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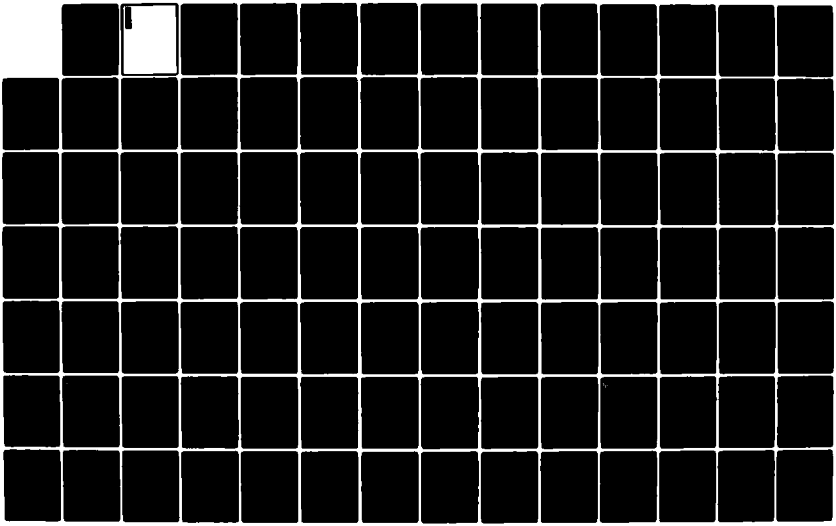

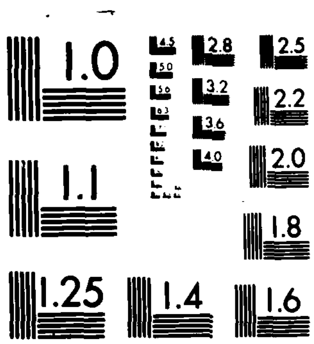

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This report describes the findings of the JPL regarding ELINT correlation algorithms used in BETA, TCATA ELINT, ITEP, QUICKLOOK and AGTELIS systems. The statistical mathematics on which the algorithms are based is examined with particular reference to assumptions. Individual algorithms are analyzed to determine whether they are performing their functions properly. Algorithms that perform the same function in different systems are compared to determine which ones are best according to various criteria.

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JET PROPULSION LABORATORY
California Institute of Technology
Pasadena, California

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Correlation Algorithm Report

1. INTRODUCTION

1.1. Purpose

This report* describes the findings of the Algorithm Analysis Subtask group working on the U.S. Army Intelligence Center and School (USAICS) Software Analysis and Management System task (USAMS) regarding Electronic Intelligence (ELINT) correlation algorithms used in five of the intelligence-gathering systems under USAICS cognizance. The statistical mathematics on which the algorithms are based is examined with particular reference to assumptions. Individual algorithms are analyzed to determine whether they are performing their functions properly. Algorithms that perform the same function in different systems are compared to determine which ones are best according to various criteria.

The algorithms examined in this report are taken from the BETA, TCATA ELINT, ITEP, QUICKLOOK, and AGTELIS systems. They were chosen from the more than 40 deployed intelligence systems for which USAICS is Combat Developer because some documentation was available and because they represented a range of ELINT applications. The ELINT correlation algorithms have been chosen since they are most nearly automatic, that is, require the least operator intervention and rely on technical parameters most amenable to statistical techniques.

1.2. Background

Each of the more than 40 intelligence systems under USAICS cognizance employs several types of algorithms to carry out its gathering and processing of intelligence data. Two important types of these algorithms, geographic transformation and correlation, have been chosen for analysis during this year. The former translates grid-zone locations, for example, from latitude-longitude to Universal Transverse Mercator (UTM), while the latter resolves many individual sitings into militarily recognizable targets and situation reports based chiefly on standard statistical procedures. It is important to develop a set

*A report on geographical transformation algorithms has been submitted in FY82 and a report on possible algorithm analysis methodologies is scheduled for FY83.

of parameters to characterize these algorithms to determine how they should be catalogued. When these activities are completed, it becomes possible to compare algorithms that perform the same function in different systems and finally to develop improved algorithms that perform the function.

For this report the JPL Algorithm Analysis Subtask group has examined ELINT radar correlation algorithms for five of the systems under USAICS cognizance, namely Battlefield Exploitation and Target Acquisition (BETA), TRADOC Combined Arms Training Activity (TCATA) ELINT Processor, Interim Tactical ELINT Processor (ITEP), Airborne Non-Communication Emitter Location and Identification System (Quicklook), and Automated Ground Transportable Emitter Location and Identification System (AGTELIS). BETA is a Test Bed program for correlating data received from several types of sensor systems and making target nominations. Both automatic correlation and aggregation techniques and interactive graphics are used in the operator's analysis. The TCATA ELINT Processor and ITEP are similar data analysis tools that integrate many sitings into various intelligence reports. AGTELIS and Quicklook are both collection and analysis systems; as collection systems they do not integrate data from as wide a range of sensor systems as do the others. These systems would generally be employed at Brigade through Corps level or at an Air Force Tactical Air Control Element (TACE) or Allied Tactical Air Force (ATAF); target nominations and tactical situation reports would be available to commanders and their staffs from Brigade through Echelons Above Corps (EAC).

USAICS has cognizance of a large number of algorithms integral to intelligence-gathering systems in various stages of development and deployment. The state of "deployment" of algorithms in the USAICS inventory ranges from that of products of research contracts not yet implemented in any system to those in fielded systems such as Quicklook. In the latter systems the algorithms are documented in design documents (narrative English and equations), and/or in machine readable design language, and in code. Often not all of these forms of documentation are available for any one system. For research algorithms not yet implemented, actual code, or even detailed flow charts, may not be available, and analysis must rely solely on mathematical descriptions.

"Algorithms" will mean any set of rules for carrying out a single conceptual operation on a set of data, such as transforming from latitude and longitude to UTM coordinates or determining a position from a number of direction measurements taken at known points.** Algorithms are often hierarchical, lower-level algorithms being used to describe higher-level algorithms and thereby illuminating the underlying logical structure. Thus results from one algorithm may be data for another. This occurs extensively for the correlation algorithms, and correctly identifying the assumptions made in linking the hierarchical levels is critical. USAICS is interested in algorithms performing intelligence data processing functions central to its systems' missions and those performing crucial support functions common to a number of systems such as geographic location. Data management or mathematical function algorithms, although vital to the efficient functioning of the systems, are not being treated in these first algorithm analyses.

1.3. User Benefits

These analyses can benefit users in several ways. First, a catalogue of existing algorithms will help USAICS avoid having algorithms redeveloped for new systems from first principles. Second, analysis of individual algorithms may, in a few cases, identify deficiencies worth correcting on the next system revision. Third, and most important, the comparison of algorithms performing the same function in different systems can lead to identifying guidelines for developing and/or selecting algorithms to include in new and revised systems. Selected algorithms from the systems studied will begin to form a library of intelligence algorithms with associated computer subroutines that will be analogous to the Collected Algorithms of the Association for Computing Machinery (ACM). The creation of such a library is in the spirit of Ada+, the Department of Defense language for embedded systems, and the Ada environment.

1.4. High-Level Schematics

There are several steps in identifying enemy locations using electronic intelligence methods. These steps are arranged in a hierarchy beginning at the bottom with observing and estimating emitter characteristics and moving

---

**These conceptual models can be presented simply and logically, but the presentation of their technical implementation is often significantly more complicated to present.

+Ada is a trademark of the Department of Defense.

through successive levels of data integration. Often one of several assumptions concerning the behavior of these observations can be chosen when moving from one level of the hierarchy to the next. How each level is modeled and what analytical techniques are chosen depend on these assumptions. Later in this report the assumptions behind some developed systems will be discussed. This section introduces a framework for the entire hierarchy, providing a context for the more detailed and technical discussions to follow.

There are four stages in the automatic processing of ELINT data as implemented in most current systems: collection, separation, self-correlation, and cross-correlation. These are illustrated schematically in Figures 1-1 and 1-2.

The first step in this "automatic" radar target acquisition and nomination process, called collection, gathers lines-of-bearing and signal parametrics associated with different emitters. Examples of signal parametrics normally collected are radar frequency, pulse repetition interval, and pulse width. During collection, error in the estimates of enemy radar location and signal parametrics enters, primarily through measurement error. Understanding this error is vital to understanding self-correlation. The different assumptions about the behavior of this error made will be discussed in detail, but the actual mathematics of collection will be deferred to a later report.

The second step, called separation, identifies which observations come from which emitters. A subset of observations is thus identified for each emitter. Each subset is a sample from the population of all possible observations of that radar by a sensor system. If the sensors are unbiased, they will gather samples whose averages will estimate the true radar characteristics. (Because the sensors are unbiased, the true radar characteristics and the mean of all possible observations are identical). The sample variance can be used to estimate the measurement error. Separating one radar's sample out of the collected observations and determining the mean and elliptical error probable (EEP) is usually called determining the "fix" for a radar. How well the observations are separated according to their populations depends on the density of the radars, the accuracy of the sensor system for both locations and signal parametrics, and the statistical techniques chosen. Examples of statistical techniques used are jackknifing and sequential searching.

