. Dr. H. Koons of the Aerospace Corporation and Mr. H. Cohen at the Air Force Geophysics Laboratory have provided us with some very interesting plasma

wave data produced by artificially injected electron and ion beams carried aboard the Scatha satellite which has geostationary orbits. One of the surprising phenomena indicated by the data is the presence of electromagnetic emissions at roughly $3/2$ times the electron gyrofrequency. We have found a direct non-resonant electron-beam generation mechanism which might be responsible for such plasma wave emissions. This work was carried out in collaboration with Ms. D. Donatelli of the Physics Department, Boston College, and Dr. W. Burke of the Air Force Geophysics Laboratory.

6. Throughout the Earth's magnetosphere and ionosphere (as well as near the bow shock and in the solar wind), low frequency electrostatic noises have been detected. A number of these emissions had been identified as ion acoustic waves. We found that when the ion temperature is high, the plasma medium generally cannot support ion acoustic waves. Some of these emissions can be identified as lower hybrid waves. Sometimes, the electric field noises are accompanied by low frequency electromagnetic fluctuations. Such emissions generally propagate almost normal to the ambient magnetic field lines. We found that they are, in fact, electromagnetic lower hybrid waves or hybrid whistlers. This work is done in collaboration with Dr. E. Marsch of the Max-Planck Institut für Aeronomie. We have published two papers in the Geophysical Research Letters and the Journal of Geophysics Research and presented a number of invited talks on this subject.

7. Using the path-integral and renormalization-group approach, we have formulated a procedure for calculating the correlation and response functions of stochastic, nonlinear, nonequilibrium systems. These ideas should have direct applications to the analytical description of strong plasma turbulence. We are currently applying these ideas to the lower hybrid turbulence and stochastic plasma heating. Dr. D. Vvedensky of the Imperial College of London and

Dr. J. F. Nicoll of the University of Maryland joined us in developing the above ideas. We have published two papers in the Physics Letters A, and Physical Review A on this subject

8. Magnetic reconnections are believed to take place at the magnetotail and on the dayside of the magnetosphere. It has been suggested that the magnetic reconnection process might be related to the onset of magnetic substorms. The particle distributions in the magnetosphere is generally believed to be anisotropic. We have studied the anisotropy-driven magnetic reconnection phenomenon. Two papers on magnetic reconnection and related instabilities have been published in the physics journals. This work is done in collaboration with Dr. B. Basu of the Space Data Analysis Laboratory at the Boston College and Dr. T. Antensen of the University of Maryland.

9. During the contract period, we organized or participated in organizing three national and international conferences:

1. AGU (American Geophysical Union) Chapman Conference on "Waves and Instabilities in Space Plasmas," held in Denver, Colorado on August 7-9, 1979.
2. "Plasma Astrophysics" sponsored by the International School of Plasma Physics and held at Varenna (Como), Italy on August 27 - September 8, 1981.
3. The 1982 "International Conference on Plasma Physics" held in Göteborg, Sweden on June 9-15, 1982.

In January each year, in cooperation with Dr. J. Jasperse of the Air Force Geophysics Laboratory, we have organized a one-day symposium on the "Physics of Space Plasmas" as one of MIT's annual IAP (Independent Activity Period) symposia. These symposia were sponsored by the Center for Space Research at MIT and its Director, Dr. H. S. Bridge. The topics discussed at the symposia covered: the planetary ionospheres and magnetospheres, the solar wind, wave-particle interactions, and particle acceleration mechanisms, auroral plasma physics, and magnetic

reconnection processes in space. The proceedings of these symposia are published as hard-covered books by the Scientific Publishers, Inc., Cambridge, Massachusetts.

During the three-year contract period, we chaired several space plasma sessions and gave a number of invited talks at the annual AGU and APS (American Physical Society) meetings as well as other national and international conferences. We have presented numerous seminars on our research findings at various major universities and scientific institutions.

The content of this report is organized as follows. In Sections II-VIII, we discuss in detail the major contributions of our research activities. In Section IX, we list the titles of the published technical and scientific papers. In Section X, we list the titles of the papers presented at scientific meetings and conferences.

II. ION CONICS IN THE TOPSIDE IONOSPHERE AND THE SUPRAAURORAL REGION OF THE MAGNETOSPHERE

A. INTRODUCTION

Recent satellite and rocket data on particle distributions collected from the topside ionosphere and the supraauroral region indicate that ions (H^+ and O^+) of ionospheric origin are accelerated transverse to the geomagnetic field lines (Sharp et al., 1977; Lysak et al., 1980; Klumpar, 1979; Ghielmetti et al., 1978; Gorney et al., 1981). The energy range of the upward flowing ions rarely extend beyond 500 eV at lower altitudes (<5,000 km) and the distributions are usually "shallow conics" with pitch angles average between 90° to 140° . (Without loss of generality, we restrict our discussions to the northern hemisphere.) At altitudes above 5,000 km, the ions can be accelerated beyond the kilovolt range (Kintner et al., 1979) and the average pitch angles become closer to 180° .

Throughout the supraauroral region, ac electric field measurements have detected plasma waves (Gurnett and Frank, 1972; Laaspere and Hoffman, 1976; Mozer et al., 1979; James, 1976) dominated generally by lower hybrid (LH) modes. At altitudes above 5,000 km, where kilovolt ion conics and beams have been observed, electrostatic ion cyclotron modes are also detected (Kintner et al., 1979; Temerin et al., 1979).

It is known that energetic electrons, injected into the supraauroral region, from the plasma sheet, are accelerated by field-aligned dc electric fields or double layers towards the ionosphere. The energies of these electron beams can reach the keV range. A sequence of electrostatic modes can be excited by wave-particle resonances involving the positive slope region of the electron distributions (Coppi et al., 1974; Maggs, 1976; Maggs and Lotko, 1981; Papadopoulos and Palmadesso, 1976). The modes in the frequency range of the ion plasma frequency

(LH modes) can resonate simultaneously with both the electron and ion populations (Coppi et al., 1976), thus transferring energy from the electrons to the ions.

This ion acceleration process is particularly efficient at altitudes between 1,000 km and 5,000 km where LH waves are generally broad-band and the wave intensity is relatively high (Chang and Coppi, 1981). It can be shown that ions (H^+ and O^+) of ionospheric origin can be accelerated upwards along the field lines and form ion conics.

B. EXCITATION OF ELECTROSTATIC LOWER HYBRID WAVES BY ENERGETIC ELECTRON BEAMS

We consider electrostatic modes with $E = -\nabla\phi$ and $\phi = \phi_k \exp(-i\omega t + ik_{\parallel}x_{\parallel} + ik_{\perp}x_{\perp})$, where the subscripts \parallel and \perp indicate the parallel and perpendicular components of the wave vector k to the geomagnetic field. In addition, $\rho_i^{-2} \ll \rho_e^{-2}$ and $\rho_i^{-2} \ll k_{\perp}^2 \ll \rho_e^{-2}$ where $\Omega_i = eB/m_i c$ and $\Omega_e = eB/m_e c$ represent the ion and electron gyrofrequencies, while $\rho_i = v_i/\Omega_i$ and $\rho_e = v_e/\Omega_e$ represent the average ion and electron gyroradii. Thus, we need only consider the electron dynamics along the magnetic field line while the ions can be treated as unmagnetized.

Consequently, we consider the following one-dimensional electron and ion model distributions:

$$F_{e\parallel} = n_{e0} (m_e/2\pi T_{e0})^{1/2} \exp(-v_{\parallel}^2/v_{e0}^2) + n_{eb} (m_e/2\pi T_{eb})^{1/2} \exp[-(v_{\parallel} - u_{eb})^2/v_{eb}^2] \quad (1a)$$

$$F_{i\perp} = n_{i0} (m_i/2\pi T_{i0})^{1/2} \exp(-v_{\perp}^2/v_{i0}^2) + n_{ih} (m_i/2\pi T_{ih})^{1/2} \exp(-v_{\perp}^2/v_{ih}^2)$$

where u_{eb} represents the electron beam velocity, $v_{eo}^2 = 2T_{e,o}/m_e$, $v_{eb}^2 = 2T_{e,b}/m_e$, $v_{io}^2 = 2T_{i,o}/m_i$, $v_{ih}^2 = 2T_{i,h}/m_i$, $v_{\perp} = k_{\perp} \cdot y/k_{\perp}$, the subscript "o" refers to the main body of both the electron and ion distributions, while the subscript "b" refers to the beam electrons, and the subscript "h" to the hot ion component of the ions produced by the excitation of the considered LH modes.

For $\omega/k_{\parallel}v_{eo} \gg 1$, and $\omega/kv_{io} \gg 1$, the dispersion relation becomes:

$$\frac{n_{eb}}{T_{eb}} W\left(\frac{\omega - k_{\parallel}u_{eb}}{k_{\parallel}v_{eb}}\right) + \frac{n_{ih}}{T_{ih}} W\left(\frac{\omega}{k_{\perp}v_{ih}}\right) + \frac{n_{eo}}{m_e} \frac{k_{\parallel}^2}{\omega^2} + \frac{n_{io}}{m_i} \frac{k_{\perp}^2}{\omega^2} = \frac{k^2}{4\pi e^2} \quad (2)$$

Here, we have assumed $\omega_{pe}^2 \ll \omega_{ce}^2$, $\omega_{pi}^2 \gg \omega_{ci}^2$ which are conditions usually satisfied in the suprauroral region. In (2), we have defined:

$$W(x) = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} dy \frac{ye^{-y^2}}{x-y} \quad (3)$$

For $(n_{eb}, n_{ih}) \ll n_{eo} \sim n_{io}$, we have:

$$\omega_r^2 \approx \omega_{pi}^2 \left[1 + \frac{k_{\parallel}^2}{k_{\perp}^2} \frac{m_i}{m_e} \right], \quad (4)$$

with $\omega = \omega_r + i\gamma$, where $\omega_{pi} = (4\pi n_{io} e^2/m_i)^{1/2}$ is the ion plasma frequency. For $\omega/k_{\parallel} \gtrsim u_{eb}$ and $\omega/k_{\perp} \gtrsim v_{ih}$, these waves can resonate simultaneously with the electrons and ions, thus transferring energy and momentum from the beam electrons to the hot ion component (Coppi et al., 1976). Thus, from (4) we conclude that ω_r of the LH modes in the suprauroral region is of the order of ω_{pi} and it is cut off from below at ω_{pi} .

We can argue that the considered instability becomes quasi-stationary when the characteristic parameters of the beam electron distribution and of the heated ions make the expression for γ obtained from the linearized stability theory vanish. Then,

$$T_{i+h} n_{eb} \operatorname{Im} W\left(\frac{\omega - k_{\parallel} u_{eb}}{k_{\parallel} v_{eb}}\right) \approx - T_{e+h} n_{ih} \operatorname{Im} W\left(\frac{\omega}{k_{\perp} v_{ih}}\right) \quad (5)$$

As indicated by the laboratory experiments that have led to identify a "slide-away" mechanism, it is likely that this process can provide a very efficient mechanism for producing a hot component in the ion distribution. Since $k_{\parallel}^2 \ll k_{\perp}^2$, and the relevant resonant condition is $\omega - k_{\perp} v_{\perp i} = 0$, the ions receive energy from the excited mode in the transverse direction to the magnetic field lines.

C. QUASILINEAR ION EVOLUTION EQUATION

Because the lower hybrid modes observed in the lower suprathermal region are generally broad-band, we can use the quasilinear expression to describe the ion heating process. For unmagnetized ions, the quasilinear wave-particle interaction term is:

$$\left. \frac{\delta f_i}{\delta t} \right|_{QL} = \frac{\partial}{\partial v} \cdot \left(\underline{D} \cdot \frac{\partial f_i}{\partial v} \right) \quad (7)$$

where

$$\underline{D} = \left(\frac{e}{m_i} \right)^2 \int \frac{d^3 k}{(2\pi)^3} \int \frac{d\omega}{2\pi} \underline{k} \underline{k} S_{\phi}(\underline{k}, \omega) \pi \delta(\omega - \underline{k} \cdot \underline{v}) \quad (8)$$

is the diffusion coefficient. In this expression, f_i is the ion distribution function, and S_{ϕ} is the spectral density of the electrostatic potential fluctuations.

To evaluate the quasilinear term, we need to know the spectral density of the lower-hybrid turbulence. A self-consistent calculation of $S_{\phi}(\underline{k}, \omega)$ is complicated by the problem of understanding the saturation mechanism of the lower

turbulence, whether it occurs by convection or by nonlinear wave-wave interactions. To sidestep that problem, we will use experimentally observed amplitudes of the lower hybrid waves. Although lower hybrid turbulence is found to be unstable

in frequency, little is known concerning the wave-vector spectrum. A variety of different assumptions for $S_\phi(\underline{k}, \omega)$, however, all lead to estimates of the diffusion coefficient which are similar in order of magnitude. To this level of accuracy, we have:

$$D_\perp \approx \left(\frac{e}{m_i}\right)^2 S_E \quad (9)$$

where S_E is the power spectrum of the electric field (in units of, e.g., $(v/m)^2/\text{Hz}$). Because the frequency spectrum typically extends from ω_{pi} to a few ω_{pi} , we take $S_E \approx |E|^2/\omega_{pi}$, where $|E|$ is the observed wave field amplitude. To calculate D_\perp as a function of velocity, we would again need the unobservable wave-vector spectrum of the turbulence; for the sake of simplicity, we will assume that D_\perp is constant above a minimum resonant velocity $v_{\phi, \min}$ and that D_\perp is zero below this velocity.

In addition to the wave-particle interaction described above, the geomagnetic field and the DC electric field also play important roles in the evolution of the ions in the supauroral region. Thus, with the approximate quasilinear term, the kinetic equation for the ion distribution function is:

$$\frac{\partial f_i}{\partial t} + v_{\parallel} \frac{\partial f_i}{\partial s} + \frac{e}{m_i} \left(E + \frac{v_{\perp} B}{c} \right) \cdot \frac{\partial f_i}{\partial \underline{v}} = \frac{\partial f_i}{\partial t} \Big|_{QL} \quad (10)$$

where s denotes position along a field line. We wish now to study the evolution of the ions by solving this equation.

D. SAMPLE ION CALCULATIONS

A Monte Carlo analysis of the spatially dependent equation (10) of the ion distribution will be considered in Sec. IIE. Here, we consider the evolution of the parallel and perpendicular energies of sample ions. In the absence of the LH modes, the ions could have been tracked along a given field line according to

their unperturbed orbits. On the other hand, the effect of the LH modes produces anomalous rates of increase of the ion perpendicular energy $\dot{W}_{i\perp})_{\text{turb}}$ and parallel momentum $\dot{P}_{i\parallel})_{\text{turb}}$. These can be estimated using the quasilinear diffusion term, $\delta f_i/\delta t)_{\text{QL}}$. Thus, we have

$$\dot{W}_{i\perp} = W_{i\perp} v_{i\parallel} \partial \ln B / \partial x_{\parallel} + \dot{W}_{i\perp})_{\text{turb}} \quad (11a)$$

$$m_i \dot{v}_{i\parallel} = eE_{\parallel} - W_{i\perp} \partial \ln B / \partial x_{\parallel} + \dot{P}_{i\parallel})_{\text{turb}} \quad (11b)$$

where $W_{i\perp}$ is the ion perpendicular energy, $v_{i\parallel}$ is the ion parallel velocity, and E_{\parallel} is the field-aligned dc electric field.

Let us consider first the evolution process of an ion in the absence of E_{\parallel} . Because of the resonance condition for excitation of the LH modes by the downward flowing electron beam, the direction of the parallel component of the phase velocity of the LH modes at generation is downward. Initially, when the value of $W_{i\perp}$ is relatively small, the wave drag $\dot{P}_{i\parallel})_{\text{turb}}$ can be larger than the mirror magnetic field term in (11b). Since the net momentum transfer rate is negative for this case, the ion will first acquire a small downward velocity (if we assume the initial parallel velocity is zero). At this time, the perpendicular energy is much larger than the parallel component. This means that the pitch angle of the ion is close to 90° . Eventually, the perpendicular energy is increased to a sufficiently high magnitude by the LH modes for the mirror force in the parallel equation of motion to overcome the wave drag term. The downward velocity will then begin to decrease in magnitude until the motion of the ion is reversed.

