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NUMERICAL ANALYSIS OF A BATTLE FROM HISTORY

by

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INTRODUCTION

→ A numerical method and computer programs for computing the probability of the outcomes of a type of heterogeneous battle model can be found in AD A129468, ~~DOAE Internal Paper 37/3(1/81)~~ → The method requires the numbers of weapons involved, their relative effectiveness, and the loss levels at which victory is attained. In order to test the method and its applicability, Professor N H Gibbs, formerly Professor of the History of War at Oxford, was asked to provide suitable numerical data from historical battles. No restriction was placed on the period from which the battles might be taken.

Data regarding three battles of the Hundred Years War was provided, namely Crécy, Poitiers, and Agincourt. These battles were of a similar type. In all cases a smaller, well disciplined, and better armed force, in a good defensive position beat a much larger force.

In this paper we consider the battle of Crécy. We give first an edited version of the material provided by Professor Gibbs. We then consider the choice of appropriate attrition functions. Some examples of the computed results are given.

The Battle of Crécy

The battle of Crécy took place on 26 August 1346. Edward II had landed his forces at Cherbourg on 12 July 1346 and commenced a pillaging march through Normandy. Because of reports of gathering French Forces he decided to retreat towards his allies the Flemings. He had difficulty in finding crossings of the Seine and the Somme but succeeded in crossing the former near Paris and the latter near its mouth. On 25 August he halted between the villages of Crécy and Wadicourt in a good defensive position. The site is close to the river Maye about 12 miles north of Abbeville.

English Manpower

When Edward III set sail from England he had with him an army of about 14,000 men (though some historians give lower estimates of 10,000 to 11,000 men). Allowing for losses in marches and skirmishes on the route to Crécy, Edward probably had a total of about 11,000 on the day of the battle. The best estimates give an army comprising:

Men-at-arms, Knights	2000
English bowmen	5000
Welsh bowmen	18000
Welsh spearmen	18000
Hobilans (mounted spearmen)	500

French Manpower

The size of the French army is much more difficult to estimate. There had been no such formal levying and commissioning as Edward III had carried out in England before the campaign began. French mobilisation (eg with communal militia) continued right up to the time of battle.

Some French estimates put the French total at little more than the English. Most English historians, then and since, have reckoned the French total at about 30,000. Something like the latter seems the more probable in view of the ground covered by the French forces on 25 August, and the time it took them to assemble. The best estimate gives:

Noblemen, knights, men-at-arms	12,000
Genoese crossbowmen	4,000 to 6,000
Communal militia and mercenaries	10,000 to 12,000,

but the last item is little more than a guess.

Weapons

Each side had essentially three types of weapon, swords, lances or spears, and bow. Weapons were common to the two sides but there was one major exception. The French archers used the crossbow (arblast) while the English archers used the long bow. Each side had shields. The balance of evidence suggests that the English had two or three small cannon which were used in the early phase of the battle.

The greater part of the English army, including the knights, men-at-arms, and many of the spearmen and archers had horses. However, all dismounted for the battle and fought on foot. On the French side knights and men-at-arms fought as mounted cavalry. Crossbow men and militia were not mounted.

There was no significant difference between the long bow and crossbow in range or in penetration. Both weapons could be very effective in the hands of trained men. But the longbow was much less cumbersome and could be 'fired' at about three times the rate of the crossbow. The longbowman usually carried 24-48 arrows and could usually be supplied with more on the battle field. Arrows would sometimes be recovered from the ground or from enemy bodies.

We know from evidence of archery contests that the longbow arrow could penetrate two layers of mail armour at optimum range of 100 to 150 yards.

Tactics

The English prepared for an essentially defensive battle in a position of natural strength. They lay on the forward slope of a hill side. To the south they were protected by a thick wood and to the north, though less effectively, by the village of Wadicourt. A sketch map, which shows the disposition and gives some indication of the terrain, can be seen in the Encyclopaedia Britannica Vol 6, page 653.