Fig. 1-1: Schematic Representation of Model Hierarchy

Target Nominations

↑ Database of ''identified'' complex entities (e.g. DIV CPs)

Cross-Correlation Algorithms ←—— Templates

↑ Database of ''identified'' simple entities

Self-Correlation Algorithms

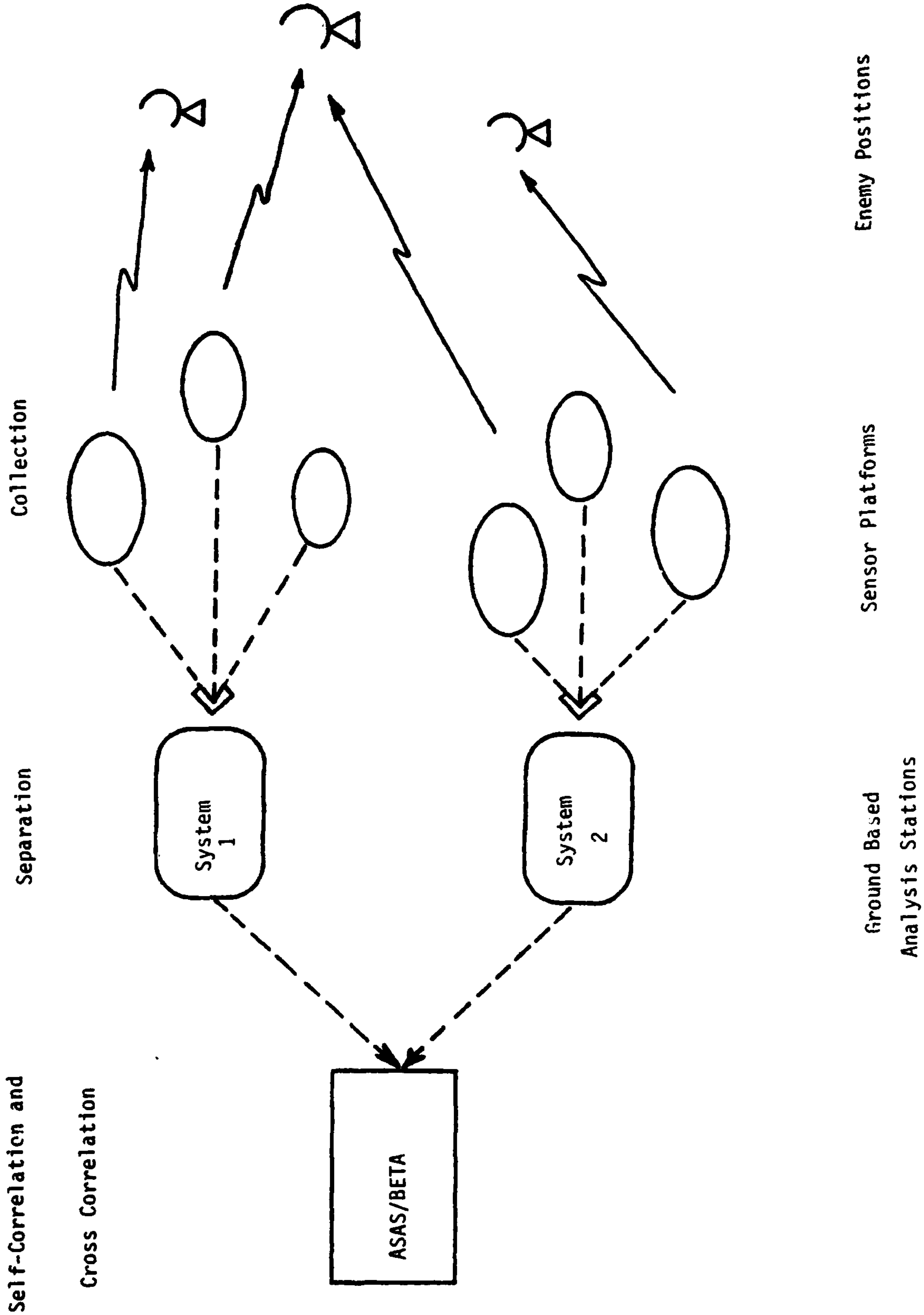
↑ candidate simple entities (e.g. radios)

Separation Algorithms

↑ Observations

Collection Algorithms ←—— Ground Truth

Fig. 1-2: Correlation Algorithms



Jackknifing determines if a subset of observations is internally consistent by evaluating how closely each of the observations can be estimated using the rest of the observations. To do this, for each observation x the location estimate of the remaining observations is calculated and the difference between its line-of-bearing and that of x is determined. If for every x in the subset this difference is less than a prespecified value, the subset is consistent and considered as coming from the same emitter.

Sequential searching usually involves taking all observations falling within some distance of, or somehow "clustering" with, another observation and determining if they are dense enough to be considered as coming from one emitter. This density may be determined by calculating the variance from the best location estimate. This process is then repeated for other points. Special rules for computing any given fix are usually applied to ensure observations are made from different locations.

Although no new random error is introduced in separation, other error is introduced if separation is not done correctly. Both jackknifing and sequential searching may fail under reasonable conditions, in which case emitter locations or signal parametrics estimates based on them may be "phantoms" and not represent existing radar characteristics. Not only can this provide a false target, but it will simultaneously hide at least one true one. Thus separation shapes much of the input to the self-correlation phase, and will be considered in its own right in another report.

The third step, and the focus of this report, is called self-correlation. Candidate radars, specified by either directly collected lines-of-bearing and signal parametrics or their estimates coming from separation, are compared with "known" radars. If the new information seems to refer to a radar already identified, it is used to refine what is known about that radar location. Many self-correlation algorithms also try to determine if the candidate represents observations of a "known" radar which has moved or one which has shifted its signal parametrics logically. A candidate which cannot be associated with any radar in the database is considered a "new" radar and is added to the database. In this manner the database of "known" radars is built; the first candidate is new by default (since the database is empty), the second is either successfully associated with the first or is added as a second database entry, and so on. When a database entry is sufficiently refined, or supported by enough candidates, it can become a target nomination.

Rules for associating candidates with database entries vary from system to system. They usually assume that location estimates are normally distributed and signal parametrics either fall between intelligence-specified bounds or have some statistical distribution whose parameters can be estimated. All systems analyzed to this point seem to assume that all measured attributes are independent. Some systems use a final "measure of correlation" for association which combines the various measures using subjective weighting factors.

The final step in this intelligence analysis, called cross-correlation, identifies more complex entities, such as Division Command Posts with the known simple entities of which they are composed. These simple entities are provided by previous processing. Either the simple or the complex entity may be the new candidate. The measures of association used rely on information contained in templates. These templates, based on intelligence estimates, indicate what specific complex targets should look like. The problems arising in cross-correlation will be studied in another report.

Turning now to a slightly more detailed view of self-correlation, figure 1-2 concentrates on its interface with separation, illustrating the mathematical assumptions made to tie the system together. These assumptions prescribe which mathematical techniques can be used for building the algorithms.

Two levels of assumptions underlie the distributions used for location and signal parametrics in the self-correlation algorithms. The first is that all observations used to calculate an estimate were made of the same emitter. Ground truth, the array of emitters with which these sensor system must actually cope, includes multiple emitters. Thus, either the separation algorithm itself or the operational capability of the collection sensor system is assumed to classify observations accurately, differentiating those coming from separate emitters.

The second level of assumptions deals with the shape of the distributions of the radar characteristics and the statistical independence of the

Fig. 1-3: General Mathematical Assumptions for Self-Correlation

ESTIMATE OF GROUND TRUTH

↑

SELF-CORRELATION ALGORITHMS

↑

- o locations normally distributed, sample drawn from a population centered on the true mean
- o signal parametrics normally, uniformly or point mass distributed, again sampled from populations centered on the true means
- o the underlying random variables are independent
- o time of observation a known constant

↑

SEPARATION ALGORITHMS

↑

GROUND TRUTH

- o single emitter with random noise
- o many emitters, each with random noise

samples. Given the validity of the first assumption, the unbiased normal distribution for location estimates can be established by classical statistical arguments. Distributions for the various signal parametrics apparently are determined by Bayesian techniques supported by intelligence-community estimates, where the term Bayesian is understood in the popular sense of relying on predetermined priors (probability estimates) which may be modified by incoming data. (All the statistical inference found in the systems surveyed is classical). The ubiquitous tacit assumption of independence among signal parametrics, underlying their joint distribution and dictating the appropriate statistical tests, seem dubious at best; even location and signal parametrics are probably made statistically dependent by tactics. Dependence among the various radar characteristics measured or inferred will be considered in greater detail in later sections, with particular attention to possible relationships with time of observation.

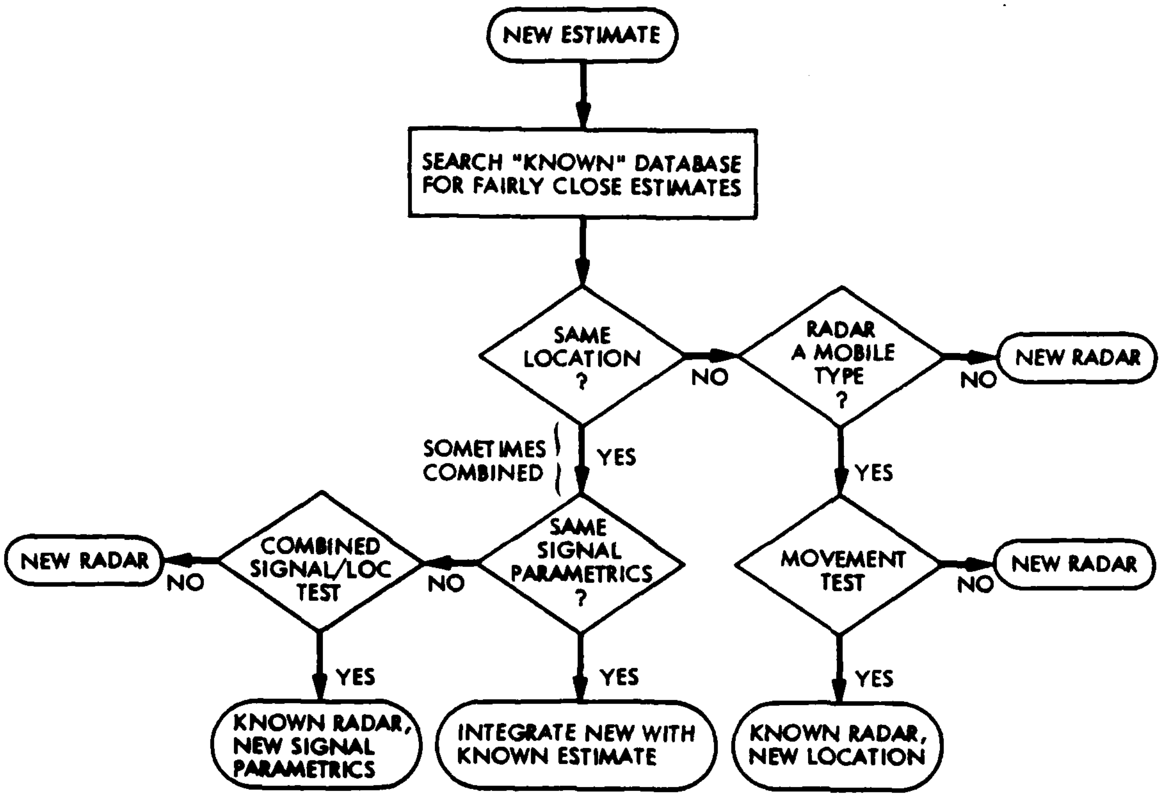
The consequences of these assumptions will be discussed for each system in Appendix E.

