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AFWAL-TR-83-4023



THE APPLICATION OF A NONLINEAR FRACTURE MECHANICS
PARAMETER TO DUCTILE FATIGUE CRACK GROWTH

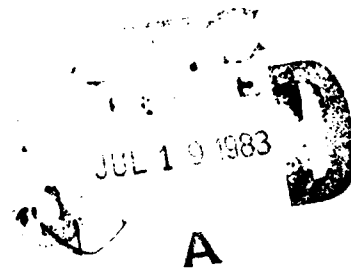
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December 1982

Final Report for Period September 1978 - December 1982

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Materials Laboratory
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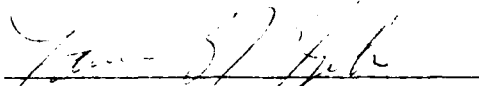


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REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER AFWAL-TR-83-4023	2. GOVT ACCESSION NO. A130402	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) The Application of a Nonlinear Fracture Mechanics Parameter to Ductile Fatigue Crack Growth		5. TYPE OF REPORT & PERIOD COVERED Final Report Sept. 1978 - December 1982
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) G. A. Hartman, III A. M. Rajendran D. S. Dawicke		8. CONTRACT OR GRANT NUMBER(s) F33615-78-C-5184
9. PERFORMING ORGANIZATION NAME AND ADDRESS University of Dayton Research Institute Dayton, Ohio 45469		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS Program Element 676100, Proj. # 2418, Task # 03, Work Unit 11
11. CONTROLLING OFFICE NAME AND ADDRESS Materials Laboratory (AFWAL/MLLN) Air Force Wright Aeronautical Laboratories, AFSC Wright-Patterson AFB, OH 45433		12. REPORT DATE December 1982
		13. NUMBER OF PAGES 114
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)		15. SECURITY CLASS. (of this report) Unclassified
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE
16. DISTRIBUTION STATEMENT (of this Report) Approved for public release; distribution unlimited.		
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)		
18. SUPPLEMENTARY NOTES		
19. KEY WORDS (Continue on reverse side if necessary and identify by block number)		
20. ABSTRACT (Continue on reverse side if necessary and identify by block number) This report describes the methodology for predicting fatigue crack growth rate response of cracked structural components wherein the assumptions of linear elastic fracture mechanics are violated. Fatigue crack growth rate tests were conducted on copper specimens using compact tension (CT), center-cracked panel (CCP), and radial-hole cracked (RHC) geometries. The CT and CCP specimens provided baseline fatigue crack growth rate data that were utilized to predict both the fatigue crack growth rate and the crack growth life behavior of the RHC test specimens.		

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The nonlinear fracture mechanics parameter chosen for extensive study on this program was the J-Integral. Crack growth rate correlations were based on J_{max} , i.e., the J-Integral evaluated at the maximum loading condition, rather than the more traditional ΔJ associated with the range of load. Three calculations of J_{max} were made; two were based on numerical results, the other on experimental results. The numerical J_{max} values were based on (a) the line integral calculated using nonlinear finite element results and (b) the Shih estimating scheme which assumes power law hardening behavior. The experimental J_{max} values were based on the method for measuring the change in the potential energy of deformation using load displacement data recorded during a fatigue test.

Crack growth rate correlations were actually developed using a pseudo maximum stress-intensity factor parameter ($\sqrt{J_{max} \cdot E}$) based on the elastic plane stress conversion relationship between J_{max} and K_{max} (the maximum stress-intensity factor). Such correlations allow for direct comparisons with correlations based on linear elastic fracture mechanics parameters. For the data generated herein, the crack growth rate correlations based on the numerical J_{max} values were shown to be similar to that achieved by the elastic K_{max} parameter. When life predictions of the RHC test specimens were made, it appeared that the numerical J_{max} analyses were slightly improved over the typically non-conservative predictions based on the elastic K_{max} parameter. The Ratwani et al., methodology was discussed and it was shown to lead to basically conservative life predictions.

As a result of symmetrical plasticity that occurred on both sides of the hole, it was not possible to accurately calculate the driving factor associated with the radial crack condition using the experimental load-displacement results. The experimental J_{max} for the CT and CCP test geometries did, however, correlate the fatigue crack growth rate data obtained from these configurations.

FOREWORD

This Technical Report was prepared by the Aerospace Mechanics Division of the University of Dayton Research Institute for the Metals and Ceramics Division, Materials Laboratory. Dr. Theodore Nicholas, AFWAL/MLLN is the project engineer. This report covers the work completed under Contract No. F33615-78-C-5184, "Nonlinear Fracture Mechanics" (Work Unit No. 24180306).

The authors wish to express their appreciation for the careful review of this report by Dr. Nicholas. We also thank Dr. J. P. Gallagher for his continuous guidance throughout the work and for his comments on the draft of this report. We also wish to thank Dr. T. Weerasooriya for his helpful suggestions during the investigation.



A

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SECTION 1 INTRODUCTION

1.1 BACKGROUND

In 1974, Dowling and Begley⁽¹⁾ presented results on correlating fatigue crack growth rate behavior with an elastic-plastic parameter based on the J-integral^(2,3). Subsequently, a substantial number of investigators have directed their attention toward further developing this J-integral parameter as well as other elastic-plastic parameters all for the purpose of describing crack growth behavior for those conditions where plasticity can be expected to occur. Recent reviews of past work are provided by References 4, 5, and 6.

To date, no single elastic-plastic fracture mechanics (EPFM) type parameter has achieved universal acceptance for its correlation capability. Most organizations continue to utilize the linear elastic fracture mechanics (LEFM) parameter K , the stress-intensity factor, to correlate crack growth rate behavior even though the conditions of LEFM are sometimes violated by the presence of plasticity. Possibly, one of the reasons for this choice is that the stress-intensity factor can be directly related to stress and geometry (structure and crack size), typically without regard to material properties. Another reason could be that the K correlations of crack growth rate behavior from different geometries and at different stress levels are (a) reasonably good or (b) sufficiently conservative for those conditions where plasticity occurs. One current concern, however, is for the localized plasticity that occurs in the region of a stress concentration or notch which is a site for crack growth.

1.2 OBJECTIVE OF RESEARCH

The general objective of the program was to develop methodology for predicting the fatigue crack growth rate (FCGR) response of cracked structural components wherein the assumptions

of LEFM might be violated. The methodology was to be developed and demonstrated for two-dimensional (planar) cracks subjected to constant amplitude loading conditions.

To accomplish the general objective in a systematic and logical fashion, we considered three specific but interrelated objectives:

- a. To establish a structural parameter (P^*) that controls fatigue crack growth (FCG) response.
- b. To use this parameter to correlate fatigue crack growth rate (FCGR) data collected in the laboratory so that the data can be used independent of geometry and stress level.
- c. To predict the behavior of cracks in typical structural components as a function of applied loading.

1.3 SCOPE OF RESEARCH

The efforts involved conducting a combined analytical-experimental program using a ductile copper alloy. The initial focus was on defining candidate structural-material parameters that could correlate FCGR behavior when the assumptions of LEFM are apparently violated. The philosophy behind the approach was based on the similarity concept; that is, when the calculated parameter " P^* " is the same for two different structures, each having a crack, then the fatigue crack growth rate at the instance of the occurrence of similar P^* will produce equal fatigue crack growth rates (da/dN). Hence, the FCGR should be represented by the empirical relationship:

$$\frac{da}{dN} = f (P^*) \quad (1)$$

where the controlling parameter, P^* , is influenced by the structural geometry, crack configuration, crack length, and stress level.

The derived crack driving parameter P^* was evaluated for its independence of crack geometry, global geometry, and the level of constant amplitude loading. Baseline data were generated using center-cracked panel (CCP) and compact (CT) type specimen geometries. Blind predictions of FCG behavior were made for a different structural crack geometry, the radial hole cracked (RHC) geometry.

SECTION 2 EXPERIMENTAL PROGRAM

2.1 MATERIAL

The material utilized in this investigation was ETP Bus Bar copper with a 101 mm (4 inch) width and thicknesses of 9.5 mm (0.375 inch) and 12.7 mm (0.50 inch). This material was chosen for its high degree of ductility. Most of the tests utilized the material in the as-received condition. In the as-received condition, the material exhibits a negligible amount of strain-hardening. One test was conducted using the material in a fully-annealed (400°C - 1.5 hr.) condition to evaluate the effect that increased strain-hardening had on the behavior.

The monotonic material properties of the as-received material were modeled as an elastic-perfectly plastic material for finite element calculations. The elastic modulus (E) was taken as 17.0×10^3 ksi and the yield point (σ_0) as 39 ksi. For calculations involving the J-integral estimation scheme, the material was modeled using the Ramberg-Osgood stress-strain relationship:

$$\frac{\epsilon}{\epsilon_0} = \frac{\sigma}{\sigma_0} + \alpha \left(\frac{\sigma}{\sigma_0}\right)^n \quad (2)$$

where

$$n = 36$$

$$\sigma_0 = 39 \text{ ksi}$$

$$E = 17.0 \times 10^3 \text{ ksi}$$

$$\alpha = 0.002/\epsilon_0$$

$$\epsilon_0 = \sigma_0/E$$

2.2 TEST SPECIMEN GEOMETRIES AND LOADING CONDITIONS

Three different types of fatigue crack growth rate specimens were employed in this investigation: the center-cracked panel (CCP) specimen, the compact (CT) specimen and the radial hole cracked (RHC) specimen. The geometries are illustrated in Figure 1.

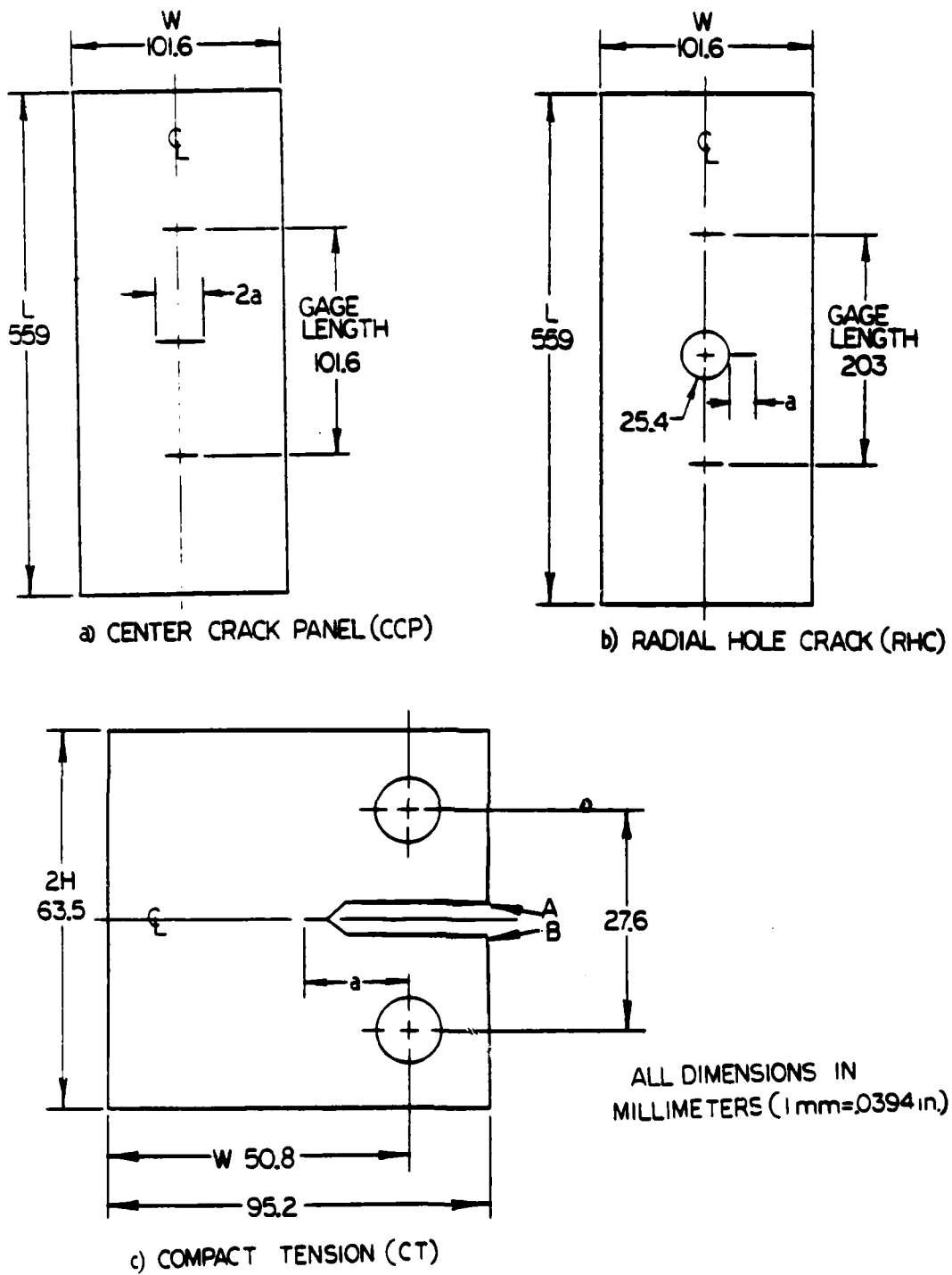


Figure 1. Specimen Geometries Employed in this Investigation. (See Table 1 for Thicknesses).

The fatigue test conditions were chosen so that the maximum stress levels induced a sufficient amount of plasticity to create localized conditions where it was expected that linear elastic fracture mechanics parameters would fail (see Figure 2). All three geometries were subjected to a stress ratio (R) of 0.1. In addition, some CCP and CT specimens were subjected to 0.5 stress ratio conditions*. One CCP specimen was subjected to a zero stress ratio condition. The test conditions, initial and final crack lengths, and specimen lives are summarized in Table 1.

2.3 EXPERIMENTAL PROCEDURES

The tests were conducted following the procedures outlined below. Generally, the specimens were first precracked according to ASTM Standard E647⁽⁷⁾ until a predetermined crack length was achieved. The test was then conducted such that crack length, cycle count, and load-displacement hysteresis loops were periodically recorded whenever the crack grew a predetermined increment (Δa).

All tests were conducted using a sinusoidal waveform under the load control mode in a servo-controlled electrohydraulic test system. Figure 3 illustrates a RHC test in progress. As shown by Figure 3, CCP and RHC specimens were loaded using hydraulic grips. Loads were measured with a 220 KN (50 kip) load cell calibrated to an NBS standard. The load cell was placed in series with the specimen. Cycle counts were determined from a mechanical cycle counter.

Crack lengths were measured optically with a traveling microscope. Surface crack lengths could be reliably determined within ± 0.025 mm (± 0.001 inch). The crack increments were chosen to give 20 to 40 data points per test. Typical Δa ranged from 0.254 mm (0.010 inch) to 0.635 mm (0.025 inch).

* These tests are not described herein because they were preliminary tests performed to evaluate extensometry and crack measurement techniques.



1 inch
(25.4mm)

Figure 2. A Photograph of a CCP Specimen in Test Showing Substantial Plastic Deformation Near the Crack Tip as the Crack Grows.

TABLE 1
TEST CONDITIONS SUMMARY

Specimen ID	Thickness (mm)	Max. Load (KN)	Min. Load (Ki)	Stress Ratio	* Frequency (hz)	a _o (mm)	a _f (mm)	N (cyc)
CCP1	12.7	200.0	0.0	0.0	0.5/.01	6.35	19.79	15980
CCP2	12.7	189.5	18.95	0.1	0.5/.01	12.62	19.94	5531
CCP3	12.7	222.4	22.24	0.1	0.5/.01	6.63	17.48	11088
CCP4	9.5	89.0	0.0	0.0	0.5/.01	6.99	20.65	27811
CCP5	9.5	166.8	16.68	0.1	0.5/.01	7.16	19.02	24920
CCP6	12.7	222.4	22.24	0.1	0.5/.01	5.74	15.57	9510
CCP7	9.5	124.5	12.45	0.1	10/.01	3.25	25.55	104500
CT1	12.7	7.50	0.750	0.1	9/.1	13.67	32.26	140047
RHC1	9.5	146.8	14.68	0.1	0.5/.01	0.89	13.77	7441
RHC2	9.5	146.8	14.68	0.1	0.5/.01	0.86	13.79	6241
RHC3	9.5	161.5	16.15	0.1	0.5/.01	1.14	9.32	2661
RHC4	9.5	161.5	16.15	0.1	0.5/.01	1.57	9.42	
RHC5	9.5	146.8	14.68	0.1	0.5/.01	0.20	15.39	12600

* First Frequency/Second Frequency

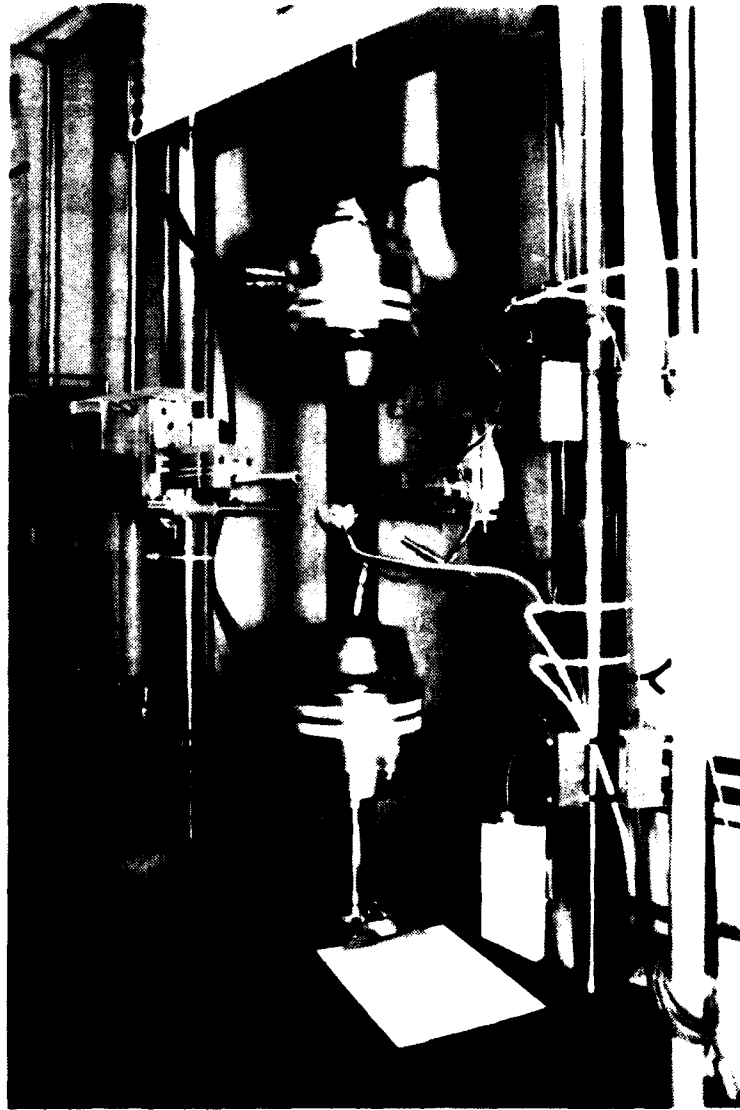


Figure 3. RHC Test in Progress with Early Style
Clip On Gage.

The displacements for the CT specimen were measured along the axis of loading with an ASTM Standard E399⁽⁸⁾ style clip-on gage. These displacements were measured at points A and B shown in Figure 1c.

Displacements on the CCP and RHC specimens were measured in several different ways. Initially, LVDT transducers were used. These produced substantial nonlinearity and were deemed unacceptable. The second method was using an ASTM 300 style clip-on gage and extension arms bolted to the specimen (see Figure 3). This method provided accurate measurements; however, when the gage length was extended to 203 mm (8 inch) the total displacements were beyond the range of this gage. The method finally used the custom built extensometer with a 203 mm (8 inch) gage length illustrated in Figure 4. This extensometer provided accurate measurements, but the measurements were made on one side of the specimen only. This introduced the possibility of inaccuracies due to specimen bending. Additional tests comparing load-displacement data collected from both sides of sample CCP-3 showed this error to be small.

Figure 5 illustrates typical load-displacement data recorded during a test of a CCP specimen (CCP-3). These data were used in conjunction with the potential energy definition of the J-integral to establish values of the operational J-integral discussed in Section 3.

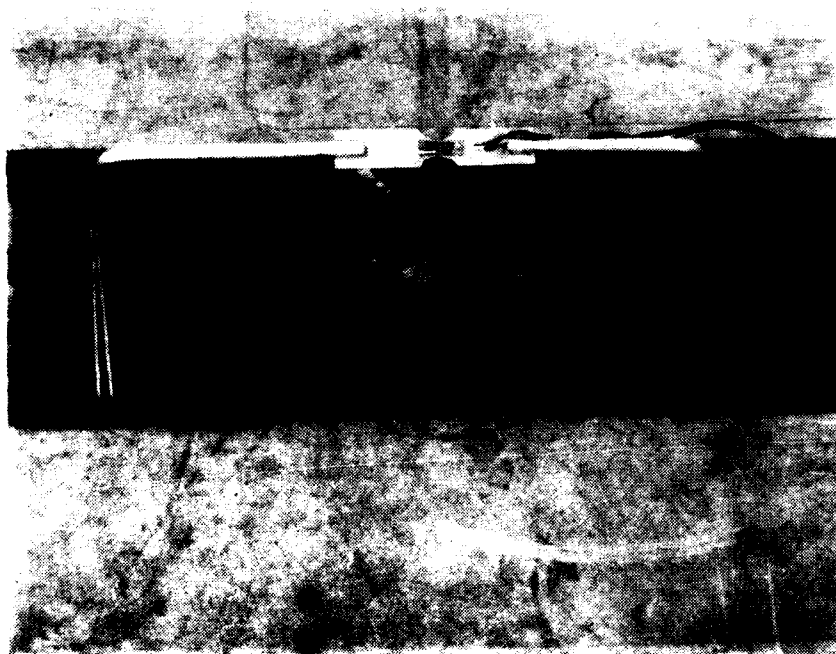


Figure 4. RHC Specimer with 203mm (8 inch) Gage Length Extensometer Attached.

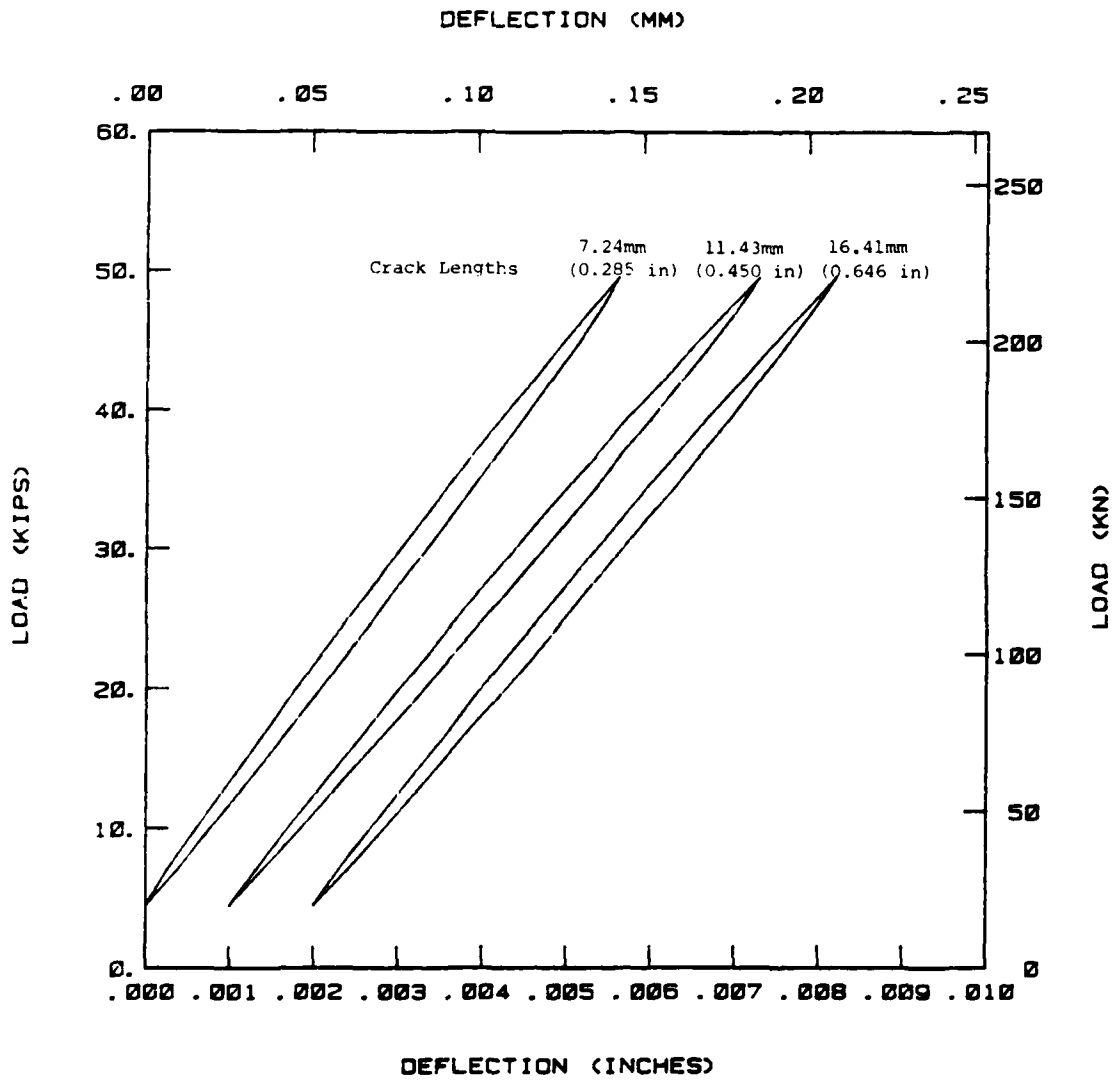


Figure 5. Load-Displacement Diagram for a CCP Specimen Subject to a Load that Ranged Between 4.5 Kips (20kN) and 50.0 Kips (222.4 kN).

SECTION 3
PARAMETER DESCRIPTIONS

Two correlation parameters were chosen for characterizing the fatigue crack growth rate (FCGR) behavior of ETP copper. One parameter is the linear elastic fracture mechanics (LEFM) parameter - the stress-intensity factor (K) and the other is a fracture mechanics parameter that has been extended into the nonlinear range - the J-integral (J). The subsections below outline the methods employed to obtain the values of these two parameters for the three specimen geometries of interest.

3.1 THE STRESS-INTENSITY FACTOR

For the standard test specimens, i.e., the center-crack panel (CCP) specimen and the compact (CT) specimen, the stress-intensity factor solutions are available from ASTM Standard E647⁽⁷⁾. The stress-intensity factor used to describe the magnitude of stress in the crack tip region of the CCP specimen is:

$$K = \sigma \sqrt{\pi a} \sec \frac{\pi a}{W} \quad (3)$$

(σ = stress, a = half crack length, and W = panel width) while that of the compact specimen is:

$$K = \frac{P(2 + \frac{a}{W})}{B\sqrt{W}(1 - \frac{a}{W})^{3/2}} [0.886 + 4.64 (\frac{a}{W}) - 13.32 (\frac{a}{W})^2 + 14.72 (\frac{a}{W})^3 - 5.6 (\frac{a}{W})^4] \quad (4)$$

(P = load, a = crack length, B = thickness, and W = width).

The evaluation of the stress-intensity factor for non-standard test geometries requires the solution of an elasticity problem in which the geometry and boundary conditions are appropriately modeled. This type of evaluation normally requires the use of numerical analysis procedures based on either the

finite element method or the boundary integral equation method. For the case of the radial hole cracked (RHC) specimen geometry utilized for this investigation, the stress-intensity factor was obtained by modeling the specimen with the finite element method. Specifically, the MAGNA⁽⁹⁾ finite element code was selected to perform the stress-strain calculations. Several additional routines⁽¹⁰⁾ were developed to calculate the stress-intensity factor based on the elastic properties of Rice's path-independent line integral, i.e., the J-integral⁽²⁾. [See subsection 3.2 for a description of the integral and how it is calculated.]

The finite element results for the RHC geometry were obtained for a number of crack lengths and the stress-intensity factor values were computed from the elastic plane stress relationship⁽²⁾

$$K = \sqrt{JE} \quad (5)$$

where E is the elastic modulus. These stress-intensity factor results are described in Figure 6 along with a least squares determined curve that describes the finite element results. The equation that describes this curve is:

$$K = \sigma \left[0.1164 + 30.99a - 164.8a^2 + 458.7a^3 - 619.4a^4 + 329.9a^5 \right]^{1/2} \quad (6)$$

where σ is the applied stress and a is the length of the radial crack. Equation 6 is valid in the range $0.01 \leq a \leq 0.65$ inch. For comparison purposes, the Bowie infinite plate radial hole crack results⁽¹¹⁾ as described by Grandt⁽¹²⁾, i.e.,

$$K = \sigma \left(0.6762 + \frac{0.8733}{0.3245 + \frac{a}{R}} \right) \sqrt{\pi a} \quad (7)$$

are also presented in Figure 6.

