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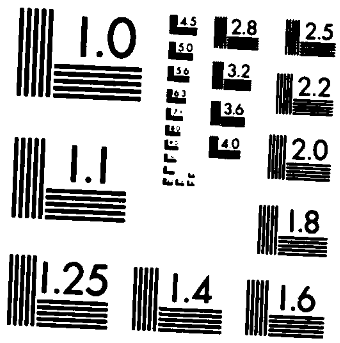
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A DIAGNOSIS ALGORITHM FOR THE BGM SYSTEM LEVEL FAULT MODEL

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ABSTRACT

A τ-diagnosable system is a system in which all faults may be identified from the test results, provided that the number of faults does not exceed τ. In this paper we present an algorithm that may be used for the diagnosis of the system level BGM fault model proposed by Barsi, Grandoni and Maestrini, whenever the system is τ-diagnosable and the number of faults is at most τ.

THE BGM SYSTEM LEVEL FAULT MODEL

Consider a system S of n units $u_1, u_2, \ldots, u_n$ and a test digraph TD, where $TD = \{ (u_i,u_j) \mid u_i \text{ tests } u_j \}$. It is assumed that no unit tests itself, that each unit is either faulty or nonfaulty, and that the state of each unit is constant during the application of the testing procedures. If (u_i,u_j) is in TD, then u_i tests u_j, and the test outcome a_{ij} is assumed to be either "0" (u_j passes the test) or "1" (u_j fails the test). The set of test outcomes $\{ a_{ij} \mid (u_i,u_j) \in TD \}$ is the syndrome of the system. In the BGM model proposed by Barsi, Grandoni and Maestrini [1], the following relationships between faults and test outcomes are assumed:

(i) if (u_i,u_j) is in TD and u_i and u_j are nonfaulty, then $a_{ij} = 0$;

(ii) if (u_i,u_j) is in TD, u_i is nonfaulty and u_j is faulty, then $a_{ij} = 1$;

(iii) if (u_i,u_j) is in TD and both u_i and u_j are faulty, then $a_{ij} = 1$;

(iv) if (u_i,u_j) is in TD, u_i is faulty and u_j is nonfaulty, then a_{ij} may take either the value 0 or 1.

Thus, if a unit u_i is tested by a unit u_j and $a_{ji} = 0$, the unit u_i is nonfaulty.

Given a set of faulty units F_S, the computation of the corresponding syndromes is not difficult, but to compute the sets of faulty units that are consistent with a given syndrome is not as easy. In this paper, we address the latter problem -- namely, syndrome decoding -- and we restrict ourselves to τ-diagnosability in the sense of Preparata, Metze and Chien [6].

Definition 1: A system S is τ-diagnosable if all faulty units within the system can be identified without replacement, provided that the number of faulty units does not exceed τ.

In the remainder of this work, $\|A\|$ will be used to denote the number of elements in the set A.

FAULT IDENTIFICATION ALGORITHM

Our approach to system diagnosis consists in defining subsets V, H_1, H_2 and H_3 that depend on the syndrome and two subsets W and X that depend only on the test digraph, and then to relate those subsets to the set F_S of faulty units in S.

The set V contains all the units in S that are tested by at least one other unit in S and found to be nonfaulty by that unit, i.e.,

$$V = \{ u_i \in S \mid u_j \text{ in } S \text{ exists so that } (u_j,u_i) \in TD \text{ and } a_{ji} = 0 \}. \quad (1)$$

Thus, if S is a BGM model, the unit u_i is nonfaulty whenever u_i is in V.

The set H_1 contains all the units in S that are tested by at least one unit u_j in V and found faulty, and all the units in S that test at least one unit u_j in V, and find it faulty, i.e.,

$$H_1 = \{u_i \in S \mid u_j \text{ in } V \text{ exists so that } (u_j,u_i) \in TD \text{ and } a_{ji} = 1 \} \cup \{u_i \in S \mid u_j \text{ in } V \text{ exists so that } (u_i,u_j) \in TD \text{ and } a_{ij} = 1 \}. \quad (2)$$

One should note that if S is a BGM model, then the sets V and H_1 are disjoint, and u_i is faulty whenever u_i is in H_1.

