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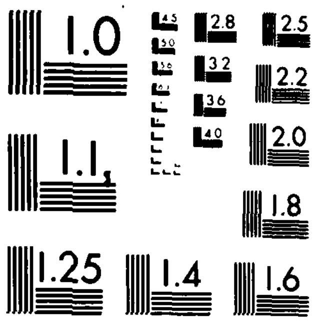

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LECTURES ON MATHEMATICAL COMBUSTION

Lecture 7: Pulsating Flames

Technical Report No. 152

J.D. Buckmaster & G.S.S. Ludford

January 1983

Theoretical and Applied Mechanics

Thurston Hall

Ithaca, New York

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Contents

Lecture 7

PULSATING FLAMES

In section 5. it was found that plane NEFs of sufficiently large Lewis number are unstable. Since $\mathrm{Im}(\alpha) \neq 0$ on the stability boundary, the instability is likely to result in either a pulsating flame or a flame that supports traveling waves. Such flames are the subject of this lecture.

One difficulty that immediately confronts us is that, in contrast to the ubiquitous nature of cellular instabilities, pulsating instabilities are not ordinarily seen. The reason seems to be the large values of L are needed; according to the theory, $Y_f \ell / T_b^2$ must exceed 32/3 (or $4(1+\sqrt{3})$ if the disturbances are one-dimensional). There is evidence that fuel-rich hydrogen-bromine mixtures might attain such values since Golovichev, Grishin, Agranat and Bertsun (1978) obtained oscillations in a numerical study, but there is no similar evidence for more commonplace gas mixtures.

For this reason we must turn from the commonplace and deal either with unusual combustible materials or else with special configurations, in order to uncover pulsating flames. Our discussion will start with thermites, which are solids that burn to form solids (a phenomenon that is appropriately called gasless combustion). There is no significant diffusion of mass, so that L is effectively infinite.

1. Solid Combustion

Experiments by Merzhanov, Filonenko, and Borovinskaya on thermites composed of niobium and boron revealed pulsating instabilities, as did numerical solutions obtained by Shkadinsky, Khaikin, and Merzhanov. The latter examined the equations

$$\partial T/\partial t - \partial^2 T/\partial x^2 = -\partial Y/\partial t = \Omega \tag{1}$$

where

$$\Omega = \mathcal{D} Y e^{-\theta/T} \quad \text{with } \mathcal{D} = DM_r^2, \tag{2}$$

and uncovered a critical value θ_c of the activation energy: for $\theta < \theta_c$ the propagation is steady, but for $\theta > \theta_c$ only pulsating propagation is seen. The prediction of pulsating propagation is consistent with the NEF analysis in section 5. , where oscillatory instability was found for sufficiently large L in the limit $\theta \to \infty$; but activation-energy asymptotics has nothing to say about a phenomenon (here the switch to steady propagation) occuring at some finite value of θ. Even though it is not observed either experimentally or numerically for large enough θ, a steady wave can nevertheless be constructed by means of activation-energy asymptotics; we shall start out discussion by doing so.

The following boundary-value problem presents itself in a frame moving with the flame sheet:

$$dT/dx - d^2T/dx^2 = -dY/dx = \mathcal{D} Y e^{-\theta/T}, \tag{3}$$

$$T \to T_f,\ Y \to T_f \text{ as } x \to -\infty,\ T \text{ bounded},\ Y \to 0 \text{ as } x \to +\infty. \tag{4}$$

The solution outside the flame sheet is

$$T = \begin{cases} T_f + Y_f e^x \\ T_b = H_f \end{cases}, \qquad Y = \begin{cases} Y_f \\ 0 \end{cases} \quad \text{for } x \lessgtr 0; \tag{5}$$

T is continuous across the flame sheet, as for the finite-L problem (cf. section 2.), but Y jumps. As a consequence, the structure of the reaction zone is somewhat different.

The structure variable is, as usual,

$$\xi = \theta x \tag{6}$$

and

$$T_0 = T_b; \tag{7}$$

but we now find the equations

$$T_b^2 d^2\phi/d\xi^2 = -dY_o/d\xi = \tilde{\mathcal{D}} Y_o e^{-\phi} \text{ with } \tilde{\mathcal{D}} = \mathcal{D} e^{-\theta/T_b}/\theta \tag{8}$$

for the temperature perturbation $T_1 = -T_b^2\phi$ and the leading term Y_o in the mass-fraction expansion. Since both ϕ and Y_o vanish as $\xi \to +\infty$, we have

$$Y_o = -T_b^2 d\phi/d\xi, \tag{9}$$

so that only the temperature equation

$$d^2\phi/d\xi^2 = -\tilde{\mathcal{D}}(d\phi/d\xi)e^{-\phi} \tag{10}$$

remains. To match with the solution outside the flame sheet, the usual boundary conditions

$$\phi = -Y_f\xi/T_b^2 + o(1) \text{ as } \xi \to -\infty, \ \phi = o(1) \text{ as } \xi \to +\infty \tag{11}$$

must be applied. The first integral

$$d\phi/d\xi = \tilde{\mathcal{D}}(e^{-\phi}-1) \tag{12}$$

of equation (10) is obtained by using the boundary condition (11b); then the boundary condition (11a) leads to the burning rate

$$M_r = \sqrt{D}\, T_b e^{-\theta/2T_b}/\sqrt{Y_f\theta}. \tag{13}$$

This result has apparently not been written down before, but others have derived it (e.g. Peters 1982).