The English army was drawn up in three battle groups. Two were placed forward to the right and left respectively, and the third was placed behind them in reserve. Baggage was kept well to the rear. Each battle group consisted of a central section of dismounted knights, men-at-arms, and spearmen, and two flanking sections of archers in diagonal formation. The two forward battle groups formed together a 'W' with its top towards the French.

The central salient and the wings consisted of archers.

Holes of about one foot square and one foot deep had been dug in front of the archers. Some of the holes were provided with pointed stakes, but they were normally used to hold a supply of arrows. Edward III made the general assumption that enemy men-at-arms would attack English men-at-arms. Archers would then pour in flanking fire.

The forces, so arranged, were trained and disciplined. They were ordered not to break ranks but to receive the enemy attack in their prepared positions. Any pursuit of the enemy was to be made only after the latter had broken against the defended position. The tactics used were not due to luck or guess-work. They had slowly developed during and since the reign of Edward I, and had been used successfully against the Scots on several occasions.

The French had no previously determined battle plan. For them it was essentially an encounter battle, with all the elements of surprise, unpreparedness and disorganisation that such battles imply.

The French advanced in a long straggling column from south to north along the Abbeville/Hesdin road, they were fully convinced that Edward III and his army were still retreating. The column was disorderly one, with new troops arriving all the time, and stretched over almost all the distance from Abbeville to Crécy. A distance of about ten miles.

It was not until late afternoon that the French king was informed by scouts of Edward III's position. He then tried to halt his army so that it could take up position for the night, reform and prepare for battle the next day. But the advance had been too disorganised to be controlled now. The only tactical arrangement which reflected any battle plan was that the Genoese crossbowmen were in front of the French army to launch the initial 'fire-power' attack. Behind them the cavalry, knights and men-at-arms, and then the communal militia infantry jostled for position in confusion without any unified control.

The Battle

The initial phase of the battle was the only one which showed any degree of planning on the part of the French. Their van reached the battlefield in the evening and the Genoese crossbowmen attacked the English army in its strong defensive position. The attack was designed to prepare the way for a cavalry charge. It was a complete failure as the crossbowmen were overwhelmed by the longbow reply at 100 to 150 yards. They retreated but the French knights behind the crossbowmen increased confusion by charging into them and attacking them for treachery.

The main battle was a series of about 15 cavalry charges. They were unco-ordinated and made by various French retinues as they arrived on the battle field.

Most of the attacks were made against the English men-at-arms and thus had to run the gauntlet of the flanking archers. As a result the flanks of the attackers were decimated. In the centre some French knights got through

and engaged in hand to hand fighting. However, no attack succeeded in breaking the English lines and the attacks ceased late in the evening when darkness fell.

There was no English pursuit either that evening or the next day. Because of the great difference in losses between the two sides complete victory had been obtained on the battle field itself.

Casualties

There is some factual evidence for the English casualties. These amounted to two knights and one squire together with about 40 men-at-arms and archers. The latter occurred among a few dozen Welsh infantry who disobeyed orders and broke ranks to go out on to the battle field to plunder dead and wounded.

French casualties are much more difficult to estimate. A battlefield count suggested that about 1500 lords and knights had fallen. For the rest it is all guess-work. Casualties could have been anything from 10,000 upwards, with losses heaviest among the mounted men-at-arms. French infantry, apart from the crossbowmen, took little part in the battle.

ANALYSIS

This historical battle can be used in more ways than one to test the numerical procedures for computing the probabilities of the different outcomes. In the following we suppose that battle has not yet commenced and consider the situation from the point of view of the English commander. The numbers, weapon types, and dispositions of the forces are known but the mode of development of the battle is a matter of conjecture.

Weapon Categories

The analysis begins by separating the weapons on the two sides into categories. Weapons in the same category need not be identical but are assumed to be equally effective and equally vulnerable. It is a reasonable simplification to regard the English army as made up of only two categories of weapon. In the first category we include the knights, men-at-arms, and spearmen. All of these fought dismounted with hand to hand weapons. The second category consists of the archers who fought with the longbow.