2. SURVEY OF SELF-CORRELATION ALGORITHMS IN EXISTING SYSTEMS

The mathematical documentation of several of the ELINT radar self-correlation algorithms has been surveyed. Self-correlation algorithms compare new intercept estimates (candidate radars) with prior estimates (known radars) of location, descriptive signal parametrics, and time of observation. If a match is found, the new information may be used to update the old; if none is found, the new estimate is added to the database of "known" radars. Some data analysis systems also take into account possible movement or change in operating signal parametrics. Figure 2-1 illustrates a general self-correlation algorithm.

Two sets of tests compare new estimates with those already in the database. One is based on radar location, the other on signal parametrics, such as frequency, pulse width, pulse repetition interval, and time of observation. Most systems assume that location estimates are normally distributed and base these tests on standard statistics. Tests based on signal parametrics are handled by several statistical and non-statistical techniques. These tests, and their assumptions, will be discussed in the remainder of this section. The focus will be on stationary radars: those that change neither their signal

Fig. 2-1: General Self-Correlation Algorithm



parametrics nor their location, and the section will conclude by discussing a specific example, the radar self-correlation algorithms in the BETA system. The discussion will be limited to general statistical forms and concepts, those interested in the underlying mathematics should consult the annotated reference list (Appendix A). The specific algorithms used in the systems surveyed will be considered in Appendix E.

As Figure 2-1 indicates, systems use both sequential and simultaneous decision tests, simultaneous tests can be based on either a joint distribution or a (usually linear) combination of tests of individual characteristics. Before looking at the mathematical form of the tests, or measurements, for individual radar characteristics (location and various signal parametrics), it will be useful to consider the mathematical implications of choosing sequential or simultaneous tests.

Sequential tests normally assume that the characteristics being tested are statistically independent. If this assumption is true, sequentially testing hypotheses based on the one-dimensional marginal distributions will eventually lead to rejection, if there are enough dimensions, whereas "averaging" the noise over all the dimensions in a test based on the joint distribution may result in acceptance. If the characteristics are correlated, however, a properly constructed test based on the joint distribution approaches one dimensionality, making the sequential approach redundantly test the same thing, so that sequential and simultaneous results are the same. Where some characteristics are correlated and some are not, which usually implies nonlinearity, the outcomes using simultaneous or sequential testing are hard to compare, unless both tests are carefully constructed to reflect the same behavior. Further, by using the joint distribution, behavior of individual characteristics is obscured. One way to retain some control over the influence exerted by individual characteristics on the outcome of the test and to provide the flexibility of easily performing a set of tests sequentially or simultaneously is to use the weighted sum of the statistics for each characteristic. This approach, and some of its possible statistical interpretations and derivations, will be considered in the section on simultaneous tests. The statistical basis for these simultaneous tests, or lack of it, will become particularly important when discussing non-stationary radars.

2.1. Location Tests for Stationary Radars

The initial, often tacit, assumption made is that each location estimate comes from a set of observations of the same radar. The consequences of relaxing this assumption will be discussed later in this report.

Most of the systems surveyed use a chi-square statistic for location tests, either as an individual hypothesis test or as part of a simultaneous test. The choice is based on established statistics, a brief heuristic discussion of which follows.

Figure 2-2 shows schematically the observational data used to determine the location estimate $\bar{x}$ and elliptical error probable. Different platforms belonging to the same sensor system take several lines-of-bearing to the radar, leading to measurement error in the angle specifying the line-of-bearing and, especially in airborne platforms, also in the platform location. These are translated into error in the location estimates which are assumed to have a bivariate normal distribution with mean $\bar{x}$ and covariance matrix S. The level curves of this joint distribution are ellipses (Figure 2-3) and not circles chiefly because the total angle of observation is small; could the sensors surround the radar, the level curves would most likely become circles.

Now letting $\bar{x}_1$ and $\bar{x}_2$ be the means, S_1 and S_2 the known population covariance matrices, and n_1 and n_2 the number of observations in two samples, the statistic

$$X^2 = (\bar{x}_1 - \bar{x}_2)^T (n_1^{-1} S_1 + n_2^{-1} S_2)(\bar{x}_1 - \bar{x}_2)$$

has a non-central chi-square distribution with non-centrality parameter

$$\lambda = \sum_i \sum_j s^{ij} \delta_i \delta_j$$

where δ_i is the difference in means for the ith characteristic and s^{ij} are the elements of $S = n_1^{-1} S_1 + n_2^{-1} S_2$ inverse. If the hypothesis that the two samples refer to the same radar is true, then $\delta_i = 0$ for $i = 1,2$ (cf. Johnson and Leone, section 17.7).

Fig. 2-2: Sensor System Geometry

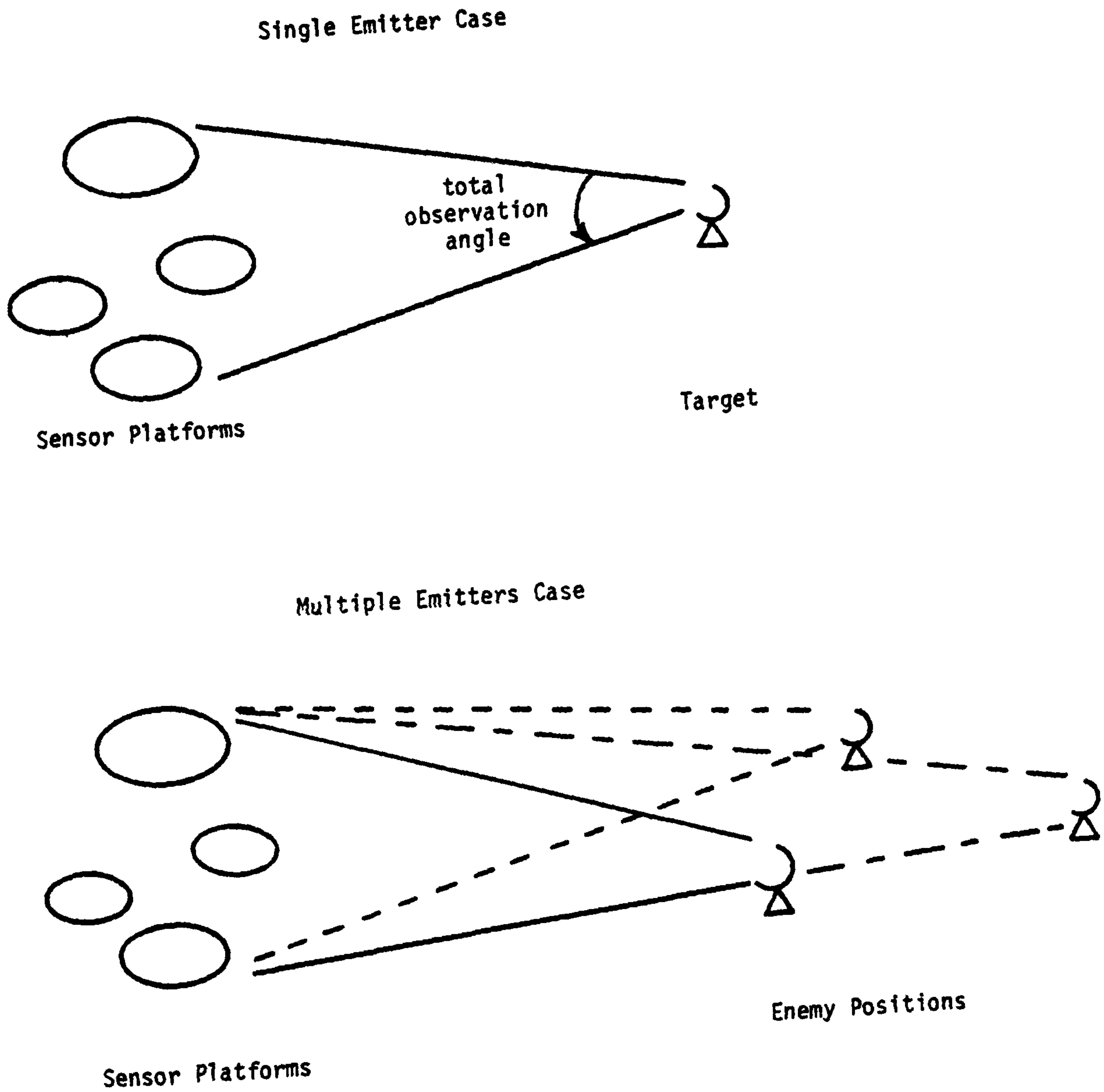
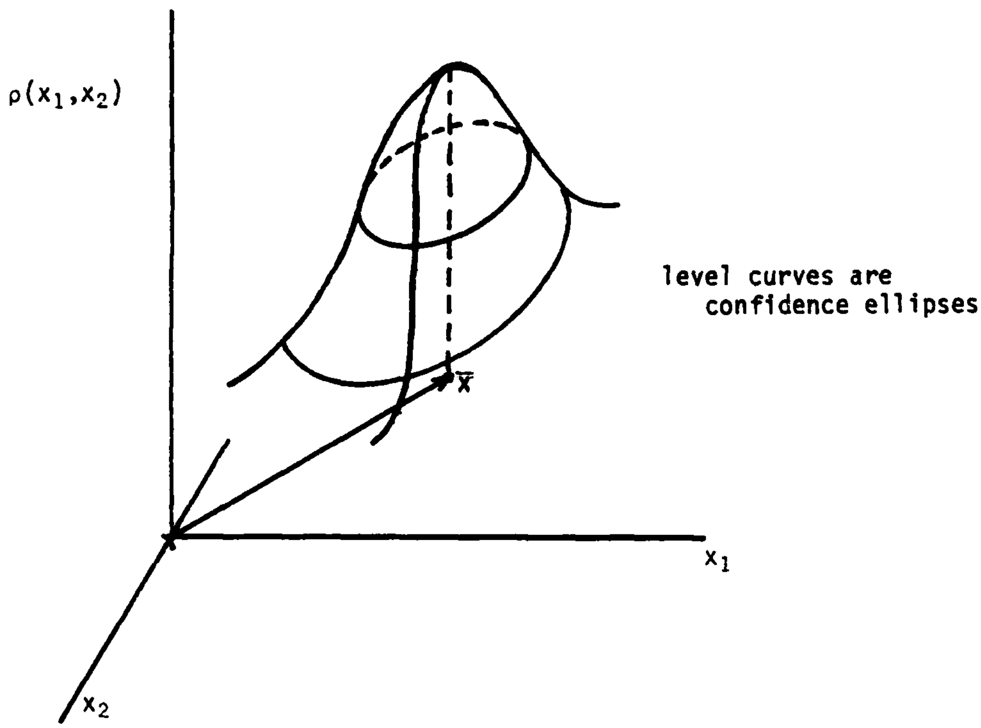