2. The Delta-Function Model

We have already noted that there is little point in investigating the stability of this solution using activation-energy asymptotics. Instead, following Matkowsky and Sivashinsky, we shall use a delta-function model suggested by the asymptotics above. Thus, the strength of the delta function that replaces the Arrhenius term will be defined so that the mass flux through the flame sheet, in quite general circumstances, is

$$M = \sqrt{D}T_* e^{-\theta/2T_*}/\sqrt{Y_f\theta}; \tag{14}$$

here T_* is the flame temperature. The dimensionless mass flux is then

$$M/M_r = (T_*/T_b)\exp[\theta(T_*-T_b)/2T_bT_*], \tag{15}$$

an expression that will be simplified before use. For θ large (but not necessarily infinite), the preexponential factor is not significant and can be replaced by 1; in addition, for small deviations of T_* from T_b (such as occur in a linear stability analysis) the exponent can be replaced by $\theta(T_*-T_b)/2T_b^2$. The formula (15) then becomes

$$W = M/M_r = \exp[\theta(T_*-T_b)/2T_b^2], \tag{16}$$

since the dimensionless density may be taken to be 1; here W is the wave speed.

The Arrhenius term (2) is now replaced by

$$\Omega = Y_f W\Delta(n) \text{ with } n = x-F(0,0,t), \tag{17}$$

where Δ is the Dirac delta function and

$$x = F(y,z,t) \tag{18}$$

is the flame sheet, written in a (fixed) coordinate system chosen so that the x-axis coincides with the normal at the point of interest at the instant considered. The equations

$$\partial T/\partial t - \nabla^2 T = -\partial Y/\partial t = \Omega, \tag{19}$$

which generalize the one-dimensional ones (1), then show that the wave speed $-F_t(0,0,t)$ is just W and that

$$\delta(T) = 0,\ \delta(\partial T/\partial n) = -Y_f W,\ \delta(Y) = Y_f. \tag{20}$$

Precisely the same formulas can be obtained by applying activation-energy asymptotics to the flame sheet in the unsteady case (i.e. with $M \neq M_r$ and a flame temperature T_* within $O(\theta^{-1})$ of T_b) if it is assumed, without justification, that there is no significant temperature gradient behind the flame sheet. What sets the delta-function method apart from activation-energy asymptotics is that, in solving equations (19), θ is treated as a finite parameter, and that there is no requirement for the outer solution to match the inner in the limit $\theta \to \infty$.

3. Stability of Thermite Flames.

Linear stability of the plane wave is investigated in the manner of section 5. for a plane NEF. Disturbances proportional to $\exp(\alpha t+iky)$ are sought, resulting in the dispersion relation

$$(2\alpha+\Theta)\sqrt{1+4(\alpha+k^2)} = \Theta(2\alpha+1) \text{ with } \Theta = Y_f\theta/2T_b^2. \tag{21}$$

The neutral stability curve

$$\Theta = [6k^2+2+(2k^2+1)\sqrt{16k^2+5}]/(4k^2+1) \tag{22}$$

is shown in figure 1; on it

$$\alpha = \pm i\ \Theta^{\frac{1}{2}}(1+4k^2)^{\frac{1}{2}}/2 \tag{23}$$

is everywhere non-zero, i.e. the neutral modes are oscillatory. Resemblance to the right stability boundary for NEFs in figure 5. is striking. Note that the extreme value

$$\Theta_e = 4. \tag{24}$$

is large enough to give credence to the notion that most of the chemical reaction is confined to a thin sheet.

The results suggest that, for the one-dimensional equations (1) steady propagation is possibly only if

$$\Theta < \Theta_c = 2 + \sqrt{5}; \tag{25}$$

otherwise pulsating combustion occurs. This is in agreement with the experimental and numerical results cited at the beginning of section 1. Additional evidence is afforded by Matkowsky and Sivashinsky's demonstration that the passage of Θ through Θ_c gives a supercritical Hopf bifurcation.

Traveling-wave instabilities, rather than pulsations, will occur if disturbances of non-zero wavenumber are permitted by the lateral boundary conditions. The effect is strikingly seen for propagation through an insulated circular cylinder of thermite (Sivashinsky 1981). Now disturbances proportional to $e^{in\phi}J_n(kr)$ are sought, where r,ϕ are polar coordinates in the y,z-plane and n is a non-negative integer. Once more the dispersion relation (21), the neutral stability curve (22), and the neutral-mode frequency (23) are obtained; but now not all values of k are admissible. Thermal insulation of the surface $r = a$ requires

$$J_n'(ka) = 0, \text{ i.e. } ka = j_{n,m}' \tag{26}$$

in a standard notation for zeros of derivatives of Bessel functions. The first seven allowable values of ka are

$$\begin{aligned} j_{o,1}' = 0,\ j_{1,1}' = 1.84,\ j_{2,1}' = 3.05,\ j_{o,2}' = 3.83, \\ j_{3,1}' = 4.20,\ j_{4,1}' = 5.32,\ j_{1,2}' = 5.33 \end{aligned} \tag{27}$$

according to Olver (1964). Some of the eigenvalues

$$k_{n,m} = j_{n,m}'/a \tag{28}$$

are marked on the neutral stability curve in figure 1 for $a = 2$ and 4. These two cases illustrate the general movement of the $k_{n,m}$-points down the curve as a increases.

Consider now what happens for cylinders of different size as Θ is increased up to the first onset of instability. (This is not easily done in a practical context.) Only the discrete points on the neutral stability curve corresponding to the values (28) of k are relevant, and which mode will be triggered first depends on the value of a. As Θ is further increased the mode becomes an admissible instability that developes a definite nonlinear form of the same general character.

