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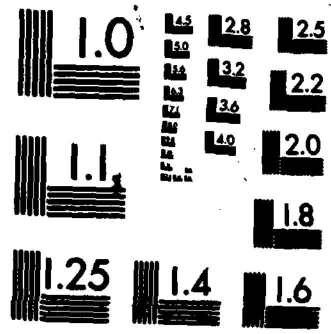

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FOREIGN TECHNOLOGY DIVISION

PHASE RADIO ENGINEERING SYSTEMS

(Selected Pages)

by

V.B. Pestryakov

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FTD- ID(RS)T-0229-83

UNEDITED MACHINE TRANSLATION

FTD-ID(RS)T-0229-83 28 April 1983

MICROFICHE NR: FTD-83-C-000538

PHASE RADIO ENGINEERING SYSTEMS (Selected Pages)

By: V.B. Pestryakov

English pages: 628

Source: Fazovyye Radiotekhnicheskiye Sistemy, Publishing House "Sovetskoye Radio", Moscow, 1968, pp. 42-234; 273-466

Country of origin: USSR
This document is a machine translation.
Requester: DRSMI/YDL
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U. S. BOARD ON GEOGRAPHIC NAMES TRANSLITERATION SYSTEM

Block	Italic	Transliteration	Block	Italic	Transliteration
А а	*А а*	A, a	Р р	*Р р*	R, r
Б б	*Б б*	B, b	С с	*С с*	S, s
В в	*В в*	V, v	Т т	*Т т*	T, t
Г г	*Г г*	G, g	У у	*У у*	U, u
Д д	*Д д*	D, d	Ф ф	*Ф ф*	F, f
Е е	*Е е*	Ye, ye; E, e*	Х х	*Х х*	Kh, kh
Ж ж	*Ж ж*	Zh, zh	Ц ц	*Ц ц*	Ts, ts
З з	*З з*	Z, z	Ч ч	*Ч ч*	Ch, ch
И и	*И и*	I, i	Ш ш	*Ш ш*	Sh, sh
Й й	*Й й*	Y, y	Щ щ	*Щ щ*	Shch, shch
К к	*К к*	K, k	Ъ ъ	*Ъ ъ*	"
Л л	*Л л*	L, l	Ы ы	*Ы ы*	Y, y
М м	*М м*	M, m	Ь ь	*Ь ь*	'
Н н	*Н н*	N, n	Э э	*Э э*	E, e
О о	*О о*	O, o	Ю ю	*Ю ю*	Yu, yu
П п	*П п*	P, p	Я я	*Я я*	Ya, ya

*ye initially, after vowels, and after ъ, ь; e elsewhere.
When written as ё in Russian, transliterate as yё or ё.

RUSSIAN AND ENGLISH TRIGONOMETRIC FUNCTIONS

Russian	English	Russian	English	Russian	English
sin	sin	sh	sinh	arc sh	$\sinh^{-1}$
cos	cos	ch	cosh	arc ch	$\cosh^{-1}$
tg	tan	th	tanh	arc th	$\tanh^{-1}$
ctg	cot	cth	coth	arc cth	$\coth^{-1}$
sec	sec	sch	sech	arc sch	sech^{-1}
cosec	csc	csch	csch	arc csch	csch^{-1}

Russian	English
rot	curl
lg	log

GRAPHICS DISCLAIMER

All figures, graphics, tables, equations, etc. merged into this translation were extracted from the best quality copy available.

PHASE RADIO ENGINEERING SYSTEMS.

(Fundamentals of statistical theory).

V. B. Pestryakov.

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§ 1.8. Initial considerations to the question about the analysis of the action of interferences in the phase systems. The problem of diagram is the detection of radio signal or the measurement of any parameter of radio signal. No matter how was perfect diagram, in it cannot be completely reduced those distortions, which introduces the interference into the utilized parameter of signal.

The basic problem of this work is the determination of the statistical characteristics of the mixture of signal and interference which determine the work of phase systems. In this case it is necessary to have in mind that the signal in phase of which is included useful information, can be very different: by continuous harmonic or modulated, pulse in the form of separate impulse/momentum/pulse, complicated, that consists of sequence or pulse packet, and finally by noise-like.

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In the subsequent chapters are examined the statistical characteristics of the phase of one interference and mixture of interference with the signal, is revealed/detected the effect of phase on the optimum detection of signal and characteristic of phase detection and finally are investigated the methods of the optimum measurement of phase and tracking it.

In order to come to light/detect/expose the statistical characteristics of mixtures which must be studied, on the basis of that presented in S 1.2, 1.3, 1.4, 1.5 and 1.6, it is necessary to consider the basic generalized versions of the diagrams, which realize an extraction of the information, which is contained in the phase of signal. The simplest generalized pattern can consist of the receiver which realizes amplification of signal and its preliminary selection, and the meter of phase which reveals/detects the information, which is contained in the phase of signal or in a phase difference. If information is embedded during the phase of modulation, then between the receiver and the meter it is necessary to include/connect detector. The block diagrams of the receiving and measuring devices/equipment for these cases are given in Fig. 1.8.1.

Signal amplitude, which enters the entrance of receiver, undergoes large changes in dependence on the power of transmitter, distance between the points of reception/procedure and transmission, etc. Meters, as a rule, require that the amplitude of the stress/voltage supplied on them would be changed within small limits. Furthermore, the cascades/stages of receiver have close margin of linearity. On these reasons in many instances block diagram must be supplemented by the blocks, which ensure insignificant changes in the signal amplitude at the output of receiver. As such blocks can be used ARU. Then block diagram takes the form, depicted in Fig. 1.8.2.

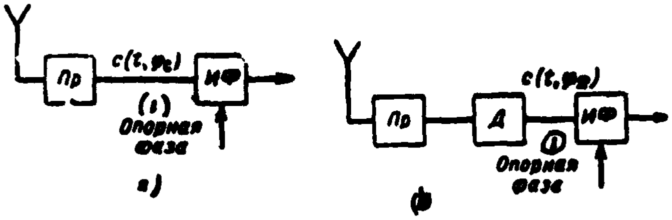

Fig. 1.8.1. The schematics of the receiving and measuring devices/equipment: Пр - receiver; ИФ - meter of phase; Д - detector. a) information is laid during the phase of radio signal; b) information is laid during the phase of modulation.

Key: (1). Supporting/reference phase.

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When information is laid during the carrier frequency or during the phase modulation on the phase of signal, it is possible for the same targets to use limiter, including it between the receiver and the meter. The corresponding block diagram is given in Fig. 1.8.3. With respect to the constancy of the signal amplitude, supplied to the meter, both these diagrams give analogous results. However, according to many important properties, in particular on how in them will function interferences, these diagrams substantially differ.

Diagrams given in Fig. 1.8.1, 1.8.2 and 1.8.3 relate to the single-channel ones. In these diagrams the information is laid during the phase of signal. Such diagrams include the systems of phase tracking (synchronization), the ranging system and radial velocity.

In many phase systems, for example difference-range finding and phase direction finders, is used the information, placed into a phase difference of two signals. The generalized schematic of this system is given in Fig. 1.8.4. This diagram is two-channel. It is obvious that in general form its properties are determined by the functions of the distribution of a phase difference of two signals, mixed with the interferences. Just as in the single-channel systems, this diagram can have versions: with the use of a phase of modulation, with the inclusion/connection of ARU and limiter. Technical fulfillment of such systems has many special features/peculiarities, for example in them it is simple to apply superheterodyne circuits, since the phase of heterodyne will not enter into the result of measurements, if it is made general/common/total for both channels.

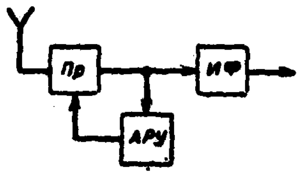

Fig. 1.8.2.

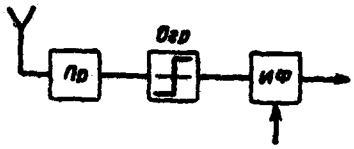

Fig. 1.8.3.

Fig. 1.8.2. Schematic of receiving and measuring device/equipment with ARU: Пр - receiver; ИФ - meter of phase; ARU - automatic gain control.

Fig. 1.8.3. Schematic of receiving measuring device/equipment with limiter: Пр - receiver; Огр - limiter; ИФ - meter of phase.

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The instability of phase displacement at equipment is evaluated only as a difference in the departure/attendance of phase in two channels and there can be much less than instability in each of them.

The complete analysis of such systems is an independent problem. As it will be shown further, many, practically important properties and special features/peculiarities of two-channel phase systems can be evaluated on the basis of the theory of single-channel systems and functions of the distribution of the mixture of signal and interference; therefore subsequently two-channel systems in general form are not examined.

Let us consider now a question about the principle of the construction of phasemeter. If useful information is laid during the phase of signal, then it can be extracted by two methods:

a) by the direct measurement of phase, it is more precise than the difference between the phase of signal and the phase of reference voltage which is accepted as initial or zero;

b) by the measurement of frequency and by the subsequent integration of result or, it is more precise, by the measurement of a difference in the frequency of the signal accepted and frequency of reference voltage with the subsequent integration of this difference.

Let us assume for simplicity, that the signal is the harmonic oscillation

$$c(t)=A_c\cos(\omega_0 t+\varphi_c).$$

Reference voltage

$$c_{\text{оп}}(t)=A_{\text{оп}}\cos\omega_0 t.$$

In the direct measurement of phase displacement, i.e., in the phase measurements we will obtain

$$\varphi_{\text{из}}=\omega_0 t+\varphi_c-\omega_0 t=\varphi_c.$$

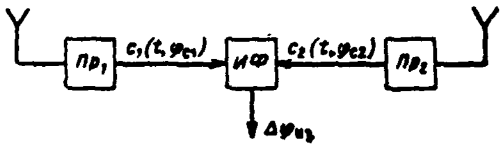

Fig. 1.8.4. The diagram of the two-channel diagram: Пр - receiver; ИФ - meter of phase.

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Let us note that according to the results of phase measurements can be found the signal frequency ω_c or its deviation $\Delta\omega$ from ω_0:

$$\omega_c = \omega_0 + \frac{d\varphi_c}{dt} = \omega_0 + \Delta\omega,$$

$$\Delta\omega = \frac{d\varphi_{из}}{dt}.$$

In the measurement of phase displacement through the measurement of frequencies, i.e., in the frequency measurements, the expressions will take the form

$$\omega_c = \frac{d}{dt}(\psi) = \frac{d}{dt}(\omega_0 t + \varphi_c) = \omega_0 + \frac{d\varphi_c}{dt},$$

$$\omega_{оп} = \omega_0.$$

Meter is determined

$$\Delta\omega = \omega_c - \omega_0, \ \Delta\omega = \frac{d\varphi_c}{dt},$$

whence

$$\varphi_{из} = \varphi_{нач} + \int_0^t \Delta\omega dt = \varphi_{нач} + \varphi_c \qquad (1.8.1)$$

with

$$\varphi_{\text{нач}}=0, \quad \varphi_{\text{из}}=\varphi_c.$$

From the point of view of final results the diagrams examined are analogous, if we have in mind that the integration constant $\varphi_{\text{нач}}$ can be considered known or equal to zero. However, according to the principle of the construction of meter and in the action of interferences, they essentially differ. Fig. 1.8.5 gives block diagrams for the cases of phase and frequency measurements. The meters utilized in the diagrams can have different characteristics. In the simplest case the meter can be considered ideal, i.e., to assume that its readings/indications do not depend on the amplitude of mixture and they instantly and accurately map the phase of mixture. Under the actual conditions the meter possesses inertness or inertness specially is introduced in order to improve results.

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In certain cases the measured phase is used for subsequent reworking of results. In this case the filtration is realized after phasemeter. During the analysis of this version it is convenient to assume that the meter is ideal, and after it is connected filter. Block diagrams examined above make it possible to come to light/detect/expose those statistical characteristics of the mixture of signal and interference, which are of interest. It is necessary to study the distribution function and the energy spectra of amplitude, phase, derived phase (frequency), mixture of signal and interference, and also functions of the distribution of zero random processes. Of great interest are also the statistical characteristics of one interference. They give the possibility to consider the behavior of receiving and measuring device/equipment in the absence of signal. This occurs in the case when signal vanished or it very weak. This state of equipment frequently is encountered during the control and its operation. Statistical properties of the mixture of signal and interference are examined in Chapters 2 and 3. Study of the statistical properties of the mixture of signal and interference yet does not give complete representation about properties and possibilities of phase systems. In the phase systems is assumed the execution of two basic operations.

The first operation is the detection of signal, with which the operator or the automatically functioning device/equipment must "make a decision", that signal there is i.e., the system functions and the receiving and measuring device/equipment is located in the zone of its action. In the systems of discrete/digital radio communication this operation is basic.

The second operation is the extraction of the information, placed into the waveform. For this must be accomplished the measurement of phase or tracking it, if in the process of observation it is changed.

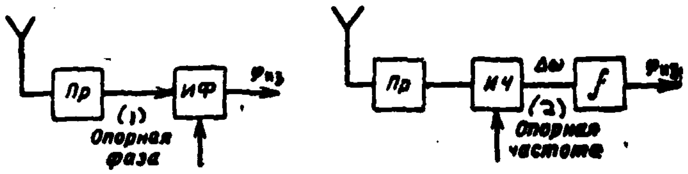

Fig. 1.8.5. Diagrams for the phase and frequency measurements: Пр - receiver; ИФ - meter of the phase: ИЧ - meter of the frequency: ∫ - integrator.

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It is obvious that there is fundamental interest in research of the possibility of the optimization of these basic procedures or operations, implemented in the phase systems, and also the properties of optimum feelers and meters. These questions are examined into 4, 5, 6, 7th chapters.

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Chapter 2.

STATISTICAL CHARACTERISTICS OF THE PHASE OF FLUCTUATING INTERFERENCE.

§ 2.1. Fluctuating interference as random process. The source of the fluctuating interferences are internally-produced noise of receiver, atmospheric and star static, and also some forms of electronic jamming.

Let us give briefly basic definitions for the interference as random process.

As everyone random process, interference ш(t) is characterized:

a) by the one-dimensional distribution, which gives the probability density of its values $w_1(ш, t)$. One-dimensional probability density can be found, if, is known the integral function of distribution $F(ш, t)$:

$$w_1(ш, t)=\frac{\partial F(ш, t)}{\partial ш},$$

where $F(ш, t)$ - probability that the value of described random process at moment of time t will not exceed level ш.

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In this case it is thought that the observation of random process at moment/torque t is realized simultaneously on a large number of realizations (one and the same random process) and that therefore value $F(ш, t)$ becomes almost constant, little that is that depending on a change in the number of workable realizations or, as they say, it acquires the statistical regularity:

b) by the two-dimensional distribution, which characterizes probability density $w_2(ш_1, ш_2, t_1, t_2)$, the specific combinations of the instantaneous values $ш_1$ and $ш_2$, observed at different moments of time t_1 and t_2. Consequently, function characterizes also the transient nature of random process. Two-dimensional probability density can be found from $F_2(ш_1, ш_2, t_1, t_2)$,

$$w_2(ш_1, ш_2, t_1, t_2)=\frac{\partial F_2(ш_1, ш_2, t_1, t_2)}{\partial ш_1 \partial ш_2},$$

where $F_2(ш_1, ш_2, t_1, t_2)$ - the two-dimensional integral distribution function, is determined probability that in each of the realizations the value of random process at moment/torque t_1 will be less than $Ш_1$, and at moment/torque t_2 - it is less than $ш_2$;

c) by the mathematical expectation $m_1(ш)$, which is determined from

$$m_1(ш, t)=\int_{-\infty}^{+\infty} ш, w(ш, t)\,dш;$$

d) by dispersion $\sigma^2_ш$ or by the second central moment of the distribution

$$\sigma^2_ш = M_2(ш, t)=\int_{-\infty}^{+\infty}[ш-m_1(ш)]^2 w(ш, t)\,dш.$$

In the majority of the cases in the description of interferences it proves to be possible to allow the series/row of the simplifications, basic from which is assumption about the stability of the random of process-interference.

Random process is stationary, if the distribution functions do not depend on time. In this case w_2 will depend only on a difference in the moments/torques of the time

$$-w_2(ш_1, ш_2, \tau),$$

where $\tau=t_2-t_1$. It is obvious that in this case $w(ш)$, $M_2(ш)=\sigma^2_ш$ and $m_1(ш)$ do not depend on time.

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The sense of this assumption lies in the fact that for those determined of the segment of time and conditions the works of property and noise characteristic are considered constants. In this case for other conditions of work or another interval of time it is possible to take another noise characteristics. This approach to the solution of problem considerably simplifies analysis and virtually completely itself it justifies in such a case, when changes in noise characteristics occur slowly. In other cases this assumption cannot be accepted and then it is necessary to consider interference as unsteady random process. In the following presentation the primary interference, which functions at the entrance of the receiving and measuring device/equipment, we will consider stationary.

It is usually considered that the law of distribution of interference is normal and the distribution functions take the form

$$w(\text{ш}) = \frac{1}{\sqrt{2\pi}\sigma_{\text{ш}}} e^{-\frac{(\text{ш} - m_1)^2}{2\sigma_{\text{ш}}^2}},$$

$$w_2(\text{ш}_1, \text{ш}_2, \tau) = \frac{1}{2\pi\sigma_{\text{ш}}^2 \sqrt{1 - R_{\text{ш}}^2(\tau)}} \times$$

$$\times e^{-\frac{(\text{ш}_1 - m_1)^2 + (\text{ш}_2 - m_1)^2 - 2R(\tau)(\text{ш}_1 - m_1)(\text{ш}_2 - m_1)}{2\sigma_{\text{ш}}^2 [1 - R_{\text{ш}}^2(\tau)]}},$$

where m_1 - average/mean value of interference; $\sigma_{\text{ш}}^2$ - dispersion of interference; $R_{\text{ш}}(\tau)$ - correlation coefficient, which shows the degree of statistical connection/communication between the values of random process, divided by the interval of time τ.

Usually interference is the random process, which has the average/mean value of m_1, equal to zero; then expressions are simplified and take the form

$$w(ш)=\frac{1}{\sqrt{2\pi}\sigma_ш}e^{-\frac{ш^2}{2\sigma_ш^2}},$$

$$w_2(ш_1,\ ш_2,\ \tau)=\frac{1}{2\pi\sigma_ш^2\sqrt{1-R_ш^2(\tau)}}e^{-\frac{ш_1^2+ш_2^2-2R_ш(\tau)ш_1,ш_2}{2\sigma_ш^2[1-R_ш^2(\tau)]}}. \quad (2.1.1)$$

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Besides the characteristics given above is very extensively used also the concept of correlation function

$$B_ш(\tau)=\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}ш_1,\ ш_2,\ w_2(ш_1,\ ш_2,\ \tau)\,dш_1dш_2.$$

To sometimes more conveniently use relative value

$$R_ш(\tau)=\frac{B_ш(\tau)}{\sigma_ш^2},$$

$$B_ш(0)=\sigma_ш^2 \text{ при } m_1=m_1(ш)=0,$$

where $R_ш(\tau)$- correlation coefficient.

It is obvious that on determination with $\tau\to0$ $B_ш(\tau)\to\sigma_ш^2$, if $m_1(ш)$ =0. Values $B_ш(\tau)$ and $R_ш(\tau)$ characterize statistical connection/communication of two values of random process, divided by the interval of time τ. When $\tau\to0$ $R_ш(\tau)\to1$; when $\tau\to\infty$ $R_ш(\tau)\to0$. The value of the interval of time τ, with which value $R_ш(\tau)$ becomes insignificant, is called time or interval of correlation $\tau_к$.

Values of the random variables, which characterize random process at the points, divided by the interval of time $\tau_к$, can be considered not virtually connected. In other words, the value of random process at one point does not determine the value of random process at another point, if these points are divided time interval, large than $\tau_к$.

Knowing $B_ш(\tau)$, it is possible to find the energy spectrum of the process

$$G_ш(\omega)=2\int_{-\infty}^{+\infty} B_ш(\tau)\,e^{-j\omega\tau}\,d\tau=4\int_{0}^{+\infty} B_ш(\tau)\cos\omega\tau\,d\tau. \quad (2.1.2)$$

Consequently, energy spectrum, i.e., the spectrum of power, which characterizes the power of random process per unit of band, is obtained by the determination of Fourier's image (spectrum) from the correlation function.

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If is known energy spectrum, then by it can be found correlation function with the help of reverse/inverse transform of Fourier:

$$B_ш(\tau)=\frac{1}{4\pi}\int_{-\infty}^{+\infty} G_ш(\omega)\,e^{j\omega\tau}\,d\omega=$$

$$=\frac{1}{2\pi}\int_{0}^{\infty} G_ш(\omega)\cos\omega\tau\,d\omega. \quad (2.1.3)$$

From these relationships/ratios it follows that

$$B_{ш}(0)=\sigma^2_{ш}=\frac{1}{2\pi}\int\limits_0^\infty G_{ш}(\omega)\,d\omega, \qquad (2.1.4)$$

$$G_{ш}(0)=4\int\limits_0^\infty B_{ш}(\tau)\,d\tau, \qquad (2.1.5)$$

where $G_{ш}(0)$ - power density at the low (theoretically zero) frequencies.

In many instances energy interference spectrum, which functions at the entrance, can be considered uniform in the limits from 0 to $\omega_{в}$, then $G_{ш}(\omega)=N_0,\ \omega<\omega_{в}$

$$\sigma^2_{ш}=\frac{1}{2\pi}N_0\omega_{в}=N_0 f_{в} \qquad (2.1.6)$$

or

$$N_0=\frac{\sigma^2_{ш}}{f_{в}},$$

where N_0 - the "one-sided" jamming density on the assumption that $\omega>0$. Here earlier than N_0 $G_{ш}(\omega)$ are given per unit of band not in the angular, but in ordinary frequencies.

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For the uniform spectrum the expression for the correlation function also is simplified

$$B_{ш}(\tau)=\frac{N_0\omega_{в}}{2\pi}\,\frac{\sin\omega_{в}\tau}{\omega_{в}\tau}=\sigma^2_{ш}\,\frac{\sin\omega_{в}\tau}{\omega_{в}\tau}. \qquad (2.1.7)$$

$$R_{ш}(\tau)=\frac{\sin\omega_{в}\tau}{\omega_{в}\tau},$$

The uniform spectrum and its correlation function are given in

Fig. 2.1.1.

Noise characteristics given above are theoretical and are based on the use of static averaging or averaging on the set. In this case it is assumed that for evaluating the random process it must be simultaneously repeatedly reproduced and all its values (for the specific moment/torque t or the specific moments/torques t_1 and t_2) then they are treated statistically.

For practical purposes this approach to noise characteristics as random process proves to be little convenient, since under the actual conditions the simultaneous reproduction of a large number of identical random processes meets many difficulties. More simply to be based on one reproduction of the random process after fixing which during the specific time interval, i.e., after obtaining its realization, to evaluate the characteristics of this process according to this one realization. Simpler this problem is solved for the stationary random processes which have their fundamental characteristics: the function of distribution and correlation, dispersion, mathematical expectation, etc., they do not depend on time.

Many random processes possess the property of ergodicity. For the ergodic random processes the averaging on the set can be substituted by averaging on the time.

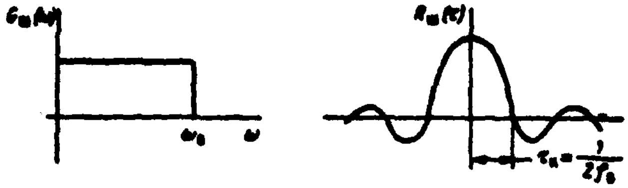

Fig. 2.1.1. Interferences with the uniform spectrum and its correlation function.

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Then fundamental characteristics can be found from the relationships/ratios

$$m_1(ш) = \lim_{T\to\infty} \frac{1}{T} \int_0^T ш(t)\,dt, \qquad (2.1.8)$$

where ш(t) - the function, which expresses the realization of the random process

$$\sigma_ш^2 = \lim_{T\to\infty} \frac{1}{T} \int_0^T ш^2(t)\,dt \text{ при } m_1(ш) = 0 \qquad (2.1.9)$$

$$B_ш(\tau) = \lim_{T\to\infty} \frac{1}{T} \int_0^T ш(t)\,ш(t-\tau)\,dt.$$

It is obvious that these relationships/ratios cannot be used for analytical obtaining of characteristics, since function ш(t) is random and cannot be expressed. In this general view these relationships/ratios make more physical sense, than analytical one, since they give demonstrative representation and the sense: $m_1(ш)$ -

as the average/mean value of random function, σ^2_m - as the power of random process and $B_m(\tau)$ - as the function, which gives representation about statistical connection/communication between the values of random process, divided by the interval of time τ. The basic positive property of the relationships/ratios, which make it possible to express the characteristics of the random process through the averaging on the time, lies in the fact that they give the possibility to switch over to the approximate relationships/ratios, which make it possible to find the unknown characteristics from the final realization of random process.

From relationship/ratio for $m_t(\text{ш})$, σ^2_m and $B_ш(\tau)$ follows that they can be obtained as a result of statistical processing of the infinitely prolonged realization of random process with the continuous fixation of its values. However, virtually realization is final, i.e., $T<\infty$.

Above were introduced the concepts of the correlation function and time of correlation. From the sense of these concepts it follows that the values of the random function, divided by a small time interval usually or most frequently are close to each other and $B_m(\tau)\simeq\sigma^2_m$. With an increase τ the readings of function, divided by this interval, acquire different values and averaging it gives low values for $B_m(\tau)$.

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When $\tau=\tau_K$ the values of function prove to be not virtually connected. It is obvious that if the duration of realization is substantially greater than τ_K, i.e., $T \gg \tau_K$, then random process for time T manages to virtually show completely all its properties. Having this realization, it is possible to find $B_{ш}(\tau)$, τ_K, $m_1(ш)$ and $\sigma^2_{ш}$, which in effect sufficiently accurately will describe the random process being investigated.

Fundamental statistical noise characteristics examined above make it possible to give its initial description and to solve some simple problems, connected with the interference effect on the work of the receiving and measuring devices/equipment. In certain cases of these characteristics it proves to be insufficiently and it is necessary to use functions of the distribution of high orders, combined distribution functions and i.e. It should be noted that the assumption about the normality of the random process, which characterizes interference, makes it possible to obtain comparatively simply the functions of the distribution of high orders, if is known the function of correlation.

§ 2.2. Passage of fluctuating interference through the receiver. Let us present the receiving and measuring device/equipment by that consisting of two parts: receiver and meter. By receiver we will understand the complex of the amplifier and selecting elements/cells, which realize amplification and selection of signal by usual methods. From the output of receiver the signal together with the interferences is supplied to the detector, or to the meter which measures the phase.

In order to consider the work of detector or meter, it is necessary to have the capability to give qualitative and quantitative evaluation of interferences (noises) and signal at the output of receiver. If at the entrance of receiver functions fluctuating interference, then at the output will function the interference, which is obtained due to the amplification and the selection. Noise at the output of receiver is also random process; however, its fundamental characteristics, and the nominal distribution function, dispersion, energy spectrum and correlation function, they will differ from the corresponding characteristics by entrance.

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In many instances the receiver can be considered linear device/equipment; therefore the special features/peculiarities of interference (random process) at the output depend on linear transformations in the receiver of the interference (random process), which enters the entrance.

In the general case the noise - random process at the output of receiver $\text{п}(t)$ is unsteady even when noise - random process at the entrance of receiver $\text{ш}(t)$ is stationary.

If are known $\text{ш}(t)$ and $K(j\omega)$, then $\text{п}(t)$ can be found with the help of convolution or Duhamel integral

$$\text{п}(t)=\int_0^t \text{ш}(T)\,\eta_{\text{в}}(t-T)\,dT,$$

where t - current time, calculated off the moment/torque of inclusion/connection with $t=0$; $\eta_{\text{в}}(t-T)$ - pulse transient function, i.e., the response of linear component/link with the frequency characteristic $K(j\omega)$ to the effect in the form of impulse function $1'(t)$ or the deltas-function $\delta(t)$.

It is known that

$$\eta_{\text{в}}(t)=\frac{1}{2\pi}\int_{-\infty}^{+\infty} K(j\omega)\,e^{j\omega t}\,d\omega. \qquad (2.2.1)$$

The physical sense of this relationship/ratio lies in the fact that if on the linear network functions the delta-function, which has the uniform spectrum, then the spectrum of response repeats frequency characteristic.

After using to the frequency characteristic inverse transformation of Fourier, we will obtain the function of time $\eta_s(t)$, describing response to the delta-function.

From the expression for $n(t)$ it follows that the random process $n(t)$ is unsteady.

From the given formula it is evident that if we take time after inclusion/connection, it is much larger than the time during which $\eta_s(t)$ differs significantly from zero, then the transient processes in the circuits, which call the transiency of random process $n(t)$, are finished and random process becomes stationary. Hence it follows if $t\to\infty$ $\eta_s(t)\to 0$, that it is possible to find this moment of time after the start of the receiver when it is admissible to not consider the transiency of random process at its output.

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For obtaining the calculated relationships/ratios we will use the fact that without effect to the result it is possible to change integration limits in the roll, for the steady state

$$n(t)=\int_{-\infty}^{+\infty} ш(T)\,\eta_s(t-T)\,dT. \qquad (2.2.2)$$

The use of this relationship/ratio makes it possible to obtain the very simple expression, which connects energy spectrum at input and output of the linear component/link

$$G_{\text{н}}(\omega)=|K(j\omega)|^2 G_{\text{м}}(\omega)=K^2(\omega)\,G_{\text{м}}(\omega). \qquad (2.2.3)$$

For evaluating the output power of interference it is convenient to use the concept of effective band width $\Delta\omega_э$ or $\Delta f_э$.

Effective band width possesses the receiver or the filter, which has ideal rectangular frequency characteristic with the frequency band from ω_1 to ω_2, moreover $\omega_1-\omega_2=\Delta\omega_э$, the effort/force of real receiver and ensuring the same amount of the output power of interferences as in the real receiver.

From this condition it follows

$$\sigma_{\text{п}}^2=N_0K^2(\omega_0)\frac{\Delta\omega_э}{2\pi}=\frac{1}{2\pi}N_0\int\limits_0^\infty K^2(\omega)\,d\omega, \qquad (2.2.4)$$

whence

$$\Delta\omega_э=\frac{1}{K^2(\omega_0)}\int\limits_0^\infty K^2(\omega)\,d\omega,$$

$$\Delta f_э=\frac{\Delta\omega_э}{2\pi}. \qquad (2.2.5)$$

where ω_0 - the midband frequency of transmission.

Real receivers can have different frequency characteristics: in the majority of their cases more convenient entire to approximate by ideal rectangular or gaussian by frequency characteristics.

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With the approximation of real frequency characteristic by ideal rectangular characteristic, which during the use of band-pass filters gives close results, we obtain:

$$K(\omega)=K_0 \text{ when } \omega_1<\omega<\omega_2 \text{ or } \omega=\omega_0\pm\frac{\Delta\omega_\text{п}}{2}$$

and

$$K(\omega)=0 \quad \text{(1) при других значениях } \omega,$$

$$G_\text{п}(\omega)=K_0^2 N_0 \quad \text{(2) при } \omega=\omega_0\pm\frac{\Delta\omega_\text{п}}{2} \text{ (3) и } G_\text{ш}(\omega_0)=N_0,$$

$$\Delta\omega_\text{э}=\Delta\omega_\text{п},$$

$$\sigma_\text{ш}^2=K_0^2 G_\text{ш}(\omega_0)\frac{\Delta\omega_\text{п}}{2\pi}=K_0^2 N_0\frac{\Delta\omega_\text{п}}{2\pi}. \qquad (2.2.6)$$

Key: (1). at other values. (2). with. (3). and.

With the approximation by gaussian function, which during the use of single oscillatory circuits gives close results, we obtain

$$K(\omega)=K_0 e^{-\frac{(\omega-\omega_0)^2}{2\beta^2}}=K_0 e^{-a\left(\frac{\omega-\omega_0}{\Delta\omega_\text{п}}\right)^2},$$

$$K(\Delta\omega)=K_0 e^{-a\left(\frac{\Delta\omega}{\Delta\omega_\text{п}}\right)^2}, \qquad (2.2.7)$$

where $\Delta\omega_{п} = \sqrt{2}\pi\beta$ - complete passband during weakening 0.46; $\Delta\omega$ - detuning to one side from ω_0;

$$G_{ш}(\omega) = N_0 K_0^2 \left[e^{-\pi\left(\frac{\omega-\omega_0}{\Delta\omega_п}\right)^2} \right]^2 =$$

$$= N_0 K_0^2 e^{-\pi\left(\frac{\omega-\omega_0}{\Delta\omega_э}\right)^2}, \qquad (2.2.8)$$

$$G_{ш}(\Delta\omega) = N_0 K_0^2 e^{-\pi\left(\frac{\Delta\omega}{\Delta\omega_э}\right)^2},$$

$$\sigma_ш^2 = \frac{1}{2\pi} N_0 K_0^2 \int_{-\infty}^{+\infty} e^{-\pi\left(\frac{\Delta\omega}{\Delta\omega_э}\right)^2} d\Delta\omega = \frac{N_0 K_0^2}{2\pi} \Delta\omega_э, \qquad (2.2.9)$$

where $\Delta\omega_э = \frac{\Delta\omega_п}{\sqrt{2}}$ corresponds to the complete band of energy spectrum to the points with weakening by 0.46.

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It is possible to show that effective band width $\Delta\omega_э$ is close to the passband with weakening 0.7 in the stress/voltage

$$e^{-\left(\frac{\Delta\omega_{0,7}}{\Delta\omega_п}\right)^2 \pi} = 0{,}7,$$

i.e.

$$\left(\frac{\Delta\omega_{0,7}}{\Delta\omega_п}\right)^2 \pi = 0{,}3,$$

whence

$$\Delta\omega_{0,7} = 0{,}97 \frac{\Delta\omega_э}{2}.$$

Knowing interference spectrum at the output of receiver, it is possible to find the correlation function

$$B_n(\tau)=\frac{1}{2\pi}\int_0^{\infty}G_n(\omega)\cos\omega\tau\,d\omega. \qquad (2.2.10)$$

Since the discussion deals with selective interference, it is convenient to switch over to the relationships/ratios, which express energy spectrum depending on detuning relative to medium frequency with the frequency of receiver response.

For this let us replace the variable/alternating

$$\omega=\omega_0+\Delta\omega,$$

then

$$B_n(\tau)=\frac{1}{2\pi}\int_{-\omega_0}^{+\infty}G_n(\omega_0+\Delta\omega)\cos(\omega_0+\Delta\omega)\tau\,d\Delta\omega=$$

$$=\frac{1}{2\pi}\int_{-\infty}^{+\infty}G_n(\omega_0+\Delta\omega)\cos\omega_0\tau\cos\Delta\omega\tau\,d\Delta\omega-$$

$$-\frac{1}{2\pi}\int_{-\infty}^{+\infty}G_n(\omega_0+\Delta\omega)\sin\omega_0\tau\sin\Delta\omega\tau\,d\Delta\omega. \qquad (2.2.11)$$

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Since in the limits of passband where $G_n(\omega_0+\Delta\omega)$ differs from zero, ($\Delta\omega<<\omega_0$), integration limit $-\omega_0$ can be substituted $-\infty$. If spectrum $G_n(\omega_0+\Delta\omega)$ (i.e., the frequency characteristic of receiver) is symmetrical and frequency ω_0 is selected in the middle of band, then the second integral is equal to zero as integral of the odd function.

Consequently, in this case

$$B_n(\tau)=\frac{1}{2\pi}\int_{-\infty}^{+\infty}G_n(\omega_0+\Delta\omega)\cos\omega_0\tau\cos\Delta\omega\tau d\Delta\omega=$$

$$=(\cos\omega_0\tau)\frac{1}{2\pi}\int_{-\infty}^{+\infty}G_n(\omega_0+\Delta\omega)\cos\Delta\omega\tau d\Delta\omega \qquad (2.2.12)$$

or

$$B_n(\tau)=\sigma_n^2R_0(\tau)\cos\omega_0\tau, \qquad (2.2.12)$$

where

$$R_0(\tau)=\frac{1}{2\pi\sigma_n^2}\int_{-\infty}^{+\infty}G_n(\omega_0+\Delta\omega)\cos\Delta\omega\tau d\Delta\omega=$$

$$=\frac{1}{2\pi\sigma_n^2}\int_{-\infty}^{+\infty}G_n^*(\Delta\omega)\cos\Delta\omega\tau d\omega\tau.$$

$G^*_n(\Delta\omega)$ - interference spectrum with $\omega_0 \to 0$.

Integration will be introduced for $\Delta\omega \gtrless 0$, consequently, $G^*_n(\Delta\omega)$ - the "two-way" spectrum for the frequency of detuning $\Delta\omega$. For those given the obtained integral can be calculated.

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For the ideal linear system with the passband $\Delta\omega_n$

$$B_n(\tau)=\sigma_n^2R_0(\tau)\cos\omega_0\tau,$$

where

$$R_0(\tau)=\frac{\sin\frac{\Delta\omega_n\tau}{2}}{\frac{\Delta\omega_n\tau}{2}}; \qquad (2.2.13)$$

$$\sigma_n^2=\frac{N_0K_0^2}{2\pi}\Delta\omega_n$$

with

$$\tau=\frac{2\pi}{\Delta\omega_n}=\frac{1}{\Delta f_n} \quad R_0(\tau)=0.$$

For the linear system, which has Gaussian frequency characteristic,

$$B_n(\tau)=\sigma_n^2 e^{-\pi\left(\frac{\tau}{\tau_0}\right)^2}\cos\omega_0\tau$$

or

$$R_n(\tau)=R_0(\tau)\cos\omega_0\tau,$$

where

$$R_0(\tau)=e^{-\pi\left(\frac{\tau}{\tau_0}\right)^2}; \qquad (2.2.14)$$

$$\tau_0=\frac{1}{\Delta f_n}$$

with

$$\tau=\frac{1}{2}\tau_0=\frac{1}{2\Delta f_n}, \quad R_0(\tau)=0{,}46.$$

The correlation function of narrow-band noise contains two factors: one - high-frequency $\cos\omega_0\tau$, that is determining by the medium frequency of the spectrum ω_0, and the second - low-frequency, that is determining by the limited noise bandwidth $\Delta\omega_n$. The narrower the band, i.e., the less is $\Delta\omega_n$, the wider curve $R_0(\tau)$. Factor of the coefficient of correlation $R_{э}(\tau)$

corresponds to the coefficient of correlation of the spectrum in $\omega_{э}\to 0$, i.e., with the shift/shear into the region of low frequencies.

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The interval of time $\tau=\frac{1}{2}\tau_{э}=\tau_{0к}=\frac{1}{2\Delta f_{э}}$ can be considered as the interval of correlation for the random process with the coefficient of correlation $R_{э}(\tau)$. The spectra and the correlation function are given in Fig. 2.2.1.

Besides energy spectrum, dispersion and correlation function of random process at the output of receiver can be determined also average/mean value and distribution functions. However, usually for this there is no necessity: the average/mean value of interferences is equal to zero, and the distribution function can be accepted normal, since even if noise at the entrance does not have normal distribution, then with its passage through narrow-band receiver occurs the normalization of distribution. In this case it is thought that the receiver does not contain nonlinear components/links.

Thus, knowing frequency receiver response $K(j\omega)$, it is possible to find all fundamental statistical noise characteristics on its output. In this case sufficient to know only amplitude-frequency characteristic, since phase-frequency characteristic does not affect the energy spectrum which describes the spectral properties of random processes. Thus, for the linear selective system can be found the correlation function of interference on its output. Knowing this function bearing in mind that the interference at the output of linear system is normal random process, it is possible to find the functions of the distribution of high orders which will be further.

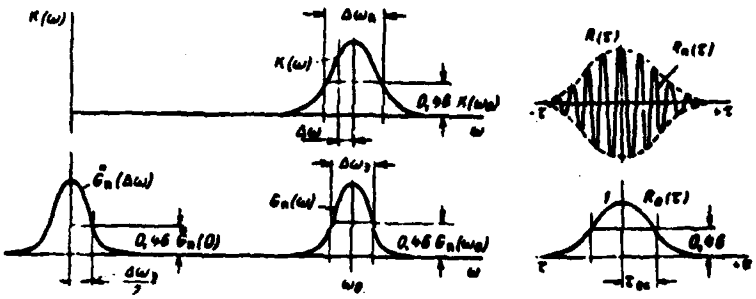

Fig. 2.2.1. Interference spectra and correlation functions with the gaussian model of filter.

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§ 2.3. Functions of the distribution of envelope and phase of selective interference. Characteristics of interference obtained previously at the output of receiver it is inconvenient to use during the evaluation of interference effect on the receiving measuring device/equipment, according to that reason, that for the extraction of the information, placed into the signal, the schematic of the receiving and measuring device/equipment usually provides for the use of the parameters of signal, its amplitude, frequency or phase.

At the output of receiver are connected the special cascades/stages - detectors or the discriminators, which make it possible to fulfill such transformations of signal, with which is revealed/detected the information placed into its parameters. These detectors can react to the amplitude, the phase and the derivative of the phase (or frequency) of signal or interference. This causes the action of interference in the receiving and measuring device/equipment.

For the solution of this problem it is necessary so converting obtained statistical noise characteristics at the output of receiver so that were revealed precisely these peculiarities of it.

To the solution of problem greatly contributes the possibility to present narrow-band random process, such as is interference at the entrance of receiver, in the form of harmonic oscillation with the random amplitude and the phase, i.e., to suppose that accordingly [2.1, 2.2, 2.4, 2.5]

$$п(t) = A_п(t)\cos[\omega_0 t - \varphi_п(t)]. \qquad (2.3.1)$$

Fig. 2.3.1 gives the graphic representation of random process at entrance and output of receiver. Experiment confirms the possibility of the representation of narrow-band random process at the output of receiver in the form of harmonic oscillation with the slowly changing amplitude and the phase. Fig. 2.3.1 clearly shows a change in amplitude. A change in the phase is possible to estimate, equating the oscillation, which corresponds to random process, with sinusoid, depicted below and as that having constants initial phase and frequency, equal to the average (resonance) frequency of the passband of receiver.

All characteristics of random process $n(t)$: the function of the distribution of different orders, correlation function, energy spectrum, dispersion, and average/mean value - they can be obtained, if are known statistical noise characteristics at the entrance of receiver and its frequency characteristic.

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If, knowing the characteristics of random process $n(t)$, it will be possible to find statistical characteristics $A_n(t)$ and $\varphi_n(t)$, then will appear the possibility of the more concrete/more specific/more actual evaluation of interference effect on the operation of the receiving and measuring device/equipment. The results, which are obtained in this case, will make it possible to consider interference effect on the receiving and measuring device/equipment which with the help of the phase measurements extracts information from the carrier frequency (distribution for φ_n) and from the phase of modulation (distribution for A_n). Distribution A_n is necessary also during the analysis of effect of ARU.

The study of interference effect on the receiving and measuring device/equipment, which extracts the information, placed during the phase, with the help of the frequency measurements and in the presence of limiter, requires the determination of another statistical noise characteristics and will be examined late.

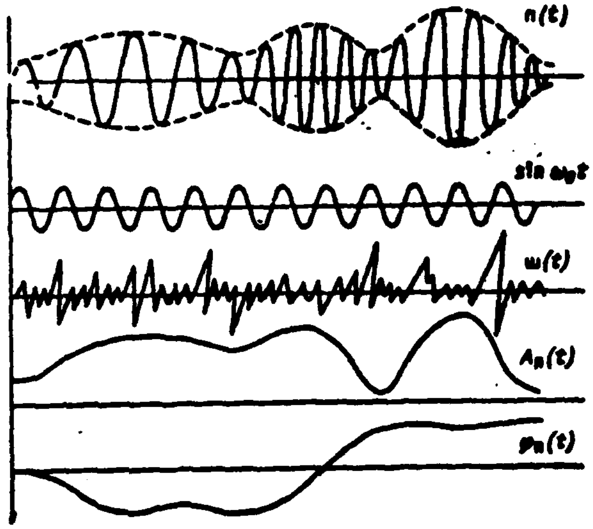

Fig. 2.3.1. Interference w(t) at the entrance and $n(t)$ at the output of the receiver: its amplitude and phase.

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For solving stated problem let us decompose the random process п(t), represented in the form of harmonic oscillation with the random amplitude and the phase, on two orthogonal narrow-band random processes:

$$\begin{aligned} \text{п}(t) &= A_{\text{п}}(t)\cos\varphi_{\text{п}}(t)\cos\omega_0 t + A_{\text{п}}(t)\sin\varphi_{\text{п}}(t)\sin\omega_0 t = \\ &= \mathcal{A}_{\text{п}}(t)\cos\omega_0 t + \mathcal{B}_{\text{п}}(t)\sin\omega_0 t, \qquad (2.3.2) \end{aligned}$$

$\mathcal{A}_{\text{п}}(t)$ and $\mathcal{B}_{\text{п}}(t)$ - random processes.

The sense of this conversion lies in the fact that the narrow-band random process n(t), considered as random process in the form of cosinusoidal oscillation/vibration the random ones by amplitude and phase, it is possible to decompose on two narrow-band random processes with the only the amplitudes.

Fig. 2.3.2 gives the interconnection between $A_n(t)$, $\varphi_n(t)$, $\mathscr{A}_n(t)$, $\mathscr{B}_n(t)$. Since

$$\mathscr{A}_n(t) = A_n(t)\cos\varphi_n(t)$$

$$\mathscr{B}_n(t) = A_n(t)\sin\varphi_n(t),$$

then

$$A_n(t) = \sqrt{\mathscr{A}_n^2(t) + \mathscr{B}_n^2(t)},$$

$$\varphi_n(t) = \operatorname{arctg}\frac{\mathscr{B}_n(t)}{\mathscr{A}_n(t)}. \qquad (2.3.3)$$

Consequently, random processes $A_n(t)$ and $\varphi_n(t)$ interesting us are expressed as random processes $\mathscr{A}_n(t)$ and $\mathscr{B}_n(t)$.

Let us consider now how it is possible to find the statistical characteristics of processes $A_n(t)$ and $\varphi_n(t)$, if they are known for the process n(t). It is at first necessary to find, as are expressed the distribution functions for $\mathscr{A}_n(t)$ and $\mathscr{B}_n(t)$ through the functions of the distribution of process n(t).

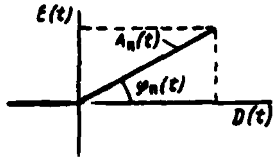

Fig. 2.3.2. Interconnection between $A_n(t)$, $\varphi_n(t)$, $\mathscr{E}_n(t)$ and $\mathscr{D}_n(t)$

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It is important to explain, is the function of the distribution of processes $\mathscr{D}_n(t)$ and $\mathscr{E}_n(t)$ normal.

For each moment/torque of time t_1 $n(t_1)$ there is a random variable with the normal distribution. It is the sum of two random variables [see (2.3.2)]: $n(t_1) = d_1 + e_1$,

$$d_1 = \mathscr{D}_n(t_1)\cos\omega_0 t_1 \overset{(1)}{\text{и}}\ e_1 = \mathscr{E}_n(t_1)\sin\omega_0 t_1.$$

Key: (1). and.

$n(t_1)$ has normal distribution, then d_1 and e_1 can have only a normal distribution. But $\mathscr{D}_n(t_1)$ differs from d_1 only in terms of the determined factor. Then the distribution function for $\mathscr{D}_n(t_1)$ [and for $\mathscr{E}_n(t_1)$] also normal.

Values d_1 and e_1 have dispersions $\sigma^2_{d_1} = \sigma^2_{\mathscr{D}}\cos^2\omega_0 t_1$ and $\sigma^2_{e_1} = \sigma^2_{\mathscr{E}}\sin^2\omega_0 t_1$. Random processes $\mathscr{D}_n(t)$ and $\mathscr{E}_n(t)$ and, therefore, the random variables d_1 and e_1 are independent. Then the distribution of sum will be also normal, with the dispersion, equal to the sum of the dispersions of the components/terms/addends

$$\sigma_n^2=\sigma_{a_1}^2+\sigma_{a_1}^2=\sigma_{\mathcal{D}}^2\cos^2\omega_0 t_1+\sigma_{\mathcal{E}}^2\sin^2\omega_0 t_1=\sigma_{\mathcal{D}}^2=\sigma_{\mathcal{E}}^2. \quad (2.3.4)$$

Thus, it is shown that random processes $\mathcal{D}_n(t)$ and $\mathcal{E}_n(t)$ have normal distribution, moreover their dispersion is equal to the dispersion of initial process σ_n^2. On the basis of this it is possible to do an assumption about the fact that processes $\mathcal{D}_n(t)$ and $\mathcal{E}_n(t)$ belong to the normal random processes, for which all distribution functions are normal. For obtaining of the functions of distribution $\mathcal{D}_n(t)$ and $\mathcal{E}_n(t)$ any order it is necessary to find their correlation function of the known correlation function of initial process. Let us begin the solution of this problem.

Let us recall that

$$n(t)=\mathcal{D}_n(t)\cos\omega_0 t+\mathcal{E}_n(t)\sin\omega_0 t. \quad (2.3.5)$$

Let us introduce the concept of random process p(t), conjugated/combined n(t), then

$$p(t)=A_n(t)\sin[\omega_0 t+\varphi_n(t)]=$$
$$=\mathcal{D}_n(t)\sin\omega_0 t+\mathcal{E}_n(t)\cos\omega_0 t. \quad (2.3.6)$$

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The concept of the conjugated/combined random process will be required for facilitating the intermediate mathematical conversions in obtaining of expressions for $B_{\mathcal{D}}(\tau)$ and $B_{\mathcal{E}}(\tau)$. From the given relationships/ratios, which show connection/communication between the processes п(t) and p(t), it follows that they have identical dispersions, correlation function and energy spectra, i.e.

$$\sigma_n^2=\sigma_p^2,\ G_n(\omega)=G_p(\omega),\ B_n(\tau)=B_p(\tau).$$

The difference between these processes lies in the fact that they are shifted relative to each other on 90°, i.e., if we for each concrete/specific/actual realization of process п(t) find Fourier's spectrum, then for the realization of the conjugated/combined process Fourier's spectrum will differ in terms of factor j, i.e., have further shift/shear for all frequency components on 90°.

Crosscorrelation function between the processes п(t) and p(t) will take the form

$$B_{n-p}(\tau)=\lim_{T\to\infty}\frac{1}{T}\int_0^T n(t)\,p(t-\tau)\,dt=$$

$$=-B_{p-n}(\tau)=\frac{1}{2\pi}\int_0^\infty G_n(\omega)\sin\omega\tau\,d\omega.$$

In detail on the concepts of crosscorrelation function we do not stop, reader if necessary can be converted, for example, to [2.1, 2.2, 2.3].

Using the concept of conjugated/combined process, it is possible processes $\mathcal{A}_n(t)$ and $\mathcal{B}_n(t)$ to express through п(t) and p(t). After multiplying the left and right sides of expressions (2.3.6) on $\cos\omega_0 t$ or $\sin\omega_0 t$, after carrying out then addition or subtraction and after leading conversions, we will obtain

$$\mathscr{D}_{n}(t)=n(t)\cos\omega_{0}t+p(t)\sin\omega_{0}t,$$

$$\mathscr{E}_{n}(t)=n(t)\sin\omega_{0}t-p(t)\cos\omega_{0}t.$$

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Now $B_{\mathscr{D}}(\tau)$ or $B_{\mathscr{E}}(\tau)$ it is possible to express through n(t) and p(t):

$$B_{\mathscr{D}}(\tau)=\lim_{T\to\infty}\frac{1}{T}\int_{0}^{T}\mathscr{D}_{n}(t)\mathscr{D}_{n}(t-\tau)\,dt=$$

$$=\lim_{T\to\infty}\frac{1}{T}\int_{0}^{T}[n(t)n(t-\tau)\cos\omega_{0}t\cos\omega_{0}(t-\tau)+$$

$$+p(t)p(t-\tau)\sin\omega_{0}t\sin\omega_{0}(t-\tau)+$$

$$+n(t)p(t-\tau)\cos\omega_{0}t\sin\omega_{0}(t-\tau)-$$

$$-p(t)n(t-\tau)\sin\omega_{0}t\cos\omega_{0}(t-\tau)]\,dt.$$

After using expression for the product of cosines and sines and bearing in mind that the integrals, which contain as factor $\cos 2\omega_{0}t$ either $\sin 2\omega_{0}t$, will be equal to zero, we will obtain

$$B_{\mathscr{D}}(\tau)=\lim_{T\to\infty}\frac{1}{T}\int_{0}^{T}\Big[n(t)n(t-\tau)\frac{1}{2}\cos\omega_{0}\tau+$$

$$+p(t)p(t-\tau)\frac{1}{2}\cos\omega_{0}\tau+n(t)p(t-\tau)\frac{1}{2}\sin\omega_{0}\tau+$$

$$+p(t)n(t-\tau)\frac{1}{2}\sin\omega_{0}\tau\Big]\,dt \qquad (2.3.7)$$

or

$$B_{\mathscr{D}}(\tau)=\frac{1}{2}\cos\omega_{0}\tau B_{n}(\tau)+\frac{1}{2}\cos\omega_{0}\tau B_{p}(\tau)+$$

$$+\frac{1}{2}\sin\omega_{0}\tau B_{n-p}(\tau)-\frac{1}{2}\sin\omega_{0}\tau B_{p-n}(\tau). \qquad (2.3.8)$$

Further, after expressing correlation function through $G_{\mathscr{F}}(\omega)$, we will obtain

$$B_{\mathscr{D}}(\tau)=\frac{1}{2\pi}\int_0^\infty G_{\text{п}}(\omega)\cos\omega_0\tau\cos\omega\tau d\omega+$$

$$+\frac{1}{2\pi}\int_0^\infty G_{\text{п}}(\omega)\sin\omega_0\tau\sin\omega\tau d\omega=$$

$$=\frac{1}{2\pi}\int_0^\infty G_{\text{п}}(\omega)\cos(\omega-\omega_0)\tau d\omega. \qquad (2.3.9)$$

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Is now expressed $B_{\mathscr{D}}(\tau)$ through factor $R_0(\tau)$ the correlation function of initial random process - $B_{\text{п}}(\tau)$.

For this let us replace the variable/alternating

$$\omega-\omega_0=\Delta\omega,\ d\omega=+d\Delta\omega,\ \omega=\omega_0+\Delta\omega,$$
$$\omega\to 0,\ \Delta\omega\to-\omega_0,\ \omega\to\infty,\ \Delta\omega\to+\infty.$$

Then

$$B_{\mathscr{D}}(\tau)=\frac{1}{2\pi}\int_{-\omega_0}^{+\infty} G_{\text{п}}(\omega_0+\Delta\omega)\cos\Delta\omega\tau d\Delta\omega=$$

$$=\frac{1}{2\pi}\int_{-\infty}^{+\infty} G_{\text{п}}(\omega_0+\Delta\omega)\cos\Delta\omega\tau d\Delta\omega=$$

$$=\frac{1}{2\pi}\int_{-\infty}^{+\infty} G^*_{\text{п}}(\Delta\omega)\cos\Delta\omega\tau d\Delta\omega. \qquad (2.3.10)$$

Here $G^*_{\text{п}}(\Delta\omega)$ - "bilateral" spectrum for the frequencies $\Delta\omega$.

Consequently [see (2.2.12)],

$$B_{\mathscr{D}}(\tau)=B_{\mathscr{E}}(\tau)=\sigma_n^2 R_0(\tau) \qquad (2.3.11)$$

or

$$R_{\mathscr{D}}(\tau)=R_{\mathscr{E}}(\tau)=R_0(\tau). \qquad (2.3.12)$$

The obtained result has important value. The autocorrelation coefficient of processes $\mathscr{D}_n(t)$ and $\mathscr{E}_n(t)$ is equal to the low-frequency factor of the autocorrelation function of initial random process [1].

FOOTNOTE [1]. For the reduction of recording in the subsequent expressions of this chapter index p for $\mathscr{D}$ and $\mathscr{E}$ we lower. ENDFOOTNOTE.

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If width of band of initial process $\Delta\omega_n$, then random processes $\mathscr{D}(t)$ and $\mathscr{E}(t)$ have a frequency band, which approximately/exemplarily corresponds $\Delta\omega_n/2$. Thus, $\mathscr{D}(t)$ and $\mathscr{E}(t)$ slow [in comparison with n(t)] the random processes whose energy spectrum corresponds approximately/exemplarily to half of the width of the spectrum of initial random process. On the basis of these information it is possible to register the multidimensional functions of the distribution of values $\mathscr{D}$ and $\mathscr{E}$

For example, the one-dimensional function of the distribution

$$w(\mathscr{D})=\frac{1}{\sqrt{2\pi}\sigma_{n}}e^{-\frac{\mathscr{D}^{2}}{2\sigma_{n}^{2}}}. \qquad (2.3.13)$$

The two-dimensional distribution function will take the form

$$w_{2}(\mathscr{D}_{1},\ \mathscr{D}_{2},\ \tau)=\frac{1}{2\pi\sigma_{n}\sqrt{1-R_{0}^{2}(\tau)}}e^{-\frac{\mathscr{D}_{1}^{2}+\mathscr{D}_{2}^{2}-2R_{0}(\tau)\mathscr{D}_{1}\mathscr{D}_{2}}{2\sigma_{n}^{2}(1-R_{0}^{2}(\tau))}}. \qquad (2.3.14)$$

Analogously are expressed

$$w(\mathscr{B})\ \overset{(1)}{\text{и}}\ w_{2}(\mathscr{B}_{1},\ \mathscr{B}_{2},\ \tau).$$

Key: (1). and.

Processes $\mathscr{D}(t)$ and $\mathscr{B}(t)$ are the independent variables at the coinciding moments of time. The functions of joint distribution easily can be found

$$w_{2}(\mathscr{D},\ \mathscr{B})=w(\mathscr{D})\,w(\mathscr{B}), \qquad (2.3.15)$$

$$w_{4}(\mathscr{D}_{1},\ \mathscr{D}_{2}\ \mathscr{B}_{1},\ \mathscr{B}_{2},\ \tau)=w_{2}(\mathscr{D}_{1},\ \mathscr{D}_{2},\ \tau)\,w_{2}(\mathscr{B}_{1},\ \mathscr{B}_{2},\ \tau). \qquad (2.3.16)$$

After are found the statistical characteristics of random processes $\mathscr{D}(t)$ and $\mathscr{B}(t)$, in the principle it is possible to find all statistical characteristics of random processes $A_{n}(t)$ and $\varphi_{n}(t)$.

Above it was shown that between $\mathscr{D}(t)$, $\mathscr{B}(t)$ and $A_{n}(t)$, $\varphi_{n}(t)$ the same connection/communication as between the rectangular and polar coordinates.

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Therefore, after finding joint distribution $w_2(\mathcal{X}, \mathcal{Y})$ or $w_4(\mathcal{X}_1, \mathcal{X}_2, \mathcal{Y}_1, \mathcal{Y}_2, \tau)$ and so forth, after fulfilling the conversion of the distribution function, which corresponds to transition from the rectangular coordinates to the polar ones, it is possible to obtain

$$w_2(A_n, \varphi_n)$$

or

$$w_4(A_{n1}, A_{n2}, \varphi_{n1}, \varphi_{n2}, \tau).$$

The conversions, connected with the transition from the the rectangular coordinates to the polar ones, carry purely mathematical character. On these reasons we for them here and subsequently will drop/omit and let us give only final result. In detail these conversions are given in [2.1, 2.2, 2.4], etc.

For the illustration of the obtained results let us consider the specific example

$$w_2(\mathcal{X}, \mathcal{Y}) = \frac{1}{2\pi\sigma_n^2} e^{-\frac{\mathcal{X}^2+\mathcal{Y}^2}{2\sigma_n^2}}.$$

Bearing in mind that $\mathcal{X} = A_n \cos\varphi_n$ and $\mathcal{Y} = A_n \sin\varphi_n$, we obtain

$$w_2(A_n, \varphi_n) = A_n w_2(A_n\cos\varphi_n, A_n\sin\varphi_n) =$$

$$= \frac{A_n}{2\pi\sigma_n^2} e^{-\frac{A_n^2\cos^2\varphi_n + A_n^2\sin^2\varphi_n}{2\sigma_n^2}} = \frac{A_n}{2\pi\sigma_n^2} e^{-\frac{A_n^2}{2\sigma_n^2}}. \quad (2.3.17)$$

By having the combined distribution functions and by realizing integration, it is possible to switch over to the distributions of the lowest orders. For example for obtaining the one-dimensional functions of distribution A_n and φ_n of that obtained is above $w_2(A_n, \varphi_n)$ it is necessary to fulfill integration.

Let us begin from the function of amplitude distribution

$$w(A_n) = \int_0^{2\pi} w_2(A_n, \varphi_n)\, d\varphi = \frac{A_n}{2\pi\sigma_n^2} e^{-\frac{A_n^2}{2\sigma_n^2}} \int_0^{2\pi} d\varphi =$$

$$= \frac{A_n}{\sigma_n^2} e^{-\frac{A_n^2}{2\sigma_n^2}}. \qquad (2.3.18)$$

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For the use of tables more conveniently to pass to the dimensionless form of the recording of the distribution function. Let us designate $\frac{A_n}{\sigma_n} = a_n$, then

$$w(a_n) = a_n e^{-\frac{a_n^2}{2}}. \qquad (2.3.19)$$

The obtained distribution is called Rayleigh.

Let us find the function of phase distribution

$$w(\varphi_n) = \int_0^\infty w_2(A_n, \varphi_n)\, dA_n = \frac{1}{2\pi}, \qquad (2.3.20)$$

since

$$\int_0^\infty \frac{A_n}{\sigma_n^2} e^{-\frac{A_n^2}{2\sigma_n^2}} dA_n = 1,$$

as integral of the distribution function in the infinite limits. Consequently, phase is distributed it is uniform about i.e., any value of phase from 0 to 2π is equally probable. Since $w(\varphi_n)w(A_n) = w_2(A_n\varphi_n)$, then it can be claimed that phase and the amplitude of the high-frequency oscillation, equivalent to noise, it is statistically independent (at the coinciding moments of time). In other words, by the value of amplitude, it cannot be judged the value of phase, and vice versa.

For obtaining the two-dimensional function of amplitude distribution and phase it is necessary to find the integral

$$w_2(A_{n1}, A_{n2}, \tau) = \int_0^{2\pi}\int_0^{2\pi} w_4(A_{n1}, A_{n2}, \varphi_{n1}, \varphi_{n2}, \tau)\, d\varphi_{n1} d\varphi_{n2} \qquad (2.3.21)$$

or

$$w_2(\varphi_{n1}, \varphi_{n2}, \tau) = \int_0^\infty\int_0^\infty w_4(A_{n1}, A_{n2}, \varphi_{n1}, \varphi_{n2}, \tau)\, dA_{n1} dA_{n2}. \qquad (2.3.22)$$

By analogous methods can be obtained other distribution functions.

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Symbolically these problems are solved simply. However, during the solution of specific problems during the calculation of integrals appear difficulties; therefore is expedient the solution of integrals to implement for each case individually.

§ 2.4. Distribution functions and fundamental statistical noise characteristics at the output of the amplitude detector of signal and ARU. One-dimensional distribution for the envelope they make it possible to consider some special features/peculiarities of the effect of interferences on receiving and measuring devices/equipment [2.8, 2.11, 2.12, 2.13, 3.2].

Enveloping or the amplitude of noise they can be showed in such a case, when on detector acts noise. In the receiver there is a detector if useful information is embedded into the amplitude modulation of radio signal and in all cases when is used ARU.

The idealized detector of useful signal can be represented as such nonlinear element which without the distortions reproduces envelope. Then the statistical characteristics of the envelope of narrow-band noise can be considered as the statistical characteristics of output potential of detector (Fig. 2.4.1).

Earlier they were obtained $w(A_{\text{ш}})$ and $w(a_{\text{ш}})$. The form of the function of distribution is given in Fig. 2.4.2. Let us consider the now in more detail statistical characteristics by which is subordinated output potential of ideal detector.

The Rayleigh distribution, to which is subordinated the envelope of noise and output potential of ideal detector, is asymmetric; therefore the average/mean value (mathematical expectation) of the envelope or constant output potential of detector is not equal to zero.

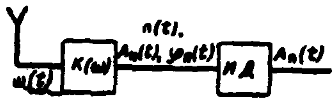

Fig. 2.4.1. Diagram with detector: ИД - ideal detector.

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For determining the constant component of detected disturbing voltage let us find the average/mean value of the envelope

$$m_1(A_ш)=\int_0^\infty A_ш w(A_ш)\,dA_ш=$$

$$=\int_0^\infty \frac{A_ш^2}{\sigma_ш^2} e^{-\frac{A_ш^2}{2\sigma_ш^2}}\,dA_ш=\sqrt{\frac{\pi}{2}}\,\sigma_ш=1{,}25\sigma_ш. \quad (2.4.1)$$

Interference on the constant component of detection is equivalent to sine voltage with an amplitude of $A_c=1{,}25\sigma_ш$ and effective value u_c:

$$u_c=\frac{1{,}25\sigma_ш}{1{,}41}=0{,}89\sigma_ш.$$

Virtually in the first approximation, it is possible to consider that the noise and sine wave with the equality of the effective values of stress/voltage give one and the same value of the constant component of the detected stress/voltage.

Besides constant component at the output of detector there will be other fluctuations whose presence can be explained by the fact that because of the Rayleigh distribution of envelope, amplitude change can be considered as the presence of amplitude modulation by the noise of the average/mean value of the amplitude of noise.

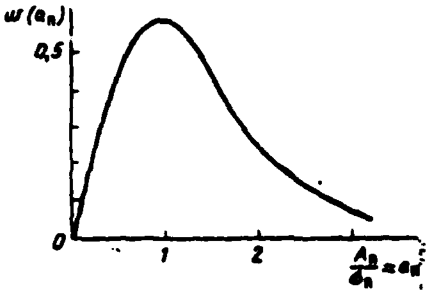

Fig. 2.4.2. Law of Rayleigh distribution.

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These fluctuations can be described by dispersion or by the second central moment of the distribution

$$\sigma_\Omega^2 = M_2(A_n) = \frac{1}{\sigma_n^2}\int_0^\infty A_n\left(A_n - \sigma_n\sqrt{\frac{\pi}{2}}\right)^2 \times$$
$$\times e^{-\frac{A_n^2}{2\sigma_n^2}} dA_n = \frac{4-\pi}{2}\sigma_n^2 = 0,43\sigma_n^2\,\sigma_A^2 \qquad (2.4.2)$$

Noise can be considered equivalent to the signal, modulated with a depth of modulation of M_n (root-mean-square).

$$M_n \simeq \frac{\sigma_\Omega}{\sigma_n}\,0,8 \simeq 0,5.$$

Directly on the load of detector functions constant and variable/alternating low-frequency stress. In many instances stress/voltage from the detector is removed/taken through the separating capacity/capacitance and in the subsequent circuits functions only variable low-frequency component. Let us find the distribution function for low-frequency component of the detected stress/voltage. For clarification of some statistical properties of fluctuations it is convenient Rayleigh distribution to approximate by normal distribution with displaced average/mean value $m(A_n)$ and by dispersion σ_n^2. Then

$$w(A_n)\approx\frac{1}{\sqrt{2\pi}\,\sigma_n}e^{-\frac{[A_n-m_1(A_n)]^2}{2\sigma_n^2}}. \qquad (2.4.3)$$

This approximation subsequently will be used with approximate solutions of some problems. The approximate distribution function for output potential of detector can be normalized relatively σ_n, which will make it possible to more conveniently be congruent/equate it with a precise (Rayleigh) distribution function.

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The function of distribution $w\left(\frac{A_n}{\sigma_n}\right)$ is given in Fig. 2.4.3 together with a precise Rayleigh distribution. For approximate solutions the coincidence can be recognized as sufficient, if points with the low density of probability do not have an effect on result. Distribution function relative to average/mean value, i.e., the function of the distribution of the fluctuations

$$w\left(\frac{A_n}{\sigma_n}\right)=\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{A_n}{\sigma_n}\right)^2}. \qquad (2.4.4)$$

Thus, when useful information is laid during the phase of modulation and to the receiving and measuring device/equipment is not supplied signal or is supplied very weak (in comparison with the interferences) signal, at the output.of detector after separating capacity/capacitance is obtained fluctuating (variable/alternating) disturbing voltage whose dispersion is equal to 0.43 from the dispersion of interference at the entrance of detector. If we have in mind only low-frequency components of the spectrum of fluctuations, then, as it will be shown further, they have relatively greater intensity. The obtained result is insufficient for a full evaluation of the work of diagrams with the detectors with the interference, since is found only dispersion and the one-dimensional function of the distribution of fluctuations. More detailed research of the action of interferences in the diagrams, which contain detectors, taking into account the filtering circuits, connected after detector, requires that would be found the spectrum of the fluctuations of the detected stress/voltage.

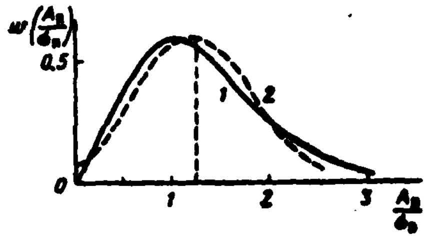

Fig. 2.4.3. The approximation of Rayleigh's function: 1 - Rayleigh's function; 2 - approximated function.

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This is done further in S 2.6. The obtained above distribution function for envelope $w(A_n)$ makes it possible to consider operation of ARU in the presence of interferences. Detector ARU can work on the diagram with the delay in the high frequency with subsequent averaging with the slow response, which removes the pulsations of the control voltage which can occur due to modulation of useful signal or fluctuations of the detected noise voltage. The diagram of detector of ARU can be carried out, also, with the delay on the detected averaged stress/voltage.

The factor of amplification of real multistage receivers in the process of their operation undergoes substantial changes because supply voltage, the temperature, the humidity, the parameters of amplifier elements/cells and the like continuously are changed. So that the smallest possible amplification would be sufficient for the reception of weak signals, during the design of receivers is provided for considerable supply on the amplification. But then noise voltage (without the signal) attains the significant magnitude, which exceeds the threshold of the limitation of detector of ARU. Consequently, typical is this work of detector of ARU with which the noise voltage exceeds the stress/voltage of delay. Under these conditions the detector of ARU develops the bias voltage, decreasing amplification up to the level, on which as a result of the detection of interference is developed the displacement indicated.

For evaluating the work of ARU in the presence of noises and effect of ARU on the action of interferences it is necessary to find the dependence of the detected stress/voltage in the circuit of ARU from the relationship/ratio between the noise voltage and the delay.

Idealizing detector and considering that at its output is reproduced the envelope, it is possible to find the constant component of the detected stress/voltage as mathematical expectation for envelope within the limits from the level delay $A_з$ to infinity. In this case it is assumed that the filtration in the circuit of detector of ARU is sufficient for the virtually complete smoothing of the pulsations of the detected stress/voltage, which usually and occurs.

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The stress/voltage, removed from the detector of ARU in the diagram with the delay to the averaging, can be expressed

$$\Delta m_{s}=\int\limits_{A_{s}}^{\infty}(A_{\text{н}}-A_{s})\frac{A_{\text{н}}}{\sigma_{\text{н}}^{2}}e^{-\frac{A_{\text{н}}^{2}}{2\sigma_{\text{н}}^{2}}}dA_{\text{н}}. \qquad (2.4.5)$$

In relative values

$$a_{\text{н}}=\frac{A_{\text{н}}}{\sigma_{\text{н}}},\ \Delta v_{s}=\frac{\Delta m_{s}}{\sigma_{\text{н}}},\ a_{s}=\frac{A_{s}}{\sigma_{\text{н}}},$$

$$\Delta v_{s}=\int\limits_{a_{s}}^{\infty}(a_{\text{н}}^{2}-a_{\text{н}}a_{s})e^{-\frac{a_{\text{н}}^{2}}{2}}da_{\text{н}}=$$

$$=\sqrt{2\pi}\,[1-F(a_{s})], \qquad (2.4.6)$$

where

$F(a_{s})=\frac{1}{\sqrt{2\pi}}\int\limits_{-\infty}^{a_{s}}e^{-\frac{t^{2}}{2}}dt$ — the tabulated integral.

The graph/diagram of the dependence of output potential of ARU is given in Fig. 2.4.4.

In the diagram with the delay on the averaged stress/voltage

$$\Delta m_з = m_1(A_ш) - A_з = 1{,}25\sigma_ш - A_з$$

or

$$\Delta v_з = 1{,}25 - a_з; \quad \Delta v_з > 0.$$

From the obtained results it follows that with the ideal work of ARU, i.e., when is sufficient very small stress/voltage, removed from the detector of ARU in order to considerably change amplification, in the diagram with the delay to the averaging it will be established/installed in such level on which the root-mean-square value of noise on the detector is 2-2.5 times less than the value of the stress/voltage of delay, since in this case Δv it composes 0.05-0.02, and in the diagram with the delay after averaging - at the level on which $\sigma_ш = A_з/1{,}3$.

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Knowing noise voltage on the output of receiver with ARU, it is possible to find constant and fluctuating components of the detected stress/voltage.

Constant detected stress is equal to:

a) in the diagram with the delay of up to the averaging

$$m_1(A_ш) = 1{,}25\sigma_ш = 0{,}6A_з; \tag{2.4.7}$$

b) in the diagram with the delay after the averaging

$$m_1(A_B) \simeq A_3.$$

The fluctuation of the detected stress/voltage they will be:

a) in the diagram with the delay to the averaging

$$\sigma_B = 0,65, \quad \sigma_B = 0,3A_3; \quad (2.4.8)$$

b) in the diagram with the delay after the averaging

$$\sigma_B = 0,55A_3.$$

stress/voltage at the different points of diagram for these cases is given on Fig. 2.4.5. Thus, in the presence of ARU stresses/voltages at the different points of diagram are determined by the stress/voltage of delay. Depending on the quality of ARU relationship/ratios can differ somewhat from those accepted earlier.

This mode/conditions virtually little is changed with a change of the factor of amplification of receiver over wide limits and with a change in the interference level at the entrance of receiver.

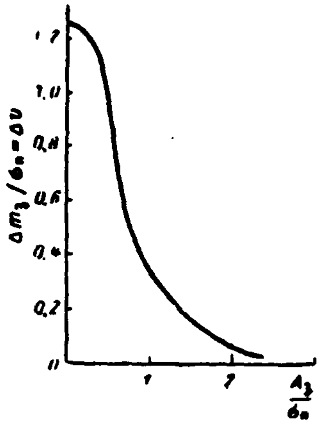

Fig. 2.4.4.

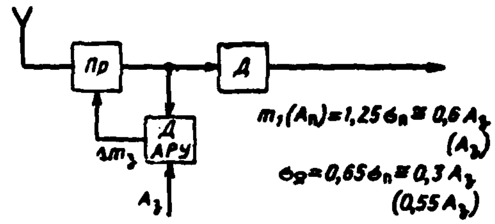

Fig. 2.4.5.

Fig. 2.4.4. Output potential of detector of ARU.

Fig. 2.4.5. Disturbing voltages in diagram with ARU: Пр - receiver; Д - detector; Д ARU - detector of ARU. In the brackets are given the values for the diagram with the delay after averaging.

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Analogous results are obtained for the receiving and measuring devices/equipment, which reveal/detect the information, placed during the carrier frequency. Disturbing voltage on the output of receiver (due to the functioning of ARU) will be established/installed such value when $\frac{A_з}{\sigma_п}=2$ or $\frac{A_з}{\sigma_п}=1,3$. This result is shown in Fig. 2.4.6. Disturbing voltage on the output of receiver will little depend on interference level at the entrance and its amplification and in essence is determined A_3.

§ 2.5. Simplest statistical characteristics of the phase of interference. Let us now move on to the phase of narrow-band noise. Earlier it was obtained that $w(\varphi_n)=\frac{1}{2\pi}$. Knowing $w(\varphi_n)$, let us find average/mean value and dispersion. Since the phase is distributed evenly, i.e., each value of phase is equiprobable in the limits from 0 to 2π or from $-\pi$ to $+\pi$ and from $-(\pi/2)$ to $3/4\pi$ and so forth, concept of average/mean value or mathematical expectation becomes conditional and it depends on that such as is selected the limit in which the value of phase is considered single-valued. To more conveniently usually use limits $-\pi$, $+\pi$, then

$$m_1(\varphi_n)=\frac{1}{2\pi}\int_{-\pi}^{+\pi}\varphi_n d\varphi_n=0.$$

For obtaining the dispersion let us compute integral.

$$\sigma_\varphi^2=\frac{1}{2\pi}\int_{-\pi}^{+\pi}\varphi_n^2 d\varphi_n=\frac{\pi^2}{3}\approx 3{,}28; \quad \sigma_\varphi\approx 1{,}8\approx 100^\circ.$$

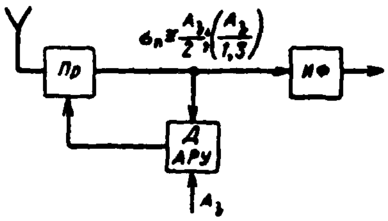

Fig. 2.4.6. Disturbing voltage in the diagram with ARU. Пр - receiver; ИФ - meter of the phase: Д ARU - detector of ARU.

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Using functional conversions, it is possible to obtain the function of the distribution of the cosine of the phase

$$w(z)=\frac{1}{\pi\sqrt{1-z^2}};\ |z|=|\cos\varphi_{п}|\leqslant 1.$$

As is evident, phasemeter in the presence of one interference must show the indefinite value of the phase (its readings/indications will substantially fluctuate).

Thus, the one-dimensional function of phase distribution makes it possible to obtain the very limited representation about how interference it affects the work of phase receiving and measuring device/equipment. For the development/detection of this effect it is necessary to find the multidimensional functions of phase distribution, correlation function and energy spectrum, and also the combined functions of phase distribution and its derivative, function of the distribution of derived phase and the function of the distribution of zero random processes. These functions which will be obtained further, will make it possible to establish/install some in principle important special features/peculiarities of interference effect on the phase of receiving and measuring device/equipment, which are substantially changed in the dependence on the principle of the construction of phasemeter.

§ 2.6. Two-dimensional function of distribution. Energy spectrum and the correlation function of the amplitude of interference. In many instances in the receiving and measuring device/equipment after amplitude detector or phasemeter stands the filter with the final passband (or amplitude detector and phasemeter possess final passband).

Then the effect of the action of interference is determined by the spectra, $G_A(\omega)$ and $G_\varphi(\omega)$ or by correlation function $B_A(\tau)$ and $B_\varphi(\tau)$.

Correlation function, energy spectra and the two-dimensional distribution functions for the envelope and the phase cannot be mixed with the characteristics of the instantaneous values of initial narrow-band random process.

In § 2.2 were resulting expressions $B_n(\tau)$ for typical forms $G_n(\omega)$. Knowing $B_n(\tau)$ or $R_n(\tau)$ and keeping in mind the normality of random process $n(t)$, it is easy to write the distribution function. For the envelope and the phase obtaining the correlation function and energy spectra is connected with the considerable mathematical difficulties.

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Can be selected any sequence of the determination of the functions

$$G_\varphi(\omega),\ G_A(\omega),\ B_A(\tau),\ B_\varphi(\tau),\ w_2(A_{n1},\ A_{n2},\ \tau),\ w_2(\varphi_{n1},\ \varphi_{n2},\ \tau),$$

since they all are connected; however, with the smallest mathematical difficulties this problem is solved through determination $w_2(A_{n1},\ A_{n2},\ \tau)$, then $B_A(\tau)$ and $G_A(\omega)$ or $w_2(\varphi_{n1},\ \varphi_{n2},\ \tau)$, then $B_\varphi(\tau)$ and $G_\varphi(\omega)$. Two-dimensional distribution for the amplitude of interference can be found from the four-dimensional distribution for the amplitude and the phase by integration for the phase. Four-dimensional distribution for the amplitude and the phase is obtained from the four-dimensional joint distribution for $\mathcal{B}$ and $\mathcal{D}$, which can be obtained from the two-dimensional distributions for $\mathcal{D}$ and $\mathcal{B}$.

Since it is accepted that $\mathcal{B}$ and $\mathcal{D}$ have normal distribution,

$$w_2(\mathcal{D}_1, \mathcal{D}_2, \tau) = \frac{1}{2\pi\sigma_n^2 \sqrt{1 - R_0^2(\tau)}} e^{-\frac{\mathcal{D}_1^2 + \mathcal{D}_2^2 - 2R_0(\tau)\mathcal{D}_1\mathcal{D}_2}{2\sigma_n^2[1 - R_0^2(\tau)]}} \quad (2.6.1)$$

analogously

$$w_2(\mathcal{E}_1, \mathcal{E}_2, \tau) = \frac{1}{2\pi\sigma_n^2 \sqrt{1 - R_0^2(\tau)}} e^{-\frac{\mathcal{E}_1^2 + \mathcal{E}_2^2 - 2R_0(\tau)\mathcal{E}_1\mathcal{E}_2}{2\sigma_n^2[1 - R_0^2(\tau)]}}. \quad (2.6.2)$$

Processes $\mathcal{D}$ and $\mathcal{E}$ can be considered independent variables, then

$$w_4(\mathcal{D}_1, \mathcal{D}_2, \mathcal{E}_1, \mathcal{E}_2, \tau) = \frac{1}{(2\pi\sigma_n^2)^2[1 - R_0^2(\tau)]} \times$$

$$\times e^{-\frac{\mathcal{D}_1^2 + \mathcal{D}_2^2 + \mathcal{E}_1^2 + \mathcal{E}_2^2 - 2R_0(\tau)(\mathcal{D}_1\mathcal{D}_2 + \mathcal{E}_1\mathcal{E}_2)}{2\sigma_n^2[1 - R_0^2(\tau)]}}. \quad (2.6.3)$$

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By knowing $w_4(\mathcal{D}_1, \mathcal{D}_2, \mathcal{E}_1, \mathcal{E}_2, \tau)$ and by using concepts of amplitude (by envelope) and phase of process, it is possible to obtain

$$w_4(A_{n1}, A_{n2}, \varphi_{n1}, \varphi_{n2}, \tau),$$

by applying the rules of the conversion of the distribution functions upon transfer from rectangular coordinates ($\mathcal{D}$ and $\mathcal{E}$) to polar ones A_n and φ_n.

After fulfilling the necessary conversions, we will obtain

$$w_4(A_{n1}, A_{n2}, \varphi_{n1}, \varphi_{n2}) = \frac{A_{n1}A_{n2}}{(2\pi\sigma_n^2)^2\sqrt{1-R_0^2(\tau)}} \times$$

$$\times e^{-\frac{A_{n1}^2+A_{n2}^2-2A_{n1}A_{n2}R_0(\tau)\cos(\varphi_{n1}-\varphi_{n2})}{2\sigma_n^2[1-R_0^2(\tau)]}}. \quad (2.6.4)$$

After fulfilling integration for φ_{n1} and φ_{n2}, we will obtain

$$w_2(A_{n1}, A_{n2}, \tau) = \int_0^{2\pi}\int_0^{2\pi} w_4(A_{n1}, A_{n2}, \varphi_{n1}, \varphi_{n2}, \tau)\, d\varphi_{n1} d\varphi_{n2} =$$

$$= \frac{A_{n1}A_{n2}}{\sigma_n^4[1-R_0^2(\tau)]} e^{-\frac{A_{n1}^2+A_{n2}^2}{2\sigma_n^2[1-R_0^2(\tau)]}} \times$$

$$\times \frac{1}{2\pi}\frac{1}{2\pi}\int_0^{2\pi}\int_0^{2\pi} e^{-\frac{2A_{n1}A_{n2}R_0(\tau)\cos(\varphi_{n1}-\varphi_{n2})}{2\sigma_n^2[1-R_0^2(\tau)]}} d\varphi_{n1} d\varphi_{n2};$$

$$w_2(A_{n1}, A_{n2}, \tau) = \frac{A_{n1}A_{n2}}{\sigma_n^4[1-R_0(\tau)]} e^{-\frac{A_{n1}^2+A_{n2}^2}{2\sigma_n^2[1-R_0(\tau)]}} \times$$

$$\times I_0\left[\frac{R_0(\tau)A_{n1}A_{n2}}{\sigma_n^2[1-R_0^2(\tau)]}\right]. \quad (2.6.5)$$

This result follows from the fact that

$$\frac{1}{2\pi}\int_0^{2\pi} e^{u\cos(\beta-\alpha)}d\beta = I_0(u),$$

where $I_0(u)$ - the modified function of zero-order Bessel.

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Consequently, after taking internal integral on φ_{n1}, we will obtain the expression, which does not depend on φ_{n2}; and the second integral on φ_{n2} after carrying out as the integral sign of all terms, which do not depend on φ_{n2}, it is led to the factor 2π.

Direct use $w_2(A_{n1}, A_{n2}, \tau)$ is hindered/hampered, since results little are demonstrative and it is difficult to interpret them.

Let us pass to $B_A(\tau)$. For obtaining $B_A(\tau)$ it is necessary to fulfill the integration

$$B_A(\tau)=\int_0^{\infty}\int_0^{\infty} A_{n1}A_{n2}w_2(A_{n1}, A_{n2}, \tau)\,dA_{n1}dA_{n2}.$$

The calculation of this integral is connected with the considerable mathematical difficulties and the bulky conversions. On these reasons we give the final result (in detail conclusion/output is given, for example, in [2.1])

$$B_A(\tau)=\frac{\pi}{2}\sigma_n^2\left\{1+\frac{R_0^2(\tau)}{4}+\sum_{k=2}^{\infty}\frac{[(2k-3)!!]^2}{2^{2k}k!}R_0^{2k}(\tau)\right\}. \qquad (2.6.6)$$

The obtained solution can be represented in the form of series/row according to degrees of $R_0(\tau)$

$$B_A(\tau)\approx\frac{\pi\sigma_n^2}{2}\left\{1+\frac{1}{4}R_0^2(\tau)+\frac{1}{64}R_0^4(\tau)+\cdots\right.$$
$$\left.\cdots+\left[\frac{(2k-3)\,1\cdot2\cdot3\cdots}{2\cdot4\cdots2k}\right]^2R_0^{2k}(\tau)\right\}.$$

If we are bounded to terms with $R^4{}_0(\tau)$, then we will obtain

$$B_A(\tau)\approx\frac{\pi}{2}\sigma_n^2\,[1+0{,}25R_0^2(\tau)+0{,}015R_0^4(\tau)+0{,}004R_0^6(\tau)]. \qquad (2.6.7)$$

If we are bounded by the first two members, since factor with $R^4_*(\tau)$ composes only 0.015 or 6% of the factor with $R^2_*(\tau)$, then we will obtain

$$B_A(\tau) \approx \frac{\pi}{2}\sigma_x^2[1+0{,}25R_*^2(\tau)]. \qquad (2.6.8)$$

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Knowing $B_A(\tau)$, we can find $G_A(\omega)$, using Fourier transform. The first term gives component with the zero frequency, i.e., the constant component of detection, then

$$G_A(\omega)=4\int_0^\infty \frac{\pi}{2}\sigma_n^2 0{,}25R_0^2(\tau)\cos\omega\tau d\tau.$$

Here $G_A(\omega)$ - "one-sided" spectrum of the fluctuations of amplitude. By ω we understand the frequency of the fluctuations of amplitude. Moreover $\omega\geq 0$. Since $B_A(\tau)$ is expressed as $R_0(\tau)$, it is possible to claim that random process $A_n(t)$ is slow [in comparison with n(t)] and is determined by the passband of receiver. In order to find the form of energy spectrum for the specific cases, we will use the approximation of frequency receiver response by ideal and gaussian filters. In this case energy interference spectrum at the output of receiver will be uniform with band $\Delta\omega_n$ or gaussian.

In the first case

$$R_0(\tau)=\frac{\sin\frac{\Delta\omega_n\tau}{2}}{\frac{\Delta\omega_n\tau}{2}},$$

where $\Delta\omega_{n}$ - complete passband of perfect filter.

In the second case

$$R_0(\tau)=e^{-\pi\left(\frac{\tau}{\tau_0}\right)^2},$$

where $\tau_0=\frac{1}{\Delta f_0}$;

Δf_0 - complete equivalent band of energy interference spectrum (before weakening by 0.46).

For the gaussian filter

$$G_A(\omega)=\frac{\pi\sigma_n^2}{2}\int_0^\infty e^{-\pi\left(\frac{\tau}{\tau_0}\right)^2\cdot 2}\cos\omega\tau d\tau=\frac{\pi^2\sigma_n^2}{2\sqrt{2}\Delta\omega_0}e^{-\frac{\pi}{2}\left(\frac{\omega}{\Delta\omega_0}\right)^2}. \quad (2.6.9)$$

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For the conclusion/output are used the formulas of § 2.2 with the transition from the designation $\Delta\omega$ (detuning of relatively carrier) to ω (frequency of the fluctuations of amplitude)

$$G_A(0)=\frac{\sigma_n^2\pi^2}{2\sqrt{2}\Delta\omega_0}=\frac{\pi\sigma_n^2}{4\sqrt{2}\Delta f_0}\approx 0,6\frac{\sigma_n^2}{\Delta f_0}. \quad (2.6.10)$$

After expressing dispersion through the energy spectrum, we will obtain

$$\sigma_A^2=\frac{1}{2\pi}\int_0^\infty G_A(\omega)\,d\omega=$$

$$=\frac{1}{2\pi}\int_0^\infty \frac{\sigma_n^2\pi^2}{2\sqrt{2}\,\Delta\omega_n}\,e^{-\pi\left(\frac{\omega}{\sqrt{2}\,\Delta\omega_n}\right)^2}d\omega\simeq 0{,}4\sigma_n^2.$$

Analogous result follows also from (2.6.8)

$$\sigma_A^2=B_A(0)-[m_1(A_n)]^2=\frac{\pi}{2}\sigma_n^2[1+0{,}25R_0^2(0)]-$$

$$-\frac{\pi}{2}\sigma_n^2=\frac{\pi}{8}\sigma_n^2\simeq 0{,}4\sigma_n^2.$$

This corresponds to the result obtained above.

For $R_0(\tau)=\frac{\sin\frac{\Delta\omega_n\tau}{2}}{\frac{\Delta\omega_n\tau}{2}}$

$$G_A(\omega)=\frac{\sigma_n^2\pi^2}{2}\int_0^\infty\left(\frac{\sin\frac{\Delta\omega_n\tau}{2}}{\frac{\Delta\omega_n\tau}{2}}\right)^2\cos\omega\tau\,d\tau=$$

$$=\frac{\pi^2\sigma_n^2}{2\Delta\omega_n}\left(\frac{\Delta\omega_n-\omega}{\Delta\omega_n}\right), \qquad (2.6.11)$$

$$G_A(0)=\frac{\sigma_n^2\pi^2}{2\Delta\omega_n}\simeq 0{,}81\,\frac{\sigma_n^2}{\Delta f_n}, \qquad (2.6.12)$$

$$\sigma_A^2=\frac{1}{2\pi}\int_0^\infty\frac{\pi^2\sigma_n^2}{2}\left(\frac{\Delta\omega_n-\omega}{\Delta\omega_n}\right)d\omega=\frac{\pi}{8}\sigma_n^2\simeq 0{,}4\sigma_n^2,$$

which also corresponds to the result, obtained earlier.

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Fig. 2.6.1 gives the energy spectra of the fluctuations of envelope or fluctuations of output potential of detector. The obtained results make it possible to do a series/row of conclusions/outputs. As is evident, the energy spectrum of the fluctuation of output potential of detector is substantially wider than half of passband on the radio frequency. This is explained by the fact that in the detector are detected the beatings between the extreme frequencies of noise spectrum, passing through the radio channel.