Fig. 2-3: Multinormal Distribution of Location Estimates



Consider the model of the observation of a single characteristic made by the kth sensor system.

$$\bar{x}_k(t)=\mu+b_k(t) + s_k(t)$$

where

$\bar{x}_k(t)$ is the sample mean (estimated location) sampled at time t

μ is the true location of the radar

$b_k(t)$ is the location bias introduced by that sensor system at time t and

$s_k(t)$ is the error, usually assumed distributed N(0,1)—normal with zero mean and unit variance

If "belonging to the same population" means "observing the same radar" - and that is what self-correlation is testing - then the non-centrality parameter is not zero, if different sensor systems introduce different biases, even though they are observing the same radar. Thus, for the statistic X^2 to have a central chi-square distribution, as is usually assumed, the observations must be unbiased, or all have the same bias. Further, the error must be invariant with respect to time. Although this may initially seem to be a harmless assumption, error probably does depend on range which changes with time. Potential dependence of measurement error on distance to the emitter usually can be accounted for, and in no case should be overlooked.

Two different methods of handling the location tests were found in the systems surveyed. The first, part of a sequence of tests, was a central chi-square test for a predetermined confidence level (1-α) using the statistic X^2. The second involved simultaneous tests with signal parametrics, needed to convert the chi-square statistic value into a value compatible with the other tests, in particular, to map it into [0,1]; the functions used for this mapping or transformation were usually of the form

$$e^{-x^2/\beta}\ ,\ \beta>0,$$

where x is the chi-squared statistic. As it is the "quadratic form" for beta equaled to 2, the above equation gives the unnormalized probability density for the bivariate normal distribution and is the standard mapping of a chi-squared distribution onto the unit interval. Either of these approaches is reasonable as long as the estimates are unbiased, the samples statistically independent, and the distributions normal; and normality is a standard assumption for distribution of sample means.

2.2. Signal Parametrics Tests for Stationary Radars

As stated above, a stationary radar is not only one that is not mobile but also has no basic changes in its operating signal parametrics. Although tests on time of observation usually appear with these signal parametrics their discussion will be postponed to the section on non-stationary emitters. Unlike those for location, there is no well-developed statistical literature for these characteristics. The three main approaches to hypothesis testing found in the systems surveyed were:

1) test whether the new characteristic estimate is within preestablished limits or preestablished bounds of the known estimate;

2) calculate a value (to be used in a simultaneous test) using a simple function of the difference between new and known characteristic estimates;

3) calculate individual measures of correlation for each characteristic (to be used in a simultaneous test) that reflect the probably that the new estimate falls within the (1-α) confidence band of the known estimate.

These three approaches will be considered separately.

Tests on whether the new estimate falls within a given interval are of two types.

1) Those for which all known operating intervals of enemy radars are predetermined, so for two observations to be of the same radar, they must both lie in the same interval;

2) Those for which the variability about a given signal parametric for enemy radars is predetermined, so the new estimate must lie within that distance of the known one.

The first type has no statistical content; it is a deterministic decision based on certain prior knowledge. The second lends itself to a statistical interpretation, especially if real-time data is used to modify the bounds, leading to a "Bayesian" approach. Neither of these need deal with the classical statistics arising from errors in measurement. The second technique may take into account priors arising from measurement methods as well as radar performance, but that can not be determined by the mathematical form of the test alone. Such motivations are known only to the creator of the "database" of predetermined bounds.

The second class of tests, those using a simple function of the difference between new and known estimates, are designed to be used in simultaneous tests, considering several signal parametrics with or without location. The functions are used to emphasize differences between estimates in certain ranges, to map the differences into [0,1] so they can be combined, or to reflect some assumed distribution for that characteristic. These distributions may reflect variability in enemy radar performance or friendly sensor observations.

The third approach assumes some distribution for the estimates, usually normal. It also assumes that the $(1-\beta)$ confidence intervals for the new and known estimates are given (they are not given in the unclassified portion of the TACELINT message format given in the referenced technical directive). The measure of correlation is

$$\int_{(1-\alpha_{new})\cap(1-\alpha_{known})} e^{-x^2}dx \Big/ \int_{(1-\alpha_{new})} e^{-x^2}dx$$

where $(1-\alpha_{new})$ indicates the closed $1-\alpha$ confidence interval for the new estimate and $(1-\alpha_{known})$ for the known estimate. Given the above assumptions and statistical independence of measurement, this equation gives the conditional probability of an estimate coming from the known population, given it is known to come from the new population, hence a *probability* that the estimates come from the same population. Unlike the tests discussed earlier that rest heavily on priors, this test is based on the variability arising in sampling (that is, in observing enemy radars). If there is variability in the enemy radar parametric *around some fixed value*, this variability will be reflected (intermingled with variability arising from the measurement technique) in this measure of correlation. To separate these two contributions to the variance requires other statistical techniques, and the relevance of the information obtained to this intelligence problem is unclear.

2.3. Simultaneous Tests for Stationary Radars

Many of the values discussed above were designed to be used in simultaneous tests. Two forms of these tests will be considered:

1) the weighted (convex) sum of values for individual characteristics, and

2) a *cumulative point test.*

These are indeed the same kind of test, cumulative point tests being a discrete version of weighted sums. Cumulative point tests use a predetermined set of values for each characteristic; among which of these values the test value falls (often the difference between estimates) determines the number of points assigned for that characteristic. These points are summed, and the decision to accept the new and known estimates as referring to the same radar is based on that sum. Determining the criteria for assigning points is similar to determining which functions will map the difference in estimates (for signal parametrics) or the chi-square value (for location) into [0,1] and to determining the set of weights.

The weighted linear sum is often known in statistical literature as a "linear discriminant". Weights and mapping functions are chosen to enhance its ability to distinguish between populations based on the characteristics being measured. Without any assumptions on the statistical properties of the characteristics, this is an often useful tool for constructing a hyperplane separating different sets of observations (those from, it is hoped, different radars) in *N dimensional space*, where N is the number of independent characteristics. Note that dependence between characteristics is reflected by fewer dimensions of the hyperplane than there are observed characteristics, and the weights assigned to dependent parameters control their contribution to the independent dimension they define.

To make statistical statements based on this discriminant, further assumptions about the characteristics are required. The classical assumptions are that the characteristics are normally distributed (true for most mean estimates based on large samples), are independent (this will be addressed later), and have equal covariance matrices. Under these assumptions a linear discriminator with usually calculable coefficients can be obtained, whose statistical behavior is known.

One observation on the functions applied to the individual characteristic differences is that these functions are the vehicle to carry distributional information about the characteristics. The functions chosen in the systems surveyed are almost invariably those classically used to represent ignorance, not prior knowledge.

Finally, note that even from this general view of their form, signal parametrics tests are seen to

1) discriminate between types of radars, but probably not individual units unless true tests are very precise, and

2) support decisions, but not make statistical inferences with confidence.

These limitations will be seen to become more important for non-stationary radars.

2.4. Non-Stationary Radars

As mentioned in Section 2.3, the tests discussed so far indicate whether a type of radar observed at a specific location is the same as that in a previous siting. With straightforward modification the tests also can identify those radars whose signal characteristics may vary in a prescribed manner as belonging to the same type. Depending on the accuracy of the weapon system to be employed this information may be sufficient for targeting, the specific piece of equipment at a location being unimportant as long as it is known that a radar of a given type is there. However, for other intelligence analyses, it may be important that the radar that was there has left and a new one of the same type taken its place.

The standard approach to testing for a moving radar is that, whenever there is no database entry close enough to the new siting measured by some chi-squared value, database entries for the same type radar (if it is a mobile type) within the movement radius of the new siting are considered as possible matches, as shown in Figure 2-1. To draw a statistical inference concerning whether the radar moved requires statistical tests for characteristics of that radar, not just that type of radar. The only truly statistical inference being drawn by the tests discussed above is for the location, and this test is abandoned with moving radars. To construct the tests required for statistically testing hypotheses about the radar itself requires

1) a statistical test of signal characteristics, including

2) the time of beam initiation.

The tests must be able to not reject the hypothesis that the new location is of the same radar while rejecting the hypothesis that the radar is in the old location, even if it has been replaced by one of the same type.

Such specific identification by radar instead of by type/location pair becomes important when results are fed into cross-correlation. Unit deployment depends heavily on terrain, so any unit occupying a given terrain is likely to deploy its radars in the same locations; this is reflected in cross-correlation templates. If a new unit takes the place of another, if its radars are of the same type and in approximately the same location as those of the former unit, new and known estimates will match in self-correlation, attaching the radar to the unit that has just moved in. It is true that this site no longer belongs to the old unit, and if sites were being kept in the database links should be broken; but the radars formerly linked to the old unit still are attached to it, at some new location. If cross-correlation is the linking of equipment, not sites, to units (and because units move, their equipment does also), such breaking and reforming of links loses continuity of information and leads to confusion.

Although the character of signal parametrics and their tests will be discussed more completely later, the particular role of observation time will be introduced here. Current incorporation of time into the tests, usually in the linear discriminant, seems mainly to be based on how long doctrine says a given radar will operate with the same characteristics in the same location. Certainly a significant time is required for one unit to leave an area and another to redeploy there, and this time is quite terrain dependent (which does not seem to be taken into account in movement rate or set-up/tear-down times). Such non-statistical decision tests, however, can only give one a "good feeling" that indeed the radar has moved from one place to another because the right amount of time has elapsed. Without statistical hypothesis tests on observed characteristics, radar movement can not be inferred with any confidence. If unit movement, and hence tactics, is being inferred from equipment movement, unsupported "good feelings" can quickly compound into disaster.