We find from (11a, b) that for a root mean square electric field $E_{\parallel} = 1 \text{ mV/m}$ and typical densities in the lower suprathermal region, both

H^+ - and O^+ -ions can be accelerated from 1 eV to tens or 100 eV in an altitudinal change of the order of 100 km. After the ions are energized above 100 eV, they begin to gather upward momentum and the mirror conversion term in (11a) can no longer be neglected. However, the drag term is now much smaller than the magnetic mirror force and can be neglected in (11b). Generally, the ions can be energized to several hundred eV before they are convected beyond the 5,000 km altitude and the entire evolution process lasts for a period between 30-40 seconds to four or five minutes. (An illustrative example is given in Fig. 1.) Of course, this acceleration process depends on the existence of LH waves. These waves may exist in an altitudinal range of tens to hundreds of km or even thousands of km.

By following the trajectories of the sample ions with different initial conditions, we can construct graphically the accelerated ion distribution as it evolves upwards. Generally, it attains a butterfly-like configuration (Fig. 2) resembling those measured experimentally (Mizera et al., 1981).

When the ions propagate across a kilovolt electrostatic shock or a series of double layers, they tend to become field-aligned. The combination of such keV ion distributions and of that of the background ions can lead to the excitation of electrostatic ion cyclotron modes (Kintner, 1980). Thus, the LH acceleration mechanism may provide a possible explanation of the simultaneous observation of the electrostatic ion cyclotron modes with the keV ion distributions in the supauroral region, particularly at altitudes above 5,000 km (Kintner et al., 1979).

E. A MONTE CARLO TECHNIQUE FOR THE ION KINETIC EQUATION

Because of the inherent importance of its spatial structure, the time-dependent solution of the kinetic equation for the ion distribution function (10) is fully a four-dimensional problem--too complicated to be solved by standard

finite-difference techniques. Instead, we adopt a model akin to a particle simulation, in which the effects of wave-particle interactions are described by using a Monte Carlo technique. Because the ions which resonantly interact with the lower hybrid waves are on the tail of the distribution function, they are few in number. Unless they excite instabilities, their effect on the rest of the particles is therefore small. Thus, we need not create a completely self-consistent model of the plasma; instead, we can impose fields and turbulence and study how these affect the resonant ions.

We begin with a segment of field line, from height $s = s_L$ to s_H . Along this segment, we specify the geomagnetic field strength $B(s)$, the dc electrostatic potential $\phi(s)$, and the magnitude of the quasilinear diffusion coefficient $D_{\perp}(s)$. The distribution of the resonant ions is approximated by using a large number of individual particles. Their velocity distribution is initially a Maxwellian. (Actually, because ions with velocities below $v_{\phi, \min}$ will not interact with the lower hybrid turbulence, we do not bother to load them into our simulation). The particle density is assumed to be a power-law function of altitude; the exponent -2 gives good agreement with Maeda's (1975) model of plasma density in the supraauroral region.

The calculation proceeds by following the orbits of the particles with time. Because we are interested in changes occurring on scales much larger than the size of a gyro orbit, we need not integrate the equations of motion in detail. Instead, we follow the motion of the particle guiding centers along the field line:

$$\frac{ds}{dt} = v_{\parallel}(s) \quad (12)$$

If for a moment, we neglect the effect of wave-particle interactions, the velocity of a particle as a function of position can be found by assuming the invariance of the particle's energy and of its first adiabatic invariant over a short time step. In this way, the equation of motion could be integrated step by step.

When we include wave-particle interactions, we need to modify this scheme slightly. In addition to the change in velocity caused by the particle's motion in the systematic fields, a change in velocity due to exchange of energy with the turbulent fields must also be included during each time step. Because the interaction is modeled as a diffusion process, the first approximation to the probability of a given change in velocity during an interval Δt is--by the central limit theorem--a Gaussian function of Δv in one dimension. The variance of this Gaussian is $2D_{\perp}\Delta t$; because there is no drag term in the kinetic equation, its mean is zero. In the present case, diffusion occurs in two dimensions, in the plane perpendicular to the magnetic field. Therefore, the increment in the square of the transverse velocity is given by:

$$\Delta(v_{\perp}^2) = 2v_{\perp} \Delta \cos(\Theta) + \Delta^2, \quad (13)$$

where Θ is a random angle, $0 < \Theta < 2\pi$, and $\Delta = (\Delta v_x^2 + \Delta v_y^2)^{1/2}$, where Δv_x and Δv_y are both Gaussian random variables with variance $2D_{\perp}\Delta t$. Such a random increment is added to the velocity of each particle following each time step.

F. THE FORMATION OF ION CONICS

It is expected that transverse heating of ions by lower hybrid modes at low altitudes will lead to the creation of conic pitch angle distributions seen at higher altitudes in the supauroral region (Chang and Coppi, 1981). The increase in perpendicular energy first creates a narrow pitch-angle distribution

peaked at 90° . Then, as the particles mirror and move up the field line, the variation of the geomagnetic field will cause some of the perpendicular energy to be transferred to parallel motion, systematically increasing the pitch angles and creating the conic distribution.

In the calculations presented here, the following parameters were adopted. Over the altitude range from 1,000 km to 5,000 km, we followed H^+ and O^+ ions, with a primordial temperature of 1 eV. No field-aligned potential drop was included. We assumed that the wave amplitude at 1,000 km was 50 mV/m, consistent with S3-3 observations (Temerin et al., 1981). The ion plasma frequency, necessary to calculate D_\perp , was taken to be $4\pi \times 10^3$ rad/s; the minimum resonant velocity was the thermal velocity. We considered two different cases for the altitudinal range of the lower hybrid turbulence: (1) narrowly distributed turbulence, with D_\perp nonzero only in the lowest 500 km of the simulation, and (2) widely distributed turbulence with D_\perp constant over 1,000 km. The actual spatial distribution is poorly known, although evidence shows (Temerin et al., 1981) that broad-band lower hybrid waves will be found below the electrostatic shock or in the double layer region whenever electron beams are detected.

As the simulation progresses, we allow a steady state to be established by replacing every particle which leaves the simulation by a particle picked at random from the primordial distribution. We will illustrate this with phase space snapshots taken after this steady state has been established.

Let us consider the model of H^+ heating with a narrow range of turbulence first. In this case, the heating and folding processes work almost independently of each other. The steady state has been reached after about 70 s of heating by the turbulence. We find that the resonant ions have been heated to a characteristic energy of a few hundred eV. Their pitch-angle distribution is

sharply collimated at all altitudes, as it folds from 90° at 1,000 km to about 150° at 5,000 km.

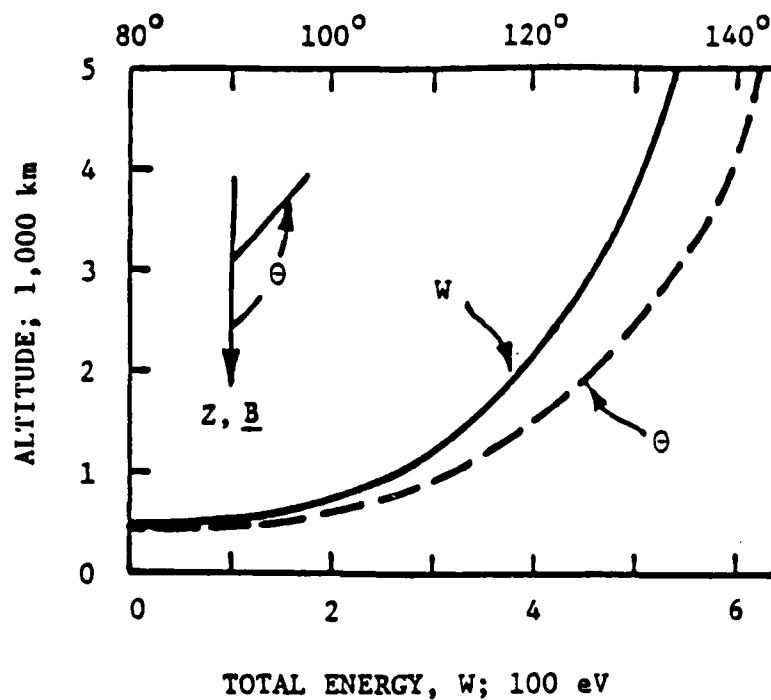
In contrast, when diffusion occurs over a wide range of altitude, the H^+ -ion conic structures are more diffuse. The continued transverse heating destroys the collimation. Because of the wider extent of the heating region, particles are also accelerated to higher energies, typically of order 1 keV for H^+ -ions in this case (Fig. 3). The increase in energy is not greater because the heating is self-limiting: as a particle gains energy, it moves out of the heating region more quickly. The broader conics calculated in the second model are much like the conics seen by S3-3 (Mizera et al., 1981). Similar results are obtained for O^+ -ions. Even though the heating rate for a given wave spectrum is lower because of the larger mass, O^+ -ions can be heated to comparable energies (as for H^+ -ions). The reason is that, because they are heavier, O^+ -ions stay in the heating range much longer than H^+ -ions.

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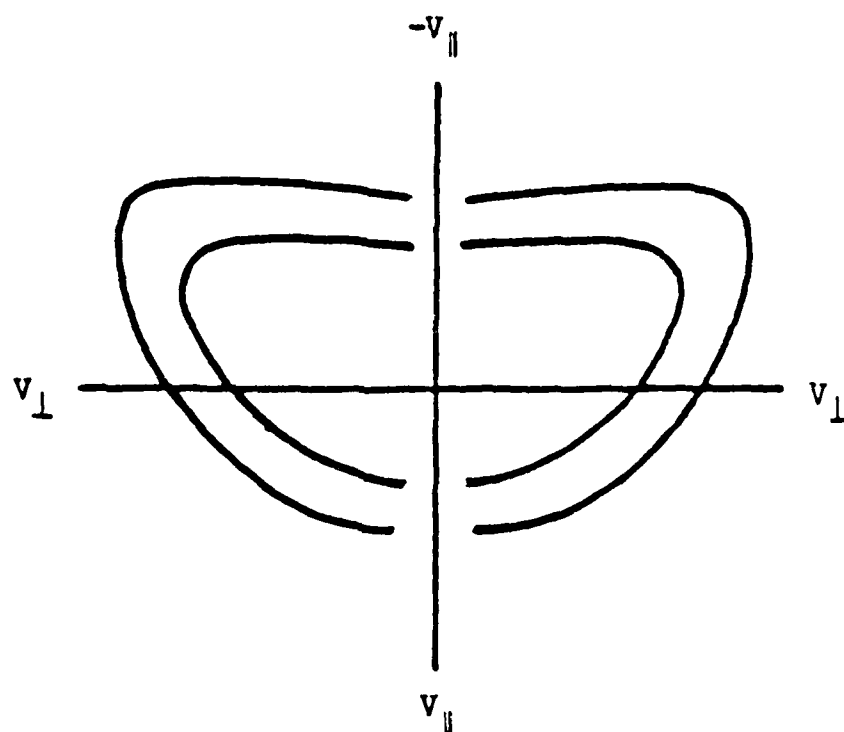
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Fig. II-1



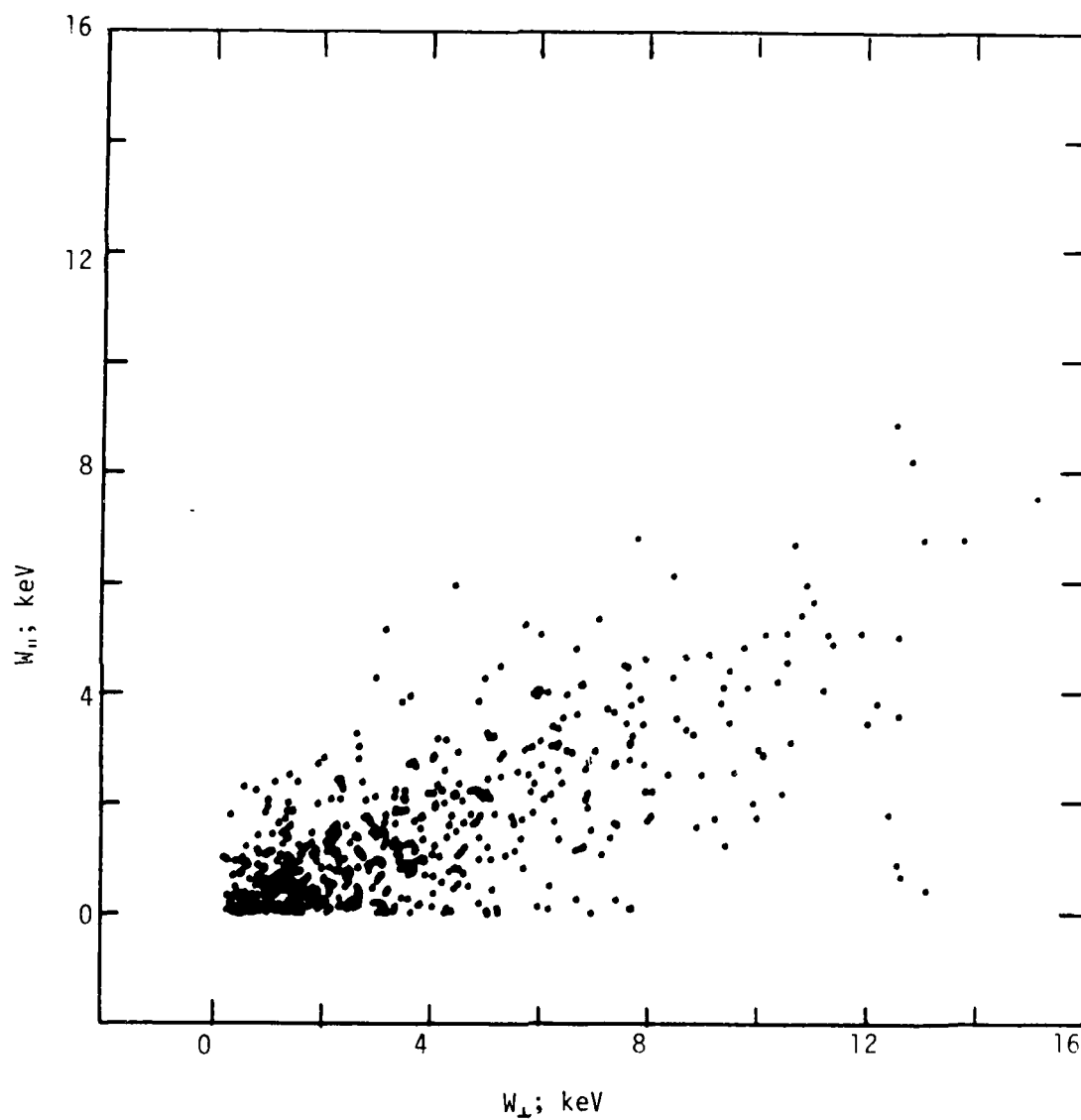
Illustrative example of the energy and pitch angle evolution of an H^+ -ion in the lower supauroral region. $S_E = 10^{-6} (V/m)^2/Hz$ and $W_0 = 1$ eV at the initial altitude $h_0 = 500$ km. S_E is assumed to decrease at the rate of 5% per 500 km increase of altitude.

Fig. 1.2



Schematic contours of the accelerated portion of a typical ion distribution function.

Fig. II-3



Monte Carlo calculation of the evolution of H^+ -ions. Acceleration range:
1,000 - 2,000 km ($S_E = 12.5 \times 10^{-7} \text{ (V/m)}^2/\text{Hz}$). Elapsed time: 50 sec.
Altitude: 3,000 km.

III. EXACT NONLINEAR SUPERSONIC PERIODIC ELECTROSTATIC ION CYCLOTRON WAVES OF ARBITRARY AMPLITUDE

A. INTRODUCTION

Spiky and sawtooth shaped nonlinear periodic electrostatic ion cyclotron (EIC) waveforms have been detected near $1 R_E$ in the suprauroral region of the magnetosphere via electric field measurements by the S3-3 satellite (Temerin et al., 1979). Approximate analysis (Chaturvedi, 1976) and direct numerical computations (Temerin et al., 1979) have shown that such waveforms are admissible traveling wave solutions to the nonlinear EIC wave equations.

We present here (Chang, 1982) a general analytical solution of nonlinear EIC waves of arbitrary amplitude. Exact periodic waveforms are obtained for supersonic and hypersonic propagations.