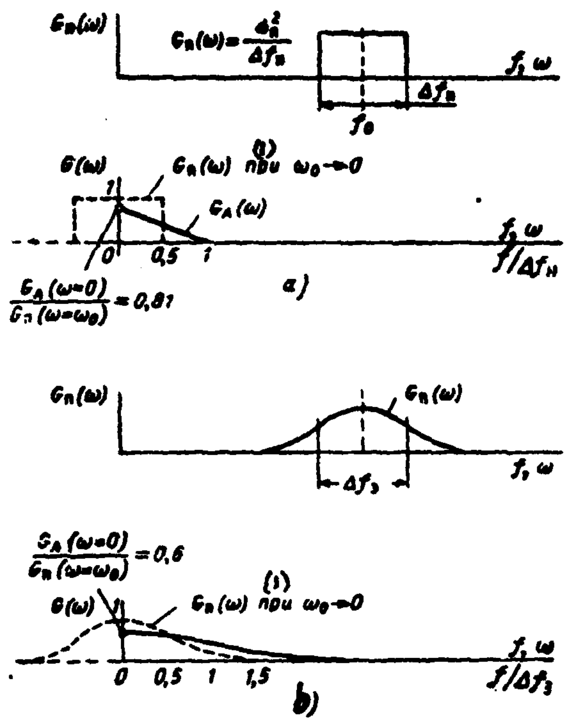

Fig. 2.6.1. The energy spectra of the fluctuations of the envelope of the interference: a) perfect filter; b) Gaussian filter.

Key: (1). with.

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It is necessary to note that the higher degrees of $R_{\bullet}(\tau)$, not considered by expression (2.6.8), give an even more considerable expansion of energy spectrum; however, the intensity of these components is negligibly small and to virtually it is possible not consider them.

Is obvious that there is no sense passband on the video frequency to make more widely than $\frac{\Delta\omega_{\text{п}}}{2}$ and $\frac{\Delta\omega_{\text{в}}}{2}$, since modulation frequencies, higher than $\frac{\Delta\omega_{\text{в}}}{2}$, must not be present in the useful spectrum of signal, since they pass through the radio channel (receiver) with the considerable weakening. If we reject/throw those parts of the spectrum of the fluctuations at the output of detector, which lie/rest above frequency $\frac{\Delta\omega_{\text{в}}}{2}$, then the dispersion of fluctuations noticeably decreases and will comprise, for example, for ideal rectangular filter 0.7 from $\sigma^2_{\text{н}}$ or $0,3\sigma^2_{\text{н}}$. However, is more substantially the fact that the spectrum of fluctuations is not uniform and the greatest power density of fluctuations is noted at the low frequencies. Power density of interference spectrum at the output of the radio-frequency part of the receiver, composing its 0.6 or 0.81 values, depending on the form of frequency characteristic.

Therefore during the calculation of the dispersion of fluctuations at the output of the narrow-band low-pass filter, connected after detector, it is necessary to consider the power density of fluctuations at the low frequencies.

The simplest correlations are obtained for the rectangular frequency characteristic. If $\Delta\Omega_{\text{в}}$ - filter pass band after detector, then the dispersion of fluctuations at the output of filter is equal to

$$\sigma^2_{\Delta\Omega} \approx \frac{1}{2\pi} G_A(0)\,\Delta\Omega_{\text{э}} = \sigma^2_{\text{ш}} \frac{\Delta F_{\text{э}}}{\Delta f_{\text{ш}}} \frac{G_A(0)}{G_{\text{ш}}(\omega = \omega_0)}$$

or

$$\sigma_{\Delta\Omega} = \sigma_{\text{ш}} \sqrt{\frac{\Delta F_{\text{э}}}{\Delta f_{\text{ш}}}} \sqrt{\frac{G_A(0)}{G_{\text{ш}}(\omega = \omega_0)}} \approx 0{,}9\sigma_{\text{ш}} \sqrt{\frac{\Delta F_{\text{э}}}{\Delta f_{\text{ш}}}},$$

where

$$\Delta F_{\text{э}} = \frac{\Delta\Omega_{\text{э}}}{2\pi}.$$

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Thus, it is virtually completely admissible during the calculation of power or rms value of fluctuations at the output of the narrow-band filter, connected after detector, to count

$$\sigma_{\Delta\Omega} \approx \sigma_{\text{ш}} \sqrt{\frac{\Delta\Omega_{\text{э}}}{\Delta\omega_{\text{ш}}}}.$$

During the contraction of band on the radio frequency it must be undertaken two times of larger than after detector, with the same passband for the modulating frequencies. The dispersion of fluctuation at the output of radio channel with that narrowed to $2\Delta\Omega$ by band has a value

$$\sigma^2_{2\Delta\Omega} = \sigma^2_{\text{ш}} \frac{2\Delta\Omega}{\Delta\omega_{\text{ш}}}.$$

After the detection of fluctuation they will have a dispersion

$$\sigma^2_{\Delta\Omega} = 0{,}43\sigma^2_{\text{ш}} \frac{2\Delta\Omega}{\Delta\omega_{\text{ш}}} \approx \sigma^2_{\text{ш}} \frac{\Delta\Omega}{\Delta\omega_{\text{ш}}},$$

i.e., the same value, as during the contraction of band to $\Delta\Omega_{\text{э}}$ after detector.

§ 2.7. Two-dimensional distribution function. Energy spectrum and the correlation function of the phase of interference. The two-dimensional function of phase distribution can be obtained from the four-dimensional function of amplitude distribution and phase.

The four-dimensional distribution function is obtained in § 2.6., then

$$w_2(\varphi_{н1}, \varphi_{н2}, \tau) = \int_0^\infty \int_0^\infty w_4(A_{н1}, A_{н2}, \varphi_{н1}, \varphi_{н2}, \tau)\, dA_{н1}\, dA_{н2} =$$

$$= \frac{1}{4\pi^2 \sigma_н^4 [1 - R_0^2(\tau)]} \int_0^\infty \int_0^\infty A_{н1} A_{н2} \times$$

$$\times e^{-\frac{A_{н1}^2 + A_{н2}^2 - 2R_0(\tau) A_{н1} A_{н2} \cos(\varphi_{н1} - \varphi_{н2})}{2\sigma_н^2 [1 - R_0^2(\tau)]}} dA_{н1} dA_{н2}. \quad (2.7.1)$$

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Let us replace variable/alternating

$$r_1 = A_{н1} / \sigma_н \sqrt{2[1 - R_0^2(\tau)]},$$
$$r_2 = A_{н2} / \sigma_н \sqrt{2[1 - R_0^2(\tau)]},$$

after transformations we obtain

$$w_2(\varphi_{n1}, \varphi_{n2}, \tau) = \frac{1 - R_0^2(\tau)}{\pi^2} \times$$

$$\times \int_0^\infty \int_0^\infty r_1 r_2 e^{-(r_1^2 + r_2^2 - 2r_1 r_2 y)} dr_1 dr_2, \qquad (2.7.2)$$

where

$$y = R_0(\tau) \cos(\varphi_{n1} - \varphi_{n2}).$$

The integral, entering in (2.7.2), is calculated (for example, see [2.1] and [2.2]) and result takes the form

$$\int_0^\infty \int_0^\infty r_1 r_2 e^{-(r_1^2 + r_2^2 - 2r_1 r_2 y)} dr_1 dr_2 = \frac{1 - \alpha \operatorname{ctg} \alpha}{4 \sin^2 \alpha}, \qquad (2.7.3)$$

where

$$\alpha = \arccos(-y). \qquad (2.7.4)$$

After substituting (2.7.4) and (2.7.3) in (2.7.2), after transformations we will obtain

$$w_2(\varphi_{n1}, \varphi_{n2}, \tau) = \frac{1 - R_0^2(\tau)}{4\pi^2} \left[\frac{1}{1 - y^2} + y \frac{\frac{\pi}{2} + \arcsin y}{(1 - y^2)^{3/2}} \right] \qquad (2.7.5)$$

with $\tau \to \infty$, $R_0(\tau) \to 0$, and, consequently

$$w_2(\varphi_{n1}, \varphi_{n2}, \infty) = \frac{1}{4\pi^2} = w(\varphi_{n1}) w(\varphi_{n2}).$$

[Page 92] As one would expect, with the large τ phases become statistically independent.

The obtained results are of definite interest; they show that the two-dimensional function of phase distribution has essential features. Probability density $w_2(\varphi_{n1}, \varphi_{n2}, \tau)$ depends not on the values of angles themselves φ_{n1} and φ_{n2}, but only from their difference. This can be explained by the fact that the function of phase distribution from $-\pi$ to $+\pi$ (or from 0 to 2π) is uniform. Therefore is not substantial the value of phases φ_{n1} and φ_{n2}, but is important how they differ from each other. Maximum value $w_2(\varphi_{n1}, \varphi_{n2}, \tau)$ will correspond $\varphi_{n1}-\varphi_{n2}=0$, most probably this combination of phases at the moments/torques, divided by any interval τ, with which initial phase remains constant/invariable. In other words, most probably this course of the random process during which it takes the form of harmonic oscillation with the constant/invariable initial phase.

For $w_{2\text{макс}}(\tau)$ when $\varphi_{n1}-\varphi_{n2}=0$ can be constructed the dependence as function of $R_0(\tau)$ or τ (with given $\Delta\omega_0$). Let us substitute into formula (2.7.5) value of $y=R_0(\tau)$ cos $0=R_0(\tau)$, we will obtain

$$w_{2\text{макс}}(\tau)=\frac{1-R_0^2(\tau)}{4\pi^2}\left\{\frac{1}{1-R_0^2(\tau)}+\right.$$

$$\left.+R_0(\tau)\frac{\frac{\pi}{2}+\arcsin R_0(\tau)}{[1-R_0^2(\tau)]^{3/2}}\right\}=$$

$$=\frac{1}{4\pi^2}\left\{1+\left[\frac{\pi}{2}+\arcsin R_0(\tau)\right]R_0(\tau)\frac{1}{\sqrt{1-R_0^2(\tau)}}\right\}. \quad (2.7.6)$$

Graphically this dependence is given in Fig. 2.7.1. Consequently, in the narrow-band noise due to the correlation of phase with the decrease τ and increase $R_0(\tau)$ probability density for values $\varphi_{n1}-\varphi_{n2}=0$ grows, and with

$$R_0(\tau)\to 1 \quad w_{2\,\text{макс}}(\tau)\to\infty.$$

According to formula $w_2(\varphi_{н1}, \varphi_{н2}, \tau)$, being assigned by different ones $\varphi_{н1}$ and $\varphi_{н2}$, it is possible to construct surface for the given value τ [or $R_0(\tau)$].

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Fig. 2.7.2 shows some plane curves, which form this surface. The more is τ, the less the maximum $w_2(\varphi_{н1}, \varphi_{н2}, \tau)$, which occurs when $\varphi_{н1}-\varphi_{н2}=0$ for any values $\varphi_{н1}$ (or $\varphi_{н2}$). $w_2(\varphi_{н1}, \varphi_{н2}, \tau)$ depends not on $\varphi_{н1}$ and $\varphi_{н2}$, but on their difference $(\varphi_{н1}-\varphi_{н2})$, and from the given formula it is possible to obtain the distribution function for difference $\Delta\varphi_{н}=\varphi_{н1}-\varphi_{н2}$. This function must show, as are distributed the probability densities of one or the other values of a phase difference, and to differ from the function examined, in which is given the probability density of different combination of phases $\varphi_{н1}$ and $\varphi_{н2}$.

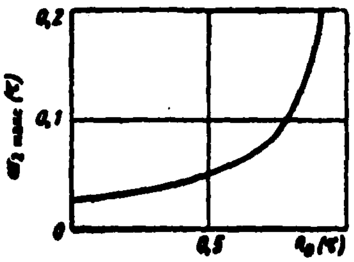

Fig. 2.7.1.

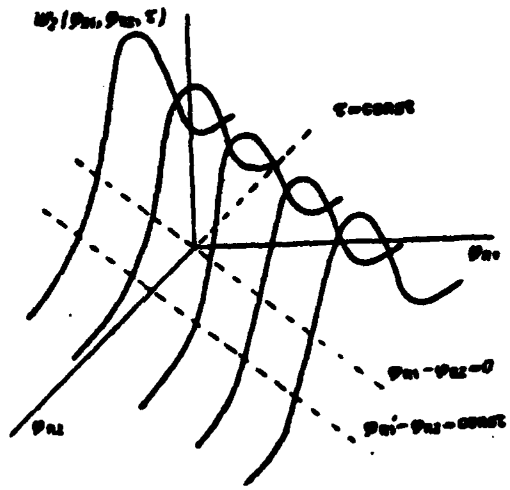

Fig. 2.7.2.

Fig. 2.7.1. Dependence $w_{2\,max}$ on $R_0(\tau)$.

Fig. 2.7.2. Surface $w_2(\varphi_{н1}, \varphi_{н2}, \tau)$ with τ=const.

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Although in this case it was established that $w_2(\varphi_{н1}, \varphi_{н2}, \tau)$ does not vary from change $\varphi_{н1}$ and $\varphi_{н2}$ if $\varphi_{н1}-\varphi_{н2}=const$, the dependence given above gives nevertheless the probability density of combination $\varphi_{н1}$ and $\varphi_{н2}$. For obtaining $w(\Delta\varphi_н)$ it is necessary to integrate probability $w_2(\varphi_{н1}, \varphi_{н2}, \tau)$ with respect to all values $\varphi_{н1}$

$$w(\Delta\varphi_{н}) = \int_0^{2\pi} w_2(\varphi_{н1}, \varphi_{н1} - \Delta\varphi_{н})\, d\varphi_{н1} =$$

$$= \frac{1 - R_0^2(\tau)}{2\pi}\left[\frac{1}{1-y^2} + y\,\frac{\frac{\pi}{2} + \arcsin y}{(1-y^2)^{3/2}}\right], \qquad (2.7.7)$$

$$y = R_0(\tau)\cos\Delta\varphi.$$

Fig. 2.7.3 give dependence $w(\Delta\varphi_{н})$ with different $R_0(\tau)$, which are the parameter, in function $\Delta\varphi_{н}$. Fig. 2.7.4 gives dependence $w(\Delta\varphi_{н})$ on $R_0(\tau)$ for different ones $\Delta\varphi_{н}$. One-dimensional probability density for $\Delta\varphi_{н}$ [depending on $R_0(\tau)$] at values of $R_0(\tau)$, close to one, i.e., for small intervals of time τ, takes very high values and passes into infinity. This means that in the short finite time intervals the probability of retaining/preserving/maintaining the initial phase, i.e., this course of the random process, during which its initial phase is not changed, is greatest.

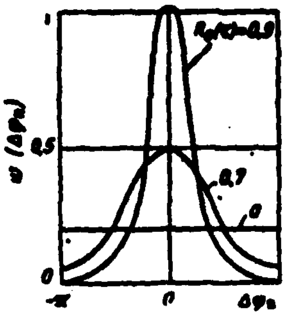

Fig. 2.7.3.

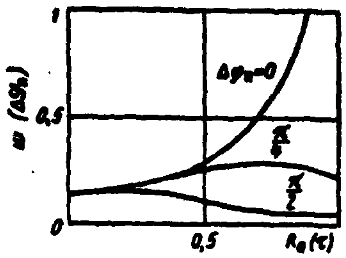

Fig. 2.7.4.

Fig. 2.7.3. One-dimensional function of distribution of phase difference.

Fig. 2.7.4. Dependence of function of distribution of phase difference on $R_0(\tau)$.

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Let us now move on to the correlation function and the energy spectrum of the fluctuations of phase.

Knowing $w_2(\varphi_{m1}, \varphi_{m2}, \tau)$, it is possible to find $B_\varphi(\tau)$:

$$B_\varphi(\tau) = \int_{-\infty}^{+\infty} \varphi_{m1}, \varphi_{m2}, w_2(\varphi_{m1}, \varphi_{m2}, \tau)\, d\varphi_{m1} d\varphi_{m2}. \quad (2.7.8)$$

After using the fact that $w_2(\varphi_{m1}, \varphi_{m2}, \tau)$ is periodic function from $\varphi_{m1} - \varphi_{m2}$, it is possible to expand it in Fourier series in terms of variable/alternating $\varphi_{m1}-\varphi_{m2}$

$$w_2(\varphi_{m1},\varphi_{m2},\tau)=\sum_{l=-\infty}^{+\infty}A_l e^{il(\varphi_{m1}-\varphi_{m2})}=$$

$$=A_0+2\sum_{l=1}^{\infty}A_l\cos l(\varphi_{m1}-\varphi_{m2}). \qquad (2.7.9)$$

where

$$A_0=\frac{1}{4\pi^2};$$

$$A_l=\frac{1-R_0^2(\tau)}{4\pi^2}\sum_{k=1}^{\infty}\frac{\Gamma^2\left(k+1+\frac{l}{2}\right)}{k!(k+l)!}R_0^{l+2k}(\tau).$$

After fulfilling integration, we will obtain

$$B_\varphi(\tau)=8\pi^2\sum_{l=1}^{\infty}\frac{1}{l^2}A_l(\tau). \qquad (2.7.10)$$

By using expressions for A_l and after carrying out an expansion of correlation function in the series/row according to degrees of $R_0(\tau)$, we will obtain

$$B_\varphi(\tau)=\frac{1}{2}\sum_{r=1}^{\infty}\sum_{k=0}^{\infty}\frac{\Gamma^2\left(k+\frac{r}{2}\right)}{k!(k+r)!}R_0^{r+2k}=$$

$$=\frac{\pi}{2}R_0(\tau)+\frac{1}{4}R_0^2(\tau)+\frac{\pi}{12}R_0^3(\tau)+$$

$$+\frac{5}{48}R_0^4(\tau)+\frac{1}{24}R_0^5(\tau)\ldots \qquad (2.7.11)$$

Page 96. More detailed derivation of formulas (2.7.9), (2.7.10), (2.7.11) is given in [2.1]. If $R_0(\tau)<0.5$, then primary meaning have the first terms of expansion. With $R_0(\tau)\to 1$ or $\tau\to 0$ the addition of the limited number of terms gives large error and it is necessary to summarize entire series/row.

The addition of entire series/row with $R_e(\tau)=1$ gives

$$B_\varphi(0)=\frac{\pi^2}{3}.$$

Graph $B_\varphi(\tau)$ in function $R_e(\tau)$ is given in Fig. 2.7.5. Coefficient of correlation

$$R_\varphi(\tau)=\frac{B_\varphi(\tau)}{B_\varphi(0)}.$$

Graph $R_\varphi(\tau)$ in function $R_e(\tau)$ is given in Fig. 2.7.6. The given formulas and graphs do not completely reveal property $B_\varphi(\tau)$ and $R_\varphi(\tau)$, since they are given not depending on τ, but depending on the low-frequency multiplier in the correlation function $R_e(\tau)$. To obtain the dependences of $B_\varphi(\tau)$ and $R_\varphi(\tau)$ on τ it is necessary to specify the form of the frequency characteristic of radio channel, which determines dependence $R_e(\tau)$ on τ.

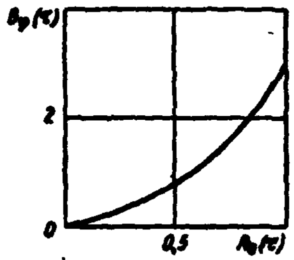

Fig. 2.7.5.

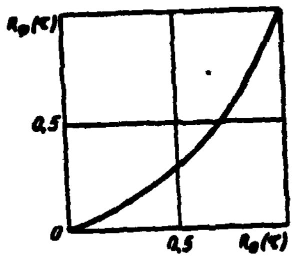

Fig. 2.7.6.

Fig. 2.7.5. Dependence $B_\varphi(\tau)$ on $R_0(\tau)$.

Fig. 2.7.6. Dependence $R_\varphi(\tau)$ on $R_0(\tau)$.

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Above were used two models of the receiver:

in the form of the perfect filter

$$R_0(\tau)=\frac{\sin\frac{\Delta\omega_n\tau}{2}}{\frac{\Delta\omega_n\tau}{2}};$$

in the form of the Gaussian filter

$$R_0(\tau)=e^{-a\left(\frac{\tau}{\tau_0}\right)^2}.$$

Using these relationships/ratios, it is possible to construct dependences $B_\varphi(\tau)$ or $R_\varphi(\tau)$ on τ/τ_m, where τ_m - interval or the time of correlation. The correlation function of phase for the case of perfect filter is constructed in Fig. 2.7.7.

From the obtained results it is possible to do important the conclusion that the correlation function of phase at point $\tau \to 0$ or $R_{\varphi}(\tau) \longrightarrow 1$ has the jump of first-order derivative. Of these reasons the determination of the correlation function of derived phase substantially hinders. $B_{\varphi}(\tau)$ is more "narrow", than the low-frequency factor of the correlation function of initial process, and the energy spectrum of the fluctuations of phase must contain more high-frequency components, i.e., it must be wider than the spectrum of initial random process. Further this question will be examined in detail.

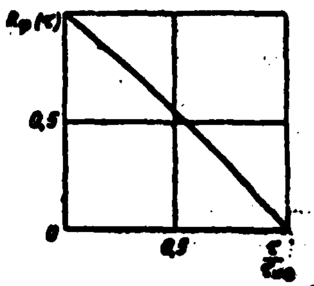

Fig. 2.7.7. Correlation function of phase.

Figure shows half of function, to $\tau/\tau_{m0}=1$; symmetrical branch for the negative τ is not given.

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Let us return to the expression for $B_\varphi(\tau)$ in the form of series/row and will take out common factor for the brackets, then

$$B_\varphi(\tau)\simeq\frac{\pi}{2}\,[R_0(\tau)+0{,}16R_0^2(\tau)+$$
$$+0{,}17R_0^3(\tau)+0{,}07R_0^4(\tau)+0{,}026R_0^5(\tau)+\ldots]; \quad (2.7.12)$$

when $\tau\to0$ $B_\varphi(\tau)=\frac{\pi^2}{3}\simeq\pi$ and sum must be equal to 2.

Let us compare this expression

$$B_A(\tau)\simeq\frac{\pi\sigma_n^2}{2}[1+0{,}25R_0^2(\tau)+0{,}015R_0^4(\tau)+\ldots].$$

with $\tau\to0$

$$B_A(\tau)=\sigma_A^2+[m_1(A_n)]^2=0{,}43\sigma_n^2+$$
$$+(1{,}25\sigma_n)^2=2\sigma_n^2$$

and the sum of the terms of series/row must be equal to

$$\frac{4}{\pi}=1{,}27.$$

from the comparison it is evident that in the series/row, through which is expressed $B_{\varphi}(\tau)$, terms with the high degrees of $R_{0}(\tau)$ have considerably larger influence, than in the series/row for $B_{A}(\tau)$. For example, for $R_{0}(\tau)=1$ the sum of coefficients with the degrees higher than second comprises in the series/row for $B_{\varphi}(\tau)$ about 0.8 or 40% of the sum of all terms of series/row, while in the series/row for $B_{A}(\tau)$ - about 0.025 or 0.7% of the sum of all terms of series/row and $\approx$7% of the term with $R_{0}(\tau)$ to the second degree. On these reasons during the study of the fluctuations of amplitude to completely admissibly disregard all terms whose degree is higher than the second. During the research of the fluctuations of phase use of term $R_{0}(\tau)$ in the second degree does not give correct representation about the processes and it is necessary to consider terms with the higher degrees.

It is obvious that the higher the degree of $R_{0}(\tau)$, the wider the energy spectrum, caused on this term.

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Since the terms with the high degrees in the series/row for $B_\varphi(\tau)$ have noticeable value, theoretically the spectrum of the fluctuations of phase must be very wide, strictly speaking, by infinitely wide, moreover the essential part of the dispersion of the fluctuations of phase must be concentrated in the high-frequency part of energy spectrum. Since with $R_0(\tau)=1$ the sum of terms with the degrees higher than second is approximately 40% of the sum of terms with the lower degree, one should expect that the noticeable part of the dispersion of phase depends on the high-frequency components of the fluctuations of the phases whose frequency is higher than $\Delta\omega_э$. Earlier repeatedly was used relationship/ratio for the transition from the correlation function to the energy spectrum

$$G_\varphi(\omega)=4\int_0^\infty B_\varphi(\tau)\cos\omega\tau d\tau,$$

then

$$G_\varphi(\omega)=2\pi\left[\int_0^\infty R_0(\tau)\cos\omega\tau d\tau+\right.$$
$$+\frac{1}{2\pi}\int_0^\infty R_0^2(\tau)\cos\omega\tau d\tau+$$
$$\left.+\frac{1}{6}\int_0^\infty R_0^3(\tau)\cos\omega\tau d\tau+\ldots\right], \qquad (2.7.13)$$

where $G_\varphi(\omega)$ - one-sided spectrum.

In order the effect of the expansion of the energy spectrum of the fluctuation of phase to determine is most clearly, let us consider the case of perfect filter.

Let us compute Fourier integral of the function

$$\left(\frac{\sin\frac{\Delta\omega_{н}\tau}{2}}{\frac{\Delta\omega_{н}\tau}{2}}\right)^{k},$$

where k - degree of the corresponding factor.

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With k=1 the result is obvious. To this degree of factor will correspond the uniform spectrum in the limits from 0 to $\frac{\Delta\omega_{н}}{2}$.

$$\int_{0}^{\infty}\frac{\sin\frac{\Delta\omega_{н}\tau}{2}}{\frac{\Delta\omega_{н}\tau}{2}}\cos\omega\tau d\tau=\frac{\pi}{\Delta\omega_{н}}. \qquad (2.7.14)$$

With k=2 the result was obtained earlier

$$\int_{0}^{\infty}R_{0}^{2}(\tau)\cos\omega\tau d\tau=\int_{0}^{\infty}\left(\frac{\sin\frac{\Delta\omega_{н}\tau}{2}}{\frac{\Delta\omega_{н}\tau}{2}}\right)^{2}\cos\omega\tau d\tau=$$

$$=\pi\frac{\Delta\omega_{н}-\omega}{\Delta\omega_{н}^{2}}. \qquad (2.7.15)$$

With k≥3 it is convenient to use the approximate relationship/ratio

$$\int_0^\infty \left(\frac{\sin\frac{\Delta\omega_x\tau}{2}}{\frac{\Delta\omega_x\tau}{2}}\right)^k \cos\omega\tau d\tau =$$

$$= \frac{1}{\Delta\omega_x}\sqrt{\frac{6\pi}{k}}e^{-\frac{6\omega^2}{k\Delta\omega_x^2}}. \qquad (2.7.16)$$

Two first terms with $R_\varphi(\tau)$ and $R^2_\varphi(\tau)$ give the components of the spectrum with a limited highest frequency of $\Delta\omega_x/2$ and $\Delta\omega_x$ respectively. Third term with $R^3_\varphi(\tau)$ and subsequent members give the components of the spectrum with the infinitely broad band, moreover their portion in the dispersion of the fluctuations of phase is very noticeable. A precise calculation of the spectrum requires the account of a large number of components. In this case qualitatively the picture will not be changed, since the account of term $R^3_\varphi(\tau)$ already gives the infinitely wide spectrum of the fluctuations of phase.

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For these reasons can represent interest the approximation of series/row by three members with an increase in the coefficient of latter/last member to the level, on which is considered the effect of all subsequent members on the dispersion.

This leads to the expression

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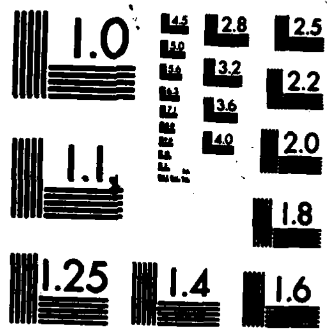

MICROCOPY RESOLUTION TEST CHART
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$$B_\varphi(\tau) \simeq \frac{\pi}{2}\left[R_0(\tau) + \frac{1}{2\pi}R_0^2(\tau) + 0{,}84R_0^3(\tau)\right], \quad (2.7.17)$$

$$R_\varphi(\tau) \simeq 0{,}5R_0(\tau) + 0{,}08R_0^2(\tau) + 0{,}42R_0^3(\tau). \quad (2.7.18)$$

During the use of the approximation/approach indicated the energy spectrum of the fluctuations of phase will take the form

$$G_\varphi(\omega) = 2\pi\left[\frac{\pi}{\Delta\omega_н}\left(\omega < \frac{\Delta\omega_н}{2}\right) + \frac{1}{2}\frac{\Delta\omega_н - \omega}{\Delta\omega_н^2} + \frac{\sqrt{2\pi}0{,}84}{\Delta\omega_н}e^{-2\frac{\omega^2}{\Delta\omega_н^2}}\right]. \quad (2.7.19)$$

With $\omega=0$

$$G_\varphi(0) = \frac{2\pi^2}{\Delta\omega_н} + \frac{\pi}{\Delta\omega_н} + \frac{2\pi\sqrt{2\pi}0{,}84}{\Delta\omega_н} = \frac{1}{\Delta\omega_н}[2\pi^2 + \pi + 0{,}84(2\pi)^{3/2}]. \quad (2.7.20)$$

Fig. 2.7.8 gives the components of the spectrum (dotted line) and spectrum $G_\varphi(\omega)\Delta\omega_н$.

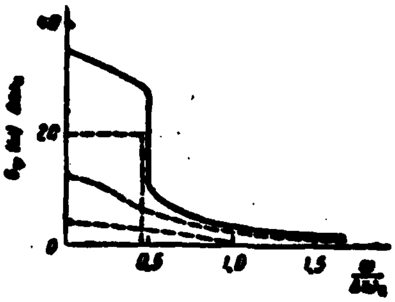

Fig. 2.7.2. The energy spectrum of the fluctuations of phase with the perfect filter.

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The presence in the energy spectrum of the phase of high-frequency components, i.e., components with the higher frequency than $\Delta\omega_{м}$, needs further explanations; these components are absent in the limits of width of band of the nar ow-band spectrum, observed at the output of perfect filter, the mutual compensation for the spectra of the fluctuations of amplitude and phase cannot occur.

It is possible to assume that the presence of high-frequency components is conditional and is defined by the fact that above phase was considered as the value whose possible values were included within the limits $\pm\pi$. Since the phase of selective interference, being random, can be changed in the wider limits, can be observed its transitions (or jumps) from $+(\pi+d\varphi_{п})$ to $-\pi$ and vice versa, which are accompanied by a change in the value, which numerically characterizes phase. However, this assumption cannot completely explain the obtained results. Let us consider this in somewhat more detail. The correlation function of phase, used for the determination, the energy spectrum, was obtained by integration within the limits $\pm\pi$ (see 2.7.8). In this case the density of fluctuations at the zero frequencies proved to be final and the process of changing the phase stationary.

If we are not interested in the high-frequency components in the spectrum of the fluctuations of the phase and its derivative, which are mapped into terms with the high degrees of $R_0(\tau)$ and cause the interruption/discontinuity of first-order derivative $B_{\dot{\varphi}}(\tau)$ on τ at point τ=0, then it is possible simply to pass to the spectrum of derived phase, using factor ω^2. Consequently, the finiteness of the density of the fluctuations of phase at the close to zero frequencies leads to the fact that must not be observed the slow fluctuations of derivative.

However, special features/peculiarities indicated above of the obtained results are not confirmed by physical representations and conclusions, which ensue/escape/flow out of more strictly the analysis of the statistical properties of the derived phase of selective interference and phase as integral of its derivative.

From the physical considerations it follows that the phase of narrow-band random process can be changed (" go away") between very wide limits, which considerably exceed $\pm\pi$. At this conclusion it is possible to arrive also from the results, obtained below in S 2.9.

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The analysis of expression 2.9.4 and 2.9.9 for the correlation function and energy spectrum of derived phase shows that the slow fluctuations of frequency have finite value. in this case the phase as integral of the frequency deviations proves to be unsteady process with the increasing dispersion, i.e., fluctuating, it will have ever more and the more increasing divergences to the different sides from the initial value, many times falling outside interval $\pm\pi$. This, probably, and causes the uniform probability density of phase in the limits $-\pi$, $+\pi$ and the stability of that random process which is described 2.7.11, 2.7.13 (and by approximations 2.7.17, 2.7.19), during conclusion/output of which the possibility of these transitions was considered.

To the determination of the sense of the presence of the high-frequency components in the energy spectrum of phase can contribute the analysis of the correlation function and energy spectrum $\cos\varphi_{\pi}$. It is obvious that $\cos\varphi_{\pi}$ does not endure changes upon transfer of phase from $+\pi$ to $-\pi$.

If high-frequency component in the energy spectrum of phase are conditional and are caused only by transitions $+\pi$, $-\pi$, then energy spectrum $\cos\varphi_{\pi}$ must have characteristic difference from the spectrum for φ_{π}.

For obtaining the expressions $B_z(\tau)$ and $G_z(\omega)$, where $z=\cos\varphi_{\pi}$, it is necessary to carry out functional transformations with $w(\varphi_{\pi1}, \varphi_{\pi2}, \tau)$, after passing to $z_1=\cos\varphi_{\pi1}$ and $z_2=\cos\varphi_{\pi2}$. After fulfilling transformations, it is possible to obtain:

$$w(z_1, z_2, \tau)=\frac{1}{\pi^2\sqrt{1-z_1^2}\cdot\sqrt{1-z_2^2}}\times$$
$$\times\left[\frac{1}{4\pi^2}+2\sum_{l=1}^{\infty}A_l\cos(l\cos z_1)\cos(l\cos z_2)\right]. \quad (2.7.21)$$

From (2.7.21) after the series/row of the transformations (see 2.1) we obtain

$$B_z(\tau)=\frac{1-R_0^2(\tau)}{2}\sum_{k=0}^{\infty}\frac{\Gamma^2\left(k+\frac{3}{2}\right)}{\pi(k+1)!}R_0^{2k+1}(\tau)=$$
$$=\frac{\pi}{8}\left[R_0(\tau)+\frac{1}{8}R_0^3(\tau)+\frac{3}{64}R_0^5(\tau)+\dots\right] \quad (2.7.22)$$

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Bearing in mind that when $w(\varphi_{\pi})=\frac{1}{2\pi}$ dispersion $\cos\varphi_{\pi}$ is equal to 0.5, from (2.7.22) we obtain, that with $R_0(\tau) \to 1$ bracketed expression approaches 1.273. Consequently, the sum of coefficients with $R_0(\tau)$ with the third and with the higher degrees must compose 0.273. Mathematical expression for $G_2(\omega)$ easily can be obtained from (2.7.22) during use (2.7.16), and we it lower. However, it is obvious that since in the expression for $B_2(\tau)$ the terms with the high degrees of $R_0(\tau)$ are considerable, energy spectrum $G_2(\omega)$ will have noticeable high-frequency components.

The aforesaid makes it possible to assume that can occur the high-frequency fluctuations of phase, which reflect the special features/peculiarities of the random process of its changes. But this assumption comes into conflict with the condition of the narrow-band characteristic of initial random process, since allows/assumes the possibility of considerable and rapid changes in the quasi-harmonic oscillation, which can be only in such a case, when it, at the finite value of amplitude, contains in the spectrum of initial oscillation further which constitute, which are located out of the limits of narrow band, which is not in the case in question. In order to avoid this contradiction, one should recall that the amplitude of selective interference is subordinated to Rayleigh distribution and there is a finite probability of its very low values. At the close to zero or very low values of amplitude abrupt changes in the phase can be observed, also, without essential changes of the spectrum of initial oscillation. Qualitatively aforesaid is clarified by Fig. 2.10.5.

In the examination of all cases with the filtration after phasemeter it is necessary to have in mind that under the effect of one interference any change in the band cannot be reflected in the dispersion of fluctuation reading/indication of phasemeter, since all values of phase remain equiprobable and dispersion $\pi^2/3$ is retained with any band. Will be changed only rate of change in readings/indications of phasemeter. Consequently, the integration of spectrum $G_\varphi(\omega)$ in close margins does not make sense.

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§ 2.8. Function of the distribution of the derived phase of interference. Above was used the representation of selective interference in the form of harmonic oscillation with the random ones by amplitude and the phase. In this case the medium frequency ω_0 was assumed/set to the equal midband frequency of the transmission of radio-frequency filters. From the obtained functions of distribution and energy spectrum of phase it is evident that the phase of this oscillation is by chance and has even distribution. The transient nature of basic changes in the phase is determined by half of passband; however, there are also more rapid changes in the phase.

Since the phase of harmonic oscillation equivalent to interference is changed, is changed its instantaneous frequency [2.6, 2.7].

It is known that if

$$n(t) = A_n(t)\cos[\omega_0 t + \varphi_n(t)],$$

then the frequency

$$\omega = \omega_0 + \frac{d\varphi_n(t)}{dt} = \omega_0 + \Delta\omega(t),$$

$$\Delta\omega(t) = \frac{d\varphi_n(t)}{dt}. \qquad (2.8.1)$$

Consequently, the instantaneous value of the frequency of equivalent harmonic oscillation is the random function of time. If the fluctuations of phase occur around some constant value, then the average/mean value of derivative is equal to zero and medium frequency $\omega_{ep} = \omega_0$.

However, since all values of phase are equiprobable, the average/mean value of phase can have monotonic change, i.e., the average/mean value of derived phase can be not equal to zero.

Thus, the obtained previously results do not give grounds for a precise estimation of average jamming frequency. It can be equal to ω_0, but it can differ from it. A precise estimation of jamming frequency is of interest in such a case, when information is embedded into the signal frequency or the derivative of phase. Phasemeter can be constructed on the principle of the direct measurement of phase displacement and then rate of change in its readings/indications will be determined derived phase. If average/mean rate of change in the phase (or the average/mean value of derived phase) is equal to zero, then readings/indications of phasemeter, fluctuating under the action of interferences, will not contain systematically changing component.

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But phasemeter can be constructed also according to the principle of the integration of a difference in the frequencies, then during the analysis of the effect on it of interference it is necessary to proceed from the concept of frequency.

These two determinations of frequency are in connection with harmonic oscillation equivalent. During the random narrow-band process these determinations give different results. Interference will differently function on the meters of phase and the frequencies, constructed on the different physical principles.

First of all let us consider the case of the direct phase measurements when by frequency is understood the derivative of phase, and then let us pass to the method of the direct measurement of frequency.

For the solution of the first problem we find the function of the distribution of derived phase for the selective interference. For obtaining the functions of the distribution of derivative it is possible to use expression for the function of the distribution of a phase difference

$$w(\Delta\varphi_{n})=\int_{-\pi}^{+\pi} w_{2}(\varphi_{n1},\ \varphi_{n1}+\Delta\varphi_{n},\ \tau)\,d\varphi_{n1}, \qquad (2.8.2)$$

where $w_{2}(\varphi_{n1},\ \varphi_{n1}+\Delta\varphi_{n},\ \tau)$ - two-dimensional distribution function. In § 2.7 this integration was carried out

$$w(\Delta\varphi_{n})=\frac{1-R_{0}^{2}(\tau)}{2\pi}\left[\frac{1}{1+y^{2}}+y\,\frac{\frac{\pi}{2}+\arcsin y}{(1-y^{2})^{3/2}}\right], \qquad (2.8.3)$$

where

$$y=R_{0}(\tau)\cos\Delta\varphi_{n}.$$

Derivative can be obtained as the limit

$$\frac{d\varphi_{n}}{dt}=\dot{\varphi}_{n}=\lim_{\tau\to 0}\frac{\Delta\varphi_{n}}{\tau}, \qquad (2.8.4)$$

then

$$w(\dot{\varphi}_{n})=\lim_{\tau\to 0} w\left(\frac{\Delta\varphi_{n}}{\tau}\right).$$

For the transition to $w(\dot{\varphi}_{n})$ it is necessary to fulfill some transformations, since in general form expression is bulky. $R_0(\tau)$ it is possible to expand in series/row according to degrees τ.

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Since function $R_0(\tau)$ is symmetrical and with $\tau=0$ has a maximum, for the points near $\tau=0$ we will obtain

$$R_0(\tau) = 1 + \frac{1}{2}\frac{d^2R_0(\tau)}{d\tau^2}\tau^2.$$

For obtaining the function of the distribution of derivative it is necessary to find expression $w(\Delta\varphi)$ for the low values τ and, therefore, $\Delta\varphi$. This allows the terms, entering in (2.8.3), to express approximately, disregarding the small second-order quantities. Then we obtain

$$1 - y^2 \simeq 1 - R_0^2(\tau)\cos^2\Delta\varphi_n \simeq \frac{d^2R_0(\tau)}{d\tau^2}\tau^2 + \Delta\varphi_n^2,$$

$$y \simeq 1 + \tau^2\frac{d^2R_0(\tau)}{2d\tau^2} - \frac{1}{2}\Delta\varphi_n^2,$$

$$\frac{\pi}{2} + \arcsin y = \frac{\pi}{2} + \arcsin \times$$

$$\times\left(1 + \frac{1}{2}\frac{d^2R_0(\tau)}{d\tau^2}\tau^2\right)\cos\Delta\varphi_n =$$

$$= \pi - \frac{1}{2}\frac{d^2R_0(\tau)}{d\tau^2}\tau^2 - \frac{1}{2}\Delta\varphi_n^2.$$

Let us designate

$$\frac{d^2R_0(\tau)}{d\tau^2} = -\delta\omega^2, \qquad (2.8.5)$$

then

$$w(\Delta\varphi_n) = \frac{-\delta\omega^2\tau^2}{4\pi}\left[\frac{1}{-\delta\omega^2\tau^2 + \Delta\varphi_n^2} + \right.$$

$$\left. + \frac{\left(1 + \frac{\delta\omega}{2}\tau^2 - \frac{\Delta\varphi_n^2}{2}\right)\left(\pi + \frac{\delta\omega^2}{2}\tau^2 - \frac{\Delta\varphi_n^2}{2}\right)}{(\delta\omega^2\tau^2 + \Delta\varphi_n^2)^{3/2}}\right]. \qquad (2.8.6)$$

Let us switch over to the function of distribution $w\left(\frac{\Delta\varphi_n}{\tau}\right)$:

$$w\left(\frac{\Delta\varphi_{п}}{\tau}\right)=\tau\frac{\delta\omega^2\tau^2}{4\pi}\left\{\frac{1}{-\delta\omega^2\tau^2+\left(\frac{\Delta\varphi_{п}}{\tau}\right)^2\tau^2}+\right.$$

$$+\frac{\left[1+\frac{\delta\omega}{2}\tau^2-\frac{1}{2}\left(\frac{\Delta\varphi_{п}}{\tau}\right)^2\tau^2\right]\times}{\left[-\delta\omega^2\tau^2+\right.}$$

$$\left.\frac{\times\left[\pi-\frac{\delta\omega^2}{2}\tau^2+\frac{1}{2}\left(\frac{\Delta\varphi_{п}}{\tau}\right)^2\tau^2\right]}{\left.+\left(\frac{\Delta\varphi_{п}}{\tau}\right)^2\tau^2\right]^{3/2}}\right\}. \quad (2.8.7)$$

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After transition to the limit with $\tau\to 0$ and transformations we will obtain

$$w(\dot{\varphi}_{п})=\frac{1}{2\delta\omega}\frac{1}{\left(1+\frac{1}{\delta\omega^2}\dot{\varphi}^2\right)^{3/2}}. \quad (2.8.8)$$

The basic parameter of the obtained distribution function it is

$$\delta\omega=\sqrt{-\left.\frac{d^2R_0(\tau)}{d\tau^2}\right|_{\text{при }\tau\to 0}}.$$

Knowing $R_0(\tau)$, it is possible to find the relationships/ratios, which connect $\delta\omega$ with the characteristics of initial random process.

For the perfect filter

$$\left.\frac{d^2R_0(\tau)}{d\tau^2}\right|_{\tau\to 0}=-\left.\frac{d^2\dfrac{\sin\frac{\Delta\omega_{п}\tau}{2}}{\frac{\Delta\omega_{п}\tau}{2}}}{d\tau^2}\right|_{\tau\to 0}=-\frac{(\Delta\omega_{п})^2}{12}$$

and

$$\delta\omega = \frac{\Delta\omega_n}{\sqrt{12}}.$$

For the Gaussian filter

$$\left.\frac{d^2R_0(\tau)}{d\tau^2}\right|_{\tau\to 0} = \left.\frac{d^2 e^{-\pi\left(\frac{\tau}{\tau_p}\right)^2}}{d\tau^2}\right|_{\tau\to 0} = -\frac{\Delta\omega_p^2}{2\pi} \quad \text{and} \quad \delta\omega = \frac{\Delta\omega_p}{\sqrt{2\pi}}.$$

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Consequently, $\delta\omega$ has the dimensionality of frequency and characterizes the width of the spectrum, in which are concentrated the basic divergences of the derived phase of interference. In [2.2] for $\delta\omega$ was accepted the name "average/mean width of the spectrum". Fig. 2.8.1 gives the curves, constructed according to formula (2.8.8). Fig. 2.8.2 gives the function of the distribution of derived phase for the perfect filter. From the obtained results it follows that there is a finite probability of large divergences of the instantaneous values of derived phase from the average/mean value, which is determined by the medium frequency of filter ω_0, since $\dot{\varphi}=\Delta\omega$ and $\omega=\omega_0+\Delta\omega$.

However, function $w(\dot{\varphi}_n)$ is symmetrical and average/mean value or mathematical expectation of it is equal to zero

$$m_1(\dot{\varphi}_n)=\int_{-\infty}^{+\infty}\dot{\varphi}_n w(\dot{\varphi}_n)\,d\dot{\varphi}_n=0.$$

Consequently, if we use direct phase measurements and to approach the concept of frequency as derived phase, then the medium frequency of selective interference will correspond to the midband frequency of transmission. Moreover, the fluctuations of frequency can contain the considerable divergences, much greater than half of passband.

Thus, the instantaneous value of the derived phase of interference is subjected to considerable changes, but its average/mean value is equal to zero.

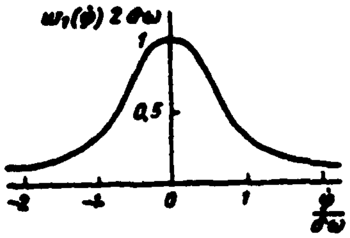

Fig. 2.8.1.

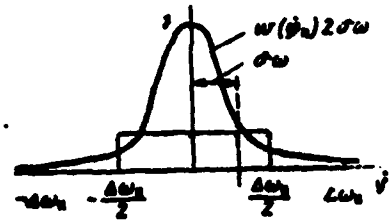

Fig. 2.8.2.

Fig. 2.8.1. Function of distribution of derived phase.

Fig. 2.8.2. Function of distribution of derived phase for perfect filter.

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It is interesting to note that the dispersion of the fluctuations of frequency cannot be obtained

$$\sigma_{\dot\varphi}^2 = \int_{-\infty}^{+\infty} \dot\varphi_n^2 w(\dot\varphi_n)\, d\dot\varphi_n.$$

This integral does not have finite value, since integrand insufficiently rapidly decreases with increase $\dot\varphi_n$. This also testifies about the finite probability of the large divergences of derived phase from its average/mean value.

The explanation of the fact that there does not exist the final dispersion of derived phase, on the research of V. I. Tikhonov [2.4, 2.9, 2.10] consists in the presence of the "migrations/jumps" of phase to the integer 2π. The noticeable probability density of the considerable divergences of derivative also does not contradict the presence noted above of the rapid fluctuations of phase.

Thus, the function of the distribution of the derived phase of selective interference they made it possible to do important conclusions the fact that in the direct phase measurements the average/mean value of derived phase is equal to zero and the presence of interference cannot give systematic error.

§ 2.9. Correlation function and the energy spectrum of the derived phase of selective interference. For evaluating the action of interferences on the measuring device, besides the function of distribution $w(\dot{\varphi}_n)$, it is necessary to know also correlation function and energy spectrum of the fluctuations of derived phase. This will make it possible to determine the transient nature of the fluctuations of derived phase and the efficiency of the action of the filters, included after meter.

For obtaining $B_{\dot{\varphi}}(\tau)$ it is possible to use two methods: simple and complicated.

Simple method provides for obtaining the correlation function of the derivative of random process by double differentiation of the correlation function of basic random process, in this case phase, i.e.

$$B_{\dot{\varphi}}(\tau) = -\frac{d^2 B_{\varphi}(\tau)}{d\tau^2}. \qquad (2.9.1)$$

However, the use of this method in the specific case of determination $B_{\dot{\varphi}}(\tau)$ meets with essential difficulties.

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The reason for these difficulties lies in the fact that $B_{\varphi}(\tau)$ does not possess continuity of first-order derivative at point $\tau=0$ and $\frac{d^2 B_{\varphi}(\tau)}{d\tau^2}$ has with $\tau=0$ infinite value. This, in particular, it is exhibited in the fact that dispersion $\sigma^2_{\dot{\varphi}}$ is infinite. Consequently, the use of a simple method of determination $B_{\dot{\varphi}}(\tau)$ proves to be impossible.

Complicated method requires the execution of entire volume of the transformations, necessary for obtaining of the two-dimensional function of distribution $\dot{\varphi}_2$ and correlation function. For this it is necessary:

to find the four-dimensional functions of the distribution

$$w_4(\Phi_1, \Phi_2, \dot{\Phi}_1, \dot{\Phi}_2, \tau)$$

and

$$w_4(\theta_1, \theta_2, \dot{\theta}_1, \dot{\theta}_2, \tau)$$

and the eight-dimensional function of the distribution

$$w_8(x_1,\ x_2,\ \dot{x}_1,\ \dot{x}_2,\ y_1,\ y_2,\ \dot{y}_1,\ \dot{y}_2,\ \tau),$$

to carry out transformation of rectangular coordinates into the polar ones, then it is obtained

$$w_8(A_{\text{п}1},\ A_{\text{п}2},\ \dot{A}_{\text{п}1},\ \dot{A}_{\text{п}2},\ \varphi_{\text{п}1},\ \varphi_{\text{п}2},\ \dot{\varphi}_{\text{п}1},\ \dot{\varphi}_{\text{п}2},\ \tau). \quad (2.9.2)$$

After carrying out integration for $A_{\text{п}1}, A_{\text{п}2}, \dot{A}_{\text{п}1}, \dot{A}_{\text{п}2}, \varphi_{\text{п}1}, \varphi_{\text{п}2}$ it is possible to obtain the unknown function. After obtaining it, it is possible to find $B_{\dot{\varphi}}(\tau)$, by using the integral

$$B_{\dot{\varphi}}(\tau)=\int\limits_{-\infty}^{+\infty}\int\limits_{-\infty}^{+\infty}\dot{\varphi}_{\text{п}1}\dot{\varphi}_{\text{п}2}w_2(\dot{\varphi}_{\text{п}1},\ \dot{\varphi}_{\text{п}2},\ \tau)\,d\dot{\varphi}_{\text{п}1}d\dot{\varphi}_{\text{п}2}. \quad (2.9.3)$$

The calculation of integrals indicated above is conjugated/combined with the bulky transformations, in connection with which we give expression for $B_{\dot{\varphi}}(\tau)$ already in final form [2.1, 2.4]

$$B_{\dot{\varphi}}(\tau)=\frac{1}{2}\left[\dot{R}_0^2(\tau)-\ddot{R}_0(\tau)R_0(\tau)\right]\times$$
$$\times\left[1+\frac{R_0^2(\tau)}{2}+\frac{R_0^4(\tau)}{3}+\ldots\right]=$$
$$=-\frac{1}{2}\left[\dot{R}_0^2(\tau)-\ddot{R}_0(\tau)R_0(\tau)\right]\frac{\ln\left[1-R_0^2(\tau)\right]}{R_0^2(\tau)},$$
$$\dot{R}_0(\tau)=\frac{dR_0(\tau)}{d\tau},\quad \ddot{R}_0(\tau)=\frac{d^2R_0(\tau)}{d\tau^2}. \quad (2.9.4)$$

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Using (2.9.4), it is possible to find $B_{\dot{\varphi}}(\tau)$ for the concrete/specific/actual models of energy interference spectrum and then to obtain expressions for the energy spectrum of the fluctuations of jamming frequency.

Let us consider this based on the example of perfect filter with band $\Delta\omega_{\text{п}}$ and medium frequency ω_0. In this case $R_0(\tau)$ is determined by formula (2.2.13).

$$\dot{R}_0(\tau)=\frac{dR_0(\tau)}{d\tau}=$$

$$=\frac{\Delta\omega_{\text{п}}}{2}\left[\frac{\cos\frac{\Delta\omega_{\text{п}}\tau}{2}}{\frac{\Delta\omega_{\text{п}}\tau}{2}}-\frac{\sin\frac{\Delta\omega_{\text{п}}\tau}{2}}{\left(\frac{\Delta\omega_{\text{п}}\tau}{2}\right)^2}\right], \qquad (2.9.5)$$

$$\ddot{R}_0(\tau)=\frac{d^2R_0(\tau)}{d\tau^2}=\frac{\Delta\omega_{\text{п}}^2}{4}\times$$

$$\times\left\{-2\frac{\cos\frac{\Delta\omega_{\text{п}}\tau}{2}}{\left(\frac{\Delta\omega_{\text{п}}\tau}{2}\right)^2}-\frac{\sin\frac{\Delta\omega_{\text{п}}\tau}{2}}{\frac{\Delta\omega_{\text{п}}\tau}{2}}\left[1-\frac{2}{\left(\frac{\Delta\omega_{\text{п}}\tau}{2}\right)^2}\right]\right\}. \qquad (2.9.6)$$

From (2.9.6) it follows that for small ones τ

$$\ddot{R}_0(\tau)\simeq-\frac{\Delta\omega_{\text{п}}^2}{12}+\frac{\Delta\omega_{\text{п}}^3}{48}\tau$$

and with $\tau\to 0$

$$\frac{d^2R_0(\tau)}{d\tau^2}\simeq-\frac{\Delta\omega_{\text{п}}^2}{12}.$$

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Using formulas (2.9.5) and (2.9.6), it is possible to fulfill the calculation of function $B_{\dot\varphi}(\tau)$. Fig. 2.9.1 gives plotted functions, entering expression (2.9.4). Fig. 2.9.2 gives intermediate graphs and graph $B_{\dot\varphi}(\tau)$; are there for the comparison given graphs $R_0(\tau)$, $R_\varphi(\tau)$, calculated according to precise and approximation formulas.

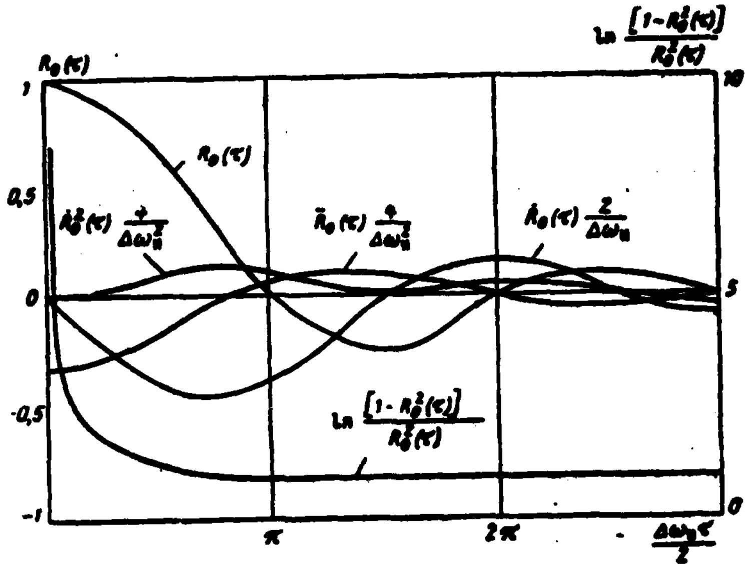

Fig. 2.9.1. Graphs of the terms, entering the expression for the correlation function of derived phase.

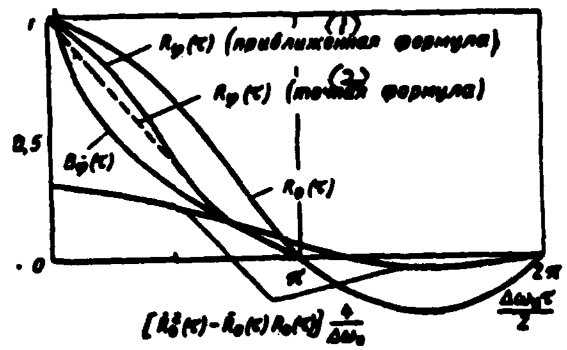

Fig. 2.9.2. Correlation function of initial random process, phase and derived phase.

Key: (1). (approximation formula). (2). (precise formula).

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From Fig. 2.9.2 it is evident that function $B_{\dot\varphi}(\tau)$ is considerably narrower than $R_0(\tau)$ and $R_\varphi(\tau)$, and, therefore, the spectrum of the fluctuations of derived phase will be considerably more widely than the spectrum of the fluctuations of phase and interference spectrum. The energy spectrum of the fluctuations of derived phase can be obtained, after carrying out a Fourier transform above $B_{\dot\varphi}(\tau)$. However, this integral with the substitution in it (2.9.4) is not expressed as known functions and can be calculated in the form of the series/row, which includes a large number of terms, which little is convenient for the calculations. In [2.4] is obtained

$$G_{\dot\varphi}(\omega) = \Delta\omega_э \sum_{k=1}^{\infty} k^{-3/2}\, e^{-1/2k\left(\frac{\omega}{\Delta\omega_э}\right)^2}$$

for $R_0(\tau) = e^{-\pi(\tau/\tau_0)^2}$; $\tau_0 = \frac{1}{\Delta f_э}$.

It is useful to find the approximations for $G_{\dot\varphi}(\omega)$, which make it possible to come to light/detect/expose the basic laws, characteristic to the energy spectrum of the fluctuations of derived phase.

From Fig. 2.9.1 it follows that in the first approximation, $\ddot R_0(\tau) \approx -\frac{\Delta\omega_э^2}{4}\frac{1}{3}R_0(\tau)$ and the effect of term $R'^2_0(\tau)$ can be disregarded/neglected.

Then

$$B_{\dot\varphi}(\tau)\simeq\frac{\Delta\omega_\text{п}^2}{24}R_0^2(\tau)\left[1+\frac{R_0^2(\tau)}{2}+\frac{R_0^4(\tau)}{3}+\ldots\right]. \quad (2.9.7)$$

To account for the terms of series/row also it is possible to allow approximation/approach. From (2.7.16) it follows that if degree $R_0(\tau)$ is higher than the third, its further increase comparatively slowly affects the form of the component of energy spectrum, caused by the appropriate term. After the grouping of terms we will obtain

$$B_{\dot\varphi}(\tau)\simeq\frac{\Delta\omega_\text{п}^2}{24}R_0^2(\tau)[1+1{,}3R_0^4(\tau)+0{,}65R_0^{10}(\tau)]=$$
$$=\frac{\Delta\omega_\text{п}^2}{12}[0{,}5R_0^2(\tau)+0{,}65R_0^6(\tau)+0{,}33R_0^{12}(\tau)]. \quad (2.9.8)$$

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Expression (2.9.8) - approximated, and cannot be used for calculations $B_{\dot\varphi}(\tau)$ for $\tau\to 0$, since gives finite value.

Energy spectrum is expressed by the formula

$$G_{\dot\varphi}(\omega)\simeq\frac{\Delta\omega_\text{п}}{4}\left[\frac{\pi}{2}\frac{\Delta\omega_\text{п}-\omega}{\Delta\omega_\text{п}}+\sqrt{\pi}\,e^{-\left(\frac{\omega}{\Delta\omega_\text{п}}\right)^2}+\sqrt{\frac{\pi}{2}}\,e^{-\frac{\omega^2}{9\Delta\omega_\text{п}^2}}\right]; \quad (2.9.9)$$

$$G_{\dot\varphi}(0)=\Delta\omega_\text{п}1{,}1. \quad (2.9.10)$$

Formula (2.9.9) - approximated. If we take into account the higher degrees of $R_0(\tau)$, then weakening energy spectrum at the high frequencies will be even slower. Fig. 2.9.3 gives the form of the energy spectrum of the fluctuations of derived phase. It is interesting to compare the energy spectra of the fluctuations of amplitude, phase and its derivative. For this Fig. 2.9.4 spectra gives to the dimensionless form. From the obtained results it follows that the spectrum of the fluctuations of derived phase is wider than the spectrum of the fluctuations of amplitude and phase.

It is essential to also note that the fluctuations of frequency have noticeable components with the frequency much of larger than width of band of initial random process.

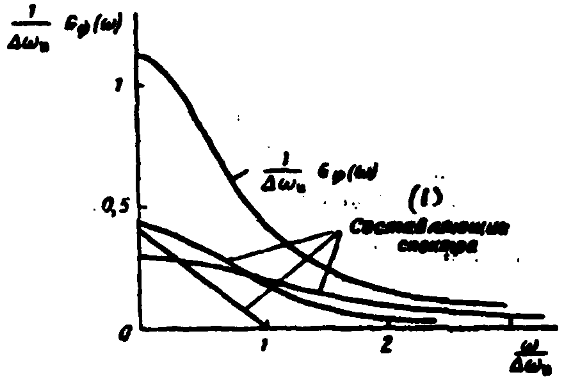

Fig. 2.9.3. The energy spectrum of the fluctuation of derived phase.

Key: (1). Components of the spectrum.

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The obtained result will be coordinated well with the fact that there does not exist final dispersion for the derived phase of interference (§ 2.8). As already mentioned earlier, these phenomena on the research of V. I. Tikhonov [2.4, 2.9, 2.10] are explained by the "migrations/jumps" of phase to the integer 2π. The expansion of the spectrum of derived phase will be coordinated also with the assumptions about the presence of the rapid fluctuations of phase.

§ 2.10. Fundamental statistical characteristics of jamming frequency. Above were examined the statistical characteristics of phase and its derivative. These characteristics make it possible to consider interference effect in the receiving and measuring device/equipment with the direct measurement of phase.

If meter is constructed according to the frequency principle, then it measures derivative of phase, but value, which characterizes frequency, i.e., a number of periods per unit time (second). Interference is random process and in connection with it it is possible to speak about the medium frequency and the current frequency. The frequency of selective interference is random process and is determined by a number of ejections per unit time. If we compute an average number of ejections in the large time interval, then this will correspond to medium frequency. Medium frequency is the simplest statistical characteristic of random frequency.

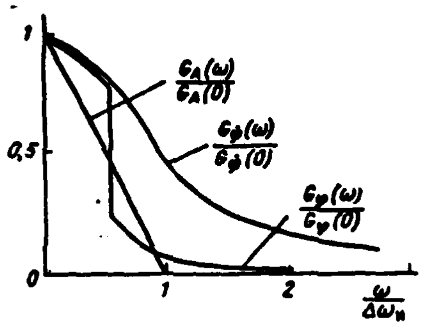

Fig. 2.9.4.

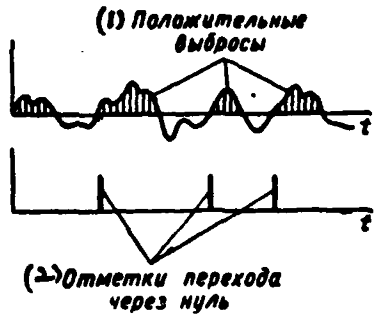

Fig. 2.10.1.

Fig. 2.9.4. Energy spectrum of fluctuation of amplitude, phase and derived phase; dimensionless form.

Fig. 2.10.1. Transitions of random process through zero.

Key: (1). Positive ejections. (2). Marks of transition through zero.

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Since the interference is random process with the zero average, it is obvious that by frequency it is necessary to understand a number of positive (or negative) ejections above the zero level. But then a number of ejections corresponds to a number of transitions of the random process through zero or briefly "number of zeros". The sense of these determinations is clarified by Fig. 2.10.1.

Thus, in the frequency measurements must be realized the calculation of a number of blips for the unit of time (or be determined the average duration of ejection) or the calculation of number of zeros (or to be determined the average duration of the interval between zero).

The technical realization of the meters, which realize these operations, does not produce fundamental difficulties. It is possible to use counters of a number of ejections (or the marking pulses of zeros) and to carry the results of calculation to the time or to use a measurement of the time interval between the impulses/momenta/pulses - marks of zeros and to determine the average value of this interval. Without examining in detail questions of the technical realization of such meters, let us note that on this basis it is possible to construct frequency meters and phasemeters. If we are be congruent/equate the measured frequency with the standard frequency and to integrate a difference in the frequencies, then the obtained value will correspond to an increase in the initial phase.

It is of interest to consider the action of interferences on the meter, constructed according to the frequency principle, and to compare with the results, which are obtained for the meter, constructed according to the principle of the direct measurement of phase.

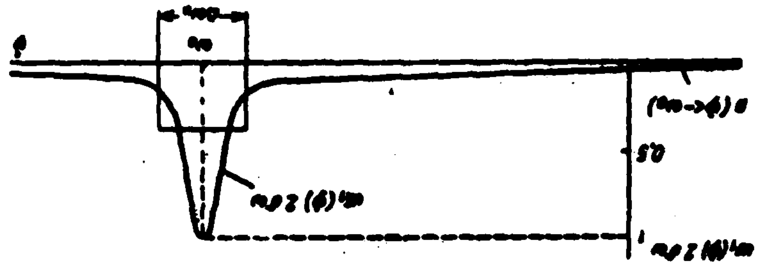

Fig. 2.10.2. Function of the distribution of derived phase and probability $p(\varphi<<\omega_{*})$.

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For this it is necessary to find the distribution functions, average/mean value, dispersion, correlation function for the jamming frequency, considered not as the derivative of phase, but as a number of ejections or "periods" per unit time. It is most expedient to begin the solution of this problem from the distribution function. In this case, as we shall see further on, there is greatest interest in the determination of the average/mean value of frequency and its comparison with the average/mean value of derived phase. For the determination of average it is possible to function by two methods: the first method - to find the distribution function for the frequency and from it to obtain average/mean value, and the second method - to directly find medium frequency, after finding an average number of "ejections", or "zero". Let us begin from the determination of expression for the function of allocation of frequencies. For this we will use the function of the distribution of derived phase. In the examination of the function of the distribution of derived phase above it was noted, that from it escape/ensues the noticeable probability of the considerable divergences of the instantaneous values of derived phase from the average/mean value. Is very substantial in this case the fact that the probability of values $\dot{\varphi}_ш$ exceeding ω_0, is not equal to zero, i.e., there is final probability that at the separate moments of time a change of the instantaneous jamming frequency will exceed the value of carrier. In this case $\dot{\varphi}_ш$ will be negative.

Fig. 2.10.2 shows the distribution function for $w(\dot{\varphi}_ш)$, from which evident that there is final probability that $\omega_0 - \dot{\varphi}_ш < 0$. For obtaining this probability it is necessary to find the integral

$$p(\dot{\varphi}_ш < -\omega_0) = \int_{-\infty}^{-\omega_0} w(\dot{\varphi}_ш)\, d\dot{\varphi}_ш.$$

Bearing in mind that with

$$\dot{\varphi}_ш > \delta\omega,\quad w(\dot{\varphi}_ш) \simeq \frac{1}{2\delta\omega}\left(\frac{\delta\omega}{\dot{\varphi}_ш}\right)^3,$$

we will obtain

$$p(\dot{\varphi}_ш < -\omega_0) = \int_{-\infty}^{-\omega_0} \frac{1}{2\delta\omega}\left(\frac{\delta\omega}{\dot{\varphi}_ш}\right)^3 d\dot{\varphi}_ш =$$

$$= \frac{\delta\omega^2}{2}\int_{-\infty}^{-\omega_0} \dot{\varphi}_ш^{-3}\, d\dot{\varphi}_ш = \frac{\delta\omega^2}{\omega_0^2}. \qquad (2.10.1)$$

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For example, with $\delta\omega=50\cdot2\pi$, $\omega_0=500\cdot2\pi$

$$p(\dot{\varphi}_n < -\omega_0) \approx 0.01 = 1\%.$$

In connection with the finite probability of negative values must be observed such phenomenon, when the radius-vector, which reflects oscillation, dwells on short period and begins to move to the reverse side.

If we approach the analysis of this phenomenon from the phase positions, then this will mean that the average speed of the rotation of vector decreases. To the equal degree there can be the instants, when $\dot{\varphi}_n$ proves to be considerable positive value and the speed of rotation of vector, which reflects oscillations, substantially will increase. On the average as this was shown earlier, the derivative of phase will be equal to zero, i.e., the average jamming frequency, determined through the derivative of phase, is equal to ω_0.

But if we approach the analysis from the frequency positions, then position in the principle is changed. Negative value $\omega_0+\dot{\varphi}_n$ indicates the decrease of average/mean rate of change in the phase and corresponds seemingly negative frequency.

But the devices/equipment, which react to the frequency, do not accept the sign of frequency and negative frequencies will be accepted just as positive. In other words, the more rapid at the separate moments of time is turned back the vector, which reflects oscillations, the more it reduces average/mean rate of change in the phase, but in this case occurs a larger value of negative frequency and a larger increase in the average/mean value of frequency. It is obvious that under these conditions the function of allocation of frequencies must differ from the velocity distribution function of a change in the phase.

The function of the distribution of derivative of phase was obtained above

$$w(\dot{\varphi}_\text{и})=\frac{\delta\omega^2}{2}\frac{1}{(\delta\omega^2+\dot{\varphi}_\text{и}^2)^{3/2}}.$$

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Then the function of the distribution of instantaneous frequency which can be defined as

$$\omega_\text{и}=\omega_0+|\dot{\varphi}_\text{и}|,$$

it can be found from the following considerations:

$$\omega_\text{и}=\omega_0+\dot{\varphi}_\text{и} \overset{(1)}{\text{или}}\ \dot{\varphi}_\text{и}^2=(\omega_\text{и}-\omega_0)^2\ \text{при}\ -\omega_0<\dot{\varphi}_\text{и}<\infty,$$

$$\omega_\text{и}=\dot{\varphi}_\text{и}-\omega_0\ \text{или}\ \dot{\varphi}_\text{и}^2=(\omega_\text{и}+\omega_0)^2\ \text{при}\ \dot{\varphi}_\text{и}<-\omega_0.$$

Key: (1). or.

Then

$$w(\omega_{\text{м}})=\frac{\delta\omega^2}{2}\left\{\frac{1}{[\delta\omega^2+(\omega_{\text{м}}-\omega_0)^2]^{3/2}}+\right.$$

$$\left.+\frac{1}{[\delta\omega^2+(\omega_{\text{м}}+\omega_0)^2]^{3/2}}\right\};\omega_{\text{м}}>0. \qquad (2.10.2)$$

Graphically obtaining $w(\omega_{\text{м}})$ is represented on Fig. 2.10.3.

From the figure one can see that average/mean value $\omega_{\text{м}}$ cannot be equal to average/mean value - ω_0. For obtaining the average/mean value of jamming frequency $\omega_{\text{м ср}}$ it is necessary, knowing the function of distribution $w(\omega_{\text{м}})$, to find mathematical expectation from $\omega_{\text{м}}$

$$\omega_{\text{м ср}}=\int\limits_0^\infty \omega_{\text{м}} w(\omega_{\text{м}})\,d\omega_{\text{м}}.$$

After computing integrals, it is possible to obtain

$$\omega_{\text{м ср}}=\omega_0\sqrt{1+\left(\frac{\delta\omega}{\omega_0}\right)^2}\approx\omega_0\left[1+\frac{1}{2}\left(\frac{\delta\omega}{\omega_0}\right)^2\right]. \quad (2.10.3)$$

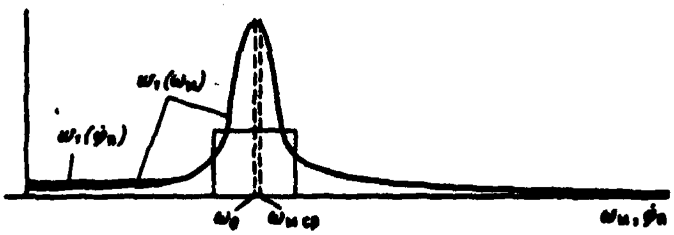

Fig. 2.10.3. Function of allocation of frequencies of interference.

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For example, for perfect filter $\delta\omega = \frac{\Delta\omega_{\text{п}}}{\sqrt{12}}$, then

$$\omega_{\text{м ср}} = \omega_0\left[1 + \frac{1}{24}\left(\frac{\Delta\omega_{\text{п}}}{\omega_0}\right)^2\right]. \qquad (2.10.4)$$

Consequently, the average/mean value of jamming frequency is greater than the medium frequency of filter, for value

$$\delta\omega_{\text{м ср}} = \omega_0\frac{1}{2}\left(\frac{\delta\omega}{\omega_0}\right)^2. \qquad (2.10.5)$$

For the perfect filter

$$\delta\omega_{\text{м ср}} = \omega_0\frac{1}{24}\left(\frac{\Delta\omega_{\text{п}}}{\omega_0}\right)^2 = \frac{\Delta\omega_{\text{п}}}{24}\left(\frac{\Delta\omega_{\text{п}}}{\omega_0}\right).$$

If process is very narrow-band, i.e., $\Delta\omega_{\text{п}} \ll \omega_0$, then $\delta\omega_{\text{м ср}} \approx 0$ and $\omega_{\text{м ср}} \approx \omega_0$. Thus, interference will differently function on the receiving and measuring device/equipment, depending on that, on which principle is based the meter. This is substantial, since the concepts of phase and frequency are connected and the values of frequency can be obtained, differentiating the measured phase, or the value of phase can be obtained, integrating the measured value of frequency. For the narrow-band random process this relationship/ratio is approximate.

As is evident, the distribution functions for derived initial phase $\dot{\varphi}_н$ and for $\delta\omega_н = \omega_н - \omega_0$ (frequency deviation from the medium frequency of filter - ω_0) differ little and difference is reduced to the small difference in the average/mean values. Correlation function and the energy spectra of derived phase and divergence of jamming frequency can be considered virtually identical. Small difference in the average/mean values can substantially change the action of interferences on the meter. If is realized the direct measurement of phase, then with the ideal equipment interference with any intensity cannot give systematic error, it will produce only the fluctuations of readings/indications of meter. If is realized the measurement of phase displacement through the integration of a difference in the frequencies, then interference will create the divergence, which systematically increases proportional to time.

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Average/mean initial phase will "run" with a speed of $\delta\omega_{н\,ср} = \omega_0 \frac{1}{2} (\delta\omega/\omega_0)^2$ radian per second. For example, for $\omega_0 = 6\cdot 10^{[illegible]}$ and $\Delta\omega_н = 600$ $\delta\omega_{н\,ср} = 2\cdot 10^{-[illegible]}$ rad/s. In the presence of weak noise signal must not give further systematic errors in the direct phase measurements and can cause large errors in the frequency measurements. Thus, in all cases where it is possible, it is necessary to use direct phase measurements.

Let us determine now average jamming frequency directly according to an average number of ejections (zero) per unit time and will compare it with the results, obtained from the analysis of the function of allocation of frequencies.

For the solution of this problem it is necessary to find an average number of intersections with the interference of some level n_0 and then, after equating $n_0=0$, to find a number of positive ejections or number of zeros.

An average number of ejections for the unit of time above the level n_0 can be computed, if will be found probability $p_\tau(n_0)$ of the intersection with the interference of this level upward in a small interval of time τ.

Then

$$f_{n}(n_0)=\frac{p_\tau(n_0)}{\tau}$$

with sufficiently small τ or, it is more precise, with $\tau\to 0$.

For obtaining $p_\tau(n_0)$ it is possible to find probability that at moment/torque t $n(t) < n_0+\Delta n/2$, and at moment/torque t-τ $n(t-\tau)$

$>n_0-\Delta n/2$. This case is depicted in Fig. 2.10.4.

This probability can be found from such condition that the random function must be in the limits from $n_0-\Delta n/2$ to $n_0+\Delta n/2$ and simultaneously it must have only positive (or only negative) derivative.

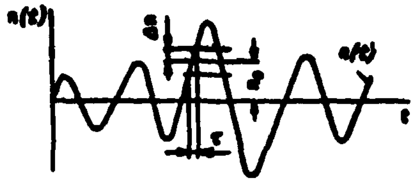

Fig. 2.10.4. Transition n(t) through the level n_0.

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For calculating this probability it is necessary to use two-dimensional, joint probability density of random function and by the derivative $w(n, \dot{n})$. Probability that the random function is within the limits from $n_0-\Delta n/2$ to $n_0+\Delta n/2$, can be found from the relationship/ratio

$$p\left(n_0-\frac{\Delta n}{2}<n<n_0+\frac{\Delta n}{2}\right)=$$
$$=\int_{n_0-\Delta n/2}^{n_0+\Delta n/2} w(n, \dot{n})\,dn=w(n_0, \dot{n})\,\Delta n. \qquad (2.10.6)$$

Probability that the random function at all positive values of derivative will be found within these limits, can be found from the following relationship/ratio:

$$p_1(n_0)=p\left(n_0-\frac{\Delta n}{2}<n<n_0+\right.$$
$$\left.+\frac{\Delta n}{2},\ \dot{n}>0\right)=\int_0^{\infty} w(n_0, \dot{n})\,\Delta n d\dot{n}, \qquad (2.10.7)$$

then

$$f_n(n_0)=\frac{1}{\tau}\int_0^{\infty} w(n_0, \dot{n})\,\Delta n\,d\dot{n}=\int_0^{\infty} w(n_0, \dot{n})\,\dot{n}d\dot{n},$$

since $\dot{n}=\Delta n/\tau$ - rate of change in the random function (derivative). Thus, for calculation $f_m(n_0)$ it is necessary to find $w(n_0, \dot{n})$, for which it is necessary to obtain expression for $w(\dot{n})$.

Let us recall that the function of the distribution of derivative can be obtained from the function of the distribution of difference Δn. For this it is necessary the function of the distribution of difference to relate to the interval of time τ, for which it is found, and to carry out a passage to the limit with $\tau\rightarrow 0$.

If there is random process $n(t)$ and its known two-dimensional function of distribution $w(n_1, n_2, \tau)$, then can be found autocorrelation coefficient $R(\tau)$.

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For a difference in two values of this process $\Delta n=n_1-n_2$ is retained the normal law of distribution, i.e., it is possible to write for it

$$w(\Delta n)=\frac{1}{\sqrt{2\pi}\,\sigma_{\Delta n}}e^{-\frac{\Delta n^2}{2\sigma^2_{\Delta n}}}, \qquad (2.10.8)$$

where $\sigma^2_{\Delta n}$— dispersion of difference.

It is known that the dispersion of a difference in two random variables is equal to

$$\sigma^2_{\Delta n} = \sigma^2_{n_1} + \sigma^2_{n_2} - 2R(\tau)\sigma_{n_1}\sigma_{n_2},$$

where $\sigma^2_{n_1}$ and $\sigma^2_{n_2}$ — dispersion of random variables; R(τ) - the correlation coefficient.

In this case, for a difference in values of one and the same random process $\sigma^2_{n_1} = \sigma^2_{n_2} = \sigma^2_n$, then $\sigma^2_{\Delta n} = 2\sigma^2_n \times$ [1-R(τ)] and the expression for w(Δn) will take the form

$$w(\Delta n) = \frac{1}{\sqrt{2\pi}\sigma_n \sqrt{[1 - R(\tau)]\,2}} e^{-\frac{\Delta n^2}{2\sigma_n^2\, 2\,[1 - R(\tau)]}}. \quad (2.10.9)$$

As one would expect, the function of the distribution of difference Δn depends also on the time τ, in which is computed this difference. After obtaining w(Δn), let us find now the distribution function for $\Delta n/\tau$.

Using the known rule of the replacement of variable/alternating in the distribution functions, we will obtain

$$w\left(\frac{\Delta n}{\tau}\right) = \frac{\tau}{\sqrt{2\pi}\sigma_n \sqrt{2[1 - R(\tau)]}} e^{-\frac{\left(\frac{\Delta n}{\tau}\right)^2 \tau^2}{2\sigma_n^2\, 2\,[1 - R(\tau)]}}, \quad (2.10.10)$$

$$w(\dot{n}) = \lim_{\tau \to 0} w\left(\frac{\Delta n}{\tau}\right). \quad (2.10.11)$$

As is evident, with $\tau \to 0$ both the numerator and denominator are converted into zero and, therefore, is obtained uncertainty/indeterminacy.

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For eliminating the uncertainty/indeterminacy let us expand R(τ) near the point τ=0 in series/row according to degrees τ. Since R(τ) - symmetrical function and the terms of the expansion higher than second can be disregarded/neglected,

$$R(\tau)=1+\left.\frac{d^2R(\tau)}{d\tau^2}\right|_{\tau\to 0}\frac{\tau^2}{2}. \qquad (2.10.12)$$

Since with τ=0 is a maximum of functions R(τ), then

$$\left.\frac{d^2R(\tau)}{d\tau^2}\right|_{\tau\to 0}<0,$$

let us designate

$$-\left.\frac{d^2R(\tau)}{d\tau^2}\right|_{\tau\to 0}=\omega_1^2. \qquad (2.10.13)$$

The physical sense of this designation will be clear from the following.

Then

$$1-R(\tau)=\omega_1^2\frac{\tau^2}{2}$$

and

$$w(\dot{n})=\frac{1}{\sqrt{2\pi}\sigma_n\omega_1}e^{-\frac{\dot{n}^2}{2\sigma_n^2\omega_1^2}}.$$

Let us switch over to relative variable/alternating $\dot{u}=\frac{\dot{n}}{\sigma_n}$

$$w(\dot{u})=\frac{1}{\sqrt{2\pi}\omega_1}e^{-\frac{\dot{u}^2}{2\omega_1^2}}. \qquad (2.10.14)$$

For obtaining w(П, П̇) we will use the fact that the random function and its derivative are independent at the coinciding moments of time.

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Then w(П, П̇)=w(П)w(П̇),

$$w(\Pi,\ \dot{\Pi})=\frac{1}{2\pi\sigma_{\Pi}^{2}\omega_{1}}e^{-\frac{1}{2\sigma_{\Pi}^{2}}\left(\Pi^{2}+\frac{\dot{\Pi}^{2}}{\omega_{1}^{2}}\right)} \qquad (2.10.15)$$

or, passing to the relative variable/alternating,

$$w(u,\ \dot{u})=\frac{1}{2\pi\omega_{1}}e^{-\frac{1}{2}\left(u^{2}+\frac{\dot{u}^{2}}{\omega_{1}^{2}}\right)}.$$

Let us find now the frequency of "ejections" or "zero" above level u_0

$$f_{\Pi}(u_{0})=\int_{0}^{\infty}w(u_{0},\ \dot{u})\,\dot{u}\,d\dot{u}=\frac{1}{2\pi\omega_{1}}e^{-\frac{u_{0}^{2}}{2}}\int_{0}^{\infty}e^{-\frac{\dot{u}^{2}}{2\omega_{1}^{2}}}\dot{u}\,d\dot{u}.$$

It is realized the replacement of the variable/alternating

$$\frac{\dot{u}}{\omega_{1}}=u_{1},\ d\dot{u}=\omega_{1}du_{1},$$

then

$$f_{\Pi}(u_{0})=\frac{1}{2\pi\omega_{1}}e^{-\frac{u_{0}^{2}}{2}}\int_{0}^{\infty}\omega_{1}^{2}u_{1}e^{-\frac{u_{1}^{2}}{2}}du_{1}=$$

$$=\frac{\omega_{1}}{2\pi}e^{-\frac{u_{0}^{2}}{2}}. \qquad (2.10.16)$$

Since in this case of basic interest is the average jamming frequency, i.e., average number of ejections above the zero level per unit time or average number of zeros per unit time, $u_0=0$.

In this case

$$f_n(0)=\frac{\omega_1}{2\pi}=f_1. \quad (2.10.17)$$

Thus, an average number of ejections is determined by the value which can be obtained from the relationship/ratio

$$\omega_1^2=-\frac{d^2R(\tau)}{d\tau^2}\bigg|_{\tau\to0}.$$

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For determining connection/communication $f_n(0)$ with other characteristics of initial random process let us return to the expression for ω_1 and will consider it in more detail.

For the narrow-band random process

$$R(\tau)=R_0(\tau)\cos\omega_0\tau,$$

then

$$\omega_1^2=-\frac{d^2R(\tau)}{d\tau^2}\bigg|_{\tau\to0}=\omega_0^2+\frac{d^2R_0(\tau)}{d\tau^2}\bigg|_{\tau\to0}. \quad (2.10.18)$$

As is evident, the medium frequency of ejections, i.e., the average jamming frequency, detected in the frequency measurements, differs from the medium frequency of the spectrum of selective interference ω_0 by value

$$\frac{d^2R_0(\tau)}{d\tau^2}\bigg|_{\tau\to0}.$$

This expression was obtained earlier during the determination of the correlation function of derived phase and carrier deviation of interference.

Earlier was introduced the designation

$$-\frac{d^2R_0(\tau)}{d\tau^2}\bigg|_{\tau=0}=\delta\omega^2,$$

then

$$\omega_1^2=\omega_0^2+\delta\omega^2 \qquad (2.10.19)$$

or

$$\omega_1=\omega_0\sqrt{1+\left(\frac{\delta\omega}{\omega_0}\right)^2}. \qquad (2.10.20)$$

Thus, calculation of average jamming frequency as an average number of ejections gave the same result, as during the determination of the average/mean value of the function of allocation of frequencies $\omega_1=\omega_{м\,ср}$ [see (2.10.3)].

Average jamming frequency differs from the midband frequency of the transmission of filter ω_0 by value $\delta\omega_1=\delta\omega_{м\,ср}$

$$\delta\omega_1=\frac{1}{2}\omega_0\left(\frac{\delta\omega}{\omega_0}\right)^2. \qquad (2.10.21)$$

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In [2.2] frequency $\omega_1=\omega_{м\,ср}$ was named "root-mean-square frequency" and it is shown that

$$\omega_1=\omega_{м\,ср}=\sqrt{\frac{1}{\sigma_п^2}\int_0^\infty\omega^2G_п(\omega)d\omega}.$$

However, on the basis of the physical sense, is better to call $\omega_1=\omega_{м\,ср}$ average jamming frequency in contrast to ω_0 - medium frequency of filter.

Let us consider the physical explanation of the obtained results. Narrow-band random process has a resemblance to the harmonic oscillation, initial phase and amplitude of which are random functions.

Amplitude changes occur slowly. The energy spectrum of the fluctuations of phase in essence is concentrated in the low-frequency region, but there are noticeable high-frequency components, i.e., together with the slow fluctuations phase completes rapid fluctuations. Due to the limited width of the spectrum the value of initial random process cannot be changed rapidly. But with small amplitudes a small change in the instantaneous value of process can be accompanied by considerable short-term changes in the phase and its derivative without the presence of corresponding components in the interference spectrum. Fig. 2.10.5 shows the realization of narrow-band random process. Solid line corresponds to the case when phase is changed so slowly as amplitude. On this line are given broken sections 1, 2, 3, in which a small change in the instantaneous value of process led to considerable changes in the instantaneous phase and frequency without the effect on the amplitude. Ejection 1 gives further transition through zero, which leads to an increase in the medium frequency, determined from a number of transitions through zero.

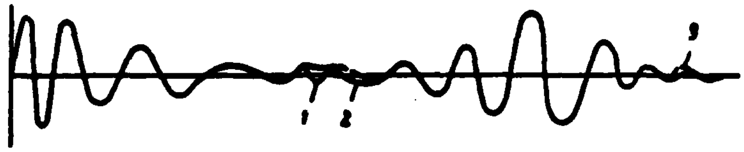

Fig. 2.10.5. To the explanation of a change in the medium frequency.

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If meter is constructed on the principle of the measurement of phase, then occurs the averaging of accelerations and retardations of a change of the phase, also, on the average these rapid fluctuations as slow, they give average/mean value for the phase and it to derivative, equal to zero. If meter is constructed on the principle of the measurement of frequency, then it determines a number of "ejections" or "zero" per unit time, they count further ejections or zero, show the value of frequency overstated relative to ω_0.

§ 2.11. Fundamental statistical characteristics of the phase of interference in the presence of limiter. In all previous paragraphs it provided that the interference passes through the linear narrow-band radio channel and it enters the meters and the cascades/stages of ARU. Therefore during the study of the action of interference on the different meters basic model was model in the form of harmonic oscillation with the random amplitude and the phase (or frequency).

In the real receiving and measuring devices/equipment is always required the guarantee of small changes in the signal amplitude, supplied to the meter with its considerable changes at the entrance of receiver. For these purposes can be used ARU whose presence affects also the passage of interference through the receiving and measuring device/equipment and what action it proves to be to the meters.

However, in the phase systems, in which the information is embedded during the phase of signal, ARU sometimes proves to be not necessary and the guarantee of constancy of signal amplitude, supplied to the meter, can be achieved/reached with the help of the limiter. In certain cases, for example, when useful information is embedded during the phase of the oscillation, which modulates basic carrier in the amplitude, the inclusion/connection of limiter is not admissible.

Therefore during the study of the action of interferences in the receiving and measuring devices/equipment of phase systems it is necessary to consider the case when interference passes through the limiter. Limiter is nonlinear element; therefore it changes the function of the distribution of the instantaneous values of random process (interference) and the previously model of interference accepted requires refinement.

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Since the limiter is nonlinear element and in transit through it simultaneously several signals and interferences can appear nonlinear transformations and mutual impositions, more expediently to include it at the output of the radio channel before the supply of signal to the meter. Then it is necessary to study such case when selective interference functions on the limiter, and, after coming to light/detecting/exposing the characteristics of the phase of the process, which is obtained at its output, to consider interference effect in this receiving and measuring device/equipment.

As the simplest model of limiter it is possible to take the "ideal limiter" whose characteristic is given in Fig. 2.11.1 and it is analytically written/recorded as follows: $y=f(x)$, $f(x)=a$, with $x\geq 0$ and

$$f(x)\equiv 0 \text{ with } x<0. \qquad (2.11.1)$$

If we to this limiter "supply" the narrow-band random process (interference), then at its output will be also random process in the form of square pulses the random ones duration and the moment/torque of emergence. Graphically this is represented in Fig. 2.11.2. From the figure one can see that the impulses/momenta/pulses at the output of limiter appear at the moments/torques, which correspond to zero initial random process, and have a duration, which corresponds to the duration of ejections. Thus, limiter reveals/detects the position of zero initial random processes.

Some statistical characteristics of this random process, namely average/mean value of number of zeros per unit time (average jamming frequency), were obtained earlier. Are of interest also such statistical characteristics, as: the function of distribution of zeros, autocorrelation function and energy spectrum of zeros. Let us consider these characteristics.

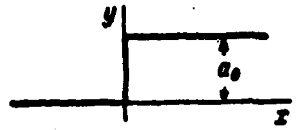

Fig. 2.11.1. Characteristic of ideal limiter.

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The function of distribution of zeros must show the probability density of transition through zero (for example, with the positive derivative) at specific moment/torque T_φ in the range of time from mT_0 to $(m+1)T_0$, where $T_0=2\pi/\omega_0$.

Fig. 2.11.3 gives the relative attitude of zero interferences. Is obvious that since the phase of interference can be any from 0 to 2π (from $-\pi$ to $+\pi$) and $w(\varphi_m)=1/2\pi$, the interval of time T_φ can be any from 0 to T_0.

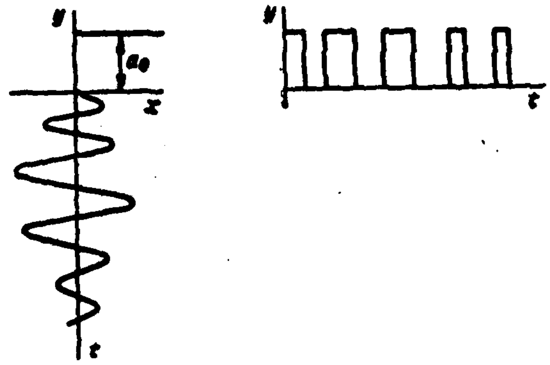

Fig. 2.11.2. Interference at the output of limiter.

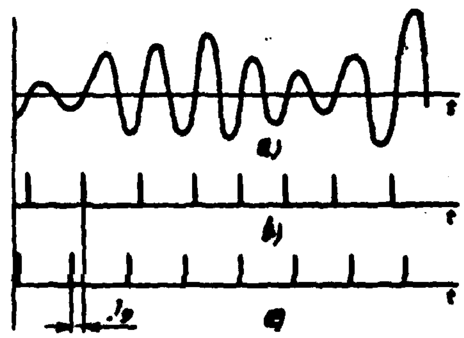

Fig. 2.11.3. Relative attitude of zero interferences: a) realization of interference; b) zero interferences; c) zero oscillations with frequency ω, and with phase, relative to which is conducted reading of phase of interference or transition of interference through zero.