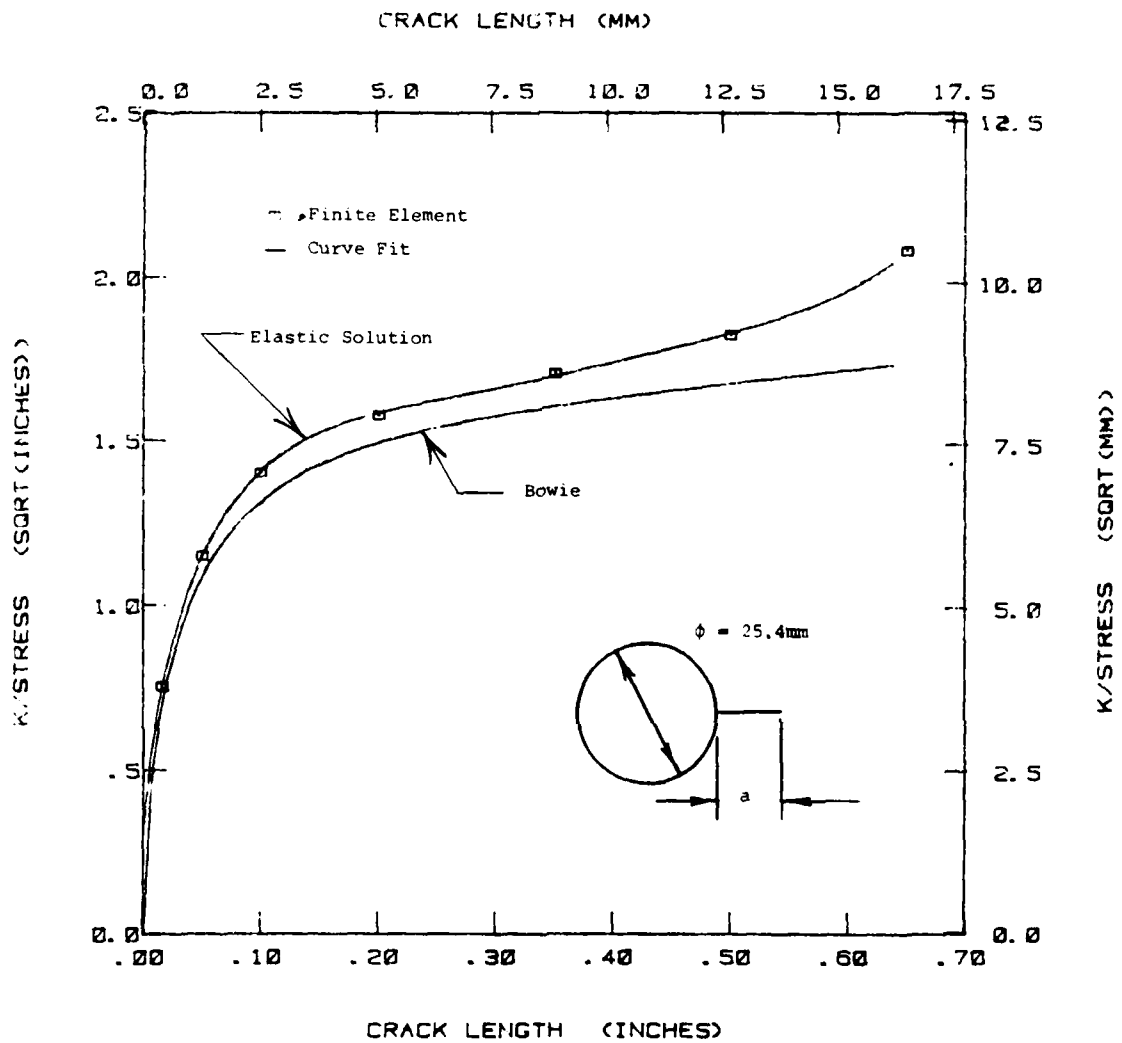


Figure 6. Comparison Between Finite Element, Finite Width Results and Bowie Radial Hole Crack Results. (Elastic Conditions Exist for Loads Below 15 kips (66.7 kN).)

3.2 THE J-INTEGRAL

Past work has shown that there are several procedures available to obtain values of the J-integral as a function of load and crack length (see References 4, 5, and 6). The procedures chosen for evaluation were based on: (a) the finite element method used in conjunction with a direct evaluation of the line integral, (b) the Shih et al.⁽¹³⁻¹⁵⁾ estimation schemes, and (c) the use of experimental load-displacement data in conjunction with the operational definition of the parameter. In this subsection, the various procedures utilized to evaluate the J-integral parameter for the three test geometries are discussed.

3.2.1 Line Integral Evaluation

A series of values for the J-integral were computed as a function of crack length from the direct evaluation of the line integral given by Rice⁽²⁾, i.e., from

$$J = \int_{\Gamma} (w dy - T_i \frac{\partial u_i}{\partial x} ds) \quad (8)$$

where

$$w = \int \sigma_{ij} d\epsilon_{ij} \quad (9)$$

is the strain energy density and Γ is any contour surrounding the crack tip, transversing in a counter clockwise direction, as described by the path in Figure 7. The figure further defines the parameters in Equation 8, i.e.,

ds = Increment of distance along the contour,

T_i = Traction vector on the contour,

u_i = Displacement vector on the contour, and

x, y = Rectangular coordinates.

To compute the value of the J-integral for a given load and crack length condition, Equation 8 has to be integrated along any path that circuits the crack tip. The integrand parameters are then based on the stresses, strains, and displacements

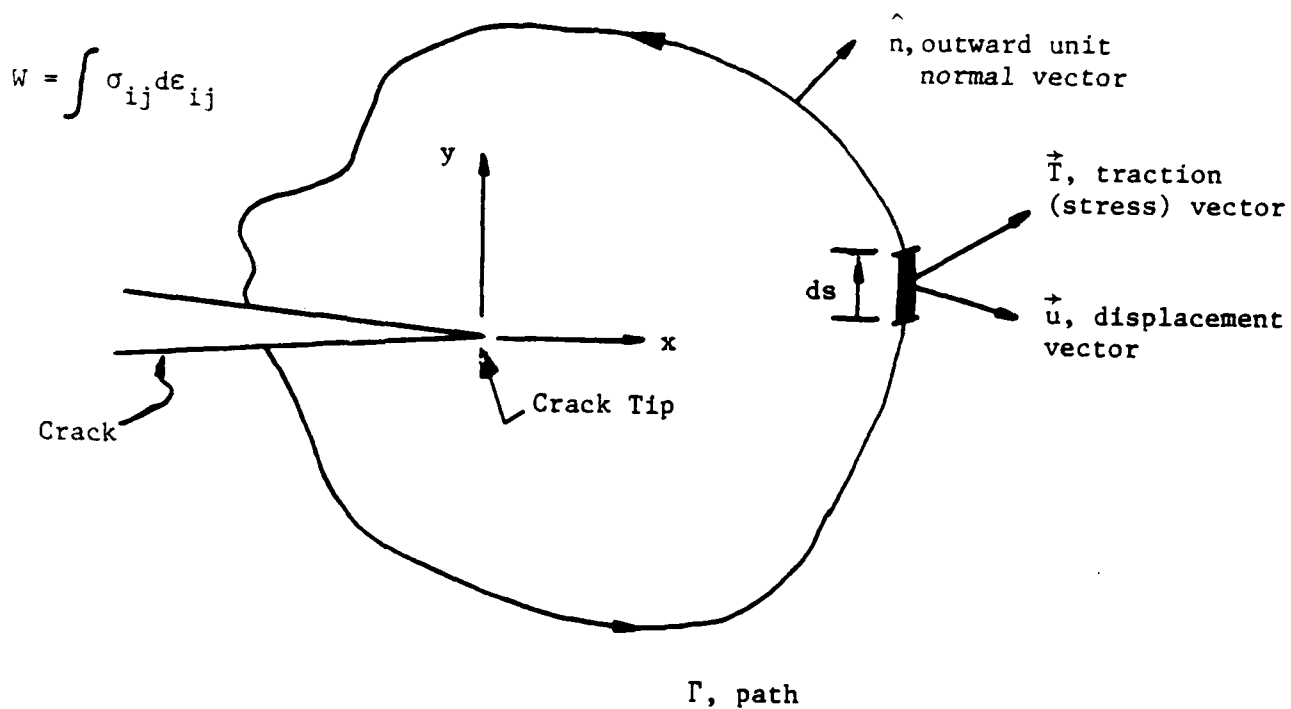


Figure 7. J-Integral Path and Associated Parameters.

evaluated for the specific path chosen. For the purpose of obtaining these mechanical parameters, the MAGNA⁽⁹⁾ finite element code was employed.

There are several ways that the line-integral path can traverse the finite elements. In the formulation added to MAGNA⁽¹⁰⁾, the J-integral paths were made to pass through the middle of each element, where smooth stress and strain data can be obtained more easily than along the element boundaries (See Figure 8 for an example illustrating three separate paths chosen for a study to evaluate path independence for the CCP geometry). The accuracy and validity of the J-integral routines in MAGNA were recently evaluated by Rajendran⁽¹⁶⁾ who compared the numerical results from MAGNA with those generated previously by Kumar et al.⁽¹⁵⁾, by Yamada and Yoshimura⁽¹⁷⁾ and by Ashbaugh and Ahmad⁽¹⁸⁾.

The finite element mesh formulations utilized for the CCP and RHC geometries are illustrated by Figures 8 and 9, respectively. These meshes were generated by an automatic mesh generator program which provides relevant mesh patterns for notch cracks as small as 0.13 mm (0.005 inch). The crack tip element sizes were always less than $a/6$, where a is the half crack size for the CCP geometry and the radial crack length for the RHC geometry. As a check on the suitability of the mesh shown in Figure 8, a second mesh with a larger number of degrees of freedom was generated and used in the analysis of RHC-1. The results from the second mesh differed by less than 2% from the results obtained using the mesh in Figure 8. For cost effectiveness, the mesh in Figure 8 was used in subsequent RHC analyses.

In the present study, the J-integral was calculated for the CCP and RHC geometries using the elastic-perfectly-plastic stress-strain model described in subsection 2.1. The results obtained for the RHC geometry compared reasonably well with the experimentally obtained load-displacement results. Figures 10 and 11 summarize some of the results for displacements measured on opposite sides of the hole along the axis of loading. The curves in these figures are from sample RHC-2.

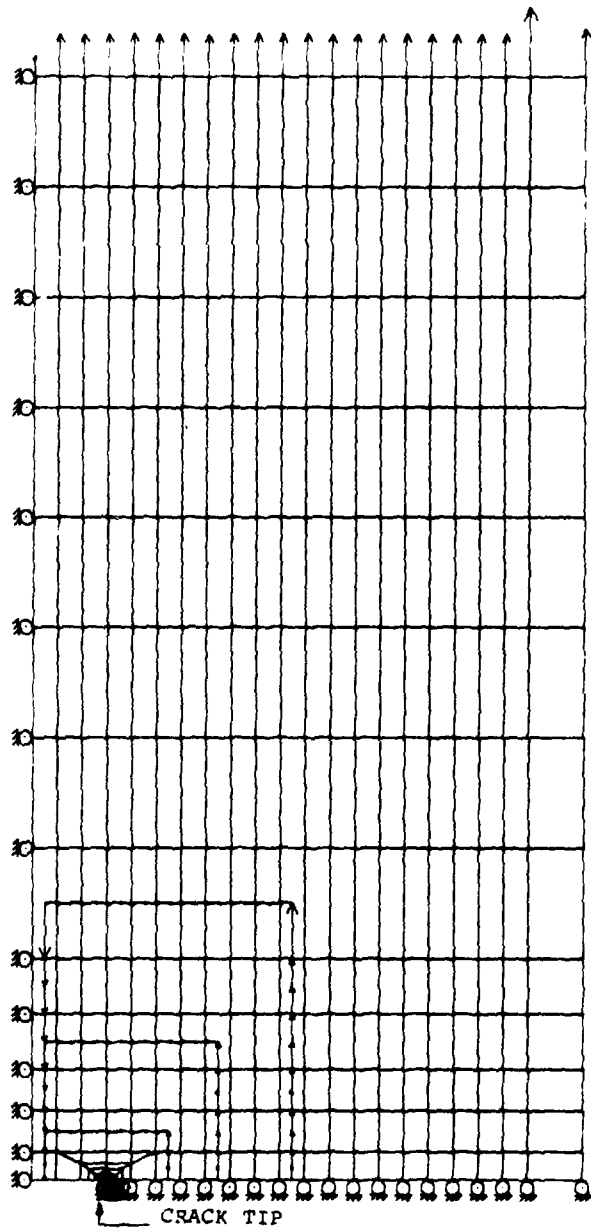


Figure 8. Three J-Integral Paths Shown Relative to the Finite Element Mesh Describing the Upper Right Quarter of the CCP Geometry and Loading.

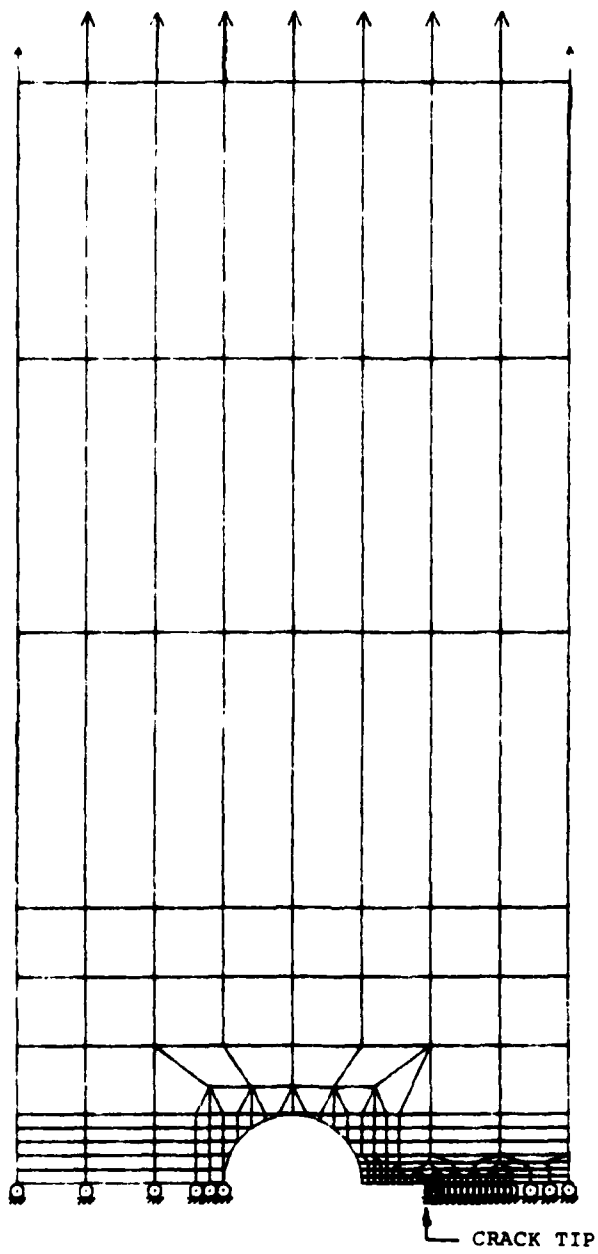


Figure 9. Upper Half Panel RHC Specimen Finite Element Mesh Used in Calculation of the J-Integral with the MAGNA Code.

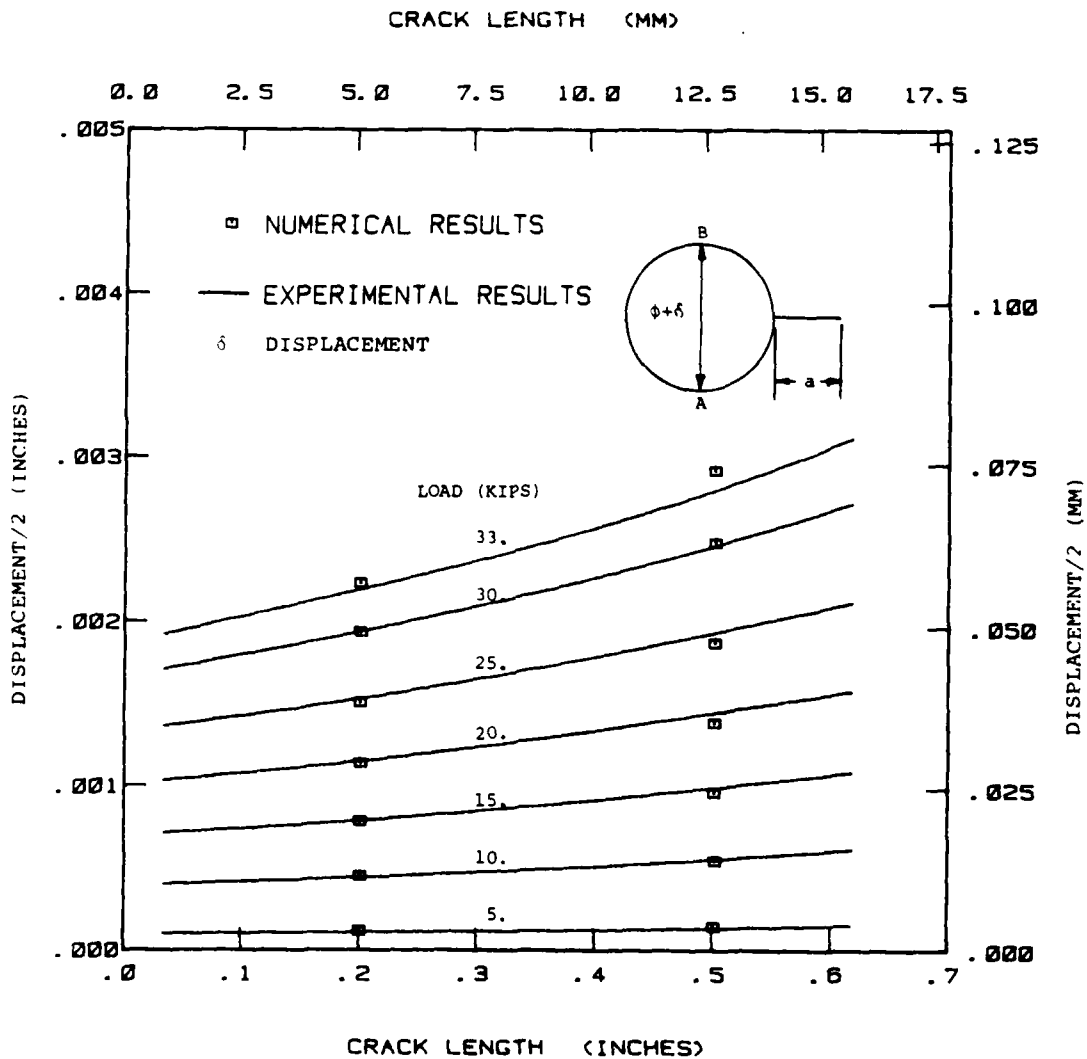


Figure 10. Comparison of Deflection Across the Hole Vs. Crack Length Between the Test and Numerical Results for Radial-Hole-Crack Specimens for Various Load Levels.

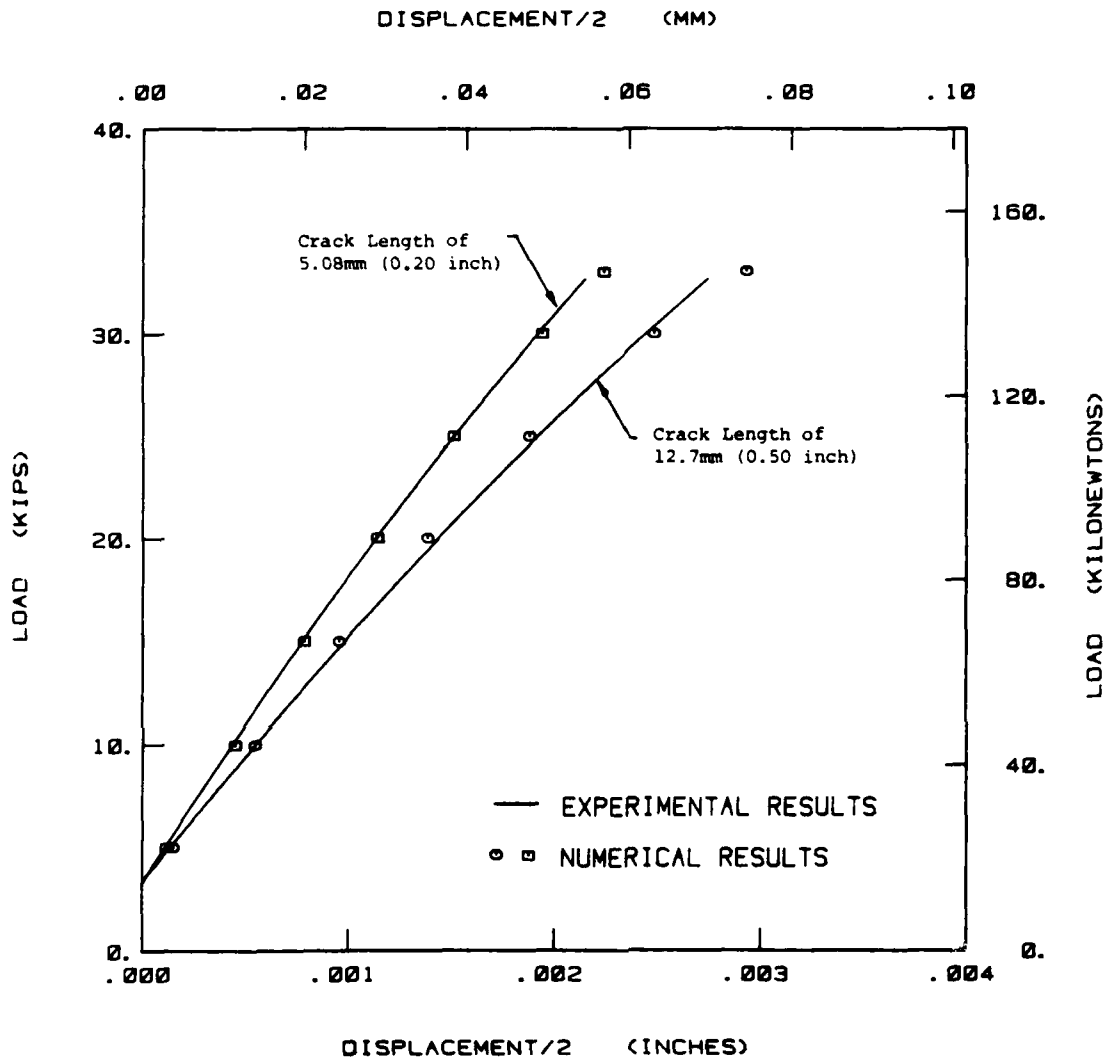


Figure 11. Comparison of Load Vs. Displacement Curves Obtained Through Experiments and Numerical Simulations for Two Different Cracks of Radial-Hole-Crack Configuration; Experimental Results from RHC-2.

The J-integral values obtained for three paths are plotted for several crack lengths in Figure 12 for a CCP specimen. It can be seen from Figure 12 that the path independent property of J-integral is well demonstrated for the crack length region of interest.

To evaluate the ability of the J-integral for correlating the fatigue crack growth rate data obtained from the CCP and RHC specimens, it was necessary to determine the values of this parameter at the specific test load levels. Figures 13 and 14 present the J-integral results for the CCP and RHC specimens, respectively, in the same format that was previously used to express the stress-intensity factor results, i.e., in terms of the ratio of stress-intensity factor (K) to stress (σ). The J-integral results obtained from the elastic-plastic finite-element analysis were converted to psuedo stress-intensity factors using Equation 5. For comparison purposes, the elastic stress-intensity factor results are also portrayed in Figures 13 and 14. We note from Figures 13 and 14 that as the load increases (and thus introduces more material nonlinearity) the psuedo stress-intensity factor is no longer linearly related to load. This is better illustrated in Figures 15 and 16 where the \sqrt{JE} results have been normalized to the elastic stress-intensity factor. Figures 15 and 16 are provided to directly show the effect that increasing the amount of plasticity has relative to the elastic result.

3.2.2 Estimated J

The J-integral can also be calculated using an estimation scheme suggested by Hutchinson, Shih, and their coworkers^(13-15, 19-20). The estimating scheme calculates the intensity of the HRR (after Hutchinson⁽²¹⁾ and Rice and Rosengren⁽²²⁾) stress-strain field at the crack tip for an elastic-plastic material. A recent study of the estimating scheme and its implementation in terms of a computer program (EST) was reported by Weerasooriya and Gallagher⁽²³⁾. The equations involved in the J-estimation scheme by program EST are briefly described below.

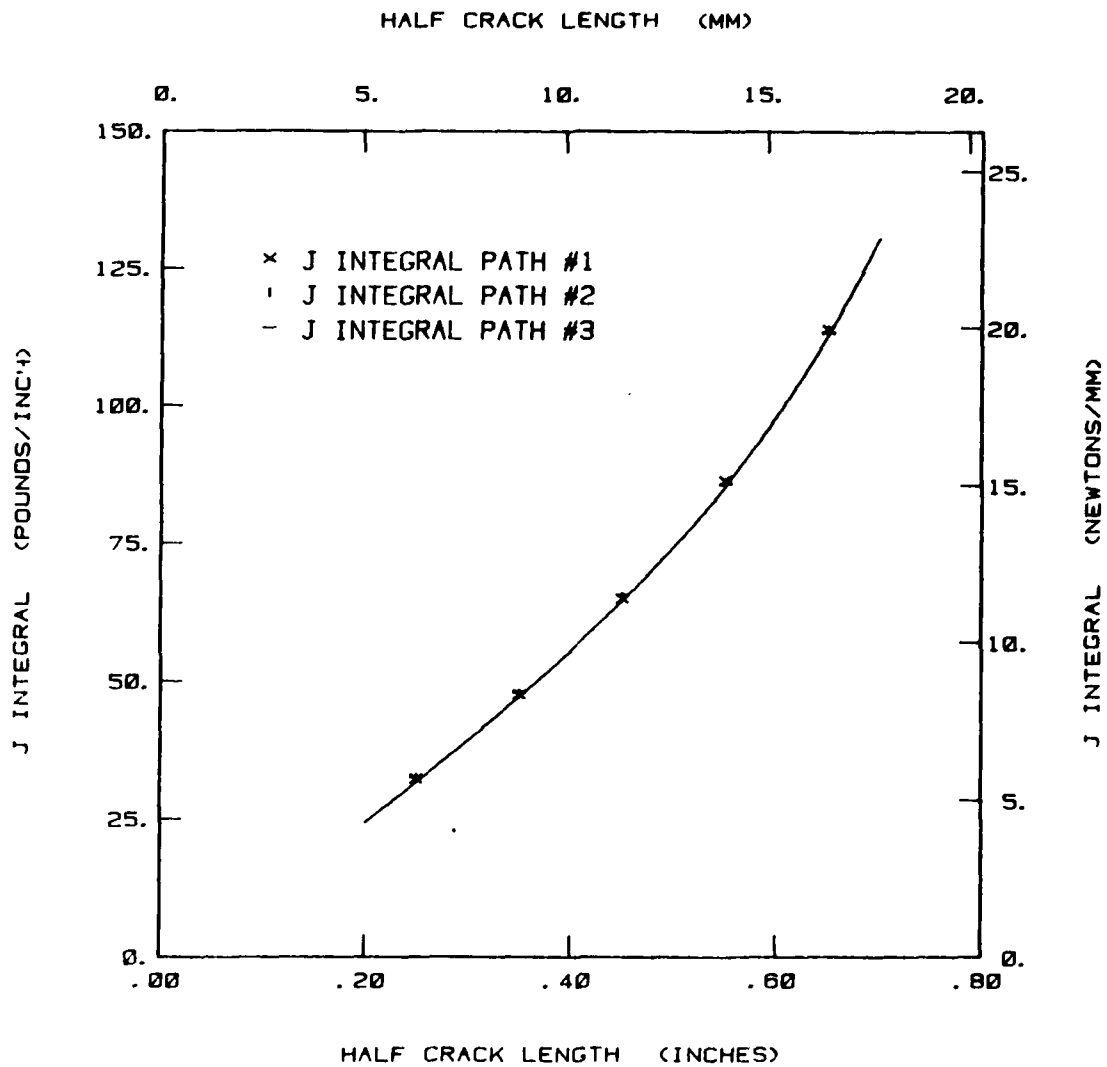


Figure 12. Elastic-Plastic J-Integral Results for the CCP Specimen Based on Finite Element Results for Three Different Paths of the Type Illustrated in Figure 8. The results presented are for a load of 50.0 Kips (222.4KN).

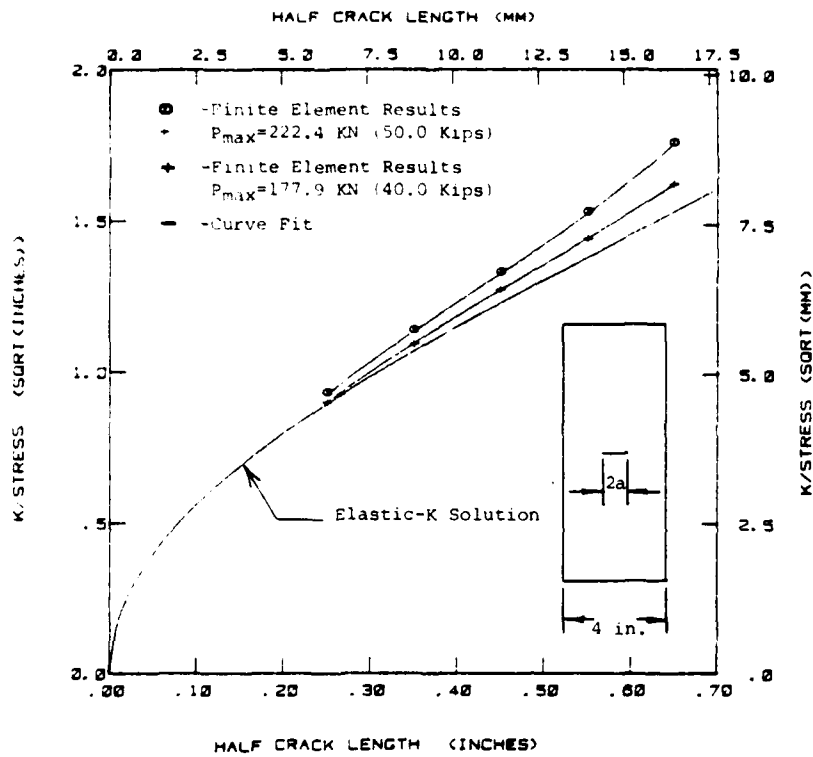


Figure 13. Comparison Between the Elastic and Elastic-Plastic Stress Intensity Factor Coefficient Results for the Center Crack Panel Tests.