On a different basis we may separate the French army into two weapon categories. The first category consists of the knights and men-at-arms all of whom fought on horseback. The second category consists of the dismounted troops, that is, the crossbowmen and militia.

A more rigorous analysis requires a separate category for the crossbowmen. When properly used they are more effective than the militia. However, they do not play a dominant role and an extra category adds to the computation.

'Valuation', Loss Level, Draw Level

At a given instant let H, M be the numbers of survivors in the respective French categories. The French 'valuation' is then $\alpha_1 H + \alpha_2 M$ where α_1, α_2 are suitably chosen constants. Similarly, the English valuation is $\beta_1 E + \beta_2 A$,

where v_i, v_j are the numbers of survivors and β_i, β_j are suitable constants. Note that 'valuation' does not necessarily determine effectiveness.

Battle ends either when the French valuation falls to the French 'loss level' or when the English valuation falls to the English 'loss level'. The end is an English victory when the French valuation has reached French 'loss level' but the English valuation remains greater than the English 'draw level'. A French victory is defined similarly and a battle end which is not a victory is a draw.

On the English side it is quite realistic to give equal value to the weapons of the two categories. On the French side we make a similar assumption but can justify it only on grounds of simplicity. Hence, in the first instance, we chose $\alpha_1 = \alpha_2 = \beta_1 = \beta_2 = 1$.

For both sides we take the loss level as 2/3 of the initial valuation and the draw level as 3/4 of the initial valuation.

Attrition Functions

The effectiveness of the French force is determined by two attrition functions $f_1(N_1, N_2)$ and $f_2(N_1, N_2)$, where N_1 and N_2 are the numbers of survivors in the first and second weapon categories respectively. The computational method allows elaborate forms for f_1 and f_2 but in the first analysis the simple linear forms $f_1 = A_1 N_1 + A_2 N_2$ and $f_2 = B_1 N_1 + B_2 N_2$, where A_1, A_2, B_1, B_2 are constants, should be used. The functions f_1 and f_2 are so defined that, in a short interval of time Δt the probability of a single casualty in category 1 or category 2 respectively of the English force is $\Delta f_1 + C(\Delta t)$ or $\Delta f_2 + D(\Delta t)$ respectively.

In a similar way the effectiveness of the English force is defined by two linear functions $\psi_1 = C_1 N_1 + C_2 N_2$ and $\psi_2 = D_1 N_1 + D_2 N_2$, where N_1, N_2 are the numbers of survivors in the first and second weapon categories, and C_1, C_2, D_1, D_2 are constants.

The coefficients $A_1, A_2, B_1, B_2, C_1, C_2, D_1, D_2$ determine the effectiveness of the individual weapons. This is shown in the following Table, where the term 'knights' includes men-at-arms and spearmen, and 'crossbows' includes militia.

A_1	is the effectiveness of the French knights against the English knights
A_2	" " " " " crossbows " " " "
B_1	" " " " " knights " " " archers
B_2	" " " " " crossbow " " " "
C_1	" " " " " English knights " " French knights
C_2	" " " " " archers " " " "
D_1	" " " " " knights " " " crossbows
D_2	" " " " " archers " " " "

Choice of the Attrition Coefficients

Good estimates of the attrition coefficients can be made only from experiments with the weapons themselves and field observations of their use. When data cannot be obtained by such means the attrition coefficients are a matter for judgement or even guesswork. Nevertheless, in a given situation the dominant weapons are almost always known and an approximate assessment of their relative effectiveness should be possible. The other weapons can be compared at least qualitatively with the dominant ones.

The terminal distribution over the survivors is not affected by multiplying all eight attrition coefficients by the same constant. Hence, any non-zero coefficient may be chosen equal to 1.0 and the others scaled appropriately. In the present case most is known about the relative effectiveness of the crossbow and longbow, hence we take A_1 , which determines the effectiveness of the French crossbows against the English knights, as 1.0.