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The distribution function for T_φ will take the form

$$w(T_\varphi)=\frac{1}{T_0}, \text{ since } T_\varphi=\frac{T_0}{2\pi}\varphi_n. \qquad (2.11.2)$$

During conclusion/output (2.11.2) it was assumed that the interference its very narrow-band and medium frequency was equal to the medium frequency of filter. Above it was shown that they differ. If we count off T_φ relative to points with period $T_1=2\pi/\omega_1$, then

$$w(T_\varphi)=\frac{1}{T_1}=\frac{\omega_1}{\omega_0}\frac{1}{T_0} \qquad (2.11.3)$$

will be coordinated with (3.9.12) when $a_c=0$. Since ω_1/ω_0 is usually close to 1, subsequently we will have in mind (2.11.2). Consequently, time interval, which characterizes position of zeros relative to zero reference voltage, has uniform even distribution.

The position of transitions through zero fluctuates.

A change in interval T_φ, of that reflecting phase, occurs with the specific transient nature; this transient nature is reflected in the autocorrelation function and the energy spectrum. Correlation function $B_T(\tau)$ must show, as are connected statistically the durations of segments T_φ, those isolated from each other by the range of time τ. Since T_φ maps phase φ_H, then in the first approximation, it is possible to consider that the function of correlation $B_T(\tau)$ and energy spectrum $G_T(\omega)$ will be similar $B_\varphi(\tau)$ and $G_\varphi(\omega)$, being characterized by only constant coefficient. Using expression (2.7.17), we obtain

$$B_T(\tau)\simeq\frac{T_0^2}{12}[0,5R_0(\tau)+0,08R_0^2(\tau)+0,42R_0^3(\tau)],$$
$$R_T(\tau)=0,5R_0(\tau)+0,08R_0^2(\tau)+0,42R_0^3(\tau) \quad (2.11.4)$$

and

$$G_T(\omega)=4\int_0^\infty\sigma_T^2R_T(\tau)\cos\omega\tau d\tau.$$

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Keeping in mind the given earlier results of calculations $B_\varphi(\tau)$ and $G_\varphi(\tau)$ and using them during the analysis $B_T(\tau)$ and $G_T(\omega)$, possible to note that interval T_φ is changed slowly. In essence these changes have a period greater than $\frac{4\pi}{\Delta\omega_H}=2T_H$, where $T_H=\frac{2\pi}{\Delta\omega_H}$ (for the perfect filter), i.e., much larger than T_0. Consequently, as a rule intervals T_φ in the adjacent periods are almost identical. However, the function of correlation $B_T(\tau)$ contains factors $R_0(\tau)$ to the third degree (in precise expression and to the higher degree), to which correspond the high-frequency components in the energy spectrum: consequently, can be observed rapid changes T_φ, leading to their difference in two adjacent periods. Example of this change T_0 is seen in Fig. 2.10.5.

Let us consider the now statistical characteristics of the period of transitions through zero. The distribution function for instantaneous frequency ω_M is given by expression (2.10.2). If we consider that the instantaneous frequency is connected with the current period of oscillations T_M through $T_M = \frac{2\pi}{\omega_M}$ and the current oscillatory period is expressed in the interval between adjacent zero, then expression (2.10.2) describes the function of the distribution of the value, reciprocal $T_M/2\pi$. Let us recall that the distribution function for $\delta\omega_M = \omega_M - \omega_0$ is close to the distribution function for $\dot{\varphi}_M$ and difference in essence is exhibited in the average/mean values.

Average for $\dot{\varphi}_M$ is equal to zero, while average for $\delta\omega_M$ is equal

$$\delta\omega_{M\,\text{cp}} = \frac{1}{2}\,\omega_0\left(\frac{\delta\omega}{\omega_0}\right)^2.$$

Correlation function and energy spectrum for $\delta\omega_M$ and φ_M can be accepted analogous. Energy spectrum $G_{\dot{\varphi}}(\omega)$ and, therefore, $G_{\delta\omega_M}(\omega)$ sufficiently wide, therefore, must be observed comparatively rapid changes T_M or ω_M. This can be explained by the presence of the "migrations/jumps" of phase on 2π [2.4, 2.9, 2.10]. The wide energy spectrum of the divergences of transition frequency through zero will be coordinated also with the presence of rapid changes T_{ϕ}.

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The measurement of phase or, it is more precise, phase displacement this is, in fact, the process of measuring of the time interval between the points, which have identical phase, which belong to two oscillations. If we take the most typical compensative and digital phasemeters, then in the first by change the phases of reference voltage attain such situation, when the average/mean value of the product of the stress/voltage of the measured signal and reference voltage is equal to zero, and the second (digital) phase displacement is measured by calculating the number of count pulses, which are placed in the time interval between zero of reference voltage and zero processes whose phase is measured.

Let us consider qualitatively the work of such meters under the influence on them of the interference, which passed through the limiter.

With the compensative meter of fluctuation of the phases of interferences they lead to the fact that the position of the moments/torques of the emergence of impulses/momenta/pulses fluctuates - it is changed according to the random law. The phase of reference voltage is changed, following these fluctuations. The changed phase of reference voltage is the useful result of measurement.

In the case of application of the compensative phasemeter, which works from the interferences, the rapid fluctuations, which appear due to the high-frequency part of energy spectrum, easily are removed as a result of the inertness of meter. During the use/application of a limiter and digital meter the position is changed.

Rapid fluctuations produce further transitions through zero (further zero) and further impulses/momenta/pulses.

With the digital meter phase displacement is determined by a number of count pulses, which are placed in the time interval from "zero" of supporting/reference oscillation to first "zero" oscillation whose phase is measured.

Hence it follows that a change in the time interval, in which is determined phase displacement, occurs in essence toward its decrease. It is obvious that the average/mean value of a number of count pulses can have the one-sided divergence which is equivalent to the distortion of phase displacement.

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Thus, the use/application of digital phase meters on the high interference levels is accompanied by further error.

§ 2.12. Special features/peculiarities of phase distribution of interference in multichannel systems. Noise characteristics examined earlier related to the single-channel phase systems, in which is measured the phase of signal with respect to the phase of reference voltage in the presence only of one interference (absence of signal). Its effect on the meter was determined above-examined statistical characteristics of phase and its derivative. On other function the interferences in multichannel systems. The simplest example of this system is the two-channel system, depicted in Fig. 1.8.4. Useful result in such systems is usually the measured difference (sum) in the phases of signals, which passed along two independent channels. In the absence of signal along the channels (receivers) are passed only the interferences and the behavior of meter depends on the statistical characteristics of a phase difference of interferences, which function in each of the channels.

During the analysis of the statistical characteristics of a difference (sum) in the initial phases of interferences it is necessary to have in mind the diverse variants. Receivers can have identical bands, medium frequency of filter and amplification and can have different characteristics indicated. The interferences, which appear at the output, can be the result of amplification and selection of internally-produced noise in each of the channels; then they prove to be not dependent on each other, but interference can be created by the external common source, which functions for two channels; then between the interferences, that are at the outputs, appears correlation. The analysis of the statistical properties of the phase of interferences and mixture of signal with the interference in multichannel systems in general form is complicated independent problem. In the presence of sufficiently strong signal as this will be shown further, the functions of phase distribution and its derivative become close to the normal ones and then the calculation of the functions of distribution and other statistical characteristics of a difference (sum) in the phases substantially is simplified, since the function of the distribution of a difference (sum) in the normal values remains normal.

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In the presence of weak signal or its absence the functions of phase distribution and their derivatives are not subordinated to normal law, the calculation of statistical characteristics is complicated and they can differ significantly from the characteristics of phase in each of the channels.

Without examining these questions in detail, let us find the function of the distribution of a phase difference in multichannel systems under the assumption of the identity of channels and independence of interferences.

In the presence of some interferences

$$w(\varphi_{\text{пк}})=\frac{1}{2\pi} \text{ and } -\pi<\varphi_{\text{пк}}<\pi,$$

where k - number of channel.

For obtaining $w(\varphi_{\Sigma})$ it is necessary to find the function of the distribution of sum to the values of those distributed evenly. With two channels the solution is comparatively simple, with the use of characteristic functions.

Lowering transformations, let us give the final result

$$w(\varphi_{\Sigma})=\begin{cases} 0 & \overset{(1)}{\text{при}}\ \varphi_{\Sigma}<-2\pi, \\ \dfrac{\varphi_{\Sigma}-2\pi}{(2\pi)^2} & -2\pi<\varphi_{\Sigma}<0; \\ \dfrac{2\pi-\varphi_{\Sigma}}{(2\pi)^2} & 0<\varphi_{\Sigma}<2\pi, \\ 0 & \varphi_{\Sigma}>2\pi. \end{cases}$$

Key: (1). with.

With a larger number of channels the solution becomes bulky. However, is important the fact that with k>4-5 the distribution function becomes close to the normal, with the dispersion

$$\sigma_{\Sigma}^{2}=(2\pi)^{2}\frac{k}{12}.$$

Since the interferences in each of the channels are independent, the distribution function will be identical for difference and sum of phases at the output. However, these results need correcting. The values of phase φ_{Σ}, greater π are smaller $-\pi$, cannot be counted off in view of the cyclic character phases.

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Consequently, probability densities, for example, for φ_{Σ} from π to 2π must be related to angles from $-\pi$ to 0. As a result, if we with the reading of phase displacement do not consider cycles, then the distribution function is returned to the uniform. Analogous result is obtained also with k>2. Root-mean-square divergence turns out to be large π. This also leads to the fact that the distribution for φ_{Σ}, that counted off in the limits $\pm\pi$, proves to be uniform.

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Chapter 3.

STATISTICAL CHARACTERISTICS OF THE PHASE OF THE MIXTURE OF SIGNAL AND INTERFERENCE.

§ 3.1. Function of the distribution of mixture. Let us examine the statistical properties of the mixture of interference and signal. Interference we will as before consider as the random process, which has normal distribution with the zero average and dispersion σ_n^2. The mixture of signal and interference takes the form

$$y(t) = c(t) + n(t).$$

In order to find the fundamental characteristics of random process y(t), we will use the rules obtained in the theory of the determination of the function of the distribution of the sum of two random variables the functions of distribution of which are known.

These results can be spread to the sum of two random processes, if we examine the one-dimensional law of random number distribution, which characterizes the values of random process at the specific moment of time.

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For the mixture of signal and interference we will obtain

$$w_2(y,\ t)=\int\limits_{-\infty}^{+\infty} w_{1\text{п}}(v)\, w_{1c}(y-v)\, dv =$$

$$=\int\limits_{-\infty}^{+\infty} w_{1\text{п}}(v)\, \delta\left[y-c(t)-v\right] dv,$$

since

$$w_{1c}(c)=\delta[c-c(t)],$$

where $w_{1\text{п}}$ — function of the distribution of interference; w_{1c} — function of the distribution of signal; v - variable/alternating of integration.

Delta-function at all points, except zero, is equal to zero, then as a result of integration we obtain

$$w(y,\ t)=w_{1\text{п}}[y-c(t)], \qquad (3.1.1)$$

For example, if

$$w_{1\text{п}}(n)=w(n)=\frac{1}{\sqrt{2\pi}\,\sigma_{\text{п}}}\, e^{-\frac{n^2}{2\sigma_{\text{п}}^2}}$$

and is signal c(t), on

$$w(y,\ t)=\frac{1}{\sqrt{2\pi}\,\sigma_{\text{п}}}\, e^{-\frac{[y-c(t)]^2}{2\sigma_{\text{п}}^2}}. \qquad (3.1.2)$$

This result escape/ensues also from the physical representations. If the chance of total process is determined only by interference, then it is obvious that for the difference between the mixture (signal and interference) and the signal will be correct the distribution, inherent in interference.

Consequently, with the addition of random stationary process and determined signal is obtained unsteady random process, since its one-dimensional distribution function depends on time, since signal is the function of time. Interference can be considered as oscillation with the random ones by amplitude and the phase. The mixture of signal and interference also can be considered as oscillation with the random ones by amplitude and the phase.

The representation of mixture, i.e., total random process in the form of oscillation with the random amplitude and phase, is very convenient for the solution of many problems by the study of the passage of the mixture of signal and noise through the receiving and measuring device/equipment.

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The functions of amplitude distribution give representation about the function of the distribution of output potential of the idealized detector, while the functions of phase distribution - about the probability of those or other significance of a deviation of the phase of signal under the effect of interferences. Latter/last distribution is of special interest for the phase systems. The study of the laws of phase distribution in the mixture of signal and noise makes it possible to come to light/detect/expose many important properties and special features/peculiarities of phase systems and must be carried out in sufficient detail.

Let us consider now the methodology of obtaining the distribution functions for the amplitude and the phases of mixture.

Let us represent narrow-band noise in the form of sinusoidal oscillation with random amplitude and phase

$$\text{ш}(t) = A_{\text{ш}}(t)\cos\varphi_{\text{ш}}(t)\cos\omega_0 t + A_{\text{ш}}(t)\sin\varphi_{\text{ш}}(t)\sin\omega_0 t = \\ = \text{Д}_{\text{ш}}(t)\cos\omega_0 t + E_{\text{ш}}(t)\sin\omega_0 t,$$

where $\text{Д}_{\text{ш}}(t)$ and $E_{\text{ш}}(t)$— narrow-band slow, random normal processes with the dispersion, equal to the dispersion of process ш(t). Their distribution functions are given in Chapter 2.

Signal so can be represented as the sinusoid, modulated on the amplitude and the phase

$$c(t) = A_c(t)\cos[\omega_0 t + \varphi_c(t)],$$

and to also break down into two orthogonal components, in which are modulated only the amplitudes

$$c(t) = \text{Д}_c(t)\cos\omega_0 t + E_c(t)\sin\omega_0 t,$$

where

$$\begin{aligned} D_c(t) &= A_c(t)\cos\varphi_c(t); \\ E_c(t) &= A_c(t)\sin\varphi_c(t); \\ A_c(t) &= \sqrt{D_c^2(t)+E_c^2(t)}; \\ \varphi_c'(t) &= \operatorname{arctg}\frac{E_c(t)}{D_c(t)}. \end{aligned} \tag{3.1.3}$$

Both the interference and signal individually and their mixture also they can be expanded to two orthogonal components.

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The functions of time, which characterize the amplitudes of orthogonal components of mixture, can be expressed through the functions, which characterize the amplitudes of orthogonal components of signal and interference:

$$y(t) = n(t) + c(t) = A_y(t)\cos[\omega_0 t + \varphi_y(t)] = \\ = D_y(t)\cos\omega_0 t + E_y(t)\sin\omega_0 t;$$

$$D_y(t) = D_c(t) + D_n(t), \quad D_y(t) = A_y(t)\cos\varphi_y(t),$$

$$E_y(t) = E_c(t) + E_n(t), \quad E_y(t) = A_y(t)\sin\varphi_y(t).$$

Knowing the distribution functions for $D_n(t)$ and $E_n(t)$, it is possible to obtain the distribution functions for $D_y(t)$ and $E_y(t)$ and then to switch over to the functions of amplitude distribution and phase of total random process (mixture).

Since $D_c(t)$ and $E_c(t)$ are functions of time, orthogonal random processes $\varnothing_y(t)$ and $E_y(t)$ in the general case will be unsteady.

If random processes $\varnothing_y(t)$ and $E_y(t)$ are unsteady, then the expressed as them random processes, which characterize amplitude and phase of mixture, it will be also unsteady.

However, in contrast to the distribution function for the instantaneous values of the mixture where any, even simplest radio signal caused transiency, since its instantaneous value was changed on the time, for the functions of amplitude distribution and phase of mixtures the simplest harmonic signal with the the noted for amplitude, by frequency and by initial phase easily it can be taken into consideration; in this case for obtaining $\varnothing_y(t)$ or $E_y(t)$, which they are used for obtaining $A_y(t)$ and $\varphi_y(t)$, to the value of orthogonal component of interference is supplemented constant value $A_0\cos\varphi_0$ or $A_0\sin\varphi_0$ respectively.

Because of the action of signal in orthogonal components of mixture is changed only average/mean value.

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In such a case, when as a result of modulation of signal on amplitude and phase values A_c and φ_0 are changed, then, after finding the functions of amplitude distribution and phase of mixture for given value A_c and φ_c, it is possible to come to light/detect/expose laws governing the change in the unknown function of amplitude distribution or phase of mixture. In this case mathematical transformations substantially are simplified. The basic reason for the possibility of simplification in the mathematical transformations lies in the fact that upon transfer to the distributions of orthogonal components, and also amplitude and phase of mixture succeeds, in to deal only concerning the slow random processes and avoiding "high-frequency nature". We will use that presented in order to determine the sequence of the transformations, implemented with obtainings of the distribution functions.

We will use the short recording

$$\mathcal{D}_c = A_c \cos\varphi_c,\ \mathcal{D}_n \quad \text{and} \quad E_c = A_c \sin\varphi_c,\ E_n,$$

understanding by A_c and φ_c a constant value or the function of time, then

$$\mathcal{D}_y = A_c \cos\varphi_c + \mathcal{D}_n,\ E_y = A_c \sin\varphi_c + E_n.$$

Since for the interference is accepted the normal distribution with the zero average and dispersion σ_n^2, the one-dimensional distribution for $\mathcal{D}_n$ and E_n is known.

Then distribution for the components of mixture will be

$$w(\O_y)=\frac{1}{\sqrt{2\pi}\,\sigma_n}e^{-\frac{1}{2\sigma_n^2}(\O_y-A_c\cos\varphi_c)^2}, \quad (3.1.4)$$

$$w(E_y)=\frac{1}{\sqrt{2\pi}\,\sigma_n}e^{-\frac{1}{2\sigma_n^2}(E_y-A_c\sin\varphi_c)^2}. \quad (3.1.5)$$

Knowing $w(\O_y)$ and $w(E_y)$, it is possible to find their joint distribution, i.e., to find the probability density of one or the other combination of values.

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Since orthogonal components of mixture are independent,

$$w_2(\O_y,E_y)=w(\O_y)w(E_y)=$$
$$=\frac{1}{2\pi\sigma_n^2}e^{-\frac{1}{2\sigma_n^2}[(\O_y-A_c\cos\varphi_c)^2+(E_y-A_c\sin\varphi_c)^2]}. \quad (3.1.6)$$

$w(A_y,\varphi_y)$ can be found from $w(\O_y,E_y)$ with the help of the rules of the functional transformations of the distribution functions.

The functions of distribution $w(A_y)$ and $w(\varphi_y)$ are obtained by integration $w(A_y,\varphi_y)$.

The analogous sequence of transformations can be used for obtaining the multidimensional functions of distribution $w_2(A_{y1},A_{y2}\tau)$, $w_2(\varphi_{y1},\varphi_{y2},\tau)$ and combined functions of distribution

$w_2(A_y, \dot{A}_y)$ and $w_2(\varphi_y, \dot{\varphi}_y)$. The basic factor, which covers obtaining results, is the fact that the interference can be described by the normal functions of distribution and function of the distribution of its orthogonal components are also normal.

The two-dimensional distribution functions for orthogonal components of mixture take the form

$$w_2(D_{y1}, D_{y2}, \tau) = \frac{1}{2\pi\sigma_n^2\sqrt{1-R_0^2(\tau)}} \times$$
$$\times e^{-\frac{(D_{y1}-A_c\cos\varphi_c)^2+(D_{y2}-A_c\cos\varphi_c)^2-2R_0(\tau)(D_{y1}-A_c\cos\varphi_c)(D_{y2}-A_c\cos\varphi_c)}{2\sigma_n^2[1-R_0^2(\tau)]}}, \quad (3.1.7)$$

$$w_2(E_{y1}, E_{y2}, \tau) = \frac{1}{2\pi\sigma_n^2\sqrt{1-R_0^2(\tau)}} \times$$
$$\times e^{-\frac{(E_{y1}-A_c\sin\varphi_c)^2+(E_{y2}-A_c\sin\varphi_c)^2-2R_0(\tau)(E_{y1}-A_c\sin\varphi_c)(E_{y2}-A_c\sin\varphi_c)}{2\sigma_n^2[1-R_0^2(\tau)]}}, \quad (3.1.8)$$

where $R_0(\tau)$ - the low-frequency factor of the correlation function of interference.

Using the independence of orthogonal components, it is possible to obtain the four-dimensional function of the distribution

$$w_4(D_{y1}, D_{y2}, E_{y1}, E_{y2}) =$$
$$= w_2(D_{y1}, D_{y2}, \tau)\, w_2(E_{y1}, E_{y2}, \tau). \quad (3.1.9)$$

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For the transition to the four-dimensional function of amplitude distribution and phases $w_4(A_{y1}, A_{y2}, \varphi_{y1}, \varphi_{y2}, \tau)$, it is necessary to use the rules of the functional transformations of the distribution functions. For obtaining $w_2(A_{\nu 1}, A_{\nu 2}, \tau)$ and $w_2(\varphi_{\nu 1}, \varphi_{\nu 2}, \tau)$ it is necessary to fulfill integrations.

Method presented above worked out by V. I. Bunimovich [2.2], has that positive side, which gives single approach to the solution of many problems of obtaining the distribution functions for the amplitude and phases of the mixture of signal and interference and their derivatives. In the case of one interference it is possible to obtain the expressions in the completed analytical form, suitable for the calculations. In the case of the presence of signal and interference during the integration frequently appear considerable difficulties and it is impossible usually to obtain such expressions.

At conclusion of this paragraph let us find the function of joint distribution $w_2(A_\nu, \varphi_\nu)$. The function of joint distribution $w_2(\mathcal{E}_\nu, E_\nu)$ was obtained earlier.

For the transition to $w_2(A_\nu, \varphi_\nu)$ it is necessary to use the rules of the functional transformations of the function of the distribution

$$w_2(A_y, \varphi_y) = \frac{A_y}{2\pi\sigma_n^2}\times$$

$$\times e^{-\frac{(A_y\cos\varphi_y - A_c\cos\varphi_c)^2 + (A_y\sin\varphi_y - A_c\sin\varphi_c)^2}{2\sigma_n^2}} =$$

$$= \frac{A_y}{2\pi\sigma_n^2} e^{-\frac{A_y^2 + A_c^2}{2\sigma_n^2}} e^{-\frac{A_y A_c}{\sigma_n^2}\cos(\varphi_y - \varphi_c)}. \qquad (3.1.10)$$

After designating $\varphi_y - \varphi_c = \Delta\varphi_y$, where $\Delta\varphi_y$ — divergence of the phase of mixture from the phase of signal, we finally obtain distribution for the amplitude and divergences of the phase of the mixture of signal and noise from the phase of the signal

$$w_2(A_y, \Delta\varphi_y) = \frac{A_y}{2\pi\sigma_n^2} e^{-\frac{A_y^2 + A_c^2}{2\sigma_n^2}} e^{-\frac{A_c A_y}{\sigma_n^2}\cos\Delta\varphi_y}. \qquad (3.1.11)$$

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In certain cases is convenient another form of recording; since the distribution function for φ_y depends only on the divergence of the phase of mixture from the phase of signal, is convenient to conduct the reading of the phase of mixture from the phase of signal and to assume $\varphi_c = 0$. Then

$$w_2(A_y, \varphi_y) = \frac{A_y}{2\pi\sigma_n^2} e^{-\frac{A_y^2 + A_c^2}{2\sigma_n^2}} e^{-\frac{A_c A_y}{\sigma_n^2}\cos\varphi_y}.$$

As can be seen from result, distribution for the amplitude of mixture and divergence of the phase of mixture from the phase of signal with the constant signal amplitude does not depend on time, i.e., random process for $\Delta\varphi_v$ is stationary for any law of change φ_c. It is obvious that the random process, which characterizes $\varphi_v = \varphi_c + \Delta\varphi_v$, is unsteady, if φ_c varies in the time. This is important result, since it shows that if we are interested not in the values of phase, to which is laid the information, but by the divergences of the phase of mixture from the value, determined by the phase of pure/clean signal, i.e., to examine a question about the distortions of the phase of signal, caused by the presence of interference, then mathematical relationships/ratios considerably are simplified.

§ 3.2. One-dimensional functions of amplitude distribution of mixture and suppression during the detection. Let us consider now the function of amplitude distribution. This function is of interest when information is embedded during the phase of modulation, and also for study of the action of ARU with the interferences. In the idealization of detector the function of amplitude distribution will correspond to the function of the distribution of output potential of detector.

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For obtaining the function of amplitude distribution it is realized the integration

$$w(A_y)=\int_0^{2\pi} w_1(A_y,\varphi_y)\,d\varphi_y=\frac{A_y}{\sigma_n^2}e^{-\frac{A_y^2+A_c^2}{2\sigma_n^2}}I_0\left(\frac{A_cA_y}{\sigma_n^2}\right), \quad (3.2.1)$$

since

$$\frac{1}{2\pi}\int_0^{2\pi} e^{-j\left(\frac{jA_cA_y}{\sigma_n^2}\right)\cos(\varphi_y-\varphi_c)}d\varphi_y=\mathcal{J}_0\left(\frac{jA_cA_y}{\sigma_n^2}\right)=I_0\left(\frac{A_cA_y}{\sigma_n^2}\right). \quad (3.2.2)$$

where $I_0\left(\frac{A_cA_y}{\sigma_n^2}\right)$ — function of zero-order Bessel (modified).

After switching over to relative values $a_y=\frac{A_y}{\sigma_n}$ and $a_c=\frac{A_c}{\sigma_n}$, we will obtain

$$w(a_y)=a_ye^{-\frac{a_y^2+a_c^2}{2}}I_0(a_ya_c). \quad (3.2.3)$$

The curves, constructed according to this formula, are given in Fig. 3.2.1. The obtained distribution function is called the generalized function of Rayleigh distribution.

For characteristic values a_c it is possible to find the simpler asymptotic approximations/approaches of formula.

With $a_c=0$ $I_0(0)=1$,

$$w(a_y)=a_ye^{-\frac{a_y^2}{2}},$$

which corresponds to Rayleigh's function.

When $a_0 < 1$, i.e., for the low values of signal amplitude

$$w(a_y) \approx a_y e^{-\frac{a_y^2 + a_c^2}{2}} (1 + 0{,}25 a_y^2 a_c^2). \qquad (3.2.4)$$

With $a_0 > 1$

$$w(a_y) = \frac{1}{\sqrt{2\pi}} e^{-\frac{(a_y - a_c)^2}{2}} \left(1 + \frac{1}{8 a_c a_y}\right) \sqrt{\frac{a_y}{a_c}}. \qquad (3.2.5)$$

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Since when $a_c \gg 1$ the probability density for $a_y \ll 1$ is very small, formula can be simplified

$$w(a_y) = \frac{1}{\sqrt{2\pi}} e^{-\frac{(a_y - a_c)^2}{2}} \qquad (3.2.6)$$

or

$$w(A_y) = \frac{1}{\sqrt{2\pi}\,\sigma_n} e^{-\frac{(A_y - A_c)^2}{2\sigma_n^2}}.$$

Consequently, with the signal, which exceeds interference, amplitude distribution of mixture is close to the normal with the average/mean value, equal to a_0 (or A_0), and by dispersion σ_n^2. The average/mean value of the amplitude of mixture or first-order moment/torque determines DC voltage on the output of detector or result of the detection

$$m_1(A_y) = \int_0^\infty A_y w(A_y) dA_y. \qquad (3.2.7)$$

Using the obtained expression for $w(A_y)$ and lowering mathematical transformations, we obtain

$$m_1(A_y)=\sigma_n\sqrt{\frac{\pi}{2}}\left[\left(1+\frac{A_c^2}{2\sigma_n^2}\right)I_0\left(\frac{A_c^2}{4\sigma_n^2}\right)+\right.$$

$$\left.+\frac{A_c}{2\sigma_n^2}I_1\left(\frac{A_c^2}{4\sigma_n^2}\right)\right]e^{-\frac{A_c^2}{4\sigma_n^2}}. \quad (3.2\ 8)$$

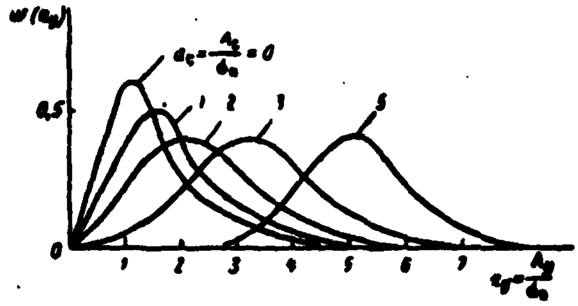

Fig. 3.2.1. The generalized function is Rayleigh distribution.

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The calculation of integral (3.2.7) is given in [3.1]

$$m_1(a_y)=\sqrt{\frac{\pi}{2}}\left[\left(1+\frac{a_c^2}{2}\right)I_0\left(\frac{a_c^2}{4}\right)+\right.$$
$$\left.+\frac{a_c}{2}I_1\left(\frac{a_c^2}{4}\right)\right]e^{-\frac{a_c^2}{4}}. \qquad (3.2.9)$$

In this form the formula little is demonstrative. For obtaining the formulas, more convenient for the calculations, let us consider the cases of weak and strong signals.

When $\frac{A_c}{\sigma_n}<1$ the formula can be simplified

$$m_1(A_y)\approx\sigma_n\sqrt{\frac{\pi}{2}}\left(1+\frac{A_c^2}{4\sigma_n^2}\right)=$$
$$=m_1(A_n)\left(1+\frac{P_c}{2\sigma_n^2}\right). \qquad (3.2.10)$$

With $\frac{A_c}{\sigma_n}=a_c>1$

$$m_1(A_y)=A_c\left(1+\frac{\sigma_n^2}{2A_c^2}\right)=A_c\left(1+\frac{\sigma_n^2}{4P_c}\right). \qquad (3.2.11)$$

A change in the average/mean value of the amplitude of mixture in comparison with the case of the presence of one interference, which can be considered as the useful result of acting the signal in the mixture, can be found from relationship/ratio $\Delta m_1 = m_1(A_y) - m_1(A_n)$. This expression gives an incremental stress on the output of ideal amplitude detector during the supplying of signal.

For the relative values of a change in the amplitude of mixture or for the useful detected stress/voltage of formula they will take the form

$$\frac{\Delta m_1}{A_c} = \frac{\sigma_n \sqrt{\frac{\pi}{2}}}{A_c}\left[1 - \left(1 - \frac{A_c^2}{2\sigma_n^2}\right) I_0\left(\frac{A_c^2}{4\sigma_n^2}\right)^2 + \right.$$

$$\left. + \frac{A_c}{2\sigma_n^2} I_1\left(\frac{A_c^2}{4\sigma_n^2}\right)\right] e^{-\frac{A_c^2}{4\sigma_n^2}}. \qquad (3.2.12)$$

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With $\frac{A_c}{\sigma_n} = a_c < 1$

$$\frac{\Delta m_1}{A_c} \approx \frac{1}{4}\sqrt{\frac{\pi}{2}}\frac{A_c}{\sigma_n} = \frac{1}{4}\sqrt{\frac{\pi}{2}} a_c. \qquad (3.2.13)$$

with $\frac{A_c}{\sigma_n} = a_c > 1$

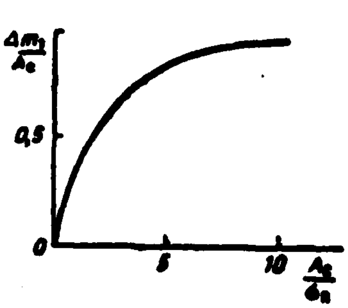
$$\frac{\Delta m_1}{A_c} \approx 1 - \frac{\sigma_n}{A_c}\sqrt{\frac{\pi}{2}} = 1 - \frac{1}{a_c}\sqrt{\frac{\pi}{2}}. \qquad (3.2.14)$$

with an increase in the signal Δm_1 asymptotically it approaches

$$\Delta m_1 \simeq A_0 - \sigma_n \sqrt{\frac{\pi}{2}}. \qquad (3.2.15)$$

The graph of the obtained law is given in Fig. 3.2.2. The given results characterize suppressions in the detector of weak signal the result of the detection of signal in the presence relative to strong interference much less than in its absence. With the voltage of signal larger than root-mean-square disturbing voltage on the detector, suppression becomes insignificant.

For the practical calculations frequently they accept, that when $A_0 > \sigma_n$ or $\frac{A_0}{\sqrt{2}} > \sigma_n$ the suppression of signal in the amplitude ceases.



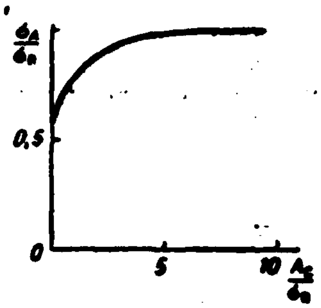


Fig. 3.2.2. Fig. 3.2.3.

Fig. 3.2.2. Dependence of useful result of detection on A_c/σ_n.

Fig. 3.2.3. Dependence of the fluctuation of the detected stress/voltage from A_c/σ_n.

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The fluctuations of the amplitude of mixture, caused by interference, can be considered with the value of dispersion, i.e., by the central moment of the second order.

For the central moment of the second order it is possible to register

$$M_2(A_y)=\sigma_A^2=m_2(A_y)-m_1^2(A_y)=2\sigma_n^2+A_c^2-m_1^2(A_y), \tag{3.2.16}$$

since

$$m_2(A_y)=\frac{1}{\sigma_n^2}e^{-\frac{A_c}{2\sigma_n^2}}\int_0^\infty A_y^2 I_0\left(\frac{A_c A_y}{\sigma_n^2}\right)\times$$

$$\times e^{-\frac{A_y^2}{2\sigma_n^2}}dA_y=2\sigma_n^2+A_c^2. \qquad (3.2.17)$$

The calculation of this integral see in [3.1].

$$\frac{A_c}{\sigma_n}<1,\ A_c\to 0 \text{ and } \frac{A_c}{\sigma_n}>1.$$

When $\frac{A_c}{\sigma_n}<1$ $m_1(A_y)$ it is given (3.2.10), then

$$\sigma_A^2=2\sigma_n^2+A_c^2-\sigma_n^2\frac{\pi}{2}\left(1+\frac{A_c^2}{4\sigma_n^2}\right)^2 \qquad (3.2.18)$$

and

$$\sigma_A^2=\sigma_n^2\left(0,43+0,2\frac{A_c^2}{\sigma_n^2}-0,1\frac{A_c^4}{\sigma_n^4}\right)^2$$

or

$$\frac{\sigma_A}{\sigma_n}=\sqrt{0,13+0,2\frac{A_c^2}{\sigma_n^2}-0,1\frac{A_c^4}{\sigma_n^4}}.$$

When $A_c\to 0$ $\sigma_A^2=0,43\sigma_n^2$; this corresponds to the results obtained previously.

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When $\frac{A_c}{\sigma_n}>1$ $m_1(A_y)$ is given (3.2.11). Then

$$\sigma_A^2 = 2\sigma_n^2 + A_c^2 - A_c^2 + \frac{\sigma_n^2}{A_c^2} - \sigma_n^2 = \sigma_n^2\left(1 - \frac{\sigma_n^2}{A_c^2}\right). \quad (3.2.19)$$

with

$$\frac{A_c}{\sigma_n} > 1$$

$$\sigma_A^2 = \sigma_n^2.$$

The obtained dependence graphic is depicted in Fig. 3.2.3. From the results it is evident that the appearance of a signal leads to an increase in the fluctuations of the amplitude of mixture, i.e., to an increase in the interference from the output of detector.

From the obtained results it follows that during the supplying of signal to the entrance of the receiving and measuring device/equipment, which contains detector, occur the complicated processes of changing the interference level and signal at the output of detector. With an increase in the intensity of signal to certain level and the useful signal and interference at the output of detector increase. According to reaching/achievement of the specific level further increase in the jamming intensity ceases, and with an increase in the signal at the entrance at the output occurs an increase only in the useful signal. A change in the relationship/ratio between the signal and the interference at the entrance of detector and at its output will characterize the effect of detectors in the diagram.

Let us find the relation of useful signal and interference on the output of detector and let us compare it with the analogous relation to the detector.

The ratio of useful signal to the interference at the output of detector will be determined by the formula

$$\frac{\Delta m_1}{\sigma_A}=\frac{m_1(A_y)-m_1(A_n)}{\sqrt{2\sigma_n^2+A_c^2-m_1^2(A_n)}}. \qquad (3.2.20)$$

In general form the expression is obtained sufficiently to bulky ones and it is expedient to be bounded to its simplified versions. We will obtain simplification in the formula and is expressed $\frac{\Delta m_1}{\sigma_A}$ through $\frac{A_c}{\sigma_n}$.

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With

$$\frac{A_c}{\sigma_n} < 1$$

$$\frac{\Delta m_1}{\sigma_A} = \frac{1}{2}\sqrt{\frac{\pi}{2}}\,\frac{\frac{A_c^2}{\sigma_n^2}}{\sqrt{0.43 + 0.2\,\frac{A_c^2}{\sigma_n^2} - 0.1\,\frac{A_c^4}{\sigma_n^4}}}. \quad (3.2.21)$$

With

$$\frac{A_c}{\sigma_n} > 1$$

$$\frac{\Delta m_1}{\sigma_A} = \frac{A_c}{\sigma_n}\left(1 - \frac{\sigma_n}{A_c}\sqrt{\frac{\pi}{2} + \frac{1}{2}\,\frac{\sigma_n^2}{A_c^2}}\right). \quad (3.2.22)$$

With

$$\frac{A_c}{\sigma_n} \gg 1$$

$$\frac{\Delta m_1}{\sigma_A} = \frac{A_c}{\sigma_n}.$$

The obtained result shows that with the decrease of relation $\frac{A_c}{\sigma_n}$ the ratio of signal to the interference at the output of detector, beginning from certain level, rapidly is reduced, which is graphically represented in Fig. 3.2.4. Thus, in the detector occurs the suppression of weak signal not only in its amplitude, but also according to the relation signal/noise.

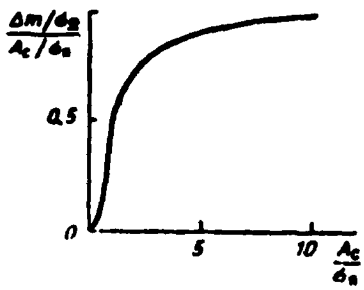

Fig. 3.2.4. Suppression of weak signal in the detector according to the ratio signal/noise.

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From Fig. 3.2.4. it is evident that when $\frac{A_c}{\sigma_{\text{ш}}} \gg 1$ the relationship/ratio between the signal and noise at the input and output of the detector of one order. When $\frac{A_c}{\sigma_{\text{ш}}} \approx 1 \div 2$ relationship/ratio the signal/noise at the output of detector begins rapidly to deteriorate.

In the practical calculations it is possible to approximately consider that when $A_c > \sigma_{\text{ш}}$ there is no suppression and relationship/ratio signal/noise in the detector barely is changed, but when $A_c < \sigma_{\text{ш}}$ begins suppression.

The obtained results have vital importance, since is determined approach to the synthesis of systems and receiving and measuring devices/equipment. The use of modulation in the systems will cause the presence of threshold, and it will not make possible to realize efficient reception of the signals the power level of which on the detector is equal or less than the power of interferences.

However, the given results yet completely do not characterize the suppression of useful signal by interference, since is not considered action of ARU.

S 3.3. Statistical characteristics of output potential of detector and the suppression of noise signal due to the action of ARU. For determining the effect of the mixture of signal and interference on the detector of ARU and th mode/conditions of the receiving and measuring device/equipment we will use the same idealization as during the analysis of effect on the detector ARU of one noise. Let us consider ARU with the "delay" in the detector and ARU with the "delay" after detection and averaging.

During the use of ARU with the delay in the detector the stress/voltage of detector ARU can be considered as the average/mean value of the amplitude of the mixture of signal and noise during the use for the averaging of the values of the amplitude of mixture, which exceed the specific threshold.

The function of amplitude distribution of mixture is given by expression (3.2.1).

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The mathematical expectation for a difference in the amplitudes of mixture and delay, which corresponds to average/mean value, and the value of output potential of detector of ARU can be found from the following relationships/ratios:

$$\Delta m_{з}=\int\limits_{A_{з}}^{\infty}(A_{y}-A_{з})\,w(A_{y})\,dA_{y}=\int\limits_{A_{з}}^{\infty}\frac{A_{y}^{2}}{\sigma_{n}^{2}}e^{-\frac{A_{y}^{2}+A_{c}^{2}}{2\sigma_{n}^{2}}}\times$$

$$\times I_{0}\left(\frac{A_{y}A_{c}}{\sigma_{n}^{2}}\right)dA_{y}-\int\limits_{A_{з}}^{\infty}\frac{A_{з}A_{y}}{\sigma_{n}^{2}}e^{-\frac{A_{y}^{2}+A_{c}^{2}}{2\sigma_{n}^{2}}}I_{0}\left(\frac{A_{c}A_{y}}{\sigma_{n}^{2}}\right)dA_{y}. \quad (3.3.1)$$

The obtained integrals are not expressed in the elementary functions; therefore let us consider the approximate relationships/ratios.

With strong signals $\frac{A_{c}}{\sigma_{n}}>1$ $w(A_{y})$ it is given (3.2.6), then

$$\Delta m_{з}=\int\limits_{A_{з}}^{\infty}(A_{y}-A_{з})\frac{1}{\sqrt{2\pi}\,\sigma_{n}}e^{-\frac{(A_{y}-A_{c})^{2}}{2\sigma_{n}^{2}}}dA_{y} \quad (3.3.2)$$

or

$$\Delta m_{з}=\sigma_{n}\int\limits_{a_{з}}^{\infty}\frac{(a_{y}-a_{з})}{\sqrt{2\pi}}e^{-\frac{(a_{y}-a_{c})^{2}}{2}}da_{y}, \quad (3.3.3)$$

where

$$a_s=\frac{A_s}{\sigma_n};\ a_y=\frac{A_y}{\sigma_n};\ a_c=\frac{A_c}{\sigma_n}.$$

Let us replace variable/alternating $a_y-a_c=b,\ a_s-a_c=b_s,$ then

$$\Delta m_s=\frac{\sigma_n}{\sqrt{2\pi}}\int_{b_s}^{\infty}(b-b_s)\,e^{-\frac{b^2}{2}}\,db=$$

$$=\frac{\sigma_n}{\sqrt{2\pi}}\left\{\int_{b_s}^{\infty}b\,e^{-\frac{b^2}{2}}\,db-b_s\int_{b_s}^{\infty}e^{-\frac{b^2}{2}}\,db\right\}. \qquad (3.3.4)$$

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It is known that

$$\int_{b_s}^{\infty}b e^{-\frac{b^2}{2}}\,db=e^{-\frac{b_s^2}{2}},$$

$$\frac{1}{\sqrt{2\pi}}\int_{b_s}^{\infty}e^{-\frac{b^2}{2}}\,db=1-F(b_s),$$

where $F(b_s)$— tabulated integral,

$$\Delta m_s=\sigma_n\left\{\frac{1}{\sqrt{2\pi}}e^{-\frac{(a_s-a_c)^2}{2}}-\right.$$

$$\left.-(a_s-a_c)\,[1-F(a_s-a_c)]\right\}. \qquad (3.3.5)$$

According to the obtained formulas can be constructed the graphs, which give the dependence of relative value of output potential of detector of ARU $\frac{\Delta m_s}{\sigma_n}$ from the relationship/ratio between $\frac{A_s}{\sigma_n}$ and $\frac{A_c}{\sigma_n}$. Fig. 3.3.1 gives the results of the calculations of value $\frac{\Delta m_s}{\sigma_n}$ in function $\frac{A_s}{\sigma_n}$ for $\frac{A_c}{\sigma_n}=5;\ 3;\ 2;\ 1.$

From the figure one can see that with the voltage of signal, equal to the stress/voltage of delay, due to the presence of noise is developed the noticeable detected stress/voltage, which activates ARU and reduces the amplification of receiver to the level, on which A_c will be less $A_з$.

Fig. 3.3.2 gives the results of calculations $\frac{\Delta m_c}{\sigma_ш}$ in function $\frac{A_c}{\sigma_ш}$ for $\frac{A_з}{\sigma_ш}=5$ and 3.

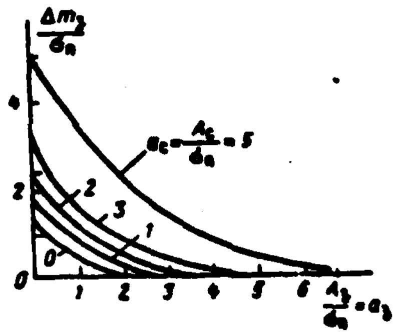

Fig. 3.3.1.

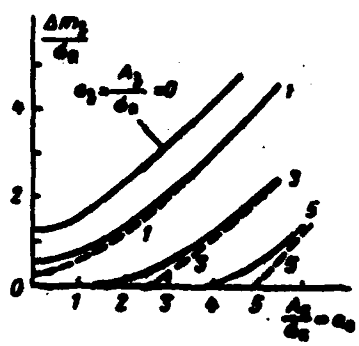

Fig. 3.3.2.

Fig. 3.3.1. Result of detection in detector of ARU.

Fig. 3.3.2. Result of detection in detector of ARU.

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From these curves it also follows that with ideal ARU the voltage of the signal, with which the detected stress/voltage virtually vanishes, it becomes less than $(0,1\div0,5)\sigma_{ш}$, approximately/exemplarily to $\sigma_{ш}$ less than the stress/voltage of delay.

In Fig. 3.3.3 (curve g) given change $\frac{\Delta m_0}{\sigma_ш}$ in the dependence on $\frac{A_0-A_з}{\sigma_ш}$, from which follows that the detected stress/voltage becomes insignificant when $\frac{A_0-A_з}{\sigma_ш}>1\div1,5$. With weak signals $\left(\frac{A_0}{\sigma_ш}=a_0<1\right)$ the function of amplitude distribution takes the complicated form.

Therefore for obtaining the simple calculated correlations it is necessary to use the simplifying approximations. Knowing from (3.2.8) and (3.2.18) dispersion and average for the mixture, it is possible to take them as the parameters of the normal distribution, which approximates a precise function.

With this simplification the formulas, used for the calculation of the result of detection for the case of strong signal, can be used also for the weak signal with the difference, however, that with the strong signal the average/mean value of the function of the distribution of mixture is defined only by signal, and dispersion - only by noise, while for the weak signal and average/mean value, and dispersion complicatedly they depend both on the signal and on noise.

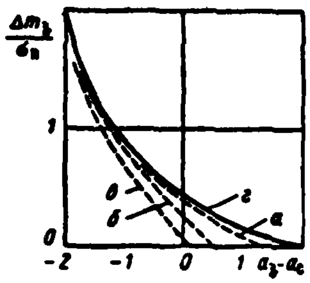

Fig. 3.3.3.

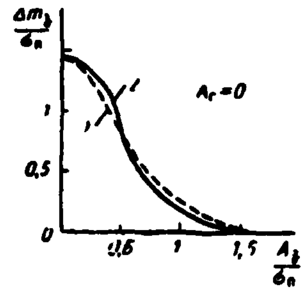

Fig. 3.3.4.

Fig. 3.3.3. Result of detection in detector of ARU.

Fig. 3.3.4. Comparison of work of detector of ARU with interferences.

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Then for each $\frac{A_c}{\sigma_ш}$ it is possible to find values $m_1(A_ш)$ and σ_A^2 from the formulas of S 3.2 and then, after substituting into the expressions for $\Delta m_э$, to compute the result of detection and to construct dependence $\frac{\Delta m_э}{\sigma_ш}$ on $\frac{A_э}{\sigma_ш}$ and on $\frac{A_c}{\sigma_ш}$

$$\Delta m_э = \sigma_A \left\{ \frac{1}{\sqrt{2\pi}} e^{-\frac{(a_{эн} - a_{cн})^2}{2}} - \right.$$

$$\left. - (a_{эн} - a_{cн}) [1 - F(a_{эн} - a_{cн})] \right\}, \qquad (3.3.6)$$

where

$$a_{эн} = \frac{A_э}{\sigma_A}; \; a_{cн} = \frac{m_1(A_ш)}{\sigma_A};$$

with

$$a_c = 0$$

$$a_{c\Omega} = \frac{1{,}25\,e_n}{0{,}65\,e_n} = 1{,}92 \approx 2$$

and

$$\Delta m_2 = e_n \left\{ \frac{1}{\sqrt{2\pi}} e^{-\frac{(a_{s\Omega}-2)^2}{2}} - \right.$$
$$\left. -(a_{s\Omega}-2)\left[1 - F(a_{s\Omega}-2)\right] \right\}. \qquad (3.3.7)$$

The results of calculation according to this approximation formula for special case ($A_c=0$) are given in Fig. 3.3.4 (curve 1). Is there given curved 2, constructed according to a precise formula, obtained in Chapter 2 for one interference (i.e., for $A_c=0$), and the confirming admissibility of the adopted simplifications.

Using the approximation formula, for different values $\frac{A_c}{e_n}<1$ it is possible to obtain $\frac{\Delta m_2}{e_n}$ in function $\frac{A_n}{e_n}$ or $\frac{A_c}{e_n}$. Fig. 3.3.1 and 3.3.2 give the results of the calculations, curves and points for $\frac{A_c}{e_n}<1$, from which is clearly evident functioning of ARU from the interferences.

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With the use of ARU with the delay on the detected stress/voltage after its averaging output potential of ARU will be determined from

$$\Delta m_s = m_1(A_y) - A_s \text{ or } \frac{\Delta m_s}{\sigma_n} = m_1(a_y) - a_s. \quad (3.3.8)$$

Expression for $m_1(A_y)$ and $m_1(a_y)$ was obtained earlier [see (3.28), (3.29)]. From them it is easy to obtain

$$\frac{\Delta m_s}{\sigma_n} = \sqrt{\frac{\pi}{2}}\left[\left(1+\frac{a_c}{2}\right)^2 I_0\left(\frac{a_c^2}{4}\right)+\right.$$

$$\left.+\frac{a_c}{2} I_1\left(\frac{a_c}{4}\right)\right] e^{-\frac{a_c^2}{2}} - a_s. \quad (3.3.9)$$

The graph/diagram of dependence $\Delta m_s/\sigma_n$ on a_c is given in Fig. 3.3.2 dotted line for $\frac{A_s}{\sigma_n} = 1; 3$ and 5. Is of interest also dependence $\Delta m_s/\sigma_n$ on $a_s - a_c$, which can be calculated by the obtained formulas and graphically given in Fig. 3.3.3 dotted curves a) $\frac{A_c}{\sigma_n} < 1$; b) $\frac{A_c}{\sigma_n} = 1$; c) $\frac{A_c}{\sigma_n} > 1$.

From that given above it follows that with ARU with the delay on the averaged detected stress/voltage the interference effect on the result of work of ARU is substantially less.

Results show that the detected stress/voltage of ARU depends substantially on the interferences. With a good quality ARU can have small value for the noticeable decrease of amplification. This change in the amplification decreases signal level at the output of radio-frequency circuit and additionally to the suppression of signal by noise in the detector will cause the suppression of signal by noise due to the work of ARU.

On the basis of the formulas given above and graphs let us analyze the effect of the suppression of noise signal due to the action of ARU.

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Let us assume that in the receiver is a sufficient gain margin, i.e., σ_n with A_s, and ARU commensurably works virtually ideally from $\frac{\Delta m_0}{\sigma_n}=0{,}05.$ Then from the graph, depicted in Fig. 3.3.3, it follows that $\frac{A_0}{\sigma_n}-\frac{A_s}{\sigma_n}=1{,}5$ for the diagram with the delay on the detector and $\frac{A_s}{\sigma_n}-\frac{A_0}{\sigma_n}=0{,}3$ for the diagram with the delay on the averaged stress/voltage. With that determined A_0/σ_n this equality can be observed only because the amplification of receiver will be changed because of the work of ARU. In this case A_0/σ_n will not be changed, since amplification to the identical degree will change signal, and noise, on will be changed A_s/σ_n due to change σ_n.

Let us consider the dependence of the voltage of signal on the output of receiver with ARU from the voltage of signal on the entrance of receiver at first for the diagram with the delay in the detector. Let us designate changed value A_{cAPY} and the initial value in the absence of ARU A_{c0}, is analogous $\sigma_{n\,APY}$ and σ_{n0}. Then

$$\frac{A_c}{\sigma_{n\,APY}} \approx \frac{A_{c0}}{\sigma_{n0}} + 1{,}5. \qquad (3.3.10)$$

As a result of the presence of noises ARU will operate/wear on the smaller signal level and the signal amplitude under the effect of ARU of interferences will be less than in the case of acting one signal without the interferences, i.e.

$$A_{c\,APY} < A_{c0}.$$

Since relation A_c/σ_n is not changed with the work of ARU,

$$\frac{A_{c\,APY}}{\sigma_{n\,APY}} = \frac{A_{c0}}{\sigma_{n0}}.$$

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To more conveniently carry signal amplitude to delay factor, then

$$\frac{A_{c\,APY}/A_з}{A_{c0}/A_з} = \frac{\sigma_{n\,APY}}{\sigma_{n0}}$$

and

$$A_{c\,APY}/A_з = \frac{\sigma_{n\,APY}}{\sigma_{n0}}\frac{A_{c0}}{A_з}$$

or

$$\frac{A_{c\,APY}}{A_з} = \frac{A_{c0}}{\sigma_{n0}}\left(\frac{1}{\frac{A_{c0}}{\sigma_{n0}} + 1{,}5}\right). \qquad (3.3.11)$$

After using these formulas and bearing in mind that the amplification of receiver conservative, i.e., $\sigma_{n0} \approx A_з$, it is possible to lead calculations and to obtain the graphs, which characterize a change in the signal amplitude at the output of radio channel with a change in the signal level at the input, i.e., with change $\frac{A_{c0}}{\sigma_{n0}}$.

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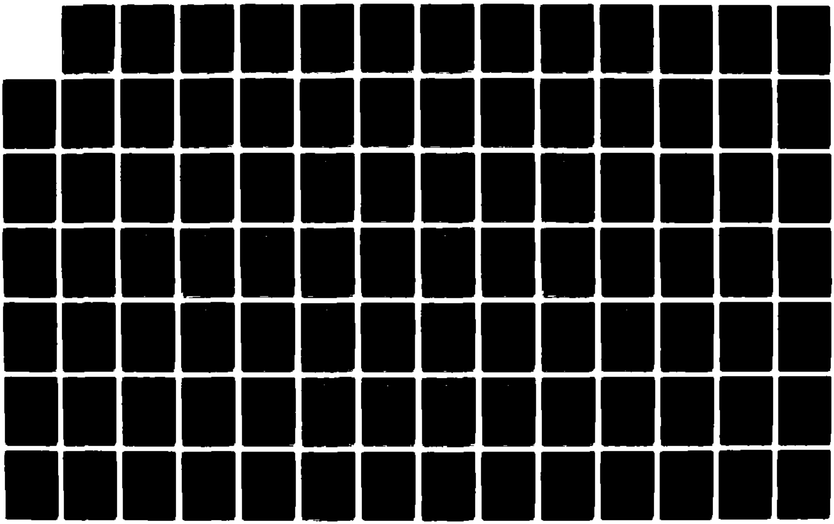

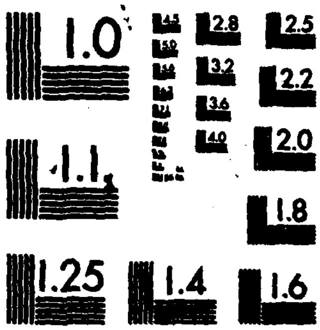

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Fig. 3.3.5 gives the results of calculations. Curve 1 corresponds to ideal work of ARU without the interferences. Curve 2 gives change $A_{c\,APY}/A_{s}$ upon consideration of the action of interferences. Curve 5 - dependence $\sigma_{c\,APY}/\sigma_{ns}$.

From the results it follows that the presence of interferences substantially changes work of ARU, causing the noticeable decrease of signal level at the output of receiver.

From that presented can be done the incorrect conclusion from that that an increase is amplification of the receiver is undesirable since it causes the operation of the ARU from interference.

If we decrease amplification, then this will be reflected in what will be increased A_{s}/σ_{ns} and ARU from the interferences will not operate/wear, but also signal level decreases.

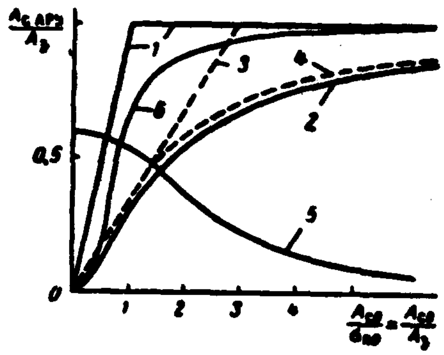

Fig. 3.3.5. Suppression of noise signal during the use/application of ARU.

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As an example in Fig. 3.3.5 are given curve 3 for the case of ideal work of ARU without the interferences with the decrease of amplification to the level when $A_s/\sigma_{m0}=3$, and curve 4 under the same conditions, but taking into account interferences. In order to consider the effect of amplification, it is possible to construct curve, given in Fig. 3.3.6, that characterizes changes in the signal level with a change in the amplification, expressed in σ_{m0}/A_s.

Consequently, during the use of ARU an increase in the initial amplification of receiver barely is manifested at the level of weak or strong signals at the output of radio channel. Hence ensues/escapes/flows out the practical advisability of an increase in the amplification of receivers so that unavoidable during the production and the operation considerable changes in the amplification would not be manifested at the level of the signal, supplied to the meters. In this case standard it is possible to consider the mode/conditions during which the nominal amplification of receiver creates the interference level, which causes functioning of ARU. Analogous calculations can be led, also, for ARU, which has delay on the detected averaged stress/voltage. From previous it follows that in this case the interferences will affect less. Lowering calculations, let us give in Fig. 3.3.5 (curve 6) dependence $A_{e\,АРУ}/A_e$ for this case.

Thus, good ARU solves the problem of the constancy of stress/voltage, supplied from the output of receiver to the meter and the detectors, but at the same time are caused negative consequences, basis of which consists of the suppression of weak noise signal through ARU.

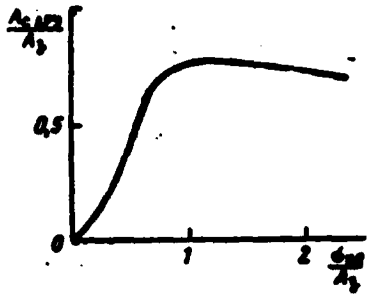

Fig. 3.3.6. Effect of initial amplification on the signal during the use/application of ARU.

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If receiving and measuring device/equipment according to one or the other reasons has wide passband to the phasemeter, in which is realized processing signal with the appropriate contraction of passband, then the ratio of interference to the signal at the output of radio-particular circuit it is possible to allow more than one, since the subsequent selection will filter out interferences.

But in such ratios of interference to the signal occurs the suppression of weak noise signal in ARU, as a result of which signal level is reduced. Ideal phasemeter works with any signal amplitude and for it the value of this relation does not have a value. In the real phasemeters for the normal operation is required the specific voltage of the signal whose decrease can cause disturbances/breakdowns, and, although the ratio of interference to the signal remains permissible, the receiving and measuring device/equipment will cease normally to work because the interferences will "drive in" (suppress) signal on its level. This "clogging" most substantially can be showed when $\frac{A_{c0}}{\sigma_{ш0}} < 1$.

If useful information is laid into modulation of radio signal, then suppression occurs both due to ARU and due to the detector.

Using the relationships/ratios obtained above, it is possible to lead calculations for the specific cases. Examining together both effects of suppression and multiplying ordinates in identical relations $A_c/\sigma_ш$ it is possible to obtain the resulting curve, which characterizes suppression. Fig. 3.3.7 gives this curve for the standard case in the case of the delay on the detector. In the case of the delay on the detected stress/voltage the suppression will be somewhat less.

From this curve it follows that when $\frac{A_c}{\sigma_ш} < 2 \div 3$ signal level sharply descends also when $\frac{A_c}{\sigma_ш} < 1$ it becomes negligibly small, composing altogether only 10-20% of the nominal level when $A_c > \sigma_ш$.

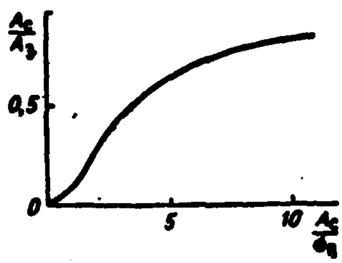

Fig. 3.3.7. Combined action of interferences on the signal in the presence of detector and ARU.

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These results have important practical value, since it proves to be that during the use of amplitude modulation, for example, in the phase systems, which work at the modulation frequency, the systems of tracking with the conical scanning where the selectivity of measuring (servo) devices can be very high and does not limit the possibility of reducing the signal of lower than the interference level, this lowering cannot be realized due to the suppression of noise signal.

§ 3.4. Functions of phase distribution in the mixture of signal and interference. Let us begin now the study of the function of phase distribution:

$$w(\varphi_y)=\int_0^\infty w_2(A_y,\varphi_y)\,dA_y=\int_0^\infty \frac{A_y}{2\pi\sigma_n^2}e^{-\frac{A_y^2+A_c^2}{2\sigma_n^2}}\times$$

$$\times e^{-\frac{A_yA_c}{2\sigma_n^2}\cos\varphi_y}\,dA_y=\frac{1}{2\pi\sigma_n^2}e^{-\frac{A_c^2}{2\sigma_n^2}}e^{\frac{A_c^2}{2\sigma_n^2}\cos^2\varphi_y}\times$$

$$\times\int_0^\infty A_y e^{-\frac{(A_y-A_c\cos\varphi_y)^2}{2\sigma_n^2}}\,dA_y=\frac{1}{2\pi\sigma_n^2}e^{-\frac{A_c^2\sin^2\varphi_y}{2\sigma_n^2}}\times$$

$$\times\int_{-A_c\cos\varphi_y}^{\infty}(A_1+A_c\cos\varphi_y)e^{-\frac{A_1^2}{2\sigma_n^2}}\,dA_1, \qquad (3.4.1)$$

where $A_1=A_y-A_c\cos\varphi_y$.

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After representing integral in the form of the sum of two integrals, we will obtain

$$w(\varphi_y)=\frac{1}{2\pi\sigma_n^2}e^{-\frac{A_c^2\sin^2\varphi_y}{2\sigma_n^2}}\times$$

$$\times\left[\int\limits_{-A_c\cos\varphi_y}^{\infty}A_1e^{-\frac{A_1^2}{2\sigma_n^2}}dA_1+\int\limits_{-A_c\cos\varphi_y}^{\infty}A_c\cos\varphi_y e^{-\frac{A_1^2}{2\sigma_n^2}}dA_1\right]=$$

$$=\frac{1}{2\pi}e^{-\frac{A_c^2}{2\sigma_n^2}\sin^2\varphi_y}e^{-\frac{A_c^2}{2\sigma_n^2}\cos^2\varphi_y}+$$

$$+\frac{1}{\sqrt{2\pi}\sigma_n}e^{-\frac{A_c^2}{2\sigma_n^2}\sin^2\varphi_y}A_c\cos\varphi_y F\left(\frac{A_c}{\sigma_n}\cos\varphi_y\right)$$

or

$$w(\varphi_y)=\frac{1}{2\pi}e^{-\frac{a_c^2}{2}}+$$

$$+\frac{a_c\cos\varphi_y}{\sqrt{2\pi}}F(a_c\cos\varphi_y)e^{-\frac{a_c^2}{2}\sin^2\varphi_y}, \qquad (3.4.2)$$

where, as before

$$a_c=\frac{A_c}{\sigma_n} \text{ и } \varphi_c=0.$$

When $a_c=0$

$$w(\varphi_y)=\frac{1}{2\pi}. \qquad (3.4.3)$$

When $\varphi_y=0$

$$w(\varphi_y=0)=\frac{1}{2\pi}e^{-\frac{a_c^2}{2}}+\frac{a_c}{\sqrt{2\pi}}F(a_c). \qquad (3.4.4)$$

The obtained expression is quite bulky for the calculations and in many instances proves to be more convenient to use the simplified approximation formulas for $a_c<1$ and $a_c>1$.

When $a_c>1$ the probability density for angles φ_y, of those differing significantly from zero, is small. Therefore an approximation can be realized for the limited range of angles and assumed not only $a_c>1$, but also $a_c \cos\varphi_y>1$.

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Then, disregarding term $e^{-\frac{a_c^2}{2}}$ and considering that

$$F(a_c \cos\varphi_y) \approx 1,$$

we obtain

$$w(\varphi_y) \approx \frac{a_c \cos\varphi_y}{\sqrt{2\pi}} e^{-\frac{a_c^2 \sin^2\varphi_y}{2}} \approx \frac{a_c}{\sqrt{2\pi}} e^{-\frac{a_c^2 \varphi_y^2}{2}}$$

or

$$w(\varphi_y) = \frac{1}{\sqrt{2\pi}\sigma_\varphi} e^{-\frac{\varphi_y^2}{2\sigma_\varphi^2}}, \text{ где } \sigma_\varphi^2 = \frac{\sigma_n^2}{A_c^2} = \frac{1}{a_c} \quad (3.4.5)$$

with an increase in the level of signal $a_c = \frac{A_c}{\sigma_n} \to \infty$ and $\sigma_\varphi^2 \to 0$, i.e., the distribution function approaches the delta-function, characteristic for phase distribution of the harmonic oscillation when is possible only one value of phase. Consequently, when $a_c>1$ phase distribution is subordinated to normal law with the dispersion

$$a_c^2 = \frac{c_c^2}{\sigma_c^2}.$$

When $a_c < 1$ the expression can be expanded in series/row according to degrees a_c; then we obtain (see [2.1])

$$w(\varphi_y) = \frac{1}{2\pi} + \frac{a_c \cos\varphi_y}{2\sqrt{2\pi}} + \frac{a_c^2 \cos 2\varphi_y}{4\pi} - \frac{a_c^3 \sin^2\varphi_y \cos\varphi_y + \dots}{4\sqrt{2\pi}} \dots =$$
$$= \frac{1}{2\pi}\left(1 + a_c\sqrt{\frac{\pi}{2}}\cos\varphi_y + \frac{a_c^2}{2}\cos 2\varphi_y - \right.$$
$$\left. - \frac{\sqrt{\pi}\, a_c^3 \sin^2\varphi_y \cos\varphi_y + \dots}{2\sqrt{2}}\right). \quad (3.4.6)$$

Use with $a_c < 1$ the distribution function in the form, given by expression (3.4.6), can cause some objections, since from the point of view of physics of process it is difficult to explain increase (with an increase a_c) in the role of term with factor a_c^2, which causes undershoots in the probability density of the phase of mixture at angles φ_y of close ones to $\pm\pi/2$.

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Since the account of terms with the higher than a_c^2, by degrees a_c is made itself expression bulky, can be recommended to use the approximation/approach, which considers only a_c.

In this case the expression is easily obtained directly from

(3.5.2), since when $a_c < 1$

$$e^{-\frac{a_c^2}{2}} \simeq 1, \quad e^{-\frac{a_c^2}{2}\sin^2\varphi_y} = 1,$$
$$F(a_c \cos\varphi_y) \simeq 0,5.$$

Then

$$w(\varphi_y) = \frac{1}{2\pi} + \frac{a_c \cos\varphi_y}{2\sqrt{2\pi}}$$

or

$$w(\varphi_y) = \frac{1}{2\pi}\left(1 + a_c\sqrt{\frac{\pi}{2}}\cos\varphi_y\right). \qquad (3.4.7)$$

The dispersion of the fluctuations of phase with the weak signal is computed from the expression

$$\sigma_\varphi^2 = \frac{\pi^2}{3} - \sqrt{2\pi}\frac{A_c}{\sigma_n}.$$

When $A_c = 0$, i.e., for one interference,

$$\sigma_\varphi^2 = \frac{\pi^2}{3}.$$

Consequently, when $a_c < 1$ the function of phase distribution corresponds to cosine curve with the constant component $1/2\pi$. Fig. 3.4.1 gives the functions of phase distribution for different values a_c. There by dotted line is shown approximation by normal law. When $a_c \geqslant 2$ the coincidence in effect complete, when $a_c = 1$ is certain disagreement. For $a_c = 0,5$ the form of the function of distribution in practice does not differ from cosine curve.

The standard deviation of phase increases with the decrease of relation $\frac{A_c}{\sigma_n}$. The graph of change σ_φ is given in Fig. 3.4.2.

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The comparison of combined amplitude distribution and phase and one-dimensional distributions for the amplitude and the phase in the mixture shows that since $w(A_y, \varphi_y) \neq w(A_y)\,w(\varphi_y)$, that the phase and the amplitude of mixture are not statistically independent variables. Statistical independence the times and amplitude at the coinciding moments of time occurs only in the absence of signal.

Of certain interest is also the integral function of phase distribution of mixture $p(\varphi_y < \varphi_{\text{утек}}) = \Phi(\varphi_{\text{утек}})$. With weak signals $\Phi(\varphi_{\text{утек}})$ will be close to the straight line, passing through point 0.5 when $\varphi_{\text{утек}} = 0$. With strong signals $\Phi(\varphi_{\text{утек}})$ it will be close to the curve of the integral normal law of distribution. In the practical problems essential interest presents probability that the divergences of the phase of mixture will not exceed specific value $\pm\varphi_{\text{доп}}$, i.e., $p(|\varphi_y| < |\varphi_{\text{доп}}|)$, which also can be found with integration $w(\varphi_y)$ and limits $\pm\varphi_{\text{доп}}$.

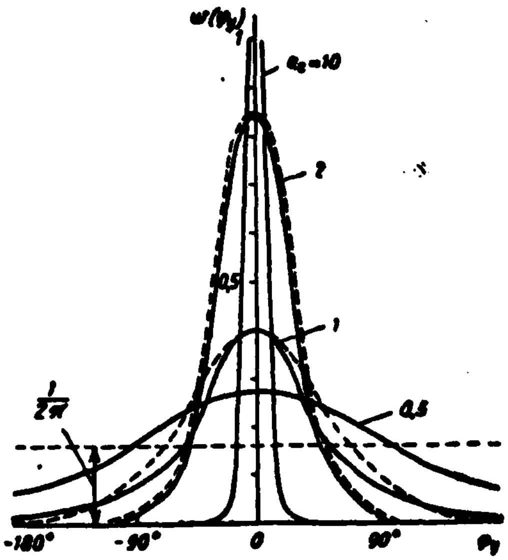

Fig. 3.4.1. Functions of phase distribution.

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With the reading of the phase of mixture relative to the phase of signal, after assuming $\varphi_0=0$, we will obtain

$$p(|\varphi_y|<|\varphi_{\text{пор}}|)=\int\limits_{-\varphi_{\text{пор}}}^{+\varphi_{\text{пор}}} w(\varphi_y)\,d\varphi_y=2\int\limits_{0}^{+\varphi_{\text{пор}}} w(\varphi_y)\,d\varphi_y=$$

$$=\frac{\varphi_{\text{пор}}}{\pi}e^{-\frac{a_c^2}{2}}+\frac{2a_c}{\sqrt{2\pi}}\int\limits_{0}^{\varphi_{\text{пор}}}\cos\varphi_y F(a_c\cos\varphi_y)\,e^{-\frac{a_c^2\sin^2\varphi_y}{2}}\,d\varphi_y.$$

This integral is not expressed as known functions. In work [2.1] it was given approximately to the expression which makes it possible to express $p(|\varphi_y|<|\varphi_{\text{пор}}|)$ in the form of the following relationship/ratio:

$$p(|\varphi_y|<|\varphi_{\text{пор}}|)=2F(a_c\sin\varphi_{\text{пор}})-1+$$
$$+\frac{\operatorname{tg}\varphi_{\text{пор}}}{\pi a_c^2}e^{-\frac{a_c^2}{2}}\left(1-\frac{1+2\cos^2\varphi_{\text{пор}}}{a_c^2\cos^2\varphi_{\text{пор}}}\right). \qquad (3.4.8)$$

For strong signals $a_c \gg 1$

$$p(|\varphi_y|<|\varphi_{\text{пор}}|)=2F(a_c\varphi_{\text{пор}})-1=2F\left(\frac{\varphi_{\text{пор}}}{\sigma_\varphi}\right)-1.$$

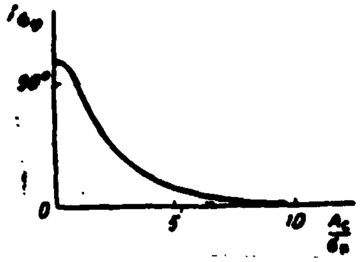

Fig. 3.4.2. Root-mean-square phase deviation depending on A_c/σ_n.

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This result is clear, since with the strong signal the distribution of the divergences of phase is subordinated to normal law with dispersion $\sigma_\varphi^2 = \frac{1}{a_c^2}$. For the normal law the probability is expressed as the tabulated integral.

For the weak signals

$$p(|\varphi_y| < |\varphi_{\text{пор}}|) \simeq \frac{\varphi_{\text{пор}}}{\pi} + \frac{a_c}{\sqrt{2\pi}} \sin \varphi_{\text{пор}}.$$

Fig. 3.4.3 gives curved functions $p(|\varphi_y| < |\varphi_{\text{пор}}|)$ for different values a_c.

§ 3.5. Four-dimensional function of amplitude distribution and phase of mixture. The analysis of the one-dimensional functions of amplitude distribution and phase of mixture made it possible to explain the series/row of the special features/peculiarities of its action on the receiving and measuring device/equipment. However, these functions do not give representation about the transient nature of random processes $A_{v}(t)$ and $\varphi_{v}(t)$. Consequently, the results obtained above are valid only for the conditions that the ideal phasemeter is inertia-free and reacts to all fluctuations of the phase of mixture - not to radio frequency or to modulation frequency. In this case the function of the distribution of the instantaneous single readings of phase, taken/removed from this phasemeter, will coincide with the distribution function in the mixture.

However, real phasemeter possesses final inertness. In many instances for the purpose of an increase in the accuracy of the measurement of phase the phasemeter is obtained satisfaction with knowingly inertia with the narrow band of transmission.

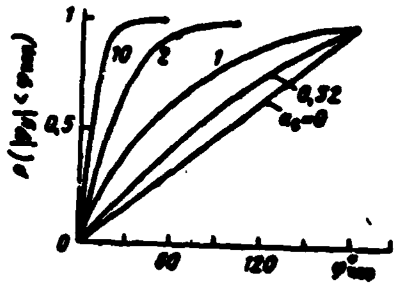

Fig. 3.4.3. Functions $p(|\varphi_y| < |\varphi_{max}|)$ for different ones a_0.

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In order to consider the action of interferences in the systems with the narrow-band phasemeter, it is convenient to use the energy spectrum of the fluctuations of phase and amplitude.

The energy spectrum of fluctuations can be found through the correlation function. Correlation function can be obtained from the two-dimensional distribution function.

The two-dimensional function of amplitude distribution or phase can be obtained from the four-dimensional function of amplitude distribution and phase $w_4(A_{y1}, A_{y2}, \varphi_{y1}, \varphi_{y2}, \tau)$, which, in turn, by functional conversions can be obtained from $w_4(\mathcal{E}_{y1}, \mathcal{E}_{y2}, E_{y1}, \dot{E}_{y2}, \tau)$.

According to general/common/total expression for the functions of the distribution of stationary normal processes it is possible to register

$$w_2(\mathcal{D}_{y1},\mathcal{D}_{y2},\tau)=\frac{1}{\sigma_n^2 2\pi\sqrt{1-R_0^2(\tau)}}\times$$

$$\times e^{-\frac{(\mathcal{D}_{y1}-d)^2+(\mathcal{D}_{y2}-d)^2-2R_0(\tau)(\mathcal{D}_{y1}-d)(\mathcal{D}_{y2}-d)}{2\sigma_n^2[1-R_0^2(\tau)]}},$$

$$w_2(E_{y1},E_{y2},\tau)=\frac{1}{\sigma_n^2 2\pi\sqrt{1-R_0^2(\tau)}}\times$$

$$\times e^{-\frac{(E_{y1}-e)^2+(E_{y2}-e)^2-2R_0(\tau)(E_{y1}-e)(E_{y2}-e)}{2\sigma_n^2[1-R_0^2(\tau)]}},$$

where

$$d=A_c\cos\varphi_c;\ e=A_c\sin\varphi_c.$$

$$B_n(\tau)=\sigma_n^2 R_0(\tau)\cos\omega_0\tau.$$

Then the four-dimensional joint distribution of components is registered in the form

$$w_4(\mathcal{D}_{y1},\mathcal{D}_{y2},E_{y1},E_{y2},\tau)=\frac{1}{(2\pi\sigma_n^2)^2[1-R_0^2(\tau)]}\times$$

$$\times e^{-\left[\frac{(\mathcal{D}_{y1}-d)^2+(\mathcal{D}_{y2}-d)^2+(E_{y1}-e)^2+(E_{y2}-e)^2-2R_0(\tau)}{2\sigma_n^2[1-R_0^2(\tau)]}\right.}$$

$$\underline{\left.[(\mathcal{D}_{y1}-d)(\mathcal{D}_{y2}-d)+(E_{y1}-e)(E_{y2}-e)]\right]}. \qquad (3.5.1)$$

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For obtaining $w_4(A_{y1},A_{y2},\varphi_{y1},\varphi_{y2},\tau)$ it is necessary to switch over from $\mathcal{D}_{y1}$, $\mathcal{D}_{y2}$, E_{y1} and E_{y2} to their expressions through A_y and φ_y, i.e.

$$\mathcal{D}_{y1}=A_{y1}\cos\varphi_{y1},\ \mathcal{D}_{y2}=A_{y2}\cos\varphi_{y2},$$

$$E_{y1}=A_{y1}\sin\varphi_{y1},\ E_{y2}=A_{y2}\sin\varphi_{y2}.$$

and to find the so-called jacobian of conversion which in this case is equal to $A_{y1}A_{y2}$, then

$$w_4(A_{y1}, A_{y2}, \varphi_{y1}, \varphi_{y2}, \tau) =$$

$$= A_{y1}A_{y2} w_4(A_{y1}\cos\varphi_{y1}, A_{y2}\cos\varphi_{y2}, A_{y1}\sin\varphi_{y1}, A_{y2}\sin\varphi_{y2}, \tau) =$$

$$= \frac{A_{y1}A_{y2}}{(2\pi\sigma_n^2)^2[1 - R_0^2(\tau)]} \exp\left\{\frac{1}{2\sigma_n^2[1 - R_0^2(\tau)]} \times\right.$$

$$\times [(A_{y1}\cos\varphi_{y1} - A_c\cos\varphi_c)^2 + (A_{y2}\cos\varphi_{y2} - A_c\cos\varphi_c)^2 +$$

$$+ (A_{y1}\sin\varphi_{y1} - A_c\sin\varphi_c)^2 + (A_{y2}\sin\varphi_{y2} - A_c\sin\varphi_c)^2 -$$

$$- 2R_0(\tau)((A_{y1}\cos\varphi_{y1} - A_c\cos\varphi_c)(A_{y2}\cos\varphi_{y2} - A_c\cos\varphi_c) +$$

$$\left. + (A_{y1}\sin\varphi_{y1} - A_c\sin\varphi_c)(A_{y2}\sin\varphi_{y2} - A_c\sin\varphi_c))]\right\}.$$

§ 3.6. Two-dimensional distribution function, the function of correlation and the energy spectrum of amplitude. For obtaining the two-dimensional function of amplitude distribution it is necessary to fulfill the integration

$$w_2(A_{y1}, A_{y2}, \tau) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} w_4(A_{y1}, A_{y2}, \varphi_{y1}, \varphi_{y2}, \tau)\, d\varphi_{y1}\, d\varphi_{y2}. \tag{3.6.1}$$

This integral was obtained earlier (for the case of the absence of signal). In the presence of signal the integration is connected with bulky conversions which we are forced to drop/omit.

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For the case of harmonic signal with the constant amplitude is obtained the following expression:

$$w_2(A_{y1}, A_{y2}, \tau)=\frac{A_{y1}A_{y2}}{\sigma_n^4[1-R_0^2(\tau)]}\times$$
$$\times e^{-\frac{A_{y1}^2+A_{y2}^2}{2\sigma_n^2[1-R_0^2(\tau)]}}e^{-\frac{A_c^2}{\sigma_n^2[1-R_0^2(\tau)]}}\times$$
$$\times\sum_{m=0}^{\infty}E_m I_m\left(\frac{R_0(\tau)A_{y1}A_{y2}}{\sigma_n^2[1-R_0^2(\tau)]}\right)\times$$
$$\times I_m\left(\frac{A_c A_{y1}}{\sigma_n^2[1+R_0(\tau)]}\right)I_m\left(\frac{A_c A_{y2}}{\sigma_n^2[1+R_0(\tau)]}\right). \qquad (3.6.2)$$

$E_m=1$ with m=0 and $E_m=2$ with m>0. For obtaining the autocorrelation function and energy spectrum, most convenient for the practical use, it is necessary to fulfill the conversions

$$B_A(\tau)=\int_0^{\infty}\int_0^{\infty}A_{y1}A_{y2}w_2(A_{y1}, A_{y2}, \tau)\,dA_{y1}\,dA_{y2}, \qquad (3.6.3)$$

$$G_A(\omega)=4\int_0^{\infty}B_A(\tau)\cos\omega\tau\,d\tau. \qquad (3.6.4)$$

These conversions are characterized by considerable complexity; therefore we will be bounded to the fact that let us give expression in the final form

$$B_A(\tau)=2\sigma_n^2[1-R_0^2(\tau)]^2 e^{-\frac{A_c^2}{2\sigma_n^2}}\times$$
$$\times\sum_{m=0}^{\infty}\frac{(2m!)\,[(2m+1)!!]^2}{2^{4m}(m!)^2}\times$$
$$\times\sum_{n=0}^{2m}\frac{R_0^{2m-n}(\tau)}{(2m-n)!\,(n!)^2}\left[\frac{A_c^2}{2\sigma_n^2}\,\frac{1-R_0(\tau)}{1+R_0(\tau)}\right]^2\times$$
$$\times\,{}_1F_1\left(n-2m-2,\ n+1,\ -\frac{1-R_0(\tau)}{1+R_0(\tau)}\,\frac{A_c^2}{2\sigma_n^2}\right). \qquad (3.6.5)$$

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Derivation of (3.6.5) is in [2.1]. The complexity of expressions, difficulty of simplification for the specific cases is made its barely suitable for practical use and obtaining $G_A(\omega)$.

The difficulties of obtaining the expressions for the autocorrelation function and energy spectrum can be partially reduced, if we use an approximation of detector in the form of the short-circuited nonlinear element with the quadratic amplitude characteristic. The solutions for this case can be obtained in the analytical form (see [2.1]). However, this approximation less accurately reproduces real detector, than accepted in this work (idealized detector). Is useful at least to qualitatively determine the form of energy spectrum $G_A(\omega)$ with different A_0/σ_n. Earlier were obtained the distribution function, dispersion and energy spectrum of the amplitude of interference.

Upon the appearance of a signal the one-dimensional function of amplitude distribution, the dispersion of the fluctuations of amplitude and energy spectrum are changed. For dispersion σ_A^2 and one-dimensional distribution function were obtained expressions (3.2.1) and (3.2.16), that make it possible produce calculations in any relations A_c/σ_m. Since autocorrelation function and energy spectrum for the envelope of mixture to accurately compute difficultly, let us attempt from the physical considerations to find the approximate relationships/ratios. With the relatively strong signal the fluctuations of envelope will depend on beatings between the signal and the components by interferences. Then the spectrum of the fluctuations of envelope must approach an initial interference spectrum (to its form of relatively carrier). With the simplest uniform high-frequency spectrum it must also approach uniform. In this case the power density will comprise

$$G_A(\omega)=2\frac{\sigma_n^2}{\Delta f_n},$$

since in this case the dispersion of fluctuations $\sigma_A^2=\sigma_n^2$.

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Thus, the spectrum of amplitude, being gradually changed with an increase in the signal, passes from the "triangular" to the "rectangular" with the narrower band with an increase of the dispersion 2.5 times and an increase in the power density at the low frequencies approximately/exemplarily 2.5 times. It is possible to assume that a change in the form of the spectrum is connected with a change in the form of the one-dimensional distribution function. The nearer it is to to the normal with the average, precisely equal to signal amplitude, the nearer the spectrum to uniform. With an increase in the signal the power density of the high-frequency fluctuations between the components of interferences is not changed and the dispersion of fluctuations increases due to the beatings between the interference and the signal, i.e., at frequencies less than $\Delta\omega_{\text{ш}}/2$. With strong signal ($A_c>\sigma_{\text{ш}}$) proceeds the effect of suppression by the signal of the detection of the high-frequency components of beatings (i.e., beatings with the frequency from $\Delta\omega_{\text{ш}}/2$ to $\Delta\omega_{\text{ш}}$), therefore with an increase in the signal power density at frequencies above $\Delta\omega_{\text{ш}}/2$ must be reduced, and at frequencies within the limits from 0 to $\Delta\omega_{\text{ш}}/2$ — increase.

Since when $\frac{A_c}{\sigma_{\text{ш}}}>5\,\sigma_A^2\approx\sigma_{\text{ш}}^2$ and the average/mean value of the distribution function becomes close to A_c, it is possible to assume that in this case the spectrum of the amplitude of mixture will be converted virtually into the uniform (for the ideal radio filter).

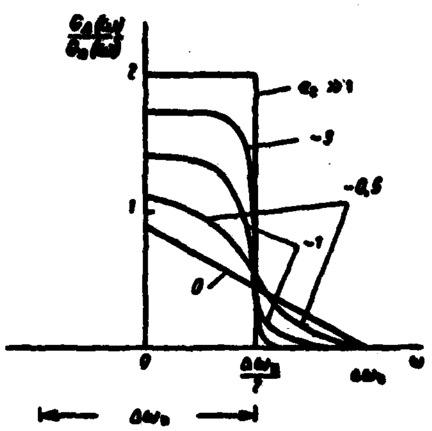

Fig. 3.6.1. The energy spectrum of amplitude.

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Fig. 3.6.1 gives the spectra for the different ones A_c/σ_m moreover their area, i.e., the dispersion of fluctuation, are obtained at a rate of formula (3.2.18) and (3.2.19), and form is constructed on the basis of the qualitative considerations, presented above.

From the obtained results it follows that the difficulties of obtaining precise relationships/ratios for $G_A(\omega)$ and $B_A(\tau)$ depend, probably, by the complicated form of the spectrum of low-frequency fluctuations, even with the simplest, uniform spectrum of high-frequency interferences.

It is possible to note that for the engineering calculations a precise calculation of the form of energy spectrum is not compulsory according to the following reasons. If measuring device is inertia-free, then on it functions entire spectrum of fluctuations. In this case it is possible to use the dispersion of fluctuations which is computed accurately. If measuring device is narrow-band, then in it, as a rule, is used the frequency band near from zero and its work does not affect the complicated form of energy spectrum. For the determination of the dependence of power density at the low frequencies on the value of signal it is possible to use the fact that with a change in the signal in essence varies the spectrum in the limits from 0 to $\Delta\omega_m/2$, growing more or less evenly in entire this frequency range.

Then in the first approximation, with an increase in the signal change $G_A(\omega)$ will be the same as increase σ_A^2, which can be calculated accurately. Fig. 3.6.2 gives dependence $\frac{\sigma_A^2}{\sigma_a^2}$ on $\frac{A_0}{\sigma_a}$, calculated according to the formulas § 3.5. On the assumption accepted the same dependence reflects change $\frac{G_A(0)}{G_A(0)_{cc}}$, where $G_A(0)_{cc}$ — power density with the strong signal. Energy spectra examined earlier completely do not reflect that the complicated processes which occur in the phase receiving and measuring device/equipment with modulation of signal, since in their obtaining was taken into consideration only carrier. Under the actual conditions functions not one carrier, but whole spectrum of modulated signal. In the simplest case it is necessary to consider carrying and two sidebands.

Obtaining precise analytical expressions for this general case is connected with the great mathematical difficulties. In the first approximation, also with modulation of noise signal can be considered by the characteristics, obtained earlier for one carrier.

Let us consider now the properties of the phase system, in which is used modulation. Block diagram and diagrams/curves of the spectra of signal and interference take the form, depicted in Fig. 3.6.3. At the output of low pass filter will be selective interference and the signal at the modulation frequency. For a blend of signal and noise the statistical characteristics were obtained earlier. Moreover it was established/installed, that they depend on the ratio of signal to the interference.

In this case they must be determined by relation $\frac{A_{c\Omega}}{\sigma_{\Delta\Omega}}$. Let us establish the relationships/ratios between $\frac{A_{c\Omega}}{\sigma_{\Delta\Omega}}$ and $\frac{A_c}{\sigma_n}$. Assuming that modulation frequency low, we can approximately consider that the jamming density near the modulation frequency is close to the power density at the zero frequency.

For the weak signal the dispersion of interference in the narrow band of filter $\Delta F_{\text{пф}}$ will be equal to

$$\sigma^2_{\Delta\Omega} = G_A(0)\,\Delta F_{\text{пф}} \simeq 0{,}8\,\frac{\sigma_n^2}{\Delta\omega_n}\,\Delta\Omega_{\text{пф}}. \qquad (3.6.6)$$

The detected voltage of signal taking into account suppression in the detector can be obtained from expression (3.2.15)

$$A_{c\Omega} = A_c M \sqrt{\frac{\pi}{32}}\,\frac{A_c}{\sigma_n},$$

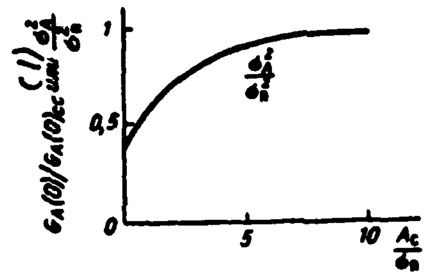

Fig. 3.6.2. Power density at the zero frequencies.

Key: (1). or.

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The ratio of signal to the interference at the output of detector (in the narrow band of filter $-\Delta F_{\text{нф}}$ or $\Delta\Omega_{\text{нф}}$)

$$\frac{A_{c\Omega}}{\sigma_{\Delta\Omega}}=\left(\frac{A_c}{\sigma_n}\right)^2\sqrt{\frac{\Delta\omega_n}{\Delta\Omega_{\text{нф}}}}\cdot M0,3. \tag{3.6.7}$$

For the strong signal:

$$A_{c\Omega}=A_cM,$$

$$\sigma^2_{\Delta\Omega}=G_A(0)\,\Delta F_{\text{нф}}=\frac{\sigma_n^2}{\Delta\omega_n}\,2\Delta\Omega_{\text{нф}},$$

$$\frac{A_{c\Omega}}{\sigma_{\Delta\Omega}}=\frac{A_c}{\sigma_n}\,M\sqrt{\frac{\Delta\omega_n}{2\Delta\Omega_{\text{нф}}}}.$$

In the system without the ratio detector is equal to A_c/σ_n. Using the obtained formulas, it is possible to find the relationship/ratio between the signal and the interference in the systems with modulation and without it.

With the strong signals

$$\frac{A_{cs}}{\sigma_{\Delta\theta}}:\frac{A_0}{\sigma_s}=\sqrt{\frac{\Delta\omega_s}{2\Delta\Omega_{s\phi}}}M. \qquad (3.6.8)$$

The depth of modulation M can have a value, close to one. Let us assume that the band is limited only to the spectrum of communication/report, which has highest frequency Ω_{cs}, then

$$\Delta\omega_s \approx 2\Omega_{cs} \quad \text{and} \quad \Delta\Omega_{s\phi}=\Omega_{cs}.$$

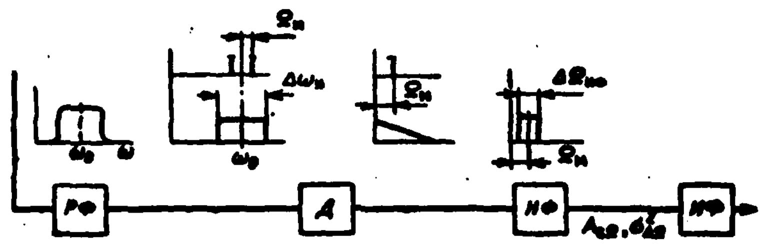

Fig. 3.6.3. The block diagram of the receiving and measuring device/equipment, in which is used the modulated signal: РФ - radio-frequency filter; Д - detector; НФ - low pass filter; ИФ - meter of phase.