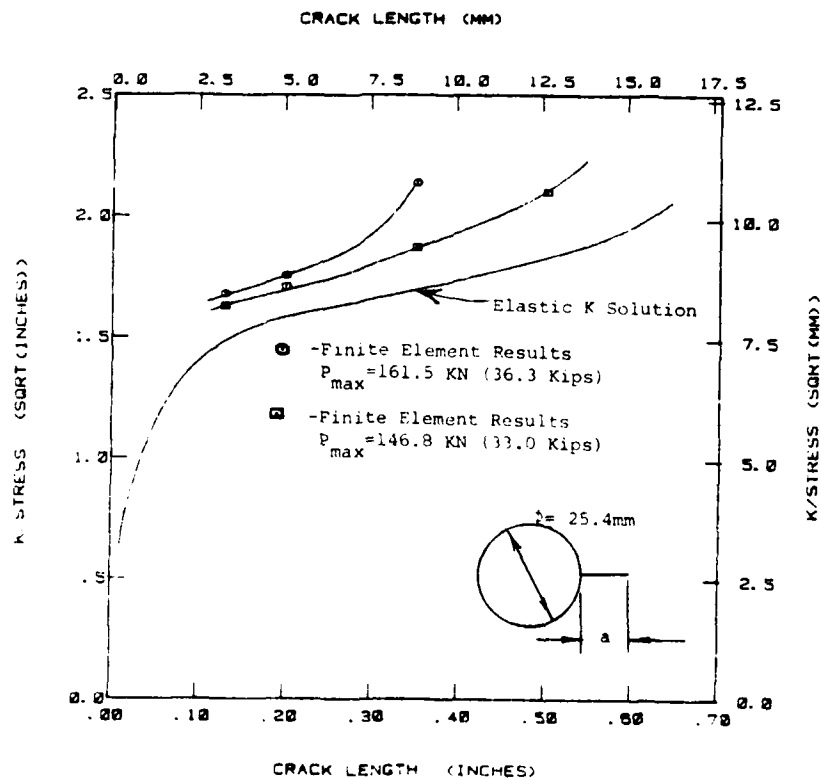


Figure 14. Comparison Between the Elastic and Elastic-Plastic Stress Intensity Factor Coefficient Results for the Radially-Cracked Hole Crack Tests.

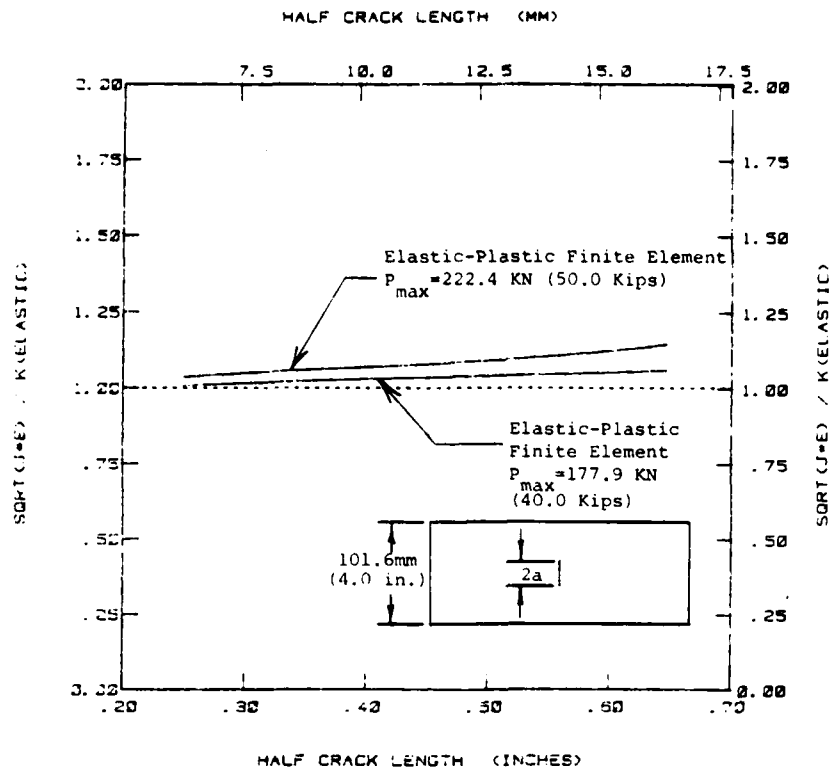


Figure 15. Impact of Including the Plastic Component in a Stress-Intensity Factor Analysis for the Center Cracked Panel Test Conditions.

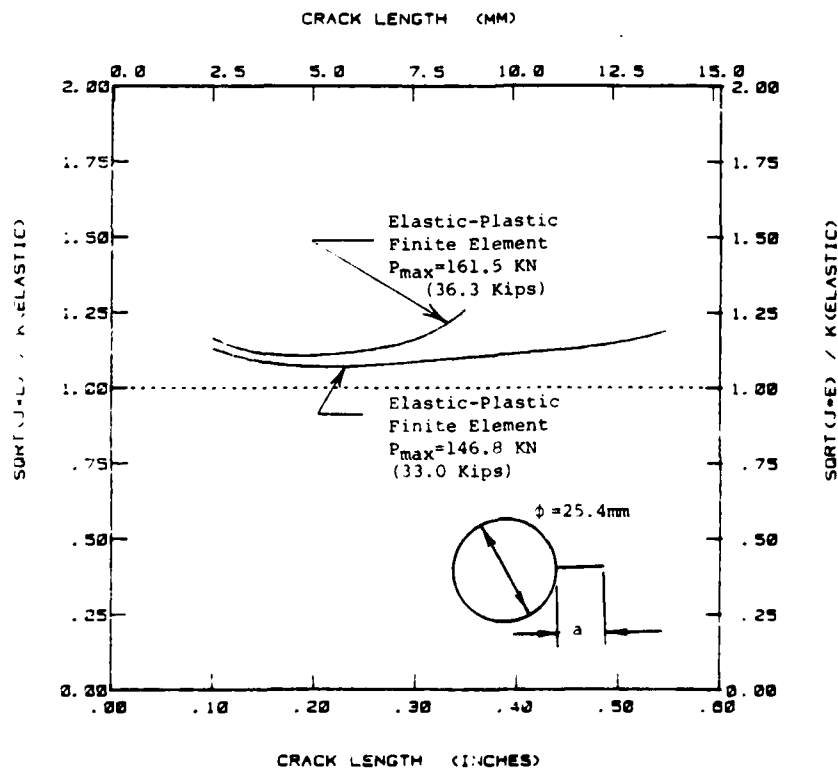


Figure 16. Impact of Including the Plastic Component in a Stress-Intensity Factor Analysis for the Radially Cracked Hole Test Conditions.

For elastic-plastic materials, the parameter J can be approximated by summing contributions due to the linear elastic and plastic parts^(13,14)

$$J = J^e + J^p . \quad (10)$$

Superscripts e and p denote the elastic and plastic components, respectively. The elastic J parameter appearing in the above equation can be expressed in the form:

$$\frac{J^e}{\sigma_o \epsilon_o a} = \left(\frac{\sigma}{\sigma_o}\right)^2 \hat{J}^e(a/b) \quad (11)$$

where σ is the remotely applied stress, and σ_o and ϵ_o are reference stresses and strains related by the expression $\sigma_o = E\epsilon_o$. Function \hat{J}^e depends only on the ratio of crack length to width (a/b). This function can be found in the literature for various finite width crack geometries.

The plastic contribution J^p can be expressed as:

$$J^p = \alpha \sigma_o \epsilon_o a f_1\left(\frac{a}{b}\right) h_1\left(\frac{a}{b}, n\right) \left(\frac{P}{P_o}\right)^{n+1} \quad (12)$$

where P and P_o are the applied and limit loads per unit thickness, respectively. f_1 is a function only of geometry and crack length while h_1 depends on geometry, crack length, and the strain hardening exponent (n) which appears in the Ramberg-Osgood stress-strain model (see Equation 2). Shih and coworkers^(13,15) have tabulated the functions f_1 and h_1 for a number of geometries.

The report by Weerasooriya and Gallagher⁽²³⁾ discusses the limitations and conditions under which the estimation procedure used for our calculations is valid.* In the current work, J -integral values were obtained for both the CCP and CT specimens using the estimation program EST⁽²³⁾ for different crack lengths.

*The Program EST was modified to include tabular f_1 and h_1 values for strain hardening exponents (n) up to 36.

An indepth comparison between the CCP specimen finite-element results reported in subsection 3.2.1 and the estimation scheme results were conducted to verify the validity of the estimation scheme and the material model on which it was based. The analytical estimates of the psuedo stress-intensity factor results created through the conversion of elastic-plastic J to K using Equation 5 are compared to the elastic-plastic finite element results at a maximum load of 222 KN (50 kips) for the CCP specimen in Figure 17. The maximum load condition of 222 KN is the largest load that the test machine could apply and it is seen that the estimation scheme provides a close approximation to the elastic-plastic finite element results shown in Figure 17. At lower load levels, the correlation was even better.

Additional comparisons were also made between the load-displacement results obtained using the EST computer code and those obtained both by finite element calculations and by experimental measurements. These comparisons showed that we could rely on the estimation scheme to calculate the J-integral as a function of load level and crack length for the two geometries used to collect the baseline crack growth rate data. The psuedo stress-intensity factor for the CT specimen is presented in Figure 18 for the load level used in the test of this specimen. The estimation scheme is numerically more efficient and thus more economical for calculating J-integral values than the finite element method. However, it must be noted that the functions embedded within the estimation scheme typically are based on finite element results; so, unless these functions are already available, the finite element method would be the most efficient method for developing the J-integral values.

3.2.3 Experimental J Definition

In Rice's original formulation of the J-integral, it was shown that the line integral definition (Equation 8) was equivalent to the negative rate of change of potential energy with respect to crack area^(2,3)

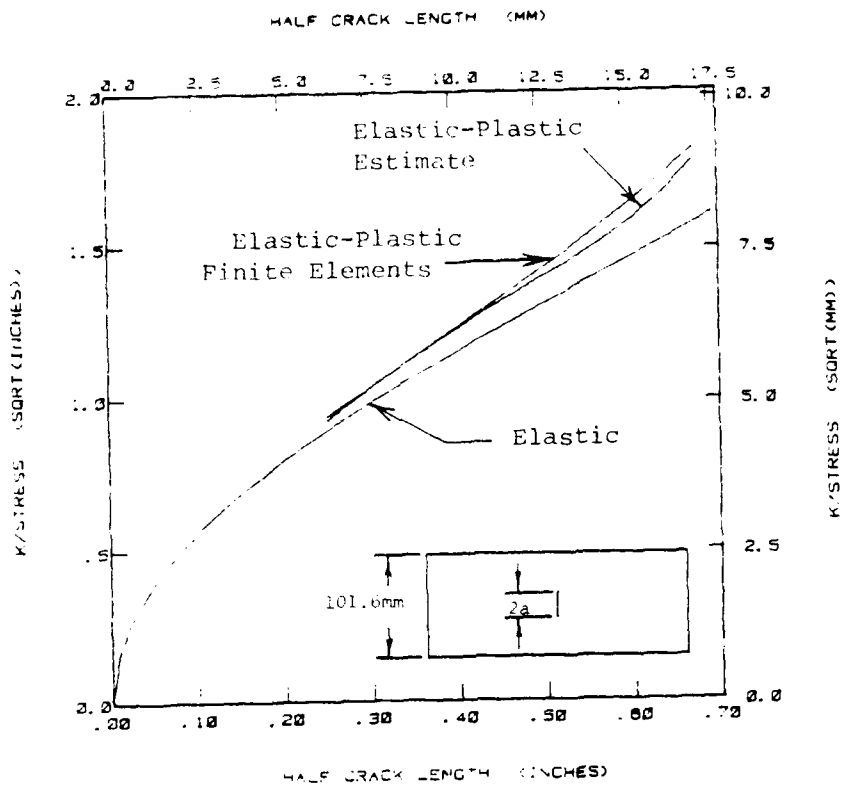


Figure 17. Comparison of Elastic-Plastic Finite Element, Elastic-Plastic Estimated and Elastic Results for a Center Crack Panel (CCP) Subjected to a Maximum Load of 50 Kips (222.4 kN).

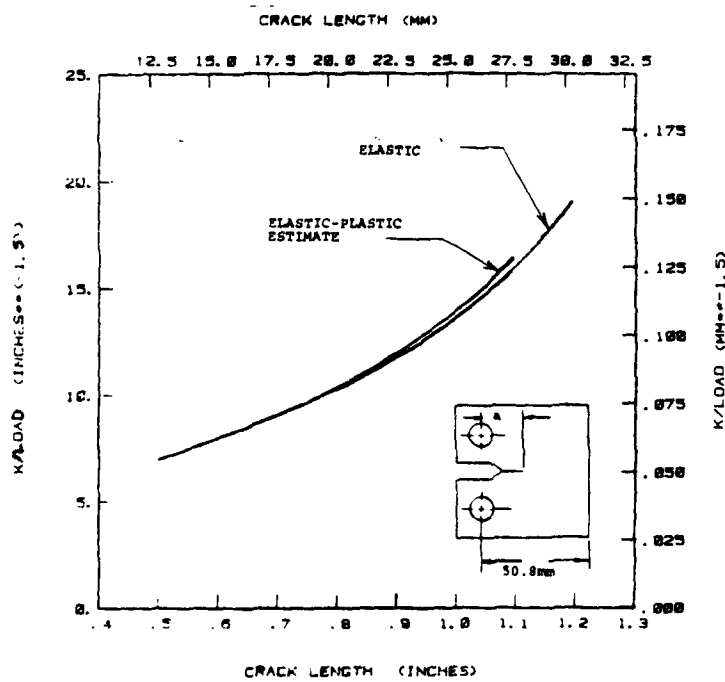


Figure 18. Comparison of Elastic-Plastic Estimated Results with the Elastic Solution for a Compact (CT) Specimen Subjected to a Maximum Load of 1.69 Kips (7.5 kN).

$$J = -\frac{1}{\alpha} \left. \frac{\partial U}{\partial A} \right|_{\delta} = \text{constant} \quad (13)$$

for a condition of fixed load point displacement. The parameter α is equal to 1 for the CT and RHC crack geometries (one crack tip) and is equal to 2 for the CCP geometry (two crack tips); the parameter A is the crack area typically taken as the product of crack length (a) and thickness (B). Experimentally, estimates of the J-integral are obtained by evaluating the area between two load-displacement curves obtained for different crack lengths, such as illustrated in Figure 19.

There are a number of potential problems that arise when the procedure suggested by Equation 13 and Figure 19 is applied to a propagating fatigue crack. One class of problems is associated with utilizing the load-displacement curves to obtain estimates of the potential energy available for cyclic crack growth while the other class is associated with experimental technique and measurement capability.

3.2.3.1 Definition of Crack Driving Factor

In attempting to define the major source of the driving force, we considered the approaches suggested by Dowling and Beiler²³ and Sadananda and Shahinian⁽²⁴⁾ who described two methods for reducing load-displacement data for non-zero minimum load conditions (See Figure 20). Each method is based on the use of the load range (maximum load minus minimum load) and allows investigators to obtain an experimental "AJ" parameter. In those cases where the crack closed prior to reaching the minimum loading, an effective load range (maximum load minus closing, or opening, load) can be utilized. Initially, we attempted to utilize the Sadananda-Shahinian method described in Figure 20c. An evaluation of this AJ parameter was conducted using load-displacement data obtained from finite element calculations for the center crack panel geometry. When the AJ results were compared to the line integral results presented in subsection 3.2.1, the AJ results were shown to be unable to predict levels of the driving force that were lower than the J-integral.

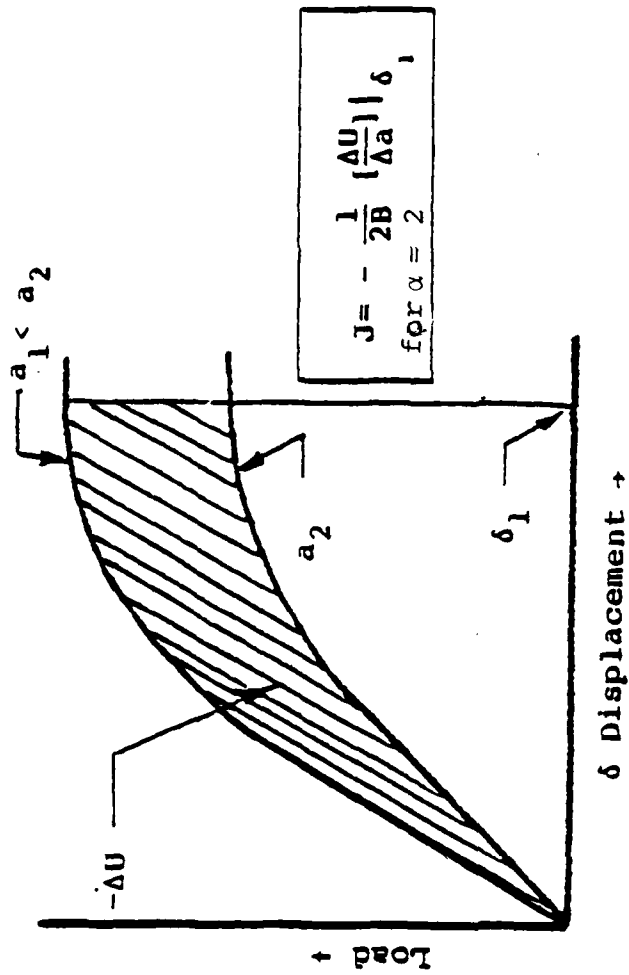
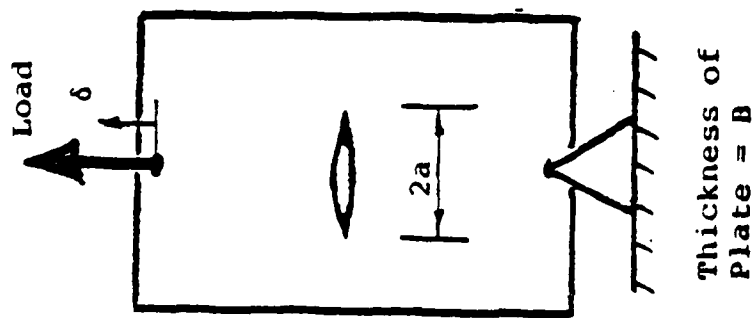


Figure 19. Procedure for Calculating Operational Values of the J-Integral.

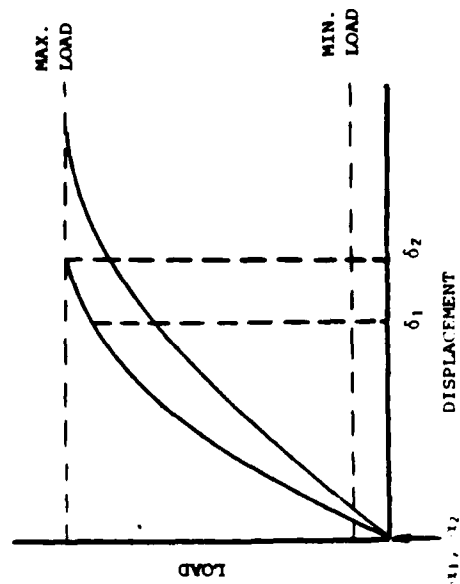
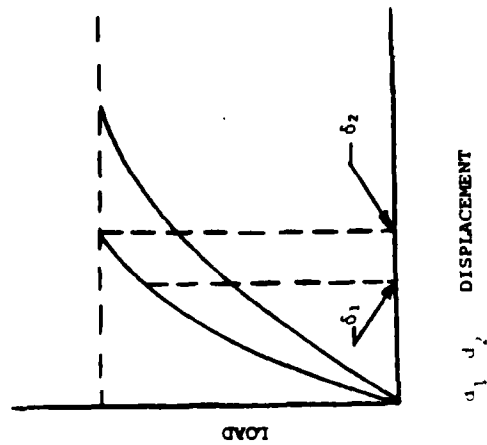
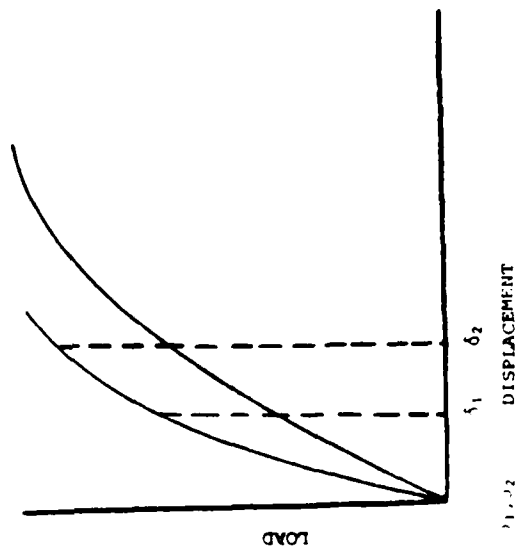
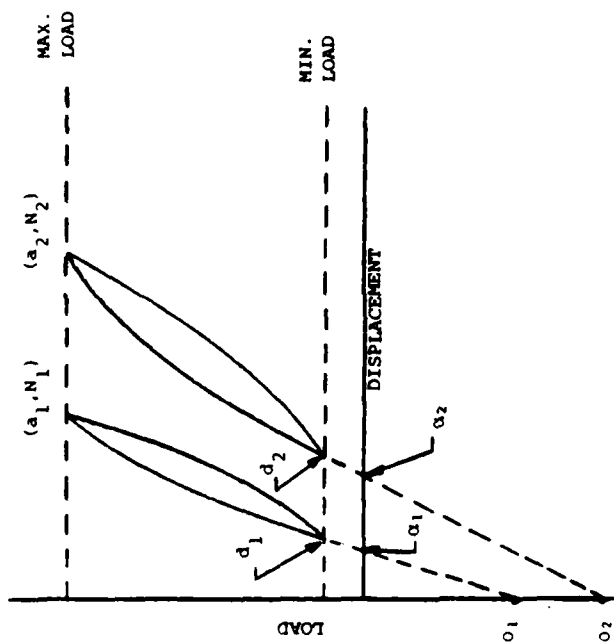


Figure 20. A Schematic Description of Approaches Used to Determine Experimental Values of the J-Integral.

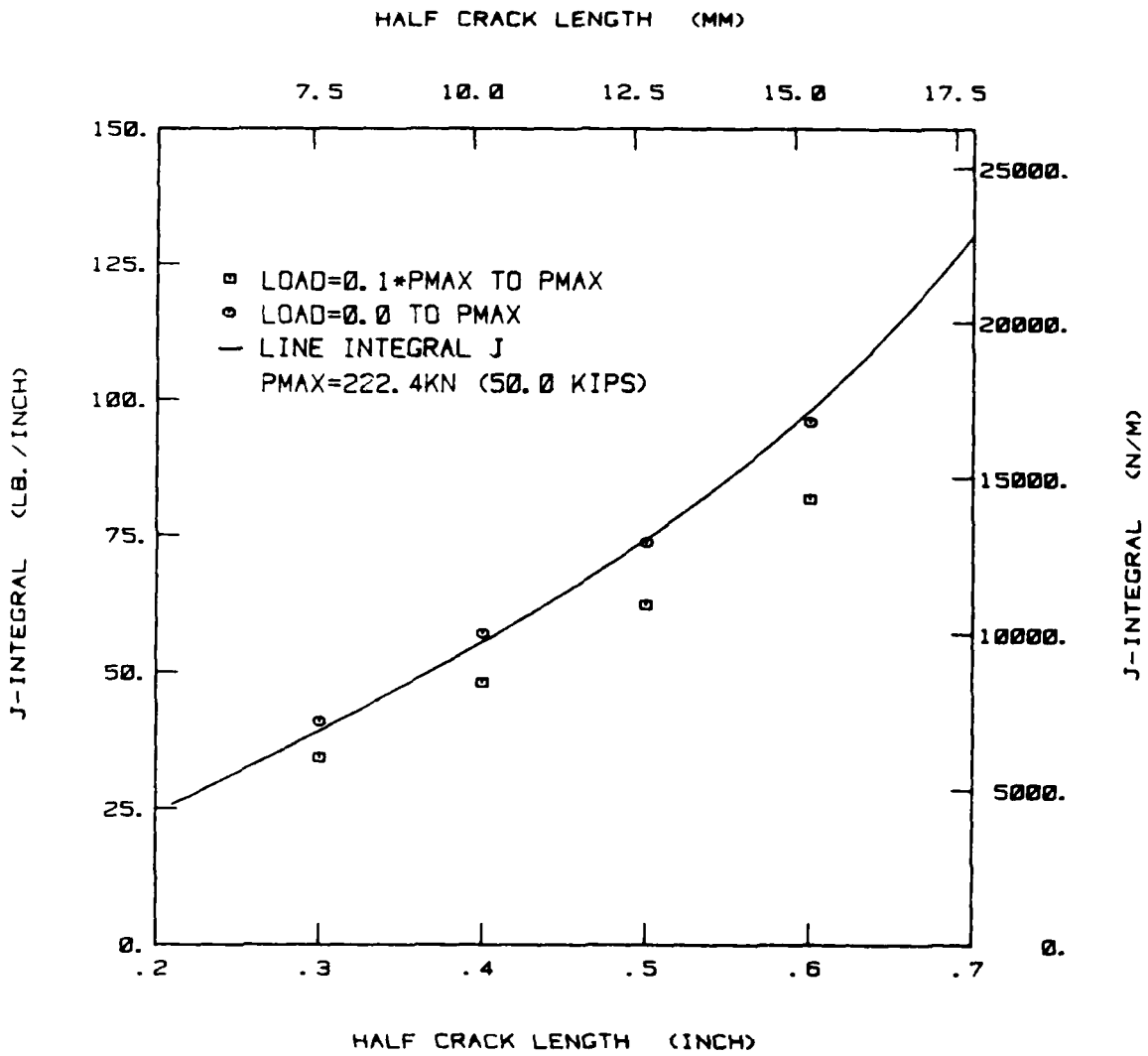


Figure 21. Comparison of J-Integral Calculations Based on Finite Element Results. The Direct Line Integral Calculation is Compared to the J-Integral Calculated Via Equation 13 wherein the Load-Displacement Data were Processed Using the Schemes Illustrated in Figure 20c and 20d.

The ΔJ results shown in Figure 21 might have been anticipated since the ΔJ parameter is based on the load range whereas the line integral results are based on the maximum load. In fact, when the zero-tension area under load-displacement curve (ala Figure 20d) was chosen for input to calculations based on Equation 13, it was found that these J-integral values (J_{\max}) compared favorably with the line-integral results (See Figure 21).

It can be shown that the Sadananda-Shahinian ΔJ parameter is related to the J_{\max} parameter by

$$\Delta J = J_{\max} (1-R)^2 \quad (14)$$

under elastic conditions and not to the difference between the maximum and minimum J-integral values (J_{\max} and J_{\min} , respectively) obtained by evaluating the line integrals at the maximum and minimum load as might initially be suspected. The ΔJ definition provided by Sadananda-Shahinian is directly related to the pseudo elastic stress-intensity factor given by the inverse of Equation 5, i.e.,

$$\Delta J = \frac{\Delta K^2}{E} = \frac{\Delta \sigma^2}{E} \left(\frac{K}{\sigma}\right)^2 \quad (15)$$

where the range of stress-intensity factor and of stress are given by ΔK and $\Delta \sigma$, respectively.

Unfortunately, we were not able to develop evidence that ΔJ is directly related to J_{\max} by Equation 14 when the load-displacement curves exhibit nonlinear behavior. It also appears that the relationship is sufficiently complicated such that it might be difficult to relate line-integral calculations of J_{\max} to the ΔJ parameter. We therefore decided to employ the J_{\max} calculation based on the experimental data evaluated using Equation 13 according to the procedure identified in Figure 20d.

3.2.3.2 Modification of Load-Displacement Data

Relative to the problems associated with experimental technique, there are two important factors: (1) the displacement (δ) changes (increases) as a function of crack length for a constant load condition, and (2) the displacement (δ) given in Equation 13 is associated with the point of loading and is measured in the direction of loading. One major experimental difficulty lies in measuring the displacement accurately enough so that the displacement change can also be accurately determined. This problem is better realized when the displacement is decomposed into two parts, δ_{NCK} and δ_{CK} , associated with the displacement of the structure without a crack and only with the crack, respectively, i.e.,

$$\delta = \delta_{NCK} + \delta_{CK} \quad (16)$$

(Note that the differences in areas under successive load-displacement curves, which are used to obtain J values, depend directly on the increase in δ_{CK} as the crack grows.)