For

$$a < 2.05 \tag{29}$$

the point $k_{1,1}$ lies to the right of $k_{o,1}$ on the neutral curve; this is exemplified by $a = 2$ in figure 1. The first manifestation of instability will be plane pulsations of frequency

$$\omega = \pm\sqrt{\theta_c}/2; \tag{30}$$

the corresponding expressions for flame temperature and location are

$$T_* = T_b + \varepsilon\cos\omega t, \quad x_* = -t-(8\varepsilon\omega/Y_f)\sin\omega t, \tag{31}$$

where ε is the (linear) disturbance amplitude. The speed of the flame is greater or less than 1 accordingly as its temperature is greater or less than T_b. The temperature gradient

$$\varepsilon\text{Re}[(\cos\omega t - i\sin\omega t)(1-\sqrt{1+4i\omega})]/2 \tag{32}$$

behind the flame does not vanish but flucturates about zero (cf. the remark at the end of the last section).

The model is based on the assumption that the reactant is consumed completely, but this is not the case in practice. Indeed, it is sometimes possible to propagate a flame through the same material twice. It is to be expected that fluctuations in temperature gradient at the reaction front will result in a layered burnt state; in the context of activation-energy asymptotics, negative temperature gradients behind the reaction zone permit reactant leakage. Merzhanov, Filonenko, and Borovinskaya noted layered structure in burnt thermites, with a layer for each pulsation.

A quite different phenomenon occurs if the first manifestation of instability is associated with $j'_{1,1}$, as is the case for

$$2.05 < a < 4.88; \tag{33}$$

now $k_{1,1}$ is the leftmost point on the neutral curve, as is exemplified by $a = 4$ in figure 1. The expressions (31) are replaced by

$$T_* = T_b + \varepsilon\cos(\omega t+\phi)J_1(k_{1,1}r) \ , \ x_* = -t-(8\varepsilon\omega/Y_f)\sin(\omega t+\phi)J_1(k_{1,1}r), \quad (34)$$

where now

$$\omega = \pm\Theta_{1,1}^{\frac{1}{2}}(1+k_{1,1}^2)^{\frac{1}{2}}/2. \quad (35)$$

The . isotherms of the flame temperature are shown in figure 2; these spin, in either direction, with the frequency (35), producing a single hot-spot traveling in helical path on the surface of the cylinder. Such hot spots were observed by Merzhanov, Filonenko and Borovinskaya.

As the radius a increases beyond 4.88 for a certain interval, $k_{2,1}$ becomes the leftmost point on the neutral curve; there are then two spinning hot spots at opposite ends of a diameter. Hot spots can also occur in the interior, for example for $k_{1,2}$, which is the leftmost point for an interval of still larger values of a.

Other cross sections give rise to their own distinctive sets of admissible wavenumbers and isotherm patterns. For rectangles, judicious choice of proportion leads to 2 or even 3 modes simultaneously characterizing the onset of instability. A small change in the proportion will cause the corresponding eigenvalue to split, leading to secondary or tertiary bifurcations. Matkowsky & Olagunju (1982) have carried out the unimodal bifurcation analysis for circular cross sections; Margolis & Matkowsky (1982) have considered the multi-modal analysis for rectangular cross sections.

The stability boundary identified here, coupled with the nature of the instabilities, shows that we are dealing with the analog of the right stability boundary in the NEF analysis. This suggests that SVFs, which lie between NEFs (with L close to 1) and thermite flames (with $L = \infty$), should also exhibit pulsations as their instability; however, for $L > 1$ their disturbances grow monotonically. Resolution of this apparent contradiction undoubtedly lies in the result (30) which suggests that the SVF analysis, by restricting attention to evolution on the time scale $t = O(\theta)$, filters out a pulsating mode. In fact, Rogg (1982) has reported numerically determined pulsations for flames that would otherwise be candidates for SVF analysis.

4. Flames Anchored to Burners

We now turn our attention to gases, which necessarily have finite Lewis numbers, almost invariably lying to the left of the right stability boundary in figure 5. . The problem is to find a mechanism that will shift this boundary to the left, making it accessible to mixtures of practical interest. Joulin and Clavin have shown that such a mechanism is the distributed heat loss of section 3. , which suggests that heat loss to a burner anchoring the flame will have the same effect. There is now experimental evidence that burner flames can indeed pulsate.

Consider a flame anchored to the so-called porous-plug burner. The mathematical problem to be solved is

$$\partial T/\partial t + \partial T/\partial x - \nabla^2 T = -\partial Y/\partial t - \partial Y/\partial x + L^{-1}\nabla^2 Y = \mathcal{D} Y e^{-\theta/T} \text{ with } \mathcal{D} = \mathcal{D} M_r^{-2}, \quad (36)$$

$$T = T_s, \; Y - L^{-1}\partial Y/\partial x = J_s \text{ at } x = 0, \quad T \text{ bounded, } Y \to 0 \text{ as } x \to +\infty. \quad (37)$$

Here M_r is the prescribed mass flux through the face of the plug at $x = 0$; while T_s and J_s are the prescribed temperature and mass-flux fraction there. (In practice, cooling coils are used to maintain T_s constant.) The physical idea underlying the boundary condition (37b) is that the porous surface inhibits the flux of reaction products, so that the flux fractions of the mixture as supplied by the burner are identical to the mass fractions in the reservoir supplying the burner.