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Accepting that $\Delta\omega_{н}=2\Omega_{в}$, we consider that the contraction of band on the radio frequency is possible. During more precise calculations it is necessary to consider the effect of narrow band on the radio frequency to the instrument/tool accuracy. It is obvious that if we assume M=1, then

$$\frac{A_{c\Omega}}{\sigma_{\Delta\Omega}}=\frac{A_c}{\sigma_n}. \qquad (3.6.9)$$

Thus, with the strong signals the use of modulation does not change relationship/ratio signal/noise. However, it is necessary to have in mind that in this case scale is changed Ω/ω_0 times and somewhat are reduced the instrument errors, expressed in the angular values. Therefore the accuracy of ranging or rates is changed, deteriorating upon transfer to the systems with modulation.

With the weak signals

$$\frac{A_{c\text{д}}}{\sigma_{\text{шд}}}:\frac{A_c}{\sigma_\text{ш}}=\frac{A_c}{\sigma_\text{ш}}\sqrt{\frac{\Delta\omega_\text{ш}}{\Delta\Omega_{\text{эф}}}}\,0{,}3\,M\approx 0{,}5\left(\frac{A_c}{\sigma_\text{ш}}\right). \quad (3.6.10)$$

Thus, with the weak signals of system with modulation they give considerable loss according to the relation signal/noise.

§ 3.7. Two-dimensional distribution function, autocorrelation function and the energy spectrum of phase. for obtaining the two-dimensional function of phase distribution it is necessary to carry out twofold integration for the amplitude of the four-dimensional function of phase distribution and amplitude

$$w_2(\varphi_{y1},\ \varphi_{y2},\ \tau)=$$
$$=\int_0^\infty\int_0^\infty w_4(A_{y1},\ A_{y2},\ \varphi_{y1},\ \varphi_{y2},\ \tau)\,dA_{y1}\,dA_{y2}. \quad (3.7.1)$$

The execution of integration is connected with the bulky mathematical conversions which we lower (see [2.1]).

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As a result of integration is obtained the expression

$$w_2(\varphi_{y1}, \varphi_{y2}, \tau) = \frac{1 - R_0^2(\tau)}{4\pi^2} e^{-\frac{A_0^2}{\sigma_n^2 [1 + R_0(\tau)]}} \times$$

$$\times e^{-\frac{A_0 \sqrt{1 - R_0(\tau)}}{\sigma_n \sqrt{1 + R_0(\tau)}} \cos \varphi_{y1}} e^{-\frac{A_0 \sqrt{1 - R_0(\tau)}}{\sigma_n \sqrt{1 + R_0(\tau)}} \cos \varphi_{y2}} \times$$

$$\times \sum_{m=0}^{\infty} 2R_0(\tau) \cos(\varphi_{y1} - \varphi_{y2})^m \frac{1}{m!} \times$$

$$\times \left\{ \left[\frac{A_0}{\sqrt{2}\, \sigma_n} \frac{\sqrt{1 - R_0(\tau)}}{\sqrt{1 + R_0(\tau)}} \cos \varphi_{y1} \times \right.\right.$$

$$\times \Gamma\left(\frac{m+3}{2}\right) {}_1F_1\left(-\frac{m}{2}, \frac{3}{2}, \frac{A_0 \sqrt{1 - R_0(\tau)}}{\sigma_n \sqrt{1 + R_0(\tau)}} \cos \varphi_{y1}\right)^2 \Big] +$$

$$+ \Gamma\left(\frac{m}{2} + 1\right) {}_1F_1\left(-\frac{m+1}{2}, \frac{1}{2}, -\frac{A_0 \sqrt{1 - R_0(\tau)}}{2\sigma_n \sqrt{1 + R_0(\tau)}} \cos \varphi_{y1}\right) \Big\} \times$$

$$\times \left\{ \left[\frac{A_0}{\sqrt{2}\, \sigma_n} \frac{\sqrt{1 - R_0(\tau)}}{\sqrt{1 + R_0(\tau)}} \cos \varphi_{y2}\, \Gamma\left(\frac{m+3}{2}\right) {}_1F_1\left(-\frac{m}{2}, \frac{3}{2}, -\right.\right.\right.$$

$$- \frac{A_0}{2\sigma_n} \frac{\sqrt{1 - R_0(\tau)}}{\sqrt{1 + R_0(\tau)}} \cos \varphi_{y2}\Big)^2 \Big] + \Big[\Gamma\left(\frac{m}{2} + 1\right) {}_1F_1 \times$$

$$\times \left(-\frac{m+1}{2}, \frac{1}{2}, -\frac{A_0}{2\sigma_n} \frac{\sqrt{1 - R_0(\tau)}}{\sqrt{1 + R_0(\tau)}} \cos \varphi_{y2}\right)^2 \Big] \Big\}, \quad (3.7.2)$$

where ${}_1F_1$ - the hypergeometric function:

$$ {}_1F_1(\alpha, \gamma, x) = 1 + \frac{\alpha}{\gamma} x + \frac{\alpha(\alpha+1)}{\gamma(\gamma+1)} \frac{x^2}{2} +$$

$$+ \frac{\alpha(\alpha+1)(\alpha+2) x^3}{\gamma(\gamma+1)(\gamma+2) 3!} + \ldots$$

Resulting expression is characterized by large unwieldiness and is not useful for the practical calculations.

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Expression for the two-dimensional function of phase distribution has high value, since must make it possible to obtain the correlation function of phase and the spectrum of the fluctuations of phase in the mixture of signal and interference. It is expedient to separately and approximately consider the cases of very weak and strong signals.

With the weak signal when its presence still little affects phase distribution, it is possible to expect that the spectrum of the fluctuation of phase will be close to that which occurs with one interference. This spectrum was obtained earlier. Let us recall the special feature/peculiarity of this spectrum. In essence the fluctuations of phase are concentrated in the frequency region from 0 to $\Delta\omega_m/2$, but there are high-frequency fluctuations in portion of which fall about 15% of "total power". This part of the fluctuations has high value for the understanding of some complicated processes, connected with the frequency of fluctuations or derived phase of fluctuations; however, during the analysis of the effect of the fluctuations of phase on phasemeter with the limited passband they will not have an essential effect, since usually they are located out of its passband.

For the high signal level the distribution functions and the spectrum of fluctuations must radically be changed. For the purpose of obtaining the relationships/ratios, which characterize the correlation function and the spectrum of the fluctuations of phase, let us derive approximation formula $w_2(\varphi_{y1}, \varphi_{y2}, \tau)$ for the strong signal. For this let us turn to expressions (3.5.2), (3.7.1) let us lead in them the series/row of simplifications.

Let us assume that the reading of phase is conducted relative to the phase of signal. Then it is possible to count $\varphi_c=0$, and

$$w_2(\varphi_{y1}, \varphi_{y2}, \tau)=\frac{1}{(2\pi\sigma_n^2)^2[1-R_0^2(\tau)]}\int_0^\infty\int_0^\infty A_{y1}, A_{y2}\times$$
$$\times\exp\left\{-\frac{1}{2\sigma_n^2[1-R_0^2(\tau)]}[(A_{y1}\cos\varphi_{y1}-A_c)^2+\right.$$
$$+A_{y1}^2\sin^2\varphi_{y1}+(A_{y2}\cos\varphi_{y2}-A_c)^2+A_{y2}^2\sin^2\varphi_{y2}-$$
$$-2R_0(\tau)((A_{y1}\cos\varphi_{y1}-A_c)(A_{y2}\cos\varphi_{y2}-A_c)+$$
$$\left.+A_{y1}\sin\varphi_{y1}A_{y2}\sin\varphi_{y2})]\right\}dA_{y1}\,dA_{y2}. \qquad (3.7.3)$$

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In the presence of signal and when $A_c>\sigma_n$ the probability of the large divergences of the phase of mixture from the phase of signal, as this follows from the one-dimensional function of phase distribution it is insignificant.

Then it is possible to assume:

$$A_{y1}\cos\varphi_{y1}\simeq A_{y1},$$
$$A_{y2}\cos\varphi_{y2}\simeq A_{y2},$$
$$A_{y1}\sin\varphi_{y1}\simeq A_{y1}\varphi_{y1}, \qquad (3.7.4)$$
$$A_{y2}\sin\varphi_{y2}\simeq A_{y2}\varphi_{y2},$$

expression will take the form

$$w_2(\varphi_{y1}, \varphi_{y2}, \tau) = \frac{1}{(2\pi\sigma_n^2)^2 [1 - R_0^2(\tau)]} \int_0^\infty \int_0^\infty A_{y1} A_{y2} \times$$

$$\times \exp\left\{-\frac{1}{2\sigma_n^2 [1 - R_0^2(\tau)]} [(A_{y1} - A_c)^2 + A_{y1}\varphi_{y1} + \right.$$

$$+ (A_{y2} - A_c)^2 + A_{y2}\varphi_{y2} - 2R_0(\tau) [(A_{y1} - A_c)(A_{y2} - A_c) +$$

$$\left. + A_{y1} A_{y2} \varphi_{y1} \varphi_{y2}]] \right\} dA_{y1}\, dA_{y2}$$

or

$$w_2(\varphi_{y1}, \varphi_{y2}, \tau) = \frac{1}{(2\pi\sigma_n^2)^2 [1 - R_0^2(\tau)]} \int_0^\infty \int_0^\infty A_{y1} A_{y2} \times$$

$$\times e^{-\frac{(A_{y1} - A_c)^2 + A_c^2 \varphi_{y1}^2}{2\sigma_n^2 [1 - R_0^2(\tau)]}} e^{-\frac{(A_{y2} - A_c)^2 + A_c^2 \varphi_{y2}^2}{2\sigma_n^2 [1 - R_0^2(\tau)]}} \times$$

$$\times e^{-2R_0(\tau) A_c^2 \varphi_{y1} \varphi_{y2}} e^{-2R_0(\tau)(A_{y1} - A_c)(A_{y2} - A_c)} dA_{y1} dA_{y2}. \quad (3.7.5)$$

In those terms in which A_{y1} and A_{y2} participate in the form of factors, in the first approximation, it is possible to suppose that $A_{y1} = A_c$ and $A_{y2} = A_c$, since the probability of the large divergences of amplitude from A_c is small.

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Then

$$w_2(\varphi_{y1}, \varphi_{y2}, \tau) = \frac{A_c}{2\pi\sigma_n^2 \sqrt{[1-R_0^2(\tau)]}} \times$$

$$\times e^{-\frac{A_c^2(\varphi_{y1}+\varphi_{y2}-2R_0(\tau)(\varphi_{y1}\varphi_{y2})}{2\sigma_n^2[1-R_0^2(\tau)]}} \times$$

$$\times \frac{1}{2\pi\sigma_n^2 \sqrt{1-R_0^2(\tau)}} \times$$

$$\times \int_0^\infty \int_0^\infty e^{-\frac{(A_{y1}-A_c)^2+(A_{y2}-A_c)^2-2R_0(\tau)(A_{y1}-A_c)(A_{y2}-A_c)}{2\sigma_n^2[1-R_0^2(\tau)]}} \times$$

$$\times dA_{y1}\, dA_{y2}. \quad (3.7.6)$$

Since the double integral expresses total probability, it is possible to place it equal to one.

Then

$$w_2(\varphi_{y1}, \varphi_{y2}, \tau) = \frac{A_c^2}{2\pi\sigma_n^2 \sqrt{1-R_0^2(\tau)}} \times$$

$$\times e^{-\frac{A_c^2(\varphi_{y1}^2+\varphi_{y2}^2-2R_0(\tau)\varphi_{y1}\varphi_{y2})}{2\sigma_n^2[1-R_0^2(\tau)]}}. \quad (3.7.7)$$

Earlier was introduced designation $\frac{\sigma_n^2}{A_c^2} = \sigma_\varphi^2$, in this case

$$w_2(\varphi_{y1}, \varphi_{y2}, \tau) = \frac{1}{2\pi\sigma_\varphi^2 \sqrt{1-R_0^2(\tau)}} \times$$

$$\times e^{-\frac{\varphi_{y1}^2+\varphi_{y2}^2-2R_0(\tau)\varphi_{y1}\varphi_{y2}}{2\sigma_\varphi^2}} \quad (3.7.8)$$

Two-dimensional distribution for the phase of the mixture of signal and interference corresponds to the distribution of the instantaneous values for the mixture under the condition for the transfer of spectrum ($\omega_0=0$) into the region of low frequencies; $R_0(\tau)$ - low-frequency factor in the coefficient of correlation of narrow-band random process.

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In this case the dispersion of the fluctuations of phase is determined by the relationship/ratio

$$\sigma_\varphi^2=\frac{\sigma_n^2}{A_c^2}.$$

In connection with this there is no necessity specially to compute $B_\varphi(\tau)$ and $G_\varphi(\omega)$, since they correspond $R_0(\tau)$ and $G*(\omega)$, taking into account factor σ_n^2/A_c^2 for the fluctuations of phase.

After using these results, it is possible to construct the energy spectra of the fluctuation of phase with the strong signal. For example, for the mixture of signal and interference with the uniform spectrum with the band from $\omega_0-\frac{\Delta\omega_n}{2}$ to $\omega_0+\frac{\Delta\omega_n}{2}$ and by dispersion σ_n^2 we will obtain the uniform spectrum of the fluctuations of phase with the band $\Delta\omega_n/2$, by dispersion σ_n^2/A_c^2 and a spectral density of

$$G_\varphi(\omega)=\frac{\sigma_\varphi^2}{\Delta f_n/2}=2\frac{\sigma_n^2}{A_c^2}\frac{1}{\Delta f_n}.$$

$$G_\varphi(\omega)=0 \text{ with } \omega>\frac{\Delta\omega_n}{2}.$$

Fig. 3.7.1 gives the spectrum of the fluctuations of the phase of signal when $\frac{\sigma_n}{A_c}=\frac{1}{\sqrt{3}}$.

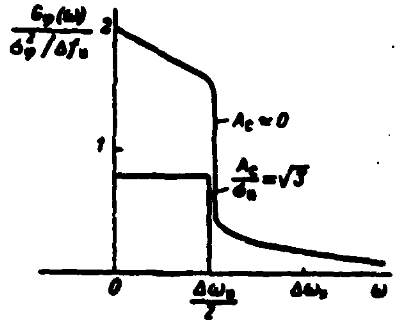

Fig. 3.7.1. The energy spectrum of phase.

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There for comparison is given the spectrum of the fluctuations of the phase of one interference. As can be seen from figure, with considerable changes in the relation the signal/noise the form of the spectrum is changed little, if we disregard/neglect the high-frequency part of the spectrum of fluctuations, which appears with the weak signal or in the absence of signal.

With change A_c/σ_n is changed the dispersion of fluctuations according to the law which was established/installed earlier. Since the width of the spectrum of fluctuations remains in the first approximation, of constant/invariable, power density (energy spectrum), including at frequencies, close to zero, can be found from the relationship/ratio

$$G_\varphi(0) = 2\frac{\sigma_\varphi^2}{\Delta f_n}.$$

§ 3.8. Suppression of weak noise signal with the demodulation of phase. In many phase systems the information is laid during the phase of high-frequency oscillation. Then the extraction of useful information from the signal, i.e., the demodulation of phase, is reduced to the measurement of the phase of the mixture of signal and interference. For an increase in the accuracy of the measurement of phase in the presence of interference it is possible to narrow band before and after demodulator-phasemeter. In certain cases it is desirable to realize reworking of the results of phase measurements. The simplest form of this working/treatment is the contraction of band after phasemeter, which simply can be realized by the start of inertia component/link or low-pass filter.

With the strong signal when $\sigma_n < A_c$,

$$\sigma_\varphi^2 = \frac{\sigma_n^2}{A_c^2} = \frac{N_0 \Delta f_n}{A_c^2}, \qquad (3.8.1)$$

where Δf_n — complete band in the high frequency.

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If filter after phasemeter passes narrow band ΔF, then

$$\sigma_{\varphi \Delta F}^2 = G_\varphi(0)\,\Delta F,$$

but

$$G_\varphi(0) = 2\frac{\sigma_\varphi^2}{\Delta f_n} = 2\frac{N_0}{A_c^2}$$

and for the ideal phasemeter

$$\sigma^2_{\varphi\Delta F} = 2\frac{N_0}{A_c^2}\Delta F = \frac{\sigma_n^2}{A_c^2}\frac{\Delta F}{\Delta f_n}2.$$

Narrowing the filter pass band, connected after phasemeter, it is possible with given one σ_n^2/A_c^2 to attain very low values $\sigma^2_{\varphi\Delta F}$. Let us consider in more detail the work of system with the narrow-band filter, connected after phasemeter in small ratios of signal to the interference.

With the weak signal phase distribution becomes uniform, the dispersion of fluctuation of phase approaches $\pi^2/3$, and the energy spectrum of fluctuations in the first approximation, can be considered uniform with a power density of

$$G_\varphi(0) = 2\frac{\sigma_\varphi^2}{\Delta f_n},$$

where Δf_n— band of interference to the phasemeter.

If on this phasemeter functions one interference, then during the contraction of band ΔF would seem must occur the decrease of the dispersion of the fluctuation of phase in accordance with the relationship/ratio

$$\sigma^2_{\varphi\Delta F} = 2\frac{\sigma_\varphi^2}{\Delta f_n}\Delta F. \qquad (3.8.2)$$

However, very average/mean value of the phase of interference is indefinite or any value of initial phase is equiprobable. Thus, during the contraction of band after phasemeter the rapid fluctuations of phase cease to affect result, the value of reading can be almost constant, but very value of phase can be any. Consequently, in this case expression (3.8.2) becomes meaningless and the work of phasemeter with the filter connected at its output on the high interference level requires special examination.

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In the ratio of interference to signal $\frac{A_0}{\sigma_n} > 1{,}5 \div 2$ the distribution function takes the form, close to the normal law with the average/mean value, to proper phase of signal, and by dispersion $\sigma_\varphi^2 = \frac{\sigma_n^2}{A_0^2}$ with the in effect zero probability of the divergences, close to $\pm\pi$. For the confirmation of this Fig. 3.8.1 gives the graph/diagram of the dependence

$$w(\varphi = \pm\pi) = \frac{1}{2\pi} e^{-\frac{a_c^2}{2}} - \frac{a_c}{\sqrt{2\pi}} [1 - F(a_c)].$$

Consequently, when $a_c = 1{,}5 \div 2$ it is possible to consider that the average/mean value of the phase of mixture will correspond precisely to the phase of signal, the probability of the divergences, close to $\pm\pi$, is negligibly small and the contraction of band after phasemeter will reduce the fluctuations of phase.

When $a_c < 1{,}5 \div 2$ the probability of the large divergences of phase becomes noticeable and the distribution function differs from normal.

In order to explain the effect of filtration after phasemeter with the weak signal, let us use the following method. If interference has high value, then it is possible to present it in the form of the sum of two components: one - п_1, the giving fluctuation phase of mixture relative to the phase of signal, and two - п_2, that gives equiprobable phase distribution then

$$\sigma_{\text{п}}^2 = \sigma_{\text{п}1}^2 + \sigma_{\text{п}2}^2 .$$

and

$$\sigma_{\text{п}2} = \sqrt{\sigma_{\text{п}}^2 - \sigma_{\text{п}1}^2} . \quad (3.8.3)$$

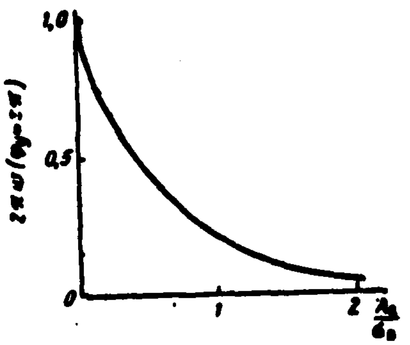

Fig. 3.8.1. Probability density with $\Delta\varphi = \pm\pi$.

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The part of the interference, which causes the fluctuations of the phase of signal, not exceeding $\pm\pi$ and undergoing averaging, is located in a certain proportion with A_c.

For it is correct

$$\frac{\sigma_{n1}}{A_c} < 0,5 \div 0,75 \quad \text{(see Fig. 3.8.1).} \qquad (3.8.4)$$

Let us designate the permissible relation by symbol k

$$\sigma_{n1}^2 = k^2 A_c^2, \qquad (3.8.5)$$

then

$$\sigma_{n2}^2 = \sigma_n^2 - k^2 A_c^2 = \sigma_n^2 \left(1 - k^2 \frac{A_c^2}{\sigma_n^2}\right). \qquad (3.8.6)$$

component n_2 can have any initial phase, and filtration after phasemeter cannot influence the distribution function and the dispersion of the fluctuations of phase, caused by this component. With any band it will remain equal to $\pi^2/3$. Thus, the fluctuations of phase at the output of the filter, connected after phasemeter, will be determined by two components: by the component, called n_1, with the dispersion

$$\sigma^2_{\varphi 1} = 2\frac{\sigma^2_{n1}}{A_c^2}\frac{1}{\Delta\omega_n}\Delta\Omega \approx \frac{\Delta\Omega}{\Delta\omega_n} \qquad (3.8.7)$$

and the component, called n_2, with the dispersion, which does not depend on the band.

Let us determine the dispersion of the divergences of phase, called component n_2. Having the uniform distribution function, this component will cause the divergences of the phase of signal, determined by relationship between A_c and n_2.

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It is possible to assume that the dispersion of these divergences to be determined from (3.4.5) depending on the relationship/ratio between A_c and σ_{n2} and with the substitution into formula A_c/σ_{n2}

$$\sigma^2_{\varphi 2} = \frac{\sigma^2_{n2}}{A_c^2}$$

or

$$\sigma^2_{\varphi_2} = \frac{\pi^2}{3} - \sqrt{2\pi}\,\frac{A_c}{\sigma_{n2}}. \qquad (3.8.8)$$

At the output of filter the resulting dispersion of the divergences of phase from a precise value, determined by the phase of signal, will be equal to

$$\sigma^2_{\varphi\Phi} = \sigma^2_{\varphi 1} + \sigma^2_{\varphi 2}.$$

Using formulas (3.8.6), (3.8.7), (3.8.8) and (3.8.9), it is possible to find $\sigma^2_{\varphi\Phi}$ with any $\frac{A_c}{\sigma_n}$, $\Delta\Omega$ and $\Delta\omega_{\varepsilon}$, after assigning k (usually it is possible to take k=0.7). In order to come to light/detect/expose the basic law, characteristic to filtration after phasemeter, let us consider the limiting case of the very narrow-band filter when $\sigma^2_{\varphi1}$ it becomes negligible value in comparison with $\sigma^2_{\varphi2}$. Fig. 3.8.2 gives graphs obtained at a rate of the given above formulas, for $\Delta\Omega \to 0$; figure gives the following designations: $\sigma_{\varphi\text{ap}}$ — root-mean-square phase error with the maximally narrow band of filter; σ_{φ}— root-mean-square significance of a deviation of the phase of the mixture of signal A_c and interference with dispersion σ^2_n.

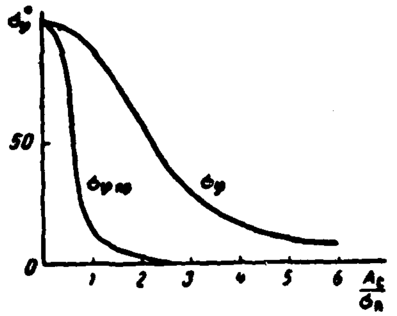

Fig. 3.8.2. Maximum fluctuation of phase.

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From the results it follows that by the contraction of passband after phasemeter it is possible to improve the accuracy of phase measurements only when $A_c > (0{,}5 \div 1)\sigma_n$. When $A_c < (0{,}5 \div 1)\sigma_n$ the contraction of passband after phasemeter cannot give accurate results in the measurement of phase. Thus, with the demodulation of phase, just as with the demodulation of amplitude, there is a threshold relation A_c/σ_n, with which it occurs the suppression of weak noise signal, and the subsequent filtration cannot give such results as provide the contraction of the passband of demodulator - phasemeter.

§ 3.9. Function of the distribution of "zero" mixture of signal and interference. In the phase systems for the reading of phase sometimes is used the time interval between "zeros" of the oscillation of mixture and supporting/reference oscillation. This method of measuring the phase makes it possible to use digital computer technology in the phasemeters, since time interval is conveniently evaluated according to a number of count pulses placed in it. In some diagrams are used the limiters. In this case basic information about the signal proves to be concentrated at the moments of the transition of the process through zero. On these and series/row of other reasons, of known interest is the function of the distribution of "zeros" and its connection/communication with the function of phase distribution. The moment/torque of transition through "zero" can be characterized by the interval of time τ_0 between the moments/torques of transition through "zero" of the stress/voltage of mixture and reference voltage as this shown on Fig. 3.9.1. Then the function of the distribution of "zeros" will be characterized by the function of timing τ_0. For the solution of this problem let us note that the transitional probability through level y_0 at some moment/torque t with the positive derivative can be expressed through the joint probability density of random process and its derivative [2.4, 3.4, 3.5, 3.6].

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$$p\left(y_0-\frac{\Delta y}{2}<y<y_0+\frac{\Delta y}{2},\ \frac{dy}{dt}=\dot{y}>0,\ t\right)=\int_0^{\infty}\int_{y_0-\Delta y/2}^{y_0+\Delta y/2} w_2(y,\dot{y},t)\,dy\,d\dot{y}, \qquad (3.9.1)$$

where y_0 - level the moment/torque of intersection of which is counted off. Subsequently this value let us place equal to zero, i.e., we will count off "zero", but for the generality let us thus far leave designation y_0. The interval Δy in value should be small, then internalizations integral is computed simply, that as in this case

$$\int_{y_0-\Delta y/2}^{y_0+\Delta y/2} w_2(y, \dot{y}, t) =$$
$$= w_2(y_0, \dot{y}, t)\,\Delta y = \dot{y} w_2(y_0, y, t)\,\Delta t \qquad (3.9.2)$$

and

$$p\left(y_0 - \frac{\Delta y}{2} < y < y_0 + \frac{\Delta y}{2},\ \dot{y} > 0,\ t\right) =$$
$$= \Delta t \int_0^\infty \dot{y} w_2(y_0, \dot{y}, t)\, d\dot{y}. \qquad (3.9.3)$$

But the transitional probability through level y_0 at moment/torque $t-t+\Delta t$ can be expressed through the probability density and the interval Δt, i.e.,

$$p(y_0 - \Delta y < y < y_0,\ \dot{y} > 0, t) =$$
$$= w(y_0, t)\,\Delta t = \Delta t \int_0^\infty \dot{y} w_2(y_0, \dot{y}, t)\, d\dot{y}, \qquad (3.9.4)$$

whence

$$w(t, y_0) = \int_0^\infty \dot{y} w_2(y_0, \dot{y}, t)\, dy. \qquad (3.9.5)$$

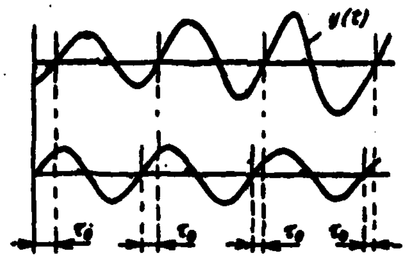

Fig. 3.9.1. The time interval between the moments/torques of transition through zero.

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Consequently, for obtaining the unknown distribution function it is necessary to find joint probability density $w_2(y, \dot{y}, t)$ and then to fulfill substitutions and integration. Interference has normal distribution with dispersion σ_n^2, then the function of the distribution of the mixture of interference with signal c(t) takes the form

$$w(y, t) = \frac{1}{\sqrt{2\pi}\,\sigma_n} e^{-\frac{[y-c(t)]^2}{2\sigma_n^2}}.$$

The derivative of interference also has normal distribution with dispersion $\sigma_{\dot{n}}^2$, then the function of the distribution of the derived mixture of interference with the signal will be

$$w(\dot{y}, t) = \frac{1}{\sqrt{2\pi}\,\sigma_{\dot{n}}} e^{-\frac{\left[\dot{y}-\frac{dc(t)}{dt}\right]^2}{2\sigma_{\dot{n}}^2}}.$$

Since the values of interference and its derivative at the coinciding moments of time are not dependent, the combined distribution function can be registered in the form

$$w(y,\ \dot{y},\ t)=\frac{1}{2\pi\sigma_n\sigma_{\dot{n}}}\times e^{-\left\{\frac{[y-c(t)]^2}{2\sigma_n^2}+\frac{\left[\dot{y}-\frac{dc(t)}{dt}\right]^2}{2\sigma_{\dot{n}}^2}\right\}}. \quad (3.9.6)$$

For the substitution in formula (3.9.5) it is necessary to take $y=y_0$ and the moment/torque of time $t=\tau$, then

$$w(\tau,\ y_0)=\int_0^\infty \frac{\dot{y}}{2\pi\sigma_n\sigma_{\dot{n}}}\times$$
$$\times e^{-\left\{\frac{[y_0-c(\tau)]^2}{2\sigma_n^2}+\frac{[\dot{y}-\dot{c}(\tau)]^2}{2\sigma_{\dot{n}}^2}\right\}}d\dot{y}, \quad (3.9.7)$$

τ - time, counted off from the moment/torque of the transition of the pure/clean signal through y_0.

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where

$$c(\tau)=c(t)\ \ \text{при}^{(1)}\ t=\tau;$$
$$\dot{c}(\tau)=\frac{dc(t)}{dt}\ \ \text{при}^{(1)}\ t=\tau.$$

Key: (1). with.

Let us take out as the integral sign the terms, which do not depend on $\dot{y}$.

$$w(\tau, y_0)=\frac{1}{2\pi\sigma_n\sigma_{\dot{n}}}e^{-\left\{\frac{[y_0-c(\tau)]^2}{2\sigma_n^2}+\frac{\dot{c}^2(\tau)}{2\sigma_{\dot{n}}^2}\right\}}\times$$

$$\times\int_0^\infty \dot{y}e^{-\frac{\dot{y}^2-2\dot{y}\dot{c}(\tau)}{2\sigma_{\dot{n}}^2}}d\dot{y}. \qquad (3.9.8)$$

Integral, entering expression (3.9.8), is calculated in [3.3]

$$\int_0^\infty \dot{y}e^{-\frac{\dot{y}^2-2\dot{y}\dot{c}(\tau)}{2\sigma_{\dot{n}}^2}}d\dot{y}=$$

$$=\sigma_{\dot{n}}^2\left[\Gamma(1)\,{}_1F_1\left(1,\frac{1}{2},\frac{\dot{c}^2(\tau)}{2\sigma_{\dot{n}}^2}\right)+\right.$$

$$\left.+\frac{\dot{c}(\tau)}{\sigma_{\dot{n}}^2}\sqrt{2\sigma_{\dot{n}}^2}\,\Gamma\left(\frac{3}{2}\right){}_1F_1\left(\frac{3}{2},\frac{3}{2},\frac{\dot{c}^2(\tau)}{2\sigma_{\dot{n}}^2}\right)\right]. \qquad (3.9.9)$$

It is known that:

$$\Gamma(1)=1,\quad \Gamma\left(\frac{3}{2}\right)=\frac{\sqrt{\pi}}{2},$$

$${}_1F_1\left(\frac{3}{2},\frac{3}{2},\frac{\dot{c}^2(\tau)}{2\sigma_{\dot{n}}^2}\right)=e^{\frac{\dot{c}^2(\tau)}{2\sigma_{\dot{n}}^2}},$$

$${}_1F_1\left(1,\frac{1}{2},\frac{\dot{c}^2(\tau)}{2\sigma_{\dot{n}}^2}\right)=$$

$$=1+\sqrt{\pi}\,e^{\frac{\dot{c}^2(\tau)}{2\sigma_{\dot{n}}^2}}\frac{\dot{c}(\tau)}{\sqrt{2}\sigma_{\dot{n}}}\frac{2}{\sqrt{\pi}}\int_0^{\frac{\dot{c}(\tau)}{\sqrt{2}\sigma_{\dot{n}}}}e^{-t^2}dt,$$

$$\frac{1}{\sqrt{\pi}}\int_0^{\frac{\dot{c}(\tau)}{\sqrt{2}\sigma_{\dot{n}}}}e^{-t^2}dt=\frac{1}{\sqrt{2\pi}}\int_0^{\frac{\dot{c}(\tau)}{\sigma_{\dot{n}}}}e^{-\frac{t_1^2}{2}}dt_1=F\left(\frac{\dot{c}(\tau)}{2\sigma_{\dot{n}}}\right)-0,5.$$

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After substituting these expressions and after producing conversions, we will obtain

$$\int_0^\infty y e^{-\frac{y^2-2y\dot{c}(\tau)}{2\sigma_{\dot{n}}^2}}\,dy=$$

$$=\sigma_{\dot{n}}^2\left\{1+e^{\frac{\dot{c}^2(\tau)}{2\sigma_{\dot{n}}^2}}\frac{\dot{c}(\tau)}{\sigma_{\dot{n}}}\sqrt{2\pi}\,F\left[\frac{\dot{c}(\tau)}{\sigma_{\dot{n}}}\right]\right\}. \qquad (3.9.10)$$

After substituting in (3.9.8) and after assuming $y_0=0$, i.e., after switching over to the function of distribution of zeros, we will obtain

$$w(\tau_0)=\frac{\sigma_{\dot{n}}}{2\pi\sigma_n}e^{-\left[\frac{c^2(\tau_0)}{2\sigma_n^2}+\frac{\dot{c}^2(\tau_0)}{2\sigma_{\dot{n}}^2}\right]}\times$$

$$\times\left\{1+e^{\frac{\dot{c}^2(\tau_0)}{2\sigma_{\dot{n}}^2}}\frac{\dot{c}^2(\tau_0)}{\sigma_{\dot{n}}^2}\sqrt{2\pi}\,F\left[\frac{\dot{c}(\tau_0)}{\sigma_{\dot{n}}}\right]\right\}=$$

$$=\frac{\sigma_{\dot{n}}}{\sigma_n}\left\{\frac{1}{2\pi}e^{-\left[\frac{c^2(\tau_0)}{2\sigma_n^2}+\frac{\dot{c}^2(\tau_0)}{2\sigma_{\dot{n}}^2}\right]}+\right.$$

$$\left.+\frac{1}{\sqrt{2\pi}}F\left[\frac{\dot{c}(\tau_0)}{\sigma_{\dot{n}}}\right]\frac{\dot{c}(\tau_0)}{2\sigma_{\dot{n}}}e^{-\frac{c^2(\tau_0)}{2\sigma_n^2}}\right\}. \qquad (3.9.11)$$

For the determination of the function of the distribution of divergence of zeros from those moments/torques which are observed with the signal without the interferences, it is convenient to take signal in the form of harmonic oscillation with intersection of zero with $\tau_0=0$.

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Then

$$c(t) = A_c \sin \omega_0 t, \quad c(\tau) = A_c \sin \omega_0 \tau,$$
$$\dot{c}(t) = \omega_0 A_c \cos \omega_0 (t), \quad \dot{c}(\tau) = \omega_0 A_c \cos \omega_0 \tau.$$

It is known that

$$\sigma_{\dot{c}}^2 = \sigma_n^2 \omega_1^2, \quad \omega_1 = \omega_0 + \delta\omega.$$

After substituting in (3.9.11) and keeping in mind $\frac{\omega_0}{2\pi} = \frac{1}{T_0}$, we will obtain

$$w(\tau_0) = \frac{\omega_1}{\omega_0} \frac{1}{T_0} \times$$
$$\times e^{-\left(\frac{a_c^2}{2} \sin^2 \omega_0 \tau_0 + \frac{\omega_0^2}{\omega_1^2} \frac{a_c}{2} \cos^2 \omega_0 \tau_0\right)} +$$
$$+ \frac{\sqrt{2\pi}}{T_0} a_c \cos \omega_0 \tau_0 F\left(\frac{\omega_0}{\omega_1} a_c \cos \omega_0 \tau_0\right) \times$$
$$\times e^{-\frac{a_c^2}{2} \sin \omega_0 \tau_0}. \qquad (3.9.12)$$

For the comparison with the function of phase distribution it is convenient to switch over to the reading of time interval τ_0 in angular units $\varphi_{y0} = \omega_0 \tau_0$.

Then

$$w(\varphi_{y0}) = \frac{\omega_1}{\omega_0} \frac{1}{2\pi} \times$$
$$\times e^{-\left(\frac{a_c^2}{2} \sin^2 \varphi_{y0} + \frac{\omega_0^2}{\omega_1^2} \frac{a_c^2}{2} \cos \varphi_{y0}\right)} +$$
$$+ \frac{1}{\sqrt{2\pi}} a_c \cos \varphi_{y0} F\left(\frac{\omega_0}{\omega_1} a_c \cos \varphi_{y0}\right) e^{-\frac{a_c^2}{2} \sin^2 \varphi_{y0}}. \qquad (3.9.13)$$

From (3.9.13) it follows that the functions of distribution of zeros and function of phase distribution [see (3.5.2)] according to their form although are analogous, they differ from each other.

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Difference in essence is determined by the divergence of factor ω_1/ω_0 from one.

In the particular case (very narrow-band process) $\omega_1/\omega_0=1$ and

$$w(\varphi_{\mu 0})=\frac{1}{2\pi}e^{-\frac{a_c^2}{2}}+$$

$$+\frac{1}{\sqrt{2\pi}}a_c\cos\varphi_{\mu 0}F(a_c\cos\varphi_{\mu 0})e^{-\frac{a_c^2}{2}\sin^2\varphi_{\mu 0}},$$

which completely coincides with (3.4.2). Consequently, for the very narrow-band processes in any ratio of signal to the interference with the function of distribution of zeros and phases they coincide. Let us consider the cases of strong and weak signals.

When $a_c \gg 1$ - strong signal

$$w(\varphi_{\mu 0})\approx\frac{a_c}{\sqrt{2\pi}}e^{-\frac{a_c^2\varphi_{\mu 0}^2}{2}}.$$

Distribution function zero normal for any width of interference spectrum with the same dispersion, as for the phase

$$\sigma^2_{\varphi_0} = \frac{\sigma^2_\text{ш}}{A^2_c}.$$

When $a_c \ll 1$ - weak signal

$$w(\varphi_{y_0}) = \frac{\omega_1}{\omega_0}\frac{1}{2\pi} + \frac{a_c \cos\varphi_{y0}}{2\sqrt{2\pi}} =$$

$$= \frac{\omega_1}{\omega_0}\frac{1}{2\pi}\left(1 + \frac{\omega_0}{\omega_1} a_c \sqrt{\frac{\pi}{2}} \cos\varphi_{y0}\right).$$

The function of distribution of zeros is close to the function of phase distribution [see (3.4.7)], but since with the broad band of interferences ω_1 is noticeably more ω_0, it is more smoothed, i.e., with the less expressed maximum.

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The obtained results have high value. They show that the fluctuations of the position of "zeros" under the action of interferences are the same as the fluctuation of phase. With the signal with the changing phase the obtained results are valid not for zeros, but for their divergence from the positions, determined by pure/clean signal. In the first approximation, it is possible to assume that the dispersion of divergence of zeros, energy spectrum and others statistical of characteristic are close to the appropriate characteristics of phase. Consequently, the passage of the mixture of signal and interference through the limiter does not affect the information, placed during the phase of signal, and is not changed interference effect on the result of measurements. Thus, for the phase systems and for the systems, in which the information about the special features/peculiarities of signal is laid during its phase, for example the phase-keyed signal, limiter appears as linear device/equipment.

§3.10. Functions of phase distribution with determinate interference, which has random phase. In the phase systems is encountered such case when, besides the useful signal whose phase carries information, into the point of reception/procedure comes the signal of the same frequency, but with another initial phase. The phase of the resulting oscillation is changed and information, laying during the phase, is distorted.

Let us consider the case, when useful and that mixes signals are the harmonic oscillations:

$c(t)=A_c\cos(\omega_0 t+\varphi_c)$ - useful signal;

$c_{отр}(t)=A_{отр}(\omega_0 t+\varphi_{отр})$ - interfering signal (reflected);

then $c_{рез}(t)=A_{c\,рез}\cos(\omega_0 t+\varphi_{рез})$ - resulting signal. (3.10.1)

Since in this case of interest is not the phase of the resulting oscillation, but its divergence from the phase of useful signal, let us place $\varphi_c=0$, the divergence of phase $\varphi_{рез}$ it will be random, since phase $\varphi_{отр}$ - random variable for which in the majority of the cases can be accepted the even distribution. It is necessary to find $w(\varphi_{рез})$. Fig. 3.10.1 depicts $\varphi_{рез}$, $\varphi_{отр}$, A_c, $A_{отр}$.

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Since $w(\varphi_{\text{отр}})$ is known, then for obtaining $w(\varphi_{\text{рез}})$ it is necessary to find the functional connection between $\varphi_{\text{рез}}$ and $\varphi_{\text{отр}}$.

Using relationships/ratios for oblique triangles, it is possible to obtain:

$$\sin\varphi_{\text{рез}} = \frac{A_{\text{отр}}}{A_c}\sin(\pi - \varphi_{\text{рез}} + \varphi_{\text{отр}}),$$

$$\operatorname{tg}\varphi_{\text{рез}} = \frac{b\sin\varphi_{\text{отр}}}{1 + b\cos\varphi_{\text{отр}}}, \quad b = \frac{A_{\text{отр}}}{A_c}. \qquad (3.10.2)$$

$$\varphi_{\text{рез}} = \operatorname{arctg}\frac{b\sin\varphi_{\text{отр}}}{1 + b\cos\varphi_{\text{отр}}} = \beta(\varphi_{\text{отр}}). \qquad (3.10.3)$$

Let us find the inverse functional connection

$$\cos\varphi_{\text{отр}} = -\frac{1}{b}\sin^2\varphi_{\text{рез}} + \cos\varphi_{\text{рез}}\sqrt{b^2 - \sin^2\varphi_{\text{рез}}},$$

$$\varphi_{\text{отр}} = \arccos\left[-\frac{1}{b}\sin^2\varphi_{\text{рез}} + \cos\varphi_{\text{рез}}\sqrt{b^2 - \sin^2\varphi_{\text{рез}}}\right] = \alpha(\varphi_{\text{рез}}). \qquad (3.10.4)$$

Function $\alpha(\varphi_{\text{рез}})$ is multiple-valued, since to one value of cosine correspond, in the limits $\pm\pi$, two values of angle $\varphi_{\text{отр}}$.

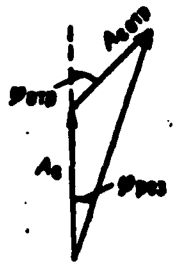

Fig. 3.10.1. Distortion of phase of signal.

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It is known that

$$w(\varphi_{\text{рез}}) = w(\varphi_{\text{отр}1}) \left| \frac{d\varphi_{\text{отр}1}}{d\varphi_{\text{рез}}} \right| + w(\varphi_{\text{отр}2}) \left| \frac{d\varphi_{\text{отр}2}}{d\varphi_{\text{рез}}} \right|, \quad (3.10.5)$$

where $\frac{d\varphi_{\text{отр}1}}{d\varphi_{\text{рез}}}$, $\frac{d\varphi_{\text{отр}2}}{d\varphi_{\text{рез}}}$ - derivatives for two values of angle $\varphi_{\text{отр}1}$ and $\varphi_{\text{отр}2}$, $w(\varphi_{\text{отр}}) = \frac{1}{2\pi}$. After fulfilling differentiation of expression (3.10.4) and after substituting in (3.10.5), after conversions we will obtain

$$w(\varphi_{\text{рез}}) = \frac{\cos\varphi_{\text{рез}}}{\pi\sqrt{b^2 - \sin^2\varphi_{\text{рез}}}} \quad (3.10.6)$$

with b≤1. Analogously we will obtain

$$w(\varphi_{\text{рез}}) = \frac{1}{2\pi} + \frac{\cos\varphi_{\text{рез}}}{2\pi\sqrt{b^2 - \sin^2\varphi_{\text{рез}}}} \quad (3.10.7)$$

with b>1.

Fig. 3.10.2 gives the distribution functions for several values of b. As can be seen from results, the functions of phase distribution of the mixture of useful signal with that mixing strongly differ from the function of phase distribution of the mixture of signal with the selective interference, although in both cases the mixing process has the random evenly distributed phase. The reason for this difference lies in the fact that in the case in question the amplitude has the specific value, and for the interference it is also by chance.

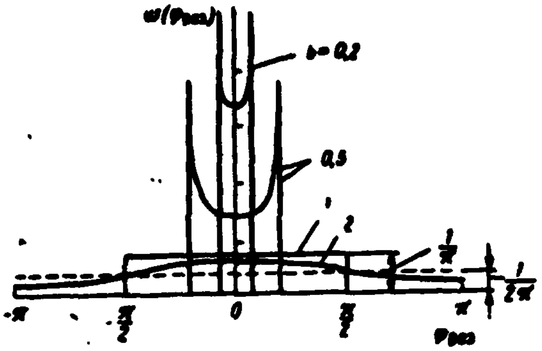

Fig. 3.10.2. Functions of distribution $\varphi_{рез}$.

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In the particular case when $A_{отр} > A_{c}$

$$w(\varphi_{рез}) = \frac{1}{2\pi}\left(1 + \frac{\cos\varphi_{рез}}{b}\right).$$

This coincides with (3.4.7), if we take

$$\frac{1}{b} = a_c\sqrt{\frac{\pi}{2}},$$

which is justified, since $a_c = \frac{\sigma_n}{A_c}$, while $b = \frac{A_c}{A_{отр}}$. Consequently, with the strong interfering signal the distribution function virtually coincides with the distribution function of phase in the mixture of signal and interference. It is interesting to note that:

- with b≤1 the average/mean value of the distribution function coincides with the function of the phase of signal and averaging on the set (or on the time, if $\varphi_{отр}$ it is changed) can improve the result of measuring the phase:

- with b>1 we have component 1/2π, which then, as this was shown in S3.8, with the averaging does not give an increase in the accuracy of measurements. On these reasons with b>1 and with any averaging will remain the error.

The dispersion of the divergences of phase can be calculated according to the formula:

$$\sigma^2_{\varphi рез}=\int_{-\infty}^{+\infty}\varphi^2_{r\,ср}\,w(\varphi_{r\,ср})\,d\varphi_{r\,ср}.$$

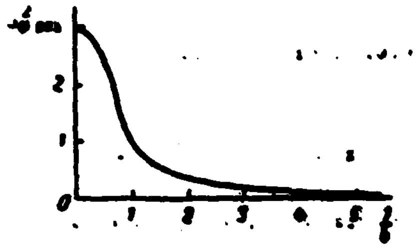

Fig. 3.10.3. Dispersion of divergences of phase.

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Lowering conversions, let us give the result of dependence $\sigma^2_{\varphi_{pca}}$ on 1/b (Fig. 3.10.3).

§3.11. Special features/peculiarities of the functions of phase distribution in the two-channel systems. In the two-channel systems the information consists in a phase difference of two signals each of which is accompanied by interferences. Despite the fact that study of two-channel phase systems is independent problem, the obtained above results can be used, also, for the analysis of some cases of two-channel systems.

The mixture of signals and interferences in the channels let us register in the form

$$y_1(t) = c(t, \varphi_{c_1}) + n_1(t) = A_{y_1}(t) \cos[\omega_0 t + \varphi_{c_1} + \varphi_{y_1}(t)]$$

and

$$y_2(t) = c(t, \varphi_{c2}) + n_2(t) =$$
$$= A_{y2}(t)\cos[\omega_0 t + \varphi_{c2} + \varphi_{y2}(t)]. \qquad (3.11.1)$$

The useful result of the work of system is a phase difference.

Without interferences $\Delta\varphi_c = \varphi_{c1} - \varphi_{c2}$. In the presence of the interferences

$$\Delta\varphi_y(t) = \varphi_{c1} - \varphi_{c2} + \varphi_{y1}(t) - \varphi_{y2}(t). \qquad (3.11.2)$$

Consequently, useful result will be distorted by fluctuations $\Delta\varphi_n(t) = \varphi_{y1}(t) - \varphi_{y2}(t)$ and interference effect will be determined by the distribution function and by the energy spectrum of random process $\Delta\varphi_n(t)$. In general form the solution of problem conjugated/combined with many difficulties (see [3, 4]) is not the purpose of this work. In the case when interferences $n_1(t)$ and $n_2(t)$ are not dependent, i.e., they are not created by one source, which functions on both channels, but they are determined by independent factors in each of the channels (for example, internally-produced noise of receivers) and signals in the channels exceed interference, statistical characteristics $\Delta\varphi_n$ can be found of that obtained above formulas. In this case $\varphi_{y1}(t)$ and $\varphi_{y2}(t)$ have normal distribution.

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Then the distribution function for $\Delta\varphi_{n}(t)$ also is normal, with the dispersion

$$\sigma_{\Delta\varphi}^{2}=\sigma_{\varphi 1}^{2}+\sigma_{\varphi 2}^{2}=\frac{\sigma_{n1}^{2}}{A_{c1}^{2}}+\frac{\sigma_{n2}^{2}}{A_{c2}^{2}}. \qquad (3.11.3)$$

Energy spectrum is the sum of the energy spectra of the fluctuations of phase in each of the channels

$$G_{\Delta\varphi}(\omega)=G_{\varphi 1}(\omega)+G_{\varphi 2}(\omega). \qquad (3.11.4)$$

If channels are identical and the relationship/ratio between the interference and the signal in them identical, then $G_{\Delta\varphi}(\omega)=2G_{\varphi}(\omega)$. Consequently, all obtained above results are valid taking into account (3.10.3) and (3.10.4), also, for the two-channel systems.

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Chapter 4.

THE ROLE OF PHASE IN THE DETECTION OF RADIO SIGNALS.

S4.1. Statistical approach to the detection problem of signal in the interferences and the criterion of optimum detection. The detection of radio signal against the background of interferences - this is the operation, with which is realized the response/answer to the question of whether there is a signal or not, or is there only interference.

The detection of signal is the first operation with which begins the functioning radio engineering of system.

With the discrete/digital methods of the transmission of information the detection of radio signal is the basic problem of the communication system. In the radar the target detection also is reduced to the detection of radio signal. In the radio navigation and in the trajectory measurements the work of system also begins from the detection of signal. If signal is discovered, this shows that the system functions and object entered into the zone of its action. In many phase systems are used pulse signals, then work begins from the search for this signal, i.e., also, actually, from its detection.

High value has the optimization of the detection of signal against the background of interferences, i.e., the determination of this diagram and operating principle of receiving indicator or this procedure either algorithm or rule of processing the mixture of signal and interference which are allowed optimally, i.e., in the best way, to discover signal.

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Interference is a random process, while the signal is a function of time with the random parameters; therefore singularly correct approach to the problem of detection is statistical, probabilistic approach.

The problem of detection is examined usually from the positions of the theory of checking or testing the statistical hypotheses and it is the part of the general theory of statistical solutions.

Let us consider problem detection from these positions.

During the detection of signal it is assumed that the signal has final duration and reception indicator device/equipment realizes certain time the observation of that process (mixture), which enters the input of radio receiver; in this case are accumulated the statistical evidence about this process.

In order to select hypothesis or to make a decision, in the receiving indicator must by some form be realized processing the statistical evidence, which acted on the input for the time of observation. Rules, algorithm of processing must be built into the reception indicator device/equipment, into its circuitry and operating principle.

On the basis of obtained information it is necessary to select one of the alternate hypotheses. One hypothesis - signal exists: let us designate by its symbol Γ_1. Another hypothesis - there is no signal; let us designate by its symbol Γ_0.

Since the reception of signal and the selection of hypotheses occurs under the conditions of the presence of interference, making of decision is accompanied by errors. Errors can be two kinds.

There can be the "passage of signal", i.e., such case when on the basis of the observed effect at the output of receiver is accepted solution - there is no signal, i.e., is accepted hypothesis

Γ_{e}, although in actuality the signal at the input was, as a result of the fact that it masked the interferences, at the output its receiving indicator they could not discover.

There can be "false detection" or "false alarm", i.e., such case when according to the results of processing mixture is accepted the solution that the signal exists, those is accepted hypothesis Γ_{c}, although in actuality there was no signal at the input, due to the action of interferences effect at the output of receiving indicator was such, as this had to be in the presence of signal.

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With the automatic reception/procedure the solution about the selection of hypotheses Γ_{c} or Γ_{e} must be accepted as reception indicator itself (its diagram) can take the form of two different output stresses/voltages - output signals. To each hypothesis corresponds its signal at the output. Errors in this case are expressed in the fact that the signal does not correspond to signal at the input.

Let us consider now a question about the criterion of optimality. Depending on the combination of the real state and the solution accepted can occur the following different situations:

1) signal was and was accepted solution about its presence, i.e., is correctly accepted hypothesis Γ_c.

2) signal was, but was accepted solution about its absence, i.e., is erroneous accepted hypothesis Γ_0.

3) there was no signal and was accepted solution about its absence, i.e., is correctly accepted hypothesis Γ_0.

4) there was no signal, but was accepted solution about its presence, i.e., is erroneously accepted hypothesis Γ_c.

Each of these situations has their probability of appearance. Let us designate them respectively p_1, p_2, p_3, p_4. Very on itself probability of one or the other error yet completely does not characterize the connected with it "harmful consequences". Errors of different character can have different "in the harmfulness" consequences.

To account for the special features/peculiarities of errors and harmful consequences, connected with their appearance, let us introduce concept "value (cost/value) of error" either "fee/pay/board for the error", or "magnitude of losses".

It is obvious that the harmful consequences of error and the real effect of error on the work of system can be considered most fully, if we take into account both the probability of erroneous solution and the value of error. This leads to the concept of "risk" ρ_i

$$\rho_i = p_i r_i.$$

The average/mean risk, connected with the entire set of errors, can be found as the sum of "risks"

$$\rho = \sum_i p_i r_i.$$

It is obvious, the optimum procedure it is possible to consider such, which ensures the minimum of the negative consequences, connected with the presence of errors.

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Thus, the optimum procedure of processing we can consider such, which ensures the minimum of average/mean risk. During the detection of radio signals erroneous situations can be two forms - passage of signal and false detection.

Then

$$\rho = p_3 r_{\text{пр}} + p_4 r_{\text{л}}, \qquad (4.1.1)$$

where $r_{\text{пр}}$ and $r_{\text{л}}$ - value of the errors of the passage of signal and false detection: p_3 and p_4 - probabilities of erroneous solutions.

The practical use of this criterion meets with some difficulties. Let us consider their reasons. For using the expression, which gives average/mean risk, it is necessary to know $r_{\text{пр}}$ and $r_{\text{л}}$. During the detection of signal in different systems the values of errors are determined by different facts. In the informational connected systems the signal and pause carry identical information - the presence of signals it corresponds to one, and its absence - zero (during the use of the binary code). Therefore the passage of signal and false detection give identical consequences. It is obvious that with this simpler anything to take the value of error for one, since the concept of value conditionally, i.e., $r_{\text{пр}} = r_{\text{л}} = 1$.

In the radar systems of the consequence of the passage of signal, i.e., passage of target, differ from the consequences of false alarm. In the phase systems (among other things of radio navigational ones) the passage of signal leads to the need of repeating the search, false detection causes the delay of search. The value of errors can be determined by time loss during the search. Thus, with that or other accuracy and persuasiveness the values of errors in some systems can be determined, in other systems their determination causes difficulties.

Probabilities p_3 and p_4 are the combined probabilities p_3 - probability that will occur two events: will be transmitted signal and it will be passed

$$p_3 = p(c, \Gamma_0).$$

$p(c, \Gamma_0)$ depends on the probability of the fact that the signal was transmitted, let us designate its $p(c)$, and from the conditional probability of the fact that in the presence of signal it was passed in connection with the interference of interferences; let us designate this probability $p(\Gamma_0/c)$.

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$p(c)$ does not depend on the procedure of processing mixture and is determined by the special features/peculiarities of the communication used. This probability is frequently called a priori or pre-experimental.

From the probability theory it is known that

$$p_3 = p(c, \Gamma_0) = p(c)\, p(\Gamma_0/c);$$

analogously it is possible to obtain $p_{ч}=p(0,\Gamma_c)=p(0)\,p(\Gamma_c/0)$,

where p(0) - the a priori probability of the absence of signal;

$p(\Gamma_c/0)$ - the conditional probability of accepting the hypothesis about the presence of signal when there is no signal at the input.

Frequently in the literature $p(\Gamma_0/c)$ designate $p_{пр}$ or $(1-D)$, and $p(\Gamma_c/0)-p_{лт}$ - or F. However, in this case designation does not stress the conditional character of these probabilities and therefore subsequently presentation will be preserved the recording, which accurately reflects the essence of concepts.

Now expression for the average/mean risk can be registered in the expanded/scanned form

$$\rho=r_{п}\,p(c)\,p(\Gamma_0/c)+r_{л}\,p(0)\,p(\Gamma_c/0). \qquad (4.1.2)$$

Let us consider the factors, which affect p(0) and p(c).

In the connected informational systems during the use of the binary code the probability of transmission 1 or 0 on the average is identical, and if to one corresponds the presence of signal, and zero its absence, then

$$p(c)=p(0)=0,5.$$

Probabilities p(0) and p(c) in the radar and radio-navigation systems to consider complicatedly. Under the specific conditions, for example in the radars, which work in the mode/conditions of the attendant survey/coverage of airspace, probability p_0 can be small. Under other conditions there can be the situation during which this probability is close to one, and therefore selection p(0) and p(c) is difficult. Thus, to use the criterion of the minimum of the average/mean risk when optimality is considered by the guarantee of the minimum ρ (see 4.1.2), in the radio navigation and the radar, little it is convenient, since selection p(0), p(c) $r_{л}$ and $r_{пр}$ causes many difficulties and frequently carries conditional character.

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Simplification in the criterion of minimum average/mean risk can be achieved when all errors are identical with respect to their negative consequences.

After placing $r_{пр}=r_{л}=1$, it is possible expression (4.1.2) to reduce to the form

$$\rho = p_{ош} = p(c)\,p(\Gamma_0/c) + p(0)\,p(\Gamma_c/0), \qquad (4.1.3)$$

where $p_{ош}$ - total probability of error.

In this case the criterion of the minimum of average/mean risk passes into the criterion of the minimum of the total probability of error. This criterion is called the criterion of ideal observer".

If, furthermore, p(0)=p(c)=0.5, then

$$p_{ош}=0,5\,[p(\Gamma_0/c)+p(\Gamma_c/0)] \qquad (4.1.4)$$

and the value of the total probability of error or average/mean risk very simply is expressed as $p(\Gamma_0/c)$ and $p(\Gamma_c/0)$.

Under the conditions for the work of radar and radio-navigation systems it proves to be possible and appropriate to be assigned by specific value $p(\Gamma_c/0)$, i.e., by the specific conditional probability of false alarm.

Basic reason lies in the fact that frequently operation of stations occurs under the conditions when or there is no signal or unknown, what it will be intensity; therefore basic error in this mode/conditions can be "false alarm". So that the false alarms would not cause the disruptions of the work of system, it is useful to assign specific value $p(\Gamma_c/0)$. This is possible to do, since $p(\Gamma_c/0)$ depends on diagram and interference level and does not depend on signal.

It is obvious that under these conditions of optimum it is possible to count the diagram which with given one $p(\Gamma_0/0)$ will ensure the minimum probability of the passage of signal $p(\Gamma_0/c)$ or the maximum probability of its correct detection $p(\Gamma_c/c)$. This makes it possible to formulate the criterion of the optimization of detection, which differs from the criterion of minimum average/mean risk. This criterion was called Neumann-Pearson criterion.

§4.2. Statistical description of interference, signal and their mixture. Statistical noise characteristics were examined earlier. However, it is useful to do one explanation.

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During the detection of signal the observation of the mixture of signal and interference or one interference alone can be continued the considerable time, substantially greater than the interval of the correlation of interference. Then for describing the process it is necessary to have not one (during the one-dimensional distribution) and not two, divided by interval τ (during the two-dimensional distribution), but many points, theoretically (during the continuous observation of mixture) - an infinite number of points. In this case the distribution function, which shows the probability density of one or the other combination of values, that characterize random process, must be infinite-dimensional.

In order to avoid the difficulties, connected with the statistical description of the prolongedly functioning interference, it is necessary from the continuous functions, which describe random process of y(t), to pass to selections y_1, y_2, ..., i.e., to use not entire set of points characterizing the continuous function of time, but their limited number, which reflects the value of the function through the specific time intervals. According to the known theorem of Kotelnikov the selection reflects all basic properties of the function of time, if the interval of the selection

$$\Delta t=\tau_{\kappa}=\frac{1}{2f_{\text{в}}},$$

where $f_{\text{в}}$ - highest frequency of the spectrum of the function: τ_{κ} - the interval of correlation.

The values of random process, undertaken through the interval of correlation, are not statistically virtually dependent between themselves. This makes it possible to radically simplify the mathematical description of random process as the functions of time. From the infinite-dimensional function of the distribution of continuous random process it is possible to switch over to the m-dimensional distribution function for m values of selection. Here

$$m=\frac{t_{н}}{\tau_{к}},$$

where $t_{н}$ - time of observation during the detection: $\tau_{к}$ - the interval of the correlation: the m-dimensional distribution function can be obtained by the simple multiplication of the one-dimensional distribution functions, valid for each of the points of selection.

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With one interference the selection is determined only by it.

Then

$$w(y_1, y_2, \ldots/п)=w(п_1, п_2, \ldots)=\prod_{i=1}^{m} w(п_i).$$

After using this expression and keeping in mind the stationary random process for which the distribution function does not depend on time, we will obtain

$$w(п_1, п_2, \ldots)=\prod_{i=1}^{m}\frac{1}{\sqrt{2\pi}\,\sigma_п}e^{-\frac{п_i^2}{2\sigma_п^2}}=$$

$$=\frac{1}{(2\pi\sigma_п^2)^{m/2}}e^{-\sum_{i=1}^{m}\frac{п_i^2}{2\sigma_п^2}} \qquad (4.2.1)$$

or

$$w(y_1, y_2, \ldots/n) = \frac{1}{(2\pi\sigma_n^2)^{m/2}} e^{-\sum_{i=1}^{m} \frac{y_i^2}{2\sigma_n^2}},$$

where σ_n^2 - dispersion of interference.

As we see, the multidimensional distribution function it was possible to obtain in the simple form what is very important result.

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In certain cases it occurs more conveniently to switch over from the sum to the integral

$$\frac{1}{2}\sum_{i=1}^{m} \frac{y_i^2}{\sigma_n^2} = \frac{1}{2\sigma_n^2 \tau_n} \sum_{i=1}^{m} y_i^2 \tau_n,$$

but

$$\sum_{i=1}^{m} y_i^2 \tau_n \approx \int_0^{t_n} y^2(t)\, dt,$$

then

$$w(y_1, y_2, \ldots/n) = \frac{1}{(2\pi\sigma_n^2)^{m/2}} e^{-\frac{1}{N_0}\int_0^{t_n} y^2(t)\, dt}, \quad (4.2.2)$$

since $\sigma_n^2 = N_0 f_n$.

The physical sense of function $w(y_1, y_2, \ldots)$ lies in the fact that it shows, which the probability density of one or the other combination $y_1, y_2, \ldots, y_m$.

Let us now move on to the statistical description of signal. If all parameters of signal are known, then the voltage (field, current) of signal on the input of receiver is the function only of time - c(t). However, this case in the practice is encountered rarely. Usually any of the parameters of signal or several parameters are unknowns. These parameters can be random variables or random processes. In the first case the parameter can be considered constant/invariable during entire interval of time when is realized detection. In the second case the random parameter of signal substantially is changed for the time of observation during the detection.

From theoretical and practical point of view these two cases differ from each other. Subsequently we will assume that the signal is a function of the time and random parameters. Briefly this signal can be registered in the form

$$c(t, \beta_1, \beta_2, \ldots), \qquad (4.2.3)$$

where β_i - random parameters of signal.

In the radio engineering systems the chance of the parameter it is possible to play different role.

If system, functioning, realizes only detection of signal, the random value of the parameter does not contain useful information, and the more such random parameters in the signal, the worse the results of its detection.

In some systems, for example radar and radio navigational, after the detection of signal usually follows the measurement of its those parameters, into which is laid useful information, for example: frequencies - for measuring of radial velocity, delay and phase - for ranging, amplitude modulation or phase displacement - for measuring the direction.

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In this case the chance of the parameter, impeding detection, as a whole in the system plays positive role, since the measurement of this parameter makes it possible to determine coordinates and elements of motion. In the signal, besides the random parameters, which carry useful information, there can be the parasitic parameters, which do not contain useful information.

It is necessary to find the methods of the analysis of the optimum detection of radio signal in general form on the assumption that the signal has the random parameters.

Now let us examine the statistical description of the mixture of interference and signal.

We will analyze the case when interference functions together with the signal and does not affect its characteristics and parameters. Such interferences are called additive. Besides the additive interferences, are even multiplicative whose special feature/peculiarity lies in the fact that they are superimposed on the signal, changing its parameters. Multiplicative interferences occur, for example, in the case when signal and interference pass through nonlinear circuits. In the majority of the cases interference can be considered as additive.

Then the mixture, which functions at the input of receiver, is simply the sum of interference and signal

$$y(t)=n(t)+c(t,\beta_1,\beta_2,\ldots). \qquad (4.2.4)$$

Since into mixture enters the interference (random process), mixture is also random process and must be described by the distribution function. If all parameters of signal are known, then the chance of mixture depends only on interference. The distribution functions for the interference were obtained earlier. In the presence of signal this expression will describe the distribution function for the mixture, if we instead of n_i register y_i-c_i.

Thus,

$$w(y_1, y_2, \ldots/c\text{-}n) = \frac{1}{(2\pi\sigma_n^2)^{m/2}} e^{-\sum_{i=1}^{m} \frac{(y_i - c_i)^2}{2\sigma_n^2}}. \quad (4.2.5)$$

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Upon transfer from the sum to the integral, we will obtain

$$w(y_1, y_2, \ldots/c\text{-}n) = \frac{1}{(2\pi\sigma_n^2)^{m/2}} e^{-\frac{1}{N_0}\int_0^{t_n} [y(t) - c(t)]^2 dt}, \quad (4.2.6)$$

where n_i, y_i, c_i - value of the selection of interference, mixture and signal, the undertaken through the interval correlations.

If signal has the random parameters β_1, β_2, ..., the chance of the values, taken by the selection of mixture, depends on the fact that the interference is random process, and fact that the signal has p of the random parameters.

The distribution function, which statistically describes this process (mixture), must be combined.

Direct obtaining of the combined distribution function causes difficulties. To it is considerably simpler obtain the conditional function of the distribution of mixture for some any combination of the values of the random parameters β_1, β_2.... It will take the form

$$w(y_1, y_2, \ldots / \beta_1, \beta_2, \ldots \text{с-п}) = \frac{1}{(2\pi\sigma_n^2)^{m/2}} \times$$

$$\times e^{-\frac{1}{N_0}\int_0^{t_n} [y(t) - c(t, \beta_1, \beta_2)]^2 \, dt} \, . \qquad (4.2.7)$$

Transition from the conditional distribution function to the combined will be examined further.

In other words, the probability density of group or combination of values of y_1, y_2, ... will be determined not only by the combination of these values, but also by the combination of the values of the random parameters of signal.

Thus, are obtained the expressions, which describe the statistical properties of interference, signal and mixture.

S4.3. Optimum procedure of processing mixture and likelihood ratio. Let us now move on to the conclusion/output of the relationships/ratios, which reveal the optimum procedure of processing mixture.

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For obtaining the expanded/scanned expressions, which show those operations which must be fulfilled in the receiver with the mixture, it is necessary to express entering the formula of average/mean risk probabilities $p(\Gamma_c/0)$ and $p(\Gamma_0/c)$ through the statistical characteristics (distribution function) of interference, signal and mixture, to find conditions, with which ρ is minimized.

Let us assume that is obtained the realization, given by selection y_1, y_2, ... it is necessary to determine, to what it corresponds. The probability (the exact differential of probability) that that the values of mixture will prove to be within the limits: from y_1 to y_1+dy_1: from y_2 to y_2+dy_2 and so forth, it can be found from the relationship/ratio

$$dp_{c\text{-}n} = w(y_1, y_2, \ldots /c\text{-}n)\, dy_1\, dy_2 \ldots \qquad (4.3.1)$$

It is analogous probability that the values only of one interference will prove to be within the same limits (from y_1 to y_1+dy_1; from y_2 to y_2+dy_2, etc.) it can be found from the relationship/ratio

$$dp_n = w(y_1, y_2, \ldots /n)\, dy_1\, dy_2 \ldots, \qquad (4.3.2)$$

$w(y_1, y_2, \ldots/c\text{-}n)$ and $w(y_1, y_2, \ldots/n)$ - conditional probability densities for the obtained combination of selections in the presence of signal and interference and only one interference.

For obtaining $p(\Gamma_c/0)$ and $p(\Gamma_0/c)$ it is necessary to carry out integration dp_{c-n} and dp_n within certain limits:

$$w(y_1, y_2, \ldots/c\text{-}n) \text{ and } w(y_1, y_2, \ldots/n),$$

is m-dimensional distribution functions, therefore, they can be depicted in the m-dimensional space. This space can be divided into two regions v_{r0} and v_{r0}. Region v_{rc} (region of signal) - this is the region, which corresponds to solution about the presence of signal. Region v_{r0} (region interference) - this is the region, which corresponds to solution about the absence of signal and the presence only of one interference. Then it is obvious that $p(\Gamma_c/0)$, i.e. probability that in the absence of signal will be accepted the solution about its presence, can be found with the m-fold integration with the limits, which correspond to region v_{rc}, i.e.

$$p(\Gamma_c/0) = \int\int_{v_{rc}} \ldots \int w(y_1, y_2, \ldots/n)\, dy_1\, dy_2 \ldots \qquad (4.3.3)$$

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Analogously it is possible to obtain

$$p(\Gamma_0/c) = \int\int_{v_{r0}} \ldots \int w(y_1, y_2, \ldots/c\text{-}n)\, dy_1\, dy_2 \ldots, \qquad (4.3.4)$$

$$p(\Gamma_c/c) = \int\int_{v_{rc}} \ldots \int w(y_1, y_2, \ldots/c\text{-}n)\, dy_1\, dy_2 \ldots, \qquad (4.3.5)$$

$$p(\Gamma_0/0) = \int\int_{v_{r0}} \ldots \int w(y_1, y_2, \ldots/n)\, dy_1\, dy_2 \ldots \qquad (4.3.6)$$

Let us recall that

$$p(\Gamma_0/c)=1-p(\Gamma_c/c) \text{ and } p(\Gamma_0/0)=1-p(\Gamma_c/0).$$

After using these relationships/ratios, it is possible expression for ρ to convert as follows:

$$\rho=r_{mp}p(c)-r_{mp}p(c)\,p(\Gamma_c/c)+r_{a}p(0)\,p(\Gamma_c/0).$$

After substituting into this expression the relationships/ratios obtained above for $p(\Gamma_c/c)$ and $p(\Gamma_c/0)$, after using the fact that these probabilities are obtained due to the integration in the limits of one and the same region of signal $v_{\Gamma c}$, we will obtain

$$\rho=r_{m}p(c)-\int\int_{v_{\Gamma c}}\cdots\int[p(c)\,r_{m}\,w(y_1, y_2, \ldots/c\text{-}n)-$$
$$-r_{a}p(0)\,w(y_1, y_2, \ldots/n)]\,dy_1\,dy_2. \qquad (4.3.7)$$

Let us consider now the conditions of obtaining the minimum ρ. Value $r_{mp}p(c)$ does not depend on that how is realized the reception of signal, but it is determined by a priori data. In order during the selection of the solution "signal to eat" or the acceptance of the hypothesis Γ_c ρ had minimum value, it is necessary that the integral on region $v_{\Gamma c}$ would have maximum value. Since p(c), p(0) r_{mp}, r_{a}, $w(y_1, y_2)$ - positive values, for guaranteeing the maximum value of integral it is necessary that the integrand would always remain positive.

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In other words, the guarantee of obtaining the minimum ρ during the selection of hypothesis Γ_c can be achieved/reached with satisfaction of the condition

$$p(c)\,r_{\text{ш}}\,w(y_1, y_2, \ldots / c\text{-}n) > r_{\text{л}}\,p(0)\,w(y_1, y_2, \ldots / n)$$

or

$$\frac{w(y_1, y_2, \ldots / c\text{-}n)}{w(y_1, y_2, \ldots / n)} > \frac{r_{\text{л}}\,p(0)}{r_{\text{ш}}\,p(c)}. \qquad (4.3.8)$$

During the acceptance of solution Γ_c it is necessary to carry out integration for region $v_{\Gamma c}$. From these concepts, and also directly from (4.3.8) it follows that the minimum average/mean risk also is ensured, if during the selection of hypothesis Γ_0 is observed the condition

$$\frac{w(y_1, y_2, \ldots / c\text{-}n)}{w(y_1, y_2, \ldots / n)} < \frac{r_{\text{л}}\,p(0)}{r_{\text{ш}}\,p(c)}. \qquad (4.3.9)$$

Relation (4.3.8) plays large role in the theory of the detection of radio signals. It is named "likelihood ratio" and it is designated $l(y_1, y_2, \ldots)$ or $l(y)$.

The principle of optimum detection lies in the fact that the reception indicator device/equipment must compute likelihood ratio $l(y)$ and compare it with the threshold

$$\Pi_1 = \frac{r_a p(0)}{r_{np} p(c)}.$$

If it proves to be that $l(y) > \Pi_1$, must be chosen hypothesis Γ_c "signal exists".

If it proves to be that $l(y) < \Pi_1$, must be chosen hypothesis Γ_0 there is no "signal".

The relationships/ratios obtained above can be used when there is an expression for $w(y_1, y_2, \ldots/c-n)$. It simply is obtained from distribution functions for the interference when all parameters of signal are known. Since, as a rule, the signal contains these or other random parameters, necessarily in more detail to stop at the methodology of obtaining $l(y)$.

Let us assume that the signal has the random parameters β_1, β_2, ... The function of the distribution of mixture takes form (4.2.7).

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For the transition to $w(y_1, y_2, .../c-п)$ we will use product rule for probabilities or probability densities

$$w(y_1, y_2, \ldots, \beta_1, \beta_2, \ldots/c\text{-п}) =$$
$$= w(y_1, y_2, \ldots/c\text{-п})\, w(\beta_1, \beta_2, \ldots/y_1, y_2, \ldots) =$$
$$= w(\beta_1, \beta_2, \ldots)\, w(y_1, y_2, \ldots/\beta_1, \beta_2, \ldots), \quad (4.3.10)$$

where $w(y_1, y_2, .../c-п)$ - the probability density of combination the values of the random parameters of signal at the condition of the presence of signal;

$w(\beta_1, \beta_2, .../y_1, y_2, ...)$ - the conditional probability density of one or the other combination of the random parameters of signal at the condition of the fact that the mixture is characterized by the realization, reflected in the specific combination, the values of selection $y_1, y_2, ...$;

$w(\beta_1, \beta_2, ...)$ - the joint probability density of different combinations of the values of the random parameters of signal;

$w(y_1, y_2, \ldots/\beta_1, \beta_2, \ldots)$ - the conditional probability of the specific combination of the values of selection $y_1, y_2, \ldots$ under the condition of the specific combination of the random parameters of signal β_1, β_2.

After multiplying all parts of equality (4.3.10) to $d\beta_1, d\beta_2, \ldots$, we will obtain the differential of probability. After carrying out then k-multiple integration for variable/alternating $\beta_1, \beta_2, \ldots, \beta_k$ and so forth and after taking integral within the limits of all possible values $\beta_1, \beta_2, \ldots$, we will obtain

$$\underbrace{\int\int}_{k\ \text{раз}}\ldots w(y_1, y_2, \ldots/\text{c-п})\, w(\beta_1, \beta_2, \ldots/y_1, y_2, \ldots)\, d\beta_1\, d\beta_2 \ldots = \\ = \underbrace{\int\int}_{k\ \text{раз}}\ldots w(\beta_1, \beta_2, \ldots)\, w(y_1, y_2, \ldots/\beta_1, \beta_2, \ldots)\, d\beta_1\, d\beta_2 \ldots \quad (4.3.11)$$

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$w_m(y_1, y_2, \ldots/\text{c-п})$ — according to the sense itself as the probability density of the random process, caused only on the interference and the presence of signal, and not depending on the random parameters of signal - is constant value for the variable $\beta_1, \beta_2, \ldots$ and can be carried out as the integral sign;

$$\underbrace{\int\int}_{k\ \text{раз}}\ldots\int w(\beta_1, \beta_2, \ldots/y_1, y_2)\, d\beta_1\, d\beta_2 \ldots = 1,$$

since the conditional probability (for some realization y_1, y_2) of all possible values β_1, β_2 is equal to 1, since it exhausts their all possible combinations. Then

$$w(y_1, y_2, \ldots/c\text{-}\pi) = \\ = \int\limits_{k\ \text{pas}}\int \ldots \int w(\beta_1, \beta_2, \ldots)\, w(y_1, y_2, \ldots/\beta_1, \beta_2, \ldots)\, d\beta_1 d\beta_2 \ldots \tag{4.3.12}$$

The obtained expression is of great interest, since it shows that for the signal with the random parameters the function of distribution $w(y_1, y_2, \ldots/c\text{-}\pi)$, necessary for calculating the relation of plausibility, can be obtained by integration, if are known the function of the joint distribution of the random parameters $w(\beta_1, \beta_2 \ldots)$ and conditional probability density the combinations of the values of selection under the condition of some value of the random parameters. As it will be shown further, in many instances these functions can be found and obtaining $w(y_1, y_2, \ldots/c\text{-}\pi)$ for these signals proves to be possible. Expression (4.3.12) can be used for the purpose of obtaining likelihood ratio for the signal with the random parameters. After substituting (4.3.12) into the expression for $l(y)$ and remembering that in the absence of signal the distribution function depends only on interference and can be introduced under the integral sign, we will obtain

$$l(y) = \int\limits_{k\ \text{pas}}\int \ldots \int w(\beta_1, \beta_2, \ldots) \frac{w(y_1, y_2, \ldots/\beta_1, \beta_2, \ldots)}{w(y_1, y_2, \ldots/\pi)}\, d\beta_1 d\beta_2; \tag{4.3.13}$$

the relation

$$\frac{w(y_1, y_2, \ldots/\beta_1, \beta_2, \ldots)}{w(y_1, y_2, \ldots/\pi)} = l(y_1, y_2, \ldots/\beta_1, \beta_2, \ldots)$$

is likelihood ratio for the specific combinations of values β_1, β_2, i.e., conditional likelihood ratio, then

$$l(y)=\underbrace{\int\int\dots\int}_{k\ \text{раз}} w(\beta_1,\beta_2,\dots)\, l(y_1,y_2,\dots/\beta_1,\beta_2,\dots)\times$$
$$\times d\beta_1\, d\beta_2\dots \qquad (4.3.14)$$

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Since usually the random parameters of signal are mutually independent,

$$w(\beta_1,\beta_2,\dots)=\prod_{i=1}^{p} w(\beta_i),$$

$$l(y)=\underbrace{\int\int\dots\int}_{k\ \text{раз}} w(\beta_1)\, w(\beta_2)\dots l(y_1,y_2,\dots/\beta_1,\beta_2,\dots)\times$$
$$\times d\beta_1\, d\beta_2\dots \qquad (4.3.15)$$

If random parameter is one, then expression is simplified

$$l(y)=\int_{\beta} w(\beta)\, l(y_1,y_2,\dots/\beta)\, d\beta. \qquad (4.3.16)$$

It is necessary to focus attention on the fact that the optimum receiver does not fulfill the function of the amplification of signal, but is realized only selection. Virtually always the signal, accepted by antenna, is very weak and must be intensified. In this case together with the signal are reinforced the interferences. Amplifier stages can possess the selectivity which must be taken into consideration. However, usually the selectivity of the cascades/stages, which realize amplification, is sufficient only for the preliminary selection of signal and is distant from the optimum.

Reception indicator device/equipment consists of: antenna, amplifier stages and the cascades/stages, which realize optimum working/treatment of mixture. The study of antennas and amplifier stages of receivers is independent theme and cannot change fundamental posing of the question about the optimum detection. Therefore in the following presentation by term reception indicator device/equipment will be understood only that part of the equipment, which realizes isolation/liberation of signal from the interferences.

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It will be considered that mixture y(t) is sufficiently intensified for the work of the end devices of receiver-indicator. Having used the obtained above general/common/total expressions, let us consider the role of phase in the detection of radio signals. For this it is convenient to take the models of signals characterized by the statistical characteristics only of phase (signal with the known, the random and fluctuating phases and the known remaining parameters). Certainly, important role in the detection plays the chance of other parameters of signal - amplitude, delay, frequency; however, into our problem does not enter the comprehensive analysis of detection. It it is possible to find in [4.1, 4.2].

§ 4.4. Optimum detection of signal with the known parameters. Using theory presented above, let us find the procedure of optimum working/treatment or the diagram of optimum reception/procedure during the detection of the signal whose all parameters are known, at this y(t)=c(t)+n(t) in the presence of signal and interference and y(t)=n(t) in the presence of one interference.

The function of the distribution of interference is given by expression (4.2.1). The function of the distribution of mixture in the presence of signal is given by expression (4.2.5). Signal has duration t_c.

Likelihood ratio

$$l(y)=e^{-\frac{1}{2\sigma_n^2}\sum_{i=1}^{m}\left[(y_i-c_i)^2-y_i^2\right]}=$$

$$=e^{-\frac{1}{2\sigma_n^2}\sum_{i=1}^{m}c_i^2}\,e^{\frac{1}{\sigma_n^2}\sum_{i=1}^{m}y_i c_i}. \qquad (4.4.1)$$

After noting that $\sigma_n^2=N_0 f_n$; $\Delta t=\frac{1}{2f_n}$, in the manner that $\Delta t=\tau_n$, where f_n — band of interference; N_0 - jamming density, we will obtain, observing signal entire possible time, i.e., taking $t=t_c$,

$$l(y)=e^{-\frac{1}{N_0}\sum_{i=1}^{m}c_i^2\Delta t}\,e^{\frac{2}{N_0}\sum_{i=1}^{m}c_i y_i\Delta t}. \qquad (4.4.2)$$

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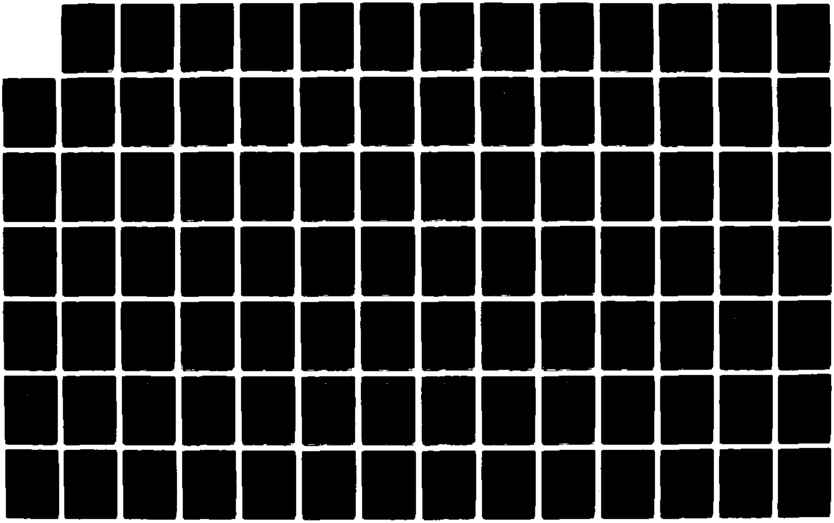

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Bearing in mind that addition m of terms is similar to integration in the limits from 0 to t_c, where

$$t_c = m\Delta t,$$

it is possible from the sums to switch over to integrals. Simultaneously we will use the fact that $c_i^2\Delta t$ is energy of the elements/cells of signal into which it is divided/marked off upon transfer to the selection. Then

$$\sum_{i=1}^{m} c_i^2 \Delta t = \mathcal{E}_c,$$

where $\mathcal{E}_c$ — energy of signal.

Upon transfer from the sums to the integrals, we will obtain

$$l(y) = e^{-\frac{\mathcal{E}_c}{N_0}} e^{\frac{2}{N_0}\int_0^{t_c} c(t)\, y(t)\, dt}, \qquad (4.4.3)$$

let us find the logarithm

$$\ln l(y) = -\frac{\mathcal{E}_c}{N_0} + \frac{2}{N_0}\int_0^{t_c} y(t)\, c(t)\, dt. \qquad (4.4.4)$$

Writing/recording limits from 0 to t_c, we assume that signal delay is equal to zero, since it is considered known.

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Is chosen hypothesis "signal is", if

$$\ln l(y) = -\frac{E_c}{N_0} + \frac{2}{N_0}\int_0^{t_c} c(t)\,y(t)\,dt > \ln \Pi_1$$

or

$$\int_0^{t_c} y(t)\,c(t)\,dt = z > \Pi_2, \qquad (4.4.5)$$

and hypothesis there is "no signal", if

$$\int_0^{t_c} y(t)\,c(t)\,dt = z < \Pi_2, \qquad (4.4.6)$$

$$\Pi_2 = \frac{N_0}{2}\ln \Pi_1 + \frac{E_c}{2}; \qquad (4.4.7)$$

with $\Pi_1=1$

$$\Pi_2 = \frac{E_c}{2}.$$

The obtained relationships/ratios are shown, which optimum procedure or the algorithm of working/treatment of mixture or which the optimum diagram of reception/procedure during the detection of signal with the known parameters. As can be seen from formulas, in the optimum diagram the mixture, supplied to the entrance of receiving indicator, must be multiplied to the signal (more precisely saying, to the copy of signal), i.e., in the receiving indicator should be created copy of signal, then after integration result is supplied to threshold or comparator, which has threshold Π_2. If value at the output of integrator exceeds threshold, we consider that the signal exists (hypothesis Γ_c); if it does not reach threshold, then we consider that there is no signal (hypothesis Γ_0). The diagram, which realizes optimum procedure, is given in Fig. 4.4.1.

The combination of multiplier and integrator is frequently called "correlator", and integral $\int_0^{t_0} y(t)c(t)dt$ is called "correlation integral". The diagram, which realizes optimum detection, consists of the elements/cells which can be simply realized. As the multiplier it can be simply realized. As the multiplier can be used, for example, phase discriminator. It is known that the output stress/voltage of phase discriminator is proportional to the product of reference voltage (here the copy of signal) and input voltage (here mixture). As the integrator can be used, for example, chain/network RC with the slow response.

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As the being congruent/equating cascade/stage can be used, for example, closed diode. Cutoff voltage (threshold) must be supplied from the external source. From the output is put out direct voltage, when receiver-indicator accepted the solution about the presence of signal by the entrance, and is put out no stress/voltage, if receiving indicator accepted the solution about the absence of signal on the entrance. The stress/voltage, removed from the output, can be used for the recording and the subsequent decoding or for the feed into the logical and computers. The greatest difficulties can arise with the generator of the copy of signal (GKS). However, in the principle, since all parameters of signal are known, the copy of signal it can be formed analogously how is formed/shaped signal itself at the transmitting end/lead. Thus, it is possible to arrive at the important conclusion about the fact that the theoretically obtained diagram of the optimum detection of the signal whose parameters are known, can be realized. The optimum diagram of the detection of signal according to the essence is constructed on the determination of the mutual correlation between the copy of signal and the mixture. In other words, optimum receiver is constructed on the principle of mutual correlation, or realizes the mutual-correlation method of reception/procedure.

In the implementation of mutual-correlation optimum receiving indicator must be used the generator of copy; therefore frequently such diagrams of the detection of signal in the interferences are called the diagrams of active filtration in contrast to the diagrams of the passive filtration, in which are used the matched filters. During the creation of practical diagrams appears the need for their addition.

The theoretical diagram, shown in Fig. 4.4.1, is the diagram of one-time action, i.e., if are known the parameters of signal, then after including/connecting this diagram at the moment of the time when can be begun signal, after observation for a period of time, which corresponds to the duration of signal, at the moment of the termination of its action is accepted the solution about presence or absence of signal on the entrance of receiver. For the reception of the following signal the diagram is not suitable, since accumulating due to the action of interferences or signal and interferences at the output of correlator stress/voltage is not equal to zero.

On the basis of the assumption about the ideality of integrator, it is possible to expect that this stress/voltage will be retained how conveniently for long.

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For guaranteeing the reception of the following signal it is necessary diagram to return into the initial state, for which is necessary value at the output of integrator to lead to zero. Consequently, for the continuous functioning of diagram it must be supplemented by the key/wrench, which closes the output of integrator to the earth after each operation of the detection of signal. Furthermore, diagram requires one more addition. It was previously noted that the comparison of value at the output of correlator with the threshold must be realized at the specific moment of time, namely at the moment of the termination of the action of signal.

Furthermore, diagram requires one more addition. It was previously noted that the comparison of value at the output of correlator with the threshold must be realized at the specific moment of time, namely at the moment of the termination of the action of signal. Consequently, stress/voltage from the output of correlator must be supplied to the threshold device/equipment not continuously, but during the short time interval (at moment/torque $t=t_c$). This role in the diagram can fulfill the key/wrench, which is closed at moment/torque $t=t_c$. The diagram of optimum detection taking into account these additions is given in Fig. 4.4.2. It is obvious that the obtained diagram in the equal degree is suitable for the simple, serrated and noise-like signals. The more complicated the signal, the more complicated its copy and the more complicatedly must be the generator of the copy of signal. In this case it is necessary to note that the copy must be reproduced with the high accuracy in all parameters of the signal: to amplitude, law of its change, to delay, law of a change in the phase, to initial phase and to frequency. Especially great difficulties appear during the reproduction in the copy of the phase of signal. To ensure this frequency stability so that phase displacement of independently workers of transmitter and generator of copy during would be preserved for a long time, proves to be very difficult problem.

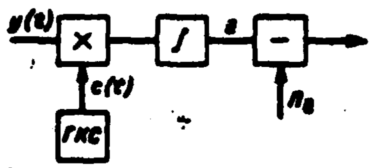

Fig. 4.4.1. Diagram of optimum reception of signal with known parameters: × - multiplier; ∫- integrator; GKS - generator of copy of signals; Π_0- threshold device/equipment.

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Therefore, even if it is possible to claim that the phase of signal is known, the technical difficulties of the realization of a precise copy of signal on the phase make it necessary in the practical diagrams to use phase synchronization with the help of the narrow-band servo all-pass filters.

Let us consider now the processes, which occur in the diagram under the influence on it of the mixture of signal and interference or only one interference. Knowing the processes, which occur at the output of correlator, it is possible to explain many special features/peculiarities of the work of diagram.

Let us consider the work of diagram under the effect only of one interference. Let us designate process at the output of correlator by symbol Z.

In the presence only of interference we will obtain

$$Z_n = \int_0^t n(t)c(t)\,dt. \qquad (4.4.8)$$

Let us pass from integral to the sum, for which we will use the selection, undertaken through the interval of correlation. Then at the moment of the termination of the action of signal $(t=t_c)$

$$Z_n = \sum_{i=1}^{m} c_i n_i \Delta t.$$

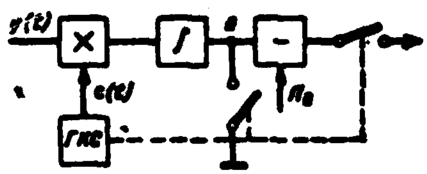

Fig. 4.4.2. The diagram of optimum reception/procedure with the further devices/equipment: ×- multipler; ∫- integrator; Π_0- threshold device/equipment; GKS - generator of the copy of signals.

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At other moments of time $(t=t_n<t_0)$

$$Z_{m,n}=\sum_{i=1}^{k} c_i n_i \Delta t,$$

where c_i —the selection of signal; n_i — selection of interference;

$$m=\frac{t_0}{\Delta t};\quad k=\frac{t_n}{\Delta t}.$$

As is evident, value $Z_{m,n}$ for any value $t=t_n$ is the random variable, which is obtained as a result of the addition of random variables $c_i n_i \Delta t$. It is known that with the addition of the random variables, which have normal distribution, the function of the distribution of sum remains normal, and the dispersion of sum is equal to the sum of dispersions.