In many structural geometries, the no crack component (δ_{NCK}) which does not change as a function of crack length, is many orders of magnitude larger than the δ_{CK} component. Thus, to assure appropriate displacement sensitivity throughout the crack length range of interest, it is important to design experiments so that the no crack contribution to the displacement is minimized relative to δ_{CK} . This means that for remotely loaded structural geometries, such as the center cracked panel and the radial hole cracked geometries, the displacement should be measured as close to the crack as possible to minimize the δ_{NCK} component.

However, the potential energy utilized in Equation 13 must be evaluated such that the displacement measurements meet other conditions. For the case of a point loading, such as for the CT geometry, the displacements must be associated with the point of loading and be in the direction of

loading. For the case of remote uniform loading such as for the CCP and RHC geometries, the displacement must be measured along the direction of loading and at a sufficient distance from the crack so that the displacement field is approximately uniform. Strictly speaking, the stresses and displacements in a remote location should be measured completely across the width of these geometries and then integrated to sense the potential energy of deformation. Thus, by approximation, the stresses are taken as the applied load divided by the area (width x thickness) and the displacement at a point is assumed representative of a uniform displacement applied across the width of the CCP and RHC geometries.

For the CT geometry, the displacements were measured at the front face of the specimen. The results reported by Hudak et al. ⁽²⁵⁾ were utilized to transfer the measured displacements at the front face of the specimen to the center line of loading. If δ_{LL} and δ_{ff} represent the displacements along the loading line and at the front face, respectively, then

$$\delta_{LL} = \delta_{ff} \frac{\frac{x_0}{W}}{\frac{x_0}{W} + 0.275} \quad (17)$$

where

$$\frac{x_0}{W} = \frac{0.0924}{1 - V_1/V_0} - 0.25 \quad (18)$$

and

$$\frac{V_1}{V_0} = \frac{\left(1 + \frac{0.1576}{a/W}\right)}{\left(1 + \frac{0.250}{a/W}\right)} \times \quad (19)$$

$$\frac{\left(2.537 + 3.904 \left(\frac{a}{W}\right) + 22.44 \left(\frac{a}{W}\right)^2 - 91.53 \left(\frac{a}{W}\right)^3 + 107.4 \left(\frac{a}{W}\right)^4 - 40.7 \left(\frac{a}{W}\right)^5\right)}{\left(1.614 + 12.68 \left(\frac{a}{W}\right) - 14.23 \left(\frac{a}{W}\right)^2 - 16.61 \left(\frac{a}{W}\right)^3 + 35.05 \left(\frac{a}{W}\right)^4 - 14.49 \left(\frac{a}{W}\right)^5\right)}$$

The use of these elastic formulas was justified on the basis that the load-displacement curves for the CT specimen were primarily linear. Figure 22 describes the relationship given by Equation 17 as a function of crack length.

For the CCP and RHC geometries, gage lengths were selected such that the displacement fields contained a sufficiently high level of δ_{CK} relative to δ_{NCK} (See Equation 16) and the displacement was reasonably uniform across the specimen width. Initially, we attempted to utilize the load-load point displacement curves generated in the experiments without giving adequate consideration to the assumption of uniformity in the displacement field. Further consideration of this assumption lead us to evaluate exactly, via finite element calculations, the displacements completely across the specimen width at the gage length associated with the measurement site along the center line of the specimens. Figure 23 describes the finite element displacement results for the CCP geometry with a half crack length (a) of 0.55 inch. As can be seen from the exaggerated scale the displacements are not exactly uniform and the measurement location yields the maximum displacement for the cross-section. Comparisons were made between the line-integral results presented in subsection 3.2.1 and J-integral values obtained using Equation 13. The Equation 13 (single value) load point displacement was approximated by the measurement site (maximum) displacement and by the average displacement across the specimen width at the gage length distance. A comparison is shown in Figure 24 for the maximum load condition for the test gage length (4 inch/101mm).

In evaluating the effects that the various displacements had on the J-integral results, it was determined that it would be better to utilize the average displacement at the gage length than that of the measurement point. Figure 25 provides the transfer function, developed on the basis of finite element results, in order to convert displacements at the measurement site to the average displacement across the width. The transfer function presented in Figure 25 was found

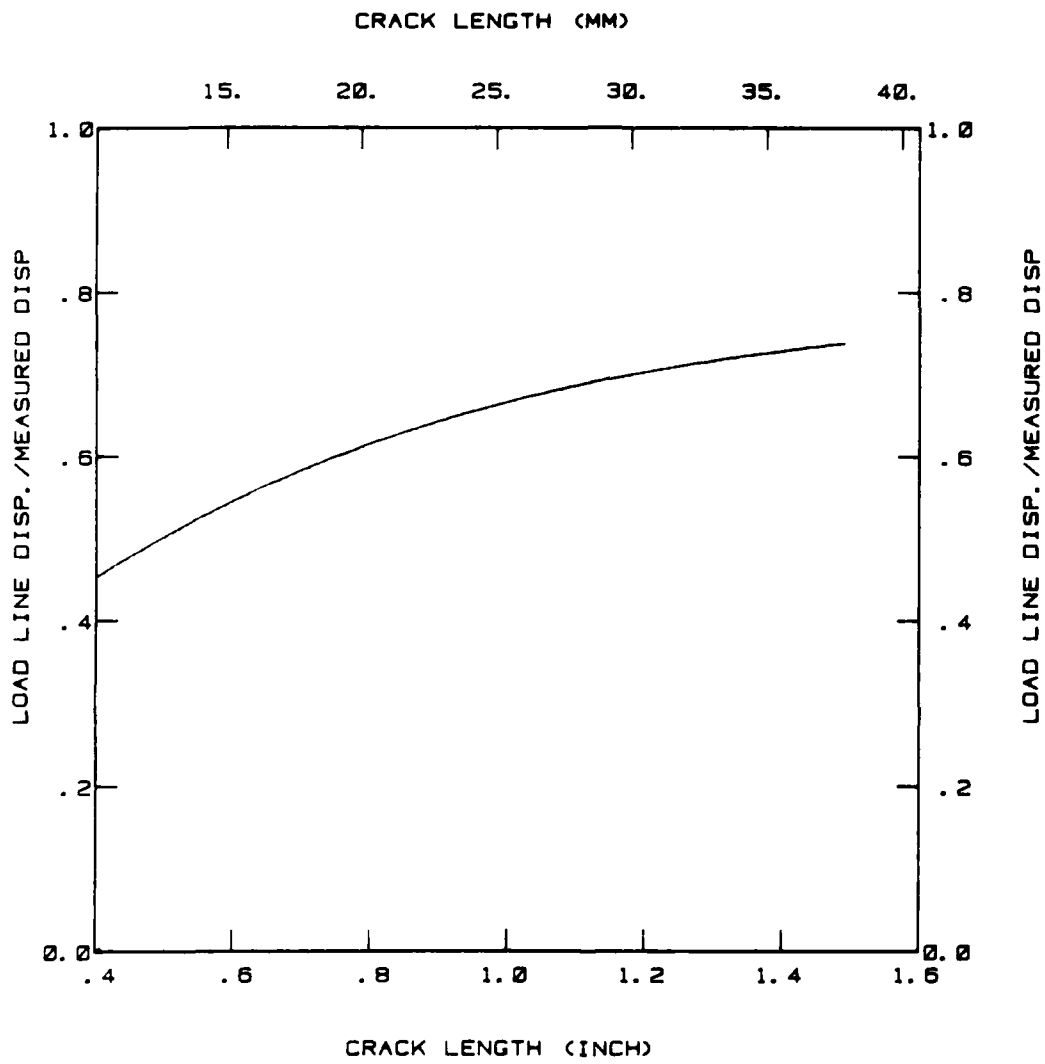


Figure 22. Factor Used to Convert the Measured Displacement to the Load Line Displacement for the CT Specimen.

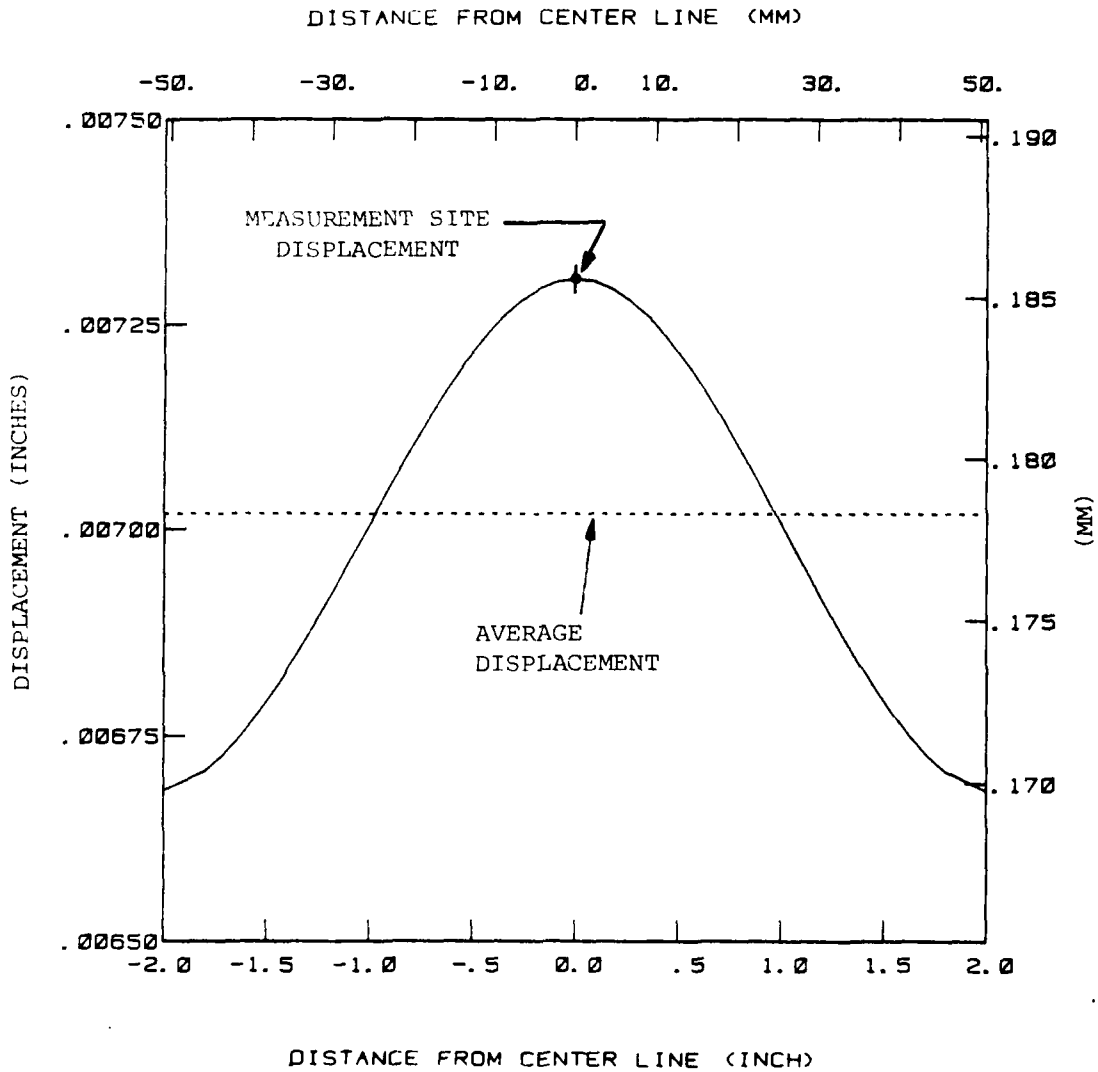


Figure 23. Variation in Displacement Across the CCP Specimen at a Distance of 2 inch (51mm) from the Crack Plane. Displacements Based on Finite Element Results for a Stress of 50.0 Kips (222.4 KN) and a Crack Length of 0.55 inch (14mm).

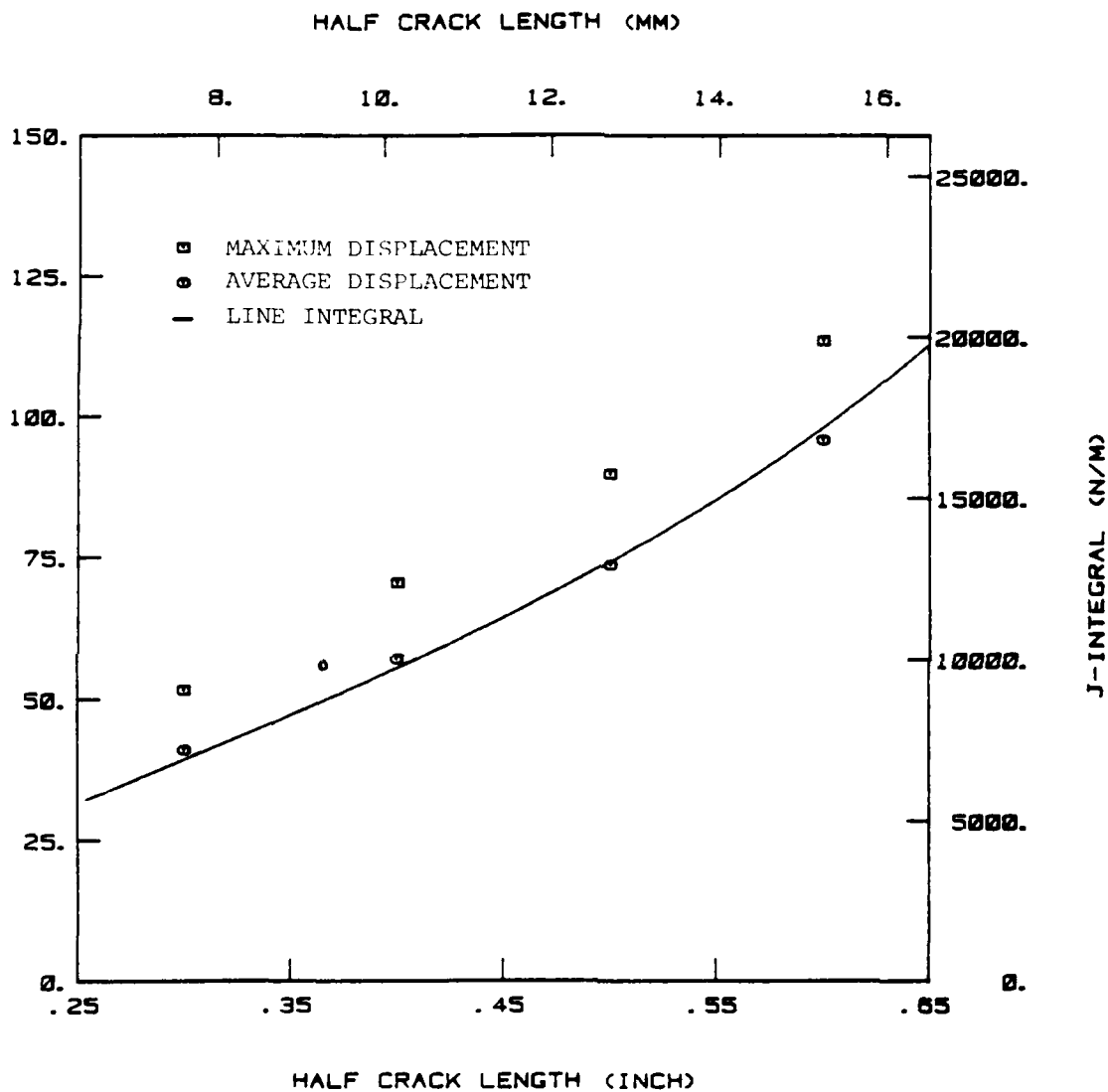


Figure 24. J-Integral Values Based on a Potential Energy Calculation Using Finite Element Load-Load Point Displacement (Gage Length = 4 inch). Results Compared to Line Integral Calculation.

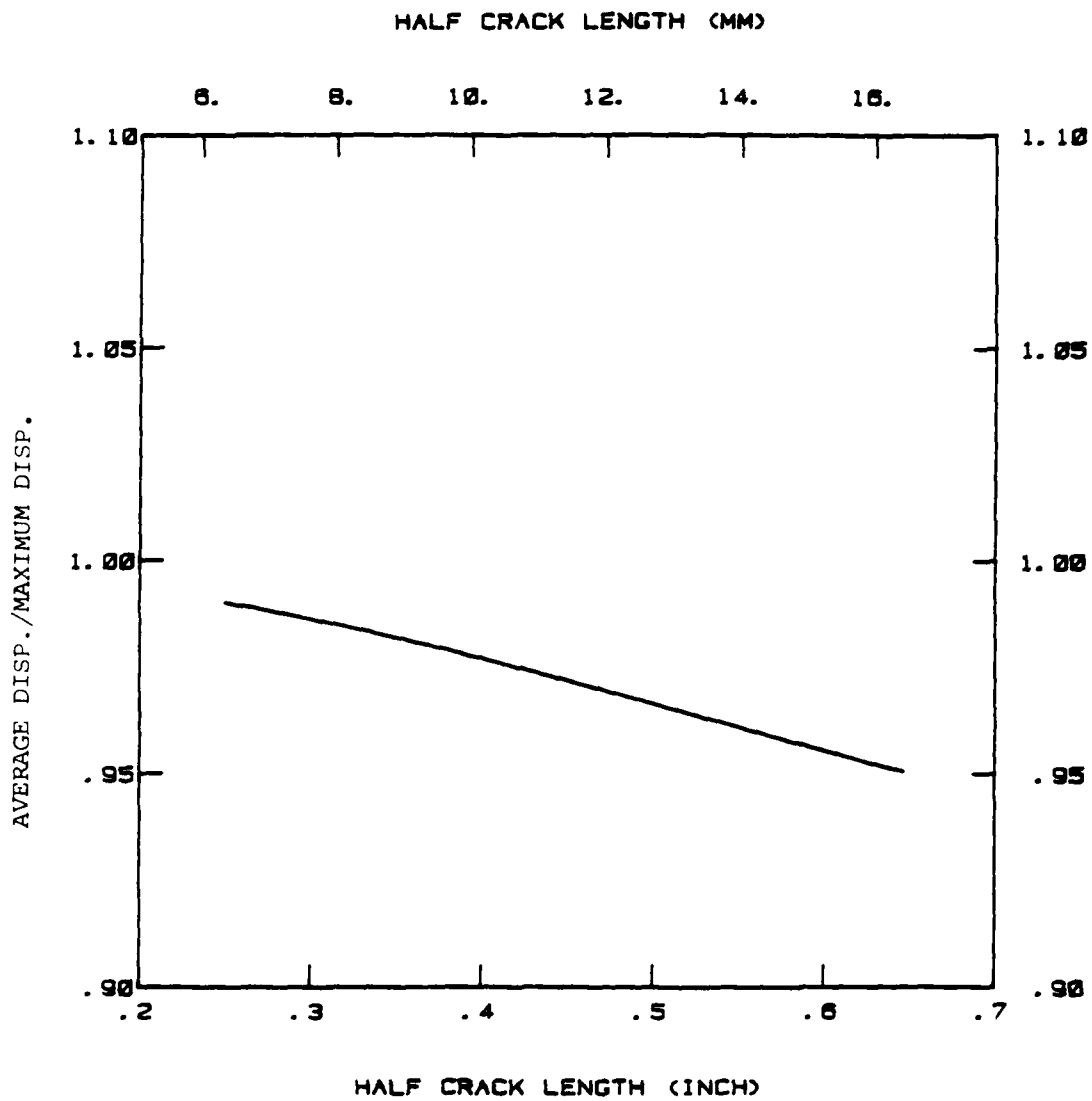


Figure 25. Correction Factor to Convert Measured Point Displacement of the CCP Specimen to the Average Displacement (Based on Finite Element Results for a Maximum Load Condition).

to be relatively independent of load (maximum differences were less than 5 percent from that shown).

A study similar to that described above was also conducted on the RHC specimen geometry. Unfortunately, due to the lack of displacement symmetry across the specimen width, it was not possible to derive a similar meaningful transfer function. For all crack lengths considered in the RHC geometry, the measurement site location gave displacements larger than at other locations across the specimen width. Thus, the displacement used to evaluate the J-integral based on Equation 13 were as measured for the RHC specimens.

3.2.3.3 Summary of Procedures and Results

As described in the above paragraphs, a certain amount of preconditioning of the load displacement information obtained during the tests was required before Equation 13 would provide meaningful results. To summarize, for the CP and CCP tests, the displacements were made compatible with the theoretical assumptions associated with Equation 13 using the transfer functions described in Figures 22 and 25, respectively. No displacement transfer function was used for the RHC test results. The transfer functions were directly applied to computer compatible displacement data obtained by digitizing the experiment load-displacement curves from the X-Y plots taken during the tests.

A linear extrapolation of these load-displacement results was utilized to obtain an estimate of displacement at zero load. Each set of load-modified displacement results for a given crack length was then described with a third order polynomial obtained using least squares results. Figure 26 shows a typical set of load-modified displacement points (obtained from the compact specimen CT-1) described by the least squares determined third order polynomial curve.

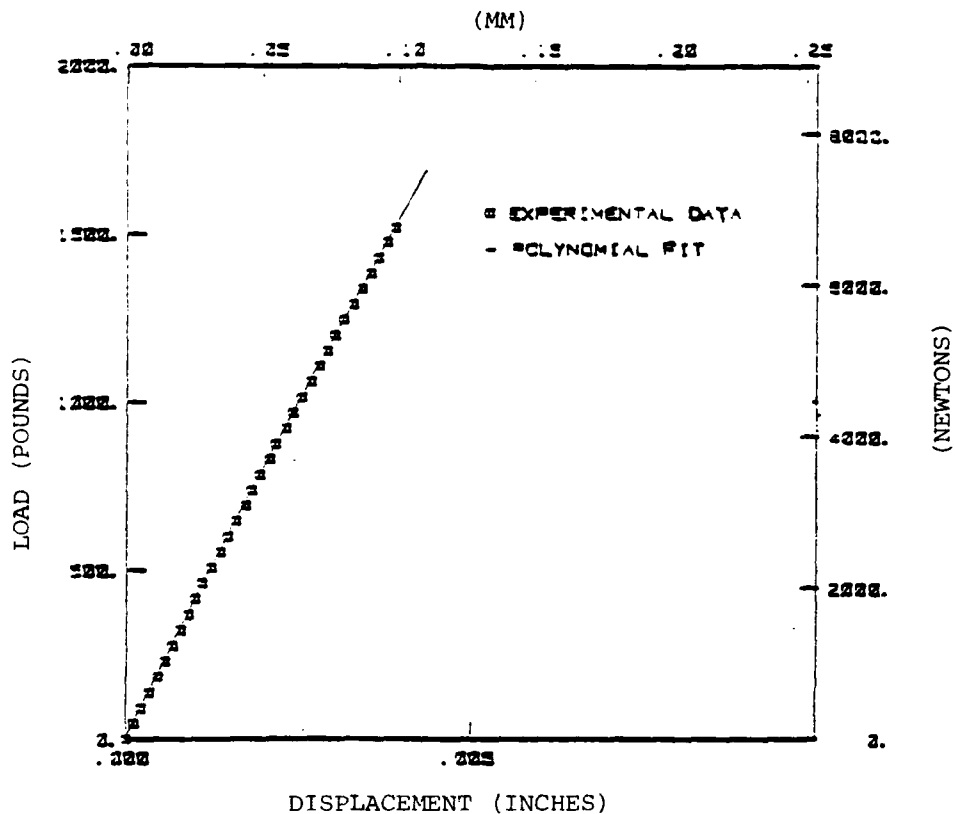


Figure 26. Load Versus Displacement Data and A Least Squares Established Curve (Compact Specimen CT1, Crack Length 13.7mm (0.538 in.)).

The resulting load-displacement curves were integrated between the appropriate displacement limits to produce the potential energy (U) for each curve.

Figure 27 presents the calculated potential energy (U) for CT-1. In Figure 27, the symbols represent the data obtained by integrating the area under the load-displacement curves for the defined values of displacement (δ). The curves illustrated in Figure 27 are obtained from least square polynomial fits to the data points shown. The leftmost point of each curve in Figure 27 corresponds to the maximum applied load at each deflection. By determining the slopes of the fitted curves at the left end, it is possible to generate J values for constant (maximum) load as a function of crack length as shown in Figure 28.

The procedure described above was applied to the load-displacement data collected on CCP, CT, and RHC specimens. The resulting J-integral values which are referred to as experimental J values, were again converted to pseudo stress-intensity factor values through the use of Equation 5. Figures 29, 30, and 31 present the experimental J-integral based values for the CCP, CT, and RHC specimens, respectively; each figure compares the experimental J based value to the elastic numerical values presented earlier in this section. It should be noted that the experimental J value differs significantly from the numerical elastic-plastic results for the CCP and RHC specimens.

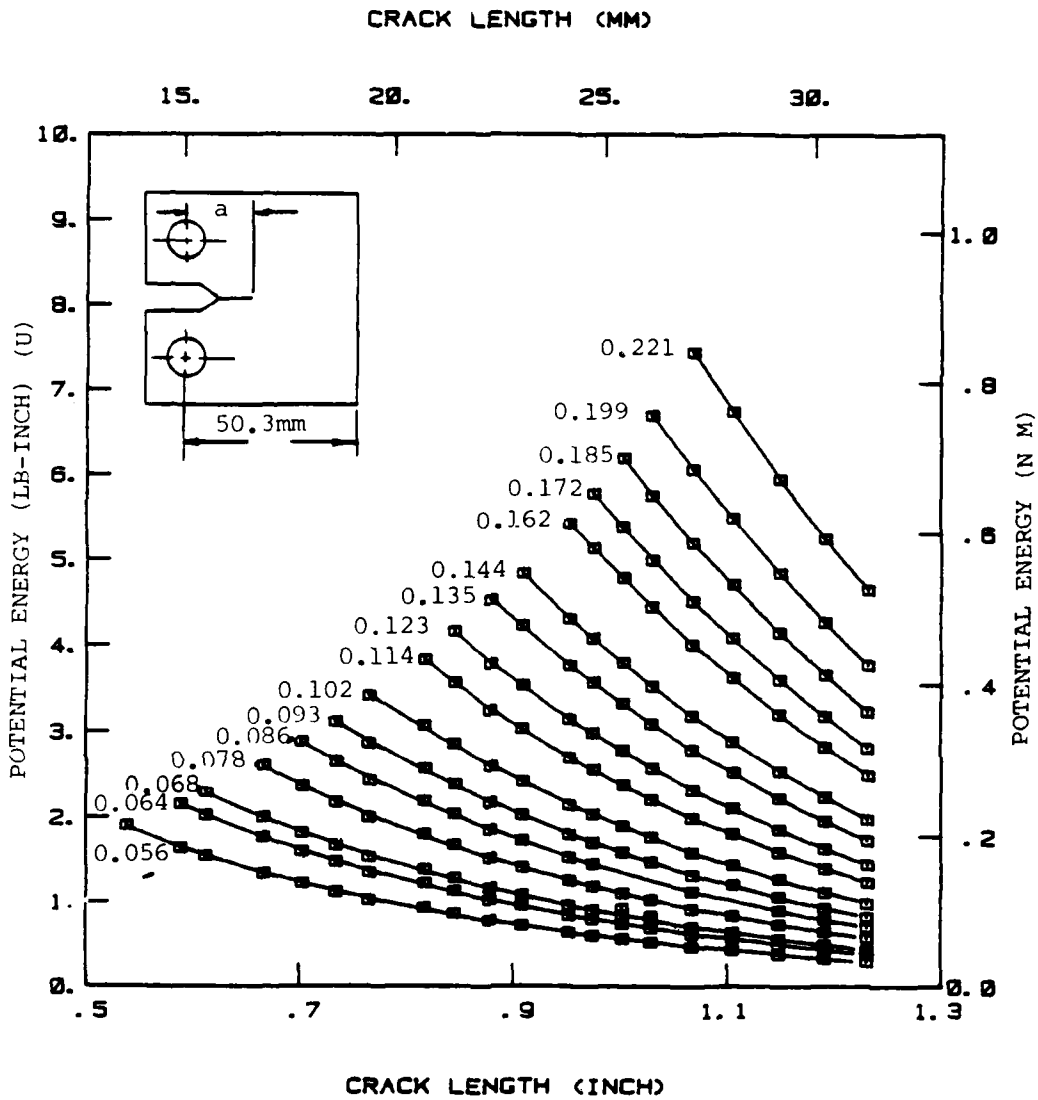


Figure 27. The Potential Energy (U) Based on the Area Under the Load-Displacement Curve up to Defined Values of Displacement (δ) as a Function of Crack Length for Specimen CT1.