B. BASIC EQUATIONS

We assume: (i) quasineutrality, $n_i = n_e = n$, (ii) cold ions, $T_i \ll T_e$, (iii) Boltzmann electron distribution, $n_e = n_0 \exp(e\phi/kT_e)$, and (iv) a constant applied uniform magnetic field, B_0 . The electrostatic ion cyclotron waves propagate essentially perpendicular to the field lines. Since the perpendicular electron motion and the parallel ion motion are not important for electrostatic ion cyclotron propagations, we need to consider only the two perpendicular equations of motion for the ions:

$$\partial \bar{u} / \partial t + \bar{u} \partial \bar{u} / \partial x = -(e/m_i) \partial \phi / \partial x + e B_0 \bar{v} / m_i c, \quad (1a)$$

$$\partial \bar{v} / \partial t + \bar{u} \partial \bar{v} / \partial x = -e B_0 \bar{u} / m_i c, \quad (1b)$$

where we have chosen a Cartesian coordinate system (x, y, z) with z along the direction of the magnetic field B_0 and x along the direction of wave propagation,

(\bar{u}, \bar{v}) are the components of the ion velocity in the (x, y) directions, respectively, m_i is the ion mass, t the elapsed time, and c the speed of light.

To complete the formulation, we need the ion continuity equation:

$$\partial n_i / \partial t + \partial(n_i \bar{u}) / \partial x = 0. \quad (2)$$

We are interested in traveling wave solutions of the form: $F(x - u_\phi t)$, where u_ϕ is the phase velocity. For such solutions, (1), (2) and (i), (iii) can be combined to yield a coupled set of nonlinear ordinary differential equations:

$$\frac{dN}{d\xi} = M^2 N^3 v / (N^2 - M^2), \quad (3a)$$

$$\frac{dv}{d\xi} = N - 1, \quad (3b)$$

where $N = n/n_0$, $M = u_\phi/c_s$, $v = \bar{v}/u_\phi$, $\xi = \Omega_i(x/u_\phi - t)$, $\Omega_i = eB_0/m_i c$ is the ion cyclotron frequency, and $c_s = (kT_e/m_i)^{1/2}$ is the ion sound velocity. Without loss of generality, we take $u_\phi > 0$.

C. ANALYTIC SOLUTIONS

For supersonic propagations, we have $M > 1$. The solutions of (3) can be discussed conveniently in terms of the integral curves in the v - N phase plane (Fig. 1a). Dividing (3b) by (3a) gives:

$$dv/dN = (N-1)(N^2-M^2)/M^2 N^3 v \quad (4)$$

There are two finite singular points for $N > 0$ located at (a) $v \neq 0$, $N = 1$, and (b) $v = 0$, $N = M$. (a) is a center and (b) is a saddle point. We note that periodic solutions are possible for $0 < N < M$. The full family of integral curves may be obtained explicitly by integrating (4), which yields

$$v^2/2 + V = K, \quad (5a)$$

with

$$V(N) = [(1-2N)/2N^2 + (\ln N - N)/M^2]. \quad (5b)$$

The general shape of $V(N)$ for $M > 1$ is depicted in Fig. (1b). There is a local minimum V_1 at $N = 1$ and a local maximum V_M at $N = M$.

Thus, the results of our calculations for supersonic propagations have the simple analogy of a "particle" in a nonlinear "potential", $V(N)$. The "kinetic energy" of the particle is $v^2/2$. The "total energy" K parameterizes the integral curves in Fig. (1a). For $V_1 < K < V_M$, the solution curves form closed loops. Since $dN/d\xi > 0$ (< 0) for $v < 0$ (> 0) when $M > N$, the direction of "time" ξ , circulates around the loops and the corresponding solutions for $N(\xi)$ are therefore periodic. For $K > V_M$, the solutions are unbounded. The "separatrix loop", which divides the periodic solutions from the unbounded solutions, is parameterized by $K = V_M$. There are no solutions for $K < V_1$. For $K = V_1$, the plasma is unperturbed.

We now complete the solution for $M > 1$ by integrating (3a) with $v(N)$ prescribed by (5a,b).

$$\xi = \int dN(N^2 - M^2)/M^2 N^3 v(N) + C. \quad (6)$$

We note that along each branch ($v < 0$ or > 0) of a closed loop with $M > N$ and $V_1 < K < V_M$, the integrand of (6) does not change sign. Thus, the integrals given by (6) along the closed loops represent the density profiles $N(\xi)$ for periodic nonlinear supersonic propagations. Since $dN/d\xi \sim \text{constant}$ near the separatrix loop, there are no soliton-like solutions for $M > 1$.

The electric field may be expressed in terms of N .

$$E \propto N^{-1} dN/d\xi = M^2 N^2 v(N)/(N^2 - M^2), \quad (7)$$

where again $v(N)$ is given by (5). Equations (5) to (7) constitute the complete analytic solution for $N(\xi)$ and $E(\xi)$ for supersonic nonlinear EIC waves. The periods and waveforms of these nonlinear waves depend on the propagating speed (u_ϕ) and the amplitude (N_{\max} or K).

One class of interesting limiting solutions is obtained for large values of M and $N_{\max} \ll M$. We have plotted the profiles of $N(\xi)$ and $E(\xi)$ for such a case for $M = 5$ in Fig. (2a). The electric field profile takes on a distinct double-spiked structure. For smaller values of M and $N_{\max} < M$, the electric field profiles vary gradually with ξ and become sawtooth-like. For $|N-1| \ll 1$, the solutions reduce to sinusoidal oscillations and the dependence on M and K disappears. These profiles bear the signatures of the linear and nonlinear EIC waves observed in the supraauroral region of the magnetosphere (Mozer et al., 1980) as was noted previously by direct numerical (Temerin et al., 1979) and small but finite amplitude calculations (Chaturvedi, 1976).

D. APPROXIMATE EXPLICIT SOLUTIONS

In fact, for sufficiently large values of M , (6) may be closely approximated by dropping the $(2N-1)/M^2$ term and the integral can be carried out explicitly. For example, for $v < 0$,

$$\begin{aligned} \xi = & [(2N-1)/N^2 + 2K]^{1/2} + \sin^{-1}[(1-N^{-1})/(1+K)^{1/2}] \\ & - N^{-2}(1+K)^{-1/2} \sin^{-1}[(2KN+1)/(1+2K)^{1/2}] + \dots \end{aligned}$$

For hypersonic propagations, $M \gg N_{\max} > 1$, the third term in (8) is also negligible. In this limit, (7) becomes:

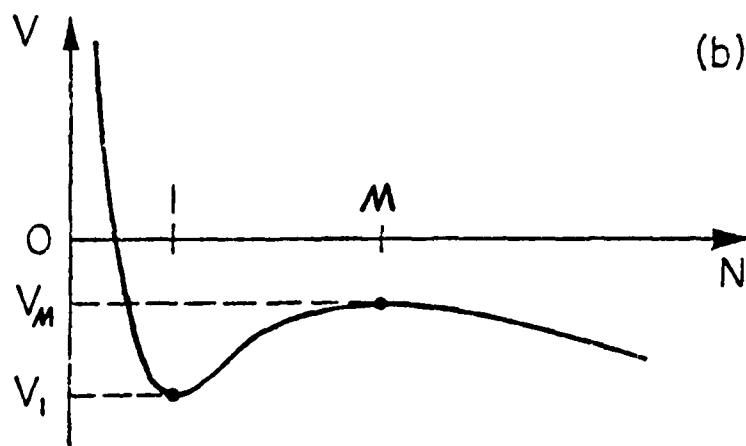
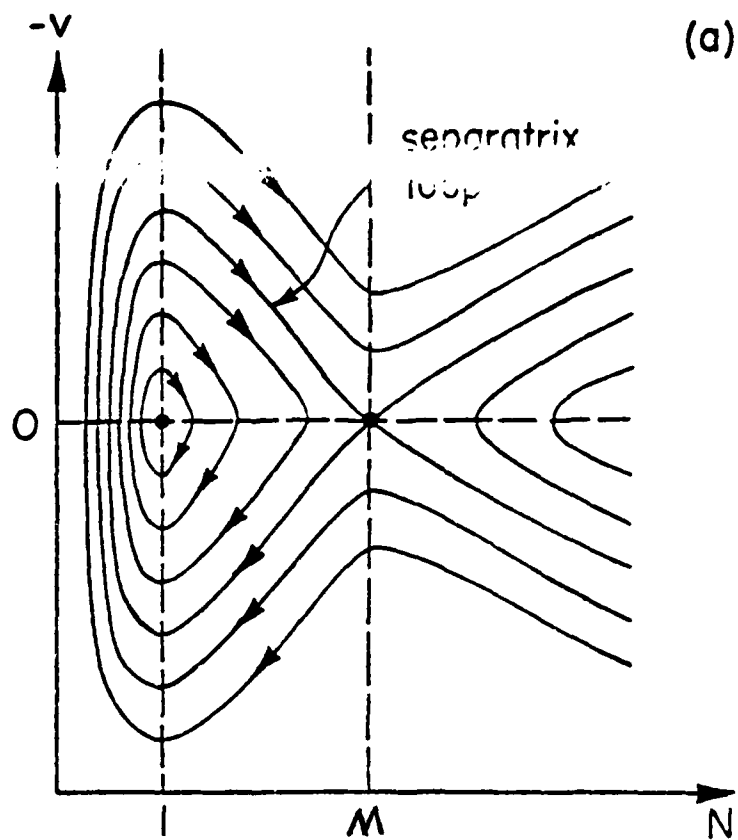
$$E \propto N [2KN^2 + 2N-1]^{1/2}. \quad (9)$$

Thus, the profiles become independent of the Mach number M explicitly. There remains, of course, the implicit dependence on the speed u_ϕ through the definition of ξ . The effective potential under this approximation is characterized by the competition of an attractive term $-N^{-1}$ and a repulsive term $1/2N^2$. There is no longer a local maximum at finite N and the saddle point has been moved to infinity. We note that in this limit, the period of propagation is always 2π units of ξ and the nonlinear profiles $N(\xi)$ and $E(\xi)$ now depend on the amplitude N_{\max} alone. We give three representative electric field profiles in Fig. (2b) for $N_{\max} = 1.25, 2.5$, and 5 . As N_{\max} is increased, the profiles are steepened exhibiting sawtooth and double-spiked structures.

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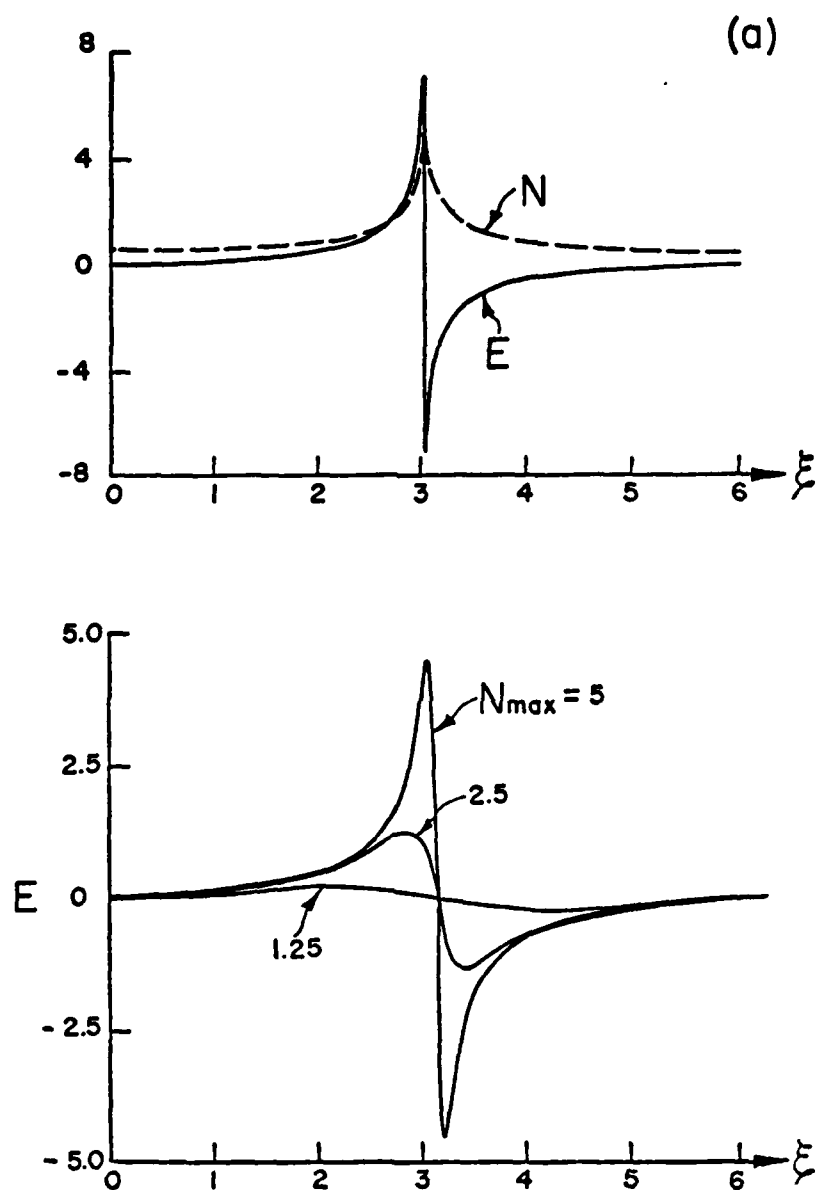
Fig. III-1



(a) Typical phase plane trajectories for $M > 1$. Arrows indicate directions of increasing t for $N > M$.

(b) Typical shape of the nonlinear potential, $V(N)$, $N > 0$.

Fig. III-2



- (a) Exact $N(\xi)$ and $E(\xi)$ profiles for $M = 5$ and $N_{\max} \approx M$. The patterns repeat approximately every 6 units of u_ϕ/Ω_i .
- (b) Hypersonic $E(\xi)$ profiles for $M \gg N_{\max} > 1$. The period of propagation is exactly 2π units of u_ϕ/Ω_i .

IV. EIC CAVITONS, PERIODIC DENSITY DIPS, AND "PAIRED SHOCKS" IN THE SUPRAAURORAL REGION

A. INTRODUCTION

Electric field measurements in the supraauroral region obtained from the S3-3 satellite revealed the existence of large paired electric fields ("shocks") oriented predominantly perpendicular to the terrestrial magnetic field (Mozer et al., 1980). Plasma density-dips were sometimes observed to occur in conjunction with the paired "dc" electric fields which were generally interspersed with coherent spectra of electrostatic ion cyclotron (EIC) waves.

In Sec. III, we have demonstrated that double-spiked and sawtooth-like EIC waves are admissible solutions to the basic EIC equations for supersonic (super-ion-acoustic) propagations, verifying the earlier numerical calculations of Temerin et al. (1979). We now show (Chang, 1982) that these same equations also admit exact nonlinear caviton-like and periodic density-dip solutions propagating at subsonic (sub-ion-acoustic) speeds. The corresponding electric field profiles exhibit paired structures with very large (infinite) peak intensities. These waves may be interspersed with each other and modified coherently by low amplitude EIC turbulence.

B. INTEGRAL CURVES FOR SUBSONIC PROPAGATIONS

The EIC equations (equations 3 and 4 of Sec. III) are still valid for subsonic ($M < 1$) propagations. Thus, the integral curves have the same general shapes as those for supersonic propagations. In the v - N phase plane for $N > 0$ (Fig. 1), we again have two singular points: (a) $v = 0$, $N = 1$ (saddle point), and (b) $v = 0$, $N = M$ (center). The integral curves can form closed loops for $0 < N < 1$. They are parameterized by $V_M < K < V_1$ and represent periodic density-dip propagating waveforms.

The separatrix loop which passes through the saddle point is given by $K = V_1$. It is easily seen from equations (5a, b) of Sec. III that the expression $V(N) - V_1 = 0$ has a double root at the saddle point for $M < 1$. Thus, the solution corresponding to the separatrix loop has an infinite period of propagation and therefore has a caviton-like structure.

C. DENSITY AND ELECTRIC FIELD PROFILES

We obtain the density profile $N(\xi)$ by integrating equations (3a) and (5) of Sec. III. We note that $dN/d\xi$ is infinite at $N = M$. Thus, ξ takes on extremum values along the line $N = M$. This means that the path of integration along each loop must jump from one branch ($v < 0$) to another ($v > 0$) at $N = M$. Referring to Fig. 1, two of such admissible path of integration are indicated by: (s) $A^+B^+C^-B^-C^+A^-$ (for the caviton-profile), and (b) $P^+Q^+R^-Q^-R^+P^-$ (for a typical periodic profile). Therefore,

$$\xi = \int dN |(N^2 - M^2)/M^2 N^3 v(N)| + C \quad (1)$$

along the path of integration indicated.