If σ_n^2—dispersion of interference, then the dispersion of each member of sum will be $\sigma_i^2=\sigma_n^2 c_i^2 \Delta t^2$.

The dispersion of value Z_n is equal to

$$\sigma_z^2=\sum_{i=1}^{m}\sigma_i^2.$$

but $\sum_{i=1}^{m} c_i^2 \Delta t = \mathcal{E}_c$ — energy of signal;

$$\Delta t = \frac{1}{2f_n}.$$

Then $\sigma_z^2 = \frac{\sigma_n^2}{2f_n}\mathcal{E}_c$ or $\sigma_z^2 = \frac{N_0 \mathcal{E}_c}{2}$. (4.4.9)

In the absence of amplitude modulation, i.e., for the simple and noise-like signals, $\mathcal{E}_c = p_c t_c$ and

$$\sigma_z^2 = \frac{N_0 p_c}{2} t_c.$$

When $t = t_n < t_c$

$$\sigma_z^2 = \frac{N_0 p_c}{2} t_n.$$

Consequently, when, at the entrance, the optimum receiver of interference is present, at the output of correlator appears interference z_n having normal distribution and dispersion, proportional to energy of signal, and within the limits of the time of action of signal proportional to the time, calculated off its beginning.

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The distribution function for value z_n will take the form

$$w(z_n) = \frac{1}{\sqrt{2\pi}\,\sigma_z} e^{-\frac{z_n^2}{2\sigma_z^2}}. \quad (4.4.10)$$

Let us consider the work of diagram during the supplying on the entrance only of signal. In this case for the moment/torque $t=t_c$

$$z_c=\int_0^{t_c} c^2(t)\,dt=\mathcal{E}_c; \qquad (4.4.11)$$

at the intermediate moments of time $t=t_n$

$$z_{c,n}=\mathcal{E}_{c,n}.$$

For the signals, which do not have amplitude modulation $\mathcal{E}_{c,n}=p_c t_n$, moreover $t_n<t_c$ and time is counted off from the beginning of signal. It is important to emphasize that no other parameters of signal, including width of the spectrum of signal, connected with the presence of any modulation, affects the output of correlator.

Let us consider the work of diagram during the supplying to the mixture of interference and signal.

In this case for $t=t_c$

$$z_y=\int_0^{t_c} c^2(t)\,dt+\int_0^{t_c} n(t)\,c(t)\,dt. \qquad (4.4.12)$$

At intermediate points will be changed limit $t=t_n$. The obtained integrals were in detail examined earlier. It is obvious that $z_y=z_c+z_n$. Consequently, value z_y is a sum of determined value z_c and random variable z_n.

Then z_y can be considered as random variable with the nonzero average/mean value and the one-dimensional function of distribution will take the form

$$w(z_y)=\frac{1}{\sqrt{2\pi}\sigma_z}e^{-\frac{(z_y-z_c)^2}{2\sigma_z^2}}. \qquad (4.4.13)$$

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If we examine not one point, but entire time interval during which can function the expected signal, then z_y will be the random process, unsteady, with the changing average/mean value and the dispersion.

From the given formulas it is possible to find the ratio of the voltage of signal to the rms value of interference at the output of correlator, at moment/torque $t=t_0$

$$\frac{z_c}{\sigma_z}=\sqrt{\frac{2E_c}{N_0}}. \qquad (4.4.14)$$

At the moment of the termination of the action of signal this relation reaches maximum.

Consequently, on the output of correlator the ratio of the voltage of signal to disturbing voltage depends only on energy of signal and jamming density.

Fig. 4.4.3 gives the curves, which characterize the work of diagram, under the effect of one interference in the cases: a - interference at the entrance; b - the copy of signal (for simplicity of image the expected signal - simple pulse and it is undertaken the corresponding to it copy); c and d - output potential of correlator. From the figure one can see that with the unfavorable confluence of the facts interference can exceed the level of (threshold d).

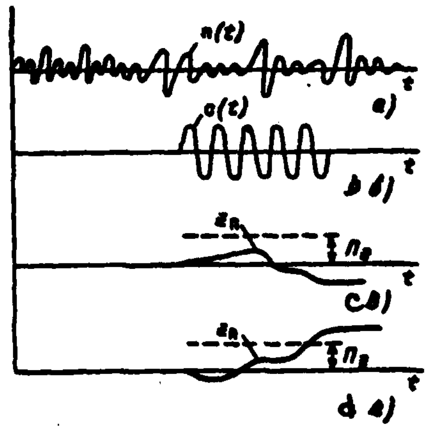

Fig. 4.4.3. Processes in diagram of correlator under effect of interference.

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Then value at the output of optimum receiver will correspond to hypothesis Γ_0—"Signal exists" although at the entrance signal it is absent. Fig. 4.4.4 gives the curves, which characterize the work of diagram under the effect of one signal in the cases: a - simple signal and its copy; b - value at the output of correlator; c - serrated signal, which consists of five impulses/momenta/pulses and its copy; d - value at the output of correlator; e - noise-like signal and its copy (for an example is undertaken five-impulse Barker code); f - value at the output of correlator.

For convenience in the comparison all signals are undertaken equal power and equal energies. At the moment of the termination of all three signals, at the output of correlator there will be one and the same value of value z_0, i.e., the stress/voltage, removed from the correlator, will be identical.

With the correctly selected threshold, which considers energy of signal and a priori data, the stress/voltage from one signal on the output of correlator will compulsorily exceed threshold and will occur correct detection.

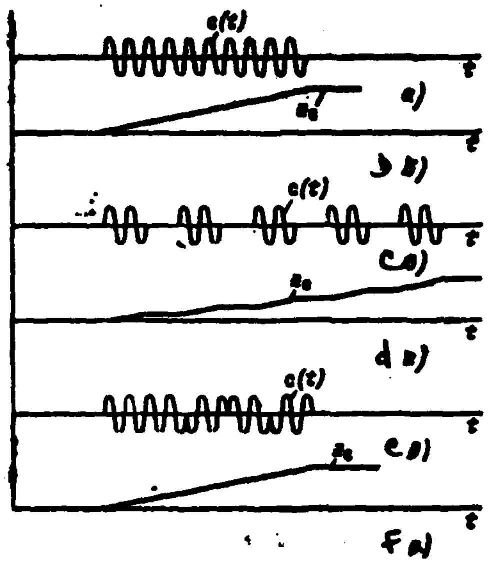

Fig. 4.4.4. Processes in diagram of correlator under effect of signal.

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Fig. 4.4.5 gives the curves, which characterize the work of diagram under the effect of the mixture of signal and interference. For simplicity of image the signal detected is undertaken simple pulse. The curves a - the mixture of signal and interference; b - copy of signal; c and d - value at the output of correlator. It is obvious that with the unfavorable confluence of facts the interference can so influence the output value of correlator, that it will not achieve threshold (curve d), in spite of the presence of signal. Will occur error in detection, since signal there will be passed. Since the dispersion of value z_{u} is determined by the jamming density N_{0} and does not depend on its dispersion σ_{s}^{2}, and value a_{0}, determined by signal, depends only on energy of signal, the results of detection will depend only on N_{0} and $\mathcal{E}_{0}$. Whatever was the signal - simple, complicated, noise-like, result will be one and the same, if their energy is identical. Consequently, from the point of view of the detection of signal in the fluctuating interferences there is no sense whatever to complicate signal, since this widens its spectrum and complicates the generator of the copy of signal.

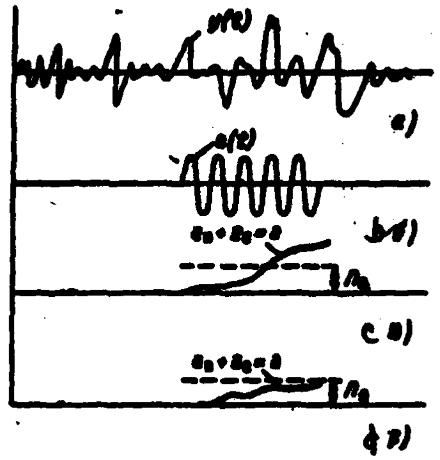

Fig. 4.4.5. Processes in diagram of correlator under effect of mixture of signal and interference.

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In all cases for the best detection at the given value of N, it is necessary to increase or the power of signal or its duration. The equivalence of these measures is observed only with respect to the natural fluctuating interferences. However, in the case of acting electronic jamming different signals behave differently, and as a result are obtained different resolutions and accuracies of measurements.

It is possible to formulate important the conclusion that in the necessary cases has the capability by any form to complicate the signal, in this case (during the correct design of receiving indicator) the conditions of the detection of signal against the background of natural fluctuating interferences do not deteriorate. For the evaluation of the quality of the detection of signal by optimum diagram it is necessary to find average/mean risk.

For the calculation ρ it is necessary to find $p(\Gamma_c/0)$ and $p(\Gamma_e/c)$. For the determination of these probabilities it is necessary, knowing distribution for values z_n and z_v, to find the probabilities of the fact that at the moment of the termination of the action of signal value z_n will exceed threshold, but value z_v will not achieve threshold.

$p(\Gamma_c/0)$ and $p(\Gamma_c/c)$ can be found with integration $w(z_n)$ within the limits from the threshold to ∞ and $w(z_v)$ in the limits from $-\infty$ to the threshold:

$$p(\Gamma_c/0)=\frac{1}{\sqrt{2\pi}\sigma_s}\int_{\Pi_s}^{\infty} e^{-\frac{z_n^2}{2\sigma_s^2}}\,dz_n, \qquad (4.4.15)$$

$$p(\Gamma_c/c)=\frac{1}{\sqrt{2\pi}\sigma_s}\int_{-\infty}^{\Pi_s} e^{-\frac{(z_v-\delta_c)^2}{2\sigma_s^2}}\,dz_v. \qquad (4.4.16)$$

Fig. 4.4.6 gives the functions of the distribution of values z_n and z_v, are shown the threshold and areas, which give $p(\Gamma_c/0)$ and $p(\Gamma_e/c)$.

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For obtaining the formulas, convenient for the numerical calculations, let us switch over to dimensionless quantities; let us designate:

$$\frac{z_0}{\sigma_z}=\xi;\ d\xi=\frac{dz}{\sigma_z};\ \frac{\Pi_z}{\sigma_z}=\xi_m=\frac{N_0}{2\sigma_z}\ln\Pi_1+\frac{\mathcal{E}_c}{2\sigma_z}$$

or, bearing in mind that $\sigma_z^2=\frac{\mathcal{E}_c N_0}{2}$, we obtain

$$\xi_m=\sqrt{\frac{N_0}{2\mathcal{E}_c}}\ln\Pi_1+\sqrt{\frac{\mathcal{E}_c}{2N_0}}.$$

After substituting dimensionless variable into expression (4.4.5), we will obtain

$$p(\Gamma_c/0)=\frac{1}{\sqrt{2\pi}}\int_{\xi_m}^{\infty}e^{-\frac{1}{2}\xi^2}d\xi=1-F(\xi_m)=$$

$$=1-F\left(\sqrt{\frac{N_0}{2\mathcal{E}_c}}\ln\Pi_1+\sqrt{\frac{\mathcal{E}_c}{2N_0}}\right). \qquad (4.4.17)$$

where $F(\xi_m)$ — tabulated integral;

$$F(\xi_m)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\xi_m}e^{-\frac{1}{2}\xi^2}d\xi.$$

For the criterion of ideal observer and with p(0)=p(c)=0.5 Π_1=1, then

$$p(\Gamma_c/0)=1-F\left(\sqrt{\frac{\mathcal{E}_c}{2N_0}}\right). \qquad (4.4.18)$$

After leading for $p(\Gamma_0/c)$ analogous conversions, we will obtain

$$p(\Gamma_0/c)=1-F\left(\sqrt{\frac{\mathcal{E}_c}{2N_0}}-\sqrt{\frac{N_0}{2\mathcal{E}_c}}\ln\Pi_1\right). \qquad (4.4.19)$$

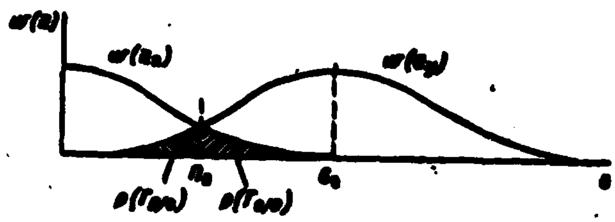

Fig. 4.4.6. Functions of distribution $z_н$ and z_y.

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For the criterion of ideal observer and with $\Pi_1=1$

$$p(\Gamma_o/c)=1-F\left(\sqrt{\frac{E_0}{2N_0}}\right). \qquad (4.4.20)$$

Most convenient in the radiolink systems for the comparison with other signals is the case $\Pi_1=1$.

In this case

$$p_{ош}=0,5[p(\Gamma_o/c)+p(\Gamma_o/0)]=1-F\left(\sqrt{\frac{E_0}{2N_0}}\right). \qquad (4.4.21)$$

The curves of dependence, which characterize the probability of errors on relation E_o/N_0, are given in Fig. 4.4.7. As can be seen from formulas, the probability of errors and minimum average/mean risk during the use of an optimum diagram of detection are determined by relation E_o/N_0. For using the obtained formulas it is necessary to know $r_з$, $r_{пр}$, p(c) and p(0). As it was noted above, in many instances, for example in the radio navigation and the radar, determination or selection of these values prove to be virtually impossible. Under these conditions the calculation ρ or $\rho_{\text{ош}}$ is also impossible, and the quality of the work of diagram must be evaluated according to another criterion.

From the relationships/ratios it follows that the a priori values, i.e., Π_1, affect only the value of threshold. The work of the remaining part of the diagram, namely correlator and the generator of the copy of signals, in any way of them does not depend. Thus, with uncertainty $r_{\text{пр}}$, $r_{\text{л}}$, $p(c)$ and p(0) cannot be realized the optimum diagram, which ensures minimum average/mean risk, only due to the uncertainty/indeterminacy of threshold. In this connection it is necessary to recall those considerations which were given above on Neumann-Pearson criterion. Initial requirement for the diagram in this case is the guarantee of given one $p(\Gamma_0/0)$.

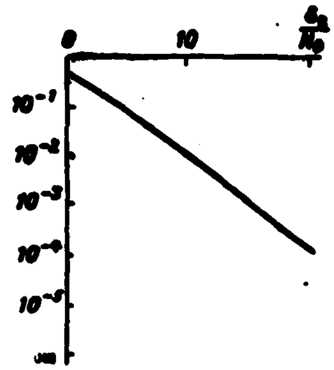

Fig. 4.4.7. Probability of errors for criterion of ideal observer.

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After using relationship/ratio (4.4.15), we will obtain

$$p(\Gamma_0/0) = 1 - F\left(\frac{\Pi_s}{\sigma_s}\right) = 1 - F\left(\sqrt{\frac{2}{N_0 E_s}}\,\Pi_s\right); \qquad (4.4.22)$$

further it is possible to obtain

$$\Pi_{s\,H-\Pi} = \sqrt{\frac{E_s N_0}{2}}\,\arg F\,[1 - p(\Gamma_0/0)], \qquad (4.4.23)$$

where argF designates inverse function; $\Pi_{s\,H-\Pi}$ — threshold during the use of Neumann-Pearson criterion on a priori data $[r_s, r_{\text{m}}, p(0), p(c)]$ does not depend.

Optimum diagram must provide maximum $p(\Gamma_0/c)$ or minimum $p(\Gamma_0/c)$ with assigned magnitude $p(\Gamma_0/0)$. $p(\Gamma_0/c)$ it is possible to compute by integration in the limits from threshold $\Pi_{s\,H-\Pi}$ to ∞, i.e.,

$$p(\Gamma_c/c) = \frac{1}{\sqrt{2\pi\sigma_c}} \int_{\Pi_0 H - \Pi}^{\infty} e^{-\frac{(z_y - \mathcal{E}_c)^2}{2\sigma_c^2}} dz_y. \quad (4.4.24)$$

After fulfilling integration, we will obtain

$$p(\Gamma_c/c) = 1 - p(\Gamma_0/c) =$$
$$= 1 - F\left\{\sqrt{\frac{2\mathcal{E}_c}{N_0}} - \arg F[1 - p(\Gamma_c/0)]\right\}. \quad (4.4.25)$$

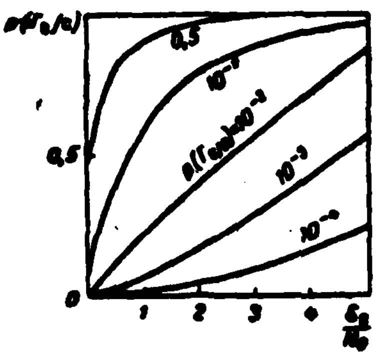

Fig. 4.4.8. Probability of detection of signal for Neumann-Pearson criterion.

Thus, the probability of the correct detection of signal is determined only by relation $\mathcal{E}_c/N_0$ and given value $p(\Gamma_c/0)$.

The curves of dependence $p(\Gamma_c/c)$ on $\mathcal{E}_c/N_0$ are given in Fig. 4.4.8. In the diagram where the threshold is determined on Neumann-Pearson criterion, the minimum of average/mean risk is not provided.

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§ 4.8. Use of matched filters in the diagrams of the optimum reception of signal with the random phase. Evaluation of the effect of the chance of phase. It is of interest to consider the possibility of replacing the two-channel correlator by matched filter. The characteristics of matched filter, form and value of the ejection of signal at its output do not depend on the phase of signal. Consequently, matched filters in the random and known phase of signal must be identical, but as a whole of diagram they must be characterized essentially by the elements/cells, included after filters.

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Matched filter has that special feature/peculiarity, that at the moment of time, which corresponds to the termination of the action of signal, it is analogous to correlator and, therefore, value for its output is also expressed by correlation integral. In the random phase of signal the diagram must reveal/detect not the instantaneous value of output potential of matched filter, but its envelope. At the output of matched filter is a stress/voltage of radio frequency whose phase is unknown, since the phase of the signal, which functions on the matched filter, is by chance. Is obvious that under these conditions is feasible only the one method of the development/detection of envelope, namely the inclusion/connection of ordinary detector. A synchronous detector cannot be used, since the phase of signal is not known. Consequently, matched filter with the detector can replace two-channel correlator. In this case it is necessary to have in mind which in the optimum correlation diagram, depicted in Fig. 4.7.2, not only is revealed/detected the envelope B, but with it are realized complicated nonlinear conversions, in accordance with function ln $I_0(2B/N_0)$.

In this respect the diagram with the matched filter and detector is convenient, since detector usually not only reveals/detects envelope, but also realizes its nonlinear conversions.

Fig. 4.8.1 gives function ln $I_0(x)$; as can be seen from figure, in the initial section it is close to the parabola and with $x \gg 1$ is close to the straight line. It is necessary to note that in many real detectors the detected stress/voltage has analogous dependence on the amplitude of subject to their entrance of the stress/voltage:

$$\Delta u = \ln I_0(b u_{\text{д}}),$$

where b - coefficient, depending on detector; Δu - detected stress/voltage; $u_{\text{д}}$— amplitude or the envelope of alternating voltage, subject on the detector.

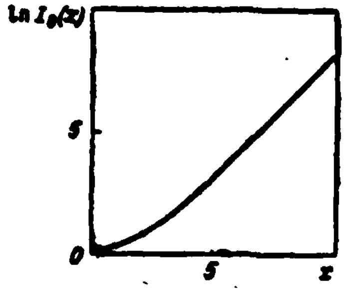

Fig. 4.8.1. Function ln $I_0(x)$.

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If we consider B as envelope (at point $t=t_0$) of radio-frequency output potential of matched filter, then expression $2B/N_0$ gives the value of this stress/voltage, led to another scale. Then it is obvious that has the capability this identification of the parameters of detector and given envelope at the output of filter, so that $bu_x=\frac{2B}{N_0}$; in this case detector will not only realize its basic function, but implement such nonlinear conversions which escape/ensue from the algorithm of the optimum processing of signal with the random phase. In this case the diagram of optimum reception/procedure will take the form, depicted in Fig. 4.8.2. However, this precise selection of the characteristics of detector and the values, supplied to it stresses/voltages, is not necessary, because, as already mentioned earlier, the indirect conversions of value B were necessary so that it would be possible during the detection with the minimum average/mean risk to produce its comparison with the simply specific threshold. To the same results it leads another version of the diagram in which the detector is taken by close one to the ideal, i.e., it simply reveals/detects the envelope of B, without its nonlinear conversions. But then threshold must be determined according to the more complicated rules. The diagram, which corresponds to this case, is given in Fig. 4.8.3. It is possible to use detectors with any characteristics, results in this case will not be changed, but will be changed the rule of the calculation of threshold. Matched filters possible to use also in the systems with the active pause.

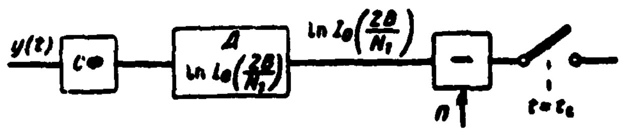

Fig. 4.8.2. Diagram with the matched filter for the optimum reception of signal with the random phase: CФ - matched filter; Д - detector; П - threshold device/equipment; $\ln I_0(2B/N_1)$ - the block of nonlinear conversions.

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The diagram of optimum recognition in this case will take the form, depicted in Fig. 4.8.4.

Let us compare now the work of diagrams with the correlators and the matched filters with the reception of signal with the random phase.

Their basic difference lies in the fact that diagram with the correlator computes one point of the autocorrelation function of signal, and matched filter at its output reproduces entire autocorrelation function on real time.

Single-channel correlator, as this follows from (4.4.5), under the effect of one signal will give

$$z_0 = \int_0^{t_c} c(t)c(t)\,dt.$$

If we create between the signal and its copy delay τ, then

$$z_0 = \int_0^{t_0} c(t)c(t-\tau)\,dt. \qquad (4.8.1)$$

Since the signal begins at moment/torque t=0 and ends at moment/torque $t=t_0$, it is possible to change the limits

$$z_0 = \int_{-\infty}^{+\infty} c(t)c(t-\tau)\,dt. \qquad (4.8.2)$$

Expression (4.8.2) with an accuracy to constant factor corresponds to expression for the autocorrelation function of signal. With the given one τ will be obtained one point of this function.

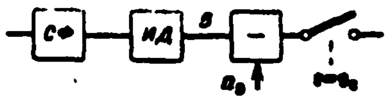

Fig. 4.8.3. Diagram with by matched filter with threshold Π_0 for the optimum reception of signals with the random phase: СФ - matched filter; ИД - ideal detector; Π_0 - threshold device/equipment.

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For example, for the pulse signal, by changing τ and by taking a large number of readings, it is possible to obtain the curve, analogous to that depicted in Fig. 2.2.1. In the two-channel correlator phase displacement of signal and copy transposes to affect result, and this diagram will reveal/detect the point of the enveloping autocorrelation function of signal.

In connection with this the correlation diagram in question for its functioning requires the knowledge of the values of signal delay, and output potential of diagram, preservable long time by constant, require "reset". The knowledge of signal delay in these diagrams is necessary also in order to manage the moment/torque of taking the reading - the acceptance by the diagram of solution. Since the delay is usually unknown or is changed, then for it is necessary to realize tracking with the help of the independent diagram (device/equipment).

The work of diagram must begin from the search and the tracking. All this complicates device/equipment and procedure of the start of system.

Diagram with the matched filter functions differently. The responses of matched filter to the signal it is described by the known expression

$$z_{\phi}(t)=\int_{0}^{t}\eta_{s}(t-T)c(T)\,dT. \qquad (4.8.3)$$

Since the signal c(t) begins at moment/torque t=0, i.e., for T≤0 c(t)≡0, lower limit can be changed on -∞.

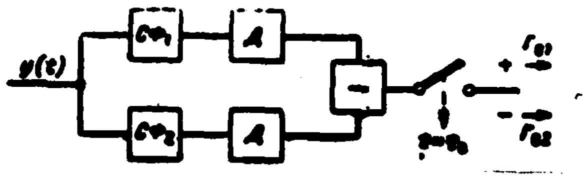

Fig. 4.8.4. Diagram with the matched filter of the optimum recognition of the signals: СФ - matched filters; Д - detectors.

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Response to the elements/cells of signal during, that follows after t=T, does not change the output stress/voltage, computed for moment/torque t. This gives the possibility to change upper limit on +∞, then

$$z_{ф}(t)=\int\limits_{-\infty}^{+\infty}\eta_{ф}(t-T)\,c(T)\,dT; \qquad (4.8.4)$$

but for the matched filter

$$\eta_{сф}(t)=c(t_0-t)$$

or

$$\eta_{сф}(t-T)=c(t_0-t+T).$$

Then

$$z_{сф}(t)=\int\limits_{-\infty}^{+\infty}c(t_0-t+T)\,c(T)\,dt, \qquad (4.8.5)$$

and for $t=t_0$ we obtain

$$z_{c\phi}(t=t_c)=\int_{-\infty}^{+\infty} c(T)c(T)dT, \qquad (4.8.6)$$

i.e., value $z_{c\phi}$ when $t=t_c$ corresponds to the value of the autocorrelation function of signal with $\tau=0$. At the intermediate moments of time from t=0 (for t=0 is accepted the moment/torque of the beginning of signal) to $t=t_c$ and when $t>t_c$ it is necessary to use general/common/total expression; however, it is more convenient to this expression to give another form. We will realize a countdown from $t=t_c$, for which let us introduce variable/alternating $t_1=t-t_c$.

Then the function, which describes a change in the response (output) of matched filter under the effect of signal, will take form,

$$z_{c\phi}(t_1)=\int_{-\infty}^{+\infty} c(T-t_1)c(T)dt, \qquad (4.8.7)$$

which coincides with the expression of the autocorrelation function of signal. However, in expression (4.8.7) t_1 - not fixed/recorded delay time of copy relative to the signal [as it takes place for τ in the expression (4.8.2)], but the current time, calculated off the moment/torque, which corresponds to the end/lead of the signal.

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Thus, the response of matched filter on real time reproduces the autocorrelation function of signal.

At the output of the detector, connected after matched filter, which corresponds to the diagram of the optimum detection of signal with the random phase, will be revealed/detected the envelope of the autocorrelation function of signal.

For the functioning of diagram with the matched filter and the detector the knowledge of signal delay is not compulsory, but the moment/torque of the maximum (peak) of signal will be changed in the dependence on signal delay. Therefore for accepting solution (i.e., the acceptance of hypotheses Γ_{0} either Γ_{0}, Γ_{c1} or Γ_{c2}) it is necessary to know signal delay and, using it, to manage the moment/torque of the switching on of the "decisive" circuit. On the diagrams (Fig. 4.8.2, 4.8.3 and 4.8.4) this is represented in the form of the key/wrench, which is closed at moment/torque $t=t_c$. Virtually difference from the correlation diagrams lies in the fact that for the beginning of the reception of signal is not required its search on the delay: it will be accepted and isolated with filter from the interferences immediately, as soon as it will appear. This simplifies the procedure of the introduction of system to action and it permits implemention of tracking the delay on the signal, observed at the output of matched filter, isolated from the interferences. For an example in Fig. 4.8.5 for the simplest noise-like signal, formed on the Barker code, they are depicted: a - signal; b - output stress/voltage on the matched filter with the detector, i.e., the envelope of the autocorrelation function of signal; dotted line there showed response with a change in signal delay on t_0; c - output stress/voltage on the two-channel correlator with $\tau=0$; d - output stress/voltage on the two-channel correlator when $\tau=t_0$. From all that has been previously stated, it follows that the matched filter "compresses" signals, if they have an autocorrelation function, characteristic for the noise-like signals (with one narrow main bang pipe), but it does not possess phase and time selection. If selection is required, then it must be realized by the diagrams, connected after filter.

Correlator does not "compress" signal, its output stress/voltage increases smoothly and it depends on energy of signal, and not on its autocorrelation function. Correlator possesses phase and temporary/time selectivity.

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With a change in signal delay the output stress/voltage of two-channel correlator at the moment of the termination of the action of reference signal (copy) will depend on the appropriate point of the enveloping autocorrelation function. Consequently, diagrams with the matched filters and the correlators will give close in essence, but results different in form.

The technical realization of filter and correlation diagrams encounters different difficulties. Filtration and accumulation in the matched filter is realized on the radio frequency. If signal complicated (pulse packet or noise-like signal), then its spectrum - amplitude-frequency or phase-frequency, and consequently, the characteristics of filter prove to be complicated and the production of such filters, especially with the large bases of signal, requires such high accuracy and stabilities of the elements of the networks which are ensured with the great technical difficulties or not at all can be achieved/reached.

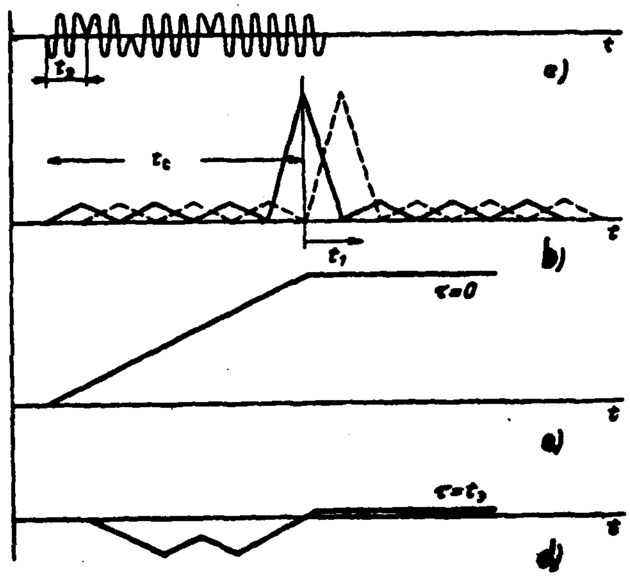

Fig. 4.8.5. Responses of matched filter and two-channel correlator to the noise-like signal.

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In the correlation diagrams the accumulation is realized in the simply realizable device/equipment - the integrator, which functions on the direct current. Basic difficulties are connected with the creation of the copy of signal, it is more precise with the control of its delay and of phase (if is used not two-channel, but single-channel correlator), or with the search for signal on the delay.

In connection with the difficulties of the realization of correlation and filter diagrams, especially for the cases of serrated and noise-like signals, considerable attention is paid by that combined - correlation-filtration diagrams. In these diagrams are used multipliers with the reference signal and matched filters.

As an example let us give the following combinations which can be used for the phase-keyed signal.

1. Is realized phase tracking of signal; on delay no search and tracking. Then in the multiplier, to which are supplied the radio code and continuous reference signal with the requiring phase, radio code is converted into alternating video code or into the packet of video pulses. Matched filter in this case with the use video-frequency delay lines is realized considerably simpler than on the radio frequency.

2. Is realized search and tracking on signal delay; on phase there is no tracking. Then in the multiplier, to which are supplied the radio code and alternating video code, radio code is converted into the prolonged unmodulated signal with the random phase, which can be treated in a comparatively simple "single-tooth" matched radio filter.

3. Search and tracking neither on delay nor on phase are realized. Then signal can be fed to the quadrature multipliers whose output stress/voltage will contain alternating video code or packet of video pulses; the amplitude of video pulses in the quadrature channels will be random and will be determined by random initial phase.

In each of the channels video codes are treated in the matched filter, after which is realized quadrature addition. The diagram indicated is given in Fig. 4.8.6.

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As the matched filters for the alternating ones it is video code, developed by quadrature multipliers, in it used the delay lines with the diversions/taps with which are connected the cascades/stages, which ensure the equality of the amplitudes of the summarized video pulses and change in their signs in the specific rule - code.

The authenticity of the detection of signal with the passive pause or the recognition of two nonzero orthogonal signals little is changed in the dependence on that, is known the phase of signals or it is by chance, i.e., it is not known, but it is constant. Loss in the energy of signal with the high authenticity, i.e., a small probability of errors p_{om} during the passive pause and the use/application of a criterion of ideal observer, small probability of false alarm $p(\Gamma_c/0)$ during the use of a criterion of Neumann-Pearson and small probability of the renaming of signals (p_{om}) in the systems with the active pause, proves to be small. An increase in the energy of signal by 10-25% in the usually utilized modes/conditions makes it possible to compensate for the deterioration in the authenticity, caused by the chance of phase. The chance of phase is substantially manifested only with weak signals ($E_c<W_0$). but these modes/conditions are not usually of interest due to the insufficient authenticity of detection (recognition), which in this case occurs.

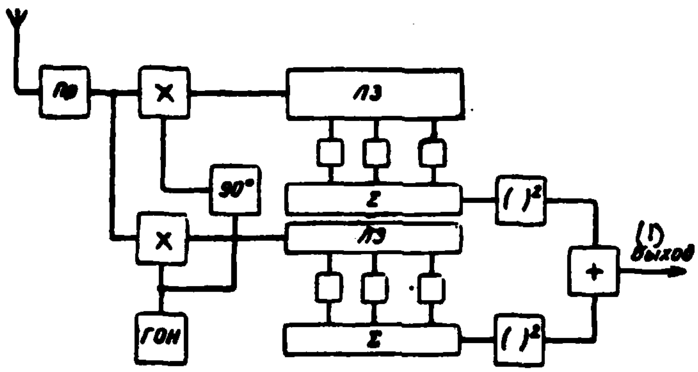

Fig. 4.8.6. The correlation-filtration diagram: Пр - receiver; X - multipliers; ГОН - reference generator; ЛЗ - delay line; ()² - square-law function generators; + - adder.

Key: (1). Output.

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The aforesaid directly escape/ensues from the curves, depicted in Fig. 4.7.4 and 4.7.9. Consequently, basic effect on the detection of signal in the noises proves to be the use of information about the constancy or known change in the phase of signal. Further information about the concrete/specific/actual value of phase does not give essential improvements in the isolation/liberation of signal from the interferences.

Since the technical realization of correlation and filter diagrams substantially is simplified in the case of the failure of the use of a concrete/specific/actual value of the phase of signal and during the orientation for the chance, since in this case is not required phase tracking, but deterioration of the quality of reception/procedure is small, the greatest use/application have the systems, in which are used the signals with the random phase.

However, to these signals in the radio communication are characteristic some limitations. In the random phase of signal it is not possible to create orthogonal signals by phase displacement to 90° and opposite signals by phase displacement on 180°. Orthogonality is achieved by shift/shear in the frequency, which widens the section of frequency band, utilized by a system. Tendency to reduce this limitation led to creation of systems with the active pause, and with the relative phase manipulation (OFM or OFT - relative phase telegraphy). In these systems phase manipulation is used also in the random phase of signal. This is reached by the fact that during transmission zero phase of message is not changed with relatively with previous, but during the transmission of one occurs a change in the phase. In the receiver occurs the comparison of the phases of two adjacent messages. Systems with OFM received wide acceptance. The properties of such systems are analyzed in the series/row of works [1.2, 1.6], and therefore on them we will not dwell. It is necessary to only note that since in these systems are used the signals with the random phase, the authenticity, ensured by these systems, will be more badly than for the case of using the signals with the known phase. Detailed analysis shows that during the use of the most appropriate solutions of system with OFM give the results, close to the results for the optimum systems of the recognition of signals with the random phase.

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§ 4.9. Optimum detection of signal with the fluctuating phase. Let us consider the now optimum detection of the signal, which has the fluctuating phase. If the phase of signal in the process of reception/procedure is changed or fluctuates, then the possibility of its use for the detection of signal hinders. From the physical considerations it is possible to assume that the more the fluctuations of phase is observed in the process of the reception of signal, the less the benefit in the detection of signal it can bring. For the mathematical solution of problem it is necessary to proceed from the fact that in this case the phase of signal is not random variable, but random process or random function of time.

Let us consider signal c(t), with fluctuating phase $\varphi_c(t)$, where $\varphi_c(t)$— the random function of time. Other parameters we consider known. The random function of time is characterized by the distribution function and by autocorrelation function. Let us assume that $w(\varphi_c) = \frac{1}{2\pi}$ is known $B_{\varphi c}(\tau)$ or energy spectrum $G_{\varphi c}(\omega)$; they are determined by the transient nature of random process, i.e., by nature of those factors which cause the fluctuations of phase. Knowing $B_{\varphi c}(\tau)$, it is possible to find time or interval of correlation $\tau_{\kappa\varphi}$. The values of phase, divided by this interval, can be considered statistically independent variables.

If $t_c < \tau_{\kappa\varphi}$, then we have case examined above in effect of constant (for the time of action of signal) phase.

If $t_c > \tau_{\kappa\varphi}$, then for the time of action of signal phase manages to take a large number of independent random values.

In this case it is possible to use the following model of signal: to consider that the signal consists of the sequence of the elementary signals (for the brevity - elements/cells), each of which has duration $t_i = \tau_{\kappa\varphi}$ and random phase.

The total number of elements/cells will comprise $k = t_c/\tau_{\kappa\varphi}$, consequently

$$c[t, \varphi_c(t)] = c(t, \varphi_{c1}, \varphi_{c2}) = \sum_{i=1}^{k} A_i \cos(\omega_0 t + \varphi_{ci}), \qquad (4.9.1)$$

$$A_i = \text{const} \quad \text{при} \quad i\tau_{\kappa\varphi} < t < (i+1)\tau_{\kappa\varphi}$$

and

$$A_i = 0 \quad \text{при} \quad i\tau_{\kappa\varphi} > t > \tau_{\kappa\varphi}(i+1).$$

Key: (1). with.

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Thus signal with the fluctuating phase can be considered as signal with k by the random parameters.

The distribution function for the mixture of signal and interference will take the form

$$w(y_1, y_2, \ldots/\varphi_{c1}, \varphi_{c2}, \ldots, \text{с-п}) = \frac{1}{(2\pi\sigma_n^2)^{n/2}} e^{-\frac{1}{N_0}\int\limits_0^{t_c}\left[y(t)-\sum\limits_{l=1}^{k} A_l \cos(\omega_l t+\varphi_{cl})\right]^2 dt}. \quad (4.9.2)$$

The function of the distribution of interference is known.

Likelihood ratio (conditional) will be registered as

$$l(y/\varphi_{c1}, \varphi_{c2}, \ldots) = e^{-\frac{1}{N_0}\int\limits_0^{t_c}\left[\sum\limits_{l=1}^{k} A_l \cos(\omega_l t+\varphi_{cl})\right]^2 dt} \times$$

$$\times e^{\frac{2}{N_0}\int\limits_0^{t_c} y(t)\left[\sum\limits_{l=1}^{k} A_l \cos(\omega_l t+\varphi_{cl})\right] dt}. \quad (4.9.3)$$

The elements/cells of signal are orthogonal; therefore

$$\frac{1}{N_0}\int\limits_0^{t_c}\left[\sum_{l=1}^{k} A_l \cos(\omega_l t+\varphi_{cl})\right]^2 dt = \frac{1}{N_0}\sum_{l=1}^{k} \mathcal{E}_l = \frac{\mathcal{E}_c}{N_0}.$$

Correlation integral can be subjected to the conversions

$$\frac{2}{N_0}\int_0^{t_c} y(t)\left[\sum_{i=1}^{k} A_i\cos(\omega_0 t+\varphi_{0i})\right]dt=$$

$$=\frac{2}{N_0}\sum_{i=1}^{k}\int_0^{t_c} y(t)\,A_i\cos(\omega_0 t+\varphi_{0i})\,dt.$$

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It is obvious that

$$\int_0^{t_c} y(t)\,A_i\cos(\omega_0 t+\varphi_{0i})\,dt=$$

$$=\int_{i\tau_{\text{ЭЛ}}}^{(i+1)\tau_{\text{ЭЛ}}} y(t)\,A_i\cos(\omega_0 t+\varphi_{0i})\,dt,$$

since A_i differs from zero only by this time interval. Consequently, each of the components/terms/addends of the sum to which is led correlation integral, it is the correlation integral, which corresponds to the reception/procedure of each of the elements/cells.

Thus, the optimum reception of signal with the fluctuating phase leads to that, then preliminarily must be obtained correlation integral for each of the elements/cells.

Each of the elements/cells is signal with the known parameters, except phase, which is random variable.

Correlation integral with the reception of this signal was obtained earlier.

Therefore, lowering conversions, immediately let us register

$$\int_{l\tau_{\kappa\varphi}}^{(l+1)\tau_{\kappa\varphi}} y(t)\, A_i \cos(\omega_0 t + \varphi_{ci})\, dt = B_i \cos(\theta_i - \varphi_{ci}), \quad (4.9.4)$$

where

$$B_i = \sqrt{\eta_i^2 + \gamma_i^2}; \quad \mathrm{tg}\,\theta_i = \frac{\gamma_i}{\eta_i},$$

$$\gamma_i = \int_{l\tau_{\kappa\varphi}}^{(l+1)\tau_{\kappa\varphi}} y(t)\, A_i \sin\omega_0 t\, dt, \quad (4.9.5)$$

$$\eta_i = \int_{l\tau_{\kappa\varphi}}^{(l+1)\tau_{\kappa\varphi}} y(t)\, A_i \cos\omega_0 t\, dt. \quad (4.9.6)$$

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Value B_i can be obtained with the help of the two-channel correlator with the quadrature channels or with the help of the filter, matched with the element/cell of signal and detector.

After substituting the obtained expressions into equation (4.9.3), we will obtain

$$l(y/\varphi_{c1}, \varphi_{c2}, \ldots) = e^{-\frac{E_c}{N_0}} \times$$
$$\times e^{\frac{2}{N_0}\sum_{i=1}^{k} B_i \cos(\theta_i + \varphi_{ci})}. \quad (4.9.7)$$

The obtained likelihood ratio is conditional, since it is correct for some specific combination of phases $\varphi_{c1}, \varphi_{c2}\ldots$

For obtaining the likelihood ratio, which does not depend on the combination of phases, let us fulfill the integration

$$l(y)=\underbrace{\int\int\ldots\int}_{k\ \text{pas}} w(\varphi_{c1},\varphi_{c2},\ldots)\times$$
$$\times l(y/\varphi_{c1},\varphi_{c2},\ldots)\,d\varphi_{c1}d\varphi_{c2}\ldots$$

The phases of elements/cells are accepted by statistically independent variables. Then

$$w(\varphi_{c1},\varphi_{c2},\ldots)=\left(\frac{1}{2\pi}\right)^k,$$

$$l(y)=\underbrace{\int\int\ldots\int}_{k\ \text{pas}} e^{-\frac{\mathcal{E}_c}{N_0}}\left(\frac{1}{2\pi}\right)^k\times$$
$$\times e^{\frac{2}{N_0}\sum_{i=1}^{k}B_i\cos(\theta_i-\varphi_{ci})}\,d\varphi_{c1}d\varphi_{c2},\ldots,=$$
$$=e^{-\frac{\mathcal{E}_c}{N_0}}\prod_{i=1}^{k}\int_{-\pi}^{+\pi}\frac{1}{2\pi}e^{\frac{2}{N_0}B_i\cos(\theta_i-\varphi_{ci})}\,d\varphi_{ci}=$$
$$=e^{-\frac{\mathcal{E}_c}{N_0}}\prod_{i=1}^{k}I_0\left(\frac{2B_i}{N_0}\right). \qquad (4.9.8)$$

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For the conversions is used the fact that the integrand can be represented in the form of product of exponential functions. In this case the variable/alternating are divided.

Let us pass to the logarithm

$$\ln l(y) = -\frac{\mathcal{E}_c}{N_0} + \sum_{i=1}^{k} \ln I_0\left(\frac{2B_i}{N_0}\right). \qquad (4.9.9)$$

Detection conditions during the acceptance of the hypothesis of signal will take the form

$$\sum_{i=1}^{k} \ln I_0\left(\frac{2B_i'}{N_0}\right) > \ln \Pi_1 + \frac{\mathcal{E}_c}{N_0}. \qquad (4.9.10)$$

The obtained expression gives the optimum algorithm of processing signal with the fluctuating phase and it makes it possible to compile diagram of optimum reception/procedure. The diagram, shown on Fig. 4.9.1, consists of: computing device (B_i) (two-channel quadrature correlator or matched with the element/cell of signal filter and detector); the block of nonlinear conversions ($\ln I_0$); the summator Σ, which accumulates the results of the reception/procedure of each of the elements/cells, and threshold device/equipment Π. Instead of the ideal the detector, which ensures nonlinear conversion during the detection.

Let us consider special cases of strong and weak signal.

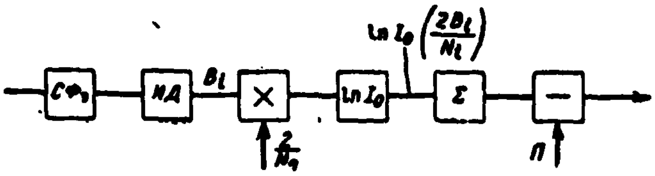

Fig. 4.9.1. The diagram of the optimum reception of signal with fluctuating phase: Сф₁ - matched filter; ИД - ideal detector; ln I_0. - block of nonlinear conversions; Σ - summator; П - threshold device/equipment.

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For strong signal $(B_i \gg N_0)$

$$\ln I_0\left(\frac{2B_i}{N_0}\right) \approx \frac{2B_i}{N_0} \qquad (4.9.11)$$

Then the condition of accepting the hypothesis Γ_0 will take the form

$$H=\sum_{i=1}^{k} B_i > \frac{N_0}{2}\ln \Pi_1 + \frac{E_0}{2} = \Pi_2. \qquad (4.9.12)$$

When $\Pi_1 = 1$ $\Pi_2 = \frac{E_0}{2}$, where Π_2 — threshold.

The diagram, which corresponds to this algorithm and which contains ideal detector, is given in Fig. 4.9.2.

For weak signal $(B_i \ll N_0)$

$$\ln I_0\left(\frac{2B_i}{N_0}\right) \approx \frac{B_i^2}{N_0^2}\left(1-\frac{B_i^2}{2N_0^2}\right). \qquad (4.9.13)$$

Using approximation/approach (4.9.13), we obtain the conditions of accepting the hypothesis Γ_0 upon consideration of term in the brackets only in the average/mean value.

$$H=\sum_{i=1}^{k} B_i^2 > N_0^2 \ln \Pi_1 + \mathcal{E}_c N_0 + 0{,}5\mathcal{E}_i\mathcal{E}_0 = \Pi_n; \qquad (4.9.14)$$

with $\Pi_1=1$

$$\Pi_n = \mathcal{E}_c N_0\left(1+0{,}5\frac{\mathcal{E}_i}{N_0}\right).$$

For obtaining B_i^2 it is possible to use a matched filter and the square law detector.

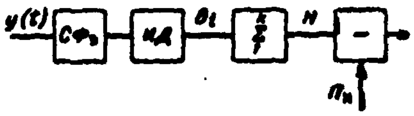

Fig. 4.9.2. The diagram of the optimum reception of signal with the fluctuating phase for threshold $П_н$: $СФ_0$ - matched filter; ИД - ideal detector; Σ - adder; $П_н$ - threshold.

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The diagram, which realizes the algorithm of optimum detection (4.9.14), is given in Fig. 4.9.3. The blocks, entering the diagrams in Fig. 4.9.2 and 4.9.3, can be realized. Matched filter $СФ_0$ must have the frequency characteristic of form $\frac{\sin x}{x}$, since envelope for each element/cell is accepted rectangular. Virtually it is possible to be satisfied by quasi-optimal filter, for example, with the gaussian characteristic, having selected its passband. Detectors with the characteristics, close to the quadratic or the ideal, are also realized. The addition of discrete/digital values B_i or B_i^2 can be obtained on the delay line with the diversions/taps. If signal with the fluctuating phase is continuous, then addition can be substituted by integration, with a change in the level of threshold.

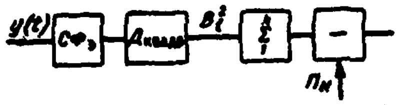

Fig. 4.9.3. Diagram of the optimum detector of a weak signal with a fluctuating phase: СФ₃ - matched filter; Дквадр - quadrature detector; Σ - adder; Пн - threshold.

For the strong signal

$$\sum_{i=1}^{k} B_i \approx \frac{1}{\tau_{кч}} \int_0^{t_c} B_c(t)\,dt.$$

Conditions for the selecting of hypothesis Γ_1

$$\int_0^{t_c} B(t)\,dt > \frac{N_0 \tau_{кч}}{2} \ln \Pi_1 + \frac{\mathcal{E}_c}{2} t_c = \Pi_L. \qquad (4.9.15)$$

For the weak signal

$$\sum_{i=1}^{k} B_i^2 \approx \frac{1}{\tau_{кч}} \int_0^{t_c} B^2(t)\,dt.$$

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The condition for the selecting of hypothesis Γ_0

$$\int_0^{t_c} B^2(t)\,dt > N_0^2 \tau_{кч} \ln \Pi_1 + \mathcal{E}_c t_c N_0 + 0{,}5\mathcal{E}_c^2 t_c. \quad (4.9.16)$$

For the understanding of the processes, which occur in the diagram, and the calculation of the probability of errors it is necessary to find the distribution functions for that value which is equal with the threshold. The distribution functions for the envelope at the output of matched filter or for the value at the output of quadrature correlator for the cases of the mixture of signal and interference and one interference were obtained earlier. With one interference distribution B_m is subordinate to Rayleigh's law.

In the presence of signal the function of distribution B_m is expressed by generalized Rayleigh's law

$$w(B_{t_y})=\frac{B_{t_y}}{\sigma_t^2}I_0\left(\frac{B_{t_y}\mathcal{E}_t}{\sigma_t^2}\right)e^{-\frac{B_{t_y}^2+\mathcal{E}_t^2}{2\sigma_t^2}}, \quad (4.9.17)$$

where $\mathcal{E}_t$— energy of the element/cell of signal;

$$\sigma_t^2=\frac{N_0\mathcal{E}_t}{2}.$$

For obtaining the functions of the distribution of the value, been congruent/equated with the threshold, it is necessary to take into account that B_t undergoes nonlinear conversions and subsequent addition.

The conclusion/output of relationships in general form is connected with cumbersome calculations. Therefore we will be bounded to special cases of strong and weak signals.

With strong signal ($\mathcal{E}_t > \sigma_t$) the generalized function of Rayleigh can be approximated by the normal distribution

$$w(B_{t_y})=\frac{1}{\sqrt{2\pi}\,\sigma_t}e^{-\frac{(B_{t_y}-\mathcal{E}_t)^2}{2\sigma_t^2}}. \quad (4.9.18)$$

In this case $B_{t_y} > N_0$ is correct expression (4.9.12). Consequently, addition undergoes directly value B_{t_y}.

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As is known with the addition of the statistically independent normal random variables the normal law of distribution is retained. In this case the dispersion and average/mean store/add up.

Then the function of the distribution of value H in the presence of signal can be easily found

$$w(H_y)=\frac{1}{\sqrt{2\pi}\,\sigma_{H_y}}e^{-\frac{(H_y-B_c)^2}{2\sigma^2_{H_y}}}, \qquad (4.9.19)$$

where

$$\sigma^2_{H_y}=k\sigma^2_{\text{ч}}=\frac{kN_0B_{\text{ч}}}{2}=\frac{N_0B_c}{2}$$

and

$$B_c=B_{\text{ч}}k.$$

In the absence of signal the distribution function for $H_{\text{ш}}$ because of the addition approaches normal. For the determination of average and dispersion after addition we will use the fact that for the Rayleigh distribution average $m_1(B_{i\text{ш}})=1{,}25\,\sigma_i$ and dispersion $\sigma^2_{Bi}=0{,}43\sigma^2_{\text{ш}}$.

Then with one interference the function of distribution $H_{\text{ш}}$ will take the form

$$w(H_{\text{ш}})=\frac{1}{\sqrt{2\pi}\,\sigma_{H_{\text{ш}}}}e^{-\frac{(H_{\text{ш}}-m_1(H_{\text{ш}}))^2}{2\sigma^2_{H_{\text{ш}}}}}, \qquad (4.9.20)$$

where

$$\sigma^2_{H_{\text{ш}}}=k\sigma^2_{Bi}\approx\frac{kN_0B_{\text{ч}}}{2}0{,}43\approx\frac{N_0B_c}{4};$$

$$m_1(H_{\text{ш}})=km_1(B_{i\text{ш}})\approx k1{,}25\sqrt{\frac{N_0B_{\text{ч}}}{2}}\approx B_c1{,}25\sqrt{\frac{N_0}{2B_{\text{ч}}}}.$$

From the obtained results it follows that the function of the distribution of parameter H, its average/mean value and dispersion in the presence of strong signal with the fluctuating phase the same as for the signals with the known and random phase. This coincidence is understandable.

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If signal is strong ($\mathcal{E}_i > N_0$), then $B_{iy} > \sigma_i$, detection occurs without the suppression and does not have a value where is realized accumulation - before the detector or afterward. In the absence of signal H_n it is distributed according to the normal law with the nonzero average. This strongly differs from distribution in the known phase (normal law with the zero average) and the random phase (Rayleigh's law).

with weak signal ($\mathcal{E}_i < N_0$) the generalized function of Rayleigh is approximated by Rayleigh's function, i.e., it remains the same as for one interference, but with the increased factor σ. Then $w(B_{iy})$ has the form

$$w(B_{iy}) = \frac{B_{iy}}{\sigma_{iy}^2} e^{-\frac{B_{iy}^2}{2\sigma_{iy}^2}}, \qquad (4.9.21)$$

$$\sigma_{iy}^2 = \sigma_i^2 + \frac{B_i^2}{2} = \sigma_i^2\left(1 + \frac{B_i^2}{2\sigma_i^2}\right) = \sigma_i^2\left(1 + \frac{\mathcal{E}_i}{N_0}\right). \quad (4.9.22)$$

The used approximation/approach escape/ensues from the following. It is known that when $B_i < N_i$

$$w(B_{i_y}) = \frac{B_{iy}}{\sigma_i^2} e^{-\frac{B_{iy}^2 + B_i^2}{2\sigma_i^2}} I_0\left(\frac{B_{iy} B_i}{\sigma_i^2}\right) \approx \frac{B}{\sigma_i^2} e^{-\frac{B_{iy}^2 + B_i^2}{2\sigma_i^2}} \times$$

$$\times \left(1 + \frac{B_{iy}^2 B_i^2}{4\sigma_i^4}\right) \approx \frac{B_{iy}}{\sigma_i^2}\left(1 - \frac{B_i^2}{2\sigma_i^2}\right) e^{-\frac{B_{iy}}{2\sigma_i^2}\left(1 - \frac{B_i}{2\sigma_i^2}\right)}.$$

Then

$$\sigma_{iy}^2 = \sigma_i^2 + \frac{B_i^2}{2}.$$

with the weak signal occurs addition B_{iy}^2 or B_{in}^2.

Consequently, the function interesting us of the distribution of the value, been congruent/equated with the threshold, can be found, if we preliminarily obtain the distribution function for B_{iy}^2.

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It is known that if any value has Rayleigh distribution, then the square of this value has the exponential distribution

$$w(B_{iy}^2) = \frac{1}{2\sigma_{iy}^2} e^{-\frac{B_{iy}^2}{2\sigma_{iy}^2}},$$

$$w(B_{in}^2) = \frac{1}{2\sigma_i^2} e^{-\frac{B_{in}^2}{2\sigma_i^2}}. \qquad (4.9.23)$$

Known also that if the components/terms/addends of sum have exponential distribution, then sum k of such components/terms/addends has χ^2_{2k} a distribution, i.e., χ^2- distribution with 2k degrees of freedom. The function of distribution χ^2 with 2k degrees of freedom takes the form

$$w_{1,2,\ldots,k}(x)=\frac{1}{2^k\Gamma(k)}\chi^{k-1}e^{-\frac{k}{2}}. \qquad (4.9.24)$$

With increase k χ^2_{2k}- distribution approaches normal with the nonzero average. For the demonstrative interpretation of results it is more convenient not to resort to χ^2-distribution should be used the approximation method, based on the approximation of this distribution with normal. It is possible to consider that values H_v and H_n will have the normal distribution whose parameters - average, and dispersion can be found, if are known average and dispersion for B^2_{4n} and B^2_{sn}.

For B^2_{4n} from the exponential law of distribution it is possible to obtain

$$\sigma^2_{B^2 n}=4\sigma^4_s,\ m_1(B^2_{sn})=2\sigma^2_s, \qquad (4.9.25)$$

then

$$\sigma^2_{Hn}=k\sigma^4_s\approx\frac{1}{k}(\mathcal{B}_0N_0)^2,$$

$$m_1(H_n)=2k\sigma^2_s=N_0\mathcal{B}_0. \qquad (4.9.26)$$

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Analogously we find for the mixture of signal and interference

$$\sigma^2_{Hy} = k\sigma_y^4\left(1+\frac{\mathcal{E}_y}{N_0}\right)^2,$$

$$m_1(H_y) = 2k\sigma_y^2\left(1+\frac{\mathcal{E}_c}{N_0}\right). \quad (4.9.27)$$

The relationship/ratio between the root-mean-square and average/mean significance of a deviation will be

$$\frac{\sigma_{Hn}}{m_1(H_n)} \approx \frac{1}{\sqrt{k}}. \quad (4.9.28)$$

Consequently, with large k average considerably exceeds standard deviation. Upon the appearance of a weak signal change undergoes average/mean value.

An increment in the average/mean value in the presence of signal is equal to $\mathcal{E}_0\mathcal{E}_c$ and its relation to the average is equal to $\frac{\mathcal{E}_c}{N_0}$

$$\Delta m_1 = \mathcal{E}_0\mathcal{E}_c. \quad (4.9.29)$$

The functions of the distribution of value H, been congruent/equated with the threshold, are obtained earlier for the case when the amplitude of copy or the amplification of matched filter are determined with signal amplitude. This form of recording is convenient for calculating the probability of errors. For the examination of the processes, which occur in the diagram, it is more correct to use distribution functions for the case of the fixed/recorded amplitude of copy.

Let us give final results, after dropping/omitting the conclusions/outputs

$$\sigma_{i0}^2 = \frac{S_{i0} N_0}{2},$$

$$w(B_{im0}) = \frac{B_{im0}}{\sigma_{i0}^2} e^{-\frac{B_{im0}^2}{2\sigma_{i0}^2}}. \qquad (4.9.30)$$

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For the strong signal

$$w(B_{iy0})=\frac{1}{\sqrt{2\pi}\sigma_{i0}}e^{-\frac{(B_{iy0}-\mathcal{E}_{i0}A_c)^2}{2\sigma_{i0}^2}}, \quad (4.9.31)$$

$$m(H_{H0})=1{,}25\mathcal{E}_0\sqrt{\frac{N_0}{2\mathcal{E}_{i0}}}=1{,}25k\sqrt{\frac{N_0\mathcal{E}_{i0}}{2}}, \quad (4.9.32)$$

$$\mathcal{E}_0=\mathcal{E}_{i0}k,$$

$$\sigma_{H\pi0}^2\approx\frac{N_0\mathcal{E}_0}{4}, \quad (4.9.33)$$

$$m_1(H_{0y})=\mathcal{E}_0A_0, \quad (4.9.34)$$

$$\sigma_{Hy0}^2=\frac{N_0\mathcal{E}_0}{2}; \quad (4.9.35)$$

threshold $\Pi_{H0}=\frac{\mathcal{E}_0A_c}{2}$ with Π=1.

The functions of distribution H_0 for this case and their comparison with the distribution functions for B_0 and Z_0 are given in Fig. 4.9.4.

For the weak signal

$$w(B_{ty0})=\frac{|B_{ty0}|}{\sigma_{ty0}}e^{-\frac{B_{ty0}^2}{2\sigma_{ty0}^2}};$$

average/mean with one interference

$$m_1(H_{H0})=N_0\mathcal{E}_0; \quad (4.9.36)$$

average with the mixture

$$m_1(H_{y0}) = N_0\mathcal{B}_0\left(1 + \frac{\mathcal{E}_{40}A_c^2}{N_0}\right); \qquad (4.9.37)$$

the threshold

$$\Pi_{H0} = N_0\mathcal{B}_0\left(1 + 0{,}5\,\frac{\mathcal{E}_{40}A_c^2}{N_0}\right); \qquad (4.9.38)$$

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the standard deviation

$$\sigma_{Hm0} \approx \sigma_{Hy0} = \frac{1}{\sqrt{k}}\mathcal{B}_0N_0; \qquad (4.9.39)$$

an increment in the average/mean upon the appearance signal

$$\Delta m = \mathcal{B}_0A_c^2\mathcal{B}_{40} = \mathcal{B}_0\mathcal{B}_4;$$

the relation of an increment in the average to the standard deviation

$$\Delta m_1/\sigma_{Hm0} = \frac{\mathcal{E}_4}{N_0}\sqrt{k};$$

the relation of increment to the average from the interference

$$\frac{\Delta m_1(H_0)}{m_1(H_{m0})} = \frac{\mathcal{E}_{40}A_c^2}{N_0} = \frac{\mathcal{E}_4}{N_0}.$$

The distribution functions for this case are given in Fig. 4.9.5; is there for the comparison given $w(B_{m0})$ and $w(B_{y0})$.

From the obtained results it follows that in the fluctuating phase the functions of the distribution of the value, been congruent/equated with the threshold, differ significantly from the function of the distribution of the corresponding values in the known and random phases, especially for the case of weak signal. In essence this difference depends on the fact that the accumulation is produced after detector on the direct current.

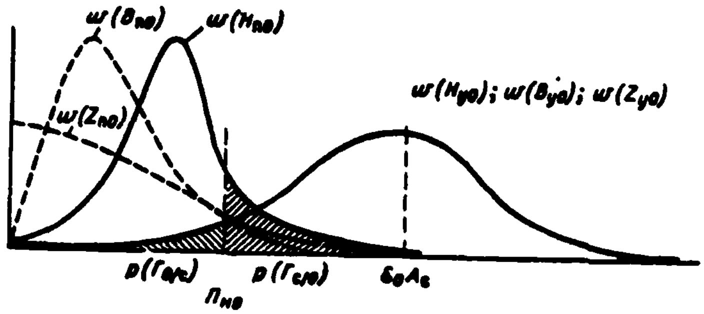

Fig. 4.9.4. Functions of distribution H_0 with the strong signal.

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Accumulation or isolation/liberation of signal from the interferences to the detector is limited to that time interval during which the phase can be considered constant. For the signal with the fluctuating phase the results strongly are changed in the dependence on the ratio of energy of the element/cell of signal $\mathcal{E}_1$, in the limits of duration of which the phase can be considered constant, to the jamming density N_0.

With the strong signal when $\mathcal{E}_1 > N_0$, a basic effect on detection has total energy of signal $\mathcal{E}_0$, but authenticity must be somewhat worse than for the signal with the known and random phases due to the greater action of interferences, caused by accumulation after detector.

With weak signal $\mathcal{E}_i < N_0$, optimum diagram and function the distributions of the value, been congruent/equated with the threshold, are changed. In contrast to the signal with the random phase the concept of strong and weak signals is connected not with the energy of entire signal, but with the energy of the element/cell in limits of which the fluctuating phase can be considered constant.

The work of diagram with the weak signal is determined not only total energy of signal, but also by ratio $\mathcal{E}_i/N_0$ or $\mathcal{E}_{i0}A_c^2/N_0$. Is substantial the fact that for small ones $\mathcal{E}_i/N_0$ the increment in the average, caused by the action of signal, in relative values is very small and equal to $\mathcal{E}_i/N_0$ or $\mathcal{E}_{i0}A_c^2/N_0$; this means that the requirements for the stability of the level of threshold prove to be very rigid. The level of threshold is determined in essence by expression $N_0/\mathcal{E}_0$; with insignificant changes in amplification or interference level it can occur its displacement/movement to the value, commensurate with the "increment".

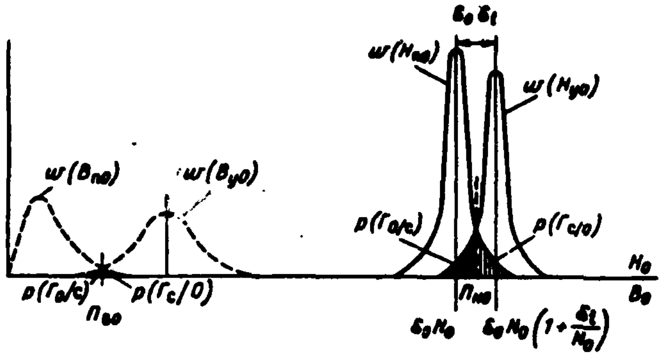

Fig. 4.9.5. Functions of distribution H, with the weak signal.

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Consequently, for the stability of the mode/conditions of diagram, to its amplification factor and to the interference level must be presented stringent requirements, in many instances difficultly feasible virtually.

It is obvious that the obtained distribution functions make it possible to consider the work of diagram, also, during the use of the fixed/recorded threshold (i.e. Neumann-Pearson criterion). In this case the level of threshold is determined by permissible value $p(\Gamma_c/0)$ and during the fixed/recorded amplitude of copy or the fixed/recorded amplification of matched filter on signal amplitude does not depend (but it depends on its duration, i.e., β_0). Signals with the fluctuating phase can be used also for the systems with the active pause; however, to stop on this question is impossible.

Let us consider now the probability of errors. Since the distribution functions for the value, been congruent/equated with the threshold, are found, the probabilities of errors can be obtained by their integration within the appropriate limits.

With the strong signal

$$p(\Gamma_c/0)=\int_{-\Pi_h}^{\infty}\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}[h_n-m_1(h_n)]^2}dh_n, \quad (4.9.40)$$

where

$$h_n=\frac{H_n}{\sigma_{Hn}};\; m_1(h_n)=\frac{m_1(H_n)}{\sigma_{Hn}}=\sqrt{k};$$

$$\Pi_{hn}=\Pi_H/\sigma_{Hn};\; \text{при } \Pi_1=1\; \Pi_{hn}=\sqrt{\frac{\mathcal{E}_c}{N_0}}.$$

The integral of a similar form was computed earlier, therefore, lowering conversions, let us register the result

$$p(\Gamma_c/0)=1-F[\Pi_{hn}-m_1(h_n)];$$

with $\Pi_1=1$

$$p(\Gamma_c/0)=1-F\left(\sqrt{\frac{\mathcal{E}_c}{N_0}}-\sqrt{k}\right). \quad (4.9.41)$$

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Let us lead analogous conversions for $p(\Gamma_0/c)$

$$p(\Gamma_0/c)=\int\limits_{-\infty}^{\Pi_{hy}}\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}(h_y-e)^2}dh_y,$$

where

$$h_y=\frac{H_y}{\sigma_{Hy}};\ e=\frac{\mathcal{E}_c}{\sigma_{Hy}};\ \Pi_{hy}=\frac{\Pi_H}{\sigma_{Hy}};$$

$$p(\Gamma_0/c)=1-F(e-\Pi_{hy}); \qquad (4.9.42)$$

with $\Pi_1=1$ $p(\Gamma_0/c)=1-F\sqrt{\frac{\mathcal{E}_c}{2N_0}}$.

For the case $\Pi_1=1$ $p_{om}=0{,}5\left\{\left[1-F\left(\sqrt{\frac{\mathcal{E}_c}{N_0}}-\sqrt{k}\right)\right]+\right.$

$$\left.+\left[1-F\left(\sqrt{\frac{\mathcal{E}_c}{2N_0}}\right)\right]\right\}. \qquad (4.9.43)$$

The probability of the passage of signal with the fluctuating phase is close to the probability of the passage of signal with the random phase and with the known phase. However, the probabilities of false detection for such signals differ substantially. Therefore let us consider in more detail $p(\Gamma_c/0)$.

We convert (4.9.41), using that the fact that $\mathcal{E}_c=\mathcal{E}_1 k$ then

$$p(\Gamma_c/0)\approx 1-F\left[\sqrt{k}\left(\sqrt{\frac{\mathcal{E}_1}{N_0}}-1\right)\right]. \qquad (4.9.44)$$

Expression is correct for the strong signal, i.e., when $\mathcal{E}_1>2N_0$. Increase k or $\mathcal{E}_1$, entering the formula for the energy of signal, causes the decrease of the probability of false reception/procedure, but it remains noticeably more than that value, which would take place during the use of a signal with the random or known phase.

In connection with the larger probability of false detection during the detection of signal with the fluctuating phase the total probability of error proves to be more than for the signal random and noted for phases.

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Fig. 4.9.6 gives the graphs of total error with

$$\Pi_1=1;\ p(0)=p(c)=0{,}5.$$

The curves a, b and c correspond to the cases, as in Fig. 4.9.6. The results, given in the figure, correspond to the case when energy of signal increases due to an increase in its duration.

For the weak signal

$$p(\Gamma_c/0)=\frac{1}{\sqrt{2\pi}\,\sigma_{H\pi}}\int\limits_{\Pi_H}^{\infty} e^{-\frac{[H_\pi-m_1(H_\pi)]^2}{2\sigma^2_{H\pi}}}\,dH_\pi, \quad (4.9.45)$$

$$p(\Gamma_c/c)=\frac{1}{\sqrt{2\pi}\sigma_{Hy}}\int\limits_{-\infty}^{\Pi_H} e^{-\frac{[H_y-m_1(H_y)]^2}{2\sigma^2_{Hy}}}\,dH_y. \quad (4.9.46)$$

Values $\sigma_{H\pi}$, $m_1(H_\pi)$, σ_{Hy}, $m_1(H_y)$ and Π_H are given by expressions (4.9.25), (4.6.26), (4.9.29).

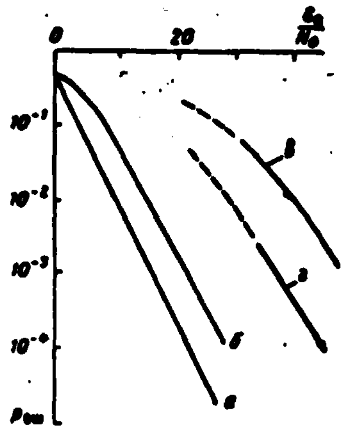

Fig. 4.9.6. Errors of the detection of signal with the fluctuating phase: a) signal with the known phase; b) signal with the random phase; c) signal with the fluctuating phase with $\mathcal{E}_1/N_0=4$; d) signal with the fluctuating phase when $\mathcal{E}_1/N_0=8$.

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The calculation of analogous integrals, was implemented earlier, therefore, lowering conversions, let us give the resulting formula, obtained with the use of the simplifying assumptions.

With $\Pi_1=1$

$$p_{\text{ош}}\approx 1-F\left(\sqrt{\frac{\mathcal{E}_c}{2N_0}\,\frac{\mathcal{E}_1}{2N_0}}\right). \qquad (4.9.47)$$

To formula (4.9.47) it is valid only when $\mathcal{E}_1\ll N_0$. Being congruent/equating it with the expression for the probability of errors with the signal with the known phase, we see that due to the fluctuations of phase equivalent energy is reduced, since $\mathcal{E}_i/N_0 \ll 1$. For Neumann-Pearson criterion

$$p(\Gamma_a/c)=1-F\left\{\frac{\sqrt{k}\,\mathcal{E}_i}{N_0}-\operatorname{arc}F\,[1-p(\Gamma_c/0)]\right\}. \quad (4.9.48)$$

The time of observation t_n for the detection with given ones $p(\Gamma_a/c)$ and $p(\Gamma_c/0)$ will be equally

$$\sqrt{t_n}=\frac{1}{\sqrt{\Delta f_c}}\frac{\mathcal{E}_i}{N_0}\left\{\operatorname{arc}F\,[1-p(\Gamma_c/0)]+\right.$$

$$\left.+\left(1+\frac{\mathcal{E}_i}{N_0}\right)\operatorname{arc}F\,[1-p(\Gamma_a/c)]\right\}, \quad (4.9.49)$$

where Δf_c - width of the spectrum of signal;

$$\Delta f_c\approx\frac{1}{\tau_{av}}.$$

Consequently, the fluctuations of phase with the weak signal considerably worsen/impair authenticity. It is interesting to note that the optimum diagram of the detection of weak signal with the fluctuating phase is close to the optimum diagram of the detection stochastic signal, i.e., the signal, similar to noise - those fluctuating by amplitude and phase. For stochastic signal the optimum diagram of energy detector contains: square-law detector, summator and threshold device/equipment.

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Consequently, weak signal with the fluctuating phase is discovered in essence due to an increase in the total power (energy) of mixture, i.e., energy feeler is close to the optimum.

However, the detection stochastic signal has some special features/peculiarities, which differ it from the detection of signal with the fluctuating phase.

§4.10. Evaluation of the effect of phase on the optimum detection of radio signals. Results presented earlier show that the characteristics of the phase of signal have a great effect on optimum circuit in the sense of the difficulty of their technical realization, and also on the authenticity of detection.

The best results on the authenticity occur in the known phase of signal. Optimum diagram in this case (without taking into account synchronization) is simple. Further energy gain of signal is achieved in the radio communication upon transfer to the systems with the active pause and the opposite signals. During the phase manipulation it is possible to economically place the communication channels over the frequency band. The complicatedly manipulated (noise-like) signals can be treated on the simpler in the realization videofilters. In this case is absent the threshold effect, the authenticity of detection gradually increases, with an increase in the energy of signal. In spite of positive sides indicated above of the detection of signal with the known phase, this version virtually is encountered rarely. In the radio navigation and the radar its use/application does not have a sense, since in these systems the phase of signal, which is capable to carry useful information, usually is not known. Before the measurements in these systems is realized the search and the detection of signal, but it must be considered as signal with the random or fluctuating phase. After initial search in the random phase is then realized the measurement of phase (for example, phase tracking), signal carried out becomes "signal with the known phase".

In the radio communication the use of signals with the known phase in the principle is possible; however, during the technical realization of such systems appear many difficulties. Since the phase of signal cannot be memorized for long, it must be tracked, which complicates equipment.

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In practice basic difficulty lies in the fact that the majority of the signals, utilized in the discrete/digital communication systems with the phase manipulation, does not have clearly expressed carrier and tracking its phase proves to be impossible. The difficulties indicated could be surmounted, if in this there was technical need.

Is very important the fact that the chance of phase (if it, being random, it is constant or it is changed according to the known law at the random, but constant value initial of phase) little worsens/impairs authenticity and is accompanied by the small energy losses of signal. Thus, search and detection of signal in the radio navigation little worsen/impair their parameters because is necessary to realize this procedure in the unknown phase of signal. In the radio communication during the orientation to the chance the phases of diagram and their technical realization considerably are simplified, moreover in this case it is possible to preserve some advantages of phase manipulation, passing to OFM. The most essential deficiency/lack proves to be the impossibility of the complete utilization of a gain of energy, connected with the opposite signals.

By these considerations is explained the fact that the greatest practical use/application obtained the systems, which realize optimum detection or recognition of signals with the random phases. Virtually the detection of signal with known phase can be important for the special systems in which the phase carries useful information, and it it is necessary to follow, and by the same channel must be transmitted communications/reports. Such conditions occur, for example in the space systems.

Optimum diagrams and properties of systems radically are changed, if the phase of the utilized signal fluctuates. Pre-detected processing either filtration or accumulation are limited here by that time interval during which the phase can be considered constant. Basic accumulation must be realized after the detector, in which the information about the phase is destroyed. As it was shown earlier, even with such strong signal with which at the output of preliminary wide-band filter, i.e., on the detector, the signal is more than interference, nevertheless occurs a noticeable deterioration in the authenticity.

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However, the basic negative property of signals with the fluctuating phase lies in the fact that their detection has clearly expressed threshold properties. With the decrease of signal amplitude, from a certain one of its levels, occurs a rapid deterioration in the authenticity and it appears much worse than that which, other conditions being equal, is observed with the signals with the random phase. However, use of diagrams, intended for the signals with the fluctuating phase, gives the simplest technical solutions, it makes it possible not to impose stringent requirements on frequency stability, methods of shaping of signals, stability and accuracy of the network elements. In connection with that presented is obvious the advisability of using the signals with the random phase. However, the conditions for generati n, propagation and reflection of radio waves create the series/row of limitations.

The improvement of the methods of formation and generation of signals makes it possible to consider that the creation of signals with the very slow fluctuations of phase is possible. Such signals usually completely can be considered as signals with the random, but constant phase.

Basic limitations appear in connection with the conditions for propagation and reflection of radio waves. In this case in the special position proves to be the radar, in which is discovered the echo signal. The presence of the rapid fluctuations of the phase of the echo signal, the need for using the impulses/momenta/pulses of short duration for the distribution of targets with the guarantee of large ratio of the power of signal (in the impulse/momentum/pulse) to the power of interferences, wide application in the first stages of the magnetron transmitters whose power to a considerable degree was limited not to pulse, but average/mean power they led to the fact that by the basic method of detection in the radar proved to be technical the simplest method of post-detector accumulation basic form signal-signal, which consists of the pulse packet with the random phases, i.e., signal with the fluctuating (in the package) phase.

The use/application of more efficient methods of detection with the use of a phase of signal is conjugated/combined with the considerable complication of equipment and the fundamental limitations, connected with the spectrum of the fluctuations of amplitude and phase of signal with reflection or, as is sometimes said, with the spectrum of the "glimmer" of target.

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With the diagrams of the detection, in which are used the packages of coherent impulses/momenta/pulses, in the principle possibly for the limited intervals of accumulation. This interval must be less than the interval of phase correlation. Since for many important targets this interval proves to be on the order of not more than 0.1-0.05 s, the methods, in which is used the phase, they are limited by the cases when storage time is substantially less.

The interval of the correlation of phase has important value for the noise-like signals. ShPS is formed/shaped in essence due to the phase manipulation (modulation). Consequently, to form/shape this signal is possible only for its duration, substantially less than the interval of phase correlation. This limits the value of the phase of such signals in the radar. But theory and practice show that it can be sufficiently large for guaranteeing the efficiency in the use/application of ShPS. On the reasons indicated in the contemporary systems in which are used ShPS, are applied the complicated devices/equipment, realizing processing of the mixture with the random, but constant initial phase of signal.

The limitations of the use of a phase of signal for an improvement in its isolation/liberation of the interferences in the radio communication are considerably less than in the radar (besides some specific cases). At the usual speeds of transmission of the information when each informational impulse/momentum/pulse is continued not more than 10^{-3}-10^{-4} s, even signals, which extend by the complicated paths (as a result of the reflection from the ionosphere, the troposphere, the belt/zone of dipoles, etc.), it is possible to consider which have a duration less than the interval of the correlation of the phase of this signal. However, in the new radiolink systems, in which are used the very small power of signals and which ensure high authenticity due to a considerable increase in the duration of signals, must be considered the limitations, assigned on the set of functions of the autocorrelation of phase and by interval of the correlation of phase, caused by the effect of conditions of propagation and reflection of radio-will. As an example of such systems can serve the information-carrying systems from deep space when it proves to be necessary to increase the duration of informational signals up to tens of seconds and even several minutes.

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It is obvious that the interval of the correlation of the fluctuations of phase due to the passage of the ray/beam through the ionosphere and the atmosphere will limit the appropriate duration of signal which it is optimally possible to develop to the detector. In the radio navigation of the fluctuation of phase and its change during the motion they can limit the time interval of observation during the initial detection. Thus, the systems, which realize detection with the use of a phase of signal, have limitations, caused by nature of radiowave propagation; however, they provide the best authenticity and minimum requirements for the power of transmitters, in many instances they can be realized, although they are guished by greater technical complexity, than the systems, in which is not used the phase of signal.

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Chapter 5.

PHASE DETECTION.

§5.1. Special features/peculiarities of phase detection. Earlier were examined basic laws governing the optimum detection of signal. In this case optimally were used all parameters of the mixture of signal and interference - namely both the phase and amplitude. It is of interest to consider a question about the detection of signal with the use of its phase and in the case of the failure of the use of amplitude. Detection during the use of a phase of signal (phase detection) is of interest from the point of view of the theoretical determination of the role of phase and signal amplitude and possibility of its practical use in the single systems.

In the implementation of this method of detection it is necessary to assume that the creation of the devices/equipment, which react only to the phase of mixture and which do not test/experience the noticeable effect of the amplitude of the mixture of signal with the interference on the possibility of measuring the phase, possibly.

In this case it is assumed that occurs the effect of signal amplitude on the fluctuation of phase from the interferences, i.e., its divergence from the value, determined by the phase of signal.

The posing of the question about the phase detection makes sense in such a case, when real devices/equipment can be with some degree of approximation approximated by ideal phasemeters. Some phase-measuring devices/equipment, for example the phasemeter of the servomechanism, measure phase displacement with amplitude change over wide limits. Consequently, broad class by phasemeter.

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This method does not assume the presence of any processing of the mixture of signal and the interference, which preliminarily destroys information about the amplitude, but it is thought that the phasemeters utilized in the diagrams are ideal and revealing/detecting the phase of mixture, they work at virtually any values of amplitude. This assumption, in fact, indicates the use of nonlinear processing of mixture in the phasemeter itself.

The independent work of phasemeter from the amplitude of mixture can be obtained also during the use of ARU; then to the phasemeter is supplied the stress/voltage of the mixture of constant amplitude, and it measures the phase of mixture, caused by the phase of signal and by its divergences from the action of interferences. It is obvious that ARU must be with sufficient speed. It is possible to visualize such schemes or the procedure of processing mixtures, in which the information about the amplitude preliminarily "is destroyed". Let us recall that the preliminary "decomposition" of the information, placed during the phase, occurs in the amplitude methods of the detection when detection makes it possible to come to light/detect/expose the information, placed only into the amplitude, and destroys the information, placed during the phase. This nonlinear processing of mixture is used in the case when phase is equiprobable and rapidly it is changed. The procedure of detection is in the essence the nonlinear operation above the mixture of signal and interference and technically very simply it makes it possible to come to light/detect/expose only the amplitude of mixture without any effect of phase on the result. This causes technical simplicity of the amplitude methods of detection. "Decomposition" of the information about the signal, placed into the amplitude of mixture, can be carried out, using nonlinear devices/equipment, the type of limiters. Actually/really, if we pass the mixture through the ideal limiter, then the process, obtained at its output, will not depend on amplitude (if the threshold of limitation is selected correctly) and instead of the sine voltage with the random ones by amplitude and the phase will occur right-angled stresses/voltages with the random moments/torques of transition through zero. In this case already it is not possible to use the distribution functions for the mixture of signal and interference, obtained under the assumption of the absence of any nonlinear processing of the mixture when the mixture of selective interference and signal was considered as process the random ones by amplitude and phase.

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The distribution function for the amplitude during the ideal limitation is converted into the delta-function, which gives infinite probability density for the amplitudes, which correspond to the threshold of limitation.

The distribution function for phases in this case passes into the distribution function for the moments/torques of the transition of the mixture through the zero level. This distribution function can differ from the function of phase distribution.

Thus, for the phase detection it is possible also to use the method which provides for diagram with the limiter. During the use of this method processing undergoes not directly the phase of mixture, but the connected with it group of "zeros" either the "transitions through zero", or "intersections of zero". From fundamental and technical point of view the methods of the phase detection, in which are used ideal phasemeter and limiter, differ from each other.

Taking into account the aforesaid, has sense to find the optimum procedure of processing phase and the diagram of optimum phase feeler corresponding to it and to determine the probabilities of errors. It is possible to present also the simple (non-optimal) processing of the mixture when is realized the measurement of phase and according to the observations of its value is accepted the solution about the selection of hypothesis about presence or absence of signal in the mixture.

At conclusion of this paragraph let us note that the phase detection in this posing of the question cannot be considered as the method, which optimizes processing mixture in any specific case, but it can rather be considered as technical capability; therefore is of interest its estimation in comparison with the optimum ones. Let us consider now the special features/peculiarities method realization of phase detection. During the analysis of phase detection let us examine the cases of signal with the known phase also of signal with the random phase.

§5.2. Simple phase detection. Let us consider the properties of simple phase detection, i.e., this procedure of processing the mixture when the structure of feeler is assigned. Feeler contains phasemeter and threshold device/equipment. Its diagram is given in Fig. 5.2.1.

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Solution about presence or absence of signal is accepted on the basis of the observation of measured phase displacement for a period of time, which corresponds to the duration of signal. In this case it is assumed that the basic parameters of the expected signal: phase, delay and duration - are known and can be used for the detection. During the simple phase detection the duration of signal it is possible to use two methods. It is possible to realize readings of phase during entire time of action of signal and then to neutralize them, using, for example, an inertness of phasemeter. It is possible to compress the filter pass band, which stands to the phasemeter to the degree to which the duration of signal allows this.

Taking into account that simple phase detection gives the very limited possibilities and a difference in the results, obtained during the use of these two methods, it is insignificant, let us consider them, using some simplifying assumptions. Depending on the duration of signal can be selected the filter pass band, established/installed before the ideal phasemeter.

In the presence of signal the dispersion of the fluctuations of phase is the less, the greater the relation A_c/σ_n, where A_c - signal amplitude, supplied to the phasemeter; σ_n - rms value of fluctuations.

It is known that a matched filter gives the maximum ratio of signal to the interference. At the moment of the termination of signal at its input the ratio of signal amplitude at the output of filter to the rms value of interference reaches maximum value and it is determined from the known relationship/ratio

$$\frac{A_{cm}}{\sigma_n}=\sqrt{\frac{2E_0}{N_0}}.$$

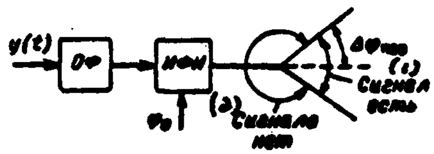

Fig. 5.2.1. The block diagram of the simple phase feeler: ОФ - optimum filter; ИФИ - ideal phase meter.

Key: (1). Signal exists. (2). There is no signal.

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Under these conditions the reading of phase, produced at the moment of the termination of the action of the signal when its value at the output of filter is maximum, will be accompanied by the smallest error and it can serve as standard/criterion during the detection. If the fixed/recorded value of phase is close to the known phase of the expected signal, then can be done the conclusion that the signal exists. If the measured value of phase proves to be that strongly differing from the known phase of the expected signal, then must be accepted hypothesis about the absence of signal. Consequently, in this case also must be introduced the concept of the threshold by which must be understood the sector of angles, which characterizes the divergences of phase displacement, in which is accepted one or the other hypothesis. For determining the value of this sector it is possible to proceed from the criterion of minimum average/mean risk or criterion of ideal observer and to use likelihood ratio. Since it is assumed that the reading of phase displacement is realized for one moment/torque of time (termination of signal), for obtaining the likelihood ratio can be used the one-dimensional functions of phase distribution of interference and mixture. These functions were obtained in the 2nd and 3rd chapters.

In their relation, easily to obtain

$$l(\varphi_y) = e^{-\frac{A_c^2}{2\sigma_n^2}} + \frac{\sqrt{2\pi} A_c}{\sigma_n} \cos\varphi_y \times$$
$$\times F\left(\frac{A_c}{\sigma_n}\cos\varphi_y\right) e^{-\frac{A_c^2}{2\sigma_n^2}\sin^2\varphi_y}; \qquad (5.2.1)$$

when $l(\varphi_y) > \Pi_1$ is accepted the hypothesis "signal is"; when $l(\varphi_y) < \Pi_1$ is accepted the hypothesis "signal is not". In the general/common/total ides the solution of this equation is cumbersome; therefore let us consider special cases - weak signal $\left(\frac{A_c}{\sigma_n} < 1\right)$ and strong signal $\left(\frac{A_c}{\sigma_n} > 1\right)$.

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Case of the weak signal

$$l(\varphi_y)=1+\sqrt{\frac{\pi}{2}}\frac{A_c}{\sigma_n}\cos\varphi_y. \qquad (5.2.2)$$

Let us take for simplicity of calculations the case of ideal observer and $p(c)=p(0)=0.5$, then $\Pi_1=1$.

Is accepted hypothesis "signal is" with

$$1+\sqrt{\frac{\pi}{2}}\frac{\sigma_c}{\sigma_n}\cos\varphi_y\geqslant 1 \quad \text{or} \quad \frac{A_c}{\sigma_n}\sqrt{\frac{\pi}{2}}\cos\varphi_y\geqslant 0. \qquad (5.2.3)$$

According to the conditions during the simple phase detection is used by that measured the value of phase displacement φ_y. The obtained expression for the development/detection of the procedure of processing phase apply we will not be, since during the simple phase detection is assumed the simple (straight/direct) use of results of measuring phase displacement; it it is possible to use for the determination of threshold value.

The threshold value of angle of displacement can be found from the condition

$$\frac{A_c}{\sigma_n}\sqrt{\frac{\pi}{2}}\cos\varphi_{y\text{пор}}=0,$$

which gives $\cos\varphi_{y\text{пор}}=0$, then

$$\varphi_{y\text{пор}}=\pm\frac{\pi}{2}. \qquad (5.2.4)$$

If the measured value of the phase angle of the mixture of signal and interference is more than 90°, then must be accepted hypothesis there is no "signal", also, at an angle less than 90°, hypothesis "signal exists".

Let us compute the errors of the detection of the signal

$$p(\Gamma_0/0) = \int_{-\pi/2}^{+\pi/2} \frac{1}{2\pi} d\varphi_y = 0,5, \qquad (5.2.5)$$

$$p(\Gamma_0/c) = 2\int_{\pi/2}^{\pi} \left(1+\frac{A_c}{\sigma_n}\sqrt{\frac{\pi}{2}}\cos\varphi_y\right) d\varphi_y < 0,5.$$

It is obvious that with such errors the detection becomes meaningless.

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Consequently, if after optimum filtration the ratio of signal amplitude to the interference remains small, i.e., relation A_c/σ_n at the input of receiver of less than one or is close to it, then simple phase detection is accompanied by large errors.

Let us consider now the case when at the input of phasemeter, i.e., at the output of matched filter,

$$\frac{A_c}{\sigma_n} = \sqrt{\frac{2E_c}{N_0}} > 1.$$

Then for φ_{ν} can be accepted normal distribution with the dispersion

$$\sigma_{\varphi}^{2}=\frac{\sigma_{n}^{2}}{A_{c}^{2}}.$$

In the absence of signal the distribution remains uniform. Then likelihood ratio

$$l(\varphi_{\nu})=\frac{w(\varphi_{\nu})}{w(\varphi_{\nu})}=\frac{\sqrt{2\pi}}{\sigma_{\varphi}}e^{-\frac{\varphi_{y}^{2}}{2\sigma_{\varphi}^{2}}}, \qquad (5.2.6)$$

with

$$\ln[l(\varphi_{\nu})]=\ln\frac{\sqrt{2\pi}}{\sigma_{\varphi}}-\frac{\varphi_{y}^{2}}{2\sigma_{\varphi}^{2}}>\ln\Pi_{1} \qquad (5.2.7)$$

must be accepted the solution that the signal exists.

In the diagram, depicted in Fig. 5.2.1, is used φ_{ν}; hence we can find the threshold value of the phase angle

$$\varphi_{\nu\text{пор}}=\sqrt{2\sigma_{\varphi}^{2}\ln\frac{\sqrt{2\pi}}{\sigma_{\varphi}}}\ \text{with}\ \Pi_{1}=1. \qquad (5.2.8)$$

Since

$$\sigma_{\varphi}=\sqrt{\frac{N_{0}}{2\mathcal{E}_{c}}},$$

then

$$\varphi_{\nu\text{пор}}=\sqrt{\frac{N_{0}}{\mathcal{E}_{c}}\ln\sqrt{\frac{4\pi\mathcal{E}_{c}}{N_{0}}}}, \qquad (5.2.9)$$

$$\frac{\varphi_{\nu\text{пор}}}{\sigma_{\varphi}}=\sqrt{2\ln\sqrt{\frac{4\pi\mathcal{E}_{c}}{N_{0}}}}.$$

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Let us find the value of the errors

$$p(\Gamma_s/0) = \int_{-\varphi_{y\,пор}}^{+\varphi_{y\,пор}} \frac{1}{2\pi}\, d\varphi_y = \frac{\varphi_{y\,пор}}{\pi} = $$

$$= \frac{1}{\pi}\sqrt{\frac{N_0}{\mathcal{E}_c}\ln\sqrt{\frac{4\pi\mathcal{E}_c}{N_0}}}; \qquad (5.2.10)$$

$$p(\Gamma_s/c) = 2\int_{\varphi_{y\,пор}}^{\infty} \frac{1}{2\pi\sigma_\varphi} e^{-\frac{\Delta\varphi_y^2}{2\sigma_\varphi^2}}\, d\varphi_y = 2\left[1 - F\left(\frac{\varphi_{y\,пор}}{\sigma_\varphi}\right)\right] \qquad (5.2.11)$$

but

$$\frac{\Delta\varphi_{y\,пор}}{\sigma_\varphi} = \sqrt{2\ln\sqrt{\frac{4\pi\mathcal{E}_c}{N_0}}}, \qquad (5.2.12)$$

then

$$p(\Gamma_s/c) = 2\left[1 - F\left(\sqrt{2\ln\sqrt{\frac{4\pi\mathcal{E}_c}{N_0}}}\right)\right], \qquad (5.2.13)$$

and

$$p_{ош} = \frac{1}{2}\left\{\frac{1}{\pi}\sqrt{\frac{N_0}{\mathcal{E}_c}\ln\sqrt{\frac{4\pi\mathcal{E}_c}{N_0}}} + \right.$$

$$\left. + \left[1 - F\left(\sqrt{2\ln\sqrt{\frac{4\pi\mathcal{E}_c}{N_0}}}\right)\right]\right\}. \qquad (5.2.14)$$

From the obtained relationships/ratios it follows that also the threshold and the probability of errors during the simple phase detection are defined only by the ratio of energy of signal to the spectral noise density, as this occurred for the optimum methods of detection the curve of dependence $p_{ош}$ on $\frac{\mathcal{E}_c}{N_0}$ is given in Fig. 5.2.2.