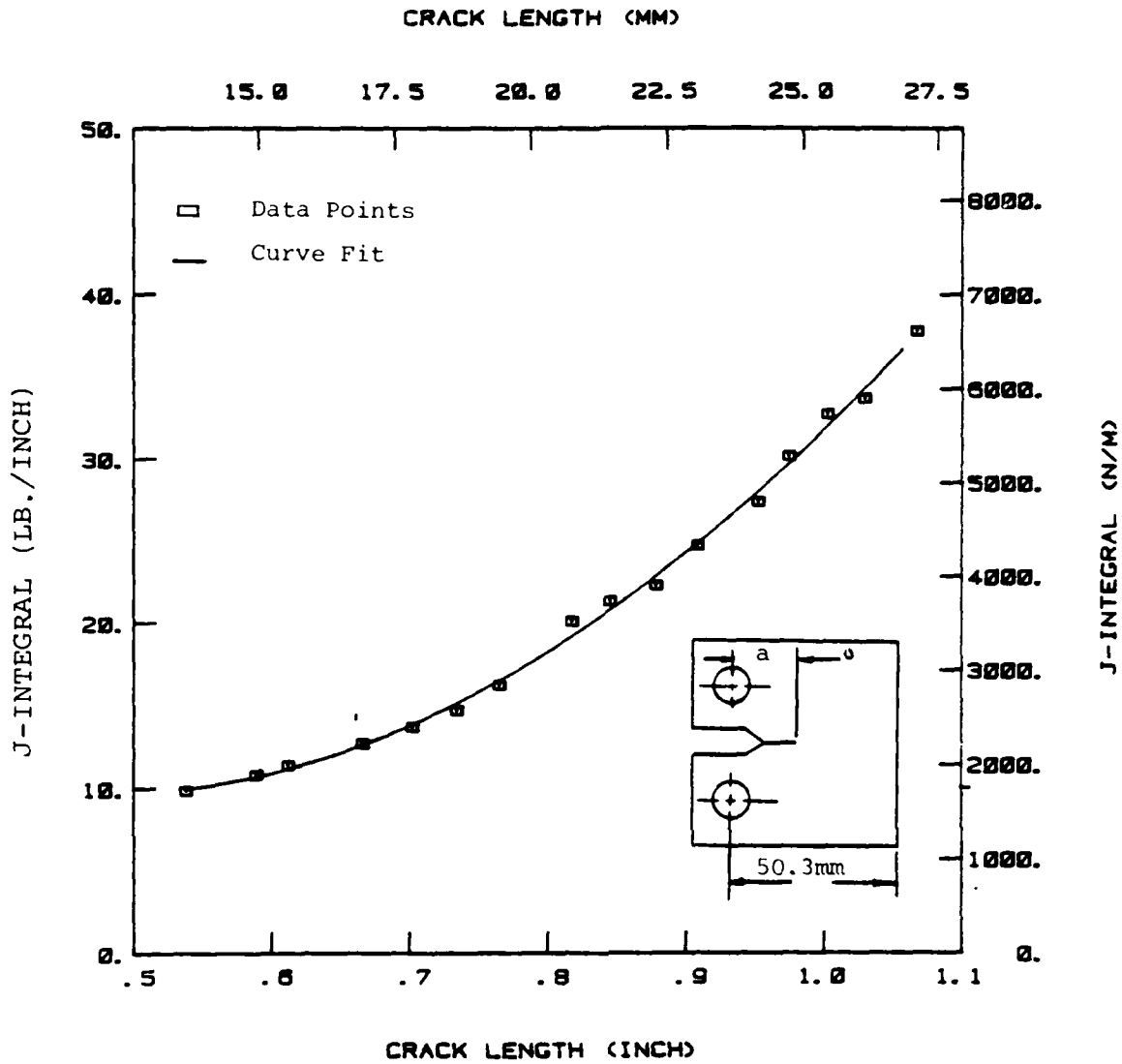


Figure 28. Experimental J-Integral for Compact (CT1) Specimen with an Applied Maximum Load of 1.686 Kips (7.5 KN) and a stress Ratio of 0.1.

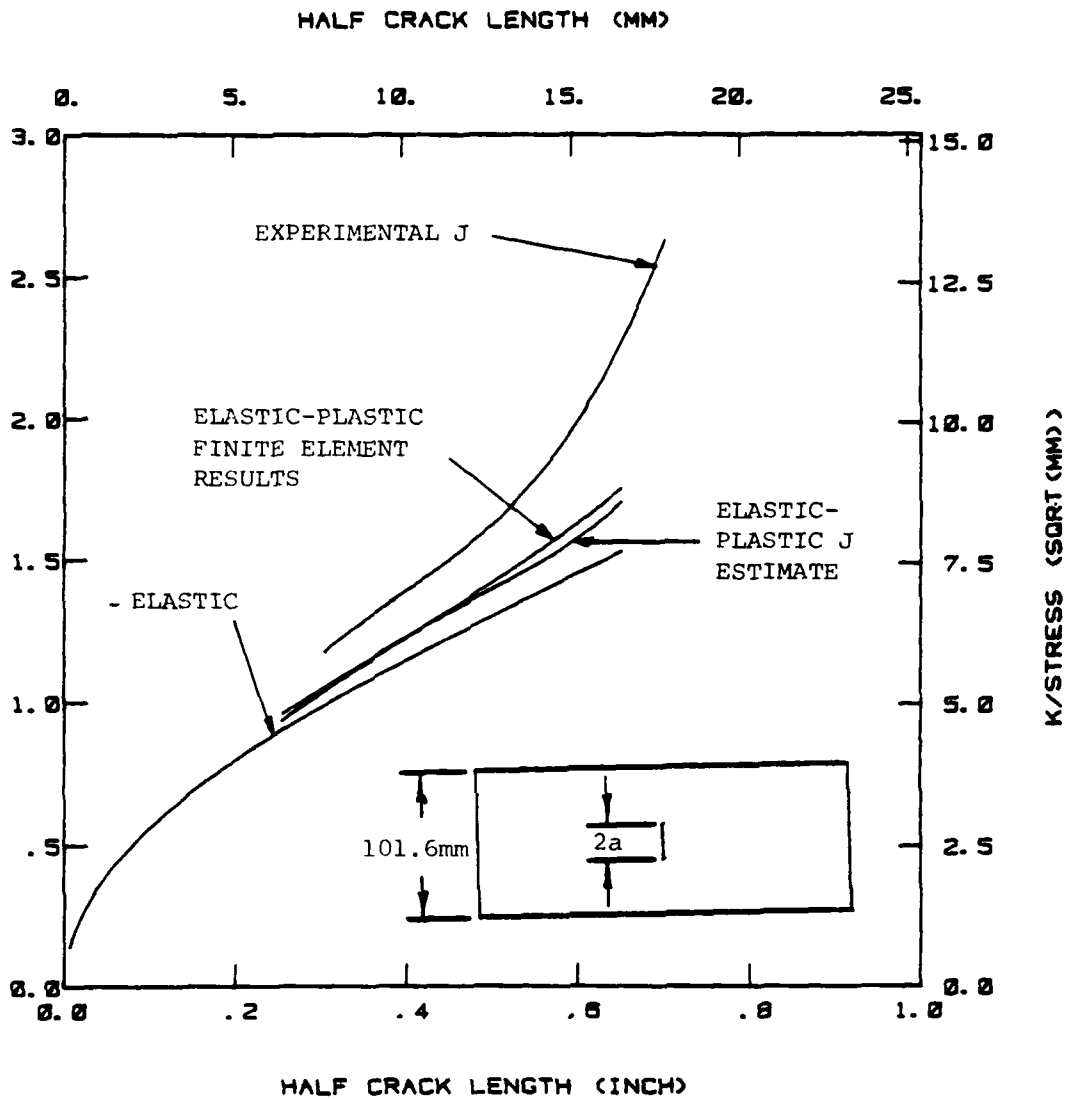


Figure 29. Experimental J-Integral Values Presented as a Pseudo Stress-Intensity Factor for the Center Cracked Panel (CCP) Specimen. Experimental J and Elastic-Plastic J Values are Established at a Maximum Load of 50.0 Kips (222.4KN).

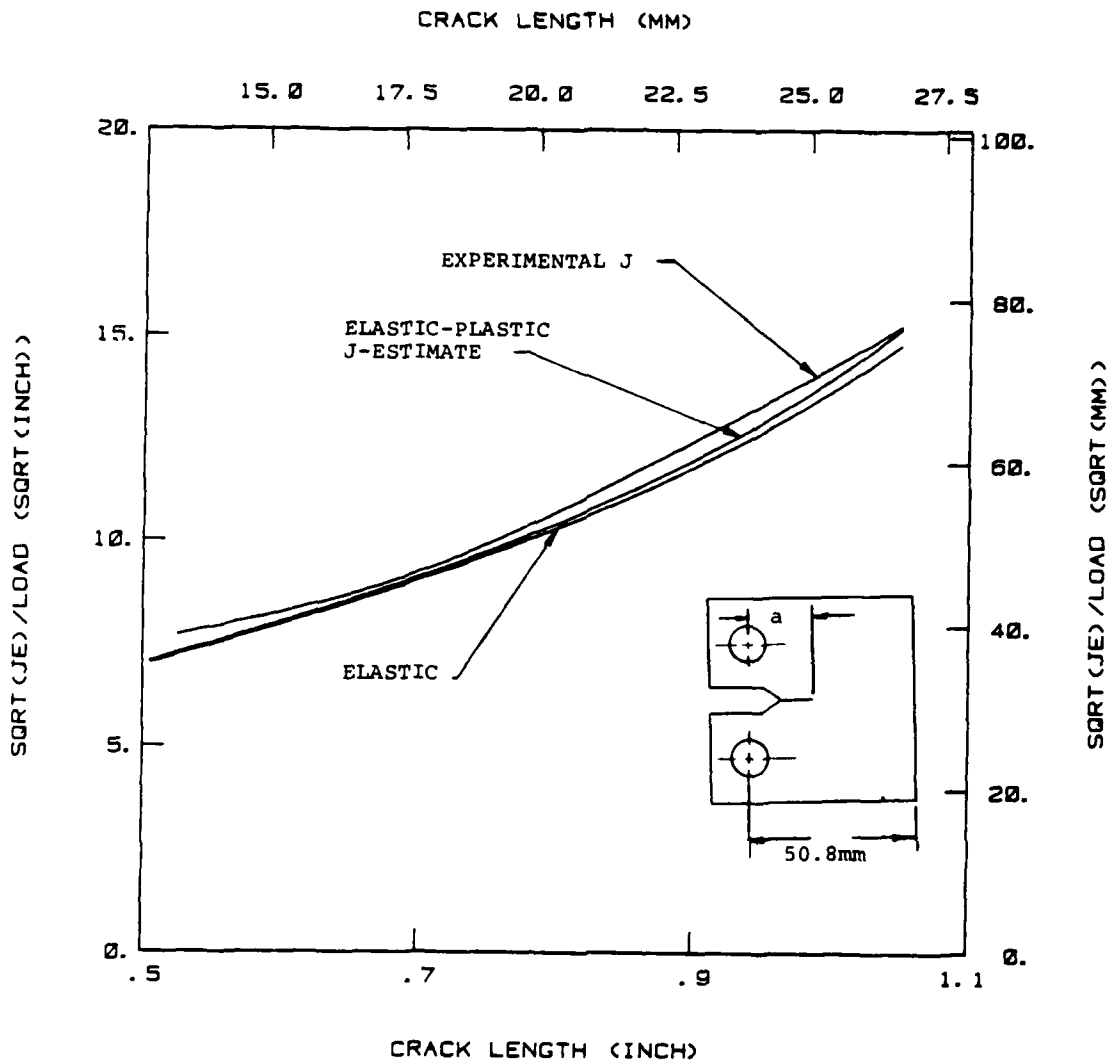


Figure 30. Experimental J-Integral Values Presented as a Pseudo Stress-Intensity Factor for the Compact (CT) Specimen. Experimental J and Elastic-Plastic J Results are Based on a Maximum Load of 1.69 Kips (7.5 KN).

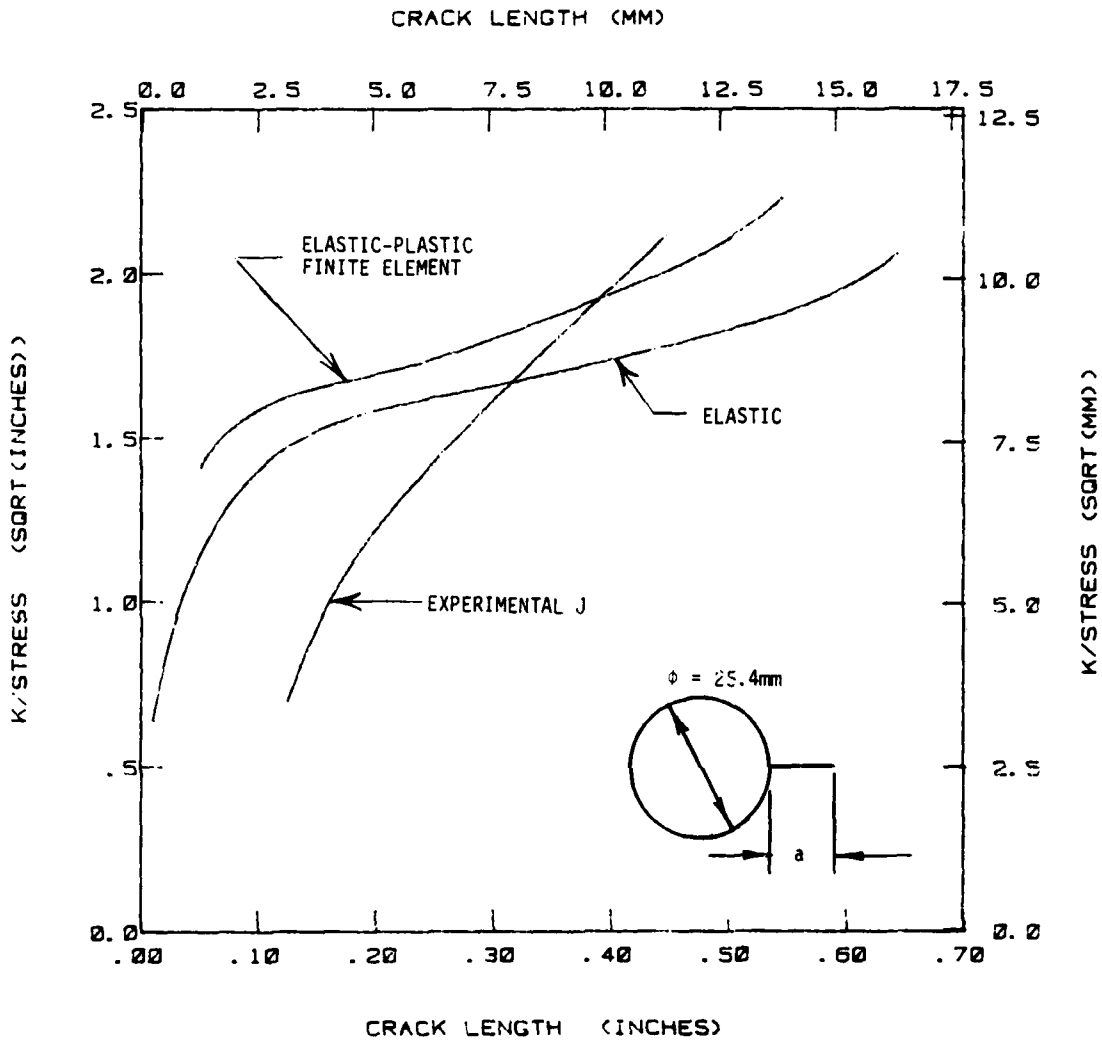


Figure 31. Experimental J-Integral Values Presented as A Pseudo Stress-Intensity Factor for the Radial Hole Crack (RHC) Specimen. Experimental and Elastic-Plastic J Results are Based on a Maximum Load of 33 Kips (146.8 KN).

SECTION 4
RESULTS AND DISCUSSION

As indicated in Section 1, the effort was focused on evaluating several P* parameters for applicability to fatigue crack growth problems where large amounts of plasticity take place. This section outlines how the parameters developed in Section 3 describe fatigue crack growth rate (FCGR) behavior and predict crack growth lives.

4.1 PARAMETER CORRELATIONS

4.1.1 Prediction Procedure

There are two basic criteria that a crack driving parameter (P*) must satisfy. These are: (a) the parameter P* must be able to correlate FCGR data via Equation 1

$$\frac{da}{dN} = f(P^*) \quad (1 \text{ Repeated})$$

such that the correlation is independent of structural geometry and stress level, and (b) the parameter P* must allow for sufficiently accurate estimates of crack growth lives (N_p) to be made from the inverse of Equation 1, i.e.,

$$N_p = \int_{a_0}^{a_f} \frac{da}{f(P^*)} \quad (20)$$

where a_0 and a_f are the initial and final crack lengths in the interval, respectively.

Relative to this program, baseline data are provided by the CCP and CT specimen geometries and the verification data were provided by the RHC specimen geometries. Fatigue crack growth rate (FCGR) data were generated using the seven point incremental polynomial method suggested by ASTM Standard E647 (7);

these FCGR data were correlated to the various parameters evaluated at the mean crack length associated with each seven point (data) interval. For purposes of evaluation, two crack growth rate plots of the type shown in Figure 32 were prepared. When the mean trend baseline curve is found to describe both the baseline data (two geometries, multiple stress levels) and the verification data (one geometry, multiple stress levels), the parameter P^* satisfies the basic similitude conditions for a crack tip driving parameter.

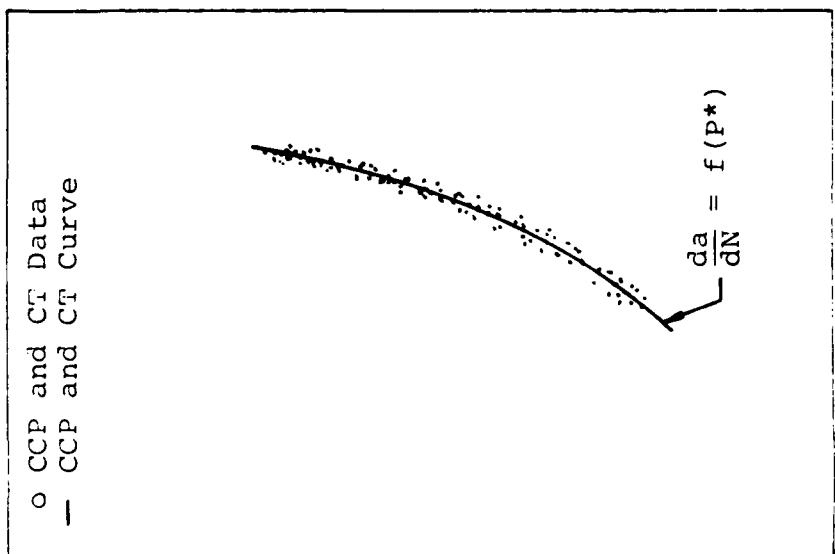
When the mean trend fatigue crack growth rate curve appears to describe the data, the life predictions given by Equation 20 are reasonably accurate. The application of Equation 20 in making blind predictions of crack growth life behavior is, however, a better discriminator of parameter correlation than the growth rate correlation since it is difficult to distinguish relatively small differences in behavior on a FCGR data plot. In making blind predictions, one tests the complete methodology associated with a fracture mechanics approach based on the given parameter. This methodology is illustrated for a numerical analysis of Equation 20 in Figure 33.

The remaining numbered paragraphs of this subsection present the FCGR data correlations as in Figure 32 and life predictions for RHC test results (if the growth rate correlations are reasonable) for the parameters given in Section 3.

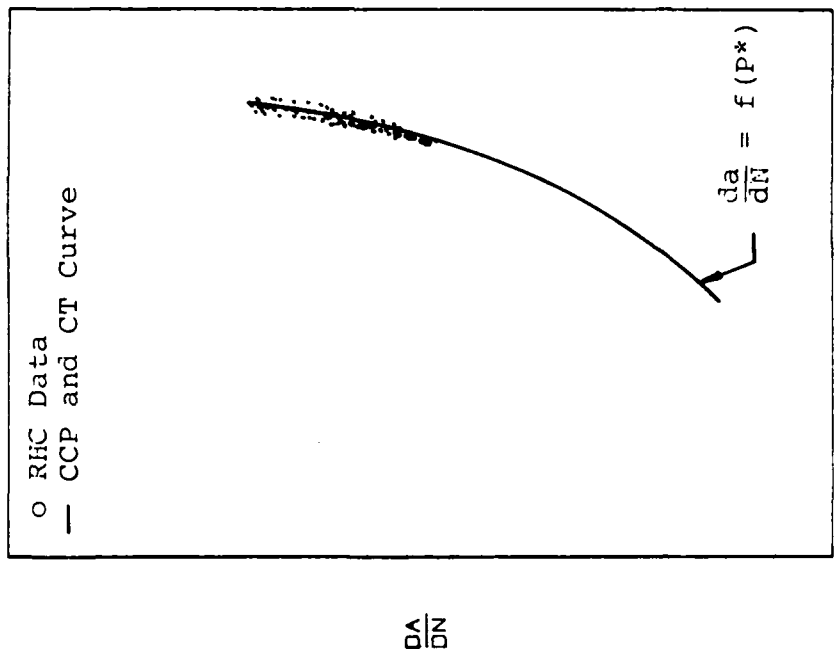
4.1.2 Stress-Intensity Factor (K) Correlations

In this paragraph, the linear elastic fracture mechanics parameter K_{max} , the maximum stress-intensity factor in the fatigue cycle, is evaluated for its ability to correlate fatigue cracking behavior where large amounts of plasticity occur. In this case, the parameter $P^* = K_{max}$, where K_{max} is evaluated from the expression

$$K_{max} = \sigma_{max} \left(\frac{K}{\sigma} \right) \quad (21)$$



(a) Baseline Data and Curve



(b) Verification Data and Baseline Curve

Figure 32. Fatigue Crack Growth Rate Verification Scheme.

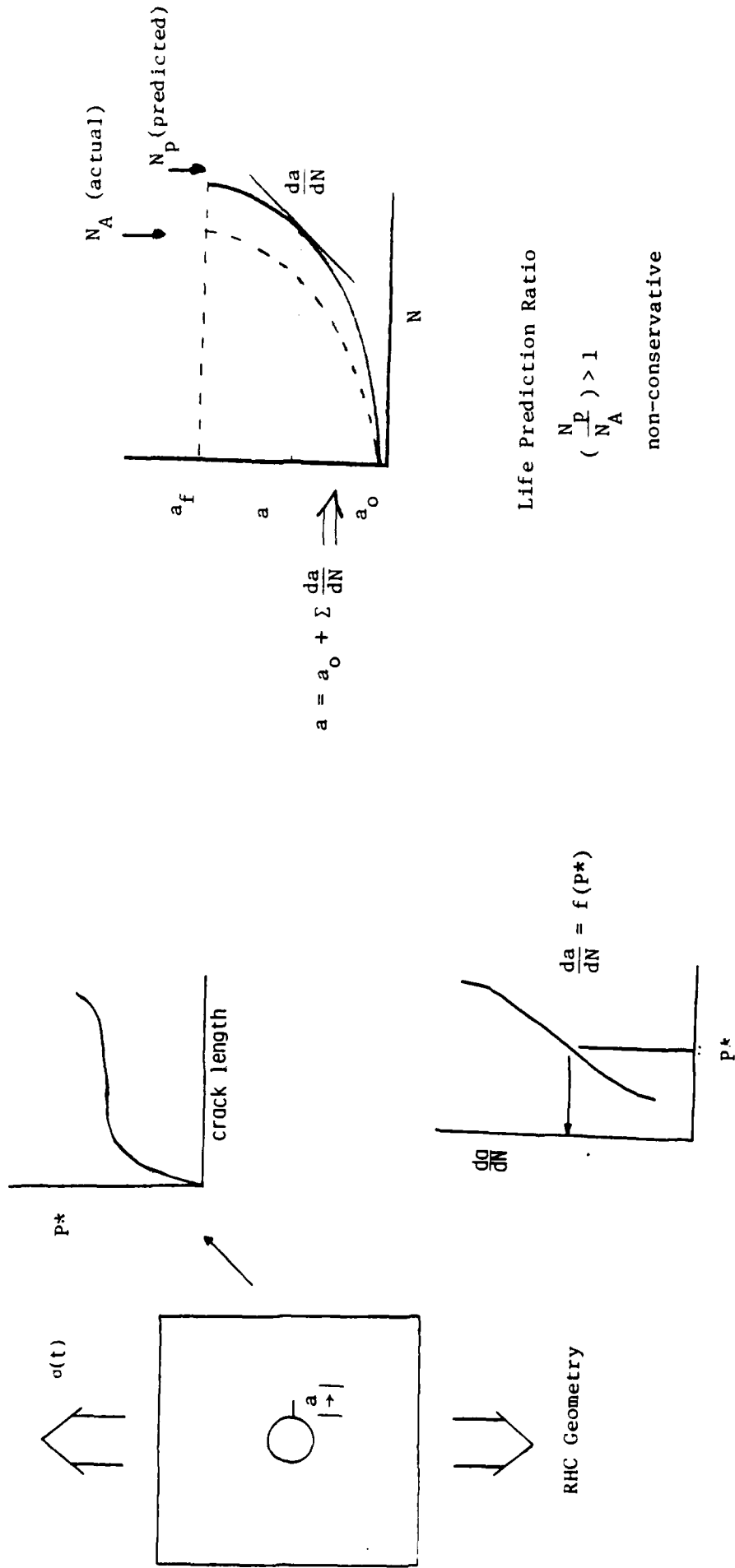


Figure 33. Verification Crack Growth Life Prediction Scheme.

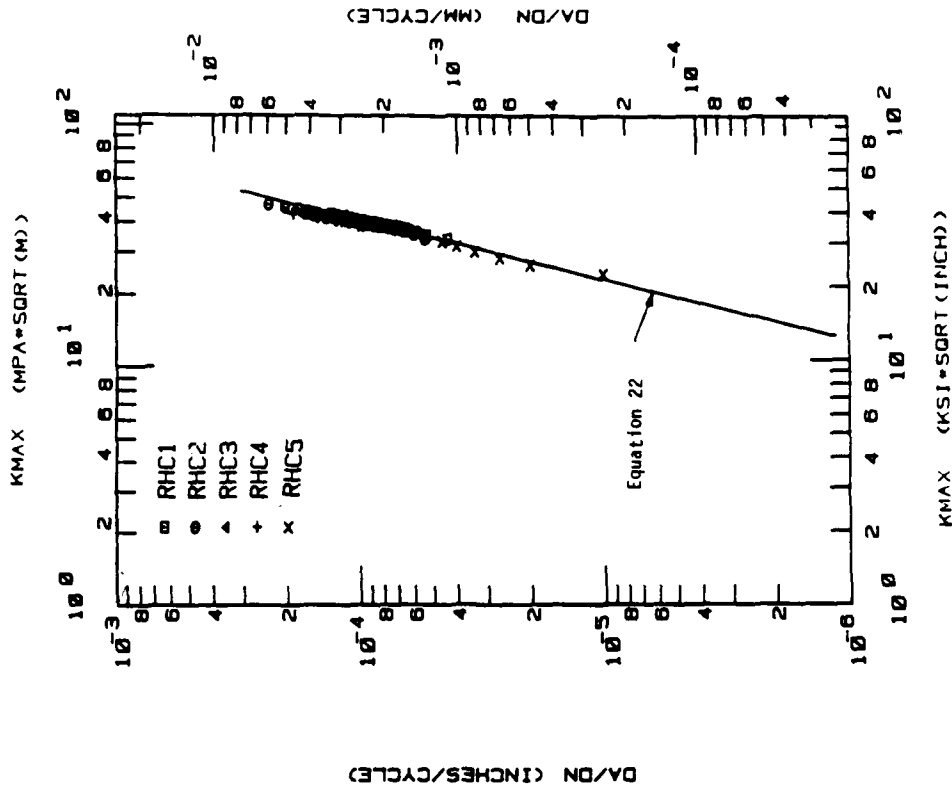
where σ_{\max} is the maximum stress in the fatigue cycle and (K/σ) is the stress-intensity factor coefficient for the given geometry of interest. We note that (K/σ) does not incorporate any influence of plasticity and is only dependent on geometry. Stress-intensity factor coefficients for the CCP, CT and RHC structural geometries were derived from Equations 3,4 and 6, respectively.