The electric field E can be calculated from equation (7) of Sec. III. We note that it is infinite at $N = M$. At $N = M$, the value of v changes sign as dictated by the condition that ξ must increase (or decrease) continuously. Thus, a vorticity layer exists within a thin sheet $\xi(M) - \Delta\xi < \xi < \xi(M) + \Delta\xi$, where the kinetic and finite gyroradius effects must be taken into consideration. This kinetic layer will reduce the peak electric field at $N = M$ from infinite to finite (but still large) values. Previous calculations (Yu, 1977) worked with the density profile alone and therefore were not aware of the jump of the velocity component v at $N = M$.

Exact density $N(\xi)$ and electric field $E(\xi)$ profiles for a typical caviton are shown in Fig. (2). The electric field profile resembles those detected in the supraauroral region of the polar magnetosphere as "paired dc electrostatic shocks" (Mozer et al., 1980). The dominant feature of the caviton structure generally spans over a characteristic distance of several units of u_ϕ/Ω_i .

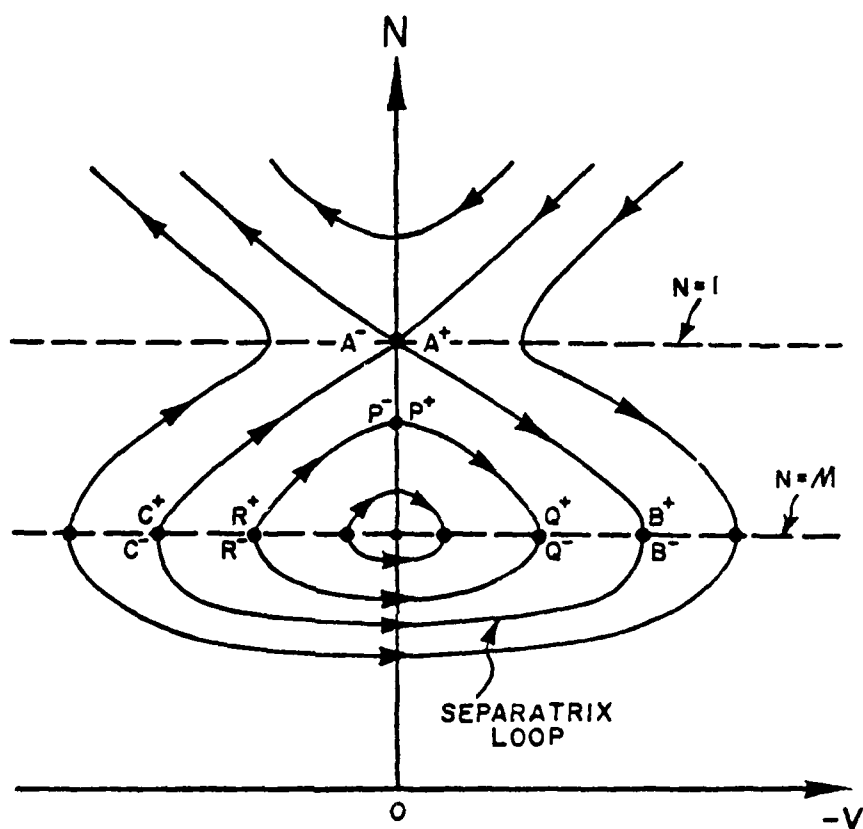
Periodic density-dip profiles have similar forms except that they have finite periods of propagation.

The fact that the solution path jumps at $N = M$ indicates that an actual solution could be several admissible solutions interspersed among themselves. When plasma wave turbulence (which is observed to co-exist with the paired shocks) and other kinetic instabilities are present, these dissipative effects can switch the path of integration to smaller and smaller loops eventually creating a density cavity behind the wave, thereby forming a bonafide shock structure.

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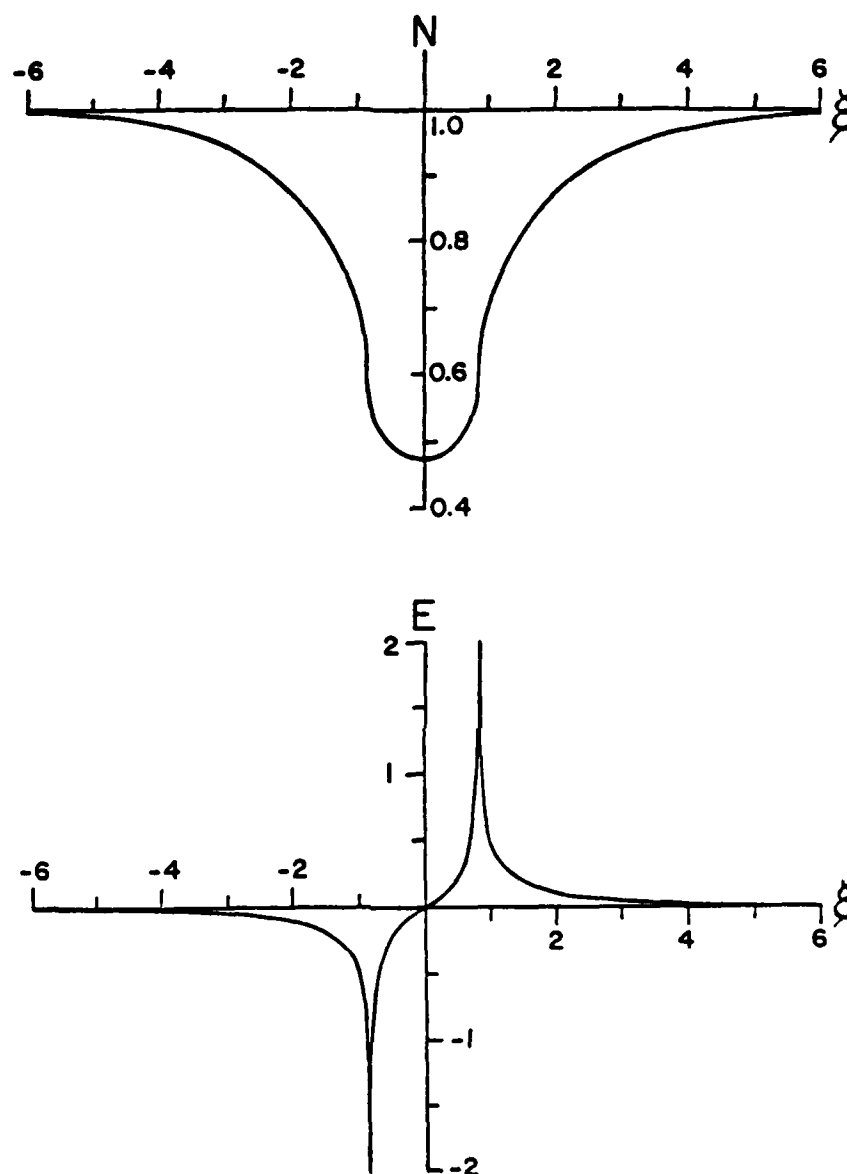
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Fig. IV-1



Typical phase plane trajectories for $M < 1$. Arrows indicate directions of increasing ξ .

Fig. IV-2



Exact caviton profiles $N(\xi)$ and $E(\xi)$ for $M = 0.6$ (E and $dN/d\xi = \infty$ at $N = M$).

V. CHARGED BEAM-PLASMA WAVE INTERACTIONS

A. INTRODUCTION

We are concerned with the generation of plasma wave radiations by a beam of charged particles (electrons or ions) artificially injected in the ionosphere. Such beam induced plasma wave experiments have been carried out in space by Cartwright and Kellogg (1974), the French-Russian ARAKS team (Delahaye et al., 1978; Cambou et al., 1978), and others (e.g., Haerendel and Sagdeev, 1981; and Moore et al., 1982). Detailed discussions of these and other related experiments can be found in a review article by Winkler (1980).

Alekhn et al. (1971) and Gendrin (1974) have shown that for an electron beam (typically 10-40 keV, 1-100 ma) injected along the magnetic field lines, the space charge effects and magnetic field strength limit the beam radius and density during the initial expansion of the beam pulse. This idea of balancing the Lorentz force with the electric force in vacuum was suggested years ago by Brillouin (1945). The resulting differential equation admits both a solution with a limiting "Brillouin radius, r_B " described above and an oscillatory solution around r_B .

For injections oblique to the magnetic field, Jones and Kellogg (1973) took a simple model of a filled cylinder. The radius of the cylinder was taken as the Larmor radius $r_L = u_{\perp}/\Omega_e$, where u_{\perp} is the perpendicular velocity of the beam, and Ω_e the electron cyclotron frequency. The length of the cylinder was estimated from the combined diffusion effects produced by the energy spread of the electrons, the angular dispersion of the beam, and the finite duration of the pulse (~1-20 ms) using simple adiabatic invariant arguments. With a warm ionospheric plasma background, they showed that a variety of electrostatic and electromagnetic plasma waves can be excited. They discussed, in particular,

the electrostatic plasma mode of a band of radiations between the electron plasma frequency ω_{pe} and the upper hybrid frequency ω_{uh} , the electrostatic Bernstein modes around $n\Omega_e$, and the electromagnetic whistler modes below ω_{uh} . Assuming the distributions are spatially uniform, they calculated the growth rates and the parallel and perpendicular components of the group velocities of the plasma and whistler modes. Using these results and the assumed "filled cylinder" beam geometry, they made rough "order of magnitude" estimates of the waves amplitudes and radiative powers of the calculated unstable modes.

Lavergnat and Pellat (1979), on the other hand, used both a hollow cylinder (radius r_L , thickness r_B), and a more realistic helix model (helix radius r_L , beam radius r_B) to calculate the plasma waves spontaneously produced (e.g., via Cerenkov and cyclotron radiations) by the oblique electron pulses. It was shown that plasma modes, Bernstein modes, and whistler modes can again be excited.

Both electrostatic and electromagnetic plasma wave radiations generated by artificially injected electron beams in the ionosphere have been observed experimentally. These include the plasma waves discussed above as well as the electrostatic lower hybrid waves and the predominantly electromagnetic VLF waves. The frequency range of these waves are characterized by $\Omega_e \ll \omega \ll \omega_{pe}$ and their wave-vectors k generally make large oblique angles with respect to the ambient magnetic field. These waves couple the dynamics of the electron beam and the ambient ions. In fact, they can effectively energize the ions in the transverse direction with respect to the ambient magnetic field lines. Experimental verifications of such type of transverse ion acceleration have been reported by Arnold *et al.* (1989).

Ion beam injections in the ionosphere are very different from those of electron beams. Firstly, we expect the ion beams to spread initially in both

larger dimensions (because of the larger Brillouin and Larmor radii) even without wave-particle interactions. This, in turn, allows the unstable modes to interact at longer time intervals and therefore possibly attain larger wave amplifications. Secondly, new unstable modes are expected from ion beam injections. These include the electrostatic ion cyclotron waves, non-resonantly excited ion cyclotron harmonics, ion acoustic waves, and the broad-band electrostatic and electromagnetic VLF waves. These waves can interact with the ion beam causing it to spread anomalously. Furthermore, the low frequency electrostatic waves can parametrically interact with the high frequency waves in the magnetosphere and convert them to electromagnetic radiations. Thirdly, ion beams are known to spread anomalously in an ambient plasma containing pre-existing fluid turbulence (e.g., those due to Kelvin-Helmholtz instability) when electrostatic ion cyclotron waves are also present. Finally, as it will be shown presently, lower hybrid waves excited by the ion beam can accelerate the electrons in the field-aligned direction. The energized electrons can, then, excite other secondary plasma waves.

A series of interesting laboratory experiments designed to simulate actual electron beam injections in space environments have been conducted by Bernstein and his co-workers (Bernstein et al., 1975, 1978, and 1979; Denig, 1982). The electron gun was generally operated dc, although sometimes a pulsed gun was also used. For a 1 keV beam with $I < 10$ ma at 10^{-6} torr in a magnetic field $B \approx 1.5$ G, the beam assumed a multi-node configuration with a node-length $L_N = 2\pi u_{||}/\Omega_e$ where $u_{||}$ is the parallel beam velocity. RF measurements indicate fluctuations near $\sim \Omega_e$ and the lower hybrid frequency.

When the current is raised (at a fixed beam energy) beyond some critical value, the RF fluctuations form (i) a band of emissions near the plasma frequency

and/or upper hybrid resonances, (ii) a broad band of whistler mode emissions with $\omega \approx \Omega_e$, and (iii) an intense broad band of VLF emissions. A column of plasma of enhanced density is observed and there is evidence that the primary electron beam is dispersed both in energy and pitch angle. The peak luminosity of the discharge becomes about ten times that of the beam alone and the radius is increased nearly three-fold.

Such type of beam-plasma-discharges (BPD) can be important in actual space experiments. The increased plasma density can produce the observed neutralization of the space vehicles by providing the required return current. The process is accompanied by strong wave-particle heating, severe energy diffusion, pitch angle diffusion, and radial expansion of the beam profile, placing practical limitations on actual beam injections in space.

It has been recognized that plasma instabilities and the ensuing wave-particle interactions are required to accelerate the charged particles in order to produce the observed anomalous ionization in BPD experiments. Earlier theoretical work (Papadopoulos, 1982) is based on the high frequency electron modes driven unstable by the electron beam. It is our contention (Chang and Casperse, 1982) that the low frequency branch of the beam-excited plasma waves can be very important in determining the coupled electron and ion dynamics of BPD. We suggest that in the presence of an external magnetic field, lower hybrid waves may play an important role in determining the critical current for the onset of BPD. In addition, gyrotron-like whistlers (or electromagnetic lower hybrid waves) are also excited. These electrostatic lower hybrid waves carry most of the energy and can resonantly accelerate the beam particles to the ionization threshold of the ambient medium, producing the enhanced ionization detected in BPD experiments.

B. LOWER HYBRID WAVES AND HYBRID WHISTLERS PRODUCED BY AN ELECTRON BEAM

It is known that in a magnetoplasma, low frequency ($\omega \ll \Omega_e$) whistler modes can be excited by an energetic electron beam. The modes that propagate at very large oblique angles with respect to the magnetic field are excited by the electron beam primarily through Landau resonance. These modes (with $k_{\parallel}^2 \ll k_{\perp}^2$) can transfer energy to the ions in the transverse direction via ion Landau- and gyro-resonances. To simplify the calculation, let us search for modes with $\Omega_i \ll \omega \ll \Omega_e$ and $\rho_i \gg k_{\perp}^{-1} \gg \rho_e$. [$\omega \gtrsim \Omega_i$ can also exist. We will return to this point later.] Thus, we work under the approximation that the electrons are strongly magnetized and the ions nearly unmagnetized.

The dispersion relation for $\omega_e \gg \Omega_e$ is then approximately (Marsch and Chang, 1982):

$$D(\omega) \equiv a_2 \omega^4 - a_1 \omega^2 + a_0 = 0, \quad (1)$$

where

$$a_0 = (\Omega_e \Omega_i)^2 M_A^2 [M_A^2 - (1 + k_{\parallel}^2/k^2)](1 - Z_e), \quad (2a)$$

$$\begin{aligned} a_1 = & (k_{\parallel}^2/k^2) \Omega_e^2 [1 - Z_e - (\omega_i/\omega_e)^2 Z_i] \\ & + (k_{\perp}^2/k^2) |\Omega_e \Omega_i| (1 - Z_i) \\ & + M_A^2 |\Omega_e \Omega_i| [(1 + k_{\parallel}^2/k^2) Z_e - 2] \end{aligned} \quad (2b)$$

$$a_2 = 1 - (k_{\parallel}^2/k^2) Z_e - (\omega_i/\omega_e)^2 Z_i, \quad (2c)$$

with

$$M_A^2 = (\omega/kv_A)^2,$$

$$v_A^2 = B^2/\Sigma 4\pi n_j m_j,$$

$$Z_e = \sum_j (n_j/n_e) [1 + (\omega/k_{||} v_{j||})^2 W(\omega/k_{||} v_{j||})],$$

$$Z_i = \sum_{\lambda} (\omega_{\lambda}/\omega_i)^2 [1 + (\omega/kv_{\lambda}(\alpha))^2 W(\omega'/kv_{\lambda}(\alpha))],$$

$$W(z) = (2\pi)^{-1/2} \int_{-\infty}^{\infty} dx \, x e^{-x^2/2} / (x-z), \quad (3)$$

$$\omega' = \omega - k_{||} u_j,$$

$$v_{j||}^2 = k_B T_{j||} / m_j,$$

$$v_i^2(\alpha) = v_{i||}^2 \cos^2 \alpha + v_{i\perp}^2 \sin^2 \alpha,$$

$$\cos^2 \alpha = k_{||}^2 / k^2, \sin^2 \alpha = k_{\perp}^2 / k^2.$$

" ω_i " is the plasma frequency of the major ion species, " u_j " the drift velocity, and all distributions are expressed as sums of Maxwellians.