Let us consider now the case when is used averaging after phasemeter. Signal is supposed strong, i.e., on the phasemeter we have $A_c > \sigma_n$.

Then the function of the distribution of the readings of phases is normal with dispersion $\sigma_\varphi^2 = \sigma_n^2 / A_c^2$.

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If passband up to the phasemeter Δf and the duration of signal t_c, the values of the measured phase will contain $m = t_c \Delta f$ the independent readings. Neutralizing the values, which correspond to these readings, it is possible to decrease the dispersion of the divergences of phase to m of times

$$\sigma_{\varphi\Phi}^2 = \sigma_\varphi^2 \frac{1}{m}.$$

The function of the distribution of the mixture of signal and interference remains normal. The function of phase distribution under the effect of one interference remains uniform with any band. Consequently, the rule of the selection of threshold (5.2.8) during the use of dispersion $\sigma_{\varphi\Phi}^2$ will be preserved.

The errors of detection will be defined by the expressions whose conclusion/output is analogous carried out earlier

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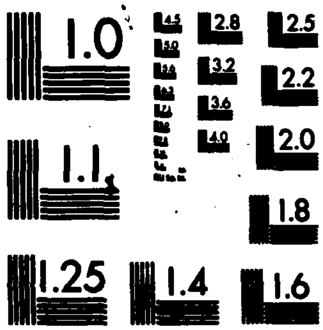

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$$p(\Gamma_c/0) = \frac{\sigma_n}{\pi A_c \sqrt{m}} \sqrt{2 \ln \frac{\sqrt{2\pi m} A_c}{\sigma_n}}, \quad (5.2.15)$$

$$p(\Gamma_c/c) = 2\left[1 - F\left(\sqrt{2 \ln \frac{\sqrt{2\pi m} A_c}{\sigma_n}}\right)\right], \quad (5.2.16)$$

and they can be given to the same form as (5.2.10) and (5.2.13). The obtained formulas are valid when $A_c > \sigma_n$. Results obtained in this case are analogous to those given in Fig. 5.2.2. Simple phase detection considerably is inferior to optimum, examined in Chapter 4. Consequently, simple observation of phase displacement during an attempt at its use for the detection of signal gives poor results. This can be explained by the fact that, for example, when $\mathcal{E}_c/N_0 = 4$, the greatest ratio of signal to the interference at the output of matched filter (A_c/σ_n) will compose 2.8.

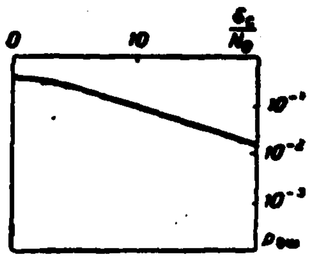

Fig. 5.2.2. Probability of errors during the simple phase detection.

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In this ratio of energy of signal to the noise density the optimum methods of the detection of signal with the known parameters will give a probability of error less than 0.07, and the rms value of the fluctuations of phase will be

$$\sigma_\varphi = \frac{\sigma_n}{A_c} = 0{,}35 \text{ rad or } \approx 20^\circ.$$

With such large fluctuations of phase, naturally, is obtained the great probability of the output of reading beyond the limits of pore sector, which leads to the great probability of false alarms.

§5.3. Optimum phase detection of weak signal with the known parameters in the diagrams with the use of ideal phasemeter. Let us consider now a question about the optimum phase detection. Problem in this case lies in the fact that using only a phase of mixture, to find the optimum procedure of its processing, i.e., to find those actions by which must undergo the measured value of phase, so that the results of detection would be obtained optimum [5.1].

By optimality of detection we will understand the guarantee of a minimum average/mean risk.

For the solution of this problem it is necessary to find likelihood ratio for the case of using only the functions of the phase of mixture.

The one-dimensional function of phase distribution of one interference is known. For obtaining the multidimensional function of phase distribution of interference we will use the fact that the random function of a change in the phase can be substituted by selection. It is most convenient to realize a selection through the interval of correlation, in this case through the interval of the correlation of phase.

Earlier it was established/installed, what energy spectrum of the fluctuations of the phase of interference in essence was concentrated in the region from 0 to $\frac{\Delta\omega_n}{2}$ (or $\frac{\Delta f_n}{2}$), where $\Delta\omega_n, \Delta f_n$ - passband of the filter. Therefore in the first approximation, it is possible to allow the selection after time $\Delta t = \frac{1}{2\Delta f_n/2} = \frac{1}{\Delta f_n}$. With this selection of phase will be missed its rapid fluctuations, which must not play the significant role with the method of detection in question. Under these conditions of the value of phase in the selection through Δt it is possible to consider it statistically independent variables.

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Then the multidimensional function of phase distribution of interference for the selection from the values of the phase through the intervals of time Δt, will take the form

$$w(\varphi_{\text{п}1},\ \varphi_{\text{п}2}, \ldots) = w(\varphi_{y1},\ \varphi_{y2}, \ldots/\text{п}) = \left(\frac{1}{2\pi}\right)^{m}. \quad (5.3.1)$$

In the presence of useful signal the one-dimensional function of phase distribution takes the form, depicted in Fig. 3.2.1.

For obtaining the multidimensional function again we will use the selection of the phase through $\Delta t = \frac{1}{\Delta f_{\text{п}}}$. Considering that at these points the phases are statistically independent, we will obtain the following expression for the multidimensional function of phase distribution of the mixture:

$$w(\varphi_{y1},\ \varphi_{y2}, \ldots/\text{с-п}) = \prod_{i=1}^{m}\left\{\frac{1}{2\pi} e^{\frac{A_i^2}{2\sigma_{\text{п}}^2}} + \right.$$

$$\left. + \frac{A_i}{\sigma_{\text{п}}\sqrt{2\pi}} \cos\varphi_{yi} F\left(\frac{A_i}{\sigma_{\text{п}}} \cos\varphi_{yi}\right) e^{-\frac{A_i^2}{\sigma_{\text{п}}^2}\sin^2\varphi_{yi}}\right\}, \quad (5.3.2)$$

where A_i - values of signal amplitude at the input of receiver with the i reading of phase; σ_n^2 - dispersion of interference at the input of receiver; φ_{yi} - divergence of the phase of mixture from the phase of signal according to the data of reading. m - sample size, which is determined by the duration of signal t_c:

$$m=\frac{t_c}{\Delta t}=t_c\Delta f_n.$$

Likelihood ratio

$$l(\varphi_y)=\prod_{i=1}^{m}\left\{e^{-\frac{A_i^2}{2\sigma_n^2}}+\frac{\sqrt{2\pi}A_i}{\sigma_n}\cos\varphi_{yi}F\times\right.$$
$$\left.\times\left(\frac{A_i}{\sigma_n}\cos\varphi_{yi}\right)e^{-\frac{A_i^2\sin\varphi_{yi}}{2\sigma_n^2}}\right\}. \qquad (5.3.3)$$

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Is accepted solution about the fact that the "signal exists", if

$$l(\varphi_y)>\Pi_1.$$

Is accepted solution that, there is no "signal", if

$$l(\varphi_y)<\Pi_1.$$

If we take the logarithm of expression for $l(\varphi_y)$, then we obtain the solution "there is a signal", if

$$\ln l(\varphi_y)>\ln\Pi_1,$$

and solution there is no "signal", if

$$\ln l(\varphi_y)<\ln\Pi_1.$$

It is obvious that

$$\ln l(\varphi_y) = \sum_{i=1}^{m} \ln \left\{ e^{-\frac{A_i^2}{2\sigma_n^2}} + \frac{\sqrt{2\pi} A_i}{\sigma_n} \times \right.$$

$$\left. \times \cos\varphi_{yi} F\left(\frac{A_i}{\sigma_n}\cos\varphi_{yi}\right) e^{-\frac{A_i^2 \sin^2\varphi_{yi}}{2\sigma_n^2}} \right\}. \qquad (5.3.4)$$

The analysis of this expression in general form leads to the cumbersome calculations, which is connected with the mathematical difficulties.

Let us consider therefore the special case: the case of weak signal $(A_0<\sigma_n)$ and the case of strong signal $(A_0>\sigma_n)$.

Examining the case of weak signal, it is necessary to keep in mind that the term "weak signal" determines the relation of the power of signal and interference. In this case the ratio of energy of signal to the jamming density, which determines the errors of optimum detection, can be large and signal can be discovered with a small probability of errors.

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Likelihood ratio for this case can be simplified, using the fact that

$$w(\varphi_{yi}) \approx \frac{1}{2\pi}\left(1+\sqrt{\frac{\pi}{2}}\frac{A_i}{\sigma_n}\cos\varphi_{yi}+\right.$$

$$\left.+\frac{A_i^2}{2\sigma_n^2}\cos 2\varphi_{yi}+\dots\right).$$

$$l(\varphi_y) \approx \prod_{i=1}^{m}\left(1+\sqrt{\frac{\pi}{2}}\frac{A_i}{\sigma_n}\cos\varphi_{yi}+\frac{A_i^2}{\sigma_n^2}\cos 2\varphi_{yi}\right)=$$

$$=\prod_{i=1}^{m} e^{-\frac{\pi}{8}\frac{A_i^2}{\sigma_n^2}+\sqrt{\frac{\pi}{2}}\frac{A_i}{\sigma_n}\cos\varphi_{yi}}, \qquad (5.3.5)$$

$$\ln l(\varphi_y) \approx \sum_{i=1}^{m}\left(-\frac{\pi}{8}\frac{A_i^2}{\sigma_n^2}+\right.$$

$$\left.+\sqrt{\frac{\pi}{2}}\frac{A_i}{\sigma_n}\cos\varphi_{yi}\right). \qquad (5.3.6)$$

Likelihood ratio is equal with the threshold Π_1 or the logarithm of likelihood ratio is equal with the logarithm Π_1.

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Is accepted the solution "there is a signal" with

$$\ln l(\varphi_y) = -\sum_{i=1}^{m}\frac{\pi}{8}\frac{A_i^2}{\sigma_n^2}+$$

$$+\sum_{i=1}^{m}\sqrt{\frac{\pi}{2}}\frac{A_i}{\sigma_n}\cos\varphi_{yi} > \ln \Pi_1$$

or

$$\sum_{i=1}^{m}\sqrt{\frac{\pi}{2}}\frac{A_i}{\sigma_n}\cos\varphi_{yi} > \ln \Pi_1 + \sum_{i=1}^{m}\frac{\pi}{8}\frac{A_i^2}{\sigma_n^2}.$$

but

$$\frac{\pi}{8}\frac{1}{\sigma_{n}^{2}}\sum_{i=1}^{m}A_{i}^{2}=$$

$$=\frac{\pi}{8\sigma_{n}^{2}}\Delta f_{n}\int_{0}^{t_{c}}A_{c}^{2}(t)\,dt=\frac{\pi 2\mathcal{E}_{c}}{8N_{0}},$$

then

$$\frac{1}{\sigma_{n}}\sum_{i=1}^{m}A_{i}\cos\varphi_{yi}>\sqrt{\frac{2}{\pi}\ln \Pi_{1}}+\frac{\pi}{4}\frac{\mathcal{E}_{c}}{N_{0}}\sqrt{\frac{2}{\pi}}$$

or

$$\frac{1}{\sigma_{n}}\sum_{i=1}^{m}A_{i}\cos\varphi_{yi}>\sqrt{\frac{2}{\pi}\ln \Pi_{1}}+\sqrt{\frac{\pi}{8}}\frac{\mathcal{E}_{c}}{N_{0}};\qquad(5.3.7)$$

from the sum let us switch over to the integral

$$\int_{0}^{t_{c}}A_{c}(t)\cos\varphi_{y}(t)\,dt>\frac{\sigma_{n}}{\Delta f}\times$$

$$\times\left(\sqrt{\frac{2}{\pi}\ln \Pi_{1}}+\frac{\mathcal{E}_{c}}{N_{0}}\sqrt{\frac{\pi}{8}}\right).\qquad(5.3.8)$$

For the simplest case when signal takes the form of sinusoidal message, $A_c(t)=A_c$, and if is used the criterion of ideal observer, then

$$\int_{0}^{t_{c}}A_{c}\cos\varphi_{y}(t)\,dt>\frac{\sigma_{n}}{\Delta f_{n}}\sqrt{\frac{\pi}{8}}\frac{\mathcal{E}_{c}}{N_{0}}$$

or

$$z_{\varphi}=\int_{0}^{t_{c}}\cos\varphi_{y}(t)\,dt>\frac{A_{c}t_{c}}{2\sigma_{n}}\sqrt{\frac{\pi}{8}}=\Pi_{\varphi},\qquad(5.3.9)$$

$$\int_{0}^{t_{c}}\sigma_{n}A_{c}\cos\varphi_{y}(t)\,dt>\sqrt{\frac{\pi}{8}}\mathcal{E}_{c}\approx 0{,}63\mathcal{E}_{c}.\qquad(5.3.10)$$

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The obtained expressions make it possible to come to light/detect/expose the optimum procedure of processing the phase of the mixture of signal and interference during the phase detection.

From these formulas it follows that for the optimum phase detection it is necessary, after measuring by ideal phasemeter phase displacement of mixture relative to the phases of the expected signal, to carry out then trigonometric transformation of this angle (to take the cosine of angle) and result to integrate for a period of time, which corresponds to the duration of signal. Value at the output of integrator is equal with the threshold which depends on the ratio of the signal to interference A_0/σ_n and of the duration of signal.

The diagram of optimum procedure is given in Fig. 5.3.1. The obtained results and diagram need some explanations. It would seem that the use of ideal phasemeter whose readings/indications do not depend on the signal amplitude or mixture, must lead to the fact that the parameters and the work of the diagram of detection must not depend on signal amplitude. However, the obtained result does not correspond to this, since threshold depends on relation $\frac{A_0 t_0}{\sigma_n}$, i.e., on the ratio of the power of signal to the power of interference and from the duration of signal. This dependence is explained by the fact that with a change in the signal-to-noise ratio is changed the intensity of the fluctuations of phase and it is necessary to establish another threshold whose level must depend on fluctuations. If is changed simultaneously A_0 and σ_n, for example, with a change of amplifying the receiver, then threshold is not required to change, but if is changed only interference or only signal, then threshold must be changed.

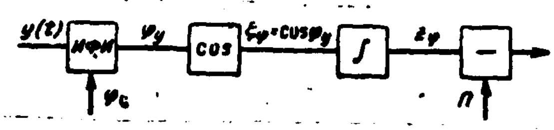

Fig. 5.3.1. The diagrams of the optimum phase feeler of the weak signal: ИФИ - ideal phase meter; ∫ - integrator; П - threshold device/equipment.

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In this respect there is certain difference with the optimum feelers; however, this difference does not give essential advantages to phase feeler. The diagram of optimum phase feeler is fairly complicated, it is substantially more complicated than the diagram of the optimum amplitude-phase feeler of signal with the known parameters. For the realization of phase feeler it is also necessary to know all parameters of signal. In this connection is of interest the detailed comparison of the procedure of processing signal in the optimum amplitude-phase and phase feelers which is carried out further.

§ 5.4. Errors during the phase detection of weak signal with the known parameters. Let us consider now the errors, characteristic to the optimum phase detection of weak signal.

Errors will be determined by the fact that there is a finite probability of excess by integral $\int_0^{t_0} \cos\varphi_u(t)\,dt$ of threshold in the absence of signal or its nonexcess in the presence of signal.

For calculating this probability it is necessary to find the distribution functions.

Let us pass from integral to the sum

$$z_\varphi = \int_0^{t_0} \cos\varphi_u(t)\,dt = \frac{1}{\Delta f_\pi} \sum_{i=1}^{m} \cos\varphi_{ui}. \qquad (5.4.1)$$

Consequently, distribution can be found, on the basis of the fact that during any distribution for $\cos\varphi_{ui}$ the distribution of sum approaches normal with the average and the dispersion, with the equal to the sum of averages and dispersions.

For the determination of the dispersion of value $\cos\varphi_{ui}$ let us recall that in the absence of signal distribution $\varphi_{ui} = \varphi_\pi$ is uniform from $-\pi$ to $+\pi$. If is known distribution for φ_{ui}, the distribution for $\cos\varphi_{ui}$ can be found from the rules of the functional ones of the conversion of random variables.

Above it was obtained

$$w(\cos\varphi_{\text{п}}) = \frac{1}{\pi\sqrt{1-\cos^2\varphi_{\text{п}}}}. \qquad (5.4.2)$$

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The first moment/torque for this distribution is equal to zero. Second moment/torque or the dispersion

$$\sigma^2_{\cos\varphi} = \frac{1}{\pi}\int_{-1}^{+1}\cos^2\varphi_{\text{п}}\frac{1}{\sqrt{1-\cos^2\varphi_{\text{п}}}}\,d\cos\varphi_{\text{п}}.$$

We use limits from -1 to +1, since the values of function beyond the limits ±1 we consider identically equal to zero. After using substitution $\cos^2\varphi_{\text{п}} = \sin x$, we will obtain

$$\sigma^2_{\cos\varphi} = \frac{1}{2\pi}\int_{-\pi/2}^{+\pi/2}(1-\cos 2x)\,dx = \frac{1}{2}, \qquad (5.4.3)$$

consequently, dispersion for value $\sum_{i=1}^{m}\cos\varphi_{\text{п}i}$ will be equal to m/2, and for value

$$z_{\varphi\text{п}} = \int_0^{t_\text{c}}\cos\varphi_\text{у}(t)\,dt = \frac{1}{\Delta f_{\text{п}}}\sum_{i=1}^{m}\cos\varphi_{\text{п}i}$$

dispersion will be equal to

$$\sigma^2_{z\varphi\text{п}} = m/2\ 1/\Delta f^2_{\text{п}}. \qquad (5.4.4)$$

In the presence of signal the distribution for $\cos\varphi_\text{у}$ is changed, since is changed the distribution for $\varphi_\text{у}$. With the weak signal the distribution function for $\cos\varphi_\text{у}$ must be obtained according to the rules of the functional conversions of random variables.

After conversions we obtain the function of the distribution

$$w(\cos\varphi_y) \approx \frac{1}{\pi\sqrt{1-\cos^2\varphi_y}}\left(1+\sqrt{\frac{\pi}{2}}\frac{A_0}{\sigma_n}\cos\varphi_y\right). \quad (5.4.5)$$

Distribution becomes asymmetric and probability density for values $\cos\varphi_y$, of close ones to +1, it increases, since increases the probability of angles φ_y, of close to zero ones.

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Under these conditions average/mean value no longer is equal to zero

$$m_1(\cos\varphi_y) = \frac{1}{2}\sqrt{\frac{\pi}{2}}\frac{A_0}{\sigma_n}, \quad (5.4.6)$$

since

$$\int_{-1}^{+1}\frac{x^2}{\sqrt{1+x^2}}dx = \frac{\pi}{2}.$$

For value $\frac{1}{\Delta f_n}\sum_{i=1}^{m}\cos\varphi_{yi} = z_{\varphi y}$ average/mean value there will be equally

$$m_1(z_{\varphi y}) = \frac{f_m A_0\sqrt{\pi}}{\Delta f_n 2\sqrt{2}\,\sigma_n} = \frac{t_0 A_0}{2\sigma_n}\sqrt{\frac{\pi}{2}} \quad (5.4.7)$$

and the dispersion

$$\sigma^2_{z\varphi y} = \frac{m}{2\Delta f_n^2} = \frac{t_0}{2\Delta f_n}. \quad (5.4.8)$$

Consequently, the obtained for value z_φ distribution functions are analogous to the functions of the distribution of value z in the amplitude-phase feeler, namely normal distribution for the values of this value in the absence of signal (with the zero average) and normal distribution with the displaced average in the presence of signal. The curves, which characterize distributions, are given in Fig. 5.4.1. The value of the threshold [see (5.3.9)] is equal to half of average/mean value as in the optimum amplitude-phase feeler.

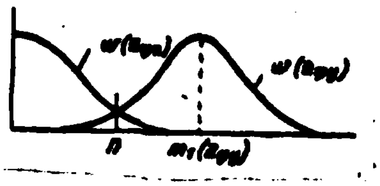

Fig. 5.4.1. Functions of the distribution of value z_φ with the weak signal.

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For the determination the probability of errors it is necessary to find the integrals

$$p(\Gamma_c/0)=\int_{\Pi_\varphi}^{\infty} w(z_{\varphi n})\,dz_{\varphi n}=\int_{\Pi_\varphi}^{\infty}\frac{1}{\sqrt{2\pi}\sigma_{z\varphi n}}e^{-\frac{z^2_{\varphi n}}{2\sigma^2_{z\varphi n}}}\,dz_{\varphi n}, \quad (5.4.9)$$

$$p(\Gamma_0/c)=\int_{-\infty}^{\Pi_\varphi} w(z_{\varphi y})\,dz_{\varphi y}=$$

$$=\int_{-\infty}^{\Pi_\varphi}\frac{1}{\sqrt{2\pi}\sigma_{z\varphi y}}e^{-\frac{[z_{\varphi y}-m_1(z_{\varphi y})]^2}{2\sigma^2_{z\varphi y}}}\,dz_{\varphi y}. \quad (5.4.10)$$

Analogous integrals were found earlier, therefore, lowering intermediate linings/calculations, let us register

$$\left.\begin{aligned} p(\Gamma_c/0)&=1-F\left(\frac{\Pi_\varphi}{\sigma_{z\varphi n}}\right),\\ p(\Gamma_0/c)&=1-F\left(\frac{\Pi_\varphi}{\sigma_{z\varphi y}}\right).\end{aligned}\right\} \quad (5.4.11)$$

With $\Pi_1=1$

$$\frac{\Pi_\varphi}{\sigma_{z\varphi m}}=\frac{\Pi_\varphi}{\sigma_{z\varphi y}}=\sqrt{\frac{\pi}{8}\frac{E_e}{N_0}}, \qquad (5.4.12)$$

then

$$p_{om}=1-F\left(\sqrt{\frac{E_e}{2N_0}\frac{\pi}{4}}\right). \qquad (5.4.13)$$

Let us recall that during the optimum amplitude-phase detection of signal with the known parameters

$$p_{om}=1-F\left(\sqrt{\frac{E_e}{2N_0}}\right).$$

The obtained result shows that optimum phase detection, if it can be technically realized, gives somewhat worse results, than optimum amplitude-phase detection. However, the obtained difference in the equivalent energy is not very great. During the phase detection equivalent energy of signal is reduced 1.25 times.

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In other words, the optimum use only of a phase of signal during the detection as by 25% reduces the energy of signal. Consequently, with the weak signal optimum phase detection gives the function of the distribution of value z_φ, of that being congruent/equating with the threshold, and authenticity, very close to the appropriate results for the optimum amplitude-phase detection. The same result is obtained during the use of Neumann-Pearson criterion, also, for the systems with the active pause. Thus basic information about the weak signal is laid in the phase.

§ 5.5. Phase detection of strong signal with the known parameters. With strong signal $A_0>\sigma_m$ then

$$w(\varphi_y)=\frac{1}{\sqrt{2\pi}\,\sigma_\varphi}e^{-\frac{\varphi_y^2}{2\sigma_\varphi^2}}.$$

Upon transfer to the selection with a volume of $m=t_0\Delta f$ we will obtain

$$w(\varphi_{y1},\ \varphi_{y2},\ldots/c\text{-}n)=\frac{1}{2\pi^{m/2}}\prod_{i=1}^{m}\frac{1}{\sigma_\varphi}e^{-\frac{\varphi_{yi}^2}{2\sigma_\varphi^2}}. \qquad (5.5.1)$$

in the absence of the signal

$$w(\varphi_{y1},\ \varphi_{y2},\ldots/n)=\left(\frac{1}{2\pi}\right)^m. \qquad (5.5.2)$$

Likelihood ratio

$$l(\varphi_y)=\frac{w(\varphi_{y1},\ \varphi_{y2},\ldots/c\text{-}n)}{w(\varphi_{y1},\ \varphi_{y2},\ldots/n)}=\prod_{i=1}^{m}\frac{\sqrt{2\pi}}{\sigma_\varphi}e^{-\frac{\varphi_y^2}{2\sigma_\varphi^2}}, \qquad (5.5.3)$$

then

$$\ln l(\varphi_y)=\sum_{i=1}^{m}\ln\frac{\sqrt{2\pi}}{\sigma_\varphi}-\sum_{i=1}^{m}\frac{\varphi_{yi}^2}{2\sigma_\varphi^2}, \qquad (5.5.4)$$

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The conditions of making a decision about the presence of signal will take the form

$$\sum_{i=1}^{m}\frac{\varphi_{yi}^{2}}{2\sigma_{\varphi}^{2}}<\sum_{i=1}^{m}\ln\frac{\sqrt{2\pi}}{\sigma_{\varphi}}-\ln\Pi, \qquad (5.5.5)$$

or

$$\frac{1}{2\sigma_{n}^{2}}\sum_{i=1}^{m}A_{ci}^{2}\varphi_{yi}^{2}<\sum_{i=1}^{m}\ln\frac{\sqrt{2\pi}A_{ci}}{\sigma_{n}}-\ln\Pi_{1}, \qquad (5.5.6)$$

$$\sum_{i=1}^{m}\varphi_{yi}^{2}<\frac{2\sigma_{n}^{2}}{A_{c}^{2}}m\ln\frac{\sqrt{2\pi}A_{c}}{\sigma_{n}}$$

when $A_{ci}=\text{const}=A_{c}$ and $\Pi_1=1$.

After switching over from the sums to integrals, we will obtain

$$\frac{1}{2\sigma_{n}^{2}}\Delta f_{n}\int_{0}^{t_{c}}A_{c}(t)\varphi_{y}^{2}(t)\,dt<$$

$$<\Delta f\int_{0}^{t}\ln\frac{\sqrt{2\pi}A_{c}(t)}{\sigma_{n}}\,dt-\ln\Pi_{1}. \qquad (5.5.7)$$

For case $A_c=\text{const}$, during the use of a criterion of the ideal observer ($\ln\Pi_1=0$) we will obtain

$$\frac{A_{c}^{2}}{2\sigma_{n}^{2}}\int_{0}^{t_{c}}\varphi_{y}^{2}(t)\,dt<\left[\ln\left(\frac{\sqrt{2\pi}A_{c}}{\sigma_{n}}\right)\right]t_{c} \qquad (5.5.8)$$

or

$$z_{\varphi}=\int_{0}^{t_{c}}\varphi_{y}^{2}(t)\,dt<2t_{c}\sigma_{\varphi}^{2}\ln\left(\frac{\sqrt{2\pi}}{\sigma_{\varphi}}\right)=\Pi_{\varphi} \qquad (5.5.9)$$

The diagram, which corresponds to the obtained optimum phase processing with the strong signal, is given in Fig. 5.5.1.

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The work of diagram has the essential feature, consisting in the fact that the response/answer "signal exists" it is given when the result of integration is less than the threshold (but not more, as it was in all previous diagrams), and the response/answer there is no "signal" - when threshold was exceeded. Physically this can be explained by the fact that in the presence of strong signal the fluctuations of the phase of mixture relative to the phase of signal become insignificant, the standard deviation of phase and the integral of squares have low value, and if they are less than the specific level, then this testifies about the presence of signal. In the absence of signal the phase of mixture is equiprobable, standard deviation has sometimes high values and integration of the square of the fluctuation of the phase of mixture it gives high values.

Let us find the distribution functions for output potential of integrator. Value φ is distributed either evenly with dispersion $\frac{\pi^2}{3}$ in the absence of signal, or according to the normal law with dispersion σ_φ^2 in the presence of signal.

For obtaining the functions of distribution φ^2 it is necessary to use the rules of functional conversions.

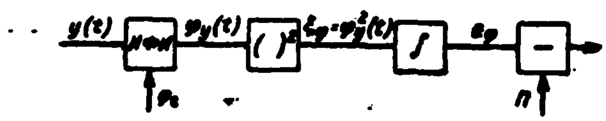

Fig. 5.5.1. The diagram of the optimum phase feeler of the strong signal: ИФИ - ideal phase meter; ()² - square-law function generator; ∫- integrator; П - threshold device/equipment.

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With

$$w_0^{\varphi}(\varphi_n) = \frac{1}{2\pi},$$

$$w(\varphi_n^2) = \frac{1}{\sqrt{\varphi_n^2}} \frac{1}{2\pi} \qquad (5.5.10)$$

and during the normal distribution for φ_y

$$w(\varphi_y^2) = \frac{1}{\sqrt{2\pi\varphi_y^2 \sigma_\varphi}} e^{-\frac{\varphi_y^2}{2\sigma_y^2}}. \qquad (5.5.11)$$

In the case in question of basic interest is not distribution function itself, but its average and dispersion, since at the output of diagram functions the sum of a large number of random variables, the function of distribution of which can be accepted by the normal with the average, equal to the sum of averages, and dispersion, equal to the sum of dispersions. Let us find averages and dispersion for the cases of absence and presence of signal.

In the absence of signal the average

$$m_1(\varphi_n^2)=\int_0^{\pi^2}\varphi_n^2\frac{1}{\varphi_n}\frac{1}{2\pi}\,d\varphi_n^2=\frac{\pi^2}{3}, \tag{5.5.12}$$

the dispersion

$$\sigma_{\varphi n}^2=\int_0^{\pi^2}\frac{1}{2\pi\sqrt{\varphi_n^2}}\,[\varphi_n^2-m(\varphi_n^2)]^2\,d\varphi_n^2=\frac{4}{45}\pi^4. \tag{5.5.13}$$

In the presence of signal the average

$$m_1(\varphi_y^2)=\frac{1}{\sqrt{2\pi}}\int_0^{\pi^2}\frac{\varphi_y}{\sigma_\varphi}e^{-\frac{\varphi_y^2}{2\sigma_\varphi^2}}\,d\varphi_y^2=$$

$$=\sigma_\varphi^2=\frac{\sigma_n^2}{A_c^2}, \tag{5.5.14}$$

the dispersion

$$\sigma_{\varphi y}^2=\frac{1}{\sqrt{2\pi}}\int_0^{\pi^2}(\varphi_y^2-\sigma_\varphi^2)^2\,\frac{e^{-\frac{\varphi_y^2}{2\sigma_\varphi^2}}}{\sqrt{\varphi_y^2}\,\sigma_\varphi}\,d\varphi_y^2=2\sigma_\varphi^4. \tag{5.5.15}$$

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Then in the absence of the signal

$$m_1(z_{\varphi n})=t_0\frac{\pi^2}{3}=\frac{m}{\Delta f}\frac{\pi^2}{3} \tag{5.5.16}$$

and

$$\sigma_{z\varphi n}^2=m\frac{4\pi^4}{\Delta f^2\,45}=\frac{4\pi^4 t_0}{45}. \tag{5.5.17}$$

In the presence of the signal

$$m_1(z_{\varphi y}) = t_0 \frac{a_s^2}{A_0^2} = t_0 a_\varphi^2 = \frac{1}{\Delta f} m a_\varphi^2, \qquad (5.5.18)$$

$$a_{z\varphi y}^2 = t_0 2 \frac{a_s^4}{A_0^4} = 2m a_\varphi^4 \frac{1}{\Delta f^2}. \qquad (5.5.19)$$

The distribution functions for the value with the output of integrator will be registered in the form

$$w(z_{\varphi n}) = \frac{1}{\sqrt{2\pi}\sigma_{z\varphi n}} e^{-\frac{(z_{\varphi n} - m_1(z_{\varphi n}))^2}{2\sigma_{z\varphi n}^2}} \qquad (5.5.20)$$

without the signal

$$w(z_{\varphi y}) = \frac{1}{\sqrt{2\pi}\sigma_{z\varphi y}} e^{-\frac{(z_{\varphi y}^2 - m_1(z_{\varphi y}))^2}{2\sigma_{z\varphi y}^2}} \qquad (5.5.21)$$

with the signal.

For determining the errors it is necessary to find the integrals

$$p(\Gamma_0/0) = \int_0^{\Pi_\varphi} w(z_{\varphi n})\, dz_{\varphi n}, \qquad (5.5.22)$$

$$p(\Gamma_0/c) = \int_{\Pi_\varphi}^{\infty} w(z_{\varphi y})\, dz_{\varphi y}. \qquad (5.5.23)$$

Fig. 5.5.2 gives the image of the distribution function for $z_{\varphi n}$ and $z_{\varphi y}$ and threshold Π_φ. The areas, which correspond $p(\Gamma_0/0)$ and $p(\Gamma_0/c)$ are shaded.

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During the calculation of errors we will be bounded to the case when $\Pi_1=1$, then

$$\Pi_{\varphi}=2t_c\sigma_{\varphi}^2\ln\frac{\sqrt{2\pi}}{\varepsilon_{\varphi}}=2\sigma_{\varphi}^2 m\ln\frac{\sqrt{2\pi}\,1}{\varepsilon_{\varphi}\Delta f}, \qquad (5.5.24)$$

$$\Pi_{\varphi}/\sigma_{z\varphi m}=\frac{\sqrt{45}}{\pi^2}\sigma_{\varphi}^2\sqrt{m}\ln\frac{\sqrt{2\pi}}{\varepsilon_{\varphi}},$$

$$\Pi_{\varphi}/\sigma_{z\varphi y}=\sqrt{2m}\ln\frac{\sqrt{2\pi}}{\varepsilon_{\varphi}}.$$

The probability of errors does not depend on whether is used in the diagram addition or integration. The integrals, analogous (5.5.22) and (5.5.23), repeatedly were computed earlier, and therefore, lowering conversions, we give the final result:

$$p(\Gamma_c/0)=1-F\left(\sqrt{\frac{45}{36}m}-\sqrt{\frac{1}{\pi^4}m}\,\sigma_{\varphi}^2\ln\frac{\sqrt{2\pi}}{\varepsilon_{\varphi}}\right)=$$

$$=1-F\left[\sqrt{m}\left(1{,}12-0{,}67\sigma_{\varphi}^2\ln\frac{\sqrt{2\pi}}{\varepsilon_{\varphi}}\right)\right],$$

$$p(\Gamma_o/c)=1-F\left[\sqrt{2m}\ln\left(\frac{\sqrt{2\pi}}{\varepsilon_{\varphi}}-\frac{1}{2}\right)\right], \qquad (5.5.25)$$

$$p_{om}=0{,}5\left[p(\Gamma_c/0)+p(\Gamma_o/c)\right]. \qquad (5.5.26)$$

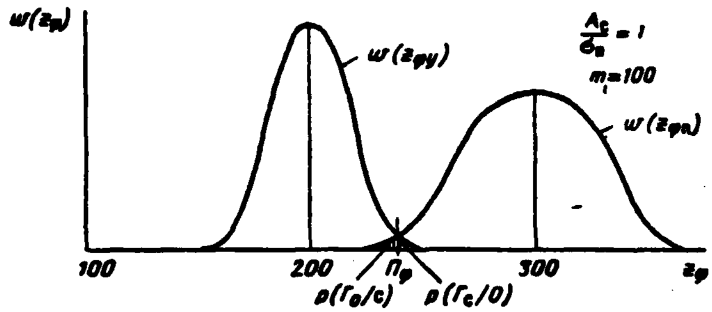

Fig. 5.5.2. Function of distribution z_φ with the strong signal.

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Formulas (5.5.25) and (5.526) can be simplified for the assigned relation

$$A_c/\sigma_n = \frac{1}{\varepsilon_\varphi}.$$

For $A_c/\sigma_n = 1,5$

$$p(\Gamma_c/0) \approx 1 - F(0,74\sqrt{m}), \qquad (5.5.27)$$
$$p(\Gamma_0/c) \approx 1 - F(1,05\sqrt{m}).$$

For $A_c/\sigma_n = 10$

$$p(\Gamma_c/0) \approx 1 - F(1,1\sqrt{m}), \qquad (5.5.28)$$
$$p(\Gamma_0/c) = 1 - F(4\sqrt{m}).$$

If formula (4.4.18) for the probability of the errors of optimum amplitude-phase feeler is converted, using the fact that

$$\mathcal{E}_e = A_e^2 \Delta t m 0{,}5,$$

then we will obtain

$$p_{ош} = [p(\Gamma_c/0) + p(\Gamma_0/c)]\,0{,}5 = 1 - F\left(0{,}5 \frac{A_c}{\sigma_n} \sqrt{m}\right).$$

When $A_c/\sigma_n = 1{,}5$ we will obtain expression, very close to (5.5.27).

From the results it follows that increase m is equally efficient both with the optimum amplitude-phase and during the optimum phase detection. An increase in the signal amplitude, i.e., increase A_c/σ_n, causes the rapid decrease of errors during the amplitude-phase detection and their comparatively slow decrease during the phase detection.

When $A_c/\sigma_n \approx 1{,}5$, when formulas (5.5.25) and (5.5.26) still can be used, during the phase detection the authenticity is obtained somewhat more badly than during the optimum amplitude-phase detection.

When $A_c/\sigma_n > 5 \div 10$ during the phase detection are obtained the noticeably greater probabilities of errors, than with the amplitude-phase.

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§ 5.6. Comparison of the procedure of processing in the optimum amplitude-phase and phase feelers of signal with the known phase. Let us compare now the procedures of the optimum processings of mixture (with the use of its amplitude and phase) and only its phase. The diagram of the optimum amplitude-phase feeler of signal with the known parameters takes the form, depicted in Fig. 4.4.1. The diagrams of optimum phase feelers are given in Fig. 5.3.1 and 5.5.1. Comparison shows that in these diagrams are general/common/total elements/cells, but there is the difference, which consists in essence in the different procedure of obtaining the value, supplied to the integrator. For the comparison of the sections of diagrams, connected to the integrator, let us decipher the operation of multiplication, utilized in the optimum amplitude-phase feeler.

Let us assume that the signal detected is the segment of sinusoid $c(t)=A_c\cos\omega_0 t$ and the copy of signal reproduces its amplitude. In the presence of signal at the entrance to multiplier is supplied the mixture, which can be represented as oscillation with the random amplitude and the phase:

$$y(t)=c(t)+n(t)=A_y(t)\cos[\omega_0 t+\varphi_y(t)],$$

where A_y - amplitude of mixture; $\varphi_y(t)$ - divergence of the phase of mixture from the phase of signal.

At the multiplier output we will obtain

$$\xi_y = y(t)c(t) = \frac{A_y(t)A_c}{2}\cos\varphi_y(t) +$$
$$+ \frac{A_y(t)A_c}{2}\cos[2\omega_c t + \varphi_y(t)].$$

Term with cos $2\omega_c t$ can be disregarded/neglected, since it does not affect the output value of integrator.

Then at the multiplier output we obtain

$$\xi_y = \frac{A_y(t)A_c}{2}\cos\varphi_y(t). \qquad (5.6.1)$$

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In the absence of signal expression (5.6.1) remains valid, but

$$A_y(t) = A_n(t), \text{ а } \varphi_y(t) = \varphi_n(t), \overset{(1)}{\text{где}}\ A_n(t)$$

Key: (1). where

and $\varphi_n(t)$ - random functions of time, which describe selective interference.

Let us compare now the procedures of processing mixture in the amplitude-phase and in the phase feelers, with the weak signal. In the phase feeler according to (5.3.9) to the integrator is supplied value $\xi_y = \cos\varphi_y(t)$.

Consequently, the procedure of processing signal during the optimum phase detection of weak signal is close to the procedure of processing in the multiplier during the optimum amplitude-phase detection. However, between them is a difference, consisting in the fact that z_v contains factor $A_v(t)$, determined by the amplitude of the mixture of signal and interference. For z_φ this factor is equal to one. This difference would not be fundamental, if $A_v(t)$ was not the random function of time. Multiplier also computes $\cos\varphi_v(t)$, but with weight coefficient $A_v(t)$, which gives large role in processing of mixture by those $\cos\varphi_v$, which are counted off at high values $A_v(t)$. Let us now compare thresholds.

In the optimum amplitude-phase feeler value

$$z=\int_0^{t_0}\frac{A_v(t)A_c}{2}\cos\varphi_v(t)\,dt \qquad (5.6.2)$$

is equal with threshold $\mathcal{B}_0/2$.

With the weak signal probable value $A_v(t)\approx\sigma_n$. In the phase feeler value

$$z_\varphi=\int_0^{t_0}\cos\varphi_v(t)\,dt \qquad (5.6.3)$$

is equal with threshold $\frac{A_c t_0}{2\sigma_n}\sqrt{\frac{\pi}{8}}$ or

$$\int_0^{t_0}\frac{A_c\sigma_n}{2}\cos\varphi_v(t)\,dt$$

is equal with the threshold

$$\frac{\varepsilon_0}{2}\sqrt{\frac{\pi}{8}}. \qquad (5.6.4)$$

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From the obtained results it is evident that if we somewhat change the diagram of phase feeler, after introducing amplification with coefficient $\frac{\sigma_m A_0}{2}$, then value ξ_φ becomes to the larger degree it is similar to value ξ_φ in the optimum feeler.

From that obtained it follows that in the essence the optimum phase feeler differs from the optimum amplitude-phase feeler in terms of the fact that the integrated value has as the factor not the random amplitude of mixture $A_\varphi(t)$, little depending on the amplitude of weak signal, but constant factor σ_m, close to probable value of the amplitude of mixture with the weak signal, and threshold, which differs little from the threshold during the optimum detection. Consequently, optimum amplitude-phase feeler with the weak signal in essence is constructed on the principle of the optimum processing of phase. It is necessary to note that the amplitude-phase feeler is simple to device/equipment and can be carried out in the form of usual phase discriminator and integrator, while in the phase feeler are required very complicated devices/equipment in the form of ideal phasemeter and block, which realizes trigonometric transformation of the measured angle. Consequently, there is no engineering sense to realize the specially optimum phase detection: it in the essence is used during the usual optimum detection of weak signal with the known parameters. The available in the literature indications of the advantage of phase feeler, in the sense of the independence of threshold from power or energy of signal, are incorrect, since in the optimum phase feeler threshold also depends on energy of signal.

Let us lead now the comparison of optimum phase and optimum amplitude-phase detection with the strong signals.

Since the signal is strong, $A_0 \approx A_y(t)$.

Then to the integrator in the amplitude-phase feeler it is supplied

$$\xi_y = \frac{A_c^2}{2}\cos\varphi_y(t). \qquad (5.6.5)$$

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Is accepted hypothesis "signal is", if

$$\int_0^{t_0} \frac{A_c^2}{2}\cos\varphi_y(t)\,dt > \frac{\theta_0}{2} \qquad (5.6.6)$$

and

$$\int_0^{t_0} \cos\varphi_y(t)\,dt > \frac{t_0}{2}. \qquad (5.6.7)$$

With the strong signal the fluctuations of phase are insignificant, then

$$\cos\varphi_v(t)\approx 1-\frac{1}{2}\varphi_v^2(t).$$

Detection condition will take the form

$$\int_0^{t_0}\varphi_v^2(t)\,dt<t_0. \qquad (5.6.8)$$

From the comparison of the obtained expression (5.5.9) it follows that the diagram of optimum amplitude-phase feeler realizes an optimum processing of phase, also, with the strong signal, and in addition to this she uses information about the presence of signal, formation, placed into the amplitude about the presence of signal, placed into the amplitude of mixture.

The optimum phase feeler of this information does not use; therefore with the strong signal the results of phase detection are noticeably worse than the optimum amplitude-phase detection. For the confirmation of this result we convert (5.5.9) so that the threshold would be expressed analogously with formula (5.6.8). Then for the optimum phase feeler of strong signal we obtain detection conditions with $\Pi_1=1$

$$\frac{A_c^2}{\sigma_n^2}\frac{1}{2\ln\left(\sqrt{2\pi}\,\frac{A_c}{\sigma_n}\right)}\int_0^{t_0}\varphi_v^2(t)\,dt<t_0.$$

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As is evident in phase feeler due to the nonutilization of information about the amplitude before the integral stands the factor, which for the case ($A_0 > \sigma_n$) in question much more than one. Consequently, for the fulfillment of inequality and detection with the identical probability of passage it is required so that the fluctuations of phase in the phase feeler would be much less than in the amplitude-phase, i.e., signal must be stronger. Just as for the weak signal, in this case usually there is no engineering sense to realize independent optimum phase feelers and strong signal. These feelers have more compound circuit than amplitude-phase. In the latter by simpler technical equipment is also realized in the essence the optimum procedure of working/treatment of phase and, furthermore, is used signal amplitude.

Let us compare now between themselves the diagrams of optimum phase feelers with the strong and weak signals. Outwardly diagrams considerably differ from each other. This fact can cause great difficulties during the practical realization, since it requires change of the diagram in the dependence on the signal amplitude. For explaining this question let us consider the work of the optimum phase feeler of weak signal with the reception of strong signal.

In the optimum diagram for the weak signal to the entrance of integrator will be supplied $\cos\varphi_v$, which can be represented as

$$\cos\varphi_v = \sqrt{1-\sin^2\varphi_v}.$$

With the strong signal angles φ_v are small and $\cos\varphi_v \approx 1-0,5\,\varphi_v^2$. Hence it follows that the optimum diagram for the weak signal, which computes $\cos\varphi_v$ and that integrating this value, with the strong signal will also realize an optimum processing according to the rules, valid for the strong signal, i.e., compute the square of angle φ_v and integrate it. Thus, all optimum diagrams of amplitude-phase and phase feeler for the signals with the known parameters realize the analogous procedure of processing the phase of mixture, which in essence is constructed during the calculation of the even function of the divergence of the phase of mixture from the phase of the expected signal.

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From this it is possible to draw the conclusion that the essential information about the signal, utilized during its processing for the detection, is concentrated in the phase of signal. Is certain, the effect of amplitude as the factor, which is determining energy of signal and authenticity of optimum detection, it remains under all conditions.

Let us compare optimum phase detection with the simple. It is necessary to note that the examined in § 5.2 simple phase detection, during which is measured the phase of mixture and result is equal with the phase of the expected signal, it gives considerably worse results, than optimum phase and optimum amplitude-phase detection. The reason for this lies in the fact that the measurement of phase with the use of a matched filter to the phasemeter or with the averaging of the results of measuring the phase after phasemeter realizes not the optimum for the detection procedure of processing the measured phase. The optimum procedure of phase detection provides for with the weak signal trigonometric transformations of measured phase ($\cos\varphi_v$), with the strong signal - squaring, which is analogous for these conditions to taking $\cos\varphi_v$. Similar operations with the phase are produced in the diagrams of optimum amplitude-phase detection. These nonlinear conversions in the optimum procedures determine the considerable difference in the results in comparison with the simple phase detection. The nonlinear conversions of the measured phase change the distribution function both for the case of one interference and for the case of the mixture of signal and interference.

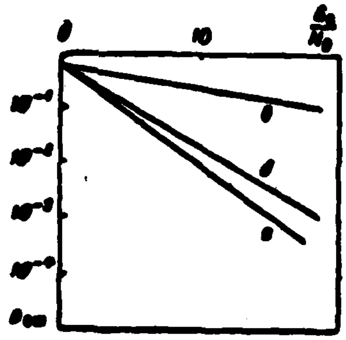

Fig. 5.6.1. Comparison of the errors of detection with the weak signal: a) for the optimum detection (formula 5.7.18); b) for the optimum phase detection (formula (5.4.13); c) for the simple phase detection, with the matched filter to the phasemeter (formula 5.2.14).

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Accumulation in integrators or filters of the lower frequencies of the results of the nonlinearly converted values of the measured phase gives considerable gain in the authenticity of detection. The physical causes for the fact that simple phase detection gives unsatisfactory results, lies in the fact that the average/mean value of the phase of noise, which is obtained as a result of integration or averaging, cannot differ from the average/mean value of the phase of mixture, since the phase of interference with any averaging remains equiprobable. In all optimum diagrams is used the even function of phase ($\cos\varphi_v$ or φ_v^2). Then the result of the accumulation of the values of readings with the mixture and the interference is different. In order to compare the authenticity of detection by different methods, let us give the result of the calculations, carried out according to the formulas obtained above. Calculations are carried out for case of p(c)=p(0)=0.5 and $r_{np}=r_n=1$. The results of comparison will be analogous and during the use of Neumann-Pearson criterion. Fig. 5.6.1 gives curves for the weak signal.

Fig. 5.6.2 gives curves for the strong signal when $A_c/\sigma_n=1$.

§ 5.7. Optimum phase detection during the use of a limiter. Earlier was examined the case when the mixture of signal and interference was supplied to the ideal phasemeter whose readings/indications did not depend on amplitude. In this case it proved to be possible to use the functions of the distribution of mixture. The use of an ideal phasemeter in the essence indicates the nonlinear transformations of mixture, because of which the effect of amplitude is removed. The elimination of the effect of amplitude it is possible to attain with the help of the direct method, after using in the limiter circuit. Then basic information about the signal there remains only in the phase, more precise in the function of "zeros".

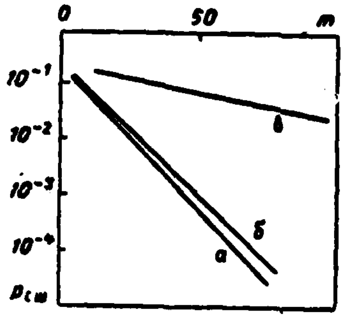

Fig. 5.6.2. Comparison of the errors of detection with strong signal $(A_0/\sigma_0=1)$: a) optimum detection; b) optimum phase detection (formulas 5.5.27 and 5.5.29); c) simple phase detection (formulas 5.2.18 and 5.2.19).

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The one-dimensional function of distribution of zeros differs little from the function of phase distribution, if the noise band is comparatively narrow. Usually in the receivers there is a filter, which realizes preliminary non-optimal filtration. On these reasons in the first approximation, it is possible to consider that the function of distribution of zeros, evaluated in the angular values, repeats the function of phase distribution. Then optimum phase feeler can be constructed on the diagram, given in Fig. 5.7.1.

To the multiplying device/equipment is supplied the stress/voltage of mixture from the output of limiter and the copy of signal. The copy of signal must have initial phase, delay and duration, that correspond to the expected signal. At the output of multiplier will appear the stress/voltage whose value is equal [see (5.6.1)]

$$\xi_{\text{огр}} \approx \frac{1{,}3 A_c A_{\text{огр}}}{2} \cos\varphi_y(t) = \text{const} \cos\varphi_y(t),$$

where $A_{\text{огр}}$ - the level of the limited stress/voltage, which is determining its amplitude; $\varphi_y(t)$ - divergence of the phase of mixture from the phase of signal.

The high-frequency component of this stress/voltage can be disregarded/neglected, since it will not affect the results of integration. It is obvious that the combination of limiter and multiplier fulfills the same function, as the combination of ideal phasemeter and trigonometric converter. During the appropriate selection of threshold this diagram must ensure the optimum processing of signal on its phase.

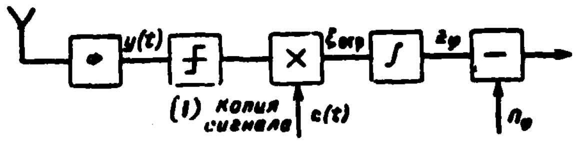

Fig. 5.7.1. The diagram of phase feeler with the limiter: Φ - filter; ⌐ - limiter; X - multiplier; ∫ - integrator; n_φ - threshold device/equipment.

Key: (1). Copy of signal.

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In the presence of signal the initial phase of mixture $\varphi_y(t)$ fluctuates with the predominant small divergences. Consequently, $\cos\varphi_y(t)$ has the prevailing values, close to one; on the output of integrator must be accumulated large stress/voltage, which usually exceeds threshold, as a result is accepted the hypothesis about the presence of signal. Due to the divergences of phase the result of integration can decrease and threshold will not be achieved/reached. In the absence of signal to the multiplying device/equipment is supplied the signal with the random equiprobable initial phase. With the sufficiently wide passband of radio channel initial phase for the storage time randomly is changed also with the equal probability they can be observed both low and high values φ_y, for which $\cos\varphi_y$ will close to zero or have negative value. This reduces the result of integration; its difference from zero is exhibited in the finite probability of false detection. In an increase in the intensity of signal the phase of mixture in essence corresponds to the phase of signal and output potential of integrator with the great probability exceeds the threshold which with an increase in the ratio of signal to the interference increases, which reduces the probability of false detection.

Assuming that the function of distribution of zeros coincides with the function of phase distribution, then the characteristics of phase feeler with the limiter will be the same as the characteristic of optimum phase feeler. On the principle examined can be constructed the feeler, in which is used Neumann-Pearson criterion. It is necessary to note that the diagram of phase feeler with the limiter and the multiplier is considerably simpler than the diagrams, given in Fig. 5.3.1 and 5.5.1. During its realization are used the limiter and the same elements/cells, as in the diagram of amplitude-phase feeler. It does not use phase measurements, also, in the essence in the technical form, close to the amplitude-phase feeler, realizes phase detection due to the loss of information about the amplitude in the limiter.

§ 5.8. Phase detection of signal with the unknown phase. Is of interest the possibility of the phase detection of the signal whose phase is not known.

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During the optimum amplitude-phase detection of signal with the random phase is used either the matched filter with the detector, or two-channel correlator with such addition of results, during which the effect of the chance of phase on the value, been congruent/equated with the threshold, is removed. Thus, in this case the phase of signal as if does not participate in the detection. However, it is possible to show that also in the random (but stable) phase the detection also to a considerable degree is realized due to the optimum processing of the phase of signal.

Let us consider the work of the optimum amplitude-phase feeler of signal with the random phase from the point of view of phase relationships/ratios. Let us take the case of the sine wave

$$c(t)=A_0\cos(\omega_0 t+\varphi_0).$$

The mixture of signal and interference can be represented as process the random ones by amplitude and the phase

$$y(t)=A_y(t)\cos[\omega_0 t+\varphi_0+\varphi_y(t)];$$

of the distribution function for $A_y(t)$ and $\varphi_y(t)$ they were obtained earlier.

Then at the multiplier output we obtain (when $A_{n}=1$)

$$\xi_{\eta}=A_{y}(t)\cos[\omega_{0}t+\varphi_{0}+\varphi_{y}(t)\cos\omega_{0}t]=$$
$$=\frac{A_{y}(t)}{2}\cos[\varphi_{0}+\varphi_{y}(t)], \qquad (5.8.1)$$

term $2\omega_{0}$ is disregarded,

$$\xi_{\gamma}=\frac{A_{y}(t)}{2}\sin[\varphi_{0}+\varphi_{y}(t)]. \qquad (5.8.2)$$

Let us examine in more detail the case of the strong signal when $A_{c}\gg\sigma_{n}$, $\varphi_{y}\ll\pi$ and terms $\sin\varphi_{y}(t)$ can be disregarded/neglected

$$\xi_{\eta}\approx\frac{A_{y}(t)}{2}\cos\varphi_{0}\cos\varphi_{y}(t), \qquad (5.8.3)$$

$$\xi_{\gamma}\approx\frac{A_{y}(t)}{2}\sin\varphi_{0}\cos\varphi_{y}(t), \qquad (5.8.4)$$

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At the output of integrators we will obtain

$$\eta=\int_{0}^{t_{c}}\frac{A_{y}(t)}{2}\cos\varphi_{y}(t)\cos\varphi_{c}\,dt, \qquad (5.8.5)$$

$$\gamma=\int_{0}^{t_{c}}\frac{A_{y}(t)}{2}\cos\varphi_{y}(t)\sin\varphi_{c}\,dt, \qquad (5.8.6)$$

then

$$B^{2}=\eta^{2}+\gamma^{2}=\left[\int_{0}^{t_{c}}\frac{A_{y}(t)}{2}\cos\varphi_{y}(t)\,dt\right]^{2}; \qquad (5.8.7)$$

since when $A_{c}\gg\sigma_{n}$

$$A_{y}(t)\approx A_{c},\ \cos\varphi_{y}(t)=1-\varphi_{y}^{2}(t),\ 0{,}5$$

then

$$B=\frac{A_{c}t_{c}}{2}-\frac{A_{c}}{4}\int_{0}^{t_{c}}\varphi_{y}^{2}(t)\,dt, \qquad (5.8.8)$$

The condition of accepting the hypothesis "signal exists" it is $\Pi_B < B$, i.e.

$$\Pi_B < \frac{A_c t_c}{2} - \frac{A_c}{4}\int_0^{t_c} \varphi_n^2(t)\,dt,$$

or, since $\Pi_B = \frac{\varepsilon_0 A_c}{2}$ (4.7.35)

$$\int_0^{t_c} \varphi_n^2(t)\,dt < t_c. \qquad (5.8.9)$$

The optimum processing of the phase of mixture in the optimum amplitude-phase feeler of signal with the random phase, revealed by formula (5.8.9), is analogous to processing in the optimum phase feeler of strong signal with the known phase, given by formula (5.5.9), and to processing in the optimum amplitude-phase feeler of signal with the known phase which in the phase interpretation is given (5.6.8).

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Thus, in the optimum diagram of the amplitude-phase feeler of signal with the random phase its phase undergoes optimum processing. Consequently, and in the random phase of signal information about it is laid during the phase, it is more precise into its constancy and the optimum processing of mixture includes the optimum processing of its phase. However, as a result of the chance of phase for its optimum processing is required the two-channel diagram, which is characterized by the increased action of interferences and by a deterioration in the authenticity.

The same results are obtained also during the analysis of the case of weak signal. Let us consider now the possibility of the realization of the phase detection of signal with the random phase. Signal with the random phase is the basic model of signal in the phase systems. Before realizing a measurement, it is necessary to discover signal, i.e., to ascertain that the receiving and measuring device/equipment is located in the zone of action of system and, if signal is pulse, to find it from the temporary situation (to reduce the uncertainty/indeterminacy of delay). If this signal it is possible to discover by phase-difference methods, then this can sometimes make it possible to simplify equipment (is removed the need for the creation of the system of amplitude-phase detection, which is in use independent of the channel of measurement). It is obvious that the simple phase detection of signal in this case in the principle is impossible, since its phase is by chance and the simple measurement of the phase of mixture cannot give response/answer to a question about presence or absence of signal. With the optimum methods of phase detection are produced the nonlinear transformations of the measured phase and the accumulation of information. Using the principles of phase detection, it is possible to create also the diagram of phase feeler, also, for the signal with the random phase.

With the realization of the optimum phase feeler of signal with the random phase it is not possible to be bounded to the single-channel diagram of processing. Let us recall that for the optimum amplitude-phase detection are necessary the two-channel diagrams with the quadrature channels. The diagram of the phase feeler of signal with the random phase also can be constructed as two-channel.

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The diagram of this feeler is given in Fig. 5.8.1. Use in one of the branches of sine converter instead of the cosine is substantiated by the fact that this channel optimally treats the phase quadrature-phase component of mixture, which is necessary in connection with the random initial phase of signal. Integration permits implemention of an accumulation in each channel, moreover in each channel the accumulated value depends on φ_0 and, therefore, it cannot be used for the comparison with the threshold. However, the subsequent transformations of the accumulated values, the analogous facts which are used in the optimum amplitude-phase feeler, make it possible to obtain value B_φ, not depending on the phase of signal and determined only by the presence of the signal:

$$\xi_{1\varphi}=\cos[\varphi_0+\varphi_s(t)],\ z_{1\varphi}=\int_0^{t_0}\xi_{1\varphi}(t)\,dt,$$

$$\xi_{2\varphi}=\sin[\varphi_0+\varphi_s(t)],\ z_{2\varphi}=\int_0^{t_0}\xi_{2\varphi}(t)\,dt,$$

$$B_\varphi^2=z_{1\varphi}^2+z_{2\varphi}^2.$$

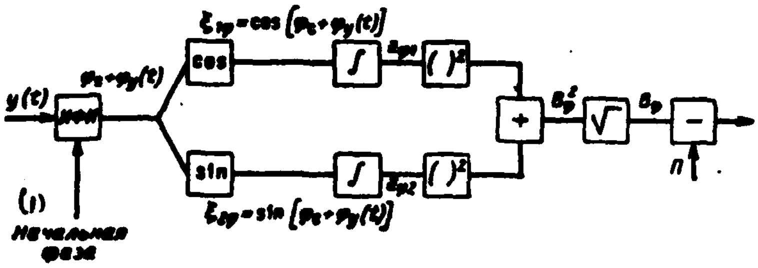

Fig. 5.8.1. The diagram of the phase feeler of signal with the random phase: ИФИ - ideal phase meter; sin and cos - trigonometric converters; $\int$ - integrators; $(\ )^2$ - square-law function generators; + - summator; $\sqrt{\ }$ - computing device of quadratic root; П - threshold device/equipment.

Key: (1). Initial phase.

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Let us consider the work of diagram during the detection of strong signal. With strong signal $(\varphi_y(t) \ll \pi)$

$$\xi_{1\varphi} \approx \cos\varphi_0 \cos\varphi_y(t), \qquad (5.8.10)$$

$$\xi_{2\varphi} \approx \sin\varphi_0 \cos\varphi_y(t). \qquad (5.8.11)$$

After integration, squaring, addition and extraction of root (Fig. 5.8.1) we will obtain

$$B_\varphi = \int_0^{t_0} \cos\varphi_y(t)\,dt \approx t_0 - 0{,}5 \int_0^{t_0} \varphi_y^2(t)\,dt.$$

The condition of making a decision about the presence of signal will take the form

$$\int_0^{t_c} \varphi_v^2(t)\,dt < 2t_c - \Pi_\varphi. \qquad (5.8.12)$$

It is obvious that the procedure of processing random phase, which corresponds to expression (5.8.12) and Fig. 5.8.1, is analogous to the procedure of processing the phase of mixture in the optimum amplitude-phase feeler of signal with the random phase, if we it examine from the point of view of phase relationships/ratios as this is done during the conclusion/output of expression (5.8.9). Analogous results are obtained also during the analysis of the detection of weak signal. Consequently, optimum phase detection of signal with the random phase to carry out possibly. It is possible to expect that the authenticity of detection or the probability of errors in the diagram of the phase feeler of signal with the random phase will be somewhat worse than in the optimum feeler of signal with the random phase and the optimum phase feeler of signal with the known phase.

Obtaining analytical expressions for the functions of the distribution of values $t_{\varphi 1}, t_{\varphi 2}$ and $z_{\varphi 1}, z_{\varphi 2}$ is connected with the bulky mathematical transformations, which determines the complexity of the mathematical expressions with the help of which it is possible to compute probability of errors $p(\Gamma_c/0)$ and $p(\Gamma_0/c)$.

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On these reasons we will be bounded to the qualitative analysis of the factors, which affect the probability of errors.

In the presence of the signal (we will assume that signal strong) phasemeter gives the little changing readings/indications, which correspond to the random phase of signal φ_c. After trigonometric transformations are obtained values $\cos\varphi_c$ and $\sin\varphi_c$. Since for the time of detection φ_c we consider as the constant value, the results of integration will be also proportional $\cos\varphi_c$ and $\sin\varphi_c$, and after transition to $B_\varphi = B_{\varphi c}$ we will obtain the value, which does not depend on φ_c, which it is to be congruent/equate with the threshold. Due to the action of interferences the value, measured by phasemeter, is changed. In this case can be observed the passage of signal in such a case, when the level of threshold will not be achieved/reached.

In the absence of signal the phasemeter measures the random phase of interference φ_n, which with the equal probability can have any value from 0 to $\pm\pi$. For the time of action of signal the value of phase undergoes changes itself. Then $\cos\varphi_n$ and $\sin\varphi_n$ are random functions. Their integration and quadratic addition will give resulting quantity $B_{\varphi n}$.

Presence $B_{\varphi n}$ creates the errors of the false detection whose probability will be determined by the probability of threshold crossing. It is obvious that with an increase in the duration of signal value $B_{\varphi c}$ at the output of integrators, caused on the action of signal, will increase more rapidly than value $B_{\varphi n}$, caused on the action of interferences.

However, the possibility of designing of the diagram of phase feeler for the signal with the random phase yet does not solve a question about the advisability of the realization of such diagrams.

It is obvious that phase feeler much more complicated than optimum amplitude-phase. Especially complicated are phasemeter and trigonometric converters. The phase feeler of signal with the random phase can be realized with the use of a limiter and two-channel quadrature correlator.

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§ 5.9. All-pass filters. In all optimum phase detectors examined earlier were used the principles of mutually correlation reception/procedure or active filtration. It is of interest to consider the possibility of the realization of passive all-pass filters. For this it is possible to use the fact that if the signal has stable initial phase, then the values of its phase at the points, divided into the integer of periods of high frequency, will be identical. For the detection of signal it is necessary to optimally develop phase displacement between the points of signal, divided by the specific delay, which is reduced to the calculation of the cosine of phase displacement and the subsequent integration. The schematic of all-pass filter, in which is realized this principle, is given in Fig. 5.9.1. Instead of the ideal phasemeter and the trigonometric converter it is possible to use limiters and multiplier, then diagram will take the form, depicted in Fig. 5.9.2. Let us dismantle/select the work of these diagrams. During the supplying to the filter of one interference the phase of disturbing voltage is the random function of time.

The values of phase, divided by the interval of time $\tau_3 = kT_0$, where $T_0 = (2\pi/\omega_0)$, and ω_0 - the midband frequency of the transmission of circuit to the all-pass filter, they differ from each other. Ideal phasemeter will give the readings of phase displacement with the equal probability in the limits from $-\pi$ to $+\pi$, if τ_3 is more than the interval of correlation for the phase. After trigonometric transformation and integration will be obtained the random function of stress/voltage with the zero average/mean and divergent dispersion. If it exceeds threshold, this can lead to the false detection.

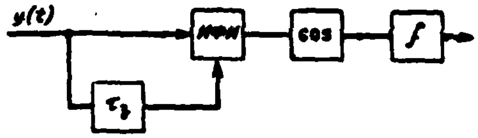

Fig. 5.9.1. The schematic of the all-pass filter: ИФИ - ideal phase meter; cos - trigonometric converter; ∫ - integrator.

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During the supplying to the filter of signal with the constant initial phase and the frequency to the phasemeter will enter the voltage component with constant phase displacement, equal to zero. Consequently, value at the output of IFI will fluctuate predominantly about the position, which corresponds to the zero shift/shear for which the cosine is close to one. As a result at the output of integrator will be accumulated the stress/voltage. Excess with this stress/voltage of threshold corresponds to the selection of hypothesis about the presence of signal. As is known, matched filter gives the same characteristics during the detection, as correlation diagram. On this basis/base it is possible to claim that during the appropriate selection of the parameters, the all-pass filter ensures the same characteristics, as correlators examined above. The work of this filter will not depend on the absolute value of the signal, supplied to its entrance, but will depend only on relation signal/noise. One of the versions of this filter was investigated in work [5.1]. It was shown that it can give the best results, than obtained during the amplitude detection. This conclusion/output can be made, also, of the theory given earlier. All-pass filters did not obtain considerable development and use/application.

§ 5.10. Estimation of phase detection. On the basis of the analysis of phase detection it is possible to do a series/row of conclusions/outputs. Basic conclusion/output lies in the fact that during the detection of signal by optimum amplitude-phase feelers the phase of signal is treated optimally. This relates both by the weak and to the strong signals. When phase is unknown (it is by chance), but it is constant, during the optimum detection also is realized the optimum processing of phase.

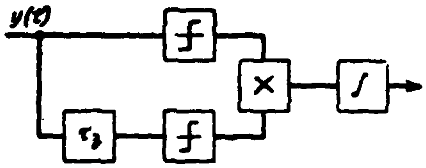

Fig. 5.9.2. The schematic of all-pass filter with the limiters: ⌐ - the device/equipment, which feeds meander; X - multiplier; ∫ - integrator.

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By this is explained the fact that the diagrams of the optimum amplitude-phase detection of signal with the unknown (but stable) phase give results close to those obtained in circuits of detection of a signal with a known phase. The basic information about the signal, utilized during the detection, consists in its phase, since the optimum amplitude-phase feeler in essence implements the operations, analogous to those implemented in the optimum phase feeler. In the optimum phase feeler is realized the measurement of phase and the subsequent transformation of the results of measurement in accordance with the optimum algorithm. Optimum phase feelers can be created for the signal with the known and random phase. In both phase feelers is optimally treated only the phase of signal. With random phase due to the two-channel construction increases the action of interferences, which causes a deterioration in the authenticity. Furthermore, the nonutilization of signal amplitude in the process of processing mixture in the phase feelers causes a deterioration in the authenticity or energy loss. These energy losses with the weak signals are insignificant. With the strong signals they prove to be more essential. Signal amplitude during the phase detection affects threshold; however, the use of mode/conditions with the active pause also of Neumann-Pearson criterion they make it possible to create the diagrams whose mode/conditions does not depend on signal amplitude. If the practical realization of the phase feelers would be simpler than amplitude-phase ones, it would be expedient raise a question about their integral use; however, the diagrams, given earlier, show that the phase feelers are more complicated than amplitude-phase. The most complicated part of the phase feelers is the meter of phase. On these reasons it is difficult to expect the independent use/application of phase feelers.

Special position in this respect can be in the phase systems. If system is intended for the extraction of information from the phase of signal, then in it usually there is a meter of phase, to a certain degree which approaches ideal. Under these conditions is in principle important the fact that in this system both the measurement of the parameter and the detection of signal can be realized on one and the same principle.

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In other words, into the phase system there is no need for introducing amplitude-phase feeler. Using information only about the phase of signal, system can solve also the problem of detection, almost without the loss of information about the signal. Optimum phase detection occurs during the use of optimum amplitude-phase feelers (into diagram of which enter the multipliers, integrators and the generators of the copy of signal) if before the mixture admission to the feeler it is normalized in the amplitude, for example, in transit through the limiter or due to the action of sufficiently high speed ARU. The aforesaid earlier, apparently, and causes interest in the theory of phase feelers whose some aspects are examined in works [5.2, 5.3]. Possibility the uses of quantization of phase with the phase-difference methods of detection are investigated in work [5.4].

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Chapter 6.

OPTIMUM MEASUREMENT OF PHASE.

§ 6.1. Formulation of the problem. After the signal is discovered (established/installed, that it there is), in many instances must be solved the problem of measuring the parameters of signal in which is contained useful information. In connection with this the value which is not smaller, than optimum detection, has the optimum measurement of the parameters of signal.

Useful information can be established in the delay, amplitude, frequency, and phase of the signal.

The measurement of each of the parameters although has its special features/peculiarities however can be established/installed general laws.

In the measurement of the parameter we consider that the parameter is the unknown random variable, in which is reflected useful information. In this case the remaining parameters of signal can be known or are also random variables, but parasitic, not carrying useful information.

In general form this is written/recorded as follows:

$$c(t, \alpha_1, \alpha_2, \ldots, \beta_1, \beta_2, \ldots), \qquad (6.1.1)$$

where t - time; α_i - random parameters, which carry information; β_i - random parameters, which do not carry information.

As the simplest example it is possible to give these combinations:

a) useful information is laid in signal delay, i.e., τ_3 - random variable, which carries information and which is subject to measurement, the remaining parameters of signal A_0, φ_0, ω_0 are random variables.

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This case corresponds to ranging to the moving/driving target by pulse radar;

b) useful information is laid during the phase of signal, i.e., φ_0 - random variable, which carries information and which is subject to measurement, the remaining parameters of signal, besides amplitude, are known. This case corresponds to the radio navigational phase system, in which is used continuous signal or pulse signal, but in this case before the measurement of phase is determined delay;

c) useful information is laid during the phase of signal, signal frequency is known; A_c and τ_3 are random variables. This corresponds to the phase system, in which are used pulse signals, if pulse delay is not determined.

The measurement of the parameters of signal is always realized under the conditions when there is not only signal, but also interference $n(t)$. For measuring the parameter during specific time t_n is observed the mixture of signal and interference. This time can be limited by the duration of signal or by any other factors, if signal lasts longer than the time during which it is possible to observe, measuring the parameters. For example, in the radio-navigation systems the time of observation can be limited not by the duration of signal, but by the time during which must be given the coordinates of observation point. Consequently, the time of observation is always limited. In the general case the measured parameter can for the time of observation be changed, and then the problem of measurement in principle is complicated. Therefore is more right the problem of measuring the parameters to solve for two cases, namely:

the measurement of the parameter, which is the random variable, which retains its value in the process of observing the signal, and

the measurement of the parameter, which is changed in the process of observation. The first case occasionally refers to as "reception/procedure of the separate values of continuous communications/reports", the second - "reception of oscillations/vibrations". Can be used also terms measurement and tracking or tracking. In this section will be examined only the measurement.

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According to observations, i.e., on the obtained realization y(t) or selection y_1, y_2, ..., this corresponding realization, must be solved a question about which value had the parameter (or the parameters), or, as they say, must be given the estimation of the parameter. Since the time of observations is limited and signal is accompanied by interferences, it is obvious that the estimation of the parameter will differ from its true value. Problem lies in the fact that to establish/install, what procedure of processing must undergo the mixture of signal and interference, so that the measurement of the parameter would be optimum, moreover it is necessary to define concretely the concept of optimality, i.e., to establish/install the criterion of optimality. After finding the procedure of optimum processing, it is useful to solve a question about the complexity and the practical feasibility of this procedure or diagram, to explain the dependence of diagram on the special features/peculiarities of signal and to establish/install the characteristics of the results of measuring the parameter. For the solution of this problem it is necessary to have the initial statistical evidence about the signal, the measured parameter and the interference.

Interference has normal distribution, it is possible to characterize: dispersions σ_n^2, by the one-dimensional function of distribution $w(n)$, by energy spectrum $G_n(\omega)$ or by autocorrelation function $B_n(\tau)$ and by the multidimensional distribution function.

The useful and parasitic random parameters of signal must be characterized by the distribution functions. For the distributions of the random parameters of signal it is not possible to take some common model. In each specific case the distribution function can be different.

In many instances it is difficult to find the function of the distribution of the random parameter. The difficulties, connected with the selection of the function of the distribution of the random parameter, are caused by the fact that frequently on the basis of previous experiment or theoretical linings/calculations is impossible to accurately formulate the law of distribution. The distribution in question is frequently called a priori, i.e., pre-test, since it does not depend on the results of experiment (observation of mixture) and must be determined preliminarily.

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However, usually the function of the distribution of the measured parameter little affects the diagram which will be found as optimum, and the random parasitic parameters have a basic effect on the procedure of processing mixture as a result of the very fact of the chance of their value. As a rule, it should be noted that the presence in signal of the parasitic, random, immeasurable parameters makes it necessary to change procedure and diagrams of optimum processing, since one or another the parameter cannot be used for the isolation/liberation of signal from the interferences and diagram must be constructed so that the random character of this parameter would not affect the result of measurement, i.e., so that would be reduced the action of this parameter. For example, in the random phase it is necessary to realize the detection which destroys information about the phase, or in the case of the random delay to pass to the output all signals, without realizing a strobing/gating, and the like. It is obvious that the model of signal and the parameter, which is subject to measurement, are determined by designation/purpose of radio engineering device/equipment and by working conditions for its. For the phase systems must be set the problem of the determination of the optimum procedure of processing in the measurement of the phase of the signal whose all remaining parameters are known, or at the presence of the random parasitic parameters. It is expedient to consider the general/common/total methodology which must make it possible to manufacture general/common/total approach to the solution of such problems.

S 6.2. Criterion of optimality. The solution of the problem of determining the optimum procedure of processing must be begun from establishing/installing, what result is to consider optimum, i.e., to develop the criterion of optimality. Questions of optimization play large role in science and technology. In each branch there are their criteria of optimality and methods of analysis and synthesis of optimum machines, devices/equipment and systems. The special feature/peculiarity of the case in question lies in the fact that is placed the problem of the optimization of measurement in the presence of interferences. The solution of this problem is based to the theory of the statistical solutions, checking of statistical hypotheses developed methods and the methods of estimations.

After using the basic results of this theory, let us explain a question about the selection of the criterion of optimality for the systems, in which is used the measurement of the parameters of radio signal. For the larger laconicism of the recording of formulas and clarity of presentation let us assume that the signal has one measured random parameter α.

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The version of the presence of the random immeasurable parameters is examined further. Supervising of mixture, it is necessary to solve a question about which valve had the parameter of signal α, i.e., to give to it estimation α.

During the solution of this problem it is possible to use instantaneous reading of mixture - y or concrete/specific/actual realization y(t), or selection from this realization $y_1, y_2, y_3, \ldots$. For obtaining evaluating the parameter it is necessary to use some rule of the solution which can be registered in the form of function g then $\hat{a}=g\,[y\,(t)]$ or is shorter $\hat{\alpha}=g(y)$. This rule of the solution can be regular, i.e., such, which to each concrete/specific/actual value of the single reading y or each concrete/specific/actual realization y(t) can correspond only the one completely specific estimation $\hat{\alpha}$, obtained with the help of one, completely specific rule of the solution. The theory of the statistical solutions makes it possible to develop this thought and examines also the randomized rules of the solution, during which different rules can be used with some probability. In the following presentation will be examined only the regular (nonrandomized) rules of the solution.

Since the reception of signal occurs in the presence of interferences, then single reading or selection are distorted by interferences and estimation $\overset{\times}{\alpha}$ differs from the true value of the measured parameter, i.e., $\overset{\times}{\alpha}-\alpha=\delta\alpha\neq0$. Value $\overset{\times}{\alpha}-\alpha$ can be named error $\delta\sigma$. In order to consider the optimality of the utilized rule of the solution, i.e., to explain, how successfully in some sense is treated mixture for obtaining the estimation, in the theory of the statistical solutions is introduced concept "loss function" or the "function of losses". Loss function must express the dependence of losses on the value of the measured parameter and its estimation.

In general form the loss function can be registered in the form $\mathcal{J}(\alpha, \overset{\times}{\alpha})$. It is most natural to consider that with $\overset{\times}{\alpha}=\alpha$ the losses can be considered as zero.

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In the case of the simple measurement of the parameter to most regularly consider that the loss function must depend only on difference $\overset{\times}{\alpha}-\alpha$ and increase the value of losses with an increase in the difference, then $\mathcal{J}(a, \overset{\times}{a}) = \mathcal{J}(\overset{\times}{a} - a)$. Most widely used are three forms of the function of the losses:

$$\mathcal{J}(\overset{\times}{a} - a) = |\overset{\times}{a} - a|,$$

$$\mathcal{J}(\overset{\times}{a} - a) = (\overset{\times}{a} - a)^2,$$

$$\mathcal{J}(\overset{\times}{a} - a) = 1 \ \overset{(1)}{\text{при}} \ |\overset{\times}{a} - a| > \Delta a, \qquad (6.2.1)$$

$$\mathcal{J}(\overset{\times}{a} - a) = 0 \ \overset{(1)}{\text{при}} \ |\overset{\times}{a} - a| < \Delta a.$$

Key: (1). with.

Graphically these loss functions are given in Fig. 6.2.1. Each of these functions has specific common sense. When $\mathcal{J}(\overset{\times}{a} - a) = |\overset{\times}{a} - a|$ it is considered that the losses increase just as error, i.e., the negative consequences of measuring error proportionals to the modulus/module of error. This dependence is characteristic for the cases, with which useful results slowly deteriorate with an increase in the errors.

When $\mathcal{J}(\overset{\times}{a} - a) = (\overset{\times}{a} - a)^2$ it is assumed that the negative consequences of error increase more rapidly than error itself, i.e., the losses, connected with greater errors, are relatively more. This loss function is used most frequently; it will be coordinated well with the intuitive representation about the fact that the small errors in close margins little affect the usefulness of the result of measurement. The third form of the function when errors within certain limits not at all cause losses (zero loss), but beginning from certain value of error the results of measurements they become little useful, corresponds to an even more critical estimation of errors.

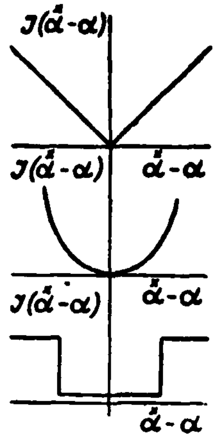

Fig. 6.2.1. Loss functions.

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As it will be shown further, the form of the function of losses little proves to be on the optimum procedure of processing the mixture of signal and interference and is reflected, in essence, in the quantitative characteristic of the consequences of inaccuracies in the estimation of the parameter. Let us formulate now the criterion of optimality. The measured parameter α is random variable and is changed from one cycle of measurements to another in accordance with the inherent in it a priori distribution $w(\alpha)$.

Value $\overset{\times}{\alpha}$ is also by chance, since it is determined by the presence of the interferences, which distort signal. Error $\delta\alpha$ and magnitude of losses, i.e., $\mathcal{J}(\overset{\times}{\alpha}, \alpha)$ are random variables, $\mathcal{J}(\overset{\times}{\alpha}, \alpha)$ can take only positive values, which with the obviousness follows of the given above standard loss functions. If $\mathcal{J}(\overset{\times}{\alpha}, \alpha)$ is random, then its one concrete/specific/actual value cannot characterize the losses, which associate measurement. For the random variables it is necessary to use the concepts of average/mean value, dispersion, functions of distribution, etc. In the case in question it is most natural to use average/mean value or mathematical expectation of magnitude of losses. Averaging must be carried out through all possible values $\overset{\times}{\alpha}$ and α taking into account the probability of their specific combinations. The average/mean value of loss function is accepted to call the average/mean risk R.

For obtaining the mathematical expectation of the value of loss function it is necessary to multiply it by the function of distribution $w(\overset{\times}{\alpha}, \alpha)$ and to fulfill integration for all possible values $\overset{\times}{\alpha}$ and α.

As a result we will obtain

$$R=\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \mathcal{J}(\overset{\times}{\alpha}, \alpha)\, w(\overset{\times}{\alpha}, \alpha)\, d\alpha\, d\overset{\times}{\alpha}. \qquad (6.2.2)$$

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The concept of average/mean risk is sufficiently to general/common/total ones and makes it possible to consider the quality of measurement and the quality of the procedure of processing the mixture of signal and the interference, used in the measurement of the parameter of the signal, which has unknown value. Therefore the optimality of one or the other procedure of the processing (one or the other rule of the solution, on which on the basis of concrete/specific/actual observation it is given estimation to the measured parameter) it is expedient to characterize on that, does give this procedure minimum average/mean risk. The procedure of processing or the rule of the solutions, which ensure minimum average/mean risk, can be recognized as optimum ones. Consequently, g(y), with which is minimized R, it is optimum - $g_{opt}(y)$.

If it is possible to find this rule, then on those mathematical operations which it contains, it is possible to determine optimum procedure or schematic of receiving and measuring device/equipment. As a result of observing the mixture is fixed/recorded selection y_1, y_2, ... or one reading y. For the determination of optimum diagram it is necessary for R to express through y_1, y_2, ... or y. Let us consider the case of using one reading y. In formula (6.2.2) let us pass from estimation $\overset{\times}{\alpha}$ to expression g(y) and reading of mixture y.

In the loss function this substitution can be carried out directly

$$\mathcal{J}(\overset{\times}{a}, a) = \mathcal{J}[g(y), a]. \qquad (6.2.3)$$

In the distribution function the replacement of variable/alternating must be realized according to the rules of the functional transformations of the distribution functions.

As a result it is possible to obtain

$$w(\overset{\times}{\alpha}, \alpha)\, d\alpha\, d\overset{\times}{\alpha} = w(y, \alpha)\, dy\, d\alpha.$$

It is obvious that $w(\overset{\times}{\alpha}, \alpha)$ and $w(y, \alpha)$ - different functions and symbol w means that both of them characterize probability densities.

Then

$$R = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \mathcal{J}[g(y), \alpha]\, w(y, \alpha)\, dy\, d\alpha, \qquad (6.2.4)$$

where $w(y, \alpha)$ - the two-dimensional function of joint distribution.

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According to the known formula of the multiplication

$$w(y,\ a)=w(a)\,w(y/a)=w(y)\,w(a/y);$$

after substitution we will obtain

$$R=\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\mathcal{J}[g(y),\ a]\,w(y)\,w(a/y)\,dy\,da=$$

$$=\int_{-\infty}^{+\infty}w(y)\left\{\int_{-\infty}^{+\infty}\mathcal{J}[g(y),\ a]\,w(a/y)\,da\right\}dy=$$

$$=\int_{-\infty}^{+\infty}w(y)R_y\,dy, \qquad (6.2.5)$$

$$R_y=\int_{-\infty}^{+\infty}\mathcal{J}[g(y),\ a]\,w(a/y)\,da, \qquad (6.2.6)$$

where R_y— conditional mean risk (at the specific value of reading y).

Since w(y) has positive values, minimization R can be provided through minimization R_y. For this it is necessary, knowing $\mathcal{J}(\overset{\times}{a},\ a)$ and w(α/y), to find the form of the function g(y), which minimizes R_y. In detail at these conversions let us pause more lately.

§ 6.3. Criterion of minimum average/mean risk during the use of finite time of observation. Earlier there was obtained general/common/total expression for R during the use of an one-time reading. In the radio engineering systems, based on the measurement of the parameters of radio signal, as a rule, has the capability to supervise of mixture during the interval of time, which substantially exceeds the interval of the correlation of interference. As a result of the fact that for a period of time of observation are stored data about the measured value, the measured value can be evaluated more accurately. On these reasons the expression for R, obtained in the previous paragraph, must be developed.

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Is more convenient for obtaining the mathematical expressions to use the concept of selection, i.e., to have in mind that for time t_n there can be made m independent readings of mixture y_1, y_2, ... through

$$\Delta t = \frac{1}{2f_s},$$

where f_s— highest frequency in the interference spectra and signal.

The rule of solution acquires the following form:

$$\overset{\times}{a} = g(y_1, y_2, \ldots); \tag{6.3.1}$$

loss function can be registered in the form

$$\mathcal{J}[g(y_1, y_2, \ldots), a]. \tag{6.3.2}$$

For obtaining the average/mean risk it is necessary to use the combined function of distribution $w(y_1, y_2, \ldots, \alpha)$ and to carry out repeated (m of times) integration for all possible values of y_1, y_2, ..., and integration for α is analogous how this is provided in (6.2.4) for the one-dimensional case.

Then expression for R will take the form

$$R=\int_{-\infty}^{+\infty}\cdots\underbrace{\iint\cdots\iint}_{m\ \text{раз}}\mathcal{F}[g(y_1, y_2, \ldots), \alpha]\times \times w(y_1, y_2, \ldots \alpha)\,d\alpha\,dy_1\,dy_2\ldots \qquad (6.3.3)$$

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Let us now move on from the joint probability density to the conditional probabilities, after using the relationship/ratio known from the probability theory:

$$w(y_1, y_2, \ldots \alpha)=w(\alpha)\,w(y_1, y_2, \ldots/\alpha)= =w(y_1, y_2, \ldots)\,w(\alpha/y_1, y_2, \ldots), \qquad (6.3.4)$$

then

$$R=\int_{-\infty}^{+\infty}w(\alpha)\,d\alpha\underbrace{\iint\cdots\iint}_{m\ \text{раз}}\mathcal{F}[g(y_1, y_2, \ldots)\alpha]\times \times w(y_1, y_2, \ldots/\alpha)\,dy_1\,dy_2\ldots \qquad (6.3.5)$$

the function

$$R_\alpha=\iint\cdots\iint\mathcal{F}[g(y_1, y_2, \ldots)\alpha]\,w(y_1, y_2/\alpha)\,dy_1\,dy_2\ldots \qquad (6.3.6)$$

can be named conditional mean risk, i.e., average/mean risk with determined value α. For obtaining the average/mean risk it is necessary to find average from all values α, i.e.

$$R = \int_{-\infty}^{+\infty} R_{\alpha} w(\alpha)\, d\alpha. \tag{6.3.7}$$

It is possible to obtain another relationship/ratio for R

$$R = \int_{-\infty}^{+\infty} \iint_{m} \cdots \iint \mathcal{F}[g(y_1, y_2, \ldots) \alpha]\, w(y_1, y_2, \ldots) \times$$
$$\times w(\alpha / y_1, y_2, \ldots)\, dy_1\, dy_2 \ldots d\alpha. \tag{6.3.8}$$

To this expression it is possible to give the following form:

$$R =$$
$$= \iint \cdots \iint w(y_1, y_2, \ldots)\, dy_1\, dy_2 \ldots \int_{-\infty}^{+\infty} \mathcal{F}[g(y_1, y_2, \ldots) \alpha] \times$$
$$\times w(\alpha / y_1, y_2, \ldots)\, d\alpha. \tag{6.3.9}$$

The function

$$R_y = \int_{-\infty}^{+\infty} \mathcal{F}[g(y_1, y_2, \ldots) \alpha]\, w(\alpha / y_1, y_2, \ldots)\, d\alpha. \tag{6.3.10}$$

also can be named conditional mean risk, i.e., average/mean risk for the specific realization y_1, y_2, ... Then the average/mean risk (unconditional) can be found, neutralizing R on all possible realizations.

Since realizations are random process, the averaging will take the form

$$R=\iint\cdots\iint R_y w(y_1, y_2, \ldots)\,dy_1\,dy_2. \quad (6.3.11)$$

The obtained expressions make it possible to do very important step.

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It is obvious that the conditional risk and probability density - value especially positive; then the minimum of average/mean risk will be provided if it is possible to ensure the minimization conditional mean risk R_y or R_a. For reasons, presented further, to more conveniently use concept R_y. Consequently, if we find such decision function $g(y_1, y_2, \ldots)$, which provides minimization R_y, then it is possible to claim that it corresponds to the optimum procedure of processing. As is evident, the solution of problem significantly was simplified because of the transition to conditional mean risk R_y and the possibility to use for its calculation a comparatively simple expression, implementing single integration. In the following presentation we will examine only conditional risk R_y and analyze the algorithms, which make it possible it to minimize.

Expression for R_y (6.3.10) can be converted, since

$$w(a/y_1, y_2, \ldots)=\frac{w(a)\,w(y_1, y_2, \ldots/a)}{w(y_1, y_2, \ldots)}, \quad (6.3.12)$$

then

$$R_y = \frac{1}{w(y_1, y_2, \ldots)} \int_{-\infty}^{+\infty} \mathcal{J}[g(y_1, y_2, \ldots)\alpha] \times$$
$$\times w(\alpha) w(y_1, y_2, \ldots/\alpha) d\alpha. \qquad (6.3.13)$$

The function of the reverse/inverse probability $w(\alpha/y_1, y_2, \ldots)$ and the function of plausibility $w(y_1, y_2, \ldots/\alpha)$ they play important role in the theory of the estimations and subsequently their use will be examined in detail. For an example let us give expression R_y for the concrete/specific/actual form of the function of losses.

Let us take quadratic loss function

$$\mathcal{J}[g(y_1, y_2, \ldots)\alpha] = [g(y_1, y_2, \ldots) - \alpha]^2, \quad (6.3.14)$$

we will obtain

$$R_y = \int_{-\infty}^{+\infty} [g(y_1, y_2, \ldots) - \alpha]^2 w(\alpha/y_1, y_2, \ldots) d\alpha$$

or

$$R_y = \int_{-\infty}^{+\infty} (\overset{\times}{\alpha} - \alpha)^2 w(\alpha/y_1, y_2, \ldots) d\alpha. \qquad (6.3.15)$$

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Analogous expressions can be obtained, also, for other loss functions; however, quadratic function permits to obtain final results and for the simplest form. Furthermore, as will be shown further, the form of the function of losses, if it remains symmetrical and increasing, does not affect the optimum procedure of processing.