The longbow is at least three times as effective as the crossbow. Hence we take C_4 , which determines the effectiveness of the English longbow against the French knights, as 3.0.

French crossbows have been included in the same category as the militia. A French knight is presumably more effective than a militiaman. Hence, we take A_1 , which determines the effectiveness of the French knight against the English knight, as 1.5. Compare this figure with $A_1 = 1.0$.

When other things are equal, an English knight is not different from a French knight. Hence we take C_1 , which determines the effectiveness of the English knights against the French knights, as 1.5. We have $A_1 = 1.0$.

A French knight is less effective against an English longbowman than he is against an English knight. He is more likely to come to close quarters with the latter. Hence we take B_1 , which determines the effectiveness of the French knights against the English longbows, as 1.2. We have $B_1 = 0.8 \times A_1$.

English knights are as effective against French crossbowmen and militia as they are against French knights. Hence, we take D_1 , which determines the effectiveness of the English knights against the French crossbowmen, as 1.5. We have $D_1 = C_1$.

The French militiaman is less effective against the English longbow than is the French knight. Hence, we take B_2 , which determines the effectiveness of the French crossbows and militia against the English longbows, as 0.8. Compare this with $A_1 = 1.0$.

The English longbow is equally effective against both categories of French weapon. Hence, we take D_4 which determines the effectiveness of the English longbows against the French crossbows and militia, as 3.0. We have $D_4 = C_4$.

No allowance has yet been made for the better deployment and better discipline of the English force. To make such allowance we multiply the coefficients C_1, D_1, C_4, D_4 by a factor in the range 1.5 to 6.0. The results of

some computer runs obtained with this range of coefficients, are given in the appendix.

Suggestions for Further Analysis

Some of the assumptions made in the above analysis do not conform well with the actual development of the battle. There were two phases and these could be analysed separately. The French made no real use of the communal militia which should be ignored. The English had no need to call on the reserve battle group. Only those weapons which were engaged should be considered.

The first phase of the battle involved a single French category, the crossbowmen, and a single English category, the longbowmen. The English loss level should correspond to a break in the English line.

The second phase of the battle involved a single French category, the mounted knights, and the two English categories. French effectiveness was weakened by attacking in groups each of which was subject to the whole of the longbow fire. This 'penny packet' effect has been observed in modern tank battles.

10.0110M = BLUE

PERMCH = 1.50

PALE-30 2.02 TERNAL DISTRIBUTION = 1000000
(GIVEN ATTRITION FUNCTIONS)