The electrostatic limit is obtained by setting $M_A = 0$. Then,

$$\omega^2 = \frac{\Omega_e^2 (k_{||}/k)^2 (1-Z_e) + |\Omega_e \Omega_i| (k_{\perp}/k)^2 (1-Z_i)}{1 - (k_{||}/k)^2 Z_e - (\omega_i/\omega_e)^2 Z_i}. \quad (4)$$

Thus, ion dynamics is important if $k_{||}^2 \ll k_{\perp}^2$ (i.e., for near perpendicular propagations). Without thermal effects, we recover the classical expression

$$\omega^2 = |\Omega_e \Omega_i| [1 + (k_{||}/k)^2 (m_i/m_e)]. \quad (5)$$

Far results are obtained for the full electromagnetic expression. With

$\omega = \omega_r + i\gamma$, we have

$$\omega_r^2 = \frac{1}{2a_2} [a_1 \pm (a_1^2 - 4a_0 a_2)^{1/2}], \quad (6a)$$

and

$$(\gamma/\omega_r)^\alpha = \text{Im } D(\tilde{\omega}), \quad (6b)$$

where

$$\begin{aligned} \text{Im } D(\tilde{\omega}) = & -\tilde{\omega}^2(\tilde{\omega}^2 - |\Omega_e/\Omega_i|) \cdot \\ & \cdot [(k_{||}/k)^2 \text{Im } Z_e + (\omega_i/\omega_e)^2 \text{Im } Z_i] \\ & - [(\tilde{\omega}^2 - 1)(1 + k_{||}^2/k^2)M_A^2 + M_A^4] \text{Im } Z_e. \end{aligned} \quad (6c)$$

We can conclude from (5) and (6) that:

- (i) the electromagnetic corrections are small on the resonance cone,
- (ii) ion damping (heating) is important for near perpendicular propagations ($k_{||}^2/k_{\perp}^2 \sim m_e/m_i$),
- (iii) the growth rate γ is large on the resonance cone.

C. THE ROLE OF LH WAVES IN BEAM-PLASMA-DISCHARGES

From the discussion of Sec. VB, we expect that both the electrostatic LH waves (electrostatic whistlers) and the electromagnetic LH waves (i.e., "Hybrid" or very obliquely propagating whistlers) will be excited by an energetic electron beam, with the former carrying most of the energy.

The primary electron beam is expected to ionize initially a small fraction of the ambient medium and at the same time to generate the LH waves. The electrostatic LH waves transfer the energy of the electrons to the ions [in fact, energizing a fraction of the ions above the thermal energy]. When some of the ions are energized above the ionization threshold, more of the ambient medium is ionized. This process (indicated by the solid lines of the schematic loop diagram in Fig. 1)

continues until that part of the lower hybrid wave energy responsible for energizing the ions just above the ionization potential is saturated. If the electron beam is maintained, the ionization will reach a sort of a stationary state. The total amount of ionization in this state is expected to be much higher than that can be produced without the LH wave-particle interaction. This process of anomalous ionization is at least partially responsible for the phenomenon of BPD.

Recent BPD experiments in the large vacuum chamber at the Johnson Space Center carried out with the Plasma Diagnostic Package (PDP) designed for the space shuttle (Denig, 1982) indicated the existence of a trapped ion population (typically with an average energy of 50-60 eV) upon the onset of BPD. There was also the indication of the appearance of a radial dc electric field with a potential well located near the beam gyroradius. These results are tentative and require further experimental verification based upon more refined diagnostic techniques and instrumentation. Furthermore, the theoretical justification of this kind of ion trapping does not appear to be immediately apparent. If the trapping does set in, however, it can be argued that the positive perpendicular gradient of the resulting ion distribution function can excite the ion cyclotron harmonic waves (Böhmer, 1976; Kintner, 1980) as well as secondary lower hybrid waves. Both of these waves can effectively energize the electrons in the field-aligned direction. These energized electrons can, in turn, ionize the ambient medium and the newly produced low energy electrons can again be accelerated by the secondary LH waves and the ion cyclotron harmonics. Beyond a certain critical density, this effect can efficiently multiply to produce a significant additional fraction of the enhanced ionization. Such a process may be quite temporal and sporadic. This may provide an explanation of the observed (10 Hz) oscillation

of the total intensity of ionization. We have indicated this "probable" process as broken lines in the loop diagram of Fig. 1.

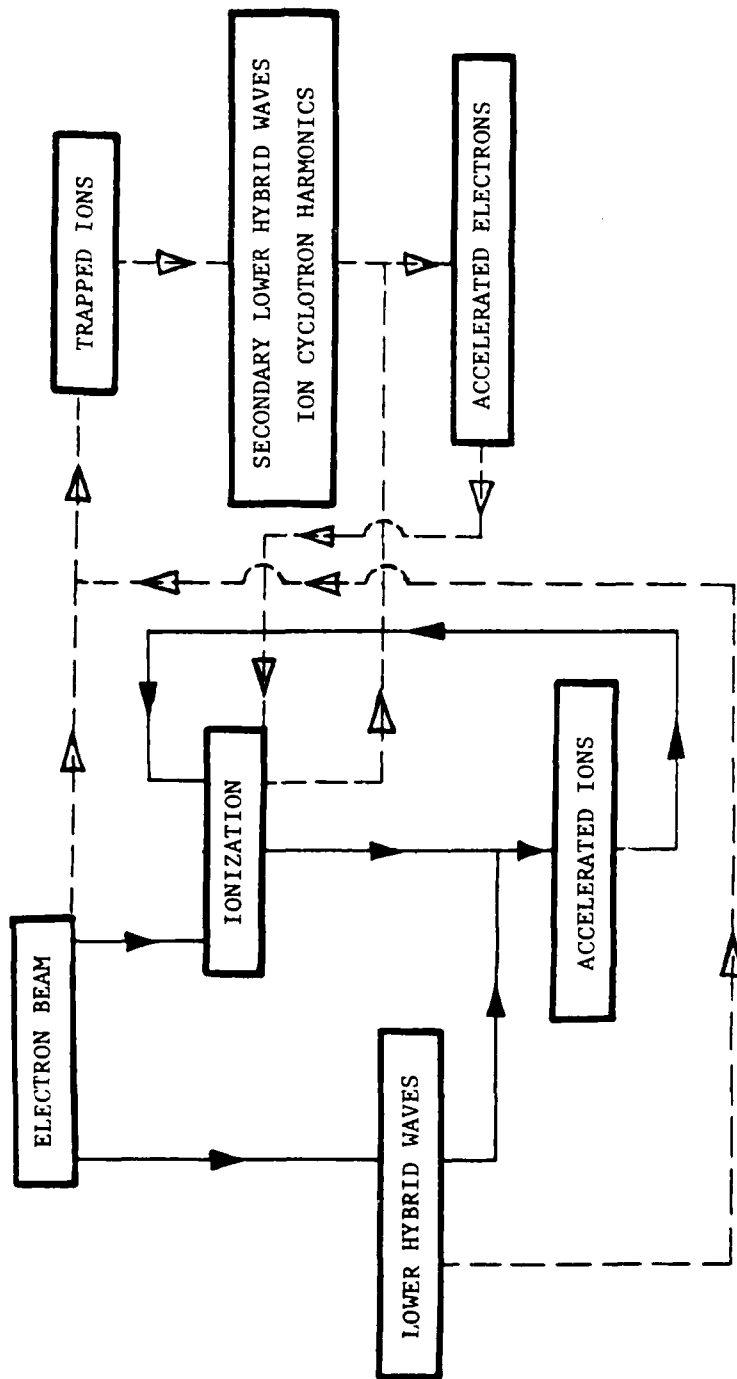
We note that a similar process also occurs for a high current, energetic ion beam (see Fig. 2). Such processes may have been observed by the PORCUPINE Xe^+ -gun experiment (Kintner, private communication), and Ar^+ -gun sounding rocket experiments (Moore et al., 1982).

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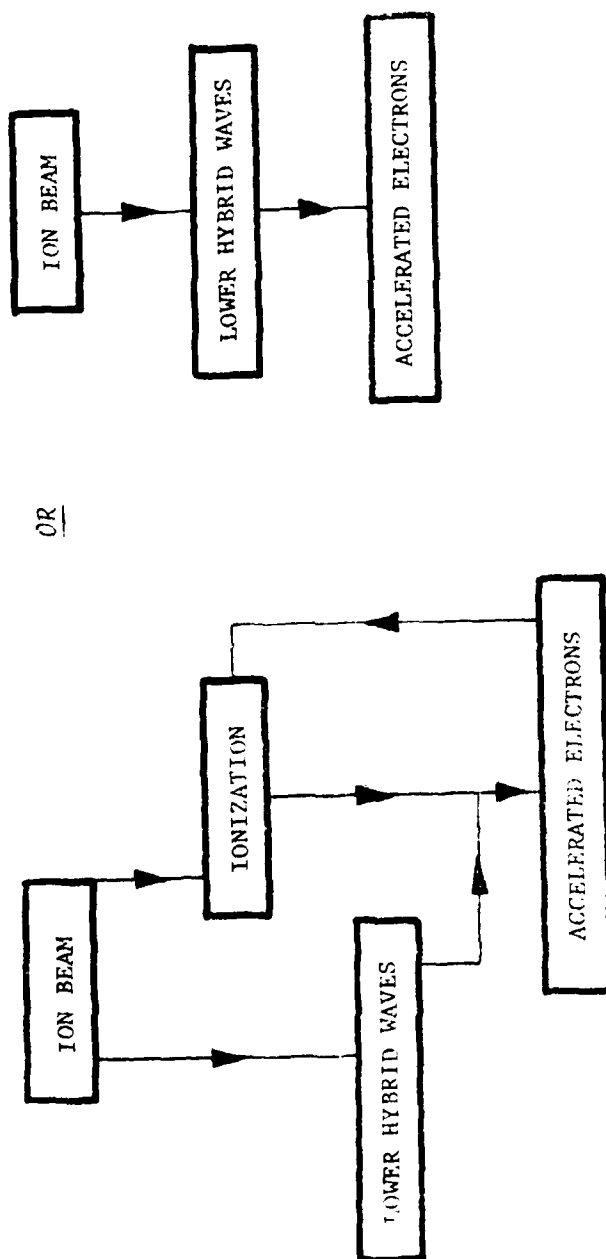
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Fig. V-1



Loop diagram depicting the suggested lower hybrid wave-particle processes in a beam-plasma discharge.

Fig. V-2



Diagrams depicting the lower hybrid wave-particle processes produced by an ion beam.

VI. THE UBIQUITOUS LOWER HYBRID MODES IN SPACE PLASMAS

A. INTRODUCTION

We have seen that the electrostatic lower hybrid waves (electrostatic whistlers) and electromagnetic lower hybrid waves (hybrid or very obliquely propagating whistlers) populate the topside ionosphere, the suprapaural region and are detected in naturally occurring or artificially created beam-plasma processes. Our investigations have shown that lower hybrid waves are omnipresent in the ionospheres and magnetospheres of the Earth and other planets, ahead of the planetary bow shocks, across interplanetary shocks, and in the solar wind. Space does not allow a full discussion of these and other phenomena involving lower hybrid wave-particle interactions in space plasmas. As an example, we will demonstrate that the frequently observed broad-band low frequency electrostatic noise in the solar wind in fact has a dominant lower hybrid component. Other similar examples are abound and have been detected in plasmas of various terrestrial and extra-terrestrial environments.

B. LOWER HYBRID WAVES IN THE SOLAR WIND (Marsch and Chang, 1982)

Plasma wave measurements in the solar wind (Gurnett and Frank, 1978; Gurnett, 1981; Scarf et al., 1981) often exhibit broad-band low frequency electrostatic turbulent noises accompanied by bursts of low frequency, obliquely propagating, hybrid whistler mode fluctuations (Beinroth and Neubauer, 1981; Coroniti et al., 1982). These electrostatic noises are usually identified as ion acoustic waves (Gurnett and Anderson, 1977; Gurnett et al., 1979). However, because of ion damping, ion acoustic waves can be excited by the solar wind particle distributions only when the electron temperature T_e is much larger than the ion temperature T_i . Most of the time this condition is not satisfied when the low frequency

electrostatic fluctuations are detected (Dum et al., 1981). Thus, an alternative explanation of these frequently observed electrostatic modes is needed. We propose here that lower hybrid waves can be excited by the halo of the electron distributions that represent the heat flux in the solar wind. These modes, Doppler-shifted, are expected to have the observed frequency range of the LF electrostatic noises.

C. DISPERSION RELATION

The solar wind electron heat flux distribution can be modeled by a thermal core and a suprathermal halo component (Feldman et al., 1975). It is known that such an electron distribution can excite electromagnetic whistler modes (Maggs, 1976). Neglecting electron damping of the core distribution for the moment, the modes that cluster around the lower hybrid resonance cone for $k_{\parallel}^2 \ll k_{\perp}^2$ and a cold plasma obey the dispersion relation

$$\omega^2 = \omega_{pi}^2 [1 + (k_{\parallel} \omega_{pe} / k \omega_{pi})^2] / [1 + (\omega_{pe} / \Omega_e)^2], \quad (1)$$

where $\omega_{pi}^2 = 4\pi n_i e_i^2 / m_i$, and $\omega_{pe}^2 = 4\pi n_e e^2 / m_e$ are the squares of the ion and electron plasma frequencies, Ω_e is the electron gyrofrequency, and $(k_{\parallel}, k_{\perp})$ are the parallel and perpendicular component of the wave vector, k . For $\omega_{pe}^2 \ll \omega_{pi}^2$, this expression reduces to that discussed by Coppi et al. (1976) and Papadopoulos and Palmadesso (1976) in the electrostatic limit and the magnitude of the frequency is of the order of the ion plasma frequency. In the solar wind, on the other hand, $\omega_{pe}^2 \gg \omega_{pi}^2$, and (1) gives $\omega \approx |\Omega_e \Omega_i|^{1/2}$, the classical lower hybrid resonance frequency. Thus, the magnitude of the frequency range of the lower hybrid modes in the solar wind can be much less than that of the ion plasma frequency with $\omega \ll \omega_{pi}$, where Ω_i is the ion gyrofrequency.

In this discussion, we consider purely electrostatic modes in an electron-ion plasma, where the ions can be treated as unmagnetized. The full dispersion relation may be written as

$$\tilde{k}^2 = - \sum_j t_j \epsilon_j(\underline{k}, \omega), \quad (2)$$

where $t_j = (k_j/k_i)^2$ is the squared ratio of Debye wave vector of the j^{th} species ($k_j^2 = 4\pi e_j^2 n_j/T_j$) to the ion Debye vector k_i and $\tilde{k} \equiv k/k_i$. In the unmagnetized case, the normalized dielectric constant for a bi-Maxwellian with a mean squared thermal speed $v_j^2 = (v_{j\parallel}^2 + 2v_{j\perp}^2)/3$ is defined by

$$\epsilon_j(\underline{k}, \omega) = [v_j/v_j(\alpha)]^2 W[\omega'/kv_j(\alpha)], \quad (3)$$

with the Doppler-shifted frequency $\omega' = \omega - k_{\parallel}u_j$ and the effective thermal velocity $v_j^2(\alpha) = v_{j\parallel}^2 \cos^2 \alpha + v_{j\perp}^2 \sin^2 \alpha$. The function $W(z)$ is related to the plasma dispersion function (Ichimaru, 1973). The angle α determines the wave propagation direction ($k_{\parallel} = k \cos \alpha$ and $k_{\perp} = k \sin \alpha$) with respect to the magnetic field. We allow for streaming along the field relative to the center of mass frame with velocity u_j for species j . The wave phase velocity refers to the same frame.