The operating history of a radar, expressed as a time history of some of its characteristics, seems a likely candidate for statistical hypothesis testing. This requires time tests emphasizing very short rather than long windows. Thus time may be crucial, although not as a characteristic in its own right treated independently of other characteristics, but crucial in its relationship to the other characteristics including location.

2.5. Self-Correlation in BETA - An Example

BETA is a testbed system for correlating reports from many different types of intelligence systems. It has self-correlation algorithms for radars, radios, "movers," "shooters," compounds, and complexes and has a cross-correlation algorithm. This discussion will center on the radar self-correlation algorithm. BETA correlation is illustrated in Figure 2-4.

BETA uses a simultaneous test that is a linear discriminant based on five characteristics: location, time, frequency, pulse repetition interval, and pulse duration. The figure of merit for each of these is defined so as to

Fig. 2-4: BETA Process Control Flow

From: TRW Document - BETA CORRELATION CENTER APPLICATIONS COMPUTER PROGRAM CONFIGURATION ITEM DEVELOPMENT SPECIFICATION - R.C. Fong

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26 Nov 1980
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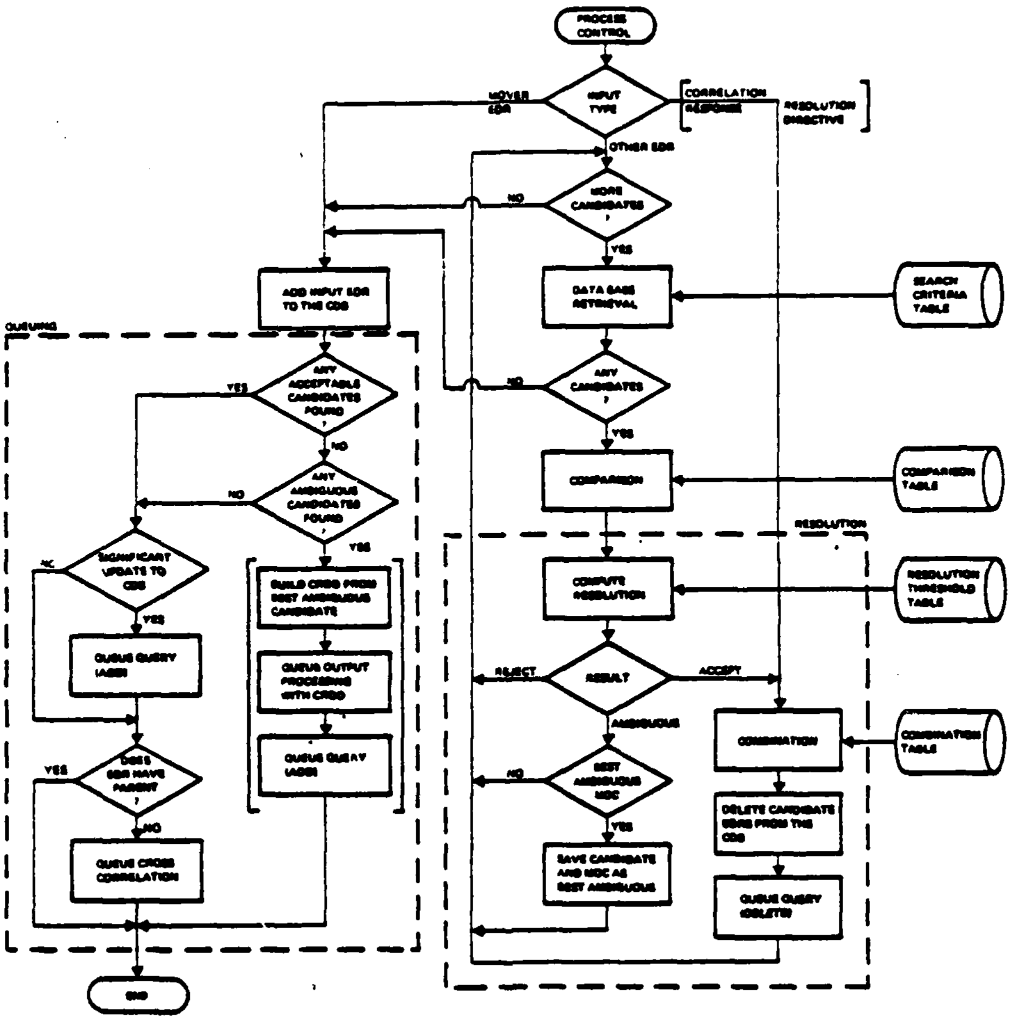

lie in [0,1], the measure of correlation (linear discriminant) being their weighted sum. The weights may be adjusted by the operator to refine the "screening power" of the test -- that is, to control the fraction of the cases handled automatically or to rebalance the probabilities of associating two estimates that really refer to different radars (type I error) and of not associating two that do (type II error) — or to respond to the tactical situation. Weights are normalized, making a convex combination of characteristics; if information is missing for some characteristics, the weights are renormalized. These factors and others affecting the choice of weights are discussed in Appendix D.

The figures of merit for the individual characteristics have three basic forms.

1) For location it is $e^{-x/2}$, where x is the chi-squared quadratic form from the multidimensional normal distribution.

2) For time it is max (0,1-d/B), where d is the linear distance between the two observation time intervals and B>0 is predetermined.

3) For signal characteristics it is 1, if the absolute difference between the closest values for the characteristic, c_1 and c_2 is smaller than some predetermined error, max $(0,1-(c_1/c_2-M)/(B-M))$ where $c_1>c_2$, and B>M are predetermined positive bounds.

For the tests for frequency and pulse repetition interval, M is one. The location test is the standard mapping of the chi-squared statistic onto the unit interval, giving a non-normalized normal density function. The time test falls off linearly as the times of observation draw farther apart; this is in fact the distribution function for a uniform distribution. The signal characteristics' measures are seen to have the same form, with atoms possible at the ends. Note that, as mentioned above, all known or suspected underlying distributions are "non-informative", those used usually to minimize the (unspecified) worst case losses in the case of ignorance. Also when they are used, the characteristics are usually assumed to be statistically independent.

A further analysis of BETA is given in Appendix D.

3. RELAXING ASSUMPTIONS ON LOCATION ESTIMATES

When many of the assumptions stated above are relaxed, or do not hold, there is no immediate statistical tool to replace the one lost. The effect of relaxing the following three assumptions will be considered in this section.

1) The variance of the location estimate is known.

2) The location estimate is unbiased.

3) The observations from which the location estimate is derived all refer to the same emitter.

Whenever possible, alternate approaches will also be considered.

3.1. Known Variance

Knowing the variance of the location estimates made if possible to use the chi-squared distribution to test the equality of the two estimates (observed sample means). This variance is passed to the self-correlation algorithm by the observing sensor system. It would usually be determined by assuming that the population variance of the observations is known, and dividing it by the number of observations. This observation variance is just the variance associated with the observation error. It may well be range dependent, but even so, knowing it for a set of ranges for a system that can be extensively field tested should be possible.

If the observation variance, and thus the variance of the sample means, is not known and its unbiased estimate used instead, the choice of statistical test depends on sample size. For an hypothesis that the mean of the population, from which one sample was drawn equals a fixed quantity, Hotelling's T^2 statistic is used for smaller samples. This statistic has an F distribution and is the multidimensional analogue of the Student's t statistic. However, an F distribution is the ratio of two chi-squared distributions divided by their respective degrees of freedom. In this case, the numerator is one half the chi-squared statistic used to test the hypothesis that the mean takes on a given value, if the variance is known, and the denominator is a distribution of sample variances. Thus, the hypothesis tested by a T-squared statistic is that the estimates come from the same distribution, that both their means and variances are the same. The hypothesis appropriate in self-correlation is that they refer to the same emitter, that is, that the means are the same. The means depend on emitter location alone, but the variance depends on the sensor system and are not the same for different sensor systems. Thus the T-squared statistic may be used only if there is only one sensor system and measurement derived variance is not range dependent.

For a multi-sensor data analysis system it is better to use the chi-squared statistic with the two known and probably unequal variances. This requires that the observations be sufficiently numerous for the unbiased estimator of the variance to be a suitable surrogate, or that the observation variance be known from field testing and be monitored (probably by a sample variance equals hypothesized population variance one dimensional F test on the angle of observation) to ensure that it does no change significantly under combat conditions.

3.2. Unbiased Mean

A fairly obvious point that should be mentioned is that the estimates must be unbiased. Unbiased is used here in the same sense as in the model mentioned above,

$$\bar{x}=\mu+b+\varepsilon$$

where b is zero, and $\bar{x}$ is the sample mean from some system, ε its error. The bias b could be nonzero from two causes:

1) hardware bias in the measurement and

2) software bias introduced by data analysis and estimation techniques.

Bias in the statistical estimation itself can be avoided by using known unbiased estimates of the mean. Note that this also requires an unbiased data integration method for refining estimates already in the database during self-correlation, as one of the samples being tested is always the database entry. Appendix D will consider such data integration methods for location estimates. Possible bias from hardware or other software, if it is well understood (again through field testing), can be adjusted for before the estimated location is passed on to self-correlation.

3.3. Single Emitter Population

Homing in on false images is not restricted to intelligence data analysis systems. The "centroid problem", an intelligent missile seeing two emitters and seeking their centroid, plagues stand-off missile design. When the environment is target dense, the targeteer has trouble deciding between so many often valuable targets, but the intelligence analyst has trouble separating observations into samples representing only one emitter so that the targeteer has a target and not a phantom. The problem is real, and current Soviet trends indicate the target density will, if anything, increase.

Separation can be based on location, or signal parameters, or both. Separation of radars of the same type based on location will be considered in this section and signal parametrics in the next. Unfortunately, the majority of the most useful information for separation is contained in the observations and has been averaged out by the time a location estimate and EEP is passed to self-correlation. The best self-correlation can do is try to identify phantoms so that "known" database entries are not contaminated, for a phantom by its very nature may correlate with one of the true targets it is hiding, thus pulling that estimate farther from its true value.