The results obtained from comparing the crack growth rate data correlated on the basis of $P^*=K_{\max}$ are described in Figure 34. The curve in Figure 34 is a power law relationship given by

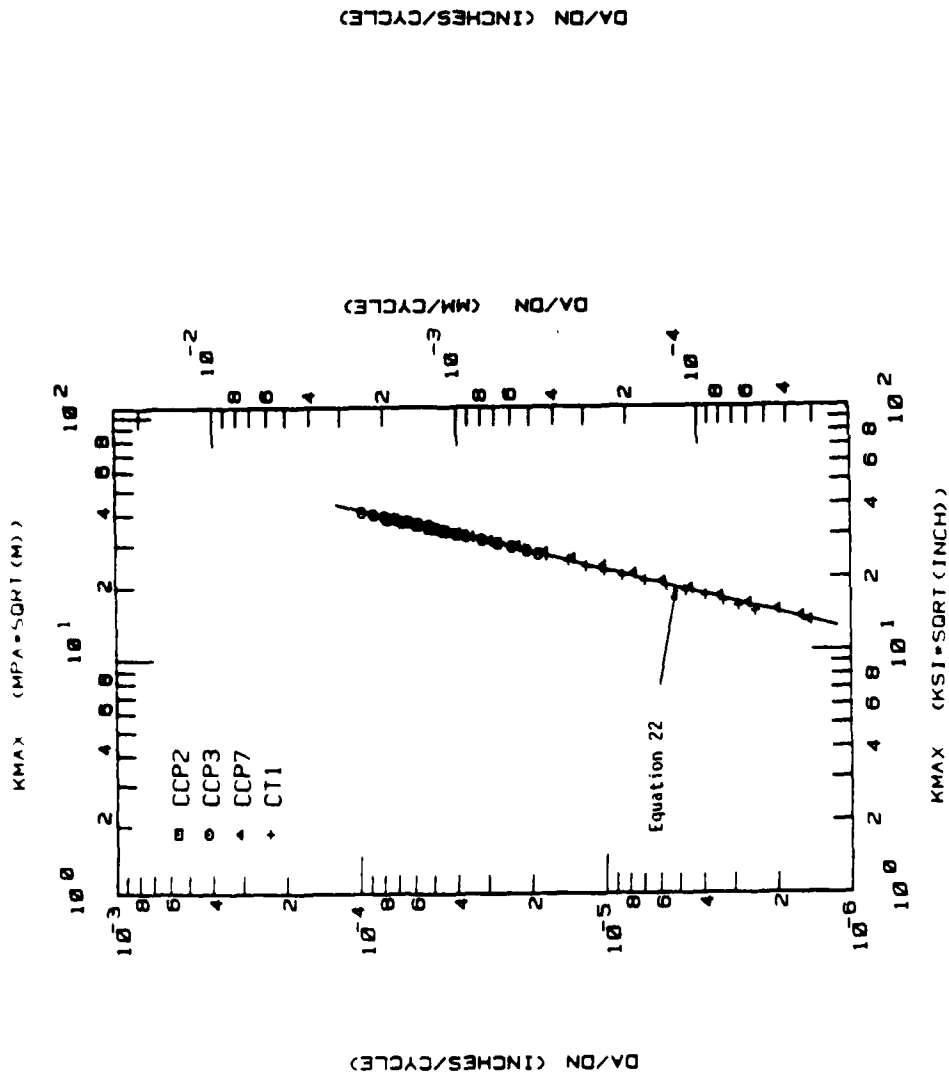
$$\frac{da}{dN} = 4.31 \times 10^{-11} K_{\max}^{4.03} \quad (22)$$

where da/dN and K_{\max} are expressed in units of inches/cycle and $\text{ksi}/\sqrt{\text{in.}}$, respectively. Since this mean trend curve appears to describe the fatigue crack growth rate behavior, Equation 22 was used in conjunction with Equation 21 to develop the life predictions listed in Table 2 for the five RHC tests. Two of the five experimental crack growth life curves are presented in Figure 35 along with the predicted curves

The results of the elastic analysis were somewhat surprising, given the considerable plasticity noted as gross specimen deformation during some tests (See Figure 3). As indicated by Figure 35, the predicted crack growth behavior paralleled the actual behavior; this was so for all the tests considered. The life predictions provided in Table 2 indicate that the elastic parameter will lead to unconservative predictions as might be expected from Figure 34b where the Equation 22 is shown to be slightly lower than the observed RHC fatigue crack growth rate data.



(b) Verification Data and Baseline Curve



(a) Baseline Data and Curve

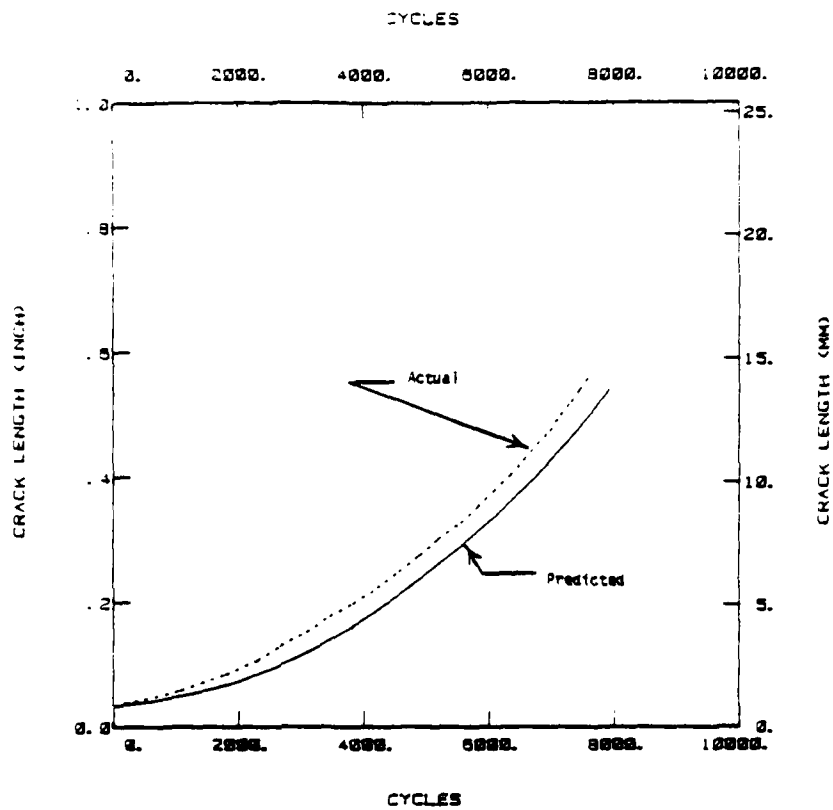
Figure 34. Fatigue Crack Growth Rate Data Described as a Function of the Maximum Stress-Intensity Factor in the Fatigue Cycle.

TABLE 2

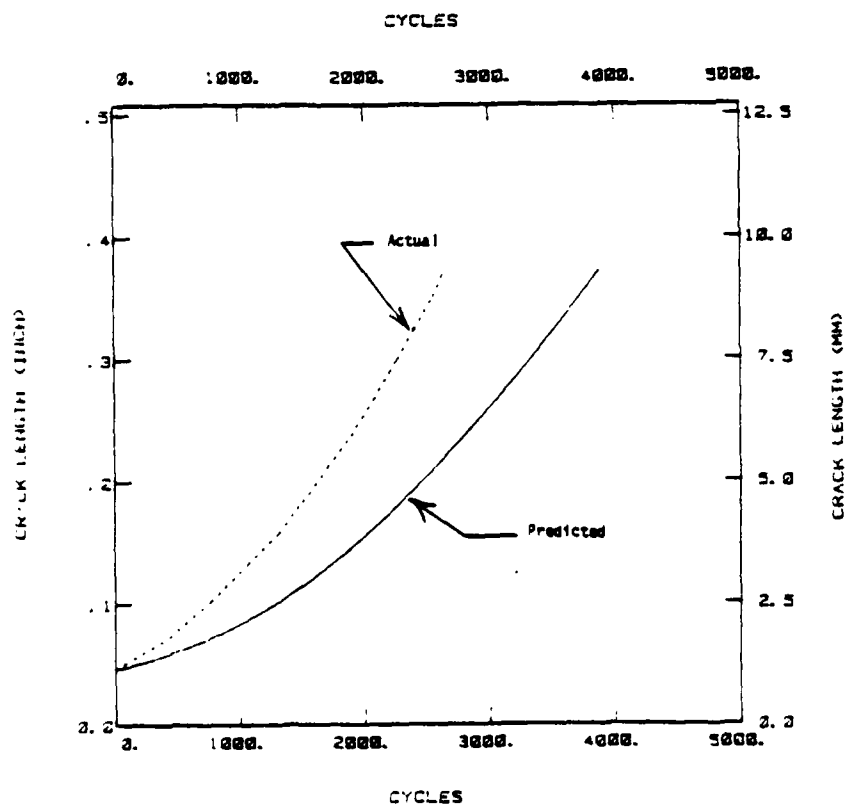
LIFE PREDICTIONS BASED ON THE ELASTIC PARAMETER K

Specimen ID	a_0 (mm)	a_f (mm)	N_f Predicted	N/A Actual	$(N_p/N_A)^*$
RHC1	0.89	13.77	7300	7441	1.06
RHC2	0.86	13.79	8010	6241	1.28
RHC3	1.14	9.32	3870	2661	1.45
RHC4	1.57	9.42	3340	2501	1.34
RHC5	0.20	15.39	14500	12600	1.15

* $(N_p/N_A) > 1$ implies Non-conservative Predictions



(a) Results for RHC1 (Low Stress)



(b) Results for RHC3 (High Stress)

Figure 35. Experimental Fatigue Crack Growth Behavior Compared to That Predicted Based on K_{max} Parameter.

4.1.3 Numerical Elastic-Plastic Parameter Correlations

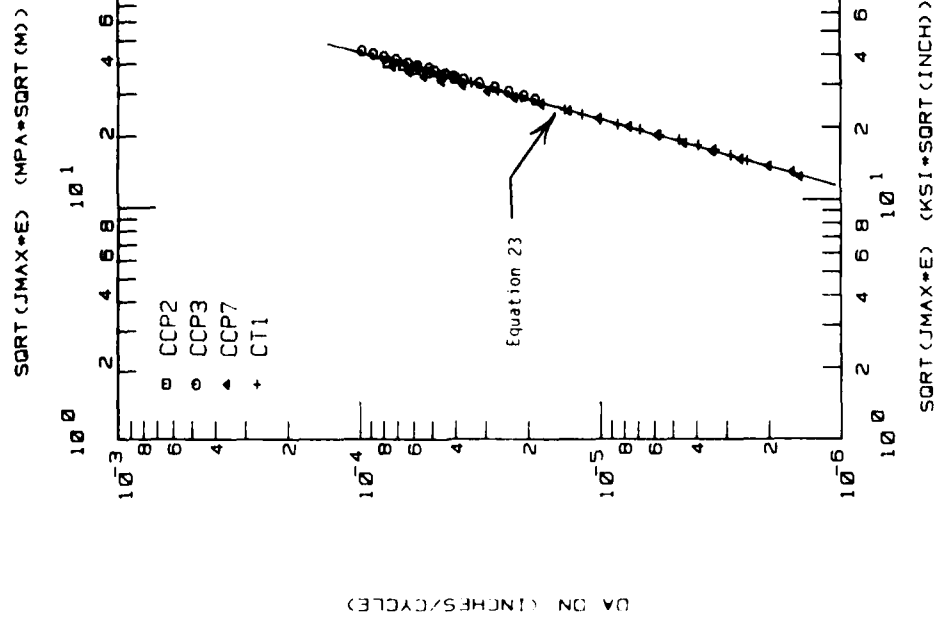
In this paragraph, the numerical elastic-plastic parameter $P^* = \sqrt{J_{\max}} \cdot E$ is evaluated for its ability to correlate fatigue cracking behavior under conditions approaching gross plasticity. The parameter $\sqrt{J_{\max}} \cdot E$ is equivalent to the K_{\max} parameter where elastic conditions dominate. For load levels that exceed those associated with small-scale yielding, the J_{\max} parameter senses the amount of potential energy available for crack growth (assuming nonlinear elasticity). The parameter $\sqrt{J_{\max}} \cdot E$ can be thought of as a pseudo maximum stress-intensity factor (obtained via Equation 5) that incorporates the influence of nonlinear behavior.

The J_{\max} values associated with a given geometry and crack-length must be evaluated for the maximum load (stress) applied in the fatigue cycle. Paragraphs 3.2.1 and 3.2.2 provide the details of the numerical procedures used to obtain these values of J_{\max} using finite element based line integral results and the Shih estimation scheme, respectively. After comparing the results for the center-cracked panel based on both the finite element and estimation schemes, it was decided that the numerical results could be used interchangeably.

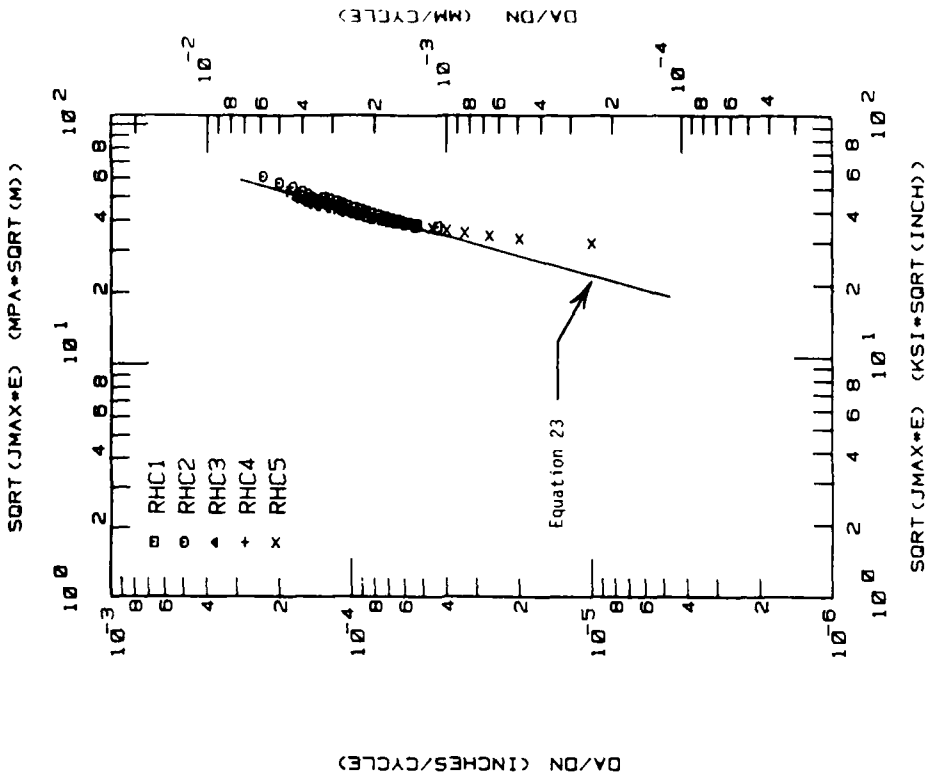
The numerical values of J_{\max} for the CCP, CT, and RHC geometries were based on finite element results, the estimation scheme, and finite element results, respectively. The specific J_{\max} values are shown in Figures 13, 18, and 14, for the maximum load (stress) conditions for the CCP, CT, and RHC geometries, respectively.

The results obtained from comparing the crack growth rate data correlated on the basis of $P^* = \sqrt{J_{\max}} \cdot E$ are described in Figure 36. The curve in Figure 36 is a power law given by

$$\frac{da}{dN} = 1.75 \times 10^{-10} \left(\sqrt{J_{\max}} \cdot E \right)^{3.56} \quad (23)$$



(a) Baseline Data and Curve.



(b) Verification Data and Baseline Curve

Figure 36. Fatigue Crack Growth Rate Data Described as a Function of the Maximum Pseudo Stress-Intensity Factor, Based on J_{max} and Derived from Numerical Results (Finite Element and Estimation Scheme).

where da/dN and $\sqrt{J_{\max}}E$ are expressed in units of inches per cycle and ksi in, respectively. Except for the lower growth rates associated with one of the radial hole cracked tests (RHC5), the mean trend baseline curve given by Equation 23 provided a reasonable description of the radial hole cracked growth rate data. When Equation 23 was used in conjunction with Equation 21, the life predictions listed in Table 3 were obtained. Figure 37 describes the crack growth behavior (actual versus predicted) for two radial hole cracked tests. As can be seen by comparing Table 3 and Figure 37 with Table 2 and Figure 35, the $\sqrt{J_{\max}}E$ parameter predicted crack growth lives and behavior to a slightly better degree than the corresponding elastic stress-intensity factor. Conversely, by comparing the crack growth rate correlations for the two parameters (Figure 34 and 36), it appears that the elastic stress-intensity factor parameter provides a better correlation. There will be more discussion on comparisons and correlations subsequently in subsection 4.2.

4.1.4 Experimental Elastic-Plastic Parameter Correlations

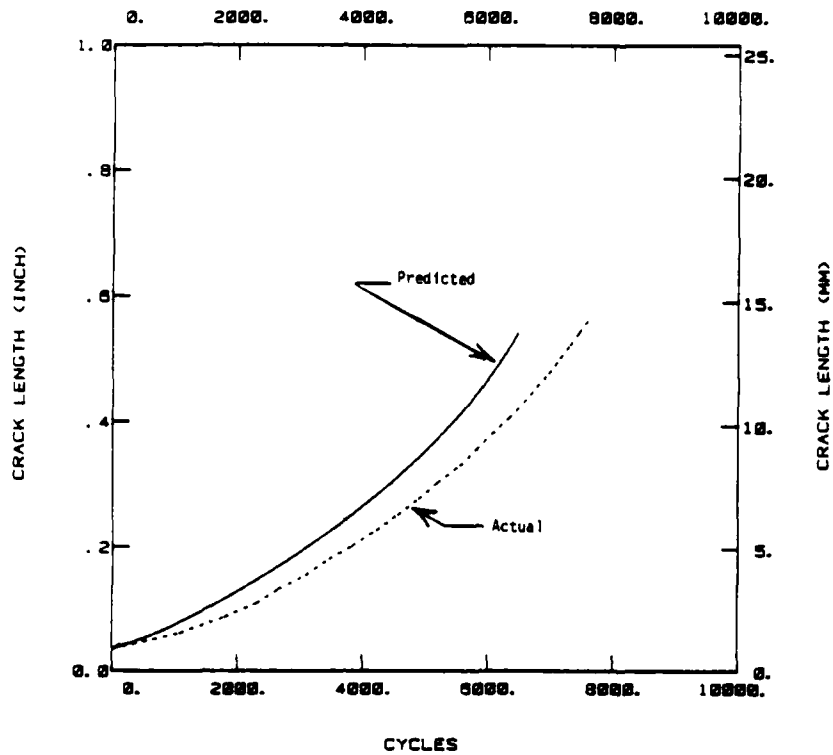
In this paragraph, the experimental elastic-plastic parameter $P^* = \sqrt{J_{\max}}E$ is evaluated for its ability to correlate fatigue cracking behavior under conditions approaching gross plasticity. From an analytical standpoint, the numerical and experimental results of J_{\max} should be equivalent; but, based on the work presented in paragraph 3.2.3, we note that they are not. The J_{\max} values associated with a given geometry and crack length were evaluated for the maximum load (stress) applied in the fatigue cycle; these were presented in Figures 29, 30, and 31 for the CCP, CT, and RHC geometries, respectively.

The results obtained from comparing the crack growth rate data correlated on the basis of $P^* = \sqrt{J_{\max}}E$ are described in Figure 38. The curve in Figure 38 is a power law given by

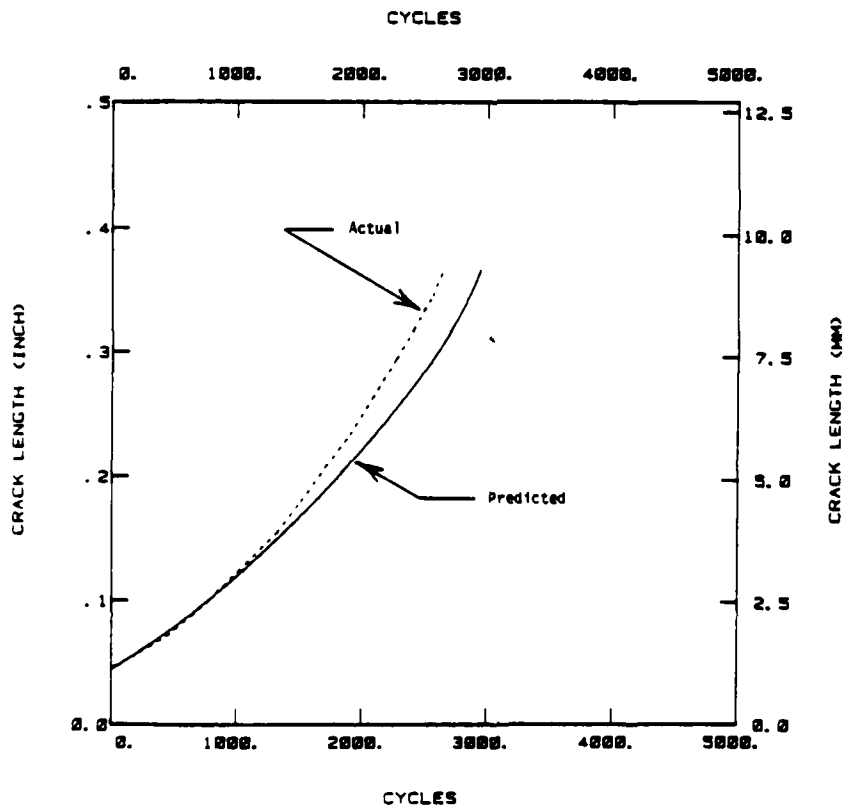
$$\frac{da}{dN} = 1.03 \times 10^{-9} (\sqrt{J_{\max}}E)^{2.93} \quad (24)$$

TABLE 3
 LIFE PREDICTIONS BASED ON THE ELASTIC-PLASTIC PARAMETER $\sqrt{J_{\max}} \cdot E$

Specimen ID	a_0 (mm)	a_f (mm)	N_p Predicted	N_A Actual	N_p/N_A
RHC1	0.89	13.77	6500	7441	0.88
RHC2	0.86	13.79	6540	6241	1.05
RHC3	1.14	9.32	2940	2661	1.10
RHC4	1.57	9.42	2690	2501	1.08
RHC5	0.20	15.39	8410	12600	0.67

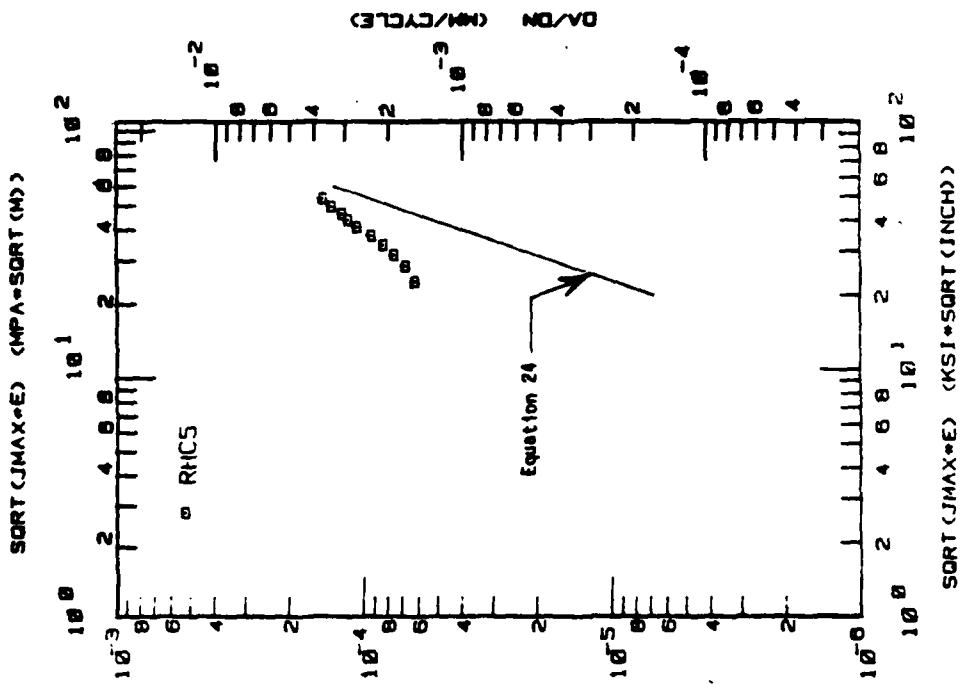


(a) Results for RHC1 (Low Stress)

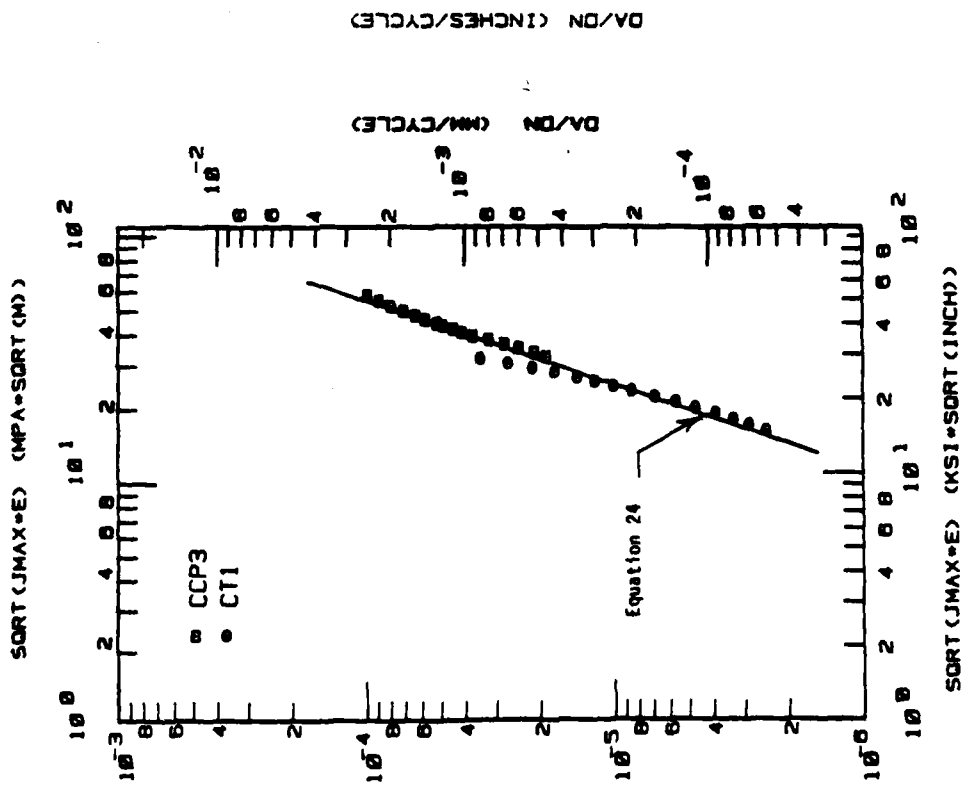


(b) Results for RHC3 (High Stress)

Figure 37. Experimental Fatigue Crack Growth Behavior Compared to that Predicted Based on Numerical $\sqrt{J_{max}}/E$ Parameter.



(a) Baseline Data and Curve.



(b) Verification Data and Baseline Curve.

Figure 38. Fatigue Crack Growth Rate Data Described as a Function of the Maximum Pseudo Stress-Intensity Factor, Based on J_{max} and Derived from Experimental Load-Displacement Results.

where da/dN and $\sqrt{J_{\max}}E$ are expressed in units of inches/cycle and ksi $\sqrt{\text{in}}$, respectively.

While Equation 24 provides an adequate description of the baseline data shown in Figure 38a, it does not describe the trend in the RHC test crack growth rate data at all. Only one RHC test (RHC-5) yielded load-displacement data which was considered sufficiently valid for J-integral analysis, i.e., the displacements were measured either directly across the hole at a 4.0 inch gage length. Because the experimental crack growth rates were so poorly correlated, no attempt to calculate life was made.

4.2 DISCUSSION OF RESULTS

The various elastic and elastic-plastic parameters considered in the previous subsection were all able to correlate the baseline fatigue crack growth rate behavior associated with center crack panel and compact geometries. Except for the experimental elastic-plastic parameter, the parameters were also found to provide adequate predictions of the behavior exhibited by the radial hole crack geometry. This subsection further considers the results in a comparative sense and then describes some of the authors' concerns as they developed the results.

4.2.1 Comparisons of Growth Rate Correlations

The pseudo stress-intensity factor parameter $\sqrt{J_{\max}}E$ can be used to provide the means for evaluating the effects that increased amounts of plasticity have on the crack growth rate. As subsection 3.2 illustrated the elastic-plastic $\sqrt{J_{\max}}E$ values were slightly higher than the elastic value of the maximum stress-intensity factor K_{\max} . In evaluating crack growth rates, it is seen in Figure 39 that the increasing amount of plasticity associated with the $\sqrt{J_{\max}}E$ parameter causes the baseline curve to shift (rotate) at the higher levels of the parameter. Thus, one would expect that a fatigue crack growth rate baseline based on linear elastic fracture mechanics would provide conservative life estimates for structures experiencing elastic-plastic strains during fatigue cycling. Figure 40 illustrates this where

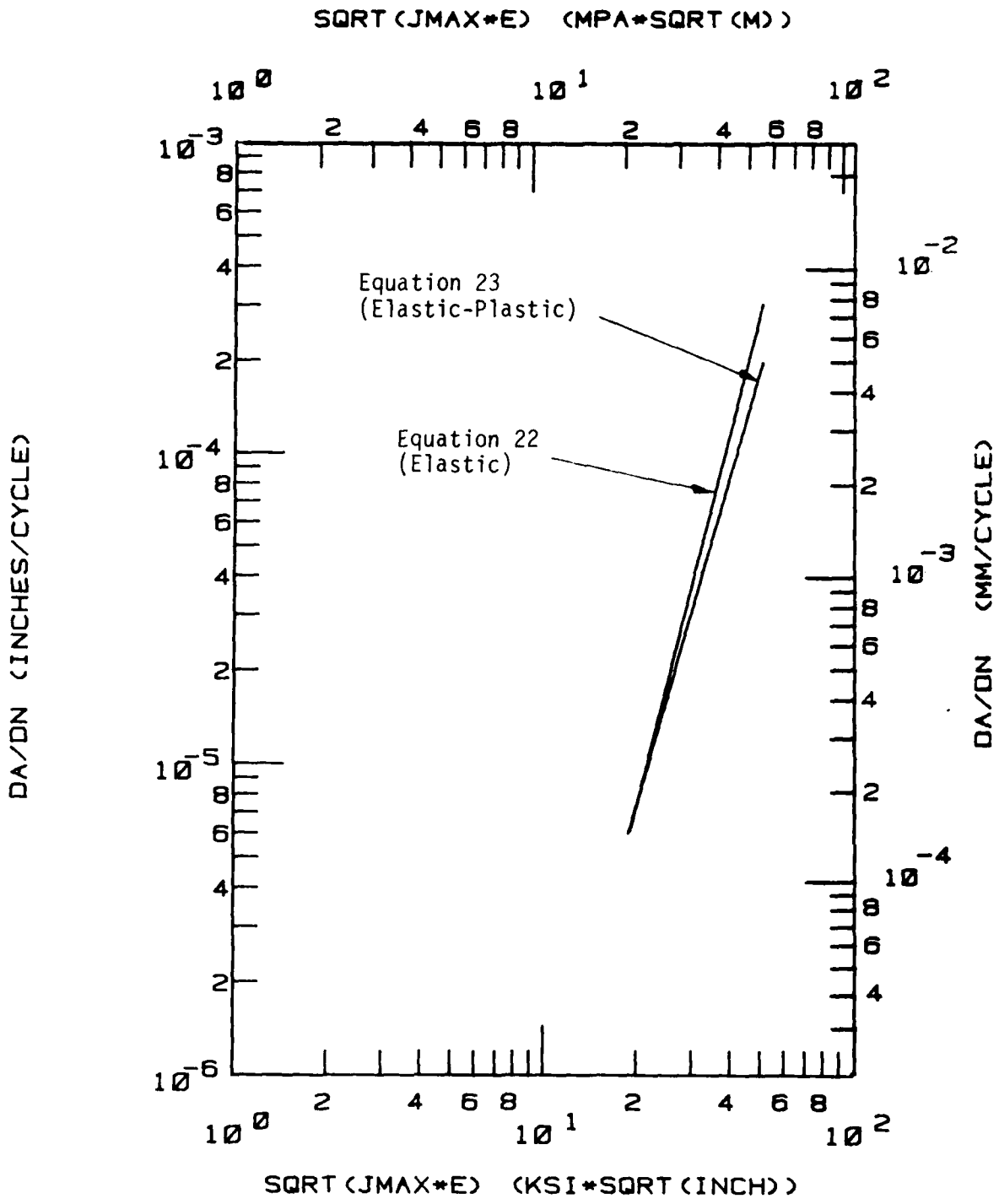


Figure 39. Comparison of the Elastic and (Numerical) Elastic-Plastic Parametric Correlations of the Baseline Fatigue Crack Growth Rate Data.