The index set H_2 depends on the cardinality of the sets $L(u_i)$, where, for every unit u_i in $S-(V \cup H_1)$, the sets $L(u_i)$ are defined by

$$L(u_i) = \{ u_j \in S-(V \cup H_1) \mid (u_i,u_j) \in TD \text{ and } a_{ij} = 1 \} \cup \{ u_j \in S-(V \cup H_1) \mid (u_j,u_i) \in TD \text{ and } a_{ji} = 1 \}.$$

Given u_i, it is possible that u_j exists so that (u_i,u_j) and (u_j,u_i) are both in TD, and $a_{ij} = a_{ji} = 1$. Obviously, in such a case u_j appears in $L(u_i)$ only once.

The set $L(u_i)$ contains all the units adjacent to the unit u_i that must be faulty if the unit u_i is actually nonfaulty. Given a scalar τ, the set H_2 consists of all the units in S, but not in $V \cup H_1$, such that the cardinality of $L(u_i)$ is strictly greater than τ, i.e.,

$$H_2 = \{ u_i \in S - (V \cup H_1) \mid |L(u_i)| \geq \tau + 1 \}. \qquad (3)$$

It is clear that if S is a BGM model and if at most τ units in S are faulty, then u_i is faulty whenever it is in H_2.

The set H_3 contains the remaining units in S, i.e.,

$$H_3 = S - (V \cup H_1 \cup H_2).$$

The definition of the sets H_1, H_2 and H_3 immediately implies the following lemma.

Lemma 1: If (i) S is a BGM model, and (ii) $|F_S| \leq \tau$, then

$$H_1 \cup H_2 \subseteq F_S \subseteq H_1 \cup H_2 \cup H_3.$$

The two subsets W and X of S that are defined now depend only on the test digraph TD and do not depend on the syndromes produced by faulty sets of units. Note that the subset W is not used in the fault identification algorithm, and is defined only to facilitate the analysis of the algorithm.

The set W contains all the units u_i in S such that: (i) the unit u_i is tested by exactly τ other units, and (ii) a unit u_j in S exists such that u_j is tested by exactly τ other units in S, and u_i and u_j test each other.

The set X be the set of all units u_i in S such that: (i) u_i is tested by exactly τ other units; (ii) a unit u_j exists such that u_j is tested by exactly τ units in S and u_i and u_j test each other; and (iii) a unit u_k in S exists such that the unit u_k tests u_j but not u_i, and u_k is tested by at least one unit that does not test u_i.

We are now ready to present our fault identification algorithm.

Algorithm 1:

Step 0: Compute the set V as in Equation (1).

Step 1: If $\|S - V\| \leq \tau$, let $F_A = S - V$ and stop; otherwise, go to Step 2.

Step 2: Compute the sets H_1 and H_2 as in Equations (2) and (3).

Step 3: If $\|H_1 \cup H_2\| = \tau$, let $F_A = H_1 \cup H_2$ and stop; otherwise, go to Step 4.

Step 4: Let $F_A = H_1 \cup H_2 \cup (H_3 \cap X)$ and stop.

ALGORITHM ANALYSIS

We start the analysis of Algorithm 1 by presenting its properties when the following assumption is satisfied.

Hypothesis 1: Every unit in S is tested by at least τ other units in S.

We now show that when Hypothesis 1 is satisfied, the set F_A generated by Algorithm 1 contains only faulty units.

Lemma 2: If (i) S is a BGM model, (ii) Hypothesis 1 is satisfied, and (iii) $\|F_S\| \leq \tau$, then $F_A \subseteq F_S$.