Examining the properties of phantoms, and the situations in which they are likely to arise, suggests a few rules-of-thumb for their identification.

1) An estimate strongly supported by at least two sensor systems is probably not a phantom.

2) Phantoms, as a class, have larger variances than true estimates.

3) When a candidate estimate associates strongly with more than one known estimate, one of them may be a phantom.

The observations leading to these will be considered separately.

Unless the mathematics of their separation algorithm is identical - and to this point no two algorithms that are sufficiently the same have been seen - different sensor systems will tend to produce different phantoms in the same situation. This follows because the phantoms being considered here are artifacts of the mathematical separation algorithms used to identify observations of the same emitter. Thus, if a location is strongly supported by at least two systems, it is probably not a phantom of either. This also points out a benefit of having different mathematics in different systems, in the absence of a phantomless algorithm whose development in a target-dense environment is unlikely.

Phantoms occur where two or more emitters of the same type are so clustered that their centroid is within sensor-system tolerance of observations of each - that is, targets are dense with respect to measurement sensitivity. Thus, the observations are drawn from at least two populations; and it is likely that the sample variance will be larger than if all observations came from only one emitter population. Also, since the controlling parameter is the angle of observation the following statements can be made:

1) the true observations are more apt to lie along the major than minor ellipse axis for standard shaped ellipses,

2) a suspiciously circular EEP is probably a plantom with true locations along its minor axis; and

3) phantoms are more apt to occur at greater ranges.

Thus, suspicion can be cast on an estimate by only knowing its EEP.

Strong association between (or among) two or more known estimates may indicate that one is a phantom. In the missile centroid problem described above, the phantom will lie at a weighted centroid of the true locations it is hiding. If there are only two, it will lie approximately on the line between them (the deviation from the line arises because all values are estimates, not true locations). Unfortunantely, for three or more true locations, the phantom can lie anywhere within their convex hull, depending on how many observations there are of each emitter. However, since many separation algorithms give true emitter estimates if at least 80% of the original set of observations comes from one emitter, and are most likely to produce phantoms, if the observations come 50% from each of two emitters, phantoms will most likely be around the centroid of the true locations. Simple terrain checking in such situations may indicate that no radars would be sited in that pattern (even taking into account EEP).

These very heuristic rules-of-thumb give some idea of the mathematical severity of the problem, if phantoms are input to self-correlation algorithms. Once their information is integrated with a known estimate, damage seems irreparable; and based solely on mathematics of location, there seems no certain way to weed out the phantoms. Some consideration based on signal parametrics will be discussed in the next section.

4. DEPENDENCE IN SIGNAL PROCESSING

The basic ELINT problem concerns the location and identification of signal parametrics for non-communications emitters based on observations made by sensors. By non-communications emitters we mean radars and certain ECM devices. Subsequent evaluation of these ELINT observations is dependent upon statistical assumptions made regarding these observations. This section considers the statistical assumptions that are, and are not, appropriate for these ELINT observations.

It is useful to first discuss the ELINT problem from the single sensor viewpoint with respect to the measurement of the signal parametrics of an emitter. Emitters to be sensed may be monostatic[1], bistatic[2], or multistatic[3] but, this analysis of emitter signal parametrics will be restricted to the monostatic case. The dependencies inherent in the bistatic and multistatic emitter cases will be deferred.

---

1 Monostatic radar - a single radar
2 Bistatic radars - a pair of cooperative radars
3 Multistatic radars - a collection of cooperative radars

The first approach to emitter signal characteristics is made here without respect to their measurement. Further, the discussion will be about only the characteristics of relatively naive emitters.

The typical emitter will radiate a formed beam of electromagnetic energy. The frequency of this emission may be fixed or varying in some fashion and may be continuous or intermittent. The beam is polarized and may be rotated, fitted, nutated, etc. Further the emitter may be mounted on a moving platform, thus continuously or intermittently changing position, velocity, and the aspect of the beam.

Since a radar's function is inferred through its signal parametrics, there is an implicit general dependence among the signal parametrics which will not be considered here. We will consider the characteristics and dependencies of some of the signal parametrics. Radar emissions (carriers) may be continuous or intermittent. Continuous emissions may be of fixed or varying frequencies depending on their purpose. Intermittent emissions may be repetitive or non-repetitive.

Repetitive emissions are characterized by their pulse repetition interval and their pulse characteristics (width, shape, uniform and non-uniform bursts). Further, the carrier frequency may be varying during each pulse. Either the pulse repetition interval or pulse width or both may vary over some range of values in a uniform or non-uniform fashion.

Non-repetitive emissions are generally characterized the same as repetitive emissions over short time intervals. The non-repetitiveness is introduced by time, carrier frequencies, and geographic diversity among a set of cooperative radars as in bistatic and multistatic systems. For our purpose, we will consider each of these cooperative radars as a separate emitter.

A reasonable subset of signal parametrics required to locate and identify a radar includes the nature of the transmission, i.e., continuous (CW) or pulsed, and the following signal characteristics:

1) carrier frequency

2) pulse width

3) pulse repetition interval

4) beam scan type

5) beam scan rate

6) beam polarization

The location is specified along with the elliptical error probable (EEP) and relative bearing of the semi-major axis. The semi-major and semi-minor axes of the EEP are both dependent on the angular accuracy of the sensor, hence, are not independent measures.

The carrier frequency is an important parameter in both CW and pulsed radars and may be either constant or modulated, as in chirp or linear FM radars. This characteristic implies the need for instantaneous as well as average frequency measurements. Since only average frequency is included in the Tactical Electronic Intelligence (TACELINT) message, the frequency characterization is incomplete.

There is an intrinsic dependence between pulse width and pulse repetition interval through the peak-to-average power ratio required for a specific radar performance factor. Even for independently varying pulse widths and intervals, this lack of independence, on the average, remains.

Pulse width measurements must take pulse multiplicity, width agility, and shape into consideration. However, only a single measure of pulse width is included in the TACELINT message.

Pulse repetition interval must take into account interval staggering with multiple stagger legs. But the TACELINT message includes only pulse repetition interval and whether it is fixed, staggered, or jittered.

The formed antenna beam is described by scan type, scan rate, and polarization. Useful measures not included in the TACELINT message are the horizontal and vertical beam width and beam multiplicity.

There are two conclusions to be drawn from this brief discussion of radar signal parametrics:

1) these signal parametrics are not all independent of one another, and certainly not of their intended function,

2) the set of signal parametrics included in the TACELINT message is insufficient to "fingerprint" a specific radar emitter as distinct from another of the same model.

The sensors measure the location and signal parametrics which lead to the production of TACELINT messages. There are four distinct cases to consider:

1) single sensor, single emitter,

2) single sensor, multiple emitters,

3) multiple sensors, single emitter,

4) multiple sensors, multiple emitters.

Since a single sensor can only provide a line-of-bearing to an emitter and a set of measures of the emitters signal parametrics, the sensor must be mounted on a moving platform to obtain a "fix" on the emitter using multiple lines-of-bearing from different locations at different times. The variances associated with each sensor of an emitter are the same because these variances derive from the sensor characteristics only. With multiple emitters, the location and signal parametrics measures may differ, but the variances associated with them remain the same.

The situation with multiple sensor systems is quite different: the "fixes" on a emitter are usually made based on the same emission in a form of time coincidence. These systems provide more refined measures of emitter location based on a Loran-like time of arrival method. However, the sensors may be of the same or different model types, so the variances associated with each sensing of the emitter may well be different. For multiple emitters the location and signal parametrics measures may differ and so will the variances associated with them.

The conclusions to be drawn from this brief discussion of ELINT sensors are that:

1) The differences in sensor-dependent variances are included in the EEP associated with each location estimate sent in the TACELINT messages. So, location data is suitable for subsequent statistical manipulation.

2) The differences in sensor-dependent variances for the signal parametrics, and indeed the variances, are not conveyed in the TACELINT messages. This implies that the signal parametrics are not amenable to subsequent statistical manipulation.

5. OBSERVATIONS AND CONCLUSIONS

Throughout this discussion we have talked interchangeably and indiscriminantly about two kinds of statistics: those for which the distribution is known and those for which it is not. On the former is based classical statistical inference and the capability to test hypotheses and make statements with some confidence. These are the basis for scientific experimental evidence. The latter are descriptive or "indicative": they may indicate something about the state of the object being studied, but can not rigorously support any such statement. They are the basis of much management and financial decision making. Most statistics used in these algorithms are indicative, yet, as in the case of mobile units, fairly sophisticated inferences are being drawn. Just as it takes a skilled and experienced manager to make good financial decisions, it takes a very skilled and knowledgeable intelligence analyst to draw the most from these analysis tools. He practices not science but an art form whose success depends on his individual talents. Automated systems that give him more and more raw data to handle, and only minimal help in handing it, are doing him a disservice. For these systems to carry with them the connotation of scientific statistics, giving an unwarranted confidence in their results and obscuring the crucial role played by the analyst does him an injustice.

To the extent tests can be put on a stronger statistical footing, the quality and credibility of the information derived from these systems, and thus the real support they provide the intelligence analyst, will increase. Several initial steps are possible and recommended to make statistically-based information more reliable.

1) Ensure the chi-squared location statistic is good by providing range-dependent population variances (perhaps obtained initially from field testing) from sensor systems, and have these variances monitored within the sensor system to verify their continued validity.

2) Develop distributions for some signal parametric statistics, perhaps using a time history. This need not replace the linear discriminate, but could provide a distribution for it.

3) Pay closer attention to distributions, especially when refining (or integrating) signal parametrics information. This may involve and developing specific prior/posterior distributions.

Implementing some of these suggestions requires additional information be carried in the TACELINT message.

Finally, it is not yet clear which or what combination of the mathematical approaches - classical or Bayesian statistics or non-statistical - will best serve the need of the intelligence analyst. It is clear that all three approaches should be pursued, in parallel, with special attention to the minimum information which will be required by cross-correlation algorithms. As stated at the outset, intelligence correlation is an hierarchical process; each part must not only be as sound as possible within itself, but also properly fulfill the information processing requirements of its role in the overall process.