The magnetized dielectric constant can be cast into the form

$$\epsilon_j(\underline{k}, \omega) = (v_j/v_{j\perp})^2 (\epsilon_{j0} + \sum_{n=1}^{\infty} \epsilon_{jn}). \quad (4)$$

The zeroth order contribution ϵ_{j0} is the only term that survives in the strongly magnetized limit,

$$\epsilon_{j0} = 1 + e^{-b_j} I_0(b_j) [(T_{j\perp}/T_{j\parallel}) W(z_0) - 1], \quad (4a)$$

and the gyroresonances are described by

$$\begin{aligned} \epsilon_{jn} = & \{z_0[(W(z_n)-1)/z_n + (W(z_{-n})-1)/z_{-n}] \\ & + [(T_{j\perp}/T_{j\parallel})-1]\{W(z_n) + W(z_{-n})\}\} e^{-b_j} I_n(b_j). \end{aligned} \quad (4b)$$

As usual, $I_n(b_j)$ is the modified Bessel function of order n with argument

$$b_j = (k_{j\perp} v_{j\perp} / \tilde{\omega}_j)^2 = \tilde{\omega}^2 (\phi_{j\perp} x_j)^2. \quad (4c)$$

For convenience, we have introduced the normalized perpendicular phase velocity $\phi_{j\perp} = \omega/k_{j\perp} v_{j\perp}$ and the frequency $\tilde{\omega} = \omega/\omega_{pi}$ normalized to the ion plasma frequency. The ratio $x_j = \Omega_j/\omega_{pi}$ determines the degree of magnetization with $\Omega_j = e_j B/m_j c$. The normalized resonant speed is $z_n = \phi_{jn}(1 - nx_j/\tilde{\omega}) - u_j/v_{j\parallel}$ whereby $t_{j\parallel} = u_j/k_{j\parallel} v_{j\parallel}$ is the parallel phase velocity in units of the thermal speed for the j^{th} species. These quantities are useful in localizing the wave-particle resonances in velocity space. For given parameters of the particle distributions, (2) depends critically on $t_j = (T_i/T_j) \cdot (n_j/n_i) \cdot (e_j/e_i)^2$ and x_j which is determined by the density and the magnetic field strength. Finally, we quote the normalized growth rate, which for small growth is given by

$$\gamma/\omega = - \sum_j t_j \operatorname{Im} \epsilon_j / \sum_j t_j \omega \operatorname{Re} \epsilon_j / \omega. \quad (5)$$

D. SOME NUMERICAL RESULTS

We briefly discuss the various possible low frequency modes derived from (2). Free energy sources for these modes are also proposed in terms of typical solar wind particle distributions. The normalized dielectric constants in (4) are then calculated as functions of their arguments (for example, $-0.285 + i\omega_{pe}/\omega$ for $t_e = 1$ and x_e ranges from 0.02 to 2.0). For $t_e \approx 1$ or $T_e \approx T_i$, both species yield comparable contributions to (2) if $\phi_{i\perp} \approx \phi_{e\perp}$. This condition implies $(k_{i\perp}^2/k_{e\perp}^2) = n_e/n_i$.

If $\phi_{i\parallel} \gg 1$ and $|\phi_{e\parallel}| \ll 1$, one has the classical undamped LH waves. Otherwise, the dispersion characteristics are modified by thermal effect and Landau damping. Considering $t_e \ll 1$ or $T_e \gg T_i$, one finds that the electron contribution to (2) can be kept small even for $|\phi_{e\parallel}| \ll 1$ and $\phi_{i\perp} < 1$. Then one obtains the usual ion sound mode of ion plasma oscillations, Debye-shielded by the background electrons. For Maxwellian distributions, this mode can exist only for $t_e < 0.285$. Realistic distributions have been discussed extensively by Dum et al. (1981).

Many plasmas are characterized by moderate ratios of $t_e > 1$. Besides the classical lower hybrid waves, another possible situation is for $|\phi_{e\parallel}| < 1$ and $\phi_{i\perp} > 1$. The last condition is necessary in order to keep the ion contribution low in the total dielectric constant. If $\phi_{i\perp}$ is large enough, the term ϵ_i in (2) can be neglected and a pure electron mode emerges.

Retaining only terms to lowest order in b_e gives the following expression for the normalized wave frequency in a bi-Maxwellian electron-ion plasma without drifts for oblique propagations

$$\omega^2 = [-z_i^2 W(z_i) - (k_{\parallel}/k)^2 (\omega_{pe}/\omega_{pi})^2 \phi_{e\parallel}^2 W(\phi_{e\parallel})] / [1 + (k_{\perp}/k)^2 (\omega_{pe}/\omega_e)^2 (1 - (T_{e\perp}/T_{e\parallel}) W(\phi_{e\parallel}))], \quad (6)$$

where $z_i = \omega/kv_i(\alpha) = \phi_{i\parallel}\phi_{i\perp}/(\phi_{i\parallel}^2 + \phi_{i\perp}^2)^{1/2}$. In the cold plasma limit $-z_i^2 W(z_i) \rightarrow 1$ and (6) reduces to the expression discussed in the introduction for $k_{\perp} \gg k_{\parallel}$. As long as $b_e < 1$, the above result is fairly general and can include the thermal effects. For given phase velocities, the frequency ω is readily obtained for (6), that yields a broad band spectrum in frequency which depends on ω_{pe}/ω_e , and the wave propagation direction. Solutions may also exist for moderate values of $k_{\parallel}/k_{\perp} < 1$.

The following numerical results, based on the full dispersion equation (2), illustrate the above qualitative arguments. For the sake of simplicity, we assume Maxwellian distributions with $n_p = n_e = 10 \text{ cm}^{-3}$, $v_p = 45 \text{ km/s}$ and $v_e = 1,250 \text{ km/s}$; these are typical densities and thermal speeds for a solar wind with a velocity of about 500 km/s (Feldman et al., 1975). The temperature ratio is then $T_p/T_e = 2.38$. The ratio $x_i = \Omega_i/\omega_{pi}$ varies to a certain extent in the solar wind depending on the macroscopic stream structure. For $x_p = 5 \cdot 10^{-4}$, $|k_{||}/k_{\perp}| = 1/25$, and $\omega/k_{||} = 3,750 \text{ km/s}$, one obtains $\tilde{\omega} \approx 0.054$ and $\gamma/\omega \approx -0.079$. The proton and electron damping are roughly equal and the value of ω is found to be very close to the value given in (6). The electrons are strongly magnetized (i.e., Landau damping dominates over cyclotron damping) with $b_e \approx 0.24$ and $x_e \approx 0.918$. Actually, for the same particle parameters used above, a solution exists for various values of x_e . For example, for $x_e = 18.4$ or $x_p = 10^{-2}$, the same value of b_e results if $\tilde{\omega}$ is increased to 0.975. It is worth emphasizing that, in contrast to ion sound, the lower hybrid waves do not crucially depend on the temperature ratio T_e/T_p .

In Table 1, we have listed some typical fast solar wind parameters with $T_p/T_e = 1$. The electron distribution is represented by two drifting bi-Maxwellians [core(c) and halo(h)]. Using these parameters and for $\omega/k_{||} = 3,500 \text{ km/s}$, $\omega/k_{\perp} = 464 \text{ km/s}$, $x_p/\omega_{pp} = 5 \cdot 10^{-4}$, $\alpha \approx 82.4^\circ$, $b_p = 167$, we find $\tilde{\omega} \approx 0.1$ and $\gamma/\omega \approx +0.031$. Leaving the phase velocities and other parameters unchanged, an increase in x_e/ω_{pe} to 18.37 increases $\tilde{\omega}$ to 2.0 with the same γ . We have chosen a value of $n_h/n_p = 0.15$ which is slightly in excess of what is usually observed. The sign of γ is changed by reducing the value n_h to $0.05 n_p$.

Finally, we consider a case with $x_p/\omega_{pp} = 10^{-3}$, a number occasionally observed at 0.3 AU. For isotropic core and halo Maxwellian distributions with $v_c = 1,200 \text{ km/s}$, $v_h = 3,600 \text{ km/s}$, $u_c = -200 \text{ km/s}$, $u_h = 3,600 \text{ km/s}$, $n_c/n_p = 0.95$, $n_h/n_p = 0.05$, $\alpha = 82.4^\circ$, $\omega/k_{||} = 3,500 \text{ km/s}$ and $\omega/k_{\perp} = 328 \text{ km/s}$, we find $\tilde{\omega} \approx 0.2$, $\gamma/\omega \approx -0.016$.

and $\alpha \cong 84.6^\circ$. By slightly increasing the halo density, this wave can be driven unstable by the halo electrons.

E. SUMMARY

We have given numerical examples indicating that typical solar wind plasmas (0.3 - 1 AU) can support electrostatic lower hybrid modes. Because $\omega_{pe} \gg \Omega_e$ in the solar wind, the cutoff frequency of these electrostatic modes is near the classical lower hybrid resonance $\omega_{LHR} \sim |\Omega_e \Omega_i|^{1/2}$. These modes are probably driven unstable by the anisotropic halos (the heat flux) of the electron distributions. This may have important consequences for transport processes.

Since these waves propagate nearly normal to the magnetic field lines, they can energize the ions in the solar wind in the transverse direction and thus produce anisotropic ion distributions. Alternatively, these waves could be excited by loss-cone type ion distributions, that may originate from interplanetary shocks.

The lower hybrid modes are generally accompanied by broad band electromagnetic whistler modes in the solar wind (Beinroth and Neubauer, 1981; Coroniti et al., 1982). These low frequency, very obliquely propagating whistler modes can be generated efficiently by the electron halos. We call these waves the electromagnetic lower hybrid or hybrid whistler modes. Unlike the ion acoustic modes, the em and es LH modes can be supported for situations with $T_i \sim T_e$ and may be ubiquitous in space plasmas. They have been found in the planetary ionospheres and magnetospheres (Temerin et al., 1981) ahead of planetary bow shocks (Anderson et al., 1981) and across interplanetary shocks (Coroniti et al., 1982).

The results of the present analysis strongly suggest that the frequently observed low frequency electrostatic noises and very obliquely propagating whistlers (in the solar wind and other space plasma environments) are probably lower hybrid modes similar to those discussed in Sections II and V.

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Table 1. Velocities in km/s and densities in cm^{-3}

Species	$v_{j\parallel}$	$v_{j\perp}$	u_j	n_j	T_p/T_j
e core	1200	1200	-953	8.5	1.03
e halo	3000	3000	+5400	1.5	0.165
protons	25	30	0	10	1

VII. STRONG PLASMA TURBULENCE AND STOCHASTIC HEATING

A. INTRODUCTION

In the previous sections, we have discussed a number of plasma instabilities and the associated wave-particle interactions using the ideas of weak turbulence and considered their possible relation to the various observed ionospheric and magnetospheric plasma phenomena. Most of the observed plasma wave data, however, display strong turbulence signatures. For strong plasma turbulence, it is well known that the correlation effects and response functions are extremely long-ranged, and therefore, cannot be adequately described by the existing classical quasilinear or nonlinear mode-mode coupling plasma theories. In a strongly turbulent plasma, charged particles can be efficiently accelerated by the long-ranged interacting, stochastic wave fields to very large energies without the classical resonant wave-particle interactions.

The concept of large scale stochasticity, strong plasma turbulence, and stochastic heating and diffusion is still in the initial developing stage. It is the consensus that these effects can be treated analytically using the newly developed tool in statistical mechanics--the renormalization-group. We have made a number of recent contributions in this direction (Chang, Nicoll and Young, 1978; Vvedensky and Chang, 1982; Vvedensky, Chang, and Nicoll, 1983). It is shown that the stochastic description of (weak and strong) plasma turbulence can be constructed in terms of a path integral formalism similar to that is employed in quantum electrodynamics. Our starting point is a generalized Langevin equation based on the Vlasov formulation coupled with a random noise term. We demonstrated that the "probability density functional" can be calculated using an appropriate functional Jacobian involving the noise fields.

To calculate the correlation and response functions, we have constructed a generating functional using the Legendre transform of the probability density functional. This generating functional may be evaluated approximately for weak turbulence using the quasilinear prescriptions or other truncated nonlinear mode-mode coupling interaction schemes provided the basic physical mechanism of the plasma instability is known. Earlier renormalization theories of Dupree (1966) and the direct-interaction approximation of Kraichnan (1964, 1970) can be derived using this path integral formalism.

B. RENORMALIZATION GROUP AND OPERATOR ALGEBRA FOR DYNAMICS

Based on this path integral formulation, we have developed differential renormalization-group techniques to evaluate the random dynamics of strongly stochastic nonlinear systems. This renormalization-group equation for dynamics can be solved most conveniently using an operator expansion technique which we developed recently (Vvedensky and Chang, 1982). Briefly, the procedure is as follows.

We start with the one-particle irreducible differential renormalization-group generator for dynamics (Chang, Nicoll, and Young, 1979).

$$\begin{aligned}
 \frac{\partial A}{\partial \lambda} = & (d+z)A + \int \frac{d\vec{k}}{(2\pi)^d} \int \frac{d\omega}{2\pi} \psi_1(k\omega) \left[\frac{1}{2}(2-\eta-d) - \mathbf{k} \cdot \nabla_{\mathbf{k}} - z\omega \frac{\partial}{\partial \omega} \right] \cdot \frac{\delta A}{\delta \psi_1(k\omega)} \\
 & + \psi_2(k\omega) \left[\frac{1}{2}(2-\eta-d) - \mathbf{z} - \mathbf{k} \cdot \nabla_{\mathbf{k}} - z\omega \frac{\partial}{\partial \omega} \right] \cdot \frac{\delta A}{\delta \psi_2(k\omega)} \\
 & + \frac{1}{2} \int \frac{d\omega}{(2\pi)^d} \int_{-\infty}^{\infty} \frac{d\omega'}{2\pi} \text{tr} \ln \hat{A}(q\omega; -q, -\omega) \\
 & - \int \frac{d\vec{p}'}{(2\pi)^d} \int \frac{d\vec{p}''}{(2\pi)^d} \int \frac{d\omega'}{2\pi} \int \frac{d\omega''}{2\pi} \hat{A}(q, \omega; \mathbf{p}'\omega') [\hat{A}^{-1}](\mathbf{p}'\omega'; \mathbf{p}''\omega'') \hat{A}(\mathbf{p}''\omega''; -q, -\omega)
 \end{aligned}
 \tag{1}$$

where A is the action integral, $\psi_1^i(k\omega)$ and $\psi_2^i(k\omega)$ are the Fourier components of the order parameter and the conjugate momentum, respectively, with $i=1\dots n$. ψ_1^i satisfies the nonlinear Langevin equation

$$\dot{\psi}_1^i(x,t) = f_i[\psi_1(x,t), x, t] + \eta_i(x,t) . \quad (2)$$

The dimension of space is d and the dynamic exponent is denoted by z . Also,

$$\hat{A}(k\omega; k'\omega') \rightarrow A_{\alpha\beta}^{ij}(k\omega; k'\omega') \rightarrow \frac{\delta^2 A}{\delta \psi_\alpha^i(k\omega) \delta \psi_\beta^j(k\omega)} . \quad (3)$$

The correlation function $G(k\omega)$ and response function $R(k\omega)$ are given by the functional derivatives:

$$\delta_{ij} G(k\omega) = \frac{\delta^2 W}{\delta h_i(-k, -\omega) \delta h_j(k, \omega)} \Big|_{h,g=0} , \quad (4a)$$

$$\delta_{ij} R(k\omega) = \frac{\delta^2 W}{\delta h_i(-k, -\omega) \delta g_j(k\omega)} \Big|_{h,g=0} , \quad (4b)$$

where the generating functional W is defined by

$$\begin{aligned} \exp W(\{h,g\}) &= \int [d\psi_1] \int [d\psi_2] \\ &\exp \{ - \int dx \int dt [iL(\psi_1(x,t), \dot{\psi}_1(x,t), \psi_2(x,t)) \\ &\quad - h(x,t)\psi_1(x,t) - g(x,t)\psi_2(x,t)] \} \end{aligned} \quad (5a)$$

with $L = L_1 + L_2$ and

$$L_1 = \psi_2(x,t) [\dot{\psi}_1(x,t) - f(\psi_1(x,t), x, t) - \frac{i}{2} \sum_i \frac{\partial f_i[\psi_1(x,t), x, t]}{\partial \psi_1^i}] \quad (5b)$$

$$\begin{aligned}
L_2 = & -\frac{i}{2} \sum_{j,k} \int dx \, dx' \int dt \, dt' S_{jk}(x,t;x',t') \psi_2^j(x,t) \psi_2^k(x',t') \\
& + \frac{1}{3!} \sum_{i,j,k} \int dx \, dx' \, dx'' \int dt \, dt' \, dt'' S_{ijk}(x,t;x',t';x'',t'') \\
& \quad \psi_2^i(x,t) \psi_2^j(x',t') \psi_2^k(x'',t'') \\
& + \dots
\end{aligned} \tag{5c}$$

"S_{jk}", "S_{ijk}", ..., are the n-point connected Green functions of η_i .