It is necessary to note the series/row of the limitations, inherent in the relationships/ratios obtained above. All conclusions were done on the assumption that the signal is the known function of time with one unknown, measured by the parameter. Under the actual conditions can stand the problem of measuring several unknown parameters α_1, α_2, ... at the presence of several random immeasurable parameters β_1, β_2, ... The analysis of the optimum processing of mixture in the simultaneous measurement of several parameters has essential features and does not enter into our problem. The optimum processing of mixture in the measurement of one parameter and at the presence in signal of the random immeasurable parameters is the rapid case. Examples of such combinations were given earlier. Therefore the obtained above general/common/total methodology of finding the optimum procedure of processing must be developed and supplemented for the case of the presence in signal of the random immeasurable parameters. Let us consider the case when there is one parasitic random parameter of signal β. For the solution of problem must be known the combined function of the distribution of that measured and immeasurable the random parameters $w(\alpha, \beta)$. Usually it is possible to consider the random parameters α and β statistically independent variables, then

$$w(\alpha, \beta)=w(\alpha)w(\beta).$$

In order to use the obtained previously results and to lead to them the solution of problem in the case of the presence in the signal of the random immeasurable parameters, it is necessary to lead such mathematical conversions which would make it possible to get rid of the effect of the random immeasurable parameter on the result. At first let us assume that is used the single reading.

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Let us recall that according to the product rule

$$w(\alpha\beta/y)w(y)=w(y/\alpha\beta)w(\alpha\beta), \qquad (6.3.16)$$

$w(\alpha\beta/y)$ - the conditional probability density of the combination of the different values of the parameters α and β with the condition of the specific reading of mixture y; w(y) - probability density of mixture values; $w(y/\alpha\beta)$ - the conditional probability density of the values of mixture under the condition of the specific value of the random parameters α and β;

$w(\alpha\beta)$ - the probability density of the joint distribution of the random parameters.

Then

$$w(\alpha\beta/y)=\frac{w(\alpha\beta)w(y/\alpha\beta)}{w(y)}=k_1w(\alpha\beta)w(y/\alpha\beta), \qquad (6.3.17)$$

since

$$\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} w(\alpha\beta/y)\,d\alpha\,d\beta=1,$$

that

$$k_1=\frac{1}{\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} w(\alpha\beta)\,w(y/\alpha\beta)\,d\alpha\,d\beta}. \qquad (6.3.18)$$

In all given earlier formulas it was used by $w(\alpha/y)$, therefore, it is necessary from $w(\alpha\beta/y)$ to pass to $w(\alpha/y)$. For this, as is known, necessary to integrate expression $w(\alpha\beta/y)$ with respect to all possible values β

$$w(\alpha/y)=\int_{-\infty}^{+\infty} w(\alpha\beta/y)\,d\beta=$$

$$=\int_{-\infty}^{+\infty} k_1 w(\alpha\beta)\,w(y/\alpha\beta)\,d\beta. \qquad (6.3.19)$$

Usually it is possible to consider the random parameters of signal as independent variables, then the obtained expression can be represented in other form:

$$w(\alpha/y)=k_1 w(\alpha)\int_{-\infty}^{+\infty} w(\beta)\,w(y/\alpha\beta)\,d\beta. \qquad (6.3.20)$$

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Thus, when, in the signal, the immeasurable random parameter is present, is possible the solution of the problem of the optimization of measurement, but optimum procedure must have essential features as a result of the need of calculating integral (6.3.20). The need of executing this integration usually significantly changes optimum procedure and, therefore, the optimum measuring circuit of the parameter.

Analogously it is possible to fulfill conversions for the case when in the measurement of the parameter of radio signal is supervised of mixture during finite time.

Function $w(\alpha/y_1, y_2, \ldots)$ can be obtained from the expression

$$w(\alpha/y_1, y_2, \ldots)=\int_{-\infty}^{+\infty} w(\alpha\beta/y_1, y_2, \ldots)\,d\beta=$$
$$=k_1 w(\alpha)\int_{-\infty}^{+\infty} w(\beta)\,w(y_1, y_2, \ldots/\alpha\beta)\,d\beta. \quad (6.3.21)$$

S 6.4. Criterion of the maximum of reverse/inverse probability. Let us consider now the sense of the obtained expressions and will formulate on this basis the version of the criterion of minimum average/mean risk, the so-called criterion of the maximum of reverse/inverse probability. From the expression for R_v it follows that for the satisfaction of the criterion of minimum average/mean risk the integral of the product of the squares of error to the conditional function of distribution $w(\alpha/y_1, y_2, \ldots)$ must be minimum. The function of distribution $w(\alpha/y_1, y_2, \ldots)$ determines the probability density of one or the other value α during the specific combination of the values of selection y_1, y_2, ..., which was obtained during the observation of mixture. This function is usually called the "function of reverse/inverse probability", since it shows the probability (is more precise, probability density) of reason α depending on corollary - selection y_1, y_2, ... Such a function usually has the form shown on Fig. 6.4.1.

In the defined realization or sampling, the distribution function at some value $\alpha=\alpha_M$ has a maximum. It is obvious that value $\alpha=\alpha_M$ is most probable.

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It is natural for the unknown value of the measured parameter to take its probable value, i.e., to place $\overset{\times}{\alpha}=\alpha_M$. Let us clarify this in more detail. Loss function is depicted in Fig. 6.4.2. Obvious that under condition $\overset{\times}{\alpha}=\alpha_M$ the area, enclosed by the curve, which is obtained as a result of multiplication $(\overset{\times}{\alpha}-\alpha)^2$ and $w(\alpha/y_1, y_2, ...)$, will be minimum, i.e., the integral, expressing average/mean risk, will be minimum. If we increase the sample size, i.e., to take a larger number of independent readings, then the function of distribution $w(\alpha/y_1, y_2)$, must become of ever of narrower, since probability that the obtained combination of readings corresponds to some other value α, will be increasingly less and it is less. The narrower there will be the distribution function, the less will be the risk, since the area, enclosed by the curve, which is obtained with multiplication

$(\overset{\times}{\alpha}-\alpha)^2$ by $w(\alpha/y_1, y_2, \ldots)$, will be reduced. Thus, when the function of distribution $w(\alpha/y_1, y_2, \ldots)$ is symmetrical relative to maximum, the criterion of minimum average/mean risk leads to the possibility to formulate the criterion of the maximum of reverse/inverse probability. Diagram will provide minimum average/mean risk, if it makes it possible to compute the function of reverse/inverse probability and to find the value α, which corresponds to the maximum of reverse/inverse probability. Then $\overset{\times}{\alpha}_{\text{опт}}=\alpha_{\text{м}}$.

It is possible to use another approach to the solution of the problem of the determination of optimum estimation.

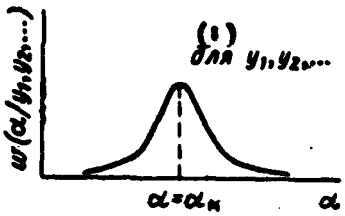

Fig. 6.4.1. Function of reverse/inverse probability.

Key: (1). for.

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Actually/really, if as a result of the realization of observation, i.e., on the obtained selection y_1, y_2, y_3, ..., it is possible to construct the function of distribution $w(\alpha/y_1, y_2, ...)$, i.e., the function of reverse/inverse probability, then the determination of optimum estimation $\overset{\times}{\alpha}_{опт}$ it is possible to approach as follows. If these the distribution function carries symmetrical character, then in accordance with the limit theorems of the probability theory with an infinite increase in the selection the average/mean value approaches true. However, observation virtually is accomplished finite time $t_н$. Then at the limited sample size the estimation of the measured parameter is nearest of all to the true, if we use an average/mean value of the estimations of the measured parameter for the time of observation. This can be made either averaging of the estimations, obtained for each of the readings, or finding of the average/mean value of the function of reverse/inverse probability (if according to observations (selection) function is constructed) with the help of the relationship/ratio

$$\overset{\times}{a}_{\text{опт}} = a_{\text{ср}} = \int_{-\infty}^{+\infty} a_1 w(a/y_1,\ y_2,\ \ldots)\, da. \qquad (6.4.1)$$

This estimation will be accompanied by error, since function $w(\alpha/y_1, y_2, \ldots)$ is constructed according to observations during the limited time. If we assume that this function is symmetrical, which usually occurs, since probable deviations to the different sides can be observed with the equal probability, then average/mean value coincides with the value at which the distribution function reaches maximum.

Thus, the rule of the selection of estimation can be formulated so

$$\overset{\times}{a}_{\text{опт}} = a_{\text{м}} = a_{\text{ср}}. \qquad (6.4.2)$$

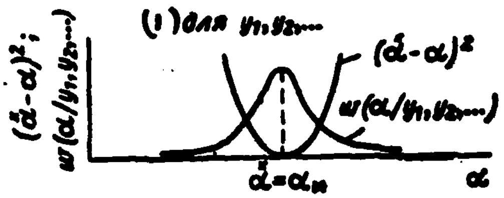

Fig. 6.4.2. Function of reverse/inverse probability and loss function.

Key: (1). for.

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§ 6.5. Criterion of the maximum of plausibility. In order to find the diagram of reception-meter, which implements the obtained optimum processing, and to consider the possibility of its simplification, it is necessary to convert the obtained expression. Into the formulas, which are determining optimum procedure, enters $w(\alpha/y_1, y_2, \ldots)$. It is necessary to obtain the expressions which connect this function with such relationships/ratios which can be simply obtained in the diagrams. For this we will use the fact that

$$w(\alpha)\,w(y_1, y_2, \ldots/\alpha) = w(\alpha/y_1, y_2, \ldots)\,w(y_1, y_2, \ldots)$$

or

$$w(\alpha/y_1, y_2, \ldots) = \frac{w(\alpha)\,w(y_1, y_2, \ldots/\alpha)}{w(y_1, y_2, \ldots)}, \qquad (6.5.1)$$

where $w(\alpha)$ - the function of the distribution of the measured parameter; $w(y_1, y_2, \ldots /\alpha)$ - the conditional distribution function, which gives the probability density of one or the other combination $y_1, y_2, \ldots$ at the specific value of the measured parameter α.

The formula, through which is written/recorded this function, must contain dependence on α. As the function α in question, this formula give the so-called "function of plausibility" of $L(\alpha)$.

Frequently the function of distribution $w(y_1, y_2, \ldots /\alpha)$ is also called the function of plausibility; $w(y_1, y_2, \ldots)$ - unconditional probability density of the obtained combination of values of $y_1, y_2, \ldots$.

Since is investigated the probability density α, i.e., the measured parameter, in the presence of the concrete/specific/actual realization $y_1, y_2, \ldots, w(y_1, y_2, \ldots)$ for this experiment or this concrete/specific/actual measurement it is possible to consider constant value, since the specific concrete/specific/actual combination of readings in the selection has the specific probability density. For determining the value $w(y_1, y_2, \ldots)$ it is possible to proceed from the fact that integration of the differential of probability within the limits of all possible values α will give one, i.e.

$$\int_{-\infty}^{+\infty} w(\alpha/y_1, y_2, \ldots)\, d\alpha = 1.$$

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After substituting for $w(\alpha/y_1, y_2, \ldots)$, expression (6.5.1) given above, we will obtain

$$\int_{-\infty}^{+\infty} \frac{w(a)\,w(y_1, y_2, \ldots/a)}{w(y_1, y_2, \ldots)}\,da = 1$$

either

$$w(y_1, y_2, \ldots) = \int_{-\infty}^{+\infty} w(a)\,w(y_1, y_2, \ldots/a)\,da, \quad (6.5.2)$$

or

$$\frac{1}{k_2} = \int_{-\infty}^{+\infty} w(a)\,w(y_1, y_2, \ldots/a)\,da. \quad (6.5.3)$$

In the principle the calculation of this integral is possible, since we assume that w(a) is known or assigned, and function $w(y_1, y_2, \ldots, /\alpha)$ or the function of plausibility, as it will be shown further, it can be found, if are known expressions for the signal and the functions of the distribution of interference. However, to compute this integral on is required, it suffices to only establish that its calculation leads to a number, but not to the function from α.

As a result of the carried out conversions the function of reverse/inverse probability can be expressed as follows:

$$w(a/y_1, y_2, \ldots) = k_2 w(a)\,w(y_1, y_2, \ldots/a). \quad (6.5.4)$$

From the results of the previous paragraph it follows that, after obtaining expression $w(\alpha/y_1, y_2, ...)$, we can find estimate $\overset{\times}{\alpha}$ by two methods: 1) find the maximum of this function of α; 2) find the average/mean value (mathematical expectation) α in the function of distribution $w(\alpha/y_1, y_2, ...)$. From expression (6.5.4) it follows that the form of the functions of reverse/inverse probability and plausibility, the position of their maximum and average/mean value, differ only due to the function of distribution $w(\alpha)$. This is very important result, since obtaining the functions of plausibility and mathematically and in circuit sense is considerably simpler than the function of reverse/inverse probability.

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The physical sense of the function of plausibility lies in the fact that it gives the form of the function of distribution for different combinations of selection (or realization) at the specific value of the parameter α. Since the observation is conducted finite time, this function the distribution function multidimensional and its graphic representation little is clearly. However, for the simplest case when is realized the measurement of the parameter with the use of one single, reading, it will be one-dimensional and it can be clearly depicted graphically. The function of distribution w(y/a) gives the probability density of one or the other value y at the given value α.

But since in this function compulsorily is contained dependence on α, it is possible to act vice versa: according to the mathematical expression for w(y/a) to construct the dependence of the probability density of any given value with a change in the parameter α. This there will be the function of plausibility. It is obvious that the less the interferences of those distorting the result of measurement, the narrower the function of plausibility. The exemplary/approximate form of the function $\omega(y/\alpha)$ is given in Fig. 6.5.1. Let us consider now the possibility of use during the optimization of the measurement not of the function of reverse/inverse probability, but the function of plausibility. For this it is necessary to explain, as it can influence by $w(\alpha)$ the difference in the form and position of maximum and average/mean value in the functions of reverse/inverse probability and plausibility. It is possible to demonstrate that for the majority of the practically important cases $w(\alpha)$ it is close to the even distribution and therefore it does not introduce essential differences into the form of the function of reverse/inverse probability and plausibility.

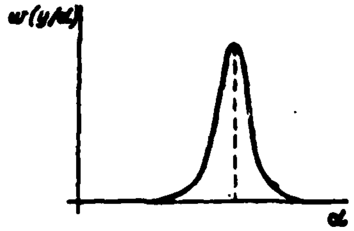

Fig. 6.5.1. Function of plausibility.

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This can be substantiated by the following considerations. Equiprobable distribution gives least favorable results, since is provided for the greatest uncertainty/indeterminacy of the value of the measured parameter. Therefore, even if it is possible to expect another distribution for α, it is useful to conduct research and calculations for the worse case. In many instances it is difficult to formulate law of $w(\alpha)$, and then it is most natural to assume that distribution equiprobable, i.e., all values of the parameter in some real limits are equiprobable. The function of distribution $w(\alpha)$ must be considerably more "smooth", than the function of plausibility $w(y_1, y_2, \ldots /\alpha)$, since otherwise becomes meaningless of conducting measurement. If, for example, function $w(\alpha)$ takes the form, depicted in Fig. 6.5.2, and is allowed for the possibility of the determination of the measured parameter in essence in the limits from α_1 to α_2, and the function of plausibility due to the larger interference level takes the form of smooth curve, depicted in Fig. 6.5.2, then it is obvious that the results of measuring for refinement of the value of the measured quantity virtually nothing give.

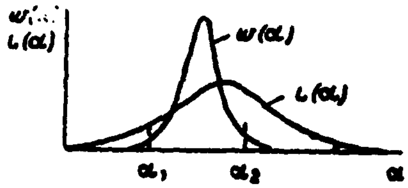

Fig. 6.5.2.

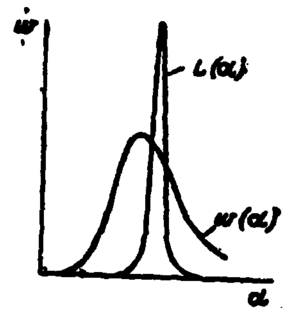

Fig. 6.5.3.

Fig. 6.5.2. Functions w(α) and L(α) for high interference level.

Fig. 6.5.3. Functions w(α) and L(α) for small interference level.

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Certainly, in this case the function of reverse/inverse probability will be clearly asymmetric. If under the same conditions, for example, due to an increase in the power of signal it is possible to decrease the "width" of the function of plausibility, then are obtained the results, depicted in Fig. 6.5.3. In this case the results of measurement make sense: they make it possible to determin the value of the measured parameter with the larger accuracy than it was known a priori. From the figure one can see that in such cases the function of a priori distribution already little can influence position and form of the function of reverse/inverse probability.

Consequently, virtually in all cases when measurements make sense, the function of reverse/inverse probability in form and position of maximum is close to the function of plausibility. On the basis of the considerations given above it follows that in the majority of the cases it is possible successfully to use the criteria of optimality, based on the function of plausibility, and to use the rule of solution which realizes this criterion.

Thus, optimum estimation can be given on the following rule:

$$\overset{\times}{a}_{\text{опт}} = a_{\text{м пр}}, \qquad (6.5.5)$$

where $a_{\text{м пр}}$ — value of the parameter of that giving the maximum of function $L(\alpha)$,

or

$$\overset{\times}{a}_{\text{опт}} = a_{\text{ср пр}},$$

where $a_{\text{ср пр}}$ — average/mean value of the parameter α in the function of plausibility $L(\alpha)$.

In the majority of the cases convenient proves to be the criterion of the maximum of the plausibility which we subsequently will use. Let us consider in general form now the algorithms of the optimum processing of mixture and the optimum measuring circuits corresponding to them of the parameters.

Let us begin from the rule of solution $\overset{\times}{a}_{опт} = a_{ср\,ап}$. If we use the function of plausibility, then algorithm will take the form

$$\overset{\times}{a}_{опт} = \text{const} \int_{-\infty}^{+\infty} a w(y_1, y_2, \ldots /a)\, da. \qquad (6.5.6)$$

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The diagram, which corresponds to this algorithm, is given in Fig. 6.5.4. Instead of the determination of the mathematical expectation for α, provided by expression (6.5.6), it is possible to realize averaging on the time. Then is computed estimation for each independent reading of mixture and then is located average. The corresponding diagram is given in Fig. 6.5.5.

Operation $\frac{1}{m}\sum_{i=1}^{m}\overset{\times}{a}_i$ can be realized by a low-pass filter.

Let us consider the diagrams, which are obtained during the use of rule of solution $\overset{\times}{a}_{опт} = a_{м\,пр}$, i.e., the criterion of the maximum of plausibility.

Diagram must compute

$$L(a) = w(y_1, y_2, \ldots /a)$$

and to find

$$\frac{dL(a)}{da};$$

$\overset{\times}{a}_{опт}$ is found from the solution of equation $\frac{dL(\overset{\times}{a}_{опт})}{da} = 0.$

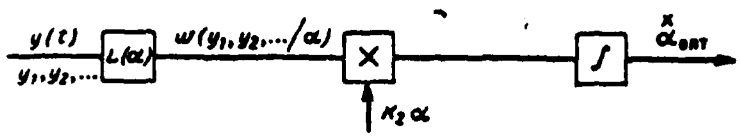

Fig. 6.5.4. The optimum diagram of the meter of the parameter in terms of the average/mean value of the function of the plausibility: L(α) - device/equipment for obtaining the function of plausibility; X - multiplier; ∫- integrator.

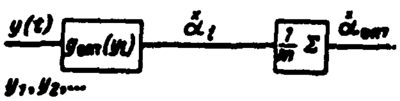

Fig. 6.5.5. Optimum scheme of a parameter measurer with the use averaging.

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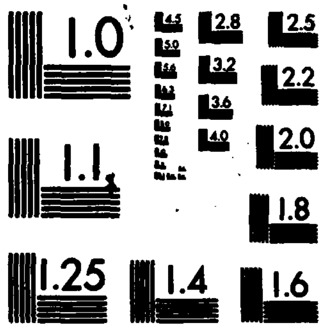

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Diagram realizing this algorithm, is depicted in Fig. 6.5.6. Earlier were obtained results in general form. They can be used for the determination of algorithms and diagrams, which ensure the optimum measurement of one or the other parameters of radio signal, which contain useful information. We use the method presented for determining the optimum procedure of the measurement of the phase of signal.

§ 6.6. Optimum procedure of the measurement of phase and measuring error. There is a signal (t, φ_c). The phase of signal φ_c is unknown and contains useful information. The reception of signal occurs against the background of additive interference. In order to find the optimum procedure of the measurement of phase, it is necessary to obtain the function of plausibility and to find estimation $\overset{\times}{\varphi}$, maximizing it.

The multidimensional conditional function of the distribution of the mixture of signal and interference takes the form

$$w(y_1, y_2, \ldots/\varphi_c) = \frac{1}{(2\pi\sigma_n^2)^{m/2}} e^{-\frac{1}{N_0}\int_0^{t_n} [y(t)-c(t,\varphi_c)]^2 dt}. \quad (6.6.1)$$

The obtained expression contains dependence on φ_c and it can be considered as the function of the plausibility

$$L(\varphi_c) = \frac{1}{(2\pi\sigma_n^2)^{m/2}} e^{-\frac{1}{N_0}\int_0^{t_n} [y(t)-c(t,\varphi_c)]^2 dt}. \quad (6.6.2)$$

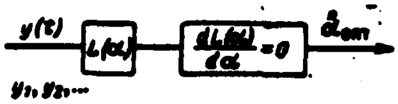

Fig. 6.5.6. Optimum scheme of a parameter measurer in accordance with the maximum function of plausibility.

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In order to represent this function in the more convenient form, which makes it possible to find the estimation, which corresponds to the maximum of plausibility, let us produce the conversions

$$L(\varphi_c)=\frac{1}{(2\pi\sigma_n^2)^{m/2}}\exp\left[-\frac{1}{N_0}\int_0^{t_u} y^2(t)\,dt-\right.$$

$$\left.-\frac{1}{N_0}\int_0^{t_u} c^2(t,\ \varphi_c)\,dt+\frac{2}{N_0}\int_0^{t_u} y(t)\,c(t,\ \varphi_c)\,dt\right]$$

σ_n^2 and N_0 - the parameters of interference, y(t) - the concrete/specific/actual realization, on which is sought the phase - they are considered known, then

$$e^{-\frac{1}{N_0}\int_0^{t_n} y^2(t)dt}=k_2$$

and

$$\int_0^{t_n} c^2(t,\ \varphi_0)\,dt=\mathcal{E}_n$$

(energy of signal for the time of observation).

After substituting into the expression for $L(\varphi_0)$, we will obtain

$$L(\varphi_0)=k_2 e^{-\frac{\mathcal{E}_n}{N_0}} e^{\frac{2}{N_0}\int_0^{t_n} y(t)c(t,\ \varphi_0)\,dt}; \qquad (6.6.3)$$

in the general case

$$\begin{aligned}c(t,\ \varphi_0)&=A_0a_0(t)\cos[\omega_0t+\psi_0(t)+\varphi_0]=\\&=A_0a_0(t)\cos\varphi_0\cos[\omega_0t+\psi_0(t)]-\\&-A_0a_0(t)\sin\varphi_0\sin[\omega_0t+\psi_0(t)].\end{aligned}$$

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In the simplest case of the sine wave

$$\begin{aligned}c(t,\ \varphi_0)=A_0\cos(\omega_0t+\varphi_0)=A_0\cos\varphi_0\cos\omega_0t+\\+A_0\sin\varphi_0\sin\omega_0t,\end{aligned}$$

then

$$L(\varphi_0)=k_0e^{-\frac{\mathcal{E}_n}{N_0}}\exp\left\{\frac{2}{N_0}\int_0^{t_n}[y(t)A_0a_0(t)\cos\varphi_0\cos[\omega_0t+\right.$$

$$\left.+\psi_0(t)]+y(t)A_0a_0(t)\sin\varphi_0\sin[\omega_0t+\psi_0(t)]\,dt\right\}.$$

For calculating some of that obtained above relationships/ratios can be used known diagrams - correlators.

It is possible to connect which computes

$$\eta=\int_0^{t_n}y(t)A_0a_0(t)\cos[\omega_0t+\psi_0(t)]\,dt,$$

$$\gamma=\int_0^{t_n}y(t)A_0a_0(t)\sin[\omega_0t+\psi_0(t)]\,dt.$$

The diagrams, which make it possible to compute these integrals, are given in Fig. 6.6.1. Keeping in mind the use of such diagrams, let us reduce expression for $L(\varphi_0)$ to the form

$$L(\varphi_0)=k_0e^{-\frac{\mathcal{E}_n}{N_0}}e^{\frac{2}{N_0}(\eta\cos\varphi_0+\gamma\sin\varphi_0)}. \qquad (6.6.4)$$

For obtaining the more convenient form of recording, let us introduce designations $\eta^2+\gamma^2=B^2$ and $\theta=$ arctg γ/η.

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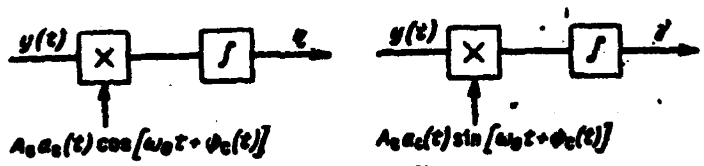

Fig. 6.6.1. Diagram of calculation of η and γ: X - multiplier, ∫ – integrator.

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Let us note that values B and θ can be obtained with the help of the mathematical conversions, realized with values η and γ, i.e., can be realized the diagrams, which give these values. Then

$$L(\varphi_c) = k_2 e^{-\frac{E_s}{N_0}} e^{\frac{2}{N_0} B\cos(\theta - \varphi_c)}, \qquad (6.6.5)$$

$$\eta = B\cos\theta,\ \gamma = B\sin\theta,$$

φ_c — the unknown initial phase of signal; θ - value which can be calculated by diagram.

The obtained expression gives the dependence of the value of the function of plausibility from the value, which can be obtained in the diagram. The evaluation of the unknown phase of signal must be realized so that, if we in expression $L(\varphi_c)$ instead of independent variable φ_c supply its estimation $\overset{\times}{\varphi}_c$, function $L(\varphi_c)$ must reach maximum. Without conducting of complicated analysis it is evident that $L(\varphi_c)$ in measurement φ_c it reaches maximum when $\varphi_c = \theta = \overset{\times}{\varphi}_{опт}$, since in this case $\cos(\theta - \varphi_c) = 1$.

Thus, the maximization of the function of plausibility is achieved during the use for evaluating the rule

$$\overset{\times}{\varphi}_{опт} = \theta = \operatorname{arctg} \frac{\int_0^{t_s} y(t) a_s(t) \sin[\omega_0 t + \psi_s(t)]\, dt}{\int_0^{t_s} y(t) a_s(t) \cos[\omega_0 t + \psi_s(t)]\, dt}. \qquad (6.6.6)$$

For the sine wave

$$\overset{\times}{\varphi}_{\text{опт}} = \operatorname{arctg} \frac{\int_0^{t_н} y(t)\sin\omega_0 t\,dt}{\int_0^{t_н} y(t)\cos\omega_0 t\,dt}.$$

The diagram, which realizes an optimum measurement of the phase of signal, must take the form, depicted in Fig. 6.6.2.

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Let us note that the diagram remains valid as optimum and with the unknown signal amplitude, since η and γ are proportional A_c, if signal amplitude is known and laid into the copy. If signal amplitude is unknown and in the copy used any amplitude, result will not be changed. Let us consider now the processes, which occur in the optimum diagram. If to the entrance is supplied signal without the interference, then earlier it was shown that

$$\eta_c = \mathcal{E}_н \cos\varphi_c \text{ и } \gamma_c = \mathcal{E}_н \sin\varphi_c, \qquad (6.6.7)$$

where $\mathcal{E}_н$ — energy of signal for the time of observation, and φ_c — phase of signal, counted off relative to value, which it is accepted as the zero and which is laid during the phase of reference generator. Then

$$\gamma/\eta = \mathrm{tg}\,\varphi_c \text{ и } \theta = \varphi_c.$$

In this case the relation of values at the outputs of two correlators corresponds precisely to the tangent of the phase of signal.

If to the input is supplied the mixture of signal and interference, then

$$\eta_{y}=\eta_{c}+\eta_{\text{ш}} \text{ и } \gamma_{y}=\gamma_{c}+\gamma_{\text{ш}},$$

where η_{y} and γ_{y} — random functions of time; η_{c} and γ_{c} — components (function), caused by the action of signal and which are determining average/mean value; $\eta_{\text{ш}}$ and $\gamma_{\text{ш}}$ — random components (function), caused by the action of interferences.

These functions have the increasing on the time dispersion and zero average.

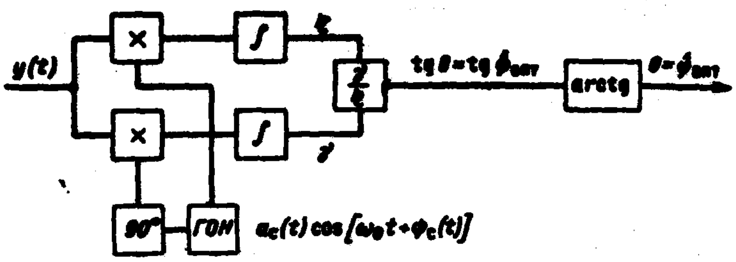

Fig. 6.6.2. The diagram of the optimum meter of the phase: X - multipliers; ГОН - reference generator; ∫- integrators; γ/η - computing device of this relation; arctg - trigonometric converter.

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At moment/torque $t=t_H$ and $\sigma_\eta^2=\sigma_\gamma^2=\sigma_\xi^2=\frac{E_H N_0}{2}$. If delay is known not accurately, then effect from the signal is reduced and the accuracy of the measurement of phase deteriorates. Decrease can be found from the correlation function of signal at τ, that corresponds to error on the delay (see S 4.8).

If delay has considerable uncertainty/indeterminacy, but reference voltage permits implemention of observation larger time than the duration of signal, then interferences will be at the output accumulated more and accuracy will deteriorate.

Consequently, at the outputs of correlators are random components, which cause the divergences of values from those values, at which follows carefully itself condition $\theta=\varphi_c$. It is obvious, being guided by algorithm with the reading of phase, we let us commit error $\delta\varphi$ and $\overset{\times}{\varphi}\neq\varphi_c$. The physical sense of this error is clarified in Fig. 6.6.3. From the figure one can see that the probable deviations of values η_N and γ_N due to the action of interferences, will cause the probable deviations of estimation $\overset{\times}{\varphi}$ from true value φ_c, i.e., random errors $\delta\varphi$. As one would expect, the optimum procedure of measurement does not free/release from the errors; however, it makes it possible to obtain their theoretically minimum.

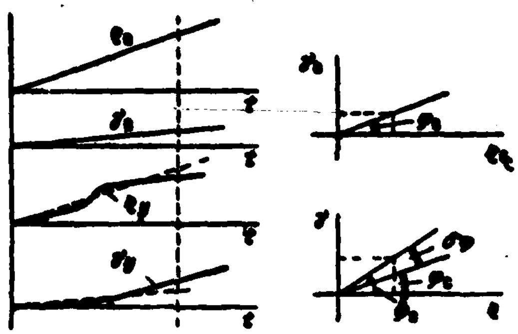

Fig. 6.6.3. Measuring error of phase.

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Let us recall that for the determination of the optimum procedure of measurement was used the rule of solution $\overset{\times}{\varphi}_{опт} = \ddot{\varphi}_{м\,пр}$. This rule provides minimum error and minimum risk; with any symmetrical loss function, depending on loss function will be changed only the numerical value of risk. It is of great interest to find the expressions, which make it possible to compute the value of measuring errors and their dependence on measuring conditions. For the solution of this problem it is necessary to have in mind that the statistical characteristics of values θ, accepted as the optimum estimation phase $\overset{\times}{\varphi}_{опт}$, which must make it possible to find the numerical values of errors, they are determined by the statistical characteristics of values η_φ and γ_φ and by the subsequent functional conversions. For the defined moment of time can be η_φ and γ_φ considered as normal random variables and distribution functions for the moment/torque of the termination of observation can be registered in the form (4.11.4)and (4.11.5)

$$w(\eta_y)=\frac{1}{\sqrt{2\pi}\sigma_1}e^{-\frac{1}{2\sigma_1^2}(\eta_y-\mathcal{E}_u\cos\varphi_c)^2}, \qquad (6.6.8)$$

$$w(\gamma_y)=\frac{1}{\sqrt{2\pi}\sigma_1}e^{-\frac{1}{2\sigma_1^2}(\gamma_y-\mathcal{E}_u\sin\varphi_c)^2}. \qquad (6.6.9)$$

For obtaining the function of distribution $w(\overset{\times}{\varphi}_0)$ it is necessary to at first find $w(\eta, \gamma)$. In Chapter 4 this function was obtained in the form

$$w(\eta, \gamma)=\frac{1}{2\pi\sigma_1^2}e^{-\frac{1}{2\sigma_1^2}[(\eta-\mathcal{E}_u\cos\varphi_c)^2+(\gamma-\mathcal{E}_u\sin\varphi_c)^2]}.$$

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Let us switch over to polar coordinates, it is analogous how this was done in Chapter 3 and 4

$$w(B, \theta)=\frac{B}{2\pi\sigma_1^2}e^{-\frac{1}{2\sigma_1^2}[(B\cos\theta-\mathcal{E}_u\cos\varphi_c)^2+(B\sin\theta-\mathcal{E}_u\sin\varphi_c)^2]}=$$

$$=\frac{B}{2\pi\sigma_1^2}e^{-\frac{B^2+\mathcal{E}_u^2}{2\sigma_1^2}}e^{\frac{B\mathcal{E}_u}{\sigma_1^2}\cos(\theta-\varphi_c)}.$$

$w(\theta)$ it is possible to obtain, after fulfilling the integration

$$w(\theta)=\int_0^\infty w(B, \theta)\,dB.$$

The calculation of a similar integral was carried out in § 3.4; therefore let us immediately register final result for $\delta\varphi = \theta - \varphi_0 = \overset{\times}{\varphi}_{0\text{MT}} - \varphi_0$, and, bearing in mind that $\sigma_\xi = \frac{\mathcal{E}_n N_0}{2}$

$$w(\delta\varphi) = \frac{1}{2\pi} e^{-\frac{\mathcal{E}_n}{N_0}} +$$

$$+ \sqrt{\frac{\mathcal{E}_n}{\pi N_0}} \cos\delta\varphi\, F\left(\sqrt{\frac{2\mathcal{E}_n}{N_0}}\cos\delta\varphi\right) e^{-\frac{\mathcal{E}_n}{N_0}\sin^2\delta\varphi}. \quad (6.6.10)$$

As can be seen from (6.6.10) the function of error distribution in the estimation of phase is expressed analogously with the function of the distribution of the divergence of the phase of mixture from the phase of signal, but the parameter of distribution is not A_0/σ_n, but $\mathcal{E}_n/N_0$. Consequently, the function of error distribution in the estimation of phase depends only on the energy of signal for the time of observation with the reading of phase and on the jamming density.

Let us consider special cases.

In the absence of signal $(\mathcal{E}_n = 0)$

$$w(\delta\varphi) = \frac{1}{2\pi},$$

then

$$\sigma^2_{\delta\varphi} = \frac{\pi^2}{3}.$$

With small energy of signal $(\mathcal{E}_n < N_0)$

$$w(\delta\varphi) \approx \frac{1}{2\pi}\left(1 + \sqrt{\frac{2\pi\mathcal{E}_n}{N_0}}\cos\delta\varphi\right). \quad (6.6.11)$$

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With the high energy of signal ($\mathcal{E}_n > N_0$)

$$w(\delta\varphi) = \frac{1}{\sqrt{2\pi}\sigma_{\delta\varphi}} e^{-\frac{\delta\varphi^2}{2\sigma_{\delta\varphi}^2}}, \qquad (6.6.12)$$

$$\sigma_{\delta\varphi}^2 = \frac{N_0}{2\mathcal{E}_n}.$$

Let us consider the now approximation method of obtaining the expression for $\sigma_{\delta\varphi}^2$. Since φ_0 is random variable and can take values from 0 to 2π, when $\mathcal{E}_n > \frac{N_0}{2}$ $\mathcal{E}_n \cos\varphi_0$ or $\mathcal{E}_n \sin\varphi_0$ can have low values.

For obtaining the approximate relationships/ratios let us assume that the function of the distribution of divergences θ from the average/mean precise value does not depend on φ_0; let us find this distribution for the case most convenient in the sense of mathematical transformations. Assumption about the independence of the function of the distribution of divergences θ from φ_0 will be coordinated well with the intuitive representations about the fact that all values of phase are equiprobable and equivalent and in its optimum measurement must not be of preference to one or the other values.

For simplification in the transformations let us consider the case when $\varphi_c \approx 0$, then: γ_v — random variable with the zero average/mean value and dispersion $\sigma_\gamma^2 = \sigma_i^2 = \mathcal{E}_n N_0/2$; $\eta_v = \mathcal{E}_n$, since the fluctuations of value η_v can be disregarded/neglected since signal strong.

Then value $\gamma/\eta = tg\theta$ at the output of divider will be the random variable, which has normal distribution with the zero average and the dispersion

$$\sigma_{tg\,\theta}^2 = \frac{\sigma_\gamma^2}{\mathcal{E}_n^2} = \frac{N_0}{2\mathcal{E}_n}. \qquad (6.6.13)$$

The rms value of the fluctuation of value $tg\theta$ will be equally

$$\sigma_{tg\,\theta} = \sqrt{\frac{N_0}{2\mathcal{E}_n}}.$$

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Since is undertaken the case when $\varphi_c = 0$ and signal is strong, then $tg\theta \ll 1$. In this case transformation arctg is simplified and $\theta = tg\theta$. Then

$$\sigma_\theta^2 \approx \frac{N_0}{2\mathcal{E}_n}. \qquad (6.6.14)$$

Since value $tg\theta$ with the adopted assumptions is distributed according to the normal law, then θ has normal distribution.

Earlier was accepted hypothesis about the independence of the divergences of the estimatión of phase from its value within the limits from 0 to 2π. Then it is possible to obtain the expression of the function of the distribution of the divergences $\delta\varphi$ of the optimum estimation of phase $\overset{\times}{\varphi}_{opt}$ from its true value φ_0 with the strong signal

$$w(\delta\varphi)=\frac{1}{\sqrt{2\pi}\sigma_{\delta\varphi}}e^{-\frac{\delta\varphi^2}{2\sigma^2_{\delta\varphi}}}, \qquad (6.6.15)$$

$$\sigma_{\delta\varphi}=\sqrt{\frac{N_0}{2E_s}},$$

which coincides with (6.6.12). The obtained results are of essential interest: they show that the dispersion of the fluctuations of phase depends only on the ratio of the jamming density to the energy of signal. Signal can be modulated by any method - on the amplitude or the phase (frequency), and if modulation is known and reproduced in the copy of signal, then the result of measurement will depend only on energy of signal. Consequently, signal can be complicated, i.e., consist of the sequence of impulses/momenta/pulses (packet) with the different shape of the envelope of each impulse/momentum/pulse and packet as a whole, or by noise-like, i.e., complicatedly modulated on the phase; the accuracy of the measurement of phase from this is not changed, if the laws of modulation are known and reproduced in the copy, and energies of signals are undertaken identical.

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This result shows that both for the detection of signal and for measuring this important parameter of signal as its phase, by basic factor, which are determining accuracy, is energy of signal for the time during which can be supervised in the measurement. Than by more complicated is undertaken signal, the more complicatedly must be the realization of diagram, since respectively is complicated the generator of the copy of signal. It would seem that under these conditions there is no sense to use serrated and noise-like signals, that as their use/application does not raise accuracy (with the same energy), but the realization of optimum diagram in this case is complicated. However, the use of such signals in the phase systems has very important value, since it must make it possible to solve series of problems - decrease of the harmful effect of "multi-beam character" on the accuracy, the time sharing of the signals, utilized for measuring the phase and, etc. In this case very importantly the fact that the signal can be chosen any, as this is required for achievement of the necessary results, and the accuracy of the measurement of its phase is not changed, if is used the optimum procedure of processing and is retained energy (with the fluctuating interferences).

There is great interest in also that the fact that the results of the optimum measurement of phase do not depend on the ratio of signal amplitude to the rms value of interferences at the entrance of the meter of phase. This depends on the fact that in the optimum phasemeter is used all information about the mixture, which consists both in its phase and in its amplitude, and the nonlinear transformations (division and the taking of arc tangent), which reveal/detect phase, are realized after the operation of accumulation (in the integrators), course of which affects both the amplitude and the phase of mixture. Since the discussion deals with phase systems, it is appropriate to raise a question about the possibility of designing and about the characteristics of such measuring circuits of the phases, in which is used the information, which consists only in the phase of mixture, and is not used the information, which consists in the amplitude of mixture. Virtually this can be fulfilled, after supplying before the optimum meter of phase the limiter which will destroy information about the amplitude of mixture. In this case the functions integrated in the correlators will be changed.

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For example, value at output of one of the quadrature correlators instead of the expression, valid for the optimum phasemeter, which uses entire information

$$\eta = \int_0^{t_n} \frac{A_y(t) A_n}{2} \cos[\varphi_n - \varphi_y(t)]\, dt,$$

will take the form (in the presence of limiter)

$$\eta_{o\,\text{огр}} = \int_0^{t_n} \frac{A_{o\,\text{огр}} A_s}{2} \cos[\varphi_n - \varphi_y(t)]\, dt.$$

The results of measuring the phase must deteriorate, since is used smaller information about the mixture. If signal is strong ($A_c > \sigma_n$), then, using approximate methodology presented above, we will obtain

$$\sigma^2_{\varphi\,\text{огр}} = \frac{N_0}{2E_n},$$

which coincides with (6.6.12). it is concealed by form, with the strong signal the amplitude of mixture is almost constant and its use with phase measurement does not affect the results of measurements. With weak signal ($A_0 \ll \sigma_n$) the divergence of the measured phase increases in $4/\pi$ the time for the gaussian interference and 2 times for the interference with the constant amplitude, i.e., with phase measurement is important the use of information about the amplitude of mixture [6.5, 6.7].

§ 6.7. Some versions of the diagrams of the optimum and quasi-optimal measurement of phase. Obtaining the maximum of plausibility, i.e., the optimum measurement of phase can be provided with other circuit solutions, which differ from diagram in Fig. 6.6.2.

Let us consider expression for the function of the plausibility

$$L(\varphi_c) = k_1 e^{-\frac{E_s}{N_0}} e^{\frac{2}{N_0}\int_0^{t_s}[y(t)c(t,\varphi_c)]dt}, \qquad (6.7.1)$$

where φ_c — the random phase of signal.

Correlation integral takes the form

$$z_\varphi = \int_0^{t_s}[y(t)c(t,\varphi_c)]dt. \qquad (6.7.2)$$

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Its maximum corresponds to maximum $L(\varphi_c)$; the realization of the diagram, which corresponds to expression (6.7.2), is hindered/hampered, since phase φ_c is unknown. If we connect of correlator and generator of the copy of signal with the variable/alternating initial phase, then the results of measurements on this diagram will make it possible to compute the integral

$$z_\varphi = \int_0^{t_s}[y(t)c_k(t,\varphi_k)]dt, \qquad (6.7.3)$$

where $c_k(t,\varphi_k)$ — copy of signal with initial phase

Changing phase φ_k it is possible for each of its values to observe mixture with the help of the correlator.

For simplicity and clarity let us take the case of the sine wave

$$y(t) = A_y(t)\cos[\omega_0 t + \varphi_0 + \varphi_y(t)]\,dt,$$

$$c_\text{к}(t,\ \varphi_\text{к}) = A_\text{к}\cos(\omega_0 t + \varphi_\text{к}),$$

$$z_\varphi = \int_0^{t_\text{н}} [y(t)\,c_\text{к}(t,\ \varphi_\text{к})]\,dt =$$

$$= \int_0^{t_\text{н}} A_y(t)\,A_\text{к}\cos[\omega_0 t + \varphi_0 + \varphi_y(t)]\cos(\omega_0 t + \varphi_\text{к}) =$$

$$= \int_0^{t_\text{н}} \frac{A_y(t)\,A_\text{к}}{2}\cos[\varphi_0 - \varphi_\text{к} + \varphi_y(t)]\,dt;$$

by term $2\omega_0$ it is disregarded, that as at the output of integrator it will give zero.

If correlation integral has maximum, then in this case there will be the maximum of plausibility, which directly follows from (6.7.1). Since the maximums of correlation integral and function of plausibility coincide, during the composition of the diagram, in which is used the principle of the maximum of plausibility, it is possible to be bounded to the fact that it must ensure the determination of the maximum of correlation integral.

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Consequently, value $\varphi_\text{км}$ with which is observed the maximum of value at the output of correlator, it can be accepted as the optimum estimation of the phase

$$\overset{\times}{\varphi}_{опт} = \varphi_{км}. \qquad (6.7.4)$$

The diagram, in which is ensured the optimum estimation of phase, can take the form, depicted in Fig. 6.7.1. If the space of a change in the phase of copy $\Delta\varphi_{к}$ will be small, then error will depend on the distortion of the result of integration by interferences and maximum will be obtained at the point where $\varphi_{км}$ differs somewhat from φ_{c}. During the practical realization of diagram it it is possible to simplify, after using instead of the ideal integrator with reset of low-pass filter or inertia component/link with the transfer function:

$$k(s) = \frac{1}{Ts+1}.$$

The diagram, which corresponds to this case, is given in Fig. 6.7.2. Just as in the diagram with the integrator, interferences will distort result and they will cause error. Fig. 6.7.3 gives dependence z_{φ} on $\varphi_{к}$. The curve a corresponds to weak interferences.

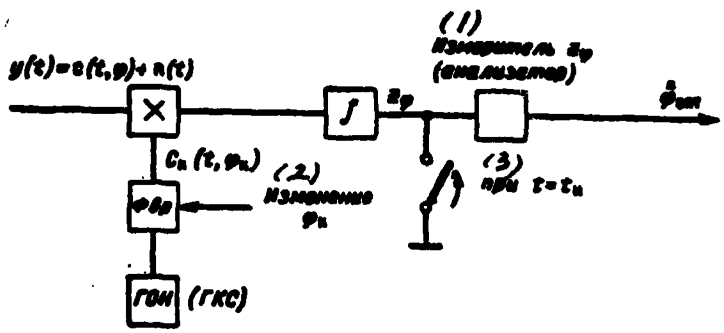

Fig. 6.7.1. The single-channel diagram of the optimum meter of the phase: X - multiplier; Фвр - phase inverter; ГОН - reference generator; ∫ - integrator.

Key: (1). Meter z_φ (analyzer). (2). Change. (3). with.

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If we consider that the diagram works ideally, then in this case maximum will be fixed virtually accurately with

$$\varphi_{\text{им}}=\overset{\times}{\varphi}_{\text{опт}}=\varphi_0.$$

In the presence of noticeable interferences the readings will have divergences; as a result of observation will be obtained, for example, the curve b. With reading $\overset{\times}{\varphi}_{\text{опт}}$ on maximum z_φ will be allowed the error $\delta\varphi$. It is obvious that in this diagram for obtaining the reading will be spent considerably more time, than in the optimum diagram with direct reading, given in Fig. 6.6.2. In the diagram (Fig. 6.6.2) the time of reading and the time of observation coincide.

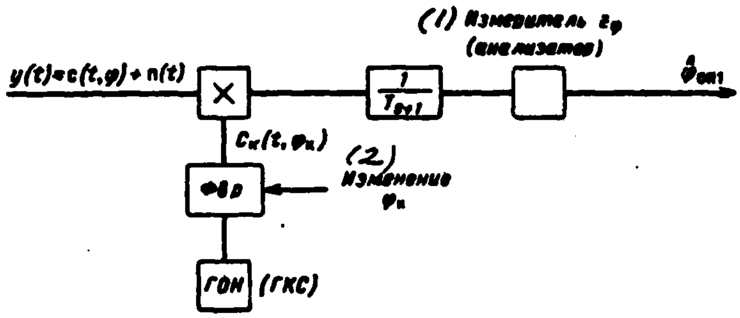

Fig. 6.7.2.

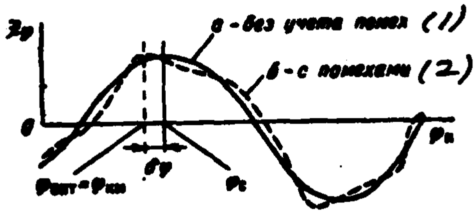

Fig. 6.7.3.

Fig. 6.7.2. Single-channel diagram of optimum meter of phase with low-pass filter: X - multiplier; Фвр - phase inverter; ГОН - reference generator; 1/(Ts+1) - inertia component/link.

Key: (1). Meter z_φ (analyzer). (2). Change.

Fig. 6.7.3. Dependence z_φ on $\varphi_{\text{и}}$.

Key: (1). a - without taking into account interferences. (2). b - with interferences.

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Diagram begins to work already in the first moments/torques after the supply of signal; output potentials of correlators begin to increase, their relation continuously giving the estimation of phase. The smallest error will be at the moment of the termination of the observation when entire/all possible energy is usefully used for evaluating the phase.

In the diagram, given in Fig. 6.7.1, 6.7.2, the time of reading will be equally

$$\frac{\varphi_с}{\Delta\varphi_к} t_н = t_{отсч}.$$

The phase of signal $\varphi_с$ — the random variable which can take values from 0 to 360° or from 0 to ±180°, and $\Delta\varphi_к$ in the precise measurement can be less than 1°.

Knowing $m_1(\varphi_с)$ and $\Delta\varphi_к$, it is possible to find $m_1(t_{отсч})$. For example, when $m_1(\varphi_с)=90°$ and $\Delta\varphi_к=1°$ $m_1(t_{отсч})=90\ t_н$.

Main disadvantages in the diagram in question: a) the time of observation at each point ($t_н$) much less than the total time which must be expended for obtaining the reading;

b) the precision determination of the weakly expressed maximum causes many technical difficulties and it can cause considerable instrument errors;

c) with the work with the diagram it is necessary to implement the series/row of the complicated operations: to change $\varphi_к$, to observe and to fix/record z_{φ}, to analyze the results of observation, by finding point $\varphi_{\kappa м}$, in which z_{φ} it is maximum. Have the capability of the improvement of the diagram examined.

Let us consider some of them. From dependence z_{φ} on φ_{κ} it follows that it has the characteristic point whose position is rigidly connected with the maximum. The observation of this point technically is considerably simpler. Such point is the transition through zero, which occurs when $\varphi_{\kappa 0}=\overset{\times}{\varphi}_{\text{осн}}\pm 90°$.

Instead of determination $\varphi_{\kappa м}$, with which is observed maximum z_{φ}, it is possible to observe angle $\varphi_{\kappa 0}$, with which $z_{\varphi}=0$. In this case $\overset{\times}{\varphi}_{\text{осн}}=\varphi_{\kappa 0}\pm 90°$. The action of interference in this case will be preserved, but requirement for it is instrument/tool accuracy and stability of those parts of the diagrams which fix/record the results of integration and reveal/detect point with the characteristic features, they will be less rigid.

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For eliminating the deficiency/lack, connected with the further expenditure of time for reading, it is possible to use multichannel diagram, depicted in Fig. 6.7.4. In this diagram all p of channels they function simultaneously. Their number is defined by requirement for the instrument/tool accuracy, that as value $\Delta\varphi_{\kappa}$ enters into the instrument error

$$p=\frac{360}{\Delta\varphi_{\kappa}}.$$

The operating principle of this diagram is obvious from previous. The time, necessary for the reading, is equal to the time of observation ($t_{отсч}=t_{н}$). The most complicated part of the diagram is the analyzer of the maximum which must "select" the channel, which gives at moment/torque $t=t_{н}$ maximum stress/voltage, and according to the number consider phase $\overset{\times}{\varphi}_{ОМТ}$. Because of this, and also due to the multichanneled effect the realization of this diagram is complicated. For the automation of the action of the diagram, depicted in Fig. 6.7.2, can be used the locked servo system, as the error signal is used the value at the multiplier output - phase discriminator. The diagram, which corresponds to this case, is given in Fig. 6.7.5.

In this diagram ПФ - the preliminary filter, which reveals/detects the constant component at the output ФД; У - amplifier, and ИО - actuating element, which uses so that the error signal would ensure a change in the position of phase inverter (Фвр). It is obvious that, as in any servo system, phase inverter will be established/installed by actuating element to the position in which the error signal vanishes.

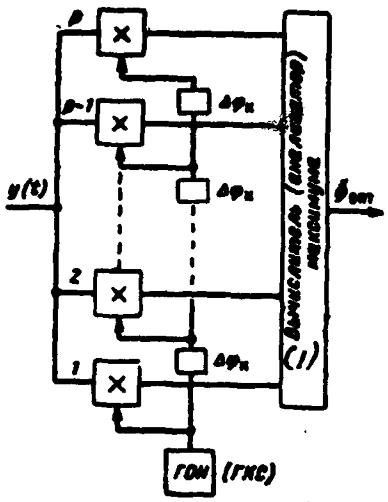

Fig. 6.7.4. Multichannel diagram of the optimum measurement of the phase: X - multipliers; ГОН - reference generator.

Key: (1). Computer (analyzer) of maximum.

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But the error signal, taken from the phase discriminator, is determined by a phase difference φ_c and φ_κ in accordance with the expression

$$CO = \text{const}\sin(\varphi_c - \varphi_\kappa + 90^\circ);$$
$$CO \to 0 \overset{(1)}{\text{при}}\ \varphi_c - \varphi_\kappa + 90^\circ \to 0.$$

Key: (1). with.

Phase displacement, caused by phase inverter, can be taken/removed from the completely calibrated scale by operator or given out in TsVM [ЦВМ - digital computer] and is accepted as the evaluation of the phase of signal taking into account correction for 90°. It is obvious that since the diagram in question is the servo closed system, with its analysis must be taken into consideration: stability, degree of astaticism of system, dynamic, instrument and fluctuating errors of system, transient processes, operating speed and characteristic of capture mode. Due to the considerable specific character of these questions to examine them in this book is not appropriate. Those interesting can with them be acquainted on literature [6.3, 6.4]. It is of interest to consider this diagram from the point of view of the optimization of the measurement of phase. In this case we will consider that all questions of the work of this diagram as servo system can be solved, i.e., can be provided its stability with the assigned degree of astaticism, and in this case are obtained the requiring passband and operating speed. Of that presented it previously follows that the diagram in question realizes the algorithm, which escape/ensues from the optimization of measurement, on the basis of the principle of the maximum of plausibility. Diagram contains all elements/cells, inherent in the optimum diagram: multiplier - the phase discriminator, narrow-band filter - the locked servo system and reference voltage - the copy of signal with the self-driving under point $z_{\varphi}=0$ phase.

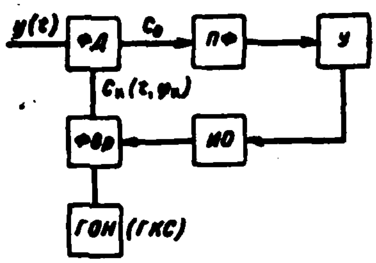

Fig. 6.7.5. The measuring circuit of phase with the servo system: ФД - phase discriminator; Фвр - phase inverter; ПФ - preliminary filter; ИО - actuating element; У - amplifier.

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The band of servo system in the locked state or its time constant T_{cc}, depending in essence on the amplification of duct/contour in the extended state, determines the time of observation $t_н \approx T_{cc}$. However, there are essential features, which differ this diagram from the optimum. The first basic special feature/peculiarity of the diagram, depicted in Fig. 6.7.5, is that that the time of reading $t_{отсч}$ and the time of observation $t_н \approx T_{cc}$ essentially they differ from each other, moreover

$$t_н < t_{отсч}.$$

In the diagram, given in Fig. 6.7.5, at the moment of supplying signal the phase inverter can be found in any position, which differs to any angle (to ±180°) from the phase of signal and its estimation, given by the position of phase inverter with the error signal, equal to zero. Consequently, servo system in the beginning of reading can prove to be in the strongly mismatched state which must be compensated, on what the system must expend considerable time.

Let us determine tentatively the relationship/ratio.between $t_{\text{отсч}}$ and $t_{\text{н}}$. Usually into the duct/contour of servo system is introduced nonlinearity of the type "saturation" so that would be ensured the permissible engine operating mode of the servomechanism, which turns phase inverter. Then large disagreement/mismatch system masters with the constant velocity. In this case the time of final adjustment proportional to initial disagreement/mismatch can be determined according to the following approximation formula:

$$t_{\text{отр}} \approx \frac{\Delta\varphi_{\text{нач}}}{\Delta\varphi_{\text{огр}}} T_{\text{сс}},$$

where $\Delta\varphi_{\text{нач}}$ — initial disagreement/mismatch, $\Delta\varphi_{\text{огр}}$ — disagreement/mismatch, during which occurs the limitation.

Virtually $\Delta\varphi_{\text{огр}} = 5 \div 10^{\circ}$, then

$$t_{\text{отр}} = 0{,}1 T_{\text{сс}} \Delta\varphi^{0}_{\text{нач}}.$$

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Since disagreement/mismatch $\Delta\varphi_{\text{нач}}$ — the value is by chance with the even distribution, it is convenient use with its average/mean value

$$m_1(\Delta\varphi_{\text{нач}}) \approx 90^{\circ},$$

then

$$m_1(t_{\text{отр}}) \approx 10 T_{\text{сс}}.$$

Into the value of time $t_{\text{отсч}}$ will enter also the time of processing in the limits of linear section, which can be calculated with the use of a methodology of the determination of the operating speed of the linear servo systems and comprises tentatively $t_{\text{отр лин}}=(3\div4)T_{\text{сс}}$. Consequently, the total time of the reading

$$t_{\text{отсч}}=t_{\text{отр}}+t_{\text{отр лин}},$$

$$m_1(t_{\text{отсч}})=m_1(t_{\text{отр}})+t_{\text{отр лин}}\approx(10\div15)t_{\text{и}}.$$

As is evident, the time of reading is substantially greater $t_{\text{и}}$, but is less than that obtained in the measurements on the diagram, depicted in Fig. 6.7.1. Consequently, for the estimation is usefully used only the part of the time of action of signal. If this time is assigned, then energy $\mathcal{E}_{\text{и}}$ during the use of a diagram, depicted in Fig. 6.7.1, will be less and accuracy under the effect of fluctuating interferences will be more badly than obtaining during the use of diagram in Fig. 6.6.2. However, this deficiency/lack is manifested only during the initial operating mode, in the beginning of measurement. Usually it is necessary to realize a measurement of the changing phase, then after the termination of initial mode/conditions and elimination of random large disagreement/mismatch the been congruent/equated diagrams work approximately/exemplarily equally, the diagrams with the servo systems can give gain in the resulting accuracy due to the use/application technically of simpler methods of decreasing some types of dynamic errors. The second fundamental special feature/peculiarity of diagram with the servo system is the fact that in contrast to the diagrams, depicted in Fig. 6.6.2 and 6.7.1, in it is used the imperfect reference voltage, that as it connected (on the phase) with the phase of the stress/voltage of mixture. The interferences, which are contained in the mixture, partially passing through the servo system, will cause the fluctuations of the phase of reference voltage.

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It is obvious that this must be accompanied by a deterioration in the freedom from interference of diagram in comparison with the theoretical level, ensured during the optimum estimation of phase (Fig. 6.6.2).

However, if is required the high accuracy of measurement, then with the given interference level and power of signal must be selected this band of servo system caused $t_{ш}$ and $\beta_{ш}$, so that the divergences of the position of phase inverter would be small, but then the fluctuations of the phase of reference voltage are also small and their effect on the freedom from interference can be disregarded/neglected. Consequently, a difference in diagram 6.7.5 from the optimum noticeably will be showed only with the realization of the rough measurements, which are not usually of great practical interest. On the reasons presented the diagram with the servo phasemeter in the majority of the practical cases can be considered as the optimum, and fluctuating measuring error which in it is inherent, close to that computed from formula (6.6.15). Let us note that in the operating principle and the characteristics the system, depicted in Fig. 6.7.5, is close to the systems of phase automatic frequency control (FAPCh) and to the servo filters which frequently are used in the phase systems. A similar analysis of systems FAPCh is given in [6.1, 6.3, 6.4, 6.8, 6.9-6.12].

Many obtained here results can be used for optimization and analysis of freedom from interference FAPCh. Let us note some special features/peculiarities of reworking of phase in the phase systems. For increasing the accuracy of phase measurements it is possible to realize reworking of the results of measurements - with the averaging, the extrapolation, etc., which especially successfully can be implemented during the use of TsVM. However, in this case it is necessary to consider some limitations. As shown in Chapter 3, with the demodulation of phase is a threshold effect or an effect of "suppression" which is exhibited in the fact that if in the measurement of the phase of its fluctuation they exceed approximately/exemplarily 50° (standard deviation), then subsequent filtration (averaging) gives substantially worse results, than to filtration to the phasemeter or in the phasemeter itself. In this case can be observed the error, not reduced with any duration of averaging.

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Consequently, it is in principle important to ensure certain accuracy during the primary processing of mixture, i.e., with phase measuring itself. These limitations are still large during the use of the servo meters. The latter are close to the optimum ones only in such a case, when they are set in the conditions, with which is ensured the high accuracy of measurements. The third special feature/peculiarity of the servo phasemeters is the fact that in many instances on the high interference level them it is necessary to consider as nonlinear systems. This substantially complicates process and their analysis.

S 6.8. Use of matched filters in the diagrams of the optimum measurement of phase. Earlier repeatedly it was indicated that in the optimum diagrams instead of the correlator can be used the matched filters. Let us recall that the matched filter independent of initial phase and delay optimally selects from the interferences the signal, with which it is matched. A change in these parameters of signal will be reflected only in phase and temporary situation of the output stress/voltage of matched filter. Correlators possess phase selectivity, that also makes it possible with their aid to create the diagrams of the optimum measurement of the initial phase of signal, examined above. Consequently, in contrast to the correlators matched filter cannot be directly used for measuring the phase; however, is possible the combination of matched filter and ideal phasemeter, depicted in Fig. 6.8.1, that makes it possible to ensure the measurement of the phase of signal. In order to come to light/detect/expose, how this a diagram is close to the theoretical optimum, let us compare the accuracy which it ensures, with the accuracy of optimum diagram.

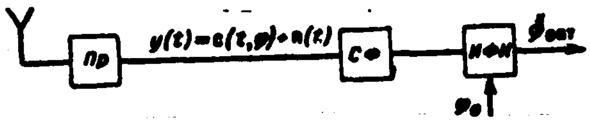

Fig. 6.8.1. The diagram of the optimum measurement of phase with the matched filter: Пр - receiver; СФ - matched filter; ИФИ - ideal meter of phase.

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With the mixture in which the signal exceeds interference, the function of phase distribution of mixture is normal and the dispersion of its fluctuations is equal to

$$\sigma_\varphi^2 = \sigma_n^2 / A_c^2,$$

where σ_n^2 — dispersion of interference; A_c — signal amplitude.

If the mixture of interference and signal is passed through the matched filter, then the ratio of signal to the interference at the moment of the termination of its action at the entrance of filter, as is known, it is equal

$$\frac{A_{\Phi m}}{\sigma_{n\Phi}} = \sqrt{\frac{2\mathcal{E}_c}{N_0}}$$

and

$$\sigma_{\varphi\Phi}^2 = \frac{\sigma_{n\Phi}^2}{A_{cm}^2} = \frac{N_0}{2\mathcal{E}_c},$$

where $A_{\Phi m}$ — maximum output potential of matched filter; $\sigma_{n\Phi}^2$ — dispersion of interferences; $\sigma_{\varphi\Phi}^2$ — dispersion of the fluctuations of phase.

If from the output of matched filter mixture is supplied to the ideal sufficiently high speed phasemeter and at the moment of the termination of the action of signal is realized a measurement of phase, then the reading of phase will be accompanied by the error which has a dispersion, which corresponds to the dispersion of the fluctuations of phase *in the mixture*. Thus, the dispersion of the divergences of the measured phase will be equal to

$$\sigma^2_{\text{иф}} = \sigma^2_{\text{вф}} = \frac{N_0}{2\mathcal{E}_c}. \qquad (6.8.1)$$

The same result for the dispersion of the divergences of the estimation of phase from its true value was obtained earlier for the optimum phasemeter. Thus, diagram *with the matched* filter and the ideal phasemeter according to the results corresponds to the diagram of optimum phasemeter. This conclusion/output will be clear, if we recall that the matched filter realizes processing entire mixture, using in this case its amplitude and phase. If the signal is simple, then the difficulties of the *realization of matched* filter in essence are determined by requirements for its passband, i.e., with the duration of signal.

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If signal consists of the sequence of the impulses/momenta/pulses whose phase carries information, then the realization of matched filter is hampered, since then it must have difficultly realizable "comb" frequency characteristic. If signal is noise-like, then matched filter must have complicated phase-frequency characteristic. In the uses/applications of matched filters for guaranteeing of optimum phase measurement and optimum detection is very essential difference.

With the optimum detection the requirements for the accuracy and the stability of phase responses are determined by the need for obtaining low losses in the amplitude of the "convoluted" signal. In this case the divergences of phases on 10-20° comparatively little affect results.

With optimum phase measurements requirements for the accuracy and stability of the phase responses of matched filters are substantially more rigid, since filter must ensure not only "convolution", but also this stability and accuracy of the phase of signal at the output of matched filter, which little would differ from the ideal case. Divergence and instability of phase will cause instrument fault of measurement of phase. In many instances of requirement for the accuracy the measurements of phase are sufficiently high, the permissible errors do not exceed 1-3°. Consequently, tuning precision and stability of matched filters in this case must be by an order higher than in the diagrams of detection. The practical difficulties of using the matched filters are explained to the high degree by the fact that the time of observation in the measurement of phase is usually considerable, and the signal used - prolonged. This leads to the fact that the passband of matched filter proves to be very narrow (order of the portion of hertz). With the narrow band insignificant instabilities and inaccuracies in the tuning will cause the essential departures/attendance of phase and error in its measurement. In connection with this for obtaining the minimum resultant error can prove to be appropriate the expansion of the filter pass band.

Let us consider effect on the accuracy of the measurements of the imperfection of filter.

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We will use the approximate relationships/ratios [see (1.14)] according to which

$$\delta\varphi_{\text{нест}} \approx \frac{2\delta f}{\Delta f_{\text{эф}}},$$

where $\delta\varphi_{\text{нест}}$ — instability of phase; δf - detuning; $\Delta f_{\text{эф}}$ — passband.

Since will be further examined the cases of matched and mismatched with the signal filters, then into the indices let us introduce the further designations: with ns.

Detuning δf depends on the instability of the frequencies of the heterodynes of receiver and transmitter, on a change in the parameters of the elements/cells of the selective ducts/contours of receiver. In practice basic effect on the detuning proves to be the instability (characteristics) of circuit elements. Losses, coupling coefficients, value of inductance and capacity/capacitance of circuit elements can significantly be changed under the effect of temperature, humidity, mechanical loads, etc.

In the first approximation, it is possible to consider that δf has the normal distribution, and to characterize instability as the standard deviation or by dispersion $\sigma^2_{\delta f}$.

Then variance of error due to the instability of the characteristics of the mismatched filter is equal to

$$\sigma^2_{\delta\varphi нс} \approx \frac{4}{\Delta f^2_{эф\,нс}} \sigma^2_{\delta f}. \qquad (6.8.2)$$

The error, caused by the action of interferences, during the use of a matched filter is equal to

$$\sigma_{\delta\varphi\,фс} = \frac{\sigma_n}{A_{ф\,m}} = \sqrt{\frac{N_0 \Delta f_{эфс}}{A_{m\,ф}}}, \qquad (6.8.3)$$

where $\Delta f_{эфс}$ — efficient band of the filter of that matched; $\Delta f_{эф\,нс}$ — efficient band of the filter of mismatched; $A_{ф\,m}$ — maximum signal amplitude at the output of matched filter.

During the expansion of the band of filter the error due to the interferences will increase, since interferences grow proportional to band, and signal amplitude can be considered constant.

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Then the dispersion of the fluctuations of phase at the output of filter with the changed band will take the form

$$\sigma^2_{\delta\varphi\,\text{физ}} = \frac{\sigma^2_{\text{шф из}}}{A^2_{\text{фм из}}} = \frac{N_0 \Delta f_{\text{шфс}}}{A^2_{\text{ф из}}} \frac{\Delta f_{\text{эф из}}}{\Delta f_{\text{шфс}}} = \sigma^2_{\delta\varphi\,\text{фс}} \frac{\Delta f_{\text{эф из}}}{\Delta f_{\text{эфс}}}; \qquad (6.8.4)$$

the dispersion of the resultant error

$$\sigma^2_{\delta\varphi\,\text{рез}} = \sigma^2_{\delta\varphi\,\text{фс}} \left[\frac{\Delta f_{\text{эф из}}}{\Delta f_{\text{эфс}}} + 4 \frac{\sigma^2_{\delta f}}{\sigma^2_{\delta\varphi\,\text{фс}} \Delta f^2_{\text{эфс}}} \frac{1}{\left(\frac{\Delta f_{\text{эф из}}}{\Delta f_{\text{эфс}}} \right)^2} \right]. \qquad (6.8.5)$$

Minimum $\sigma^2_{\delta\varphi\,\text{рез}}$ in a change in band $\Delta f_{\text{эф из}}$ corresponds to the condition when

$$\frac{d(\sigma^2_{\delta\varphi\,\text{рез}})}{d\frac{\Delta f_{\text{эф из}}}{\Delta f_{\text{эфс}}}} = 0,$$

which gives

$$\left(\frac{\Delta f_{\text{эф из}}}{\Delta f_{\text{эфс}}} \right)_{\text{опт}} = \sqrt[3]{\frac{8\sigma^2_{\delta f}}{\sigma^2_{\delta\varphi\,\text{фс}} \Delta f^2_{\text{эфс}}}}, \qquad (6.8.6)$$

then

$$\sigma^2_{\delta\varphi\,\text{рез мин}} = \sigma^2_{\delta\varphi\,\text{фс}} \cdot 3 \sqrt[3]{\frac{\sigma^2_{\delta f}}{\sigma^2_{\delta\varphi\,\text{фс}} \Delta f^2_{\text{эфс}}}}. \qquad (6.8.7)$$

Expression 6.8.6 is correct only for $8\frac{\sigma^2_{\delta f}}{\Delta f^2_{\text{эфс}}} > \sigma^2_{\varphi\text{фс}}$, i.e., it is unsuitable for the high relative stabilities when $\sigma_{\delta f} < \Delta f_{\text{эфс}}$, which is caused by the assumption, used for obtaining by 6.8.4.

From the obtained results it follows that in many instances during the use in the phase systems of selective elements/cells on the radio frequency the instability of the characteristics of radio-frequency filter can prove to be the factor, which reduces the accuracy and the freedom from interference of system. If we proceed from the permissible deterioration in the accuracy, then it is possible to find the requiring stability.

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For example, if the error from the interferences, which is obtained in the optimum measurement of phase, is equal to $\sigma_{\text{ш}\varphi\text{фс}} = 0{,}05 = 2{,}5 \div 3^\circ$ and its dispersion $\sigma^2_{\text{ш}\varphi\text{фс}} = 0{,}0025$, then for guaranteeing its increase upon transfer to the real radio channel not more than three times, is required, in order to

$$\frac{\sigma^2_{\delta f}}{\Delta f^2_{\text{эфс}}} = 0{,}0025$$

or

$$\sigma_{\delta f} = 0{,}05 \Delta f_{\text{эфс}}.$$

In this case the passband of system must be increased 2 times in comparison with the band of matched filter. Band $\Delta f_{эфс}$ is determined in essence by the time of observation $t_н$. Let us assume that it comprises, for example, 1 Hz, then the requiring instability must be less than 0.05 Hz (standard deviation), which usually little is actual. If we proceed from the real instability of tuning, then it is possible to determine a deterioration in the accuracy of system.

As an example let us take the case when: $\sigma_{\Delta f}=1$ Hz and the conditions of measuring the phase allow in the optimum version (without taking into account instability) to realize band $\Delta f_{эфс}=1$ Hz; precision in the optimum measurement is considered by dispersion $\sigma^2_{\Delta\varphi\, фс}=0{,}0025$. After leading calculation, we will obtain that the real accuracy will deteriorate approximately/exemplarily 50 times (on the dispersion) and the band of radio channel must be undertaken 30 times of of wider than in matched filter. The aforesaid previously makes it possible to do important the conclusion that the use/application of matched filters on the radio frequency in the phase systems usually little is expedient. It is more correct to use optimum and quasi-optimum correlation diagrams, including "tracking". In these diagrams the accumulation of information, which ensures the narrow passband of interferences, is realized direct current, little affects the instrument errors and is realized technically simply (even for the very narrow bands). Considerable interest present also combined correlation-filtration diagrams.

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Chapter 7.

OPTIMIZATION OF PHASE TRACKING.

§ 7.1. Formulation of the problem. In many instances the observation of the phase of signal is continued relatively for long, and it for this time can substantially be changed. Phase in this case must be considered as random process. It must be noted that the results obtained previously in this case are not applicable and problem must be solved independently.

Sometimes a change in the phase in time can be described by any determined function; however, results obtained in this case have particular interest. It is more correct to assume that the measured phase is by chance and is changed as the random function of time. During the analysis of the results of measuring the changing phase arise specific complicated questions. In the measurement of constant phase the errors can depend on the imperfection of the elements of network (instrument/tool) and on the action of interferences. Earlier was found the optimum diagram, which gives the minimum of errors due to the interferences. With the tracking the changing phase appear the even more dynamic errors, caused by the fact that the measuring (servo) device, having the limited operating speed, cannot immediately reflect or reproduce all changes in the measured phase. To decrease the dynamic errors is possible by an increase in the operating speed of meter. But in this case is widened its passband and, therefore, increases the action of interferences. Contradiction between the dynamic and fluctuating errors (due to the interferences) is the basic special feature/peculiarity of such meters.

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Usually by optimization of phase tracking is understood the determination of modes/conditions or conditions under which the resultant error will be minimum. It is possible to use the more general/more common/more total and stricter criteria of optimality, however, as confirmed practice, it usually proves to be sufficient to use the simplest criterion of optimality. It is obvious that for the analysis of optimum procedure it is necessary to find or to assign the fundamental characteristics of phase as random process. To them should be related the distribution function and the energy spectrum or correlation function. Let us consider the basic factors, which are determining the statistical characteristics of the measured phase, which carries useful information.

§ 7.2. Statistical characteristics of the phase of signal. For the interferences with a sufficient accuracy can be accepted the model of normal stationary random process with the uniform energy spectrum. The distribution functions and the energy spectrum of the divergences of the phase of signal under the action of interferences are examined in Chapter 3.

The statistical characteristics of the changing phase of signal (communication/report) cannot be evaluated so simply and it is uniform for different actual conditions and problems.

Let us recall that during the phase it can be laid: a) any communication/report or the information about a change in the conditions of propagation and stability of phase shifts in the equipment, used for the creation of the stress/voltage of supporting/reference phase, which ensures the interference-free reception of information in the communication systems; b) information about the position and the motion of object or information about a change in the carrier frequencies.

These two cases essentially differ from each other in the statistical characteristics of phase and must be examined independently.

Let us consider the at first statistical characteristics of the phase of the signal when its change is caused by communication/report or changes in the conditions for propagation and passage of signals in the circuits of equipment.

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These changes in the phase of signal occur in the final limits and usually depend on many, independently functioning factors, for example by change the temperatures, the humidity and pressure in different the points of the route of the extending radio wave, by a change in tuning ducts/contours under the effect of the humidity, the temperature, aging of parts, mechanical effects, fluctuations of supply voltages, etc.

In the first approximation, under such conditions the random process of changing the phase it is possible to consider stationary and the function of distribution - normal.

Since the average/mean value of the phase of signal can be recognized for the "zero" phase, the function of phase distribution will take the form

$$w(\varphi_c)=\frac{1}{\sqrt{2\pi}\sigma_{\varphi c}}e^{-\frac{\varphi_c^2}{2\sigma_{\varphi c}^2}}, \qquad (7.2.1)$$

where $\sigma^2_{\varphi c}$ — dispersion of changes in the phase.

However, there is basic interest in the energy spectrum of changes in the phase.

The intensity of the slow fluctuations of phase is usually more than rapid ones. This is caused by the fact that many factors, which call a change in the phase, are slowly changing.

The most convenient model of the energy spectrum of changes in the phase is the Gaussian or "bell-shaped" spectrum

$$G_{\varphi c}(\omega)=G_{\varphi c}(0)\,e^{-\pi\left(\frac{\omega}{2\Delta\omega_{\vartheta\varphi c}}\right)^2}, \qquad (7.2.2)$$

where $G_{\varphi c}(0)$ — value of the energy spectrum of the fluctuations of phase at the zero frequencies.

In this case the correlation function takes the form

$$B_{\varphi c}(\tau)=\sigma^2_{\varphi c}\,e^{-\pi\left(\frac{\tau}{2\tau_{0\varphi c}}\right)^2}, \qquad (7.2.3)$$

where

$$\tau_{0\varphi c}=\frac{1}{4\Delta f_{\vartheta\varphi c}}=\frac{2\pi}{4\Delta\omega_{\vartheta\varphi c}}.$$

$\tau_{\kappa\varphi c}=\tau_{0\varphi c}$ — the interval of correlation; $\Delta f_{\vartheta\varphi c}$ — equivalent band of the energy spectrum

$$G_{\varphi c}(0)=\frac{\sigma^2_{\varphi c}}{\Delta f_{\vartheta\varphi c}}.$$

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Fig. 7.2.1 gives: the realization of the random function of a change in phase $B_{\varphi c}(\tau)$ and $G_{\varphi c}(\omega)$.

Knowing $G_{\varphi c}(\omega)$, $B_{\varphi c}(\tau)$ and $w(\varphi_c)$, it is possible to find the distribution functions, correlation and energy spectrum for the derived phase.

The function of the distribution of derived phase remains normal

$$w(\dot{\varphi}_c)=\frac{1}{\sqrt{2\pi}\sigma_{\dot{\varphi}c}}e^{-\frac{\dot{\varphi}_c^2}{2\sigma^2_{\dot{\varphi}c}}}, \qquad (7.2.4)$$

where

$$\sigma^2_{\dot{\varphi}c}=\sigma^2_{\varphi c}\frac{\pi}{2\tau^2_{0\varphi c}}=\sigma^2_{\varphi c}\frac{2\pi}{4}4\Delta f^2_{э\varphi c}=\sigma^2_{\varphi c}\frac{\Delta\omega^2_{э\varphi c}}{2\pi}. \qquad (7.2.5)$$

Correlation function can be found from the expressions

$$B_{\dot{\varphi}c}(\tau)=-\frac{d^2B_{\varphi c}(\tau)}{d\tau^2}=-\frac{d^2\left[\sigma^2_{\varphi c}e^{-\pi\left(\frac{\tau}{2\tau_{0\varphi c}}\right)^2}\right]}{d\tau^2}=$$

$$=\sigma^2_{\varphi c}\frac{1}{2}\frac{\pi}{\tau^2_{0\varphi c}}e^{-\pi\left(\frac{\tau}{2\tau_{0\varphi c}}\right)^2}\left[1-\frac{\pi}{2}\left(\frac{\tau}{\tau_{0\varphi c}}\right)^2\right]=$$

$$=\sigma^2_{\dot{\varphi}c}R_{\dot{\varphi}c}(\tau), \qquad (7.2.6)$$

$$R_{\dot{\varphi}c}(\tau)=\left[1-2\pi\left(\frac{\tau}{2\tau_{0\varphi c}}\right)^2\right]e^{-\pi\left(\frac{\tau}{2\tau_{0\varphi c}}\right)^2}. \qquad (7.2.7)$$

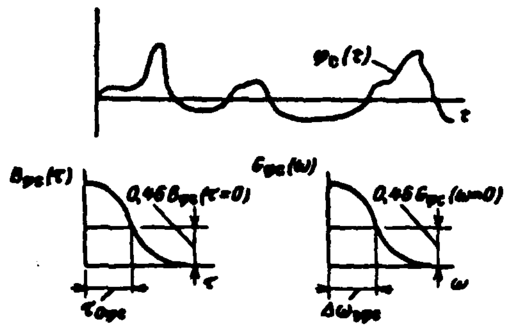

Fig. 7.2.1. Realization of the random process of changing the phase, $B_{\varphi c}(\tau)$ and $G_{\varphi c}(\omega)$.

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Energy spectrum we find, after using Fourier transform to $B_{\dot\varphi c}(\tau)$, or we determine him with respect to the frequency characteristic of the differentiator

$$G_{\dot\varphi c}(\omega)=G_{\varphi c}(\omega)\,\omega^2=\frac{4\pi^2\sigma^2_{\varphi c}}{\Delta\omega^3_{э\varphi c}}\,\omega^2 e^{-\pi\left(\frac{\omega}{2\Delta\omega_{э\varphi c}}\right)^2}. \quad (7.2.8)$$

Plotted functions $R_{\dot\varphi c}(\tau)$ and $G_{\dot\varphi c}(\omega)$ are given in Fig. 7.2.2.

From the obtained results it is possible to draw essential conclusions.

If we assume that the spectrum of changes in the phase is gaussian, and random process - stationary, then the energy spectrum of the fluctuations of derived phase or the energy spectrum of the frequency deviations has important special feature/peculiarity - it does not contain the components of very slow changes in the frequency.

In many instances above functions of phase distribution accepted and simplest model of the spectrum of changes in the phase require further explanations. Under the actual conditions during the stationary random process of changing the phase can be observed the case when the divergences of phase considerably exceed $\pm\pi$ or $\pm 2\pi$, as is shown in Fig. 7.2.3.

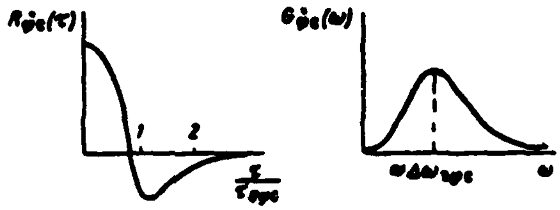

Fig. 7.2.2. Correlation function and the energy spectrum of derived steady state.

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Since phase meter by itself cannot divide the values of phase, which differ on 2π, readings/indications of ideal phasemeter in this case will be changed in the manner that it is shown in Fig. 7.2.3.

In this case the function of phase distribution (in the limits $\pm\pi$) is changed and in the first approximation, can be accepted uniform to $w(\varphi_c)=\frac{1}{2\pi}$. As is known, dispersion in this case comprises $\sigma^2_{\varphi c}=\frac{\pi^2}{3}$.

It must be noted that the use of a uniform function of phase distribution with the limited dispersion for some problems is incorrect. The system, which tracks after the phase, that is changed in this law, sometimes must master changes in the phase within those limits in which the latter actually is changed in the signal. But in order to use these limits, it is necessary to conduct the reading of the intervals of the variation in the phase on 2π. With the interruption in the measurements the results are broken. Since to phase measurements is characteristic the multiformity, with modulation of phase by communication/report, if it is revealed/detected through the measurement of phase, it is desirable to be limited to a change in the phase to $\pm\pi$ or to introduce the permission/resolution of ambiguity with the help of further more coarse readings.

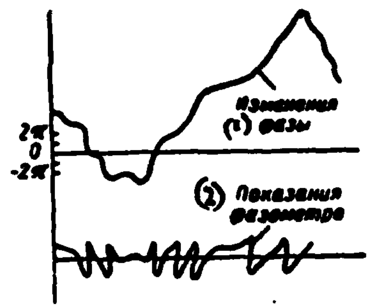

Fig. 7.2.3. Stationary process of changing the phase within the limits, which exceed 2π.

Key: (1). Changes in the phase. (2). Readings/indications of phasemeter.

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If we do not realize a reading of cycles or permission/resolution of ambiguity, then usual phase meter when will not give perfect information about the changing phase. However, this information sometimes proves to be sufficient, for example in the case of tracking the phase of signal for the creation of the copy of signal in the systems, which realize the optimum reception of signal with the known parameters.

It is interesting to note that the statistical characteristics of the phase of signal examined are similar to the characteristics of the phase of selective interference, but between the signal with the random phase and the interference there is a vital difference. The width of the spectrum of the signal (has in mind the width of the spectrum to one side from the carrier) is more than the width of the spectrum of the fluctuations of phase, which is evident from (7.2.8). In contrast to this the width of interference spectrum is always narrower than the width of the spectrum of the fluctuations of its phase (see Chapter 2). this occurs because in the interference is by chance also the amplitude.

Let us consider the statistical characteristics of the measured phase with the change or the instabilities of the carrier frequencies.

With this more conveniently at first to consider the statistical characteristics of frequency shift Δf_0 since this random process in many instances can be acknowledged stationary, while the random process of changing the phase proves to be unsteady.

From the physical considerations it is obvious that typical is the case when shift/shear in the frequency can take different values, beginning from the zero ones, and the probability density of the frequency deviations with an increase in the amount of deflection is reduced. In many instances it is possible to take the normal function of the distribution of the frequency deviation with dispersion $\sigma^2_{\Delta fc}$.

The energy spectrum of the frequency deviations must have low-frequency components, since can occur very slow changes in the frequency. This fact is in principle important, since its corollary is the transiency of the random process of changes in the phase.

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In many instances can be accepted the Gaussian model of the energy spectrum of the deviations of the frequency or derived phase

$$G_{\Delta fc}(\omega)=G_{\Delta fc}(0)\,e^{-\pi\left(\frac{\omega}{2\Delta\omega_{\vartheta\Delta fc}}\right)^2}, \qquad (7.2.9)$$

where $\frac{\Delta\omega_{\vartheta\Delta fc}}{2\pi}=\Delta f_{\vartheta\Delta fc}$ — equivalent band of the frequency deviations;

$$G_{\Delta fc}(0)=\frac{\sigma^2_{\Delta fc}}{\Delta f_{\vartheta\Delta fc}} \qquad (7.2.10)$$

The correlation function of the deviations of the frequency or derived phase will take the form

$$B_{\Delta fc}(\tau)=\sigma^2_{\Delta fc}\,e^{-\pi\left(\frac{\tau}{2\tau_{0\Delta fc}}\right)^2}, \qquad (7.2.11)$$

where

$$\tau_{0\Delta fc}=\frac{1}{4\Delta f_{\vartheta\Delta fc}}.$$

The statistical characteristics examined occur, for example, with the tracking the phase of the signal, which has certain instability of the carrier frequency.

Knowing the technical characteristics of heterodynes, utilized on the radio link, it is possible to find the energy spectrum of the frequency deviations and the dispersion of these divergences. Depending on the taken measures for stabilization, the standard deviation of frequency usually is from 10^{-3} to 10^{-9} from the value of carrier.

The energy spectrum of divergences usually very narrow, and value $\Delta f_{эф/c}$ can be 10^{-2}-10^{-3} Hz.

However, in the use/application of constructions/designs and the diagram of the heterodynes, weakly defended from the mechanical effects and the pulsations of supply voltages, there can be the cases, when $\Delta f_{эф/c}$ has a value to 100 Hz.

Let us pass from the statistical characteristics of the frequency deviations to the statistical characteristics of changes in the phase.

The random function of phase can be expressed in the following form:

$$\varphi_c(t) = 2\pi \int_0^t \Delta f_c(t)\, dt, \qquad (7.2.12)$$

and it is the random function of time with its functions of distribution and correlation.

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The basic special feature/peculiarity of the random function of time $\varphi_c(t)$ in the case in question will be its transiency, of what it is possible to be convinced, on the basis of the following considerations.

Upon transfer to the selection the integral is replaced by the sum

$$\varphi_{cm} = 2\pi \sum_{i=1}^{m} \Delta f_{ci}\, \Delta t,$$

where

$$m = \frac{t}{\Delta t},$$

$$\Delta t = \tau_{\kappa\Delta fc} = \tau_{o\Delta fc}.$$

Since φ_{cm} is obtained as a result of the addition of a large number of random variables $\Delta f_{ci}\Delta t$, the distribution function is normal.

Dispersion $\sigma^2_{\varphi c}$ can be found as the sum of the dispersions of the summarized random variables

$$\sigma^2_{\varphi c} = (2\pi)^2 \sum_{l=1}^{m} \sigma^2_{\Delta f c}\, \Delta t^2 = 4\frac{(2\pi)^2 \sigma^2_{\Delta f c}}{\Delta f_{э\Delta f c}}\, t, \qquad (7.2.13)$$

where

$$\Delta f_{э\Delta f c} = \frac{1}{4\tau_{к\Delta f c}}.$$

Consequently, the random process, which describes phase, is normal nonstationary process with the increasing dispersion. In this case, if we count off phase in the limits $\pm\pi$, the function of phase distribution will be uniform to $(1/2\pi)$. The energy spectrum of the fluctuations of phase will narrower than the spectrum of signal, and at the zero frequencies approach infinity.

Let us consider the now statistical characteristics of phase with its changes due to the displacement/movement of objects. The random process of changing the phase, as it will be evident from that following, is unsteady. Let us find the statistical characteristics of changes in the frequency deviation.

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The speed of relative motion, which is determining the frequency deviation from the nominal value, can be changed in close margins; therefore the random process of changing the frequency deviations proves to be stationary. The function of the distribution of divergences usually takes the complicated form, and sometimes in the first approximation, it can be accepted uniform or normal; however, it does not play large role, since phase is an integral of the frequency deviation and, therefore, the function of phase distribution will be usually close to the normal. It is necessary to only know the dispersion of the deviations of frequency $\sigma^2_{\Delta f_c}$.

There is basic interest in the energy spectrum of the frequency deviations. From the physical considerations it follows that rapid changes in the frequency must be observed more rarely than slow. Thus, it is possible to expect that the energy spectrum of the frequency deviations will have a density of "power", which is reduced with an increase in the frequency. However, these qualitative explanations it is insufficient, and for the calculations is required obtaining the quantitative characteristics of the random process in question, in the first place, the energy spectrum of the frequency deviations. Let us consider this question in more detail.

For obtaining the energy spectrum of the frequency deviations, caused by motion, must be accepted any hypothesis of motion. The simplest motion is uniform. It is possible to present motion with the determined maneuver, for example development on 180° with the assigned radius or according to the more complicated programs.

The hypotheses, in which are used the determined situations, can be of certain interest, and for them it is possible to calculate the characteristics, which describe effect. However, this gives the possibility to obtain only particular solution and to carry out an optimization of the mode/conditions of tracking for the specific, specific case.

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There is basic interest in the study of the hypotheses of motion, which give the possibility to carry out a statistical approach to the problem. As the models of such motions let us take the trajectory of the "maneuvering" object and "transport" assembly of the trajectories. An example of the trajectory of the "maneuvering" object is given in Fig. 7.2.4; there also is given the graph of changes in the relative speed of the motion of object. In this case for simplicity we assume that the speed of motion is changed abruptly.

Usually it is possible to consider the parameters of the motion of object, i.e., its speed and the average duration of motion without maneuver τ_{cp}.

After considering that the random process of changing the relative speed is stationary, it is possible to find correlation function and energy spectrum.

Let us consider the general case when the moments/torques of change and value of speeds are by chance. In this case $v(t)=v_i$ when $t_i<t<t_{i+1}$, where v_i — random values of relative speed; $\Delta t_i=t_{i+1}-t_i$ the random intervals in limits of which does not occur the velocity discontinuity.