Analysis of the steady problem in the limit $\theta \to \infty$ proceeds as in section 2. for the unbounded flame, except that the burnt-gas temperature T_b is the fundamental quantity to be determined, not M_r. Integration of the steady version of equation (36a) from $x = 0$ to ∞ yields

$$T_s' = T_s + J_s - T_b, \tag{38}$$

i.e. the heat received by the plug in terms of T_b. This enables us to write the solution in the form

$$T = \begin{cases} T_b - J_s + (T_s + J_s - T_b)e^x \\ T_b \end{cases}, \quad Y = \begin{cases} J_s - J_s^{1-L}(T_s + J_s - T_b)^L e^{Lx} \\ 0 \end{cases} \quad \text{for } x \lessgtr x_*, \tag{39}$$

where

$$x_* = \ln[J_s/(T_s + J_s - T_b)]. \tag{40}$$

Consistency requires x_* to lie between 0 and ∞, so that we must have

$$T_s < T_b < T_s + J_s. \tag{41}$$

The right inequality shows that T_s' is necessarily positive, i.e. the plug must be a heat sink. Finally, a flame-sheet analysis gives

$$M_r = \sqrt{2LD}\, T_b^2 \exp(-\theta/2T_b)/\theta J_s, \tag{42}$$

from which T_b can be determined. (The result (2.) is recaptured on replacing J_s with Y_f.) Ferguson and Keck have made satisfactory comparisons between experiment and a theoretical result essentially equivalent to the determination (40) of the stand-off distance x_* as a function of the injection rate M_r. (Note that M_r is also used in making x_* nondimensional.)

The inequalities (41) define limits on M_r. When the injection rate is decreased (increased) beyond its limiting value a surface (remote) flame is obtained, requiring a different asymptotic analysis. We shall be concerned only with injection rates within the limits.

5. Stability of Burner Flames

In considering the stability of the solution in the last section it is natural to turn to a NEF analysis. However, such an analysis requires not only ℓ to be sufficient close to 1 but also that the boundary conditions permit T + Y to be constant to leading order. In general, the conditions (37) do not satisfy the second requirement.

One way out of the dilemma is to abandon activation-energy asymptotics and adopt a suitable modification of the delta-function model used in the discussion of thermites. (The strength of the delta function is again proportional to $\exp(-\theta/2T_*)$.) Such an approach was used by Margolis who, by means of a numerical investigation of the complicated dispersion relation obtained from a linear stability analysis, was the first to demonstrate the leftward displacement of the stability boundary alluded to at the beginning of the last section. He also carried out a complete numerical simulation of a fuel-rich hyrogen-oxygen flame, thereby demonstrating pulsations; these were apparently confirmed by experiments performed at Sandia-Livermore, although there is no published account of them.

The NEF requirement that T+Y should be constant to leading order is a sufficient but not necessary condition for the flame temperature to vary by $O(\theta^{-1})$ only. Thus, Buckmaster (1982) also approached the question by means of a delta-function model but then, a posteriori, justified the model through activation-energy asymptotics. This amounts to identifying the circumstances under which the dispersion relation is asymptotically meaningful, and for which there is a flame structure linking the states on the two sides of the flame sheet obtained by the equivalent jump conditions. Some care is necessary because of the sensitivity of the solution to variations in flame temperature; the disturbance field is O(1) on the θ-scale, which entails calculating the flame-temperature perturbations correct to $O(\theta^{-1})$. In turn, this entails deriving jump conditions (from the flame-sheet structure) correct to the same order. Buckmaster found

$$\delta(T) = -\delta(Y) = \varepsilon T_{*1}/\theta, \tag{43}$$

$$\delta(\partial T/\partial x) = -\delta(\partial Y/\partial x) + \ell J_s W/\theta = -J_s W + q/\theta \text{ with } W = 1+\varepsilon T_{*1}/2T_b^2. \tag{44}$$

Here ℓ has the same meaning (4.) as in NEF analysis, ε is the small parameter characterizing the size of the disturbance, the flame temperature is $T_b + \varepsilon T_{*1}/\theta$, and q is a quantity (calculated from details of the flame structure) that is never needed. The term $-J_s W$ is the linearized form of the term $-Y_f W$ appearing in the jump condition (20b), with J_s replacing Y_f.

With the jump conditions in hand, it is a straightforward matter to carry out the analysis of unstable disturbances proportional to $\exp(\alpha t + iky)$. Circumstances justifying the delta-function model are then found to be

$$\theta e^{-\kappa x_*} = O(1) \tag{45}$$

where κ has the definition (5.). An asymptotically self-consistent dispersion relation

$$2\kappa^2(1-\kappa^2)+\bar{\ell}(1+\kappa)[(1-\kappa)^2-4k^2] = 8\Theta\kappa^3 e^{-\kappa x_*} \tag{46}$$

is obtained, the corresponding combustion field matching the structure used to deduce the jump conditions (43), (44); here $\bar{\ell}$ and Θ have the definitions (4.) and (21b) with J_s in place of Y_f.

A similar dispersion relation (but free of θ and with $k = 0$) was derived by Matkowsky and Olagunju for a somewhat artifical burner whose boundary conditions are compatible with NEF analysis. They do not discuss the full *ramifications of their results.*

What emerges from Buckmaster's analysis is essentially a NEF. To get a hint of this, note that the boundary conditions imply that the disturbance satisfies

$$T_1 + Y_1 = L^{-1}\partial Y_1/\partial x \quad \text{at} \quad x = 0. \qquad (47)$$

Now the disturbance field decays rapidly ahead of the flame sheet, because Λx_* is logarithmically large in θ. It follows that $\partial Y_1/\partial x$ is small at the plug, small enough for the condition (47) and the near-equality of thermal and mass diffusions to ensure that T_1+Y_1 is, at most, $O(\theta^{-1})$ throughout the combustion field.

Finally, it should not be thought that the *requirement* (45) is a constraint on x_*. Insofar as the right stability boundary is concerned, the relation (46) implies that as x_* is decreased, $Re(\kappa)$ increases (through an increase in the frequency of pulsations) so as to keep the term on the right balanced. When x_* is $O(1)$ the frequency is logarithmically large in θ.

The displacement of the right stability boundary is illustrated by figure 3, which shows how the point $k = 0$ on it varies with x_*. (Note that $\bar{l} \to 2(1+\sqrt{3})$ as $x_* \to \infty$, in agreement with the result in section 5. for unbounded flames.) As x_* is decreased the boundary first moves to the left, by an amount that increases with θ. But eventually this motion is halted and the boundary moves back to the right.

6. Pulsations for Rear-Stagnation Point Flow.

It seems likely that there are other mechanisms that will make the pulsations accessible. One that has been suggested by theory (but not confirmed experimentally) is negative strain, such as is experienced by a flame in a rear stagnation-point flow.

For moderate Reynolds numbers, flames can be stabilized behind the closed laminar wake at the rear of a thin plate or rod. Figure 4 shows the configuration, which Mikolaitis & Buckmaster (1981) treated with a NEF formulation based on the equations

$$(\partial/\partial t+\varepsilon y\partial/\partial y-\partial^2/\partial y^2)(T,h) = \ell\epsilon y\partial(0,T)/\partial y \qquad (48)$$

and the jump conditions (4.27-29). These equations are the unsteady version of equations (4.40) with the sign of ϵ changed because we are dealing with a rear instead of a front stagnation point. The problem is completed by the boundary conditions

$$T = T_f, \quad h = 0 \quad \text{at} \quad y = 0, \qquad (49)$$

the latter corresponding to the prescription $Y = Y_f$ at $y = 0$.

The steady solution can be written in a closed form similar to that in section 4. for a front stagnation point. Its stability to one-dimensional disturbances was carefully explored using a combination of Galerkin's method and the method of weighted residuals. The results are illustrated by the two curves in figure 5, which shows variations in the stand-off distance y_* of the flame with (negative) straining rate. All responses have the form of a backward C, so that there is a maximum straining rate beyond which the flame must blow off.

For values of $\bar{\ell}$ ($=\ell Y_f/2T_b^2$) less than about 0.91 the response is characterized by the curve (a); the lower branch is stable and the upper branch is unstable, with a real eigenvalue crossing through the origin as the turning point T is traversed. For other values of $\bar{\ell}$, as characterized by the curve (b), part of the lower branch is also unstable, with a complex conjugate pair of eigenvalues crossing the imaginary axis as the point P (for pulsations) is traversed. This raises the possibility that, for sufficiently large values of the Lewis number, blow-off will in practice be preceded by pulsations. (The critical value of $\bar{\ell}$ is quite accessible.)

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Figure Captions

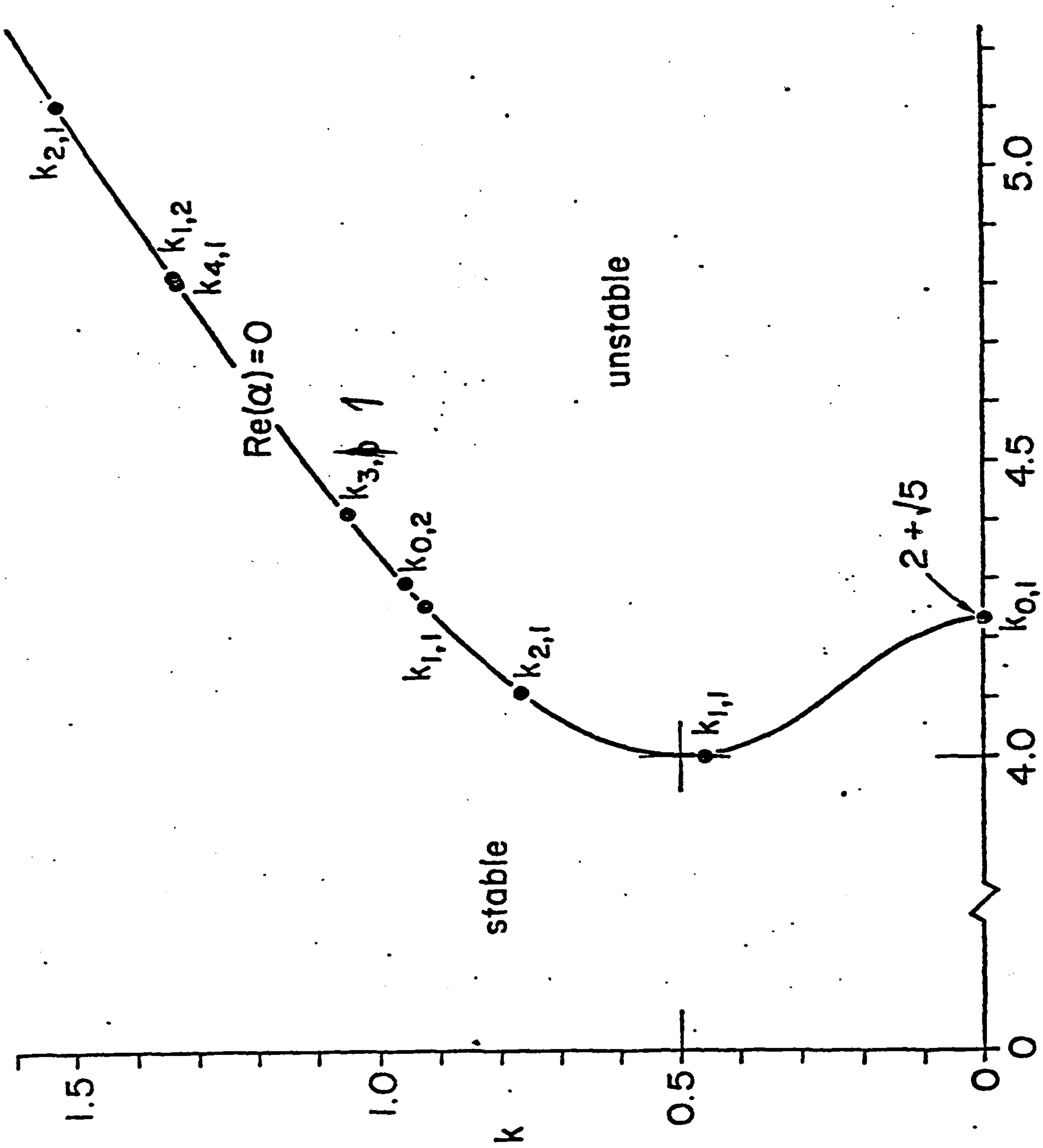
7.1 Linear stability regions for thermite flames, with admissible values of k when confined to insulated circular cylinder. Labels to the right (left) of the neutral stability curve correspond to $a = 4(2)$.

7.2 Isotherms of spinning thermite flame, with +/- denoting the hot/cold sides.

7.3 Displacement of the right stability boundary in figure 6.8 due to the presence of a burner, for various values of θ . The purely imaginary numbers are values of α .

7.4 NEF in a rear stagnation-point flow.

7.5 Variation of stand-off distance y_* with straining rate ϵ for NEFs in rear stagnation-point flows.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1.0
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0
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5.0
Θ
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$k_{4,1}$
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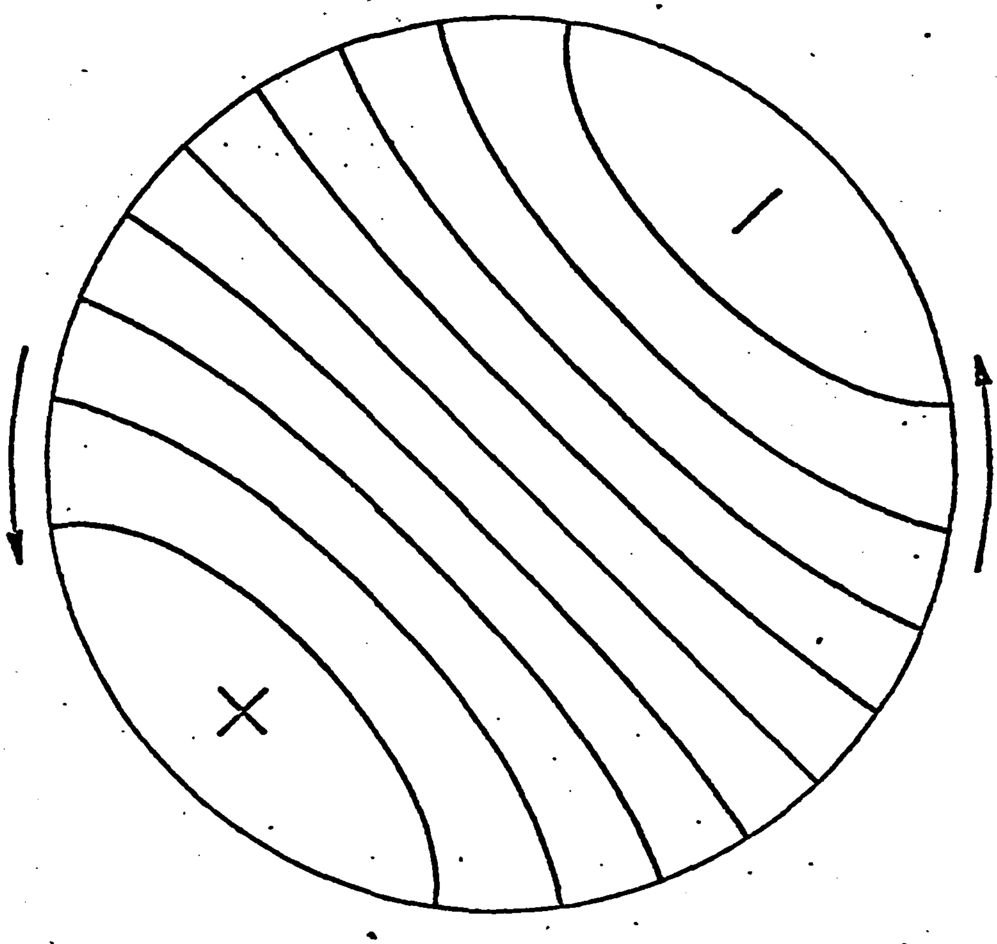

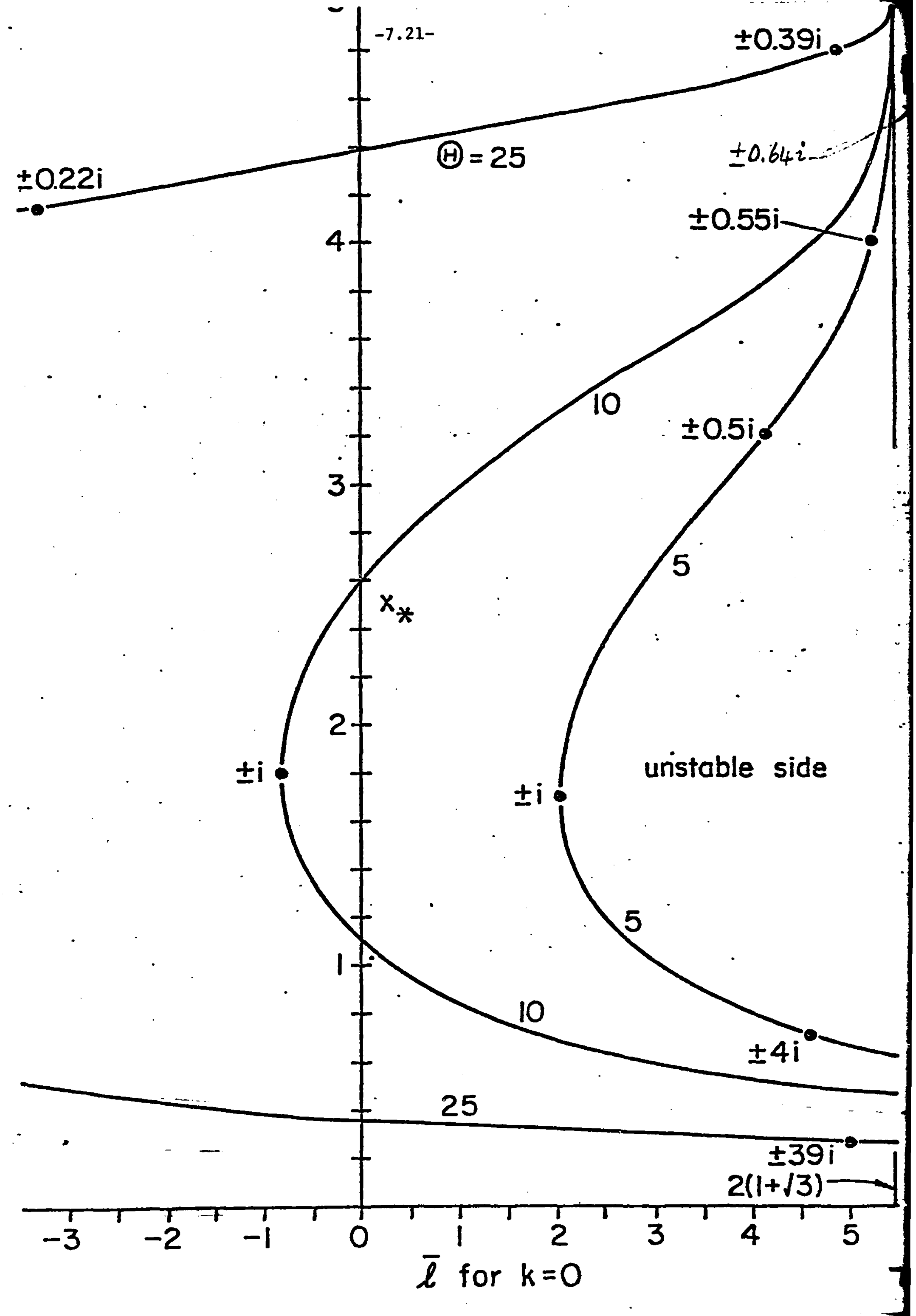
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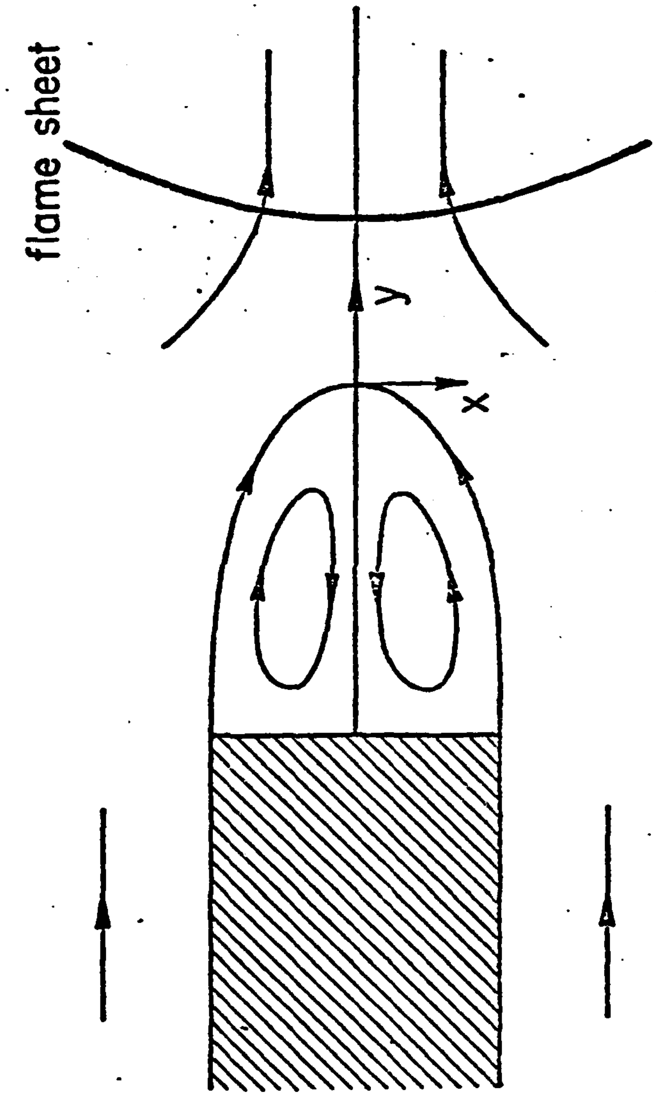
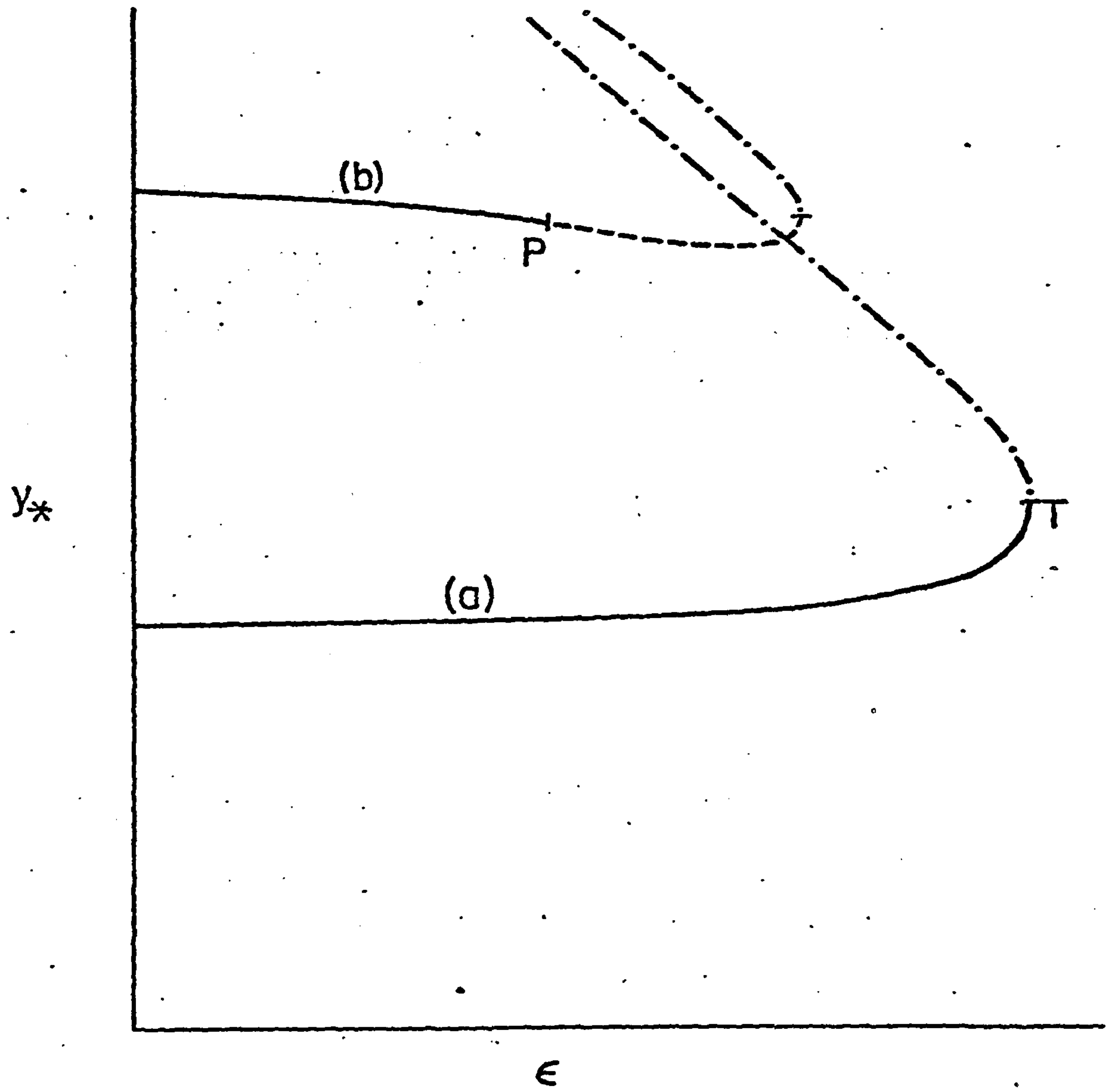
x*
2
±i
±i
unstable side
5
1
10
±4i
25
±39i
2(1+√3)
-3
-2
-1
0
1
2
3
4
5
$\bar{\ell}$ for k=0


flame sheet
y
x


(b)
P
y_*
T
(a)
ϵ

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Pulsating flames, traveling waves, thermites, delta-function model, spinning hot-spot, secondary and tertiary bifurcations, burner flame and rear stagnation-point pulsations.

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

In section 5. it was found that plane NEFs of sufficiently large Lewis number are unstable. Since $\mathrm{Im}(\alpha) \neq 0$ on the stability boundary, the instability is likely to result in either a pulsating flame or a flame that supports traveling waves. Such flames are the subject of this lecture.

One difficulty that immediately confronts us is that, in contrast to the ubiquitous nature of cellular instabilities, pulsating instabilities are not ordinarily seen. The reason seems to be the large values of $\mathcal{L}$ are needed; according to the theory, $Y_{s}\ell/T^{2}$ must exceed 32/3 (or $4(1+\sqrt{3})$ if the

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SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered) disturbances are one-dimensional). There is evidence that fuel-rich hydrogen-bromine mixtures might attain such values since Golovichev, Grishin, Agranat, and Bertsun (1978) obtained oscillations in a numerical study, but there is no similar evidence for more commonplace gas mixtures.

For this reason we must turn from the commonplace and deal either with unusual combustible materials or else with special configurations, in order to uncover pulsating flames. Our discussion will start with thermites, which are solids that burn to form solids (a phenomenon that is appropriately called gasless combustion). There is no significant diffusion of mass, so that L is effectively infinite.

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