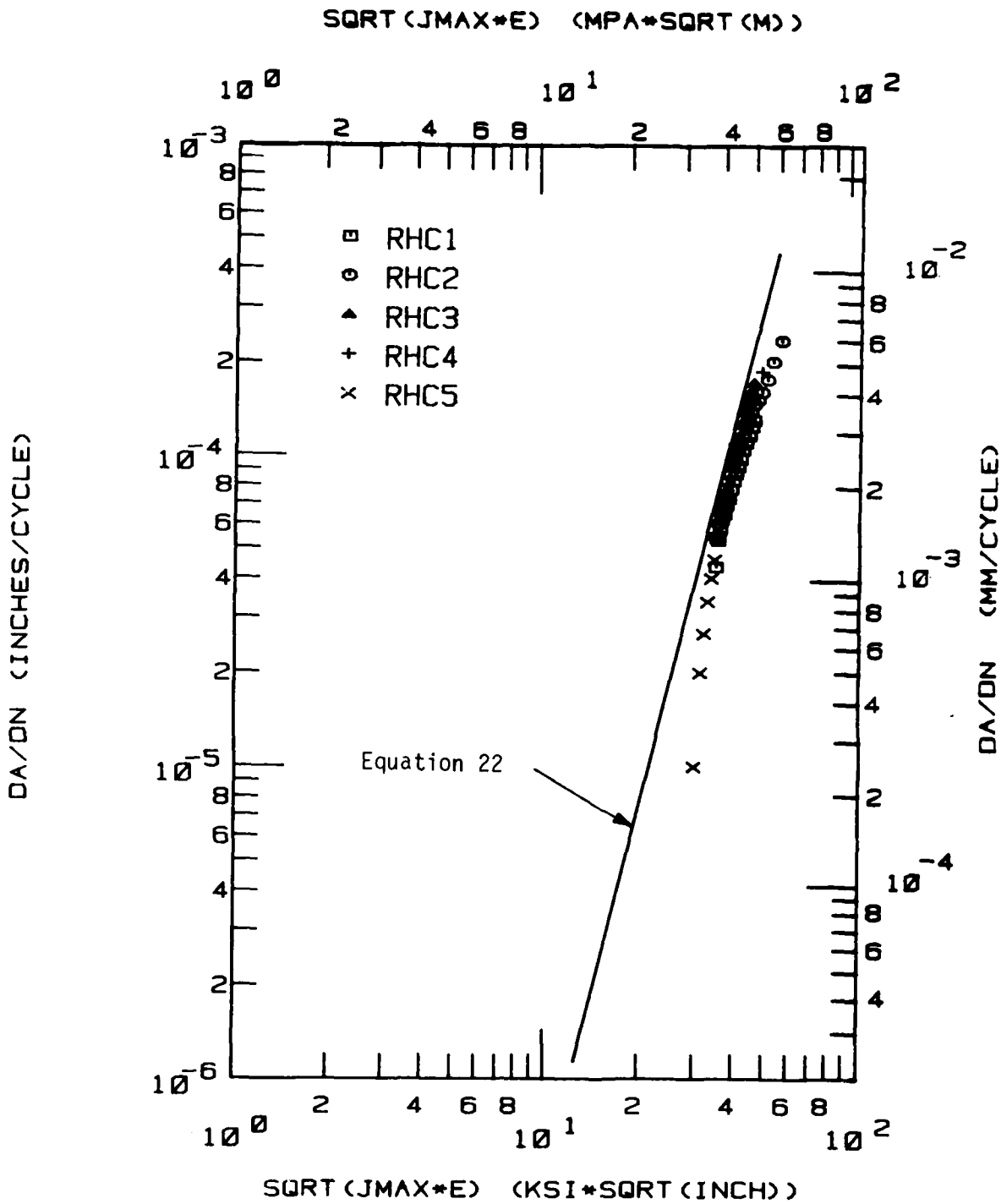


Figure 40. Elastic Baseline Crack Growth Rate Curve Compared to RHC Data Correlated Using Numerical Elastic-Plastic Parameters.

the elastic baseline (Equation 22) is compared to the RHC fatigue crack growth rate data correlated using the numerical elastic-plastic $\sqrt{J_{\max}^T E}$ parameter. Table 4 provides the corresponding life predictions based on this approach. Just how conservative this approach is relative to life can be determined by comparing the life prediction ratios in Table 4 to those in Table 3.

An approach based on an elastic baseline and on an elastic-plastic parameter for the structure was previously suggested by Ratwani et al. ⁽²⁶⁾ for a structural airframe problem. Ratwani et al. studied the crack growth behavior at large semi-circular notches subjected to a substantial amount of localized plasticity and developed a predictive crack growth life analysis based on a J-integral approach. While the application of the J-integral approach was in itself not new, they provided a systematic approach to developing the structural parameter J via analysis and estimating crack growth rate through

$$\frac{da}{dN} = f (K_{\max}^T, R). \quad (25)$$

which was derived using long crack data collected under nominally elastic conditions. For the structural geometry of interest, the parameter of K_{\max}^T was obtained from

$$K_{\max}^T = K_{\max} \left(\frac{J_T}{J_{el}} \right)^{\frac{1}{2}} \quad (26)$$

where K_{\max} is the maximum stress-intensity factor obtained in the traditional way and where the terms J_T AND J_{el} represent the J-integral values obtained from a finite element analysis of the structure assuming elastic-plastic and elastic material behavior, respectively. Figure 41 provides an evaluation of the Ratwani et al. $(J_T/J_{el})^{\frac{1}{2}}$ factor for a crack growing from a 0.50 inch radius semi-circular notch in 7075-T7351. Using their approach, reasonable estimates of the variable amplitude fatigue life data were obtained whereas a purely elastic based approach (elastic baseline and elastic structural parameter) resulted in very non-conservative life predictions.

TABLE 4

LIFE PREDICTIONS USING THE ELASTIC BASELINE DATA AND
THE ELASTIC-PLASTIC PARAMETER $\sqrt{J_e}$

Specimen ID	a_o (mm)	a_f (mm)	N_p Predicted	N/A Actual	(N_p/N_A)
RHC1	0.89	13.77	4790	7441	0.64
RHC2	0.86	13.79	4830	6241	0.77
RHC3	1.14	9.32	2070	2661	0.78
RHC4	1.57	9.42	1390	2501	0.76
RHC5	0.20	15.39	6030	12600	0.48

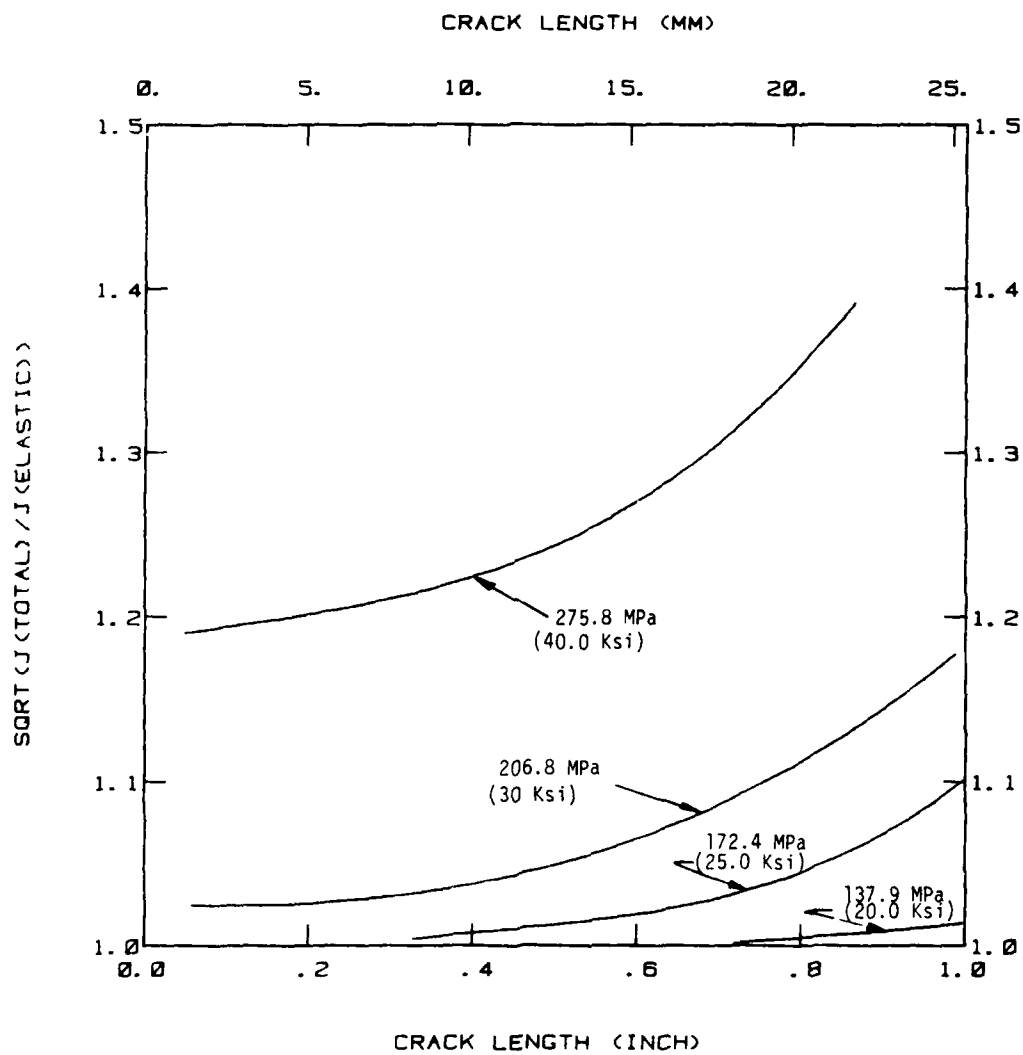


Figure 41. Plastic Correction Factor for 7075-T7351 Aluminum with a Semi-Circular (0.50 Inch Radius) Notch, Derived from Reference 26.

One of the strongest enticements for using the Ratwani, et al. approach is that one can use the current constant amplitude data base as the starting point for a life analysis. However, if one has to compute the J-integral values for every geometry and every material of interest via a finite element procedure, the initial enticement is quickly lost. Shih and his co-workers have noticed this particular drawback and have developed procedures to obtain estimates of J_T for various crack geometries. These estimates are expressed in functional forms that allow one to generalize the finite element results obtained for one material. General Electric recently prepared a report for the Electric Power Research Institute that summarized their work on this technique⁽¹⁵⁾. We have followed the work of Shih and his co-workers since we believe that their methodology is applicable to the subcritical crack growth problem. During an earlier phase of our evaluation, a report was issued which described this approach to estimating the J-integral and other related field parameters⁽²³⁾.

4.2.2 Concerns and Assumptions

During the course of this investigation there were a series of assumptions made relative to the modeling of the material behavior. This paragraph reviews and discusses the assumptions and some of the authors' concerns.

4.2.2.1 Choice of Material

The ETP copper was chosen for the study because it exhibited almost elastic perfectly plastic stress-strain behavior. It was anticipated that this material would exhibit substantial cyclic plasticity throughout the fatigue crack growth test. In particular, we were anticipating that large cyclic plastic zones would be created at the edge of the hole in the RHC specimens. A preliminary set of finite element calculations based on the monotonic stress-strain behavior showed (See Figures 42 and 43) that a substantial amount of plasticity would occur at the hole (uncracked configuration).

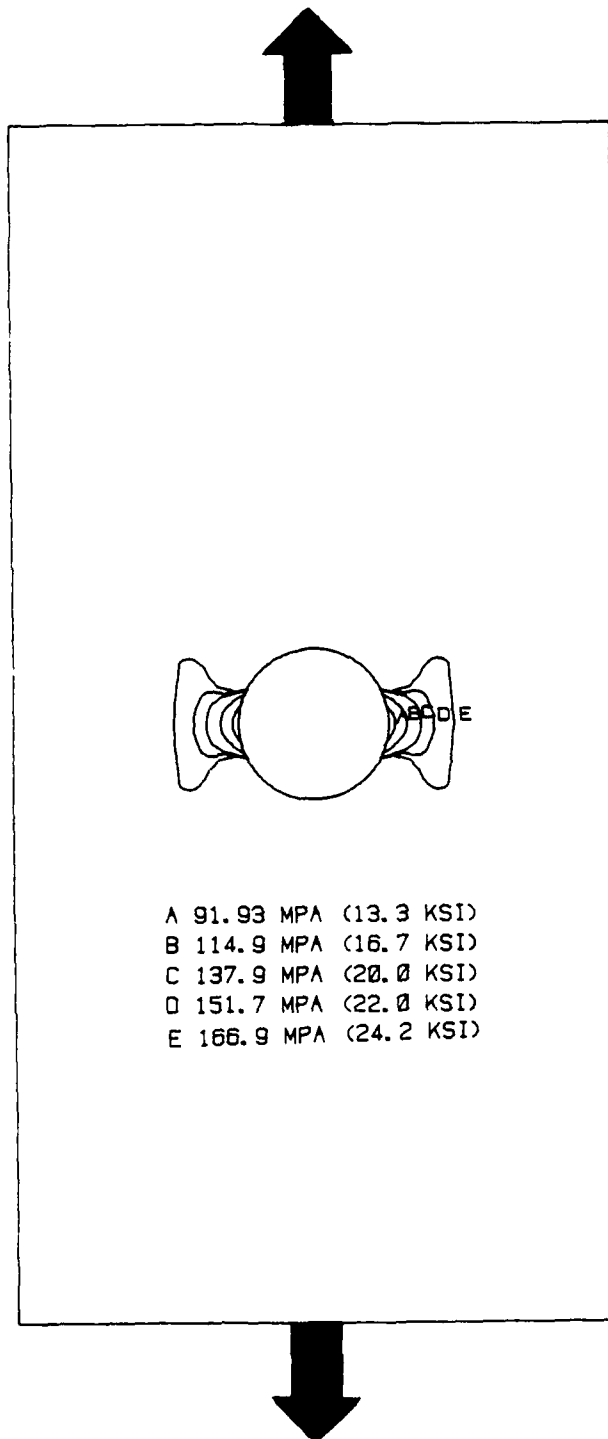


Figure 42. Extent of Plastic Zone Size for a 1-inch Diameter Hole in a 4-inch Wide Copper Plate Subjected to the Loading Indicated (Based on Monotonic Loading).

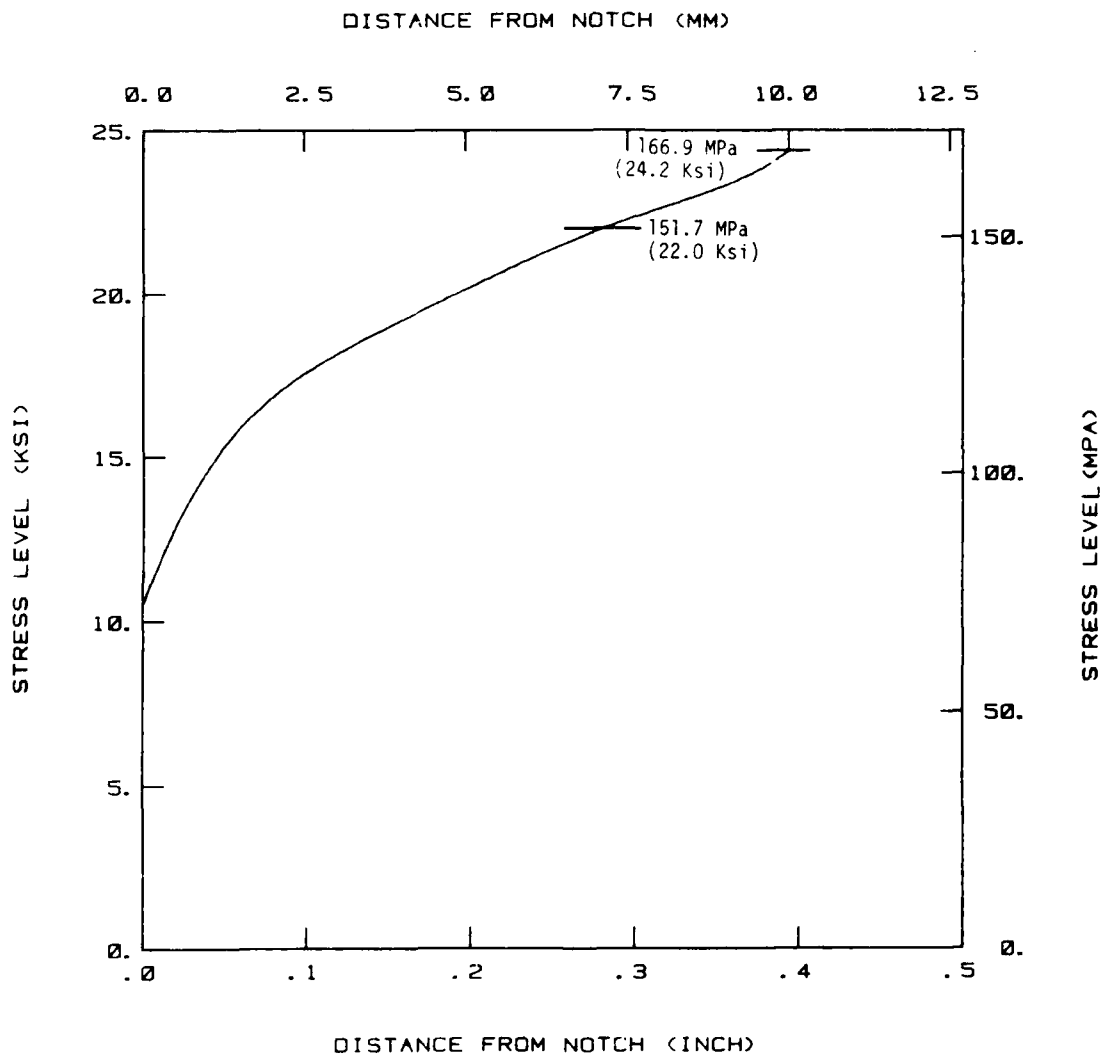


Figure 43. Extent of Plastic Zone Size Along the Expected Path of the Crack (Based on Monotonic Loading).

During the course of experiments, we noted that the material exhibited linear load-displacement behavior for the most part with the exception of test start-up and for crack length conditions approaching critical levels.

In fact, when experiments were performed to monitor the strains at the notch during first and subsequent cyclic loading, we noted that significant reduction of notch strain occurred. The results of this test are shown in Figure 44 for a strain gage located 1.27mm (0.05 inch) from the hole edge. As expected, the cyclic strains were much smaller than the monotonic strains because of the redistribution of stresses following first yielding. The finite element calculations simulated the monotonic strains to within 5 percent of the experimental results but substantially overestimated the strains during cycling.

Despite this overestimation of cyclic plastic strains, the life predictions obtained from the finite element based elastic- $\sqrt{J_{\max}E}$ were good. (Refer to Table 3 for a summary of the life prediction results.) By incorporating the residual stress behavior into the calculations of J_{\max} , the correlation parameter would give a mean trend curve that is bounded by the elastic and elastic-plastic correlations shown in Figure 39.

One additional difficulty worth noting is that due to the toughness of the copper relative to the yield strength, failures occurred by net section yielding rather than by a fracture criteria. This made it difficult to obtain crack growth rate data in the region above 10^{-4} inch/cycle in the center crack panel specimens.

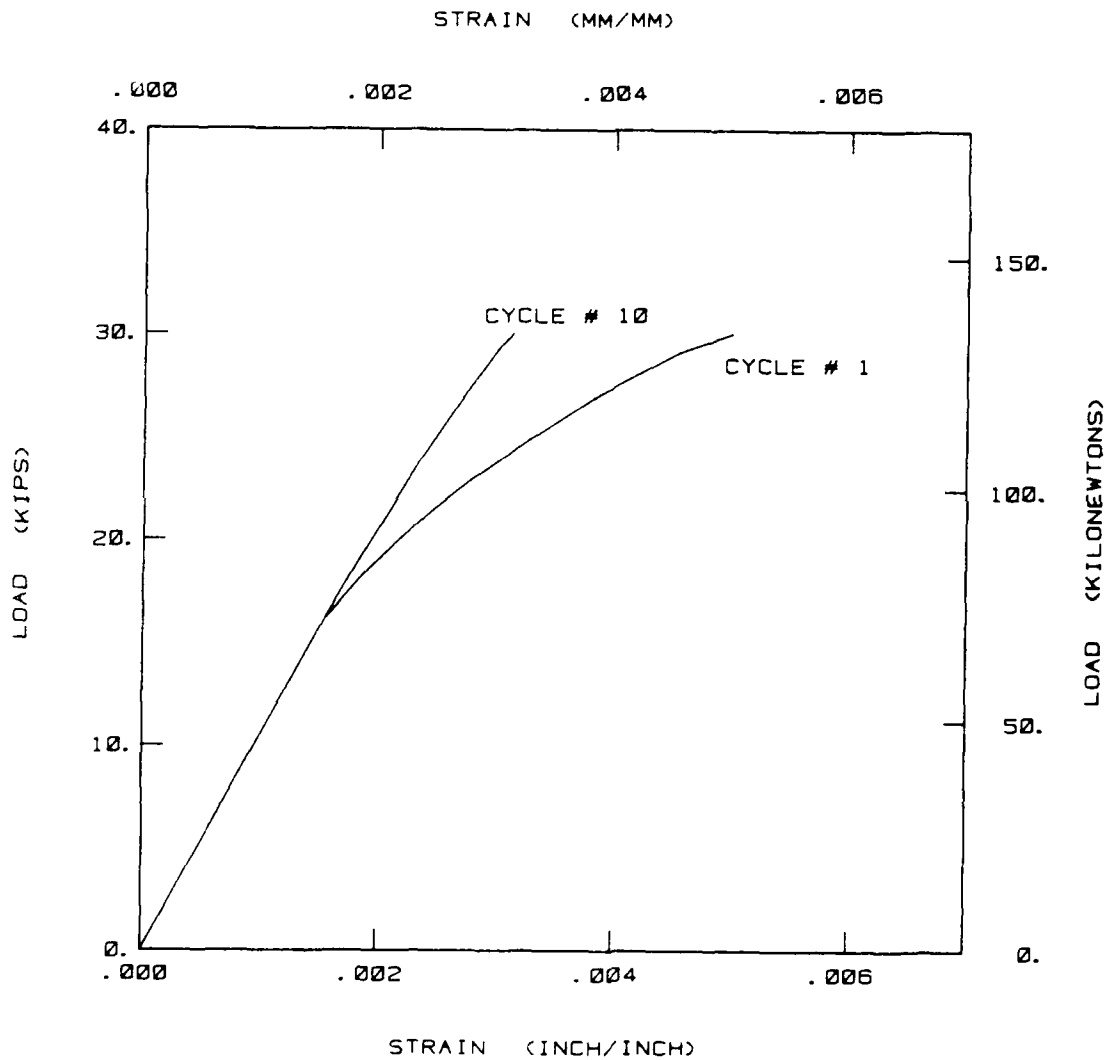


Figure 44. Surface Strains at a Distance of 0.05 Inch (1.27 mm) from the Hole Edge. Strains were Measured with Conventional Foil Strain Gages.

4.2.2.2 Choice of J as a Driving Force

There is one basic, but overriding, assumption made relative to using the J-integral as the crack driving force. This assumption is that the J-integral calculations primarily measure the change in the potential energy of deformation as a function of crack length. As long as the material remains primarily elastic, the assumption is valid. Our concerns are that the load-displacement behavior measured during the cyclic crack extension senses recoverable as well as non-recoverable energy associated with crack extension. Equation 13 was initially based on nonlinear elastic material behavior, and thus only senses the change in recoverable (or stored) energy during the crack extension process. Our concerns were heightened as a result of the RHC test results.

In the RHC tests, there was a non-recoverable cyclic plastic deformation process occurring on both sides of the hole while only the radial crack was extending. We believe that this is the major part of the reason why the crack growth correlations failed for the experimental $\sqrt{J_{\max}} \cdot E$ parameter. Since the experimental method measured non-recoverable (cyclic plasticity) work associated with the uncracked side of the hole, and this non-recoverable work was not associated with the cracking process, it identified a potential problem associated with measurements of non-recoverable work in general. Notwithstanding the previous arguments of Paris ⁽³⁰⁾, Rice ⁽³¹⁾ and Parks ⁽³²⁾ based on steady state crack movements, we are concerned about incorporating the non-recoverable work in the calculations of the driving force without a clear understanding of their contribution to the cracking process.

The above remarks are made even though it has been shown here and elsewhere ^(1, 27-29) that incorporating nonlinear inelastic material behavior for J-integral calculation will provide good crack growth rate correlation when the inelastic behavior is directly associated with the crack tip region as in CT and CCP geometries. While the numerical line integral results were also shown to correlate the RHC geometry cracking rates, it would

be advisable to further investigate the specific contribution that the cyclic non-recoverable work processes have on the fatigue crack growth behavior.

4.3 RECOMMENDATIONS

The following recommendations for additional study are out-growths of difficulties encountered in the present study.

1. Additional experimental J-parameter data should be collected using pin-loaded specimens. This would eliminate potential nonsymmetric loading conditions and provide unambiguous points to measure load point deflection. Any materials chosen for subsequent studies should include consideration of the ratio of toughness to yield strength and cyclic properties.

2. Most of the available elastic-plastic fracture mechanics parameters are in some way related to the localized stress-strain field at the tip of the crack. The magnitude of elastic-plastic parameters in terms of the remote loading and geometrical conditions are heavily influenced by the material properties (described by constitutive models). For application, it will be necessary to generate convenient handbook procedures for calculating the parameters, in the manner suggested by Reference 15.

3. Improved characterization of the fatigue cracking process should be obtained by detailed analytical crack tip studies where various crack tip criteria for crack extension are evaluated. These analytical growth studies should be compared with corresponding fatigue crack growth rate data.

SECTION 5
CONCLUSIONS

The following conclusions have been drawn from analysis of the test data and results from other investigations cited herein:

1. The maximum stress-intensity factor, based on a linear elastic fracture mechanics approach was observed to correlate fatigue crack growth rate behavior reasonably well. Life predictions of the radial hole cracked (RHC) geometry were slightly nonconservative based on crack growth rate data obtained from center cracked panel (CCP) and compact (CT) specimen geometries.

2. The pseudo maximum stress-intensity factor parameter ($\sqrt{J_{\max}} \cdot E$) based on finite element calculations and on the Shih et al. estimation scheme provided a good correlation of the crack growth rate behavior and improved life estimates of the RHC test geometries.

3. The experimentally based pseudo maximum stress-intensity factor parameter $\sqrt{J_{\max}} \cdot E$ correlated the crack growth rate behavior of the CCP and CT geometries but not of the RHC test specimens. No life predictions were made here. The inability of this parameter to correlate the fatigue crack growth rate behavior from the RHC geometry is probably due to the measured inelastic behavior occurring on the non-cracked side of the hole.

4. Coupling an elastically based crack growth rate description with an elastic plastic (pseudo elastic) parameter evaluated for a given structure will result in conservative fatigue crack growth life predictions.