APPENDIX A

A. ANNOTATED REFERENCE LIST

References will be listed in the categories: mathematics, radars, and military systems. Each will be followed by a brief indication of its formal level (if appropriate) and applicable areas of this report. Most works listed include good reference lists.

<u>Mathematics</u>

Box, G. E. P, and Tiao, G. C. <u>Bayesian Inference in Statistical Analysis</u>. Reading, Massachusetts: Addison-Wesley, 1973.

A mathematically thorough applications-motivated senior/graduate text on Bayesian inference, accessible to those in other technical fields.

Chakravarti, I. M., Laha, R. G., Roy, J. <u>Handbook of Methods of Applied Statistics</u>, Volume I. John Wiley and Sons, New York: 1967.

A well organized guide to descriptive and inferential statistical techniques, with clearly stated assumptions and examples, a good section on multivariate analysis.

Deutsch, R. <u>Estimations Theory</u>. Englewood Cliffs, New Jersey: Prentice-Hall, 1965.

Standard book on location estimation, confidence ellipses, and mathematical estimation arising especially in radar problems.

Hoel, P. G. <u>Introduction to Mathematical Statistics</u>. New York: John Wiley and Sons, 1971.

Provides necessary background for more advanced books.

Hoel, P. G., Port, S. C., and Stone, C. J. Introduction to Probability Theory. Boston: Houghton Mifflin, 1971.

____________________. Introduction to Statistical Theory. Boston, Houghton Mifflin, 1971.

These two volumes provide a thorough contemporary mathematical introduction to probability and statistics.

Johnson, H. L., and Leone, F. C. Statistical and Experimental Design. New York: John Wiley and Sons, 1977.

Another strong section on multivariate analysis, with examples, assumptions not as explicitly stated, more applications oriented.

Kendall, M. G. The Advanced Theory of Statistics, Vols. I and II. London: Charles Griffin and Company Limited, 1948.

Classical treatment of statistics, multivariate analysis presented in second volume, more analysis-oriented development.

Lass, H., and Gottleib, P. Probability and Statistics. Reading, Massachusetts: Addison-Wesley, 1971.

A unified introduction to probability and statistics with some focus on engineering requirements.

Scheffe', H. The Analysis of Variance. New York: John Wiley and Sons, 1959.

Radars

Barton, David K. *Radar System Analysis*. Englewood Cliffs, New Jersey: Prentice-Hall, 1965.

This graduate-level text treats measurement errors especially well. Multistatic radar systems and their characteristics are also covered.

Barton, David K., and Ward, Harold. *Handbook of Radar Measurement*. Englewood Cliffs, New Jersey: Prentice-Hall, 1969.

This handbook emphasizes radar measurement errors and discusses those due to digital signal processing of radar signals.

Cook, Charles E., and Bernfeld, Marvin. *Radar Signals*. New York: Academic Press, Inc., 1967.

The most comprehensive source for radar signal design.

Oppenheim, Alan V., ed. *Applications of Digital Signal Processing*. Englewood Cliffs, New Jersey: Prentice-Hall, 1978.

This relatively recent compendium contains an excellent chapter on applications of digital signal processing to radar from a non-hardware viewpoint.

Rabiner, Lawrence R., and Gold, Bernard. *Theory and Application of Digital Signal Processing*. New York: McGraw-Hill, 1975.

This engineering level text devotes an entire chapter to digital signal processing applications to radar with a strong hardware emphasis.

Skolnik, Merrill I. Introduction to Radar Systems. New York: McGraw-Hill, 1962.

An excellent general reference on radar systems although dated.

Military Systems

Beta Correlation Center Applications Computer Program Configuration Item Development Specification (No. SS42-43E Part I). Los Angeles: TRW, 1980.

Beta Correlation Center Applications Computer Program Configuration Item Development Specification, Volume I and Appendix II [Correlation Processing CPC] (No. SS22-43 Part II). Los Angeles: TRW, 1981.

Eustace, Lake, and Hartman, eds. The International Countermeasures Handbook, 7th Edition [1981-1982]. Palo Alto, California: EW Communications, 1982.

Isky, D. C. Weapons and Tactics of the Soviet Army. London: Jane's, 1981.

Technical Directive 005 for TCAC(D)ASAS-SEWS ADM Message Processing Tactical ELINT Message (No. CAC-TDM-005). Burlington, Massachusetts: RCA Corporation, 1981.

Intelligence Message Formatting and Procedures, User Handbook, Army Test Unit, DRSEL-SEI-ATU, Fort Monmouth, New Jersey, 21 September 1981.