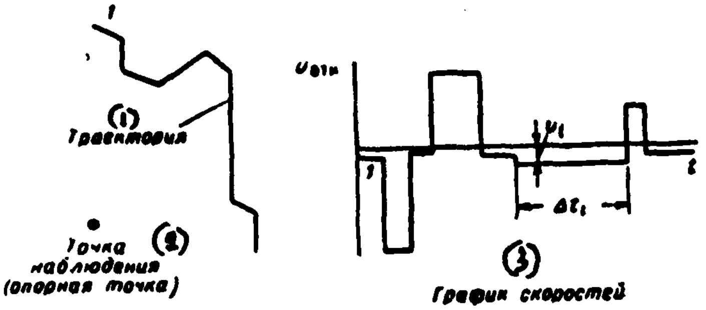

Fig. 7.2.4. Trajectory and the graph of the speeds of the "maneuvering" object.

Key: (1). Trajectory. (2). Observation point (reference point). (3). Graph of speeds.

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Probability k of changes in the speed for the time τ will be determined from the Poisson distribution [7.4]

$$p(k)=\frac{(\nu_{cp}\tau)^k}{k!}e^{-\nu_{cp}\tau}; \qquad (7.2.14)$$

with k=0, i.e., for the case when changes does not occur,

$$p(0)=e^{-\nu_{cp}\tau},$$

where ν_{cp} — average number of changes per second (unit of time)

$$\nu_{cp}=\frac{1}{\tau_{cp}}. \qquad (7.2.15)$$

Depending on whether are located the moments of time, divided by interval τ, within limits of one and the same interval Δt_i or different, we will obtain

$$v(t)\,v(t+\tau)=v_{i}^{2},$$

or

$$v(t)\,v(t+\tau)=v_{i}v_{i+k},$$

where k - difference in a number of intervals.

Correlation function can be found as mathematical expectation (average/mean value) from product v(t)v(t+τ) for all possible combinations of incidence/impingement into one and the same or different intervals Δt_i.

For the random process with the discrete/digital values

$$B_v(\tau)=\lim_{m\to\infty}\frac{1}{m}\sum_{i=1}^{m}v_iv_{i+k},$$

with k=0, 1, 2.....

Correlation function can be expressed in the form of two sums:

$$B_v(\tau)=\lim_{m\to\infty}\frac{1}{m}\sum_{i=1}^{m}v_i^2P(0)+$$

$$+\lim_{m\to\infty}\frac{1}{m}\sum_{i=1}^{m}v_iv_{i+k}\,[1-P(0)].$$

The first term corresponds to the cases when in the interval speed is not changed.

We consider the values of speed, after the moment/torque of jump, statistically independent variables, then

$$\lim_{m\to\infty}\frac{1}{m}\sum_{i=1}^{m} v_i v_{i+k} = 0 \quad \overset{(1)}{\text{при}}\ k \geqslant 1.$$

Key: (1). with.

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At the same time

$$\lim_{m\to\infty}\frac{1}{m}\sum_{i=1}^{m} v_i^2 = \sigma_v^2$$

and

$$B_v(\tau) = P(0)\sigma_v^2 = \sigma_v^2 e^{-\nu_{cp}\tau}. \qquad (7.2.16)$$

Knowing $B_v(\tau)$, we find $G_v(\omega)$,

$$G_v(\omega) = 4\int_0^\infty \sigma_v^2 e^{-\nu_{cp}\tau}\cos\omega\tau\, d\tau =$$

$$= \Big| _0^\infty 4\sigma_v^2 \frac{e^{-\nu_{cp}\tau}}{\nu_{cp}^2 + \omega^2}(\nu_{cp}\cos\omega\tau + \omega\sin\omega\tau) =$$

$$= 4\sigma_v^2 \frac{\nu_{cp}}{\nu_{cp}^2 + \omega^2} = 4\frac{\sigma_v^2}{\nu_{cp}}\frac{1}{1 + \frac{\omega^2}{\nu_{cp}^2}}. \qquad (7.2.17)$$

From the functions for the relative speed it is easy to switch over to functions for the deviation of the frequency

$$\Delta f_c = f_0 \frac{v}{c}, \quad v = \frac{c\,\Delta f_c}{f_0},$$

then

$$B_{\Delta fc}(\tau) = \sigma^2_{\Delta fc} e^{-v_{\text{ср}}\tau},$$

where

$$\sigma^2_{\Delta fc} = \sigma^2_v \frac{f_0^2}{c^2};$$

If we suppose that all speeds from 0 to $v_{\text{макс}}$ are equiprobable, then

$$\sigma^2_v = \frac{v^2_{\text{макс}}}{3}, \quad \sigma^2_{\Delta fc} = \frac{v^2_{\text{макс}}}{3} \frac{f_0^2}{c^2},$$

where $v_{\text{макс}}$ — the maximum speed of object.

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Analogous conversions can be fulfilled for the energy spectrum

$$G_{\Delta fc}(\omega) = 4 \frac{\sigma^2_{\Delta fc}}{v_{\text{ср}}} \frac{1}{1+\frac{\omega^2}{v^2_{\text{ср}}}} = \frac{4}{3} \frac{v^2_{\text{макс}}}{c^2} \frac{f_0^2}{v_{\text{ср}}} \frac{1}{1+\frac{\omega^2}{v^2_{\text{ср}}}} =$$

$$= \frac{4}{3} \frac{v^2_{\text{макс}}}{c^2} \frac{f_0^2}{v^2_{\text{ср}}} \frac{v_{\text{ср}}}{1+\frac{\omega^2}{v^2_{\text{ср}}}} = G_{\Delta fc}(0) \frac{1}{1+\frac{\omega^2}{v^2_{\text{ср}}}}. \quad (7.2.18)$$

Density of "power" at the low frequencies

$$G_{\Delta fc}(0) = \frac{4}{3} \frac{v^2_{\text{макс}}}{c^2} \frac{f_0^2}{v_{\text{ср}}} = \frac{4\sigma^2_{\Delta fc}}{v_{\text{ср}}}.$$

Let us consider the example

$$v_{макс} = 3600 \text{ км/час}^{(1)}, \quad f_0 = 1 \text{ Мгц}^{(2)}, \quad v_{ср} = 0{,}2 \frac{1}{сек}^{(3)}$$

$$\sigma^2_{\Delta fc} = 3 \frac{1}{сек^2}{}^{(3)} \quad G_{\Delta fc}(0) = 60 \frac{1}{сек}{}^{(3)}$$

Key: (1). km/h. (2). MHz. (3). s.

Graphs $B_{\Delta fc}(\tau)$ and $G_{\Delta fc}(\omega)$ for the example in question are given in Fig. 7.2.5.

The obtained results make it possible to do important the conclusion that the energy spectrum of the frequency deviations for the frequencies, close to zero, has finite value. This means that the phase will be described by unsteady random process.

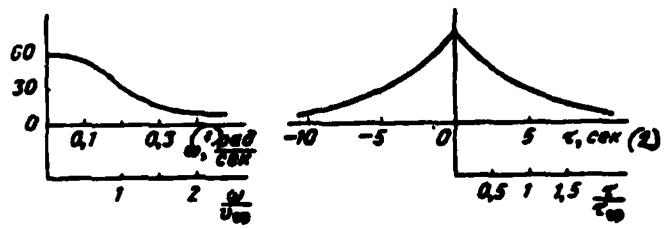

Fig. 7.2.5. Energy spectrum and the correlation function of the frequency deviations for the maneuvering object.

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During the conclusion/output of expressions for the "maneuvering" object was done the series/row of assumptions. Very essential assumption is assumption about the possibility of an abrupt change in the speed of object, that under the actual conditions cannot be fulfilled, since in this case will be required infinite accelerations.

Under the actual conditions the "maneuvers" of object must be smooth, which will lead to the more rapid decrease of the intensity of high-frequency components. On these reasons use (7.2.17) and (7.2.18) for the calculations is not always justified.

Let us now move on to the analysis of the spectrum which occurs with the "transport" problem. In this case the object can follow past the reference point by any routes. Let us take the model of rectilinear routes, then their combination n take the form, depicted in Fig. 7.2.6.

Virtually the range of radio engineering system is always limited and in the model accepted this is expressed in the form of circle/circumference with a radius of R_{max}.

Considering all routes equiprobable and bearing in mind that the spectrum of the information, which is contained in the phase of signal, does not depend on direction of motion, we can switch over to the model, represented in Fig. 7.2.7, in which is given the discrete/digital set of trajectories.

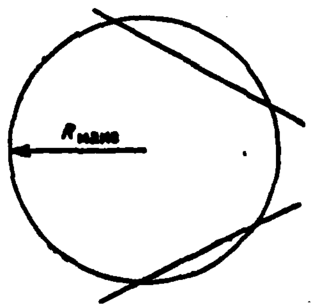

Fig. 7.2.6.

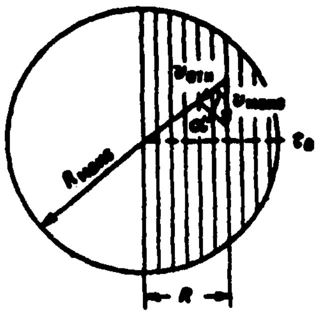

Fig. 7.2.7.

Fig. 7.2.6. Random routes.

Fig. 7.2.7. Model of random routes.

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Since $v_{\text{отн}} = v_{\text{макс}} \cos\alpha$, $\operatorname{tg}\alpha = \frac{R}{v_{\text{макс}}\Delta t_0}$, where Δt_0 - time, calculated off moment/torque t_0,

$$v_{\text{отн}} = v_{\text{макс}} \cos\left(\operatorname{arctg}\frac{R}{\Delta t_0 v_{\text{макс}}}\right),$$

the values $v_{\text{отн}}$ making sense only in limits $\Delta t_{0\,\text{макс}}$, which can be calculated from the formula

$$R_{\text{макс}} = \sqrt{v^2_{\text{макс}}\Delta t^2_{0\,\text{макс}} + R^2}.$$

For each concrete/specific/actual trajectory can be constructed the corresponding curve of a change in the relative speed. However, each of their trajectories is by chance, and curves can be alternated in any sequence. Considering that the object during the motion passes of one region to another, it is possible to construct the realization of the random process of change $v_{\text{отн}}$. consisting of the determined segments of curves with the random variables of the jump of relative speed. The example to this realization is given in Fig. 7.2.8.

The random process of changing in the relative speed and frequency switch can be considered as stationary. This process to a certain extent is analogous to that which was obtained for the "maneuvering" object. Difference lies in the fact that each velocity discontinuity is accompanied by a change in the sign and relative speed has both jumps and smooth transitions. In the first approximation,, by assuming that the speed is changed with jumps, it is possible to consider that the energy spectrum is determined by the expression, analogous (7.2.16), the spectrum in this case will be wider.

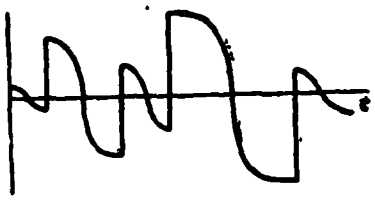

Fig. 7.2.8. Graph of random speeds.

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Lowering conclusion/output, let us write expression for $G_v(\omega)$:

$$G_v(\omega)=\frac{\sigma_v^2\cdot 2}{v_{cp}}\,\frac{1}{1+\frac{\omega^2}{4v_{cp}^2}}. \qquad (7.2.19)$$

If one assumes that the velocity distribution function is close to the uniform, we will obtain

$$\sigma_v^2=\frac{v_{макс}^2}{3}.$$

It must be noted that the assumption about even distribution $v_{отн}$ within the limits from 0 to $v_{макс}$ is rough approximation. Although the velocity distribution function and this case differs from uniform however the use/application of a more precise analysis will only somewhat change the numerical value of dispersion. The basic laws, important for the optimization, in this case remain valid.

The duration of each ejection of speed is within the limits from 0 to $R_{\text{макс}}/v_{\text{макс}}$.

Average/mean value of the duration of the ejections

$$\tau_{\text{ср}} = \frac{1}{\nu_{\text{ср}}} \approx 0{,}7\frac{R_{\text{макс}}}{v_{\text{макс}}}.$$

Because of the presence of the sections of a smooth change in the speed the real spectrum will contain fewer high-frequency components than this follows from (7.2.16). Observing under the actual conditions for the "maneuvering" object and the "transport" problem and in other analogous cases a smooth change of the speed leads to the fact that the spectrum, given by expression (7.2.16), can at the high frequencies differ significantly from that obtaining from (7.2.18) and (7.2.19).

In this connection let us consider the possibility of using the gaussian model of the spectrum of the frequency deviations.

Fig. 7.2.9 gives plotted functions

$\dfrac{1}{1+\dfrac{\omega^2}{\nu_{\text{ср}}^2}}$ (curve a) and

$e^{-s\left(\frac{\omega}{\Delta\omega_{\text{ЭАЖ}}}\right)^{2}}$ (curve b) for the case when $\Delta\omega_{\text{ЭАЖ}}=v_{ep}$.

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As is evident, curves noticeably diverge at the high frequencies. The evenness of a change in the relative speed or frequency switch it is possible by the easiest method to take into account, using the inertia factor

$$\frac{1}{T^2\omega^2+1}.$$

After assuming $T=\frac{1}{2}\tau_{c1}$, i.e. assuming that the object maneuvers on the average after the time, two times it is larger than its inertness, and after taking into account the correction, given to them, we will obtain the curve c in Fig. 7.2.9.

The energy spectrum of changes in the relative speed upon consideration of inertia factor virtually completely coincides with the gaussian model, besides the high frequencies where the intensity of the spectrum is insignificant.

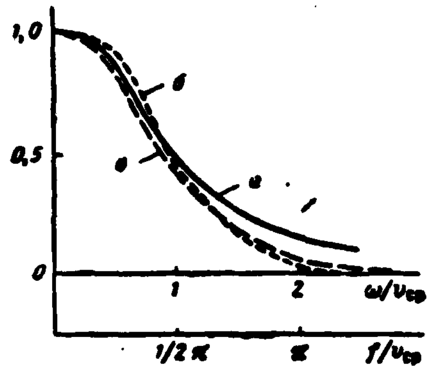

Fig. 7.2.9. Approximations of the spectrum of phase.

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This gives grounds for the cases of the "maneuvering" object and the "transport" situation to take the Gaussian model of the spectrum

$$G_{\Delta f/c}(\omega)=G_{\Delta f/c}(0)\,e^{-\pi\left(\frac{\omega}{2\Delta\omega_{э\Delta f/c}}\right)^2}, \qquad (7.2.20)$$

where

$$G_{\Delta f/c}(0)=\frac{\sigma^2_{\Delta f/c}}{\Delta f_{э\Delta f/c}};$$

$$\sigma^2_{\Delta f/c}\approx\frac{1}{3}f_0^2\frac{v^2_{макс}}{c^2},$$

and for the "maneuvering" object we have

$$\Delta\omega_{э\Delta f/c}=v_{ср}=2\pi\,\Delta f_{э\Delta f/c}.$$

For the previously examined case the Gaussian model of the spectrum will be registered in the form

$$G_{\Delta/c}(\omega) \approx 100 e^{-\frac{\pi}{4}\left(\frac{\omega}{0.2}\right)^2}.$$

Equivalent band

$$\Delta f_{\text{эф}/c} = \frac{0.2}{2\pi} = 0{,}035 \text{ (1) Гц}.$$

Key: (1). Hz.

The energy spectrum of the frequency deviations in the case in question at the zero frequencies has finite value. This will lead to the fact that the random process of changing the phase proves to be unsteady.

Summing up the result to the results, obtained in the present paragraph, it is possible to note the following.

In some phase systems the phase of signal, which carries information, can it is described by the stationary random process for which can be accepted the fundamental statistical characteristics, examined earlier. In this case appears the problem of measuring the changing phase, which is stationary random process. This version can be called the case of "steady state".

In the series/row of the phase systems of change the phases are characterized by unsteady random process and the analysis of the optimization of measurement is complicated. In this case it is possible simply to find the statistical characteristics of derived phase or deviation of the frequencies, the random process of changing which in many instances is stationary.

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By analogous methods it is possible to find the statistical characteristics not of the phase of signal, but phase difference of two signals. In connection with phase navigational and trajectory systems these characteristics must describe the statistical properties of the phase difference, measured by navigational receiving and measuring device/equipment in the hyperbolic phase system or phase direction finder in the system of trajectory measurements. The obtained previously statistical characteristics of phase described a change in the phase of each of the signals during the motion of object (maneuver) within the limits of the area of action of one system or during the intersection of the areas of action of several systems.

Basic operating region in navigation aids is the region in which the distance of the station is commensurated with the basis. In this range of change in the phase of the signal of each of the stations it is possible to consider it virtually statistically independent variables. Then the function of the distribution of a difference in the deviation of frequency or phase difference can be found from the rules of the determination of the function of the distribution of a difference in two independent variables, which have specified distribution. The methodology of optimization is based on the determination of the minimum of variance of error from the energy spectra; therefore first of all must be found energy spectrum. It is obvious that with the independence of a change in the deviation of the frequencies or phases the energy spectrum of a difference in the deviation of frequencies or phase difference will be equal to the sum of the energy spectra of a change in the deviation of the frequencies or phases. the resulting spectrum preserve the same form, dispersion will be equal to the sum of dispersions.

More complicatedly is solved the problem, when basis much less than distance of observation points. This version requires separate research. With the relatively small basis changes in the phase of two signals prove to be correlated, and the result of measuring the phase difference contains information about the angular coordinate α or β (see Fig. 1.4.1, 1.4.2). Then are more convenient the statistical characteristics of a phase difference to consider directly through the statistical characteristics of angular velocity and angle.

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Since the object possesses final linear velocity and cannot be located in immediate proximity of measuring point that the random process of changing the angular velocity it proves to be stationary, while the angle is described by unsteady process. This follows from Fig. 7.2.4 if to examine not distances, but angles.

After fulfilling conversions, analogous by that given in the beginning of paragraph, we will obtain

$$G_{\dot{\alpha}}(\omega)=\frac{4\sigma^2_{\dot{\alpha}}}{\nu_{cp}}\,\frac{1}{1+\frac{\omega^2}{\nu^2_{cp}}},$$

$$\sigma^2_{\dot{\alpha}}\approx\frac{\dot{\alpha}^2_{\text{макс}}}{3}. \qquad (7.2.21)$$

Object possesses inertness; therefore maneuvers cause not an abrupt, but smooth change in the angular velocity. With this better to use the gaussian model of the energy spectrum

$$G_{\dot{\alpha}}(\omega)=G_{\dot{\alpha}}(0)\,e^{-\frac{\pi}{4}\left(\frac{\omega}{\Delta\omega_{\text{э}\dot{\alpha}}}\right)^2},$$

$$\Delta\omega_{\text{э}\dot{\alpha}}=2\pi\,\Delta f_{\text{э}\dot{\alpha}}=\nu_{cp},\quad G_{\dot{\alpha}}(0)=\frac{\sigma^2_{\dot{\alpha}}}{\Delta f_{\text{э}\dot{\alpha}}}. \qquad (7.2.22)$$

From the characteristics of angular velocity it is possible to pass to the characteristics of a difference in the increases in the frequencies [Δ(Δf)]. In this case will be changed the dispersion. Thus, for instance, for the plane and the central sector

$$\sigma^2_{\Delta(\Delta l)} = \sigma^2_{a}\left(\frac{Б}{\lambda}\right)^2.$$

where Б - base.

The energy spectrum of the derivative of angle and difference in the increases in the frequencies has finite value at the zero frequencies, therefore, the random process of changing in angles and phase difference proves to be unsteady.

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S7.3. Measurement of phase during the reproduction of the changes, which correspond to basic part of the spectrum.

Let us assume that the statistical characteristics of a change in the phase are known. Then the continuous process of changing the phase can be characterized by the selection, realized through interval $\Delta t = \tau_{кор}$, as shown in Fig. 7.3.1. In this formula $\tau_{кор}$ — the interval of correlation for the random process, which characterizes a change in the phase.

In this case some parts of process will be missed; however, the basic picture of its change will be preserved, since all "independent" values of process, i.e., statistically independent of the previous values, will be taken into consideration.

In the first approximation, the smooth process of changing the phase can be replaced with the set of the discrete/digital values, which retain their value for a period of time $\tau_{\kappa\varphi c}$. Under this assumption the optimum measurement of the changing phase can be separated for the optimum measurement of the random values of phase, constants for a period of time of observation $\Delta t = \tau_{\kappa\varphi c}$.

The optimization of the measurement of random phase was in detail examined earlier and here it is possible to use these results.

Theoretical diagram for the case in question is given in Fig. 7.3.2.

Integration must be realized for a period of time Δt, after which they must be fixed/recorded phase and values η and γ to be set to zero.

Consequently, the procedure of the measurement of the changing phase provides for the use of correlators, but the interval of accumulation in them (integration) is determined not by the time interval of the observation of signal $t_{\text{н}}$, but by the interval (time) of correlation $\tau_{\kappa\varphi c}$.

The effective value of energy of signal in this case is reduced and is determined not by its duration and power but by the width of the energy spectrum of phase or with the time of correlation and by power of signal.

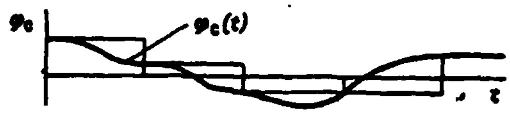

Fig. 7.3.1. Change in the phase and the discrete/digital representation of function.

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It should be noted that the measuring errors of phase in the optimum diagram can be determined, using expression (6.6.15), from the formula

$$\sigma^2_{\delta\varphi} = \frac{N_0}{2\mathscr{E}_э}, \qquad (7.3.1)$$

where $\mathscr{E}_э$ -- energy of informational element/cell.

Energy of the informational element/cell

$$\mathscr{E}_э = P_{ср}\tau_{эφc},$$

where $P_{ср}$ — average/mean power of signal.

The obtained relationships/ratios make it possible to find theoretical limit of accuracy of the measurement of the phase of the signal when it has very large duration or is in effect continuous (for example, in navigation aids or systems of trajectory measurements) and when accuracy is determined not by the duration of signal, but by speed of a change in the measured phase.

From the diagram it follows that the signal can be of any kind - simple, complex, noise-like - result from this will not be changed, since the law of modulation on the amplitude or the phase can be reproduced in the copies.

The obtained results make it possible in the first approximation, correct to consider the potential possibilities of measuring the changing phase.

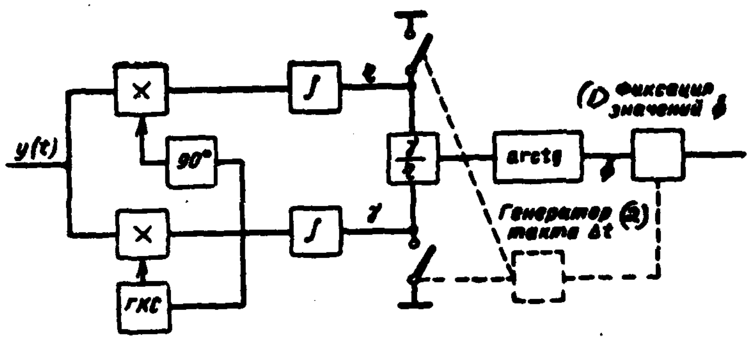

Fig. 7.3.2. The diagram of phase tracking: × - multipliers; ГКС - generator stock signal; ∫ - integrators; γ/η - device/equipment, which computes data relation; arctg - trigonometric converter.

Key: (1). fixation of values. (2). cadence generator.

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However, they have one important fundamental limitation.

During the conclusion/output of the relationships/ratios, which are determining procedure, it was assumed, that necessary condition is the reproduction of the basic information, placed during the phase. This was reflected in what the duration of the informational element/cell of signal was accepted $\tau_{\text{кφс}}$. With this interval of the selection of the values of phase in it (to selection) are reflected basic (but not all) changes. With the weak interferences can be unsuitable the loss of the part of the information about changes in the phase. This leads to the dynamic errors, since the energy spectrum of changes in the phase slowly decreases with the frequency and the part of the spectrum of changes, which lies higher than $1/2\tau_{\text{кφс}}$, in the measurements it is not considered. With the intense interferences there can be such conditions, with which the errors, caused by interferences, much more than the errors, which appear, when the part of the energy spectrum of phase is not considered. in this case for an improvement in the results of measurements it can be useful to use increased time interval during which the phase is received as constant. In this case will be increased the errors in the reproduction of communication/report, but due to an increase in the storage time and energy of the informational element/cell of signal will be reduced errors from the interferences. The methodology in question does not give the possibility to find optimum from the accuracy, of what and consists its important fundamental limitation.

In connection with that presented it is necessary to consider for the giving of phase tracking in the wider setting, which has as a goal to obtain optimum results, i.e., the minimization of the resultant error, which consists of the fluctuating and dynamic errors.

§ 7.4. Optimum phase tracking with the use of linear filtration. In order to solve the problem of the optimization of the characteristics of the meter of the changing phase, let us consider measurement for the case when filtration is realized before the phasemeter or in the phasemeter itself with the servo narrow-band system under the condition of guaranteeing the high quality of measurement. Useful "signal" for the phasemeter is the random function of time, which characterizes the phase of signal $\varphi_c(t)$, for which must be known the distribution function and energy spectrum.

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Interference creates the fluctuation of the phase of mixture $\varphi_{\Sigma}(t)$. With the strong signal the function of the distribution of the fluctuations of phase is normal with dispersion $\sigma^2_{\varphi y}$ and energy spectrum repeats energy interference spectrum with the displacement to the "zero" carrier frequency.

Diagram corresponding to this case is given in Fig. 7.4.1. Problem in this case is reduced to the filtration of "signal" from the mixture with the noise, with the fact in order to "signal" at the output of filter $\varphi_{\phi}(t)$ most accurately reproduced "signal" $\varphi_{c}(t)$. Word "signal" is included in the quotation marks, since in the phase system useful result are readings/indications of phasemeter, i.e., the value, counted off in the angular units.

Such a formulation of the problem is sufficiently common. This problem was solved theoretically by Kolmogorov-Wiener.

Let us consider basic stages and results of the solution in connection with the case of the optimization of phase tracking. We consider that $\varphi_c(t)$ and $\varphi_y(t)$ - random functions of time whose energy spectra $G_{\varphi c}(\omega)$ and $G_{\varphi y}(\omega)$ are known.

It is necessary to find the complex frequency characteristic of optimum linear filter $k_{\phi\,опт}(j\omega)$, which ensures the best reproduction of useful "signal" $\dot{\varphi}_c(t)$. By the best or optimum reproduction of the useful signal we will understand such, with which the rms error is minimum.

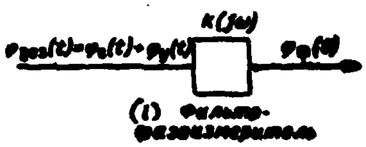

Fig. 7.4.1. Start of the linear filter of phasemeter.

Key: (1). filter-phasemeter.

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It is known that

$$k_{\text{ф}}(j\omega)=\int\limits_{-\infty}^{+\infty}\eta_{\text{и}}(t)\,e^{-j\omega t}\,dt$$

where $\eta_{\text{и}}(t)$ - pulse transient function or weight function of filter; $k_{\text{ф}}(j\omega)$ - complex frequency characteristic of filter and

$$\eta_{\text{и}}(t)=\frac{1}{2\pi}\int\limits_{-\infty}^{+\infty}k_{\text{ф}}(j\omega)\,e^{j\omega t}d\omega.$$

Since $k_{\text{ф}}(j\omega)$ and $\eta_{\text{и}}(t)$ are connected with Fourier transform, it is theoretically unimportant which will be found for optimum filter $k_{\text{ф опт}}(j\omega)$ or $\eta_{\text{и опт}}(t)$. As is known, the response of the component/link, which has weight function $\eta_{\text{и}}(t)$, to effect $\varphi_{\text{рез}}(t)$ can be found with the help of the convolution integral or Duhamel integral

$$\varphi_{\text{ф}}(t)=\int\limits_{0}^{\infty}\varphi_{\text{рез}}(t-\tau)\,\eta_{\text{и}}(\tau)\,d\tau.$$

In the case in question the effect takes the form

$$\varphi_{\text{вх}}(t)=\varphi_c(t)+\varphi_y(t), \qquad (7.4.1)$$

then

$$\varphi_\Phi(t)=\int\limits_0^\infty[\varphi_c(t-\tau)+\varphi_y(t-\tau)]\,\eta_n(\tau)\,d\tau. \qquad (7.4.2)$$

In the absence of interferences and distortions in the filter

$$\varphi_\Phi(t)=\varphi_c(t).$$

Now we can register expression for the error

$$\delta\varphi(t)=\varphi_c(t)-\varphi_\Phi(t). \qquad (7.4.3)$$

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After computing the square of error and after substituting value $\varphi_\Phi(t)$, we will obtain

$$\delta\varphi^2(t)=\varphi_c^2(t)-2\varphi_c(t)\int\limits_0^\infty[\varphi_c(t-\tau)+\varphi_y(t-\tau)]\,\eta_n(\tau)\,d\tau+$$

$$+\left\{\int\limits_0^\infty[\varphi_c(t-\tau)+\varphi_y(t-\tau)\,d\tau]\right\}^2=$$

$$=\varphi_c^2(t)-2\varphi_c(t)\int\limits_0^\infty[\varphi_c(t-\tau)+\varphi_y(t-\tau)]\,\eta_n(\tau)\,d\tau+$$

$$+\int\limits_0^\infty\int\limits_0^\infty\{[\varphi_c(t-\tau)+\varphi_y(t-\tau)][\varphi_c(t-\tau_1)+$$

$$+\varphi_y(t-\tau_1)]\,\eta_n(\tau)\,\eta_n(\tau_1)\}\,d\tau\,d\tau_1.$$

Average/mean value of the square of error or dispersion of error

$$\sigma_{\delta\varphi}^2=\lim_{T\to\infty}\frac{1}{T}\int\limits_0^T\delta\varphi^2(t)\,dt.$$

After substituting into this expression value $\delta\varphi^2(t)$, we will obtain

$$\sigma^2_{\Delta\varphi} = \sigma^2_{\varphi c} - 2\int\limits_0^{\infty} B_{c\gamma_1}(\tau)\,\eta_{в}(\tau)\,d\tau +$$

$$+\int\limits_0^{\infty}\int\limits_0^{\infty} \eta_{в}(\tau)\,\eta_{в}(\tau_1)\,B_{1\,cs}(\tau-\tau_1)\,d\tau\,d\tau_1, \qquad (7.4.4)$$

since

$$\lim_{T\to\infty}\frac{1}{T}\int\limits_0^{\infty} \varphi_c(t)\,\varphi_{1+cs}(t-\tau)\,dt = B_{c\gamma}(\tau),$$

i.e., the function of the mutual correlation between signal and mixture of signal and interference and

$$\lim_{T\to\infty}\frac{1}{T}\int\limits_0^{\infty} \varphi_{1+cs}(t-\tau)\,\varphi_{1+cs}(t-\tau_1)\,dt = B_{y}(\tau-\tau_1),$$

i.e., the autocorrelation function of mixture.

Obtained integral equation (7.4.4) must be investigated to the minimum. For this it is possible to use the methods of variation analysis. Let us drop/omit detailed conclusion/output, since it is in SS 7.3 and 7.4.

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After the series/row of transformations it is possible to obtain

$$G_{\varphi c}(\omega) = 2\int\limits_{-\infty}^{+\infty}\int\limits_{-\infty}^{+\infty} \eta_{вопт}(t)\,[B_{\varphi c}(\tau-t) +$$

$$+ B_{\varphi y}(\tau-t)]\,e^{-j\omega\tau}\,d\tau\,dt, \qquad (7.4.5)$$

where $\eta_{\text{в опт}}(t)$ - the pulse transient function of optimum filter.

Further solution of equation is possible substantially to simplify, if integration limits are changed 0-∞ on -∞-+∞. With a change in integration limits is realized also for the moments/torques of time t<0, i.e., in obtaining of expression for the characteristics of optimum filter it is assumed that the response of optimum filter can be, also, to that moment/torque, as is applied effect, or, in other words, for the realization of this optimum filter can be required the introduction to its composition of the physically unrealizable components/links in advance. The analysis of a stricter solution and characteristics of optimum filter, obtained under the stipulated simplifying assumptions, shows that for the practically important forms of spectra $G_{\varphi c}(\omega)$ and $G_{\varphi y}(\omega)$ this simplification can be used. On these reasons let us examine in more detail simple solution.

Having changed integration limits and after producing replacement of variables

$$\tau - t = \tau_1, \quad e^{-j\omega\tau} = e^{-j\omega\tau_1} e^{-j\omega t},$$

we will obtain

$$G_{\varphi c}(\omega) = 2 \int_{-\infty}^{+\infty} \int_{-\infty}^{\infty} \eta_{\text{в опт}}(t) \times$$
$$\times \left[B_{\varphi c}(\tau) + B_{\varphi y}(\tau)\right] e^{-j\omega t} e^{-j\omega\tau_1}\, dt\, d\tau_1.$$

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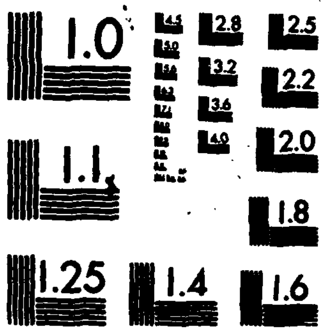
1.0
1.1
1.25
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2.5
2.8
3.2
3.6
4.0

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During this transformation the variable/alternating are divided and

$$G_{\varphi c}(\omega)=2\int_{-\infty}^{+\infty}\eta_{\text{в опт}}(t)e^{-j\omega t}dt\int_{-\infty}^{+\infty}[B_{\varphi c}(\tau_2)+$$
$$+B_{\varphi y}(\tau_2)]e^{-j\omega\tau_2}d\tau_2, \qquad (7.4.6)$$

but

$$\int_{-\infty}^{+\infty}\eta_{\text{в опт}}(t)e^{-j\omega t}dt=k_{\Phi\,\text{опт}}(j\omega),$$

where $k_{\Phi\,\text{опт}}(j\omega)$ - complex frequency characteristic of optimum filter.

Then

$$G_{\varphi c}(\omega)=k_{\Phi\,\text{опт}}(j\omega)[G_{\varphi c}(\omega)+G_{\varphi y}(\omega)]$$

or

$$k_{\Phi\,\text{опт}}(j\omega)=\frac{G_{\varphi c}(\omega)}{G_{\varphi c}(\omega)+G_{\varphi y}(\omega)}=\frac{1}{1+\frac{G_{\varphi y}(\omega)}{G_{\varphi c}(\omega)}}. \qquad (7.4.7)$$

Consequently, optimum filter must have amplitude-frequency characteristic, which depends on the relationship/ratio of the energy spectra of changes in the phase of signal, caused by communication/report, and its fluctuations under the action of interferences, and the phase-frequency characteristic, which foresees phase displacement, equal to zero. Taking into account that expression (4.7.7) contains only real part, subsequently we will consider it as expression for the modulus/module of the frequency characteristic of optimum filter.

With the weak "signal" $[G_{\varphi c}(\omega) \ll G_{\varphi y}(\omega)]$

$$k_{\phi\,opt}(\omega) = \frac{G_{\varphi c}(\omega)}{G_{\varphi y}(\omega)}.$$

If, as so often is the case, the spectrum of the fluctuations of phase due to the interferences it is possible to consider it uniform, then

$$k_{\phi\,opt}(\omega) = \mathrm{const}\, G_{\varphi c}(\omega).$$

i.e., the frequency characteristic of optimum filter must repeat the energy spectrum of "signal".

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With the strong "signal" $[G_{\varphi c}(\omega) \gg G_{\varphi y}(\omega)]$

$$k_{\phi\,opt}(\omega) = 1,$$

i.e., the frequency characteristic of optimum filter in entire frequency band of the energy spectrum of "signal" must ensure flat gain.

Let us find now expression for the minimum rms error. At the output of filter will occur the spectrum of the mixture

$$G_{\varphi\phi}(\omega) = k^2_{\phi\,opt}(\omega)\,[G_{\varphi c}(\omega) + G_{\varphi y}(\omega)]. \qquad (7.4.8)$$

Error in the reproduction of "signal" at the output will have the energy spectrum

$$G_{i\varphi}(\omega)=G_{\varphi c}(\omega)[1-k^2_{\Phi\,\text{опт}}(\omega)]+G_{\varphi y}k^2_{\Phi\,\text{опт}}(\omega)=$$
$$=G_{i\varphi д}(\omega)+G_{i\varphi п}(\omega), \qquad (7.4.9)$$

where $G_{i\varphi д}(\omega)$ and $G_{i\varphi п}(\omega)$ - energy spectra of dynamic and fluctuating components of error.

Variance of error

$$\sigma^2_{i\varphi}=\frac{1}{2\pi}\int_0^\infty G_{i\varphi}(\omega)\,d\omega=\sigma^2_{i\varphi д}+\sigma^2_{i\varphi п}, \qquad (7.4.10)$$

where $\sigma^2_{i\varphi д}$ and $\sigma^2_{i\varphi п}$ - dispersion of dynamic and fluctuating components of error.

during the use of formula (7.4.7) for $k_{\Phi\,\text{опт}}(\omega)$ we have

$$\sigma^2_{i\varphi}=\frac{1}{2\pi}\int_0^\infty\left[G_{\varphi c}(\omega)-\frac{G^3_{\varphi c}(\omega)}{[G_{\varphi c}(\omega)+G_{\varphi y}(\omega)]^2}\right]d\omega+$$
$$+\frac{1}{2\pi}\int_0^\infty\frac{G^2_{\varphi c}(\omega)\,G_{\varphi y}(\omega)}{[G_{\varphi c}(\omega)+G_{\varphi y}(\omega)]^2}\,d\omega. \qquad (7.4.11)$$

with the strong "signal" $[G_{\varphi c}(0)\gg G_{\varphi y}(0)]$

$$\sigma^2_{i\varphi}\approx\frac{1}{2\pi}\int_0^\infty G_{\varphi y}(\omega)\,d\omega\approx\sigma^2_{\varphi y}.$$

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The theoretical possibility of designing of the optimum measuring systems of the changing phase, or the systems of optimum phase tracking, is the important factor, which permits implemention of analysis and synthesis of real systems, it is sufficient close ones to the optimum ones.

§ 7.5. Special features/peculiarities of the realization of the systems of optimum tracking of phase. For the determination of the basic special features/peculiarities of the characteristics of optimum filters we will use the results of § 7.4. As it was shown earlier, the spectrum of changes in the phase can be approximated by the gaussian function [see (7.2.2)].

$$G_{\varphi c}(\omega)=G_{\varphi c}(0)\,e^{-\pi\left(\frac{\omega}{2\Delta\omega_{\text{эфс}}}\right)^2},$$

$$G_{\varphi c}(0)=\frac{\sigma^2_{\varphi c}}{\Delta f_{\text{эфс}}}, \qquad (7.5.1)$$

where $\sigma^2_{\varphi c}$ - dispersion of changes in the phase of signal, which must track (measure) the system.

The fluctuations of phase under the action of interference have in the first approximation, the uniform spectrum

$$G_{\varphi y}(\omega)=G_{\varphi y}(0)=\frac{\sigma^2_{\varphi y}}{\Delta f_{\text{п}}}=\frac{\sigma^2_{\text{п}}}{A_c^2\,\Delta f_{\text{п}}} \qquad (7.5.2)$$

$$(1)\ \text{при}\ \omega<\Delta\omega_{\text{п}}=2\pi\Delta f_{\text{п}},$$

Key: (1). with.

where $\Delta f_{\text{п}}$ - to passband of interferences in the section to the servo phasemeter (on the radio frequency band $2\Delta f_{\text{п}}$).

It is necessary to note, that $G_{\varphi c}(\omega)$ and $G_{\varphi c}(0)$ do not depend on signal amplitude and are determined only by the special features/peculiarities of changes in its phase. Spectrum $G_{\varphi y}(\omega)$ depends on relation A_c/σ_n. In Chapters 2 and 3 it was shown that the energy spectrum of the fluctuations of phase with the decrease of relation A_c/σ_n and with one interference acquires complex form and the dispersion of the fluctuations of phase in this case approaches limit $\pi^2/3$. However, since it is assumed that the measurement of phase is realized during the use of linear filtration, at the "entrance" of phasemeter relation σ_n/A_c, to which corresponds conditional variance $\sigma_n^2/A_c^2=\sigma_{\varphi y}^2>\frac{\pi^2}{3}$, it is possible to allow/assume large.

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Knowing $G_{\varphi c}(\omega)$ and $G_{\varphi y}(\omega)$, it is possible, using (7.4.7), to find the frequency characteristic of optimum filter. As a result of calculation can be obtained the frequency characteristic realization of which will require the compound circuit of filter. In this connection is virtually important the fact that the optimum frequency characteristic in the first approximation, can be approximated by the frequency characteristic of inertia component/link. Fig. 7.5.1 as an example gives dependence $k_{\Phi\,opt}(\omega)$ (curve a) for:

$$\Delta f_{сиг} = 1 \text{ гц}, \quad \Delta f_{ш} = 5 \text{ гц}, \quad \sigma_{ш}/A_c = 1 = \sigma_{\varphi y},$$

$$\sigma^2_{\varphi c} = 3, \quad G_{\varphi c}(0) = 3, \quad G_{\varphi c}(\omega) = 3e^{-\pi\left(\frac{f}{2}\right)^2}, G_{\varphi y}(0) = 0{,}2$$

and dependence for the inertia component/link (curve b).

Inertia component/link in practice easily is realized; therefore it are conveniently used as the filter, but this filter will be quasi-optimal.

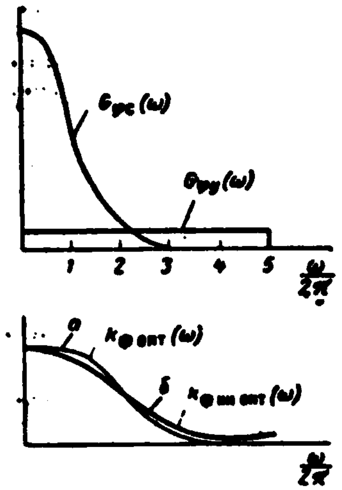

Fig. 7.5.1. Spectrum of changes (fluctuations) in the phase and the frequency characteristic of the optimum filter: a) for the optimum filter; b) for the inertia component/link.

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The frequency characteristic of inertia component/link takes the form

$$k_{\Phi\,\text{зв}}(\omega)=\frac{k_{\Phi\Phi}}{\sqrt{\omega^2 T^2_{\Phi\,\text{зв}}+1}},$$

where $T_{\Phi\,\text{зв}}$ - time constant of component/link;

$$T_{\Phi\,\text{зв}}=\frac{1}{\omega_{\Phi\,\text{c}\,\text{зв}}},$$

where $\omega_{\Phi\,\text{c}\,\text{зв}}$ - frequency of coupling inertia component/link, which corresponds to effective band width of component/link $\Delta\omega_{\Phi\,\text{э}}$ multiplied by $2/\pi$.

It is convenient to mate characteristics at point with relative amplification by 0.5 (Fig. 7.5.1)

$$k_{\phi\,\text{нч опт}}(\omega_{\phi 0,5\,\text{нч опт}}) = k_{\phi\,\text{опт}}(\omega_{\phi 0,5\,\text{опт}})$$

but

$$\omega_{\phi 0,5\,\text{нч опт}}\,0,6 \approx \omega_{\phi\,\text{сн опт}}.$$

Detuning for the optimum frequency characteristic at the level 0.5 can be calculated in accordance with the expression which is obtained from (7.5.1), (7.5.2), (7.4.7):

$$f_{\phi 0,5\,\text{опт}} = \frac{2\Delta f_{\text{эфс}}}{\sqrt{\pi}} \ln\left[\frac{G_{\varphi c}(0)}{G_{\varphi y}(0)}\left(1 + 2\frac{G_{\varphi y}(0)}{G_{\varphi c}(0)}\right)\right]. \quad (7.5.3)$$

After computing $f_{\phi 0,5\,\text{опт}}$, it is easy to find the frequency of coupling the quasi-optimal filter of lower frequencies (inertia component/link)

$$f_{\phi\,\text{с нч опт}} = 0,6\,f_{\phi 0,5\,\text{нч опт}} = 0,6\,f_{\phi 0,5\,\text{опт}}.$$

The passband of the quasi-optimal matched filter, which consists of the inertia component/link, depends on the ratio of "signal" to the "interference", i.e., on $G_{\varphi c}(0)/G_{\varphi y}(0)$.

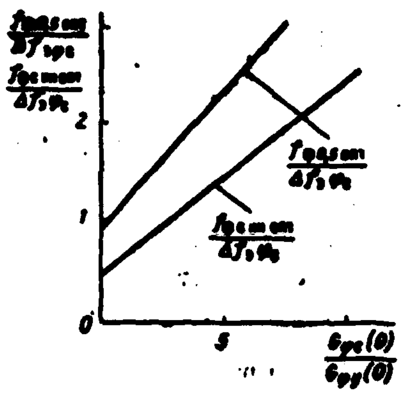

Fig. 7.5.2. Dependence of the band of filter on the interference level.

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The curves of dependence $f_{\phi.s\,opt}$ and $f_{\phi\,c\,n\,opt}$ on $\frac{G_{\varphi c}(0)}{G_{\varphi y}(0)}$ are given in Fig. 7.5.2.

With the work of equipment basic changes undergoes relation A_c/σ_n. Then $G_{\varphi c}(0)/G_{\varphi y}(\omega)$ it is possible to express through A_c/σ_n:

$$\frac{G_{\varphi c}(0)}{G_{\varphi y}(0)}=\frac{\sigma^2_{\varphi c}\Delta f_n A_c^2}{\Delta f_{\text{эфс}}\sigma_n^2}=\sigma^2_{\varphi c}\frac{\Delta f_n}{\Delta f_{\text{эфс}}}\,\frac{A_c^2}{\sigma_n^2}. \qquad (7.5.4)$$

Results for example examined earlier are given in Fig. 7.5.3, in which is given the dependence

$$\frac{f_{\phi\,c\,n\,opt}}{\Delta f_{\text{эфс}}} \text{ on } \frac{A_c}{\sigma_n}$$

with $\sigma_{\varphi}^{2}\frac{\Delta f_{n}}{\Delta f_{\text{эфф}}}=15.$

From the results it is evident that with the weak "signal" the band narrow, $f_{\text{фазмод}}$ composes approximately/exemplarily 0.6 from $\Delta f_{\text{эфф}}$; with an increase in the intensity of "signal", i.e., with increase $G_{\varphi s}(0)/G_{\varphi\varphi}(0)$ or $A_{0}/\sigma_{ш}$, occurs the expansion of band, and it can several times exceed $\Delta f_{\text{эфф}}$.

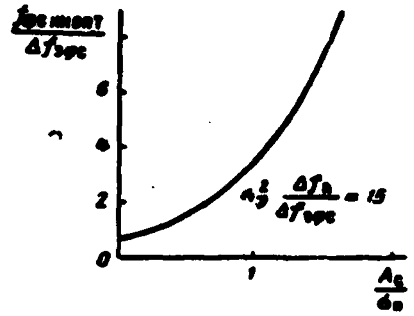

Fig. 7.5.3. Dependence of band on A_0/σ_n.

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The need for band change is the important factor, which limits the use of optimum modes/conditions with the phase tracking, since in this case is complicated filter and proves to be necessary introduction to the schematic of the difficultly feasible measuring device, which records the relationship/ratio between the interference and the signal. For this reason it is desirable to consider the possibility of using the filter with the previously selected fixed/recorded band, which will be done at the end of the present paragraph.

Let us consider the factors, which are determining following error the phase. Expression for error (7.4.11) with the optimum tracking is obtained earlier.

Error in the reproduction of phase can be divided into two parts - fluctuating of the interference and dynamic (7.4.9) and (7.4.10). If is known the frequency characteristic of filter $k_{\text{ф}}(\omega)$, we obtain

$$\sigma^2_{\delta\varphi\text{п}} = \frac{1}{2\pi}\int_0^\infty G_{\varphi y}(\omega)\,k^2_{\text{ф}}(\omega)\,d\omega, \qquad (7.5.5)$$

$$\sigma^2_{\delta\varphi\text{д}} = \sigma^2_{\varphi\text{с}} - \frac{1}{2\pi}\int_0^\infty G_{\varphi\text{с}}(\omega)\,k^2_{\text{ф}}(\omega)\,d\omega; \qquad (7.5.6)$$

It is thought that $k_{\text{ф}}(0)=1$, since phasemeter reacts to the phase, without changing its absolute value. If is used the quasi-optimal filter, made in the form of inertia component/link, or the optimum filter, approximated by inertia component/link, then calculations are simplified.

Then, bearing in mind that effective band width of interferences for the inertia component/link

$$\Delta\omega_{\text{ф э ш}} = \frac{\pi}{2}\omega_{\text{ф с ш}}, \qquad (7.5.7)$$

we will obtain

$$\sigma^2_{\delta\varphi\text{п}} = \frac{1}{2\pi}G_{\varphi y}(0)\frac{\pi}{2}\omega_{\text{ф с ш опт}} = \frac{G_{\varphi y}(0)\,\omega_{\text{ф с ш опт}}}{4} =$$

$$= 0{,}6\sqrt{\pi}\,\sigma_{\varphi\text{с}}\frac{G_{\varphi y}(0)}{G_{\varphi\text{с}}(0)}\ln\left\{\frac{G_{\varphi\text{с}}(0)}{G_{\varphi y}(0)}\left[1+2\frac{G_{\varphi y}(0)}{G_{\varphi\text{с}}(0)}\right]\right\}, \qquad (7.5.8)$$

$$\sigma^2_{\delta\varphi\text{д}} = \sigma^2_{\varphi\text{c}} - \frac{1}{2\pi}\int\limits_0^\infty G_{\varphi\text{c}}(0)\, e^{-\pi\left(\frac{\omega}{2\Delta\omega_{\text{эφc}}}\right)^2} \frac{d\omega}{1+\omega^2/\omega^2_{\phi\text{ c нн опт}}} =$$

$$= \sigma^2_{\varphi\text{c}} - G_{\varphi\text{c}}(0)\frac{1}{2\pi}\int\limits_0^\infty e^{-\pi\left(\frac{\omega}{2\Delta\omega_{\text{эφc}}}\right)^2} \frac{d\omega}{1+\frac{\omega^2}{\Delta\omega^2_{\text{эφc}}}\frac{\Delta\omega^2_{\text{эφc}}}{\omega_{\phi\text{ c нн опт}}}}.$$

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Let us designate

$$\alpha = \frac{\omega}{\Delta\omega_{\text{эφc}}},\quad \beta = \frac{\Delta\omega_{\text{эφc}}}{\omega_{\phi\text{ c нн опт}}}. \qquad (7.5.9)$$

then

$$\sigma^2_{\delta\varphi\text{д}} = \sigma^2_{\varphi\text{c}} - G_{\varphi\text{c}}(0)\frac{\Delta\omega_{\text{эφc}}}{2\pi}\int\limits_0^\infty e^{-\frac{\pi}{4}\alpha^2}\frac{d\alpha}{1+\alpha^2\beta^2} = \sigma^2_{\varphi\text{c}}[1 - I(\beta)],$$

where

$$I(\beta) = \int\limits_0^\infty e^{-\frac{\pi}{4}\alpha}\frac{1}{1+\alpha^2\beta^2}\,d\alpha = \frac{\pi}{2\beta}\left[1 - \frac{2}{\sqrt{\pi}}\int\limits_0^{\sqrt{\frac{1}{4\pi}}} e^{-x^2}dx\right] e^{\frac{\pi}{2\beta^2}}.$$

Dependence I(β) and 1-I(β) on β is given in Fig. 7.5.4. The given results make it possible to obtain relative dynamic error in general form

$$\frac{\sigma^2_{\delta\varphi\text{д}}}{\sigma^2_{\varphi\text{c}}} = 1 - I(\beta).$$

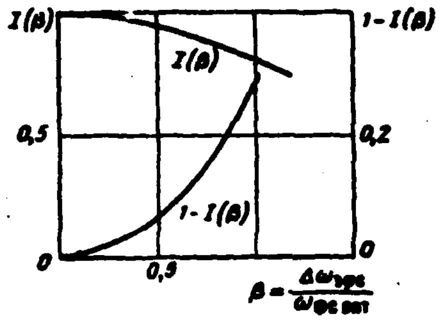

Fig. 7.5.4. Value of integral I(β).

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From 7.5.3 it follows that

$$\beta=\frac{\sqrt{\pi}\,0{,}3}{\ln\left[\frac{G_{\varphi c}(0)}{G_{\varphi y}(0)}\left(1+2\frac{G_{\varphi y}(0)}{G_{\varphi c}(0)}\right)\right]},$$

then

$$\frac{\sigma^2_{i\varphi д}}{\sigma^2_{\varphi c}}=1-I\left\{\frac{\sqrt{\pi}\,0{,}3}{\ln\left[\frac{G_{\varphi c}(0)}{G_{\varphi y}(0)}\left(1+2\frac{G_{\varphi y}(0)}{G_{\varphi c}(0)}\right)\right]}\right\} \quad (7.5.10)$$

The curve of dependence $\sigma^2_{i\varphi д}/\sigma^2_{\varphi c}$ on $G_{\varphi y}(0)/G_{\varphi c}(0)$ is given in Fig. 7.5.5.

After obtaining $\sigma^2_{i\varphi д}$ and $\sigma^2_{i\varphi ш}$, it is possible to find [on $\sigma_{i\varphi}$ (7.4.10)].

Using formulas (7.5.7), (7.5.9), (7.5.10) and (7.4.10) and by the curves, depicted in Fig. 7.5.4, 7.5.5 and 7.5.2, it is possible to fulfill the calculations of errors with the accuracy, sufficient for many practical purposes.

as can be seen from results, error in the optimum measurement of the changing phase depends on relation $G_{\varphi\nu}(0)/G_{\varphi c}(0)$, i.e. on the ratio of interference to the signal, and the statistical characteristics of randomly changing phase [7.5.4]. From a fundamental point of view it is useful to come to light/detect/expose the dependence of error with the optimum phase tracking on the energy of signal and jamming density.

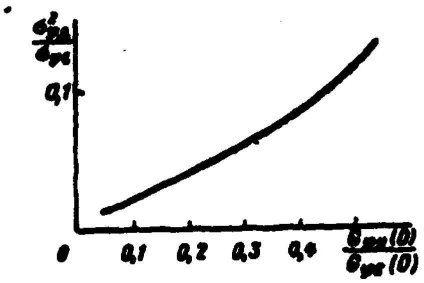

Fig. 7.5.5. Dependence of dynamic error on the interferences (with optimum band).

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Formula (7.5.7) can be converted as follows:

$$\sigma^2_{\phi\,\text{фш}} = \frac{1}{4}\frac{\sigma_n^2}{A_c^2}\frac{\omega_{\Phi\,\text{c вх опт}}}{\Delta f_n} = \frac{N_0\pi}{16\mathcal{E}_0}\frac{1}{\beta}, \qquad (7.5.11)$$

where $\mathcal{E}_0$ - energy of the element/cell of the signal, which has duration, equal to the interval of the correlation of the phase of signal.

But earlier it was shown that $\beta = \Delta\omega_{\text{эφс}}/\omega_{\Phi\,\text{с вх опт}}$ depends on $G_{\phi\text{c}}(0)/G_{\phi\text{y}}(0)$ (see 7.5.3), but

$$\frac{G_{\phi\text{c}}(0)}{G_{\phi\text{y}}(0)} = \frac{\sigma^2_{\phi\text{c}}}{\Delta f_{\text{эφс}}}\frac{A_c^2\Delta f_n}{\sigma_n^2} = \sigma^2_{\phi\text{c}}\frac{8\mathcal{E}_0}{N_0}. \qquad (7.5.12)$$

Consequently, relation $\frac{G_{\phi\text{c}}(0)}{G_{\phi\text{y}}(0)}$ depends also on $\mathcal{E}_0/N_0$.

Thus, in the final analysis fluctuating component of following error the phase depends on ratio $\mathcal{E}_0/N_0$ and $\sigma^2_{\varphi c}$. Let us consider from this point of view expression for $\sigma^2_{\varphi\Sigma}$ (7.5.10). Dynamic component of error depends on β, but β is determined by relation $G_{\varphi c}(0)/G_{\varphi y}(0)$, which, in turn, depends on $2\mathcal{E}_0/N_0$ and $\sigma^2_{\varphi c}$. Thus, dynamic component of error in the final analysis is also determined by ratio $\mathcal{E}_0/N_0$ and $\sigma^2_{\varphi c}$.

From the aforesaid it is possible to do important the conclusion that in the optimum measurement of the changing phase the basic factor, which are determining accuracy, is the relation of energy of the element/cell of signal $\mathcal{E}_0$, in the limits of duration of which the phase of signal can be considered constant, to the jamming density. For the optimum phase tracking is required the start of filter with the changing frequency characteristic, in the simplest case - with the changing band. In this connection are of interest questions about the criticality of band and form of the frequency characteristic of filter and about the possibility of using the filter with the fixed/recorded band.

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Examining the criticality of the band of filter, we can note that with divergence $\omega_{\text{ф ош}}$ from $\omega_{\text{ф ош опт}}$ must occur decrease of one both an increase in another component of error and an optimum it must be "washed away". Calculations can be fulfilled according to formulas

(7.5.7) and (7.5.8), after replacing $\omega_{\phi\,эк\,опт}$ by $\omega_{\phi\,опт}$. Fig. 7.5.6 gives the results of calculation for the case

$$G_{\varphi c}(0)/G_{\varphi y}(0)=5,\ \Delta\omega_{п}/\Delta\omega_{эφc}=100,\ \sigma^2_{\varphi c}=1.$$

From them it follows that the band of filter is not susceptible. This substantially facilitates the practical.realization of optimum filters, justifies the use of an approximation of the frequency characteristic of optimum filter by a frequency characteristic of inertia component/link and gives proof for applying the quasi-optimal low-pass filters. Examining a question about the use of a filter with the fixed/recorded band, it is necessary to have in mind that on some signal level and during the use of an optimum filter the errors become so/such considerable, that becomes meaningless the use of results. Usually in the practical problems it is possible to be assigned by maximum error $\sigma_{\text{пред}}$.

Optimum filter can be selected for this relationship/ratio between the signal and the interference, for which with the optimum band will be provided $\sigma_{\text{пред}}$ and in the process of functioning the parameters of filter changes do not undergo.

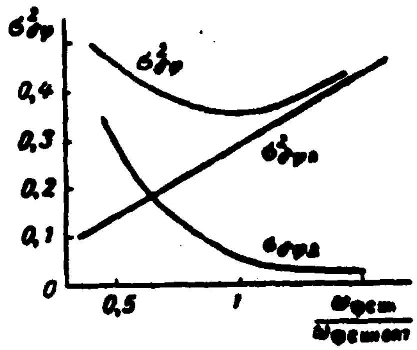

Fig. 7.5.6. Effect of band on the accuracy of the measurement of phase.

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It is obvious that in this case there will be the losses in the accuracy, since during an improvement in the relation signal/noise it would be possible to widen band and to reduce dynamic errors, and with a deterioration in the relation signal/noise to narrow down band and to attain smaller error due to the decrease of fluctuating errors.

However, the detailed examination of this question, which cannot be given due to the limited size of the book, it shows that in many instances the objective parameters of the accuracy of the measuring system of phase during the use of a filter with the fixed/recorded band deteriorate virtually insignificantly.

For the illustration of this position Fig. 7.5.7 gives changing the variance of error with a change in relation $G_{\varphi y}(0)/G_{\varphi c}(0)$ during the selection of optimum filter for the conditions when maximum variance of error comprises $\sigma^2_{\text{допуск}} = 0{,}25$. The remaining parameters correspond to the previous example. From the curves it follows that for the conditions of an example to in practice admissibly use the fixed/recorded band.

Thus, in a number of cases has the capability to use filters with the fixed/recorded band for the compromise solution of the problem of guaranteeing the minimum error of the reproduction of phase. In this case the practical execution of filter is simplified, since is not required to measure the relation signal/noise and to change the band of filter, but errors somewhat increase.

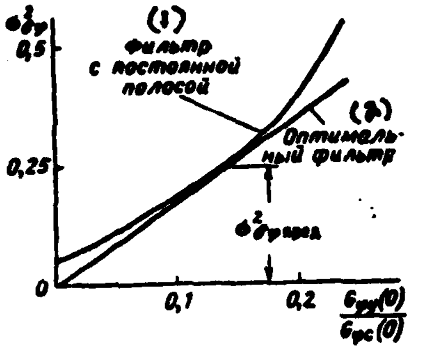

Fig. 7.5.7. Errors with the constant band.

Key: (1). Filter with the constant band. (2). Optimum filter.

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The obtained results show that it is possible to find the characteristics of optimum filter. The passband of such filters is commensurated with spectrum band of changes in the measured phase. Earlier it was noted, that in many phase systems this spectrum is comparatively narrow, which makes it possible to create narrow-band interference-free phase systems. However, the theoretical possibility of designing of narrow-band systems always cannot be realized.

Let us consider the basic versions of technical realization, close to the optimum ones, meters of variable/alternating phase. The basic versions are: a) direct-read phasemeter; b) the combination of matched filter and high speed phasemeter; c) phasemeter with the servo system; d) phasemeter with the connected at its output filter.

Versions a and b can ensure the optimum measurement of the changing phase in any ratios of interference to the signal. However, their practical realization meets many difficulties. In accordance with considerations presented above it is possible to fit the parameters of direct-read phasemeter, which works on the diagram, depicted in Fig. 7.3.2. The creation of narrow band does not cause in this case technical difficulties. Considerable difficulties appear with the creation of the copy of signal, especially with the serrated signals, and because of the need for the realization before the measurement of search and tracking the signal on the delay. Important deficiency/lack is also the difficulty of eliminating the dynamic errors with the standard determined dynamic effects and the difficulty of a precise execution of mathematical transformations (of fission mode and the taking of arc tangent). From the reasons presented this version did not find wide practical use/application.

Version with the matched filter has that special feature/peculiarity, that in it are imposed the minimum requirements on phasemeter. All basic difficulties of the optimum processing of mixture in this case are transferred to the matched filter. This causes many difficulties with the realization of such filters, if the spectrum, $G_{\varphi\varphi}(\omega)$ narrow-band.

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If radio signal is continuous, which occurs in some navigation aids and systems of trajectory measurements, then its spectrum is determined by modulation of phase due to the information. The spectrum of radio signal complicatedly depends on the spectrum of phase. Without examining these dependences, let us note that in the phase systems it usually proves to be narrow. The passband of matched filter must be narrow. To in practice carry out on the radio frequency a passband of the order of several ones hertz or with the portion of hertz is very difficult. Moreover requirements for the stability of the elements/cells of filter will be very rigid, since change in the tuning to 2% of the band gives the departure/attendance of phase approximately/exemplarily on 1°, which causes the appropriate instrument error. The creation of matched filter additionally is complicated, if is used serrated signal, for example in the form of burst of pulses. In this case the spectrum is widened in accordance with the duration of each pulse and consists of separate "teeth", which are located on the "distance" $\frac{1}{T_r}$ from each other where T_r - pulse repetition period. Width of band of "tooth" is determined with a finite number of impulses/momenta/pulses in the packet by the duration of packet, and with the continuous sequence of impulses/momenta/pulses - by spectrum of changes in the phase. Consequently, matched filter must have several narrow-band channels and very stringent requirements for the stability of the elements/cells of filters. On the reasons presented the version with the matched filters is also used rarely. The greatest use/application has version with the servo system, depicted in the diagram Fig. 7.5.8. By simple technical equipment - by a change in the parameters of servo system - reaches narrow band in the servo phasemeter (СлФИ). For the continuous signal during the selection of the passband of servo system in accordance with the given formulas will be provided the optimum measurement of the changing phase.

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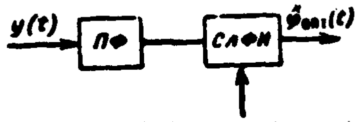

Fig. 7.5.8. Quasi-optimal diagram of phase tracking of the simple signal: ПФ - band-pass filter of radio signal; СлФИ - servo phasemeter.

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The filter for the radio signal (ПФ in the diagram Fig. 7.5.8), connected to the phasemeter, can have relatively broad band, which raises instrument/tool accuracy and simplifies its construction/design. From a theoretical point of view the ratio of interference to the signal at the entrance of phasemeter does not play role and error will be determined only by the ratio of energy of signal for the time, which corresponds to the interval of the correlation of phase, to the jamming density. However, it is virtually desirable to reduce the level of the latter for preventing the "clogging" of cascades/stages by interferences. With the serrated signals the use of the servo phasemeters is complicated. Reference stress/voltage, supplied to the phase discriminator of servo system, must have the same modulation on the amplitude and the phase as useful signal, with the guarantee of agreement of modulations on time. In certain cases it is expedient to use correlation-filtration diagrams. An example of this diagram for the burst of pulses is given in Fig. 7.5.9. In this diagram $C\Phi_1$ - the matched filter for processing of separate impulse/momentum/pulse, multiplier and generator of the sequence of video pulses realize a preliminary processing of mixture, and narrow band reaches in the servo phasemeter. The basic limitation of diagrams with the servo phasemeter is the fact that they put out the results, clcse to the optimum ones, until the fluctuations of readings/indications are insignificant.

Version with the selection after phasemeter is of interest during the use of reworking of phase measurements and has limitations, caused on the effect of suppression with the demodulation of phase, noted earlier.

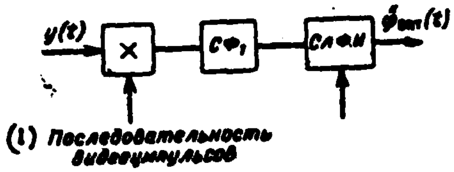

Fig. 7.5.9. The quasi-optimal diagram of phase tracking of the serrated signal: X - multiplier; $СФ_1$ - matched filter; СлФИ - servo phasemeter.

Key: (1). sequence of video pulses.

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From that presented previously it follows that are most simple in the technical solution the phasemeters with the servo system, with the appropriate selection of their band.

§ 7.6. Optimization of tracking with the transiency of the random process, which characterizes phase. In paragraph 7.2 were found some statistical characteristics of derivative of phase and phase for the cases when with the displacement/movement of object or change in the frequency information is embedded during the phase.

If the purpose of the work of system is the direct measurement of the deviations of the frequency (velocities), then for the tracking the frequency are used frequency discriminators. For the optimization of the measuring systems of the changing frequency proves to be valid the theory of optimization, presented in § 7.4, in this case is used the spectrum of the derivative of the fluctuations of phase, caused by the action of interferences.

More complicated for the analysis is the case, when the basis of the work of system is phase tracking. For the optimization of this system it is necessary to know the statistical characteristics of phase. These characteristics can be obtained from the statistical characteristics of its derivative. As it was shown in S 7.2, phase is characterized by unsteady random process.

This fact creates fundamental difficulties in the optimizations of such systems.

In the solved in S 7.4 problem optimization were assumed that the filter would ensure the simple reproduction of useful "signal".

In the unsteady phase it is necessary to find the optimum filter which in the presence of interferences reproduces phases (unsteady) with the minimum errors. Consequently, filter must not only minimize error, but also realize a conversion of random stationary process, in the case - integration in question.

At the output of filter will be obtained the function of time $\varphi_\phi(t)$.

Error in the reproduction of "signal" taking into account conversion will be expressed

$$\delta\varphi(t) = \varphi_\bullet(t) - 2\pi\int_0^t \Delta f_\bullet(t)\,dt. \qquad (7.6.1)$$

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Variance of error

$$\sigma^2_{\delta\varphi} = \lim_{T\to\infty}\frac{1}{T}\int_0^T\left[\varphi_\bullet(t) - 2\pi\int_0^t \Delta f_\bullet(t)\,dt\right]^2 dt. \qquad (7.6.2)$$

Integral equation (7.6.2) must be investigated in such a way as to come to light/detect/expose weight function $\eta_\bullet(t)$, minimizing dispersion $\sigma^2_{\delta\varphi}$. Mathematically the solution of this problem is considerably more complicated than for the filter with the simple reproduction of "signal", which was examined in S 7.4.

In connection with this it is expedient to lead the study of the quasi-optimal filter, in which with the assigned form of frequency characteristic is realized the selection of the passband, with which is provided obtaining the minimum resultant error, which consists of the dynamic and fluctuating (from the interferences) errors. this solution is simpler, gives demonstrative physical interpretation of results and in many instances corresponds to real requirements, since the selection of optimum band little is critical and it usually gives the results, close to the theoretically optimum ones.

It is necessary to note that the optimization of the system of tracking the unsteady phase cannot be solved with the help of the optimization of this system with the tracking derived phase. Sensing element with the phase tracking reacts to the phase of the mixture of signal and interference, and the error signal is determined by phase displacement between the reference voltage and the stress/voltage of mixture. The error signal is equal to zero at that moment/torque when phase displacement is equal to zero, although in this case the frequencies (i.e., the derivatives of complete phase) can differ from each other.

Consequently, during the optimization of tracking the unsteady phase it is necessary to investigate diagram, bearing in mind that it reacts to phase displacement.

In the case in question, just as in previous, is most expedient the use of phasemeters with the servo system. During the analysis of the optimization of the measurement of unsteady phase we will be oriented toward this version.

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The behavior of the linear servo system in the presence of the managing and perturbing effects can be analyzed with the help of the transfer functions and the frequency characteristics. We will use these known methods of the analysis of servo systems [7.2] for obtaining the expressions, which make it possible to find value (rms value) of the errors of dynamic ones and fluctuating in the dependence on the band and then to determine the band with which the accumulated error will be minimum. The interference, which distorts phase, is applied to the "entrance" of servo system, and its passage "on output", i.e., its effect on the result of measuring the phase, it will be determined by transfer function $\Phi_y(s)$ or complex frequency characteristic $\Phi_y(j\omega)$ of the control channel

$$\Phi_y(s)=\frac{N_\phi(s)}{1+N_\phi(s)},\quad \Phi_y(j\omega)=\frac{N_\phi(j\omega)}{1+N_\phi(j\omega)},$$

where s - complex number; $N_\phi(s)$ — transfer function; $N_\phi(j\omega)$ — complex frequency characteristic of servo system in the extended state.

Knowing the spectrum of the fluctuations of phase from interferences $G_{\varphi y}(\omega)$ it is possible to find spectrum and dispersion of fluctuations of readings/indications of phasemeter from the interferences

$$G_{\text{вуп}}(\omega)=|\Phi_y(j\omega)|^2 G_{\varphi y}(\omega),$$

$$\sigma^2_{\text{вуп}}=\frac{1}{2\pi}\int_0^\infty|\Phi_y(j\omega)|^2 G_{\varphi y}(\omega)\,d\omega.$$

For calculating the dynamic error it is possible to use the fact that in the servo system is a signal, which corresponds to error. It is developed the afterward being congruent/equating (subtracting) device/equipment (Fig. 7.6.1).

Dynamic error is determined from the transfer function by mistake $\Phi_{\text{ош}}(s)$

$$\Phi_{\text{ош}}(s)=\frac{1}{1+N_{\Phi}(s)}. \qquad (7.6.3)$$

With the study of dynamic errors for the random effects it is necessary to pass to the frequency characteristic on the error signal

$$\Phi_{\text{ош}}(j\omega)=\frac{1}{1+N_{\Phi}(j\omega)}$$

and its modulus/module

$$\Phi_{\text{ош}}(\omega)=|\Phi_{\text{ош}}(j\omega)|. \qquad (7.6.4)$$

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It is known that

$$G_{\delta\varphi\text{д}}(\omega)=[\Phi_{\text{ош}}(\omega)]^2 G_{\varphi c}(\omega), \qquad (7.6.5)$$

where $G_{\delta\varphi\text{д}}(\omega)$— the energy spectrum of dynamic error; $G_{\varphi c}(\omega)$— the energy spectrum of changes in the phase.

This relationship/ratio cannot be used, since in our case phase is unsteady and its spectrum $G_{\varphi c}(\omega)$ is changed in the time. Since in unsteady phase $G_{\varphi_c}(\omega=0)$ it has infinite values, from (7.6.5) it follows that $G_{\delta\varphi}(\omega=0)$ so can have infinite values, i.e., error will be characterized by unsteady random process.

Let us examine the conditions, with which dynamic error will be expressed by stationary random process. In general form of Laplace's image the effects [changing phase $\varphi_c(s)$] and response [error $\delta\varphi(s)$] are connected with the relationship/ratio

$$\delta\varphi(s)=\Phi_{om}(s)\varphi_c(s)$$

or

$$\delta\varphi(s)=\frac{\Phi_{om}(s)}{s}\varphi_c(s)s, \qquad (7.6.6)$$

where $\varphi_c(s)s$ — image of the derivative of the effect: $\frac{\Phi_{om}(s)}{s}$ — the transfer function, which connects the image of the derivative of effect and the image of error.

The obtained expression makes it possible to find the image of error not from the image of effect, but from the image of its derivative.

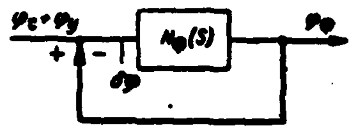

Fig. 7.6.1. Schematic of servo system.

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If we pass to the frequency characteristics, then it is possible to find the spectrum of response (error) from the spectrum of the derivative of effect and the changed frequency characteristic on the error signal

$$\delta\varphi_{д}(j\omega)=\frac{\Phi_{ош}(j\omega)}{j\omega}\varphi_{c}(j\omega)\,j\omega.$$

During the analysis of random effects it is necessary from Fourier's spectrum to pass to the energy spectrum.

Then

$$G_{\delta\varphi д}(\omega)=\left|\frac{\Phi_{ош}(j\omega)}{j\omega}\right|^{2}G_{\Delta fc}(\omega). \qquad (7.6.7)$$

During the calculation of the spectrum of response (error) of the spectrum of the derivative of effect transfer function and frequency characteristic endure fundamental changes.

In the initial transfer function of the error signal is supplemented the factor 1/s, while to the complex frequency characteristic factor $1/j\omega$.

The frequency characteristic according to which it is possible to compute the spectrum of error, if is known the spectrum of the derivative of effect, it will take the form

$$|\Phi'_{om}(j\omega)| = \left|\Phi_{om}(j\omega)\frac{1}{j\omega}\right|. \qquad (7.6.8)$$

If effect is characterized by the finite value of the energy spectrum of derivative at the zero frequencies, then the integrating factor will stipulate that the fact that the error will be unsteady random process with the increasing dispersion. Consequently, the first space of optimization must be the determination of the conditions under which the derivative of effect with the finite value of energy spectrum at the zero frequencies, will not give the increasing on the time dynamic error. It is obvious, this is determined by the form of transfer functions $\Phi_{om}(s)$ and $N_{\phi}(s)$.

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The transfer function of system in the extended state can contain the integrating factors

$$N_{\phi}(s) = \frac{R(s)}{s^{\nu}D(s)}, \qquad (7.6.9)$$

where R(s) and D(s) - the polynomials: p - degree of astaticism of system.

For static system (p=0)

$$\Phi_{om}(s)=\frac{1}{s^{0}}\frac{D(s)}{D(s)+R(s)}. \qquad (7.6.10)$$

In this case in (7.6.8) is an integrating factor, and the optimization of system is impossible.

For astatic system (p=1),

$$N_{\phi}(s)=\frac{R(s)}{sD(s)},$$

$$\Phi_{om}(s)=\frac{sD(s)}{sD(s)+R(s)}$$

and

$$\Phi'_{om}(s)=\frac{D(s)}{D(s)s+R(s)}. \qquad (7.6.11)$$

In this case the integrating factor vanishes and the optimization of system it proves to be possible. The physical sense of this result consists of the following. If the spectrum of changes in the frequency is final at the zero frequencies and the random process of changing the phase is nonstationary, then, by using a static system (for the phase), it is not possible to obtain the minimum of error, since transient effect, i.e., unsteady phase at the entrance, will cause error with increasing dispersion.

In the astatic system unsteady change of the phase at the entrance does not give the increasing error, since system is astatic and error is caused not by the value of effect, but by its change.

All further actions we will undertake, keeping in mind astatic system.

Let us find the energy spectrum of the dynamic error

$$G_{t\varphi д}(\omega)=G_{\Delta f c}(\omega)\,|\Phi'_{ош}(j\omega)|^2=G_{\Delta f c}(\omega)\left|\frac{D(j\omega)}{R(j\omega)+j\omega D(j\omega)}\right|. \tag{7.6.12}$$

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As can be seen from the obtained result, in the astatic system at all frequencies the intensity of the energy spectrum of error is final and dispersion $\sigma^2_{t\varphi д}$ so must have finite quantity.

For facilitating the understanding of some special features/peculiarities of the optimization of such systems we convert the obtained expression

$$\Phi_{ош}(j\omega)=\Phi_{y}(j\omega)\,\frac{1}{N_{\phi}(j\omega)},$$

then

$$\frac{\Phi_{\text{ош}}(j\omega)}{\xi/\omega}=\frac{\Phi_{y}(j\omega)}{j\omega N_{\phi}(j\omega)},$$

$$G_{\text{ϕд}}(\omega)=G_{\Delta fc}(\omega)\left|\frac{\Phi_{y}(j\omega)}{j\omega N_{\phi}(j\omega)}\right|^{2}. \qquad (7.6.13)$$

Formula (7.6.13) can be used also for calculating the energy spectrum of error with to steady state.

In the steady state the energy spectrum of derivative $G_{\dot{\varphi}c}(\omega)$ has at the zero frequencies zero intensity, error in this case can be final at all frequencies and in the static system.

It is important to emphasize that the spectrum of dynamic error will be different in the cases of stationary and unsteady phase with the same form of the frequency system characteristics in the locked state, i.e., with identical $\Phi_{y}(j\omega)$ and $N_{\phi}(j\omega)$.

This difference depends on the form $G_{\dot{\varphi}c}(\omega)$ for the stationary and $G_{\Delta fc}(\omega)$ for the unsteady phase essentially differs from each other. Here and subsequently $G_{\Delta fc}(\omega)$— the spectrum of changes of the frequency: $G_{\dot{\varphi}c}(\omega)$— the spectrum of the derived phase: $\omega_c=2\pi\Delta f_c$; from these expressions follows that in the steady state initial is the random process of changing the phase, and in the unsteady phase - process of changing the frequency.

In order to compare the spectra of errors, let us consider an example.

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Let us assume that

$$N_{\phi}(j\omega)=\frac{N_{\phi 0}}{j\omega},$$

i.e., in the extended state the system is led to the integrating component/link, what is first approximation for many real systems: then complex frequency characteristic along the control channel will take the form:

$$\Phi_y(j\omega)=\frac{1}{j\omega/N_{\phi 0}+1}. \qquad (7.6.14)$$

$$|\Phi_y(j\omega)|^2=\Phi_y^2(\omega)=\frac{1}{\omega^2/N_{\phi 0}^2+1}; \quad N_{\phi}(j\omega)\,j\omega=N_{\phi 0},$$

then

$$\left|\frac{\Phi_y(j\omega)}{j\omega N_{\phi}(j\omega)}\right|^2=\frac{1}{N_{\phi 0}^2}\,\frac{1}{1+\omega^2/N_{\phi 0}^2}=\frac{1}{N_{\phi 0}^2+\omega^2}. \qquad (7.6.15)$$

The frequency characteristics of the system of phase tracking in question in the locked state is similar to frequency characteristic of inertial component/link with the amplification, equal to one, and by the inertness, which depends on amplification $N_{\phi 0}$ in the extended state. The frequency system characteristics on the error signal for the derived phase analogous, but amplification is expressed by relation $1/N_{\Phi 0}$.

After assigning the gaussian models of the spectrum of the phase (for the steady state) and the spectrum of the deviations of the frequency (for the unsteady phase) and using the obtained expressions for the frequency characteristics, it is possible to calculate the energy spectra of dynamic error. The results of calculation are given in Fig. 7.6.2 (a - for the steady state; b - for the unsteady phase).

As is evident, the spectra of dynamic error for two cases in question strongly differ. On these reasons the form of the characteristic of filter and its band will differently affect dynamic error.

In the servo systems to manage the form of frequency characteristic in the locked state along the channel of control $[\dot{\Phi}_y(\omega)]$ is difficult, since system it is possible to have a tendency toward the instability.

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We will use now expression (7.6.13) for obtaining the relationships/ratios, which make it possible to find the optimum band of the servo system, which performs the role of filter.

In these reasons the form of the frequency characteristic of filter must be close to the form of the frequency characteristic of inertia component/link and during the optimization it is desirable to be limited to the selection of the filter pass band (system), bearing in mind that in the extended state the system contains the integrating component/link. The modulus/module of frequency characteristic will be equal to

$$|\Phi_y(j\omega)| = \frac{1}{\sqrt{\omega^2/\omega^2_{\phi c m} + 1}},$$

where $\omega_{\phi c m}$ — frequency of coupling the inertia component/link to which is given the servo system in the locked state.

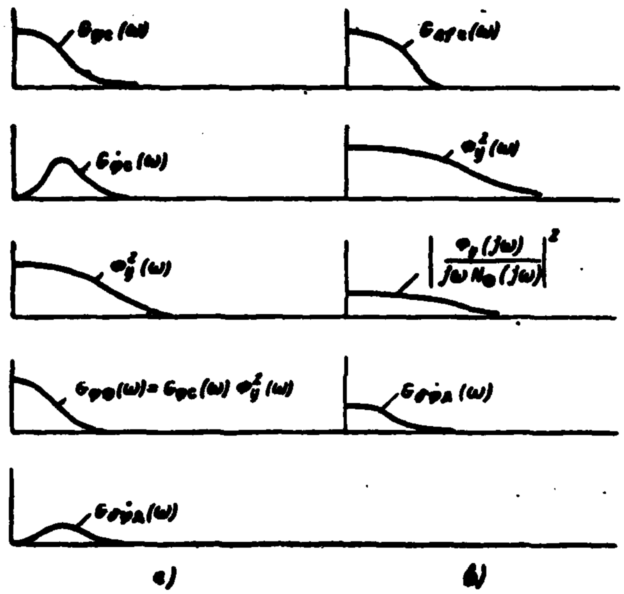

Fig. 7.6.2. Energy spectra: a) for steady state; b) for unsteady phase.

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Then

$$\left|\frac{\Phi_y(j\omega)}{j\omega N_\phi(j\omega)}\right|^2=\frac{1}{\omega^2_{\phi c m}}\,\frac{1}{\left(\frac{\omega^2}{\omega^2_{\phi c m}}+1\right)}.$$

After substituting in (7.6.13) and after carrying out integration of energy spectrum for all frequencies, we will obtain the dispersion of the dynamic error

$$\sigma^2_{\delta\varphi A}=\frac{1}{2\pi}\int_0^\infty G_{\Delta fc}(\omega)\frac{1}{\omega^2_{\phi c m}+\omega^2}\,d\omega. \qquad (7.6.16)$$

With the approximation of the spectrum by gaussian model we will obtain

$$\sigma^2_{\delta\varphi A} = \frac{G_{\Delta f c}(0)}{2\pi} \int_0^\infty e^{-\pi/4\left(\frac{\omega}{\Delta\omega_{\vartheta\Delta f c}}\right)^2} \frac{d\omega}{\omega^2 + \omega^2_{\phi c m}}. \quad (7.6.17)$$

For facilitating the calculations it is possible to switch over to dimensionless quantities. For this we convert expression (7.6.17), after introducing variable/alternating $a = \frac{\omega}{\Delta\omega_{\vartheta\Delta f c}}$ and parameter $\beta = \Delta\omega_{\vartheta\Delta f c}/\omega_{\phi c m}$:

$$\sigma^2_{\delta\varphi A} = \frac{G_{\Delta f c}(0)\,\Delta\omega_{\vartheta\Delta f c}}{\omega^2_{\phi c m} 2\pi} \int_0^\infty e^{-\frac{\pi}{4}a^2} \frac{1}{1 + a^2\beta^2}\, da. \quad (7.6.18)$$

The integral, entering the expression, was computed earlier. The graph, on which is determined the value of this integral, is given in Fig. 7.5.4; then

$$\sigma^2_{\delta\varphi A} = \frac{G_{\Delta f c}(0)\,\Delta\omega_{\vartheta\Delta f c}}{2\pi\omega^2_{\phi c m}} I(\beta) = \frac{\sigma^2_{\Delta f c}}{\Delta\omega^2_{\vartheta\Delta f c}} \beta^2 I(\beta).$$

Coefficient $\sigma^2_{\Delta f c}/\Delta\omega^2_{\vartheta\Delta f c}$ characterizes the special feature/peculiarity of the spectrum of changes (divergences) in the frequency: $\beta^2 I(\beta)$ it depends on the relationship/ratio between the band of the filter of servo system and the width of the spectrum of changes in the frequency.

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Variance of error from the interferences is equal to

$$\sigma^2_{\text{ɜφп}} = \frac{G_{\varphi п}(0)}{2\pi} \frac{\pi}{2} \omega_{\phi\,c\,пп} = \frac{\sigma_п^2}{A_c^2} \frac{f_{\phi\,c\,пп}}{\Delta f_п} \frac{\pi}{2} =$$

$$= \frac{\sigma_п^2}{A_c^2} \frac{\pi}{2} \frac{\Delta\omega_{\text{ɜ}\Delta f c}}{\Delta\omega_п} \frac{1}{\beta}.$$

Coefficient $\sigma_п^2/A_c^2 \frac{\Delta\omega_{\text{ɜ}\Delta f c}}{\Delta\omega_п} \frac{\pi}{2}$ characterizes interference spectrum $\Delta\omega_п$ and their intensity.

Fig. 7.6.3 gives the dependences $\beta^2 I(\beta)$ and $1/\beta$ on $1/\beta$, on which it is possible simply to construct curves for the different values of coefficients and to find the optimum band of filter.

For an example Fig. 7.6.4 gives error functions, resultant error and optimum band for the case

$$\sigma^2_{\Delta f c} = 1 \frac{1}{сек^2}, \ \Delta\omega_{\text{ɜ}\Delta f c} = 1, \ \Delta\omega_п = 100,$$

$$A_c/\sigma_п = 0{,}22.$$

Then

$$\frac{\sigma_п^2}{A_c^2} \frac{\Delta\omega_{\text{ɜ}\Delta f c}}{\Delta\omega_п} = 0{,}2; \ \frac{\sigma^2_{\Delta f c}}{\Delta\omega_{\text{ɜ}\Delta f c}} = 1.$$

In the example in question the optimum filter pass band is 2.5 rad/s, i.e., is approximately/exemplarily three times wider than $\Delta\omega_{\text{ɜ}\Delta f c}$.

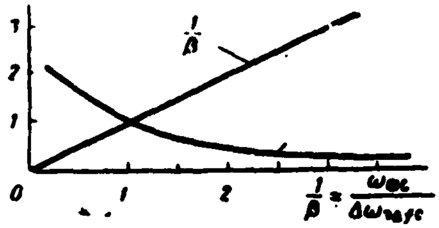

Fig. 7.6.3. Effect of band on components of errors.

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With the decrease of relative interference level the band of filter will be widened, with an increase - become narrow.

Thus, in the unsteady phase it is possible to optimize phasemeter. The passband of this meter is commensurated with $\Delta\omega_{зб/с}$; in the phase systems it it is frequently narrow, which gives the possibility to create narrow-band interference-free phase systems.

S 7.7. Measurement of the changing phase difference of two signals. In the phase systems is encountered the case, when it is necessary to realize a measurement of a phase difference of two signals $\Delta\varphi_c=\varphi_{c1}-\varphi_{c2}$ accepted.The measurement of a phase difference of two signals has some special features/peculiarities which in essence consist in the fact that the distribution functions and the energy spectrum of a phase difference of signals can differ from the appropriate functions of the phase of signals and each of the signals a phase difference of which is measured, is accepted against the background of interferences, i.e., their phase under the action of interferences fluctuates. Interferences in each of the channels can be independent variables or correlated. In connection with that presented in general form the theory of the two-channel phase systems requires separate examination.

However, some cases, which have vital importance for the practice, can be examined on the basis of methods presented here.

In S 7.2 were explained the conditions, with which the energy spectrum of the phase difference being subject to measurement of two signals can be easily obtained by the addition of the energy spectra of changes in the phase of each of them.

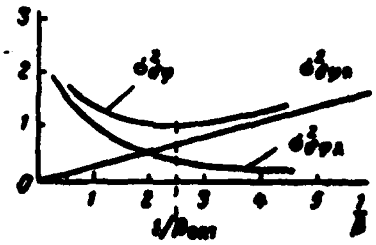

Fig. 7.6.4. Example error function.

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In this case the optimum measuring circuit of a phase difference can be constructed during the optimum measurement of the phase of each of the signals relative to supporting/reference, that has any phase, and the subsequent calculation of difference. The corresponding diagram is given in Fig. 7.7.1. Earlier it was established/installed, which optimum phasemeters is technically most expedient to construct with the use of servo systems. If phases $\varphi_{c1}(t)$ and $\varphi_{c2}(t)$ are unsteady, then servo systems must have astaticism first-order minimum.

Using methodology presented earlier, it is possible to find the optimum parameters of each of the meters, for example the optimum passband of servo systems, and variance of error.

If phases $\varphi_{c1}(t)$ and $\varphi_{c2}(t)$ essence the independent random processes, and the interferences, which function in each of the channels, then are not dependent, i.e., are not dependent the fluctuations of phase under the action of interferences $\varphi_{v1}(t)$ and $\varphi_{v2}(t)$, then the dispersion of resultant error will be equal to sum the dispersion of the resulting measuring error in each of the channels.

The dispersion of dynamic errors and fluctuation errors from the interferences also they are summarized.

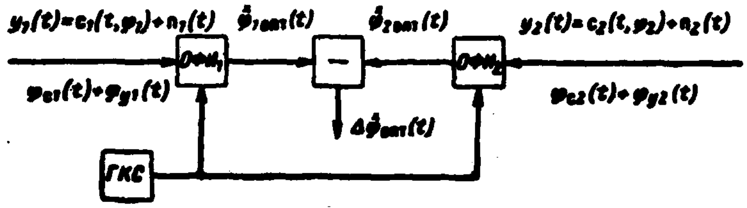

Fig. 7.7.1. Optimum measuring circuit of phase. difference of two signals: ОФИ - optimum phasemeters; ГКС - generator of copy of signals.

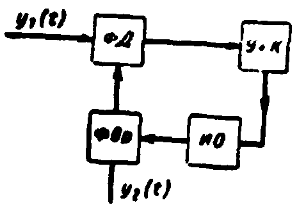

Fig. 7.7.2. Quasi-optimal measuring circuit of phase difference of two signals: ФД - phase discriminator; Фвр - phase inverter; ИО - actuating element; У+К - amplifier and correcting terms.

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Consequently, methods presented earlier and results can be used during the analysis of some versions of two-channel phase systems.

It is possible to create the meter of a phase difference with the servo system in which as the supporting/reference is applied one of the signals. The diagram of this meter is given in Fig. 7.7.2. The reading of the position of phase inverter is used for evaluating the phase difference

$$y_1(t)=A_{y1}(t)\cos[\omega_0 t+\varphi_{c1}+\varphi_{y1}(t)];$$
$$y_2(t)=A_{y2}(t)\cos[\omega_0 t+\varphi_{c2}+\varphi_{y2}(t)].$$

After the phase discriminator we will obtain

$$0{,}5A_{y1}(t)\,A_{y2}(t)\cos[(\varphi_{c1}-\varphi_{c2})+\varphi_{y1}(t)-\varphi_{y2}(t)],$$

if we disregard/neglect fluctuations of phase in the phase inverter in comparison with the fluctuations of phase in the mixture.

Consequently, diagram works analogously to that examined earlier.

The fluctuations of a phase difference are determined by the properties of the random process

$$\Delta\varphi_y(t)=\varphi_{y1}(t)+\varphi_{y2}(t).$$

If $\varphi_{y1}(t)$ and $\varphi_{y2}(t)$ are distributed according to the normal law, then this distribution is retained also for $\Delta\varphi_y(t)$.

The dispersion of the fluctuations of a phase difference will be equal to the sum of the dispersions

$$\sigma^2_{\Delta\varphi}=\sigma^2_{\varphi1}+\sigma^2_{\varphi2}.$$

Upon the identical dispersions it doubles. It is obvious that for this servo system also it is possible to find optimum passband, using the methodology, presented earlier. For this it is necessary to know the distribution function, the energy spectrum of a phase difference and interference level.

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