MUC 2.02 CEFY C SCALING 1000

SEL PARAMETERS

•E2PO'S KNIGHTS

INITIAL UNITS M1 = 12 M2 = 12

VALUATION COEFFICIENT = 1.00 LOSS VALUATION RATIO = 0.40 DEAN VALUATION RATIO = 0.75

INITIAL VALUATION = 30.00 LOSS VALUATION = 12.00 DEAN VALUATION = 22.50

BLUE PARAPETEGS

LE3PONS KNIGHTS

INITIAL UNITS N1 = 6 N2 = 7

VALUATION COEFFICIENT = 1.00 LOSS VALUATION RATIO = 0.50 DEAN VALUATION RATIO = 0.75

INITIAL VALUATION = 11.00 LOSS VALUATION = 5.50 DEAN VALUATION = 7.75

ATTRITION FUNCTIONS RECS EFFECTIVENESS

PA1=1+0.1*2*MU2 AGAINST BLUE KNIGHTS

A1 = 1.500 A2 = 1.000

PA2=1+0.1*2*MU2

B1 = 1.200 B2 = 0.800

ATTRITION FUNCTIONS BLUE'S EFFECTIVENESS

PS1=C2*MU1+C4*MU2 AGAINST RED KNIGHTS

C3 = 4.500 C4 = 3.000

PS2=C3*MU1+C4*MU2 AGAINST RED CROSSBOWS

D3 = 4.500 D4 = 3.000

PROBABILITY OF BLUE VICTORY 0.250000

PROBABILITY OF BLUE VICTORY 0.250000

PROBABILITY OF BLUE 0.250000

PROBABILITY OF BLUE 0.250000

SOME NUMERICAL RESULTS

APPENDIX

1

200 T. ...

200 T. ... SCALING 1000

SEE PARAMETERS

WEAPONS KNIGHTS CROSSBOWS

INITIAL UNITS M1 = 12 M2 = 18

VALUATION COEFFICIENT = 1.00 LOSS VALUATION RATIO = 0.60 DRAW VALUATION RATIO = 0.75

INITIAL VALUATION = 30.00 LOSS VALUATION = 18.00 DRAW VALUATION = 27.00

BLUE PARAMETERS

WEAPONS KNIGHTS

INITIAL UNITS M1 = 4 M2 = 7

VALUATION COEFFICIENT = 1.00 LOSS VALUATION RATIO = 0.60 DRAW VALUATION RATIO = 0.75

INITIAL VALUATION = 11.00 LOSS VALUATION = 6.60 DRAW VALUATION = 8.25

ATTRITION FUNCTIONS RED'S EFFECTIVENESS

P1=A1*M1+A2*M2 AGAINST BLUE KNIGHTS

A1 = 1.500 A2 = 1.000

P2=B1*M1+B2*M2

B1 = 1.250 B2 = 0.500

ATTRITION FUNCTIONS BLUE'S EFFECTIVENESS

P3=C1*M1+C2*M2 AGAINST RED KNIGHTS

C1 = 2.250 C2 = 4.500

P4=D1*M1+D2*M2

D1 = 2.250 D2 = 4.500

MT = 273 HT = 213

PROBABILITY OF RED VICTORY 0.698175

PROBABILITY OF BLUE VICTORY 0.037014

PROBABILITY OF DRAW 0.273111

TOTAL PROBABILITY 1.000000

PROBABILITY OF RED FAVOURED DRAW 0.136236

PROBABILITY OF BLUE FAVOURED DRAW 0.043175

RED PARAMETERS

WEAPONS KNIGHTS
 INITIAL UNITS N1 = 12 N2 = 1
 VALUATION COEFFICIENT = 1.00 LOSS VALUATION RATIO = 0.60 DRAW VALUATION RATIO = 0.75
 INITIAL VALUATION = 50.00 LOSS VALUATION = 1.00 DRAW VALUATION = 24.50

WEAPONS KNIGHTS
 INITIAL UNITS N1 = 4 N2 = 7
 VALUATION COEFFICIENT = 1.00 LOSS VALUATION RATIO = 0.60 DRAW VALUATION RATIO = 0.75
 INITIAL VALUATION = 11.00 LOSS VALUATION = 6.60 DRAW VALUATION = 6.25

ATTRITION FUNCTIONS RED'S EFFECTIVENESS
 P1 = A1 * N1 + A2 * N2 AGAINST BLUE KNIGHTS
 A1 = 1.500 A2 = 1.000
 P2 = B1 * N1 + B2 * N2 AGAINST BLUE LONGFONS
 B1 = 1.200 B2 = 0.500

ATTRITION FUNCTIONS BLUE'S EFFECTIVENESS
 Q1 = C1 * N1 + C2 * N2 AGAINST RED KNIGHTS
 C1 = 2.000 C2 = 1.000
 Q2 = D1 * N1 + D2 * N2 AGAINST RED CROSSBOWS
 D1 = 0.800 D2 = 1.000

PROBABILITY OF RED VICTORY 0.041921
 PROBABILITY OF BLUE VICTORY 0.64705
 PROBABILITY OF DRAW 0.311028
 TOTAL PROBABILITY 1.000000