Proof: (i) Assume that $\|H_1 \cup H_2 \cup H_3\| \leq \tau$. In that case, Algorithm 1 stops in Step 1 and $F_A = S - V = H_1 \cup H_2 \cup H_3$. Two cases are possible: either $\|F_S\| < \tau$ or $\|F_S\| = \tau$.

(i.a) Assume that $\|F_S\| < \tau$. Let u_i be a nonfaulty unit. By assumption, every unit is tested by at least τ other units and the fact that $\|F_S\| < \tau$ implies that u_i is tested by at least one nonfaulty unit, say u_j. Thus, if u_i is nonfaulty, a unit u_j exists so that (u_j, u_i) is in TD, $a_{ji} = 0$, and it follows that V contains the indices of all the nonfaulty units in S. Now let u_i be a faulty unit.

Hypothesis 1 and the fact that $\|F_S\| < \tau$ imply that u_l is tested by at least one unit u_j in V and $a_{jl} = 1$. It follows that $H_1 = F_S$, V and H_1 form a partition for S, $S - V = H_1$, and thus $F_A = F_S$.

(i.b) Assume now that $\|F_S\| = \tau$. In that case, Lemma 1 implies immediately that $F_S = H_1 \cup H_2 \cup H_3 = S - V = F_A$.

(ii) Assume now that $\|H_1 \cup H_2 \cup H_3\| > \tau$. By assumption, $\|F_S\| \leqslant \tau$, and thus, using Lemma 1, we may conclude that $\|H_1 \cup H_2\| \leqslant \tau$. Thus, once again, two cases are possible: either $\|H_1 \cup H_2\| = \tau$ or $\|H_1 \cup H_2\| < \tau$.

(ii.a) Assume that $\|H_1 \cup H_2\| = \tau$. In that case, Algorithm 1 stops in Step 3 and $F_A = H_1 \cup H_2$. Lemma 1 implies immediately that $\|F_S\|$ must be equal to τ and that $F_A = F_S$.

(ii.b) Assume now that $\|H_1 \cup H_2\| < \tau$. In that case, Algorithm 1 stops in Step 4 and $F_A = H_1 \cup H_2 \cup (H_3 \cap X)$. Let u_l be in $H_3 \cap X$ and let u_j and u_k be units that satisfy part (ii) and (iii) in the definition of the set X. Suppose that u_l and u_k are both nonfaulty. All the units that test u_l and those that test u_k are then faulty. The unit u_k is tested by at least one unit that does not test u_l and therefore, the assumption that both u_l and u_k are nonfaulty implies that at least $\tau + 1$ units are faulty. This is impossible, and thus, if u_l is nonfaulty, u_k must be faulty. The unit u_k does not test the unit u_l, and the fact that u_l is in H_3 implies that u_l does not test u_k. Thus, if u_l is assumed to be nonfaulty, we must conclude that at least $\tau + 1$ units are faulty. Once again, this is impossible; Therefore, the unit u_l must be faulty -- i.e., u_l must be in F_S. We already know from Lemma 1 that H_1 and H_2 are subsets of F_S, and thus, we may conclude that $H_1 \cup H_2 \cup (H_3 \cap X)$ is in F_S, i.e., that F_A is a subset of F_S.

In their 1976 paper, Barsi, Grandoni and Maestrini [1] proposed a condition on the test digraph TD that insures τ-fault diagnosability. Using our notation, we will now repeat that assumption and show that it may be used to insure that F_S is found by Algorithm 1

Hypothesis 2: If the units u_i and u_j are in W and if u_i and u_j test each other, then u_i or u_j or both are in X.

Lemma 3: If (i) S is a BGM model, (ii) Hypotheses 1 and 2 are satisfied, (iii) $\|F_S\| \leq \tau$, (iv) $\|H_1 \cup H_2 \cup H_3\| > \tau$ and (v) $\|H_1 \cup H_2\| < \tau$, then $H_3 \subseteq W$, $\|F_S\| = \tau$ and $H_1 \cup H_2 \cup (H_3 \cap X) = F_S$.