SECTION 6
REFERENCES

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APPENDIX A
CRACK GROWTH DATA

CRACK GROWTH DATA

THE FOLLOWING CRACK GROWTH DATA WERE COLLECTED FROM TESTS OF ETP COPPER SAMPLES AT ROOM TEMPERATURE AND IN LABORATORY AIR. THE FOLLOWING DATA APPLY TO ALL SAMPLES OF THE TYPE LISTED

CCP

WIDTH= 4.00 INCHES

LENGTH= 22.0 INCHES

CRACK LENGTHS LISTED ARE HALF CRACK LENGTHS CALCULATED BY DIVIDING THE TOTAL CRACK LENGTH BY 2.

RCH

WIDTH= 4.00 INCHES

THICKNESS= .375 INCHES

LENGTH= 22.0 INCHES

HOLE DIAMETER= 1.00 INCHES

CRACK LENGTHS LISTED ARE MEASURED FROM THE EDGE OF THE HOLE TO THE CRACK TIP.

CT

WIDTH= 2.00 INCHES

THICKNESS= .500 INCHES

CRACK LENGTHS LISTED ARE MEASURED FROM THE LOAD LINE TO THE CRACK TIP.

1
SPECIMEN ID= CCP2 # POINTS= 12 GROSS SECTION STRESS= 21.3 KSI
THICKNESS= .500 INCHES

CYCLES	A (INCH)
0.	.4970
871.	.5280
1371.	.5360
2051.	.5630
2701.	.5940
3522.	.6380
4061.	.6640
4431.	.6860
4791.	.7120
5081.	.7360
5291.	.7580
5531.	.7850

SPECIMEN ID= CCP3 # POINTS= 22 GROSS SECTION STRESS= 25.0 KSI
THICKNESS= .500 INCHES

CYCLES	A (INCH)
0.	.2610
390.	.2670
1600.	.2850
2190.	.2960
3340.	.3210
4635.	.3450
5510.	.3690
6305.	.3950
7190.	.4240
7730.	.4500
8140.	.4660
8300.	.4740
8700.	.4930
9080.	.5130
9410.	.5330
9720.	.5540
9951.	.5730
10250.	.5960
10560.	.6230
10751.	.6460
10945.	.6670
11088.	.6880

SPECIMEN ID= CCP7 # POINTS= 24 GROSS SECTION STRESS= 18.7 KSI

THICKNESS= .375 INCHES

CYCLES	A (INCH)
95000.	.1280
100000.	.1390
105000.	.1460
110000.	.1500
120000.	.1630
130000.	.1780
140000.	.2030
150000.	.2320
160000.	.2720
170000.	.3280
175000.	.3630
180000.	.4110
185000.	.4700
189000.	.5350
192000.	.6040
194000.	.6630
195000.	.7000
196000.	.7460
197000.	.7980
197500.	.8280
198000.	.8610
198500.	.8990
199000.	.9460
199500.	1.0060

1
SPECIMEN ID= CT1 # POINTS= 21 APPLIED LOAD= 1.689 KIPS

CYCLES	A (INCH)
5957.	.5380
36150.	.5890
47630.	.6120
70210.	.6660
94410.	.7020
93770.	.7340
102540.	.7650
114610.	.8170
119810.	.8450
125050.	.8780
128320.	.9080
133000.	.9520
135000.	.9740
137051.	1.0020
138900.	1.0290
140637.	1.0670
142385.	1.1050
143810.	1.1480
144800.	1.1910
145516.	1.2310
146004.	1.2700

1
SPECIMEN ID= RCH1 # POINTS= 23 GROSS SECTION STRESS= 22.0 KSI

CYCLES	A (INCH)
0.	.0350
1070.	.0600
1790.	.0860
2294.	.1080
2650.	.1310
3071.	.1530
3445.	.1790
3811.	.2000
4201.	.2250
4551.	.2500
4881.	.2760
5181.	.3020
5541.	.3290
5811.	.3540
6051.	.3790
6321.	.4040
6551.	.4290
6761.	.4540
6961.	.4760
7141.	.5010
7302.	.5220
7441.	.5420
7581.	.5610

SPECIMEN ID= RCH2 # POINTS= 27 GROSS SECTION STRESS= 22.0 KSI

CYCLES	A (INCH)
0.	.0340
591.	.0570
1151.	.0810
1641.	.1080
2101.	.1400
2521.	.1610
2861.	.1870
3211.	.2120
3541.	.2390
3831.	.2630
4121.	.2880
4361.	.3140
4631.	.3400
4901.	.3680
5131.	.3930
5371.	.4220
5571.	.4470
5741.	.4730
5921.	.5010
6091.	.5220
6241.	.5430
6381.	.5710
6481.	.5850
6631.	.6180
6723.	.6440
6861.	.6790
6971.	.7250

1

SPECIMEN ID=	CT1	# POINTS=	21	APPLIED LOAD=	1.689 KIPS
CYCLES	A (INCH)				
5957.	.5380				
36150.	.5890				
47630.	.6120				
70210.	.6660				
94410.	.7020				
93770.	.7340				
102540.	.7650				
114610.	.8170				
119810.	.8450				
125050.	.8780				
128320.	.9080				
133000.	.9520				
135000.	.9740				
137051.	1.0020				
138900.	1.0290				
140837.	1.0670				
142385.	1.1050				
143810.	1.1480				
144800.	1.1910				
145516.	1.2310				
146004.	1.2700				

SPECIMEN ID= RCH3	# POINTS= 13	GROSS SECTION STRESS= 24.2 KSI
CYCLES	A (INCH)	
21.	.0450	
451.	.0710	
781.	.0990	
1081.	.1290	
1331.	.1540	
1521.	.1790	
1721.	.2070	
1911.	.2330	
2091.	.2620	
2281.	.2940	
2421.	.3180	
2571.	.3450	
2661.	.3670	

SPECIMEN ID= RCH4	# POINTS= 14	GROSS SECTION STRESS= 24.2 KSI
CYCLES	A (INCH)	
0.	.0620	
31.	.0640	
371.	.0910	
801.	.1290	
1031.	.1560	
1311.	.1930	
1511.	.2200	
1721.	.2490	
1911.	.2790	
2111.	.3120	
2221.	.3280	
2341.	.3490	
2431.	.3710	
2501.	.3960	

SPECIMEN ID= RCH5	# POINTS= 24	GROSS SECTION STRESS= 22.0 KSI
1000.	.0080	
5000.	.0170	
6070.	.0290	
6570.	.0355	
6970.	.0430	
7370.	.0565	
7770.	.0640	
8100.	.0790	
8400.	.0935	
8700.	.1105	
9000.	.1245	
9500.	.1545	
10000.	.1895	
10500.	.2185	
11000.	.2610	
11300.	.2915	
11600.	.3175	
11900.	.3515	
12200.	.3895	
12500.	.4310	
12800.	.4655	
13100.	.5065	
13400.	.5655	
13600.	.6060	

APPENDIX B
LOAD-DEFLECTION PRESENTED FOR GIVEN CRACK LENGTHS

LOAD-DEFLECTION DATA

THE FOLLOWING DATA WERE COLLECTED FROM TESTS ON ETP COPPER. DETAILS OF THE TEST CONDITIONS ARE DESCRIBED IN APPENDIX A.

RCH-5

CURVE #1 A= 0.1245 INCHES

DEFL. LOAD
(INCH) (POUNDS)

.0 0.
.00104 2915.
.00237 6655.
.0037 10395.
.00511 14195.
.00687 19195.
.00871 24195.
.01055 29170.
.01079 29645.

CURVE #2 A= 0.1545 INCHES

DEFL. LOAD
(INCH) (POUNDS)

.0 0.
.00146 4120.
.00324 9125.
.00502 14130.
.00687 19135.
.00873 24140.
.01066 29145.
.01084 29605.

CURVE #3 A= 0.1895 INCHES

DEFL. LOAD
(INCH) (POUNDS)

.0 0.
.00141 4090.
.00322 9080.
.00506 14100.
.00686 19090.
.00876 24080.
.01087 29590.

CURVE #4 A= 0.2185 INCHES

DEFL. LOAD
(INCH) (POUNDS)

.0 0.
.0015 4085.
.00323 9070.
.00504 14125.
.00692 19100.
.0088 24110.
.01097 29575.

CURVE #5 A= 0.261 INCHES

DEFL. LOAD
(INCH) (POUNDS)

.0 0.
.00148 4045.
.00326 9060.
.00504 14040.
.00697 19065.
.00883 24065.
.01096 29550.

CURVE #6 A= 0.2915 INCHES
DEFL. LOAD
(INCH) (POUNDS)
.0 0.
.00149 4055.
.00328 9060.
.00516 14065.
.00693 19070.
.00891 24075.
.01106 29550.

CURVE #7 A= 0.3175 INCHES
DEFL. LOAD
(INCH) (POUNDS)
.0 0.
.00139 3995.
.00318 9015.
.0051 14045.
.00692 19000.
.00884 24020.
.01106 29520.

CURVE #8 A= 0.3515 INCHES
DEFL. LOAD
(INCH) (POUNDS)
.0 0.
.00146 4025.
.00324 9020.
.0051 14015.
.00692 19010.
.00893 24045.
.0109 29015.
.01111 29510.

CURVE #9 A= 0.3895 INCHES
DEFL. LOAD
(INCH) (POUNDS)
.0 0.
.00148 4005.
.00332 9040.
.00515 14035.
.00706 19030.
.00903 24025.
.01124 29495.

CURVE #10 A= 0.431 INCHES
DEFL. LOAD
(INCH) (POUNDS)
.0 0.
.00143 4045.
.00333 9070.
.00516 14065.
.00705 19060.
.00906 24055.
.0113 29535.

CURVE #11 A= 0.4655 INCHES

DEFL. LOAD
(INCH) (POUNDS)

.0 0.
.00148 4035.
.00331 9020.
.00525 14030.
.00713 19040.
.00916 24050.
.0115 29515.

CURVE #12 A= 0.5065 INCHES

DEFL. LOAD
(INCH) (POUNDS)

.0 0.
.00148 4080.
.00338 9095.
.0052 14110.
.00721 19125.
.00922 24105.
.01161 29595.

CURVE #13 A=0.5655 INCHES

DEFL. LOAD
(INCH) (POUNDS)

.0 0.
.00157 4135.
.00346 9145.
.00543 14155.
.00737 19140.
.00944 24160.
.01195 29695.

CURVE #14 A=0.606 INCHES

DEFL. LOAD
(INCH) (POUNDS)

.0 0.
.00158 4155.
.00351 9185.
.00548 14195.
.0075 19205.
.00958 24185.
.01221 29655.

07-1

CURVE #1 A= 0.538 INCHES

DEFL. (INCH)	LOAD (POUNDS)
.00000	0.
.00011	46.
.00022	92.
.00034	138.
.00047	184.
.00055	230.
.00066	277.
.00078	323.
.00089	369.
.00099	415.
.00110	461.
.00123	507.
.00136	553.
.00146	599.
.00159	645.
.00172	691.
.00181	737.
.00193	783.
.00207	830.
.00216	876.
.00231	922.
.00241	968.
.00254	1014.
.00267	1060.
.00280	1106.
.00290	1152.
.00302	1198.
.00315	1244.
.00328	1290.
.00341	1336.
.00354	1383.
.00365	1429.
.00377	1475.
.00388	1519.

CURVE #2 A= 0.589 INCHES

DEFL. (INCH)	LOAD (POUNDS)
.00000	0.
.00011	48.
.00024	96.
.00037	144.
.00050	192.
.00064	240.
.00076	288.
.00089	336.
.00102	384.
.00117	432.
.00129	481.
.00141	529.
.00155	577.
.00167	625.
.00180	673.
.00193	721.
.00208	769.
.00220	817.
.00235	865.
.00248	913.
.00260	961.
.00275	1009.
.00286	1057.
.00299	1105.
.00313	1153.
.00326	1201.
.00340	1249.
.00353	1297.
.00367	1345.
.00383	1393.
.00397	1442.
.00410	1490.
.00420	1520.

CURVE #3 A= 0.612 INCHES

DEFL. (INCH)	LOAD (POUNDS)
.00000	0.
.00011	46.
.00024	91.
.00037	137.
.00050	182.
.00063	228.
.00076	273.
.00089	319.
.00102	364.
.00115	410.
.00128	455.
.00141	501.
.00153	546.
.00166	592.
.00179	637.
.00192	683.
.00205	728.
.00218	774.
.00231	819.
.00244	865.
.00259	910.
.00271	956.
.00283	1001.
.00297	1047.
.00310	1093.
.00324	1138.
.00337	1184.
.00351	1229.
.00364	1275.
.00378	1320.
.00392	1366.
.00406	1411.
.00420	1457.
.00433	1502.
.00440	1520.

CURVE #4 A= 0.566 INCHES

DEFL. (INCH)	LOAD (POUNDS)
.00000	0.
.00014	50.
.00030	101.
.00046	151.
.00062	201.
.00078	251.
.00094	302.
.00110	352.
.00126	402.
.00142	453.
.00158	503.
.00174	553.
.00190	604.
.00206	654.
.00220	704.
.00237	754.
.00251	805.
.00267	855.
.00284	905.
.00300	956.
.00317	1006.
.00333	1056.
.00350	1107.
.00366	1157.
.00383	1207.
.00399	1257.
.00416	1308.
.00432	1358.
.00449	1408.
.00465	1459.
.00482	1509.
.00495	1519.

CURVE #5 A= 0.702 INCHES

DEFL. (INCH)	LOAD (POUNDS)
.00000	0.
.00017	51.
.00033	102.
.00049	153.
.00069	203.
.00084	254.
.00103	305.
.00121	356.
.00139	407.
.00157	458.
.00175	509.
.00189	559.
.00207	610.
.00225	661.
.00243	712.
.00261	763.
.00277	814.
.00293	865.
.00313	915.
.00330	966.
.00350	1017.
.00367	1068.
.00383	1119.
.00402	1170.
.00421	1221.
.00439	1272.
.00456	1322.
.00474	1373.
.00491	1424.
.00509	1475.
.00526	1520.

CURVE #6 A= 0.734 INCHES

DEFL. (INCH)	LOAD (POUNDS)
.00000	0.
.00016	47.
.00033	93.
.00050	140.
.00066	186.
.00083	233.
.00101	279.
.00118	326.
.00135	372.
.00152	419.
.00169	465.
.00186	512.
.00203	558.
.00220	605.
.00237	651.
.00254	698.
.00271	744.
.00288	791.
.00305	837.
.00322	884.
.00339	930.
.00356	977.
.00373	1023.
.00390	1070.
.00407	1116.
.00425	1163.
.00444	1209.
.00461	1256.
.00478	1302.
.00495	1349.
.00512	1395.
.00530	1442.
.00549	1488.
.00560	1520.

CURVE #7 A= 0.765 INCHES

DEFL. (INCH)	LOAD (POUNDS)
.00000	0.
.00024	61.
.00048	122.
.00072	183.
.00096	244.
.00120	305.
.00144	366.
.00168	427.
.00189	488.
.00214	549.
.00239	610.
.00262	671.
.00285	732.
.00308	793.
.00334	854.
.00358	915.
.00381	976.
.00406	1037.
.00429	1098.
.00455	1159.
.00479	1220.
.00504	1281.
.00528	1341.
.00553	1402.
.00578	1463.
.00604	1520.

CURVE #8 A= 0.817 INCHES

DEFL. (INCH)	LOAD (POUNDS)
.00000	0.
.00024	64.
.00053	127.
.00081	193.
.00109	260.
.00137	323.
.00165	391.
.00193	455.
.00221	521.
.00249	587.
.00277	653.
.00305	715.
.00333	777.
.00361	839.
.00389	904.
.00417	968.
.00445	1036.
.00473	1096.
.00501	1163.
.00529	1224.
.00557	1282.
.00585	1349.
.00613	1407.
.00641	1467.
.00663	1520.

CURVE #9 A= 0.845 INCHES	
DEFL. (INCH)	LOAD (POUNDS)
.00000	0.
.00020	51.
.00039	92.
.00059	134.
.00078	175.
.00098	219.
.00117	262.
.00137	306.
.00156	346.
.00176	392.
.00195	437.
.00215	476.
.00234	515.
.00254	559.
.00273	601.
.00293	643.
.00312	689.
.00332	730.
.00351	770.
.00371	814.
.00390	856.
.00410	903.
.00429	937.
.00449	980.
.00468	1021.
.00488	1061.
.00507	1102.
.00527	1142.
.00546	1183.
.00566	1223.
.00585	1264.
.00605	1304.
.00624	1345.
.00644	1388.
.00663	1429.
.00683	1465.
.00702	1506.
.00709	1520.

CURVE #10 A= 0.878 INCHES

DEFL. (INCH)	LOAD (POUNDS)
.00000	0.
.00024	50.
.00047	98.
.00071	145.
.00094	192.
.00118	240.
.00141	285.
.00165	330.
.00188	380.
.00212	429.
.00235	474.
.00259	523.
.00282	571.
.00306	615.
.00329	664.
.00353	713.
.00376	755.
.00400	803.
.00423	850.
.00447	897.
.00470	944.
.00494	992.
.00517	1036.
.00541	1081.
.00564	1125.
.00588	1170.
.00611	1218.
.00635	1265.
.00658	1308.
.00682	1358.
.00705	1401.
.00729	1447.
.00752	1490.
.00767	1521.

CURVE #11 A= 0.908 INCHES	
DEFL. (INCH)	LOAD (POUNDS)
.00000	0.
.00025	49.
.00049	95.
.00074	141.
.00098	188.
.00123	234.
.00147	280.
.00172	325.
.00196	373.
.00221	420.
.00245	469.
.00270	514.
.00294	559.
.00319	604.
.00343	652.
.00368	700.
.00392	745.
.00417	791.
.00441	837.
.00466	882.
.00490	928.
.00515	973.
.00539	1019.
.00564	1066.
.00588	1111.
.00613	1157.
.00637	1202.
.00662	1250.
.00686	1293.
.00711	1341.
.00735	1383.
.00760	1429.
.00784	1472.
.00811	1520.

CURVE #12 A= 0.952 INCHES

DEFL. (INCH)	LOAD (POUNDS)
.00000	0.
.00027	47.
.00054	94.
.00081	137.
.00108	183.
.00135	231.
.00162	272.
.00189	319.
.00216	367.
.00243	416.
.00270	465.
.00297	514.
.00324	558.
.00351	599.
.00378	646.
.00405	693.
.00432	742.
.00459	790.
.00489	837.
.00518	889.
.00548	941.
.00577	989.
.00607	1040.
.00636	1088.
.00666	1136.
.00695	1190.
.00725	1237.
.00754	1288.
.00784	1337.
.00813	1385.
.00843	1433.
.00872	1479.
.00897	1521.

CURVE #13 A= 0.974 INCHES

DEFL. (INCH)	LOAD (POUNDS)
.00000	0.
.00043	69.
.00086	138.
.00129	207.
.00172	276.
.00215	351.
.00258	421.
.00301	494.
.00344	563.
.00387	633.
.00430	702.
.00473	771.
.00516	842.
.00559	911.
.00602	980.
.00645	1049.
.00688	1118.
.00731	1187.
.00774	1256.
.00817	1322.
.00860	1389.
.00903	1455.
.00948	1520.

CURVE #14 A= 1.002 INCHES

DEFL. (INCH)	LOAD (POUNDS)
.00000	0.
.00039	58.
.00077	114.
.00116	174.
.00154	234.
.00193	292.
.00231	352.
.00270	413.
.00308	470.
.00347	529.
.00385	589.
.00424	648.
.00462	708.
.00501	763.
.00539	823.
.00578	883.
.00616	941.
.00655	998.
.00693	1058.
.00732	1115.
.00770	1169.
.00809	1228.
.00847	1286.
.00886	1341.
.00924	1397.
.00963	1452.
.01001	1508.
.01010	1521.

CURVE #15 A= 1.029 INCHES

DEFL. (INCH)	LOAD (POUNDS)
.00000	0.
.00036	54.
.00073	105.
.00110	157.
.00149	209.
.00182	259.
.00216	308.
.00249	354.
.00283	404.
.00316	451.
.00350	501.
.00383	550.
.00417	597.
.00450	647.
.00484	694.
.00517	742.
.00551	789.
.00584	834.
.00618	885.
.00651	929.
.00685	976.
.00718	1024.
.00752	1071.
.00785	1118.
.00819	1163.
.00852	1211.
.00886	1256.
.00919	1301.
.00953	1347.
.00986	1392.
.01020	1437.
.01053	1481.
.01082	1520.

CURVE #16 A= 1.067 INCHES

DEFL. (INCH)	LOAD (POUNDS)
.00000	0.
.00037	48.
.00074	95.
.00111	143.
.00148	191.
.00185	240.
.00222	288.
.00259	337.
.00296	385.
.00333	434.
.00370	483.
.00407	533.
.00444	580.
.00481	630.
.00518	676.
.00555	727.
.00592	773.
.00629	823.
.00666	872.
.00703	916.
.00740	966.
.00777	1014.
.00814	1060.
.00851	1107.
.00888	1153.
.00925	1199.
.00962	1246.
.00999	1292.
.01036	1336.
.01073	1380.
.01110	1426.
.01147	1470.
.01189	1520.

CURVE #17 A= 1.105 INCHES

DEFL. (INCH)	LOAD (POUNDS)
.00000	0.
.00045	52.
.00089	105.
.00134	157.
.00178	207.
.00223	262.
.00267	315.
.00312	368.
.00356	422.
.00401	476.
.00445	529.
.00490	583.
.00534	635.
.00579	689.
.00623	743.
.00668	796.
.00712	847.
.00757	899.
.00801	950.
.00846	1002.
.00890	1055.
.00935	1104.
.00979	1156.
.01024	1205.
.01068	1257.
.01113	1305.
.01157	1355.
.01202	1406.
.01246	1454.
.01291	1502.
.01305	1520.

CURVE #18 A= 1.148 INCHES

DEFL. (INCH)	LOAD (POUNDS)
.00000	0.
.00046	47.
.00091	94.
.00137	142.
.00182	189.
.00228	238.
.00273	286.
.00319	335.
.00364	384.
.00410	433.
.00455	481.
.00501	528.
.00546	578.
.00592	624.
.00637	675.
.00683	723.
.00728	769.
.00774	819.
.00819	869.
.00865	913.
.00910	959.
.00956	1005.
.01001	1052.
.01047	1098.
.01092	1144.
.01138	1190.
.01183	1234.
.01229	1281.
.01274	1327.
.01320	1372.
.01365	1414.
.01411	1458.
.01456	1504.
.01474	1520.

CURVE #19 A= 1.191 INCHES

DEFL. (INCH)	LOAD (POUNDS)
.00000	0.
.00059	54.
.00118	108.
.00177	164.
.00236	219.
.00295	275.
.00354	330.
.00413	388.
.00472	443.
.00531	500.
.00590	557.
.00649	611.
.00708	669.
.00767	725.
.00826	777.
.00885	834.
.00944	889.
.01003	943.
.01062	996.
.01121	1050.
.01180	1101.
.01239	1156.
.01298	1205.
.01357	1260.
.01416	1310.
.01475	1362.
.01534	1412.
.01593	1464.
.01663	1521.

CURVE #20 A= 1.231 INCHES

DEFL. (INCH)	LOAD (POUNDS)
.00000	0.
.00059	49.
.00117	97.
.00176	146.
.00234	196.
.00293	247.
.00351	294.
.00410	345.
.00468	394.
.00527	444.
.00585	494.
.00644	544.
.00702	594.
.00761	643.
.00819	692.
.00878	742.
.00936	790.
.00995	838.
.01053	887.
.01112	935.
.01170	981.
.01229	1029.
.01287	1075.
.01346	1121.
.01404	1167.
.01463	1212.
.01521	1260.
.01580	1304.
.01638	1348.
.01697	1391.
.01755	1436.
.01814	1478.
.01875	1521.

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CURVE #1 A= 0.261 INCHES

DEFL. LOAD
(INCH) (POUNDS)

.0 0.
.000555 4980.
.0011475 9960.
.001755 15005.
.002385 20050.
.0029975 25050.
.0036325 30050.
.00429 35050.
.0049475 40050.
.00565 45025.

CURVE #2 A= 0.285 INCHES

DEFL. LOAD
(INCH) (POUNDS)

.0 0.
.0005525 4935.
.0011625 9960.
.001755 14935.
.0023725 19910.
.0029975 24960.
.0036425 29965.
.004305 34970.
.00494 39975.
.0056475 44980.

CURVE #3 A= 0.296 INCHES

DEFL. LOAD
(INCH) (POUNDS)

.0 0.
.00058 4990.
.001185 10030.
.00179 15020.
.002415 20010.
.00304 25000.
.003665 29990.
.004325 34980.
.004985 40020.
.0056825 45060.

CURVE #4 A= 0.321 INCHES

DEFL. LOAD
(INCH) (POUNDS)

.0 0.
.00058 5015.
.0011825 10030.
.0018075 15045.
.002425 20010.
.0030425 24975.
.0036975 29995.
.0043525 35015.
.00502 40035.
.005735 45055.

CURVE #5 A= 0.345 INCHES

DEFL. LOAD
(INCH) (POUNDS)

.0 0.
.000595 4995.
.00121 9990.
.0018325 15010.
.002475 20030.
.0031175 25050.
.00376 30005.
.00442 34960.
.00511 39980.
.005815 45000.

CURVE #6 A= 0.369 INCHES

DEFL. LOAD
(INCH) (POUNDS)

.0 0.
.0006 5010.
.00123 10020.
.00186 15030.
.00249 20040.
.00315 25050.
.00381 30060.
.00447 35070.
.00516 40080.
.0058875 45090.

CURVE #7 A= 0.395 INCHES

DEFL. LOAD
(INCH) (POUNDS)

.0 0.
.0006025 4975.
.00124 10005.
.00187 14990.
.00252 19975.
.00317 24960.
.003845 29995.
.0045425 35030.
.0052175 40000.
.0059675 44970.

CURVE #8 A= 0.424 INCHES

DEFL. LOAD
(INCH) (POUNDS)

.0 0.
.00063 5005.
.00126 10010.
.0019025 15015.
.0025625 20020.
.0032375 25025.
.0039125 30030.
.0046075 35035.
.0053025 40040.
.00606 45045.

CURVE #9 A= 0.450 INCHES
 DEFL. LOAD
 (INCH) (POUNDS)
 .0 0.
 .0006225 5000.
 .001245 10000.
 .0018975 15000.
 .00257 20000.
 .003225 25000.
 .00391 30000.
 .0046225 35000.
 .005345 40000.
 .0061175 45000.

CURVE #10 A= 0.466 INCHES
 DEFL. LOAD
 (INCH) (POUNDS)
 .0 0.
 .0006325 5030.
 .0012875 10060.
 .0019425 15050.
 .002615 20040.
 .0032875 25030.
 .0039775 30020.
 .004695 35010.
 .0054125 40000.
 .0061775 44990.

CURVE #11 A= 0.474 INCHES
 DEFL. LOAD
 (INCH) (POUNDS)
 .0 0.
 .00061 4995.
 .0012725 9990.
 .0019225 14985.
 .0025925 19980.
 .003285 24975.
 .003975 30005.
 .0046925 35035.
 .0054325 40065.
 .00619 45005.

CURVE #12 A= 0.493 INCHES
 DEFL. LOAD
 (INCH) (POUNDS)
 .0 0.
 .000645 5000.
 .00129 10000.
 .0019675 15000.
 .002645 20000.
 .00334 25000.
 .004035 30000.
 .00475 35000.
 .005485 40000.
 .0062625 45000.

CURVE #13 A= 0.513 INCHES

DEFL. LOAD
(INCH) (POUNDS)

.0 0.
.0006475 5030.
.0013225 10060.
.0019975 15090.
.0026825 20045.
.0033675 25035.
.0040825 30025.
.00481 35015.
.005555 40005.
.0063525 44995.

CURVE #14 A= 0.533 INCHES

DEFL. LOAD
(INCH) (POUNDS)

.0 0.
.000645 4995.
.0013275 9990.
.00199 14985.
.00269 19980.
.00339 24975.
.004105 29970.
.0048475 34965.
.00562 39960.
.006435 44960.

CURVE #15 A= 0.554 INCHES

0.554
(INCH) (POUNDS)

.0 0.
.0006425 5055.
.0013125 10020.
.002005 14985.
.0026975 19950.
.0034125 24995.
.00417 30040.
.004915 35020.
.005695 40000.
.0065225 45045.

CURVE #16 A= 0.573 INCHES

DEFL. LOAD
(INCH) (POUNDS)

.0 0.
.0006824999999999 4980.
.0013325 9960.
.0020475 15020.
.002745 20000.
.0034675 24980.
.0042125 29960.
.0049575 34940.
.0057525 39950.
.0066175 44980.

CURVE #17 A= 0.596 INCHES
 DEFL. LOAD
 (INCH) (POUNDS)

.0	0.
.0006775	5070.
.0013675	10015.
.002085	15025.
.0028025	20035.
.00354	24995.
.004285	30010.
.005065	35025.
.005875	40040.
.0067525	45055.

CURVE #18 A= 0.623 INCHES
 DEFL. LOAD
 (INCH) (POUNDS)

.0	0.
.0006925	4995.
.001385	9990.
.0021225	14985.
.00286	19980.
.0035975	24975.
.00437	29970.
.00517	34965.
.00599	39960.
.0068675	44955.

CURVE #19 A= 0.646 INCHES
 DEFL. LOAD
 (INCH) (POUNDS)

.0	0.
.0006925	5025.
.0014225	10050.
.0021175	14975.
.00288	19960.
.003615	24945.
.004415	29930.
.005215	34980.
.0060525	39970.
.0069725	44960.

DEFL. LOAD