We restrict our discussion to the special situation of a time-dependent-Ginsburg-Landau (TDGL) model with

$$f_i = -\Gamma(x,t) \frac{\delta H}{\delta \psi_i(x,t)}, \tag{6}$$

$$\langle \eta_i(x,t) \eta_j(x',t') \rangle = 2\Gamma(x,t) \delta(x-x') \delta(t-t') \delta_{ij}. \tag{7}$$

Then, the action integral is:

$$\begin{aligned}
A = & \sum_{\alpha,\beta} \int \frac{d\vec{k}}{(2\pi)^d} \int \frac{d\omega}{2\pi} r_{\alpha\beta}(k,\omega) \psi_\alpha(k,\omega) \psi_\beta(-k,-\omega) \\
& + \sum_{\alpha,\beta,\gamma,\delta} r_{\alpha\beta\gamma\delta} \int \frac{d\vec{k}_1}{(2\pi)^d} \dots \int \frac{d\vec{k}_2}{(2\pi)^d} \int \frac{d\omega_1}{2\pi} \dots \int \frac{d\omega_2}{2\pi} \\
& \quad \cdot u_{\alpha\beta\gamma\delta}(k_1\omega_1; \dots; k_2'\omega_2') \psi_\alpha(k_1\omega_1) \psi_\beta(k_1'\omega_1') \psi_\gamma(k_2\omega_2) \psi_\delta(k_2'\omega_2') \\
& \quad \cdot \delta(k_1 + \dots + k_2') \delta(\omega_1 + \dots + \omega_2') \\
& + \dots
\end{aligned} \tag{8}$$

where the $n_{\alpha\beta\gamma\delta}$'s are combinatorial factors, and

$$\begin{aligned} r_{12} &= i(r+k^2+i\omega/\Gamma_k) \quad , \\ r_{21} &= i(r+k^2-i\omega/\Gamma_k) \quad , \\ r_{22} &= 1/2\Gamma_k \quad . \end{aligned} \quad (9)$$

We write $A = A_G + A'$, with the Gaussian fixed point

$$\begin{aligned} A_G &= \int \frac{d\vec{k}}{(2\pi)^d} \int \frac{d\omega}{2\pi} [r_{12} \vec{\psi}_1(k, \omega) \cdot \vec{\psi}_2(-k, -\omega) \\ &\quad + r_{21}(k, \omega) \vec{\psi}_2(k, \omega) \cdot \vec{\psi}_1(-k, -\omega) + r_{22}(k) \vec{\psi}_2(k, \omega) \cdot \vec{\psi}_2(-k, -\omega)] \quad , \end{aligned} \quad (10)$$

[From here on, we shall explicitly display the inner products, indices, and summation signs.]

Associated with the Gaussian fixed point are the exponents

$$\eta_G = 0 \text{ and } z_G \equiv \bar{z} = \begin{cases} 4, & \text{if } \Gamma_k = \gamma k^2 \\ 2, & \text{if } \Gamma_k = \Gamma. \end{cases} \quad (11)$$

Then, defining

$$\Gamma_\alpha^i(k, \omega) = \psi_\alpha^i(k, \omega) \left[\frac{1}{2}(2-d) - \bar{z} \delta_{\alpha 2} - k \partial_k - \bar{z} \omega \frac{\partial}{\partial \omega} \right], \quad (12)$$

the linearization of (1) about A_G reads,

$$\begin{aligned} \frac{\partial A'}{\partial \ell} &= (d+\bar{z})A' + \int \frac{d\vec{k}}{(2\pi)^d} \int \frac{d\omega}{2\pi} \sum_{\alpha=1}^2 \sum_{i=1}^n \Gamma_\alpha^i(k\omega) \frac{\delta A'}{\delta \psi_\alpha^i(k\omega)} \\ &\quad + \frac{1}{2} \int \frac{d\Omega}{(2\pi)^d} \int \frac{d\omega}{2\pi} \text{tr} [\hat{A}_G^{-1}(q, \omega; -q, -\omega) \hat{A}'(q, \omega; -q, -\omega)] \end{aligned} \quad (13)$$

We denote the RHS of (13) by L and define eigenfunctions ρ with eigenvalues y by

$$L\rho = y\rho. \quad (14)$$

If the last term of (13) was absent, the solutions to (14) would follow immediately. Thus, invoking the identity

$$e^A B e^{-A} = B + [A, B] + \frac{1}{2}[A, (A, B)] + \dots \quad (15)$$

for operators A and B , we consider a transformation of (14) generated by an operator y

$$(e^y L e^{-y})(e^y \rho) = L' \rho' = y \rho' = y e^y \rho, \quad (16)$$

such that the second functional derivative terms are eliminated. Such a transformation is obtained by choosing y to satisfy the commutator relations,

$$\left[y, \int \frac{d\omega}{(2\pi)} \int \frac{d\omega}{2\pi} \text{tr} [A_G^{-1}(q, \omega; -q, -\omega) \hat{A}(q, \omega; -q, -\omega)] \right] = 0, \quad (17a)$$

$$\begin{aligned} & \left[y, \int \frac{d\mathbf{k}}{(2\omega)} \int \frac{d\omega}{2\pi} \sum_{\alpha} \frac{\psi_{\alpha}^i(\mathbf{k}, \omega)}{\delta \psi_{\alpha}^i(\mathbf{k}, \omega)} \right] \\ &= - \frac{1}{2} \int \frac{d\omega}{(2\pi)} \int \frac{d\omega}{2\pi} \text{tr} [A_G^{-1}(q, \omega; -q, -\omega) \hat{A}(q, \omega; -q, -\omega)], \end{aligned} \quad (17b)$$

where $\hat{A}(q, \omega; -q, -\omega)$ is the matrix with entries $\hat{A}_{\alpha\beta}^{ij}(q, \omega; -q, -\omega)$ and

$$\hat{A}_{\alpha\beta}^{ij}(q, \omega; -q, -\omega) = \frac{\delta^2}{\delta \psi_{\alpha}^i(q, \omega) \delta \psi_{\beta}^j(-q, -\omega)}. \quad (18)$$

For y of the form,

$$y = \int \frac{d\vec{k}}{(2\pi)^d} \int \frac{d\omega}{2\pi} \text{tr} [\hat{c}(k, \omega) \hat{\Delta}(k, \omega; -k, -\omega)], \quad (19)$$

where $\hat{c}(k, \omega)$ is the matrix with entries $c_{\alpha\beta}^{ij}(k, \omega) = \delta_{ij} c_{\alpha\beta}(k, \omega)$, the relation (17a) is immediately satisfied for any choice of $c_{\alpha\beta}$. Thus, \hat{c} is completely determined by the second commutator in terms of the partial differential equations:

$$(2 + \bar{z} + k \partial_k + \bar{z} \omega \partial_\omega) c_{11}(k, \omega) = - \frac{1}{2} \frac{r_{22}(q)}{|r_{12}(q, \omega)|^2} \delta(k-1), \quad (20a)$$

$$(2 + k \partial_k + \bar{z} \omega \partial_\omega) c_{12}(k, \omega) = \frac{1}{2} \frac{r_{12}(q, \omega)}{2|r_{12}(q, \omega)|^2} \delta(k-1), \quad (20b)$$

$$(2 + k \partial_k + \bar{z} \omega \partial_\omega) c_{21}(k, \omega) = \frac{1}{2} \frac{r_{21}(q, \omega)}{|r_{12}(q, \omega)|^2} \delta(k-1), \quad (20c)$$

$$(2 - \bar{z} + k \partial_k + \bar{z} \omega \partial_\omega) c_{22}(k, \omega) = 0. \quad (20d)$$

We require that any entry $c_{\alpha\beta}$ of \hat{c} vanish when the corresponding entry of $(\hat{r}_G^{-1})_{\alpha\beta} = (\hat{A}_G^{-1})_{\alpha\beta}(q, \omega; -q, -\omega)$ vanishes. Thus, the solution to (20) may be represented in the form,

$$\hat{c}(k, \omega) = \frac{1}{2} O(1-k) (\hat{r}_T^{-1})(k, \omega) \quad (21)$$

The generalization to any number of internal fields is immediate. The transformed Eq. (14) thus reads,

$$(d + \bar{z}) \rho' + \int \frac{d\vec{k}}{(2\pi)^d} \int \frac{d\omega}{2\pi} \sum_i \Gamma_\alpha^i(k, \omega) \frac{\delta \rho'}{\delta \psi_\alpha^i(k, \omega)} = y \rho'. \quad (22)$$

The ϕ' so determined are homogeneous polynomials in ψ_1 and ψ_2 . Label the degree of the ϕ' by $2n$, $n = 1, 2, \dots$ and ψ_2 by m . The degree of ϕ in ψ_1 is therefore $2n-m$. The ϕ' are of the general form

$$\phi'_{2n,m} = n_{\alpha_1 \dots \alpha_n} \int \frac{dk_1}{(2\pi)} \int \frac{d\omega_1}{2\pi} \dots \int \frac{dk_{2n}}{(2\pi)} \int \frac{d\omega_{2n}}{2\pi} v_{2n,n}(k_1, \omega_1; \dots; k_{2n}, \omega_{2n})$$

$$\cdot \hat{\psi}_{\alpha_1}(k_1, \omega_1) \cdot \hat{\psi}_{\alpha_2}(k_2, \omega_2) \dots \hat{\psi}_{\alpha_{2n-1}}(k_{2n-1}, \omega_{2n-1}) \cdot \hat{\psi}_{\alpha_{2n}}(k_{2n}, \omega_{2n})$$

$$\delta(k_1 + \dots + k_{2n}) \delta(\omega_1 + \dots + \omega_{2n}), \quad (23)$$

where $n_{\alpha_1 \dots \alpha_n}$ is a normalization constant depending on (n, m) and the pairing, and v_{2n} satisfies

$$[d + \bar{z} + n(2-d) - m\bar{z} - \sum (k_i \partial_{k_i} + \bar{z} \omega_i \partial_{\omega_i})] v_{2n,m} = y v_{2n,m}. \quad (24)$$

The $v_{2n,m}$'s are thus homogeneous polynomials in the k_i and the $\omega_i^{1/2}$ of degrees n and s , respectively, with the associated eigenvalues

$$y_{2n,m}^{r,s} = 2n + d(1-n) + \bar{z}(1-m) - r - s. \quad (25)$$

Having developed the eigenoperators, we can now expand A in terms of $\phi = e^y \phi'$:

$$A = A_0 + \sum_{n,m} \sum_{r,s} a_{2n,m}^{r,s} \phi_{2n,m}^{r,s}. \quad (26)$$

When taking inner product, we have

$$\frac{da_{2n,m}^{r,s}}{dz} = [y_{2n,m}^{r,s} - n\delta_1 - (m-1)\delta_2] a_{2n,m}^{r,s}$$

$$+ a_{2n,m}^{r,s} R[\bar{z}] a_{2n',m'}^{r',s'} a_{2n'',m''}^{r'',s''}, \quad (27)$$

where $\delta\eta$ and δz represent corrections to the respective Gaussian values and A' represents the nonlinear terms in A .

We now suppose in $d = 4$ dimensions, the fixed point value of $a_{4,1}^{*\eta,\eta}$ is of order ϵ while all other $a_{2n,m}^{*r,s}$ are of higher order in ϵ . We further suppose that the leading corrections to $\delta\eta$ and δz are of order ϵ^2 . Thus,

$$a_{4,1}^{*0,0} = -\epsilon (a_{4,1;4,1;4,1}^{0,0;0,0;0,0})^{-1}, \quad (28a)$$

$$a_{2n,m}^{*r,s} = -\epsilon^2 a_{2n,m;4,1;4,1}^{r,s;0,0;0,0} (a_{4,1;4,1;4,1}^{0,0;0,0;0,0}) + 6 \quad (28b)$$

where $a_{ijk} = \partial_i Q(\partial_j, \partial_k)$ and Q is the quadratic part of R .

We obtain the expressions for $\delta\eta$ and δz to order ϵ^2 in terms of a_{ijk} :

(i) Conserved order parameter

$$(n=1, m=1, r=2, s=0)$$

$$\begin{aligned} \delta\eta &= a_{2,1;4,1;4,1}^{2,0;0,0;0,0} (a_{4,1;4,1;4,1}^{0,0;0,0;0,0})^{-2} \epsilon^2 \\ &= [(n+2)/2(n+8)] \epsilon^2, \end{aligned} \quad (29a)$$

$$(n=1, m=2, r=-2, s=0)$$

$$\delta z = a_{2,2;4,1;4,1}^{-2,0;0,0;0,0} (a_{4,1;4,1;4,1}^{0,0;0,0;0,0})^{-2} \epsilon^2 - \delta\eta = -\delta\eta. \quad (29b)$$

(ii) Non-conserved order parameter

$$(n=1, m=1, r=2, s=0)$$

$$\delta\eta = [(n+2)/2(n+8)] \epsilon^2, \quad (30a)$$

$$(n=1, m=2, r=0, s=0)$$

$$\delta z = [6 \ln(4/3) - 1] \delta \eta. \quad (39b)$$

C. REMARKS

We have demonstrated that the problem of large-scale stochasticity of a classical nonlinear dynamical system can be discussed conveniently in terms of a path integral formalism. To understand the full stochastic behavior of the system $\{\psi_1\}$, it is necessary to introduce a set of conjugate momentum field variables $\{\psi_2\}$. The relevant correlation and response functions G and P are obtained as functional derivatives of the generating functional W [Eqs. 4 and 5].

For classical problems in critical dynamics, a one-particle-irreducible differential renormalization-group generator is introduced [Eq. (1)]. This equation can be solved conveniently in terms of a complete set of Gaussian operators. The readers are urged to consult the Physical Review article by Vvedensky, Chang, and Nicoll (1983) for additional details.

Large-scale stochasticity in Hamiltonian systems with multi-resonant terms (e.g., the problem of stochastic heating of a magnetized plasma by large amplitude coherent waves) can be studied in terms of Poincaré maps. The threshold of large-scale stochasticity has been shown recently to be amenable to a renormalization scheme (Escande, 1982) akin to that is discussed in VII A and B.

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VIII. FIELD SWELLING INSTABILITY IN ANISOTROPIC PLASMAS (BASU AND COPPI 1982)A. INTRODUCTION

In plasma regimes where the degree of collisionality is relatively low, the particle distribution in velocity space can depart considerably from a Maxwellian. Then plasma collective modes can be excited and, as a result, induce a substantial change in the particle distribution in velocity space. Here we consider the case where the electron distribution is characterized by an anisotropic temperature. The plasma is imbedded in a relatively weak magnetic field such that the particle pressure is of the same order as the magnetic pressure and the electron temperature anisotropy ($T_{e\perp} \neq T_{e\parallel}$) is relative to the direction of the magnetic field. We consider collective modes that can be described by moment (fluid-like) equations, and so do not rely on wave-particle resonance processes which depend on the evolution of a small portion of the particle distribution in velocity space. The plasma motion is allowed to be decoupled from the magnetic field lines. That is, the so-called "frozen-in condition" is not imposed. Thus, a new kind of instability is found to be excited when $T_{e\perp} \neq T_{e\parallel}$. In this case the magnetic field is perturbed from its equilibrium state, and transverse electron thermal energy is transported toward the regions where the magnetic field is weakened, both by a particle flow and an effective thermal conductivity along the magnetic field lines. The local increase in the particle transverse pressure tends to make the magnetic field "swell" further locally and the excited mode is amplified as a consequence. We point out that, as shown in a parallel paper¹, modes of the type treated here can be important in producing so-called magnetic reconnection when a plasma is confined in a magnetic field configuration that contains a ("neutral") surface where the field

vanishes. Magnetic reconnection corresponds to a change of the field topology, as in the case where magnetic "islands" are formed out of a configuration that initially has straight and parallel field lines. In fact, the formation of these "islands" would not occur if the field lines were constrained to move together with the plasma. The analysis of neutral sheet configurations where magnetic reconnection takes place is of special interest in space physics.