Proof: (i) Let u_i be a unit in H_3. Every unit in S that tests u_i find u_i faulty (otherwise u_i would be in V). No unit in V tests u_i (otherwise u_i would be in H_1). The unit u_i cannot be tested by more than τ other units (otherwise u_i would be in H_2), and thus, Hypothesis 1 implies that u_i is tested by exactly τ other units. We may conclude that every unit in H_3 is tested by exactly τ other units that must be in $H_1 \cup H_2 \cup H_3$.

We have assumed that $\|H_1 \cup H_2\| < \tau$, and therefore, if u_i is in H_3, u_i must be tested by at least one other unit u_j also in H_3. If u_i does not test u_j, then u_j is tested and found faulty by a set of τ units that does not include u_i, and u_j tests and finds u_i faulty. It is clear that $L(u_j) > \tau$, and thus u_j must be in H_2. This contradicts the fact that u_j is in H_3 and therefore we may conclude that u_i tests u_j. Every unit in H_3 is in W and thus $H_3 \subseteq W$.

(ii) Suppose that $\|F_S\| < \tau$. Part (i.a) of the proof of Lemma 2 shows that $H_1 = F_S$ and V and H_1 form a partion for S. Thus, H_2 and H_3 are empty and $\|H_1 \cup H_2 \cup H_3\| < \tau$. This contradicts the fact that $\|H_1 \cup H_2 \cup H_3\| > \tau$, and we may conclude that $\|F_S\| = \tau$.

(iii) Suppose that a unit u_i exists so that u_i is in H_3 but is not in X. The unit

u_i is tested by exactly τ other units that must be in $H_1 \cup H_2 \cup H_3$. It follows that u_i is tested by at least $\tau - \|H_1 \cup H_2\|$ units in H_3. Hypothesis 2 then implies that all those units in H_3 (and thus in W) that test u_i must be in X. We may then conclude that $\|H_3 \cap X\| = \tau - \|H_1 \cup H_2\|$. The fact that the sets H_1, H_2 and H_3 are disjoint then implies that $\|H_1 \cup H_2 \cup (H_3 \cap X)\| = \tau$. We have proved that $\|F_S\| = \tau$, from Lemma 2, we know that $\| H_1 \cup H_2 \cup (H_3 \cap X)\| \subseteq F_S$, and it follows that $H_1 \cup H_2 \cup (H_3 \cap X) = F_S$.
(iv) Suppose that all units in H_3 are also in X. In that case, $H_3 \cap X = H_3$ and Lemma 1 implies immediately that $H_1 \cup H_2 \cup (H_3 \cap X) = F_S$.

Using Lemmas 2 and 3, we may then obtain the following result.

Theorem 1: If (i) S is a BGM model, (ii) Hypotheses 1 and 2 are satisfied, and $\|F_S\| \leqslant \tau$, then the set F_A generated by Algorithm 1 is equal to F_S.

It is known that if a BGM model is τ-diagnosable, then Hypotheses 1 and 2 are satisfied [1]. Hence, we obtain the main result of the paper.

Theorem 2: Let S be a τ-diagnosable BGM fault model and let F_S be the set of faulty units in S. If $\|F_S\| \leqslant \tau$, then the set F_A generated by Algorithm 1 is equal to F_S.

Reference [5] contains a comprehensive bibliography concerning system level fault models and some additional results concerning Algorithm 1. For example, it is shown that if S is a BGM model, Hypothesis 1 is satisfied and no two units test each other [2, Theorem 1], then Hypothesis 2 is automatically satisfied and Algorithm 1 always stop in either Step 1 or 3 whenever $\|F_S\| \leqslant \tau$. Note that Holt has obtained diagnosability results and some diagnosis algorithms for a system level fault model that is related to the BGM model [3], [4].

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