APPENDIX B

B. ALGORITHMS IN STANDARD FORM

```
package EDR_Package is

        type TacelIntMsg is private;
        type EDR         is  private;
        type Radar_T     is (type_a,type_b,type_c);
        type REAL is digits  7;

        function  GET_EDR (Msg:TacelIntMsg)  return  EDR;
        function  RADAR_TYPE (Rec:EDR)  return  Radar_T;
        function  FETCH (First:Boolean;  Radar: Radar_T)
                        return  EDR;
        function BOX (OldRec,NewRec: EDR; Radar:Radar_T)
                                     return  Boolean;
        procedure  CORRELATE (OldRec,NewRec: in  EDR;
                                   Radar : in  Radar_T;
                                   Moc   : out  Real);
        procedure  STORE  (NewRec:  in  EDR;  Radar  :   in  Radar_T;
                                 OK :  out  Boolean);
        procedure  REPLACE (OldRec,NewRec:  in  EDR; OK:  out  Boolean);
        function  INTEGRATE (Oldrec,Newrec:  in  EDR)  return  EDR;

 private
     --full type declarations for TacelIntMsg and EDR
 end EDR_Package;

--Inside the package body EDR_Package, the
--procedures STORE, REPLACE, and FETCH will be
--implemented by calling appreciate entries of a
--"monitor" task (which serves as a synchronization
--agent for accesssing the common data base).

 with  EDR_Package ;  use  EDR_Package;
 procedure  MAIN  is

   Some_Condition   :Boolean;
   OldRec,NewRec    :EDR;

   Procedure Keep_Best_EDR is separate;

 begin

     NewRec := GET_EDR (Msg);
     Radar  := RADAR_TYPE (NewRec);
     while  Some_Condition    loop
          OldRec := FETCH (First, Radar);
          if  BOX (OldRec, NewRec< Radar) then
               Keep_Best_EDR;
          end  if;
          end  loop;
          if  Moc > Max then
               STORE (NewRec, Radar, OK);
          elsif Moc < Min then
               REPLACE (OldRec, INTEGRATE(OldRec,NewRec),OK);
          else
               GO_TO_OPERATOR;
          end if;
end main;
```

```
MODULE ACSRET (INPUT,OUTPUT);

PROCEDURE ACSRET;

(NAME: ACSRET-(COMPARISON PROCESS CONTROL))

(PURPOSE: COMPARISON DETERMINES THE SIMILARITY BETWEEN SUBJECT AND
(CANDIDATE EDRS.  SUBJECT AND CANDIDATE EDRS ARRIVE AT
(COMPARISON FOLLOWING DATA BASE RETRIEVAL VIA PROCESS CONTROL.
(A MEASURE OF CORRELATION (MOC) DEFINING THE LIKELIHOOD THAT THE
(SUBJECT EDR AND THE CANDIDATE EDR REFER TO THE SAME ENTITY IS
(COMPUTED.  THIS MEASURE IS COMPUTED AS A LINEAR COMBINATION OF THE
(RESULTS OF FIELD COMPARISON TESTS FOR CERTAIN CORRESPONDING FIELDS
(OF THE SUBJECT AND CANDIDATE EDRS

(INCLUDE 'ACCEDR.COM')
(INCLUDE 'ACCPDR.COM')
(INCLUDE ' IMPRR.COM')
(INTEGER J CK,PCFLAG')
(DIMENSION TSTNAM (5 )


VAR

(LOCAL VARIABLES))

I       : INTEGER;
INERR   : INTEGER;
LENTOL  : INTEGER;
I SUBT  : INTEGER;
PCFLAG  : INTEGER;
IWEIGH  : INTEGER;
MOC     : REAL;
SUM     : REAL;
PNT     : REAL;

(COMMON CMPLOC/FOMCMP(5), IVC (5), ITERR(5 )

(GLOBAL VARIABLES)

          IRADAR  : INTEGER;
          IVCK    : ARRAY [1..5] OF INTEGER;
          ITERR   : ARRAY [1..5] OF INTEGER;
          FOMCMP  : ARRAY [1..5] OF REAL;

(EXTERNAL PROCEDURES)

          PROCEDURE ACSPLT; EXTERN;
          PROCEDURE ACSRFT; EXTERN;
          PROCEDURE ACSRFF; EXTERN;
          PROCEDURE ACSRFI; EXTERN;
          PROCEDURE ACSRFD; EXTERN;

(EXECUTION TIME)

BEGIN

(INITIALIZE DATA)

FOR I := 1 TO 5 DO
    BEGIN
        FOMCMP[I] := 0.0
          [illegible]
```

```
    END;
IXERR := 0;
MOC   :=0.0;
SUM   :=0.0;

{SET TABLE LENGTHS BY TYPE}

LENTBL:=ZRADAR;

{START COMPARISON PROCESSING}

{CHECK INDEX TO TABLES}

IF (IXSUBJ > 0) AND (IXSUBJ <= LENTBL)
    THEN BEGIN

{PERFORM FOM TEST FOR LOCATION}

        ACSRLT;
        PCFLAG := ITERR[1];
        IF (IVCK[I] = 0) AND (PCFLAG = 0)
            THEN BEGIN

{PERFORM FOM TEST FOR TIME}

                ACSRFT;

{PERFORM FOM TEST FOR RADAR FREQUENCY}

                ACSRFF;

{PERFORM FOM TEST FOR PRI #2}

                ACSRFI;

{PERFORM FOM TEST FOR PULSE DURATION}

                ACSRFD;

{CALCULATE MOC}

                FOR I := 1 TO 5 DO
                    BEGIN
                        IF (IVCK[I] = 0) AND (ITERFL[I] = 0)
                            THEN BEGIN
                                {RWT := FLOAT(IWEIGH[IXSUBJ,I]);
                                MOC :=  MOC + RWT + FOMCMP[I];
                                SUM :=  SUM + RWT
                            END;
                    END;

                IF (SUM <> 0) THEN MOC := MOC/SUM

            END
{ NO ELSE, IVCK, PCFLAG}
    END
    ELSE IXERR := 1
END; {OF PROCEDURE ACSRET}

END. {OF MODULE ACSRET}
```

```
MODULE ACSRLT (INPUT, OUTPUT);


PROCEDURE ACSRLT;

{NAME:  ACSRLT - FOM TEST FOR LOCATION}

{PURPOSE:  THIS FIGURE OF MERIT TEST MEASURES THE LIKELIHOOD THAT THE}
{SUBJECT & CANDIDATE EDRS REFER TO THE SAME ENTITY ON THE BATTLEFIELD.}
{THIS FIGURE OF MERIT IS COMPUTED AS PROBABILITY GIVEN THEIR LOCATION}
{& LOCATION ERRORS THAT THE SUBJECT & CANDIDATE EDRS ARE COLOCATED}

{{PARAMETER EDFLT = 20,000,000, NDFLT = 10,000,000}
{PARAMETER MAXDIS = 18.42     !EXP(-.5*DIST) < .0001}
{INCLUDE "ACCEDR.COM"}
{COMMON /CMPLOC/FOMCMP(5),IVCK(5),ITERR(5)}


CONST

EDFLT = 20000000;
NDFLT = 10000000;
MAXDIS = 18.42;
VAR

{LOCAL VARIABLES}
DIFFE     : REAL;
DIFFN     : REAL;
ADD11     : REAL;
ADD12     : REAL;
ADD22     : REAL;
DET       : REAL;
DIST      : REAL;

{GLOBAL VARIABLES}
SLOC    : ARRAY [1..5] OF REAL;
CLOC    : ARRAY [1..5] OF REAL;
CSIGMA  : ARRAY [1..5] OF REAL;
SSIGMA  : ARRAY [1..5] OF REAL;
IVCK      : ARRAY [1..5] OF INTEGER;
ITERR     : ARRAY [1..5] OF INTEGER;
FOMCMP  : ARRAY [1..5] OF REAL;

BEGIN

{CHECK FIELDS FOR VALID DATA}

IF ((ABS(SLOC[1]) > EDFLT) OR
     (ABS(CLOC[1]) > EDFLT) OR
     (ABS(SLOC[2]) > NDFLT) OR
     (ABS(CLOC[2]) > NDFLT))
   THEN IVCK[1] := 1
   ELSE IF (SSIGMA[1] < 0 ) OR (SSIGMA[3] < 0) OR
             (CSIGMA[1] < 0) OR (CSIGMA[3] < 0))
      THEN IVCK[1] := 1;

     {COMPUTE DETERMINNANT (DET) IF DATA IS VALID}

IF (IVCK[1] = 0)
   THEN BEGIN
      {DIFFE := FLOAT(CLOC[1] - SLOC[1]) ...
      {DIFFN := FLOAT(CLOC[2] - SLOC[2]) ...
      ADD11 := SSIGMA[1] + CSIGMA[1];
      ADD12 := SSIGMA[2] + CSIGMA[2];
      ADD22 := SSIGMA ...
```

```
    {CHECK FOR VALID CONVARIANCE MATRICES}

       IF ((ADD11 >= 0) AND (DET > 0))
          THEN BEGIN

    {COMPUTE FOM FOR LOCATION}

             DIST := (ADD22 * (DIFFE**2) - 2.0 * DIFFN * DIFFN * ADD12
                      + ADD11 * (DIFFN **2)) / DET;
             IF( DIST <= MAXDIS )
                THEN FOMCMP[1] := EXP(-0.5 * DIST)

          END
          ELSE ITERR[1] := 1;
    END
    {NO ELSE AS FOMCMP PRESET TO ZERO}

 END; {OF PROCEDURE ACSRLT}

 END. {OF MODULE ACSRLT}
```

```
MODULE ACSRFT (INPUT,OUTPUT);

PROCEDURE ACSRFT;

{NAME: ACSRFT-(FOM TEST FOR TIME)}

{PURPOSE: THIS FIGURE OF MERIT TEST MEASURES THE LIKELIHOOD BASED ON}
{THE TIME OF THEIR FIRST AND LATEST OBSERVATIONS THAT THE SUBJECT AND}
{CANDIDATE EDRS REFERS TO THE SAME ENTITY ON THE BATTELFIELD.  THIS}
{FIGURE IF MERIT IS BASED ON THE RELATIVE POSITION OF THE OBSERVATION}
{INTERVALS (TIME BETWEEN THE FIRST AND LATEST OBSERVATIONS) OF THE}
{SUBJECT AND CANDIDATE EDRS.}

{RESRTICTIONS: NONE}

{INCLUDE "ACCEDR.COM"}
{INCLUDE "ACCPDR.COM"}
{INCLUDE "ACIMPR.COM"}
{COMMON  CMPLOC FOMCMP (5),IVCK(5), ITER(5)}


TYPE
   SHORT    = (ONE,TWO,THREE,FOUR,FIVE,SIX,SEVEN,EIGHT,UNKNOWN);

VAR

{LOCAL VARIABLES????}
SFOT      : REAL;
SLOT      : REAL;
CFOT      : REAL;
CLOT      : REAL;
BIG       : REAL;
RSLOT     : REAL;
RSFOT   : REAL;
RCFOT     : REAL;
RCLOT     : REAL;

{GLOBAL VARIABLES}
IXSUBJ : SHORT;
IVCK     : ARRAY [1..5] OF INTEGER;
ITERR    : ARRAY [1..5] OF INTEGER;
FOMCMP : ARRAY [1..5] OF REAL;

{PERHAPS A FUNCTION PROCEDURE???}
VALUEOF: ARRAY [SHORT,SHORT] OF REAL;

BEGIN

{CHECK FIELDS FOR VALID DATA}

IF  (SFOT >= 0) AND (SLOT >=0) AND (CFOT >= 0) AND (CLOT >= 0)
   THEN BEGIN
      BIG := VALUEOF [IXSUBJ,ONE]:
      IF (BIG > 0)
         THEN BEGIN

{CALCULATE FOM FOR TIME}

                 {RSFOT := FLOAT( (SFOT);???}
                 {RSLOT := FLOAT( (SLOT);???}
                 {RCFOT := FLOAT( (CFOT);???}
                 {RCLOT := FLOAT( (CLOT);???}

{IF THERE IS ANY OVERLAP IN TIME INTERVALS, SET FOM TO 1}
```

```
{IS A VALUE BETWEEN [0,1]}

                  IF ((CFOT >= SFOT) AND (CFOT <= SLOT ))
                     THEN FOMCMP [2] := 1.0
                     ELSE IF ((SFOT >= CFOT) AND (SFOT <= CLOT))
                        THEN FOMCMP [2] := 1.0

                        ELSE IF ((RCFOT > RSLOT) AND (RCFOT < (RSLOT + BIG)))
                           THEN FOMCMP [2] := 1.0 - (RCFOT - RSLOT)/BIG
                           ELSE IF ((RCLOT > RSFOT - BIG) AND (RCLOT < RSFOT))
                              THEN FOMCMP [2] := 1.0 - (RSFOT - RCLOT)/BIG
            END
            ELSE ITERR[2] := 1
      END
      ELSE IVCK[2] := 1

END; {OF PROCEDURE ACSRFT}

END. {OF MODULE ACSRFT}
```

```
[illegible]

PROCEDURE ACSRFF;

{NAME: ACSRFF-(FOM TEST RADAR FREQUENCY)}

{PURPOSE: THIS FIGURE OF MERIT TEST MEASURES THE LIKELIHOOD THAT THE}
{SUBJECT AND CANDIDATE EDRS REFER TO THE SAME ENTITY ON THE BATTLEFIELD}
{BASED ON THEIR FREQUENCIES.  THIS FIGURE OF MERIT IS COMPUTED AS A}
{FUNCTION OF THE ABSOLUTE DIFFERENCE BETWEEN THE SUBJECT AND CANDIDATE}
{FREQUENCIES.}

{RESTRICTIONS: NONE}

{INCLUDE "ACCEDR.COM"}
{INCLUDE "ACCRDP.COM"}
{INCLUDE "XCMPRP.COM"}
[illegible] CMPLOC/FOMCMP [illegible] ITERR [illegible]


TYPE

SHORT      = (ONE,TWO,THREE,FOUR,FIVE,[illegible]);

VAR

{LOCAL VARIABLES????}
NFS        : INTEGER;
NFC        : INTEGER;
[illegible] : INTEGER;
SMALL      : REAL;
[illegible] : REAL;
DIFF       : REAL;
TDIFF      : REAL;
D          : REAL;
[illegible] : REAL;

{GLOBAL VARIABLES}
SFREQ  : ARRAY [1..5] OF REAL;
CFREQ  : ARRAY [1..5] OF REAL;
[illegible] : ARRAY [1..5] OF INTEGER;
ITERR  : ARRAY [1..5] OF INTEGER;
FOMCMP : ARRAY [1..5] OF REAL;
{PERHAPS A FUNCTION PROCEDURE}
VALUEOF: ARRAY [SHORT,SHORT] OF REAL;

{BEGIN FOM TEST FOR RADAR FREQUENCY}

BEGIN

{DETERMINE # OF VALID FREQUENCIES}

NFS := 0;
NFC := 0;

FOR I := 1 TO 5 DO
    BEGIN
        IF SFREQ[I] [illegible] 0.0
            THEN NFS := NFS + 1;

        IF CFREQ[I] [illegible] 0
            THEN NFC := NFC + 1;
    END;

[illegible]
```

```
IF ( [illegible]S   0  AND (NF[illegible] > 0
   THEN BEGIN

{CHECK FOR VALID TABLE VALUES}

        SMALL := VALUEOF [IXSUBJ,THREE];
        RMAX := VALUEOF [IXSUBJ,TWO];

        IF ((SMALL > 0.0) AND (RMAX > 1.0))
           THEN BEGIN

{FIND THE PAIR OF FREQUENCIES CLOSEST TOGETHER}

              K := 1;
              L := 1;
              DIFF := ABS (SFREQ[1] - CFREQ[1]);

              FOR I := 1 TO NFS DO
              FOR J := 1 TO NFC DO
                 BEGIN
                    TDIFF := ABS (SFREