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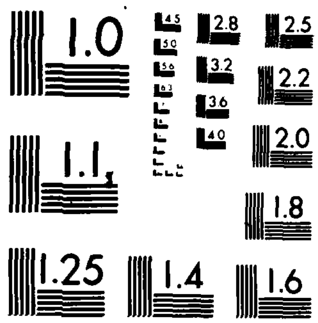

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Final Report

on

DEVELOPMENT OF A NONLINEAR PROGRAMMING SYSTEM

by

James E. Falk
Anthony V. Fiacco
Garth P. McCormick

The George Washington University
School of Engineering and Applied Science
Department of Operations Research

Contract N00014-75-C-0611
Office of Naval Research

Serial 18218
6 June 1983

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1. INTRODUCTION

The title of the contract for which this is a final report is "Development of a Nonlinear Programming System." More fundamentally, the research is concerned with factorable functions, functions which are complicated compositions of transformed sums and products of transformed sums and products . . . of functions of a single variable.

For the past twenty years considerable effort has been expended on the development of nonlinear programming theory and algorithms for solving nonlinear programming problems. Some of these algorithms have been implemented on digital computers. It is fair to say, however, that solving a complicated nonlinear programming problem by a computerized nonlinear programming algorithm is not an automatic process. Unlike linear programming, where computerized algorithms (variations on the simplex method, usually) have long been able to solve problems of large size, nonlinear programming is still in its infancy as regards its ability to guarantee solutions to problems of even moderate size.

A major barrier to solving nonlinear programming problems is the lack of a computationally oriented way of representing nonlinear functions of several variables. Examples of real world problems which give rise to complicated nonlinear functions are weapon/target allocation, optimal inventory policy determination, minimum weight structural design, and parameter estimation through nonlinear least squares regression and maximum likelihood estimation.

Algorithms for solving problems such as those above are often not as efficient as they could be because of the inability to compute automatically quantities related to the complicated nonlinear functional relationships that describe the models. For example, the accurate and speedy computation of first derivatives is a usual requirement for algorithms which solve systems of nonlinear equations.

The primary objective of this project was to provide a language that is user-oriented and which establishes an interface between optimization problems stated algebraically and computer coded algorithms for solving them.

The basis for this language is a new approach for representing nonlinear functions of several variables. The class of functions to which this approach applies is called "factorable functions." The basic definition is as follows:

> a function $f(x_1,\ldots,x_n)$ is a *factorable function* of several variables if it can be represented as the last in a finite sequence of functions $\{f_j(x)\}$ that are composed as follows:
>
> $$f_j(x) = x_j\ , \quad \text{for} \quad j = 1,\ldots,n\ .$$

For $j > n$, $f_j(x)$ is either of the form

$$f_k(x) + f(x)_\ell \ , \quad k,\ell < j \ ,$$

$$f_k(x) \cdot f(x)_\ell \ , \quad k,\ell < j \ ,$$

or

$$T[f_k(x)] \ , \quad k < j \ ,$$

where $T(f)$ is a function of a single variable.

A brief general description of the factorable programming approach is given in [McCormick 1979], where its relevance to problems in engineering design is emphasized. A more general description is contained in Chapter 3 of the recent textbook [McCormick 1983]. Details of the various developments contained in the papers in the cumulative bibliography at the end of this report have been touched on in previous progress reports. Here just a brief summary of the work accomplished will be given. In the fall of 1983 a book giving the integrated details of this and current work will be begun by McCormick while he is on sabbatical leave.

2. COMPUTER CODES AND APPLICATIONS PAPERS

The first computer code implementing the concept of factorable functions was written by Robert Pugh and two of his associates. This was described in the publication [Pugh 1972]. The automatic computation of first and second derivatives and the dyadic structure of the Hessian matrices of factorable functions was an important feature of this code.

The current version of the program implementing the factorable function approach to solving problems is documented in [Ghaemi and

McCormick 1979]. Publications relevant to this are [Ghaemi 1979], [Monaco 1979], and [Ghaemi and McCormick 1980].

Two user oriented applications papers which show examples of optimization problems and how they could be encoded in the factorable way are [McCormick 1975] and [McCormick 1977].

3. SENSITIVITY ANALYSIS

Automatic computation of first order post optimality sensitivity analysis based on factorable functions was pursued in the dissertation [DeSilva 1978] and in the paper [DeSilva and McCormick 1978]. The capability of doing this sensitivity computation automatically was implemented in the code described in [Ghaemi and McCormick 1979].

4. USE IN MATRIX METHODS

The implications of the factorable representation of nonlinear functions for matrix methods used in nonlinear programming algorithms is great. Some numerical techniques based on the outer product representation of the Hessian matrix of factorable functions have been developed. For the unconstrained case a stable method was developed in the dissertation [Emami 1978] and is described also in [Emami and McCormick 1978].

Numerical techniques implementing algorithms for the linearly constrained problem were developed in the dissertation [Ghotb 1980].

5. ALGORITHM COMPARISON

The factorable approach provides a new way to evaluate competing algorithms in a rigorous way, particularly by providing a point of view to make comparable the amount of work required to calculate function values, first and second derivatives. A start in this area is in the dissertation [Shayan 1979], with draft papers [McCormick and Shayan, 1981a, 1981b].

6. COMPUTATION OF EIGENVECTORS AND EIGENVALUES

The natural form of Hessian matrices, that is, the dyadic form, has implications for the computation of eigenvectors and eigenvalues. Some investigations into this are in [Falk, *et al.*, 1974] and in [McCormick 1983, Chapter 2].

7. OBTAINING AND VERIFYING GLOBAL SOLUTIONS FOR NONLINEAR OPTIMIZATION

The simple structure of factorable functions allows for two computations that have implications for nonconvex programming problems (i.e., problems for which local solutions may exist, which are not global solutions). One step in the determination of a global solution is the ability to form convex programming problems that underestimate the original one. The use of factorable programming problems in this regard is in [McCormick 1976]. The second step is the ability to verify that a local minimizer is a global minimizer in a hyperrectangle. This is discussed for the unconstrained case in [Mancini and McCormick 1976] and [Mancini and McCormick 1979]. For the general nonlinearly constrained case this is discussed in Section 18.3 of [McCormick 1983].

For functions of one variable the material in [McCormick 1981] combined with the developments in [McCormick and Shayan 1981] and [Shayan 1979] present an interesting way of obtaining a global solution.

Global solutions can also be obtained by converting factorable problems to equivalent separable problems. When the functions involved are factorable, this can be an automatic procedure (see [McCormick 1972, 1981]).

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