We note that fluid-like instabilities driven by a plasma temperature anisotropy had been found earlier, and the one which is closest to that considered in this paper is usually referred to as the "mirror" instability.² However, this is derived under the assumptions that: a) the "frozen-in condition" is valid and b) the effective thermal conductivity along the magnetic field is negligible. In particular, the threshold value of $T_{e\perp}/T_{e\parallel}$ above which this instability can be excited is six times that for the instability we describe in the present paper.

B. ANALYSIS

We write the equilibrium electron distribution as

$$f_e = \frac{n}{(2\pi T_{e\perp}/m_e)^{3/2}} \left(\frac{T_{e\perp}}{T_{e\parallel}}\right)^{1/2} \exp\left(-\frac{m_e v_{\perp}^2}{2T_{e\perp}} - \frac{m_e v_{\parallel}^2}{2T_{e\parallel}}\right),$$

and the uniform equilibrium magnetic field as $\underline{B}_0 = B_0 \hat{z}$. We analyze perturbations of the form $\underline{\tilde{B}} \propto \exp(-i\omega t + ik_{\perp}x + ik_{\parallel}z)$ with $\omega < \Omega_i < \Omega_e$ and

$$v_{thi} < \frac{\omega}{|k_{\parallel}|} < v_{the},$$

where $\Omega_i = eB_0/m_i c$, $\Omega_e = eB_0/m_e c$, $v_{the} = (2T_{e\parallel}/m_e)^{1/2}$, and $v_{thi} = (2T_{i\parallel}/m_i)^{1/2}$.

We introduce the vector and scalar potentials by $\underline{\tilde{B}} = \underline{\nabla} \times (\underline{\tilde{A}})$, and $\underline{E} = (i\omega/c)\underline{\tilde{A}} - \underline{\nabla}\phi$ and consider the quasi-neutrality condition $\tilde{n}_i = \tilde{n}_e$ (i.e., $\underline{\nabla} \cdot \underline{\tilde{A}} = 0$). We assume

for simplicity that the ion temperature is negligible in comparison to the electron temperature. Thus, the longitudinal (cold) ion momentum conservation equation yields

$$\tilde{u}_{i\parallel} = \frac{ie}{\omega m_i} \tilde{E}_z . \quad (1)$$

From the transverse (cold) ion momentum conservation equation we obtain, for $\omega < \Omega_i$,

$$\tilde{u}_{ix} \approx \frac{i\omega}{B_0} \tilde{A}_y + \frac{\omega^2}{B_0 \Omega_i} \tilde{A}_x - \frac{\omega}{\Omega_i} \frac{ck_{\perp} \tilde{\phi}}{B_0} , \quad (2)$$

$$\tilde{u}_{iy} \approx \frac{ick_{\perp} \tilde{\phi}}{B_0} + \frac{\omega^2}{B_0 \Omega_i} \tilde{A}_y - \frac{i\omega}{B_0} \tilde{A}_x . \quad (3)$$

Then the ion mass conservation equation gives

$$\frac{\tilde{n}_i}{n} \approx \frac{ik_{\perp}}{B_0} \tilde{A}_y + \frac{iek_{\parallel}}{m_i \omega^2} \tilde{E}_z , \quad (4)$$

for $\tilde{u}_{ix} \approx i\omega \tilde{A}_y / B_0$ that corresponds to $|k_{\parallel}/k_{\perp}| > |\omega/\Omega_i|$. In order to proceed to evaluate \tilde{n}_e we write the electron pressure tensor as

$$\underline{\underline{p}}_e = p_{e\perp} \underline{\underline{I}} + (p_{e\parallel} - p_{e\perp}) \underline{\underline{b}} \underline{\underline{b}} ,$$

where $\underline{\underline{I}}$ is the unit dyadic, $\underline{\underline{b}} = \underline{B}/B$, and the p_e 's are the scalar components.

The relevant linearized longitudinal electron momentum conservation equation is

$$0 = -en \tilde{E}_z - ik_{\parallel} \tilde{p}_{e\parallel} + ik_{\parallel} (p_{e\parallel} - p_{e\perp}) \frac{\tilde{B}_z}{B_0} .$$

Thus

$$\tilde{p}_{e_{\parallel}} = T_{e_{\parallel}} \tilde{n}_e = \frac{ien\tilde{E}_z}{k_{\parallel}} + ik_{\perp} (p_{e_{\parallel}} - p_{e_{\perp}}) \frac{\tilde{A}_y}{B_0},$$

as $\tilde{T}_{e_{\parallel}} = 0$ given that $\omega/k_{\parallel} < v_{the}$, and

$$\frac{\tilde{n}_e}{n} = \frac{ie}{k_{\parallel} T_{e_{\parallel}}} \tilde{E}_z + \frac{ik_{\perp}}{B_0} (1 - \frac{T_{e_{\perp}}}{T_{e_{\parallel}}}) \tilde{A}_y. \quad (5)$$

Then the quasi-neutrality condition gives

$$(1 - \frac{\omega_s^2}{\omega^2}) \frac{e\tilde{E}_z}{k_{\parallel} T_{e_{\parallel}}} = \frac{T_{e_{\perp}}}{T_{e_{\parallel}}} \frac{k_{\perp}}{B_0} \tilde{A}_y, \quad (6)$$

where $\omega_s^2 = k_{\perp}^2 T_{e_{\perp}}/m_i$. From the transverse electron momentum conservation equation we find

$$\tilde{u}_{ex} = \frac{i\omega}{B_0} \tilde{A}_y - \frac{ck^2}{eB_0^2} (T_{e_{\parallel}} - T_{e_{\perp}}) \tilde{A}_x. \quad (7)$$

Then, from the Ampere's law

$$\tilde{J}_x = \frac{ck^2}{4\pi} \tilde{A}_x$$

we obtain

$$[1 - \frac{\omega_s^2}{k^2 v_A^2} - \frac{1}{2} (R_{\parallel} - R_{\perp})] \tilde{A}_x = - \frac{\omega_s^2}{k^2 v_A^2} (\frac{ck_{\perp}}{\omega}) \tilde{A}_y, \quad (8)$$

where $k^2 = k_{\parallel}^2 + k_{\perp}^2$, $v_A^2 = B_0^2 / (4\pi n m_i)$, $R_{\parallel} = 8\pi p_{e_{\parallel}}/B_0^2$, and $R_{\perp} = 8\pi p_{e_{\perp}}/B_0^2$.

Next, we consider

$$\tilde{J}_y = \frac{ck^2}{4\pi} \tilde{A}_y, \quad (9)$$

and derive \tilde{J}_y from the total momentum conservation equation

$$- i\omega n m_i \tilde{u}_{ix} = - ik_{\perp} \tilde{p}_{e_{\perp}} - ik_{\parallel} (p_{e_{\parallel}} - p_{e_{\perp}}) \frac{\tilde{B}_x}{B_0} + \frac{1}{c} \tilde{J}_y B_0. \quad (10)$$

Combining Eqs. (2), (9) and (10) we obtain

$$\left[1 - \frac{\omega^2}{k^2 v_A^2} - \frac{k^2}{2k^2} (\beta_{||} - \beta_{\perp})\right] \tilde{A}_y = i \frac{4\pi k_{\perp}}{k^2 B_0} \tilde{p}_{e\perp} \quad (11)$$

We derive $\tilde{p}_{e\perp}$ from the linearized equation of state

$$\frac{\partial \tilde{p}_e}{\partial t} + 2\tilde{p}_{e\perp} \nabla \cdot \tilde{\mathbf{u}}_e - \tilde{p}_{e\perp} \nabla_{||} \tilde{u}_{e||} = \nabla \cdot (\hat{z} \tilde{q}_{e||}^{\perp}) \quad (12)$$

where $\nabla_{||} \equiv (\mathbf{B}_0 \cdot \nabla)/B_0$ and

$$\tilde{q}_{e||}^{\perp} \equiv \frac{1}{2} m_e \int d\mathbf{v} v_{\perp}^2 (v_{||} \tilde{f}_e - \tilde{u}_{e||} f_e) \quad (13)$$

\tilde{f}_e being the perturbed electron distribution.

Since we have not found an easy way to derive $\tilde{q}_{e||}^{\perp}$ from a 'fluid' approach, we calculate it directly from Eq. (13) after solving the perturbed linearized Vlasov equation. Introducing the polar coordinates $\theta, v_{||}$ and v_{\perp} in velocity space we obtain the moment

$$\begin{aligned} \int_0^{2\pi} d\theta \tilde{f}_e = \frac{2\pi e i}{T_{e\perp}} \left\{ \frac{\tilde{E}_z}{k_{||}} - \frac{\omega - (1 - \alpha_e) k_{||} v_{||}}{\omega - k_{||} v_{||}} \left[\frac{\tilde{E}_z}{k_{||}} - \frac{v_{\perp}}{2c} \frac{k_{\perp} v_{\perp}}{\Omega_e} \tilde{A}_y \right] \right. \\ \left. - \frac{v_{||}}{2c} \frac{k_{\perp} v_{\perp}^2}{\Omega_e^2} (1 - \alpha_e) B_y \right\} \tilde{f}_e \quad (14) \end{aligned}$$

that is relevant to the evaluation of $\tilde{u}_{e||}$ and $\tilde{q}_{e||}^{\perp}$. Here $\alpha_e = T_{e\perp}/T_{e||}$ and we find

$$\tilde{q}_{e||}^{\perp} = - i n T_{e\perp} \frac{\omega}{k_{||}} \frac{T_{e\perp}}{T_{e||}} \frac{k_{\perp} \tilde{A}_y}{B_0} - i n T_{e\perp} (T_{e||} - T_{e\perp}) \frac{c k_{\perp} \tilde{B}_y}{e B_0^2} \quad (15)$$

Noting that $2p_{e\perp} \nabla \cdot \tilde{u}_e - p_{e\perp} \nabla_{\parallel} \tilde{u}_{e\parallel} = i\omega p_{e\perp} (\tilde{n}_e/n) + ik_{\perp} p_{e\perp} \tilde{u}_{ex}$, we obtain, using Eqs. (5), (7) and (15) in (12),

$$\frac{p_{e\perp}}{p_{e\parallel}} = \frac{ie}{k_{\parallel} T_{e\parallel}} \tilde{E}_z + 2i \left(1 - \frac{T_{e\perp}}{T_{e\parallel}}\right) \frac{k_{\perp} \tilde{u}_{ex}}{B_0}, \quad (16)$$

and, since $p_{e\perp} = T_{e\perp} \tilde{n}_e + n T_{e\perp}$,

$$\frac{T_{e\perp}}{T_{e\parallel}} = \left(1 - \frac{T_{e\perp}}{T_{e\parallel}}\right) \frac{\tilde{B}_z}{B_0}. \quad (17)$$

Then combining Eqs. (6), (11) and (16) we finally arrive at the dispersion relation

$$\left(\frac{\omega^2}{\omega_s^2} - 1\right) \left[1 - \frac{\omega^2}{k^2 v_A^2} + \frac{k_{\perp}^2}{2k^2} \frac{T_{e\perp}}{T_{e\parallel}} (2\beta_{\parallel} - \beta_{\perp}) - \frac{k_{\parallel}^2}{2k^2} (\beta_{\parallel} - \beta_{\perp})\right] + \frac{\beta_{\perp}}{2} \frac{k_{\perp}^2}{k^2} \frac{T_{e\perp}}{T_{e\parallel}} = 0. \quad (18)$$

It should be mentioned that the above dispersion relation can be obtained from the linearized Vlasov-Maxwell equations by evaluating \tilde{n}_e from Eq. (14) and \tilde{u}_{ey} from

$$\begin{aligned} \frac{2\pi}{\omega} \sin \theta \tilde{f}_e &= \frac{\pi e i}{T_{e\perp}} [\omega - (1 - \alpha_e) k_{\parallel} v_{\parallel}] \left\{ \frac{k_{\perp} v_{\perp}}{\Omega_e} \frac{\tilde{A}_x}{\omega - k_{\parallel} v_{\parallel}} \right. \\ &\quad \left. - \frac{k_{\perp} v_{\perp}}{\Omega_e} \left(1 - \frac{k_{\perp}^2}{k_{\parallel}^2} \frac{k_{\parallel} v_{\parallel}}{\omega - k_{\parallel} v_{\parallel}}\right) \frac{\tilde{A}_x}{ck_{\perp}} \right. \\ &\quad \left. + i \frac{v_{\perp}}{c} \left(\frac{\omega - k_{\parallel} v_{\parallel}}{\Omega_e} - \frac{k_{\perp}^2 v_{\perp}^2}{2\Omega_e^2} \frac{1}{\omega - k_{\parallel} v_{\parallel}}\right) \tilde{A}_x \right\} \tilde{f}_e. \end{aligned}$$

We first consider $k_{\parallel}^2/k^2 \ll 2/\beta_{\parallel}$. Then one of the roots of Eq. (18) corresponds to be slow magnetosonic mode with $\omega^2 \sim \omega_s^2$, and is given by

$$\frac{\omega^2}{\omega_s^2} \left[1 + \frac{\beta_{\perp}}{2} \left(2 - \frac{T_{e\perp}}{T_{e\parallel}} \right) \right] \approx 1 + \beta_{\perp} \left(1 - \frac{T_{e\perp}}{T_{e\parallel}} \right) . \quad (19)$$

This becomes unstable if

$$1 < \frac{T_{e\perp}}{T_{e\parallel}} \frac{\beta_{\perp}}{1+\beta_{\perp}} < 2 .$$

The other root, corresponding to the fast magnetosonic mode with $\omega^2 \sim k^2 v_A^2$, is given by

$$1 - \frac{\omega^2}{k^2 v_A^2} + \frac{\beta_{\perp}}{2} \left(2 - \frac{T_{e\perp}}{T_{e\parallel}} \right) \approx 0 ,$$

and become unstable if

$$\frac{T_{e\perp}}{T_{e\parallel}} \frac{\beta_{\perp}}{1+\beta_{\perp}} > 2 .$$

This we call the field-swelling instability.

In the other limit, $k_{\parallel} \gg k_{\perp}$, we have

$$\omega^2 \approx k_{\parallel}^2 v_A^2 \left[1 - \frac{1}{2} (\beta_{\parallel} - \beta_{\perp}) \right] ,$$

the instability criterion being

$$\beta_{\parallel} - \beta_{\perp} > 2 ,$$

and this is the so-called 'firehose' instability.² In this case, $\tilde{E}_z = 0$,

$\tilde{B}_z = 0$ implying $\tilde{n}_e = \tilde{n}_i = 0$ and $\tilde{T}_{e\perp} = 0$.

Another well known instability driven by an electron temperature anisotropy is the Weibel mode.³ This differs from the one we present here in that it develops only in zero or weak magnetic fields, for which $\omega > \Omega_e$. It does not involve the ion population, it does not produce density perturbations, and is characterized by $\underline{k} \cdot \underline{\tilde{E}} = 0$. Another related kind of instability reported in the literature⁴ involves resonant interaction of the mode having $\omega = k_{||} v_A$ and $k_{||} \gg k_{\perp}$ with the tail of the ion distribution in velocity space. However, in this case the driving temperature anisotropy is that of the ion population.

Finally, we refer to Eq. (19) and note that for $T_{e\perp} = T_{e\parallel}$ and $\beta > 1$ we have $\omega^2 + \omega_S^2(2/\beta) = k_{||}^2 v_A^2$. Thus, $\omega \rightarrow 0$ as $B_0 \rightarrow 0$, and we may argue that this is the limit from which the familiar "tearing" mode, that is driven by the (spatial) gradient of the current density, emerges in a plane "neutral" sheet configuration. In fact the influence of an electron temperature anisotropy, adding the effects of the Weibel instability to the theory of the tearing mode, was analyzed in Ref. 5. In Ref. 1 it is pointed out that, for $T_{e\perp} \neq T_{e\parallel}$ the root of Eq. (19) $\omega^2 = \omega_S^2(1 - T_{e\perp}/T_{e\parallel})/[1 - T_{e\perp}/(2T_{e\parallel})]$ does not vanish as $B_0 \rightarrow 0$ and a new kind of "fast" magnetic reconnection process associated with this mode can take place in a neutral sheet configuration.

C. CONCLUSION

In summary, we have shown that collisionless plasmas imbedded in a magnetic field and with the electron temperature transverse to the magnetic field larger than the longitudinal temperature can be subject to an instability that produces transport of transverse electron thermal energy into the regions where the magnetic field is weakened by the perturbation while the plasma motion is decoupled from that of the magnetic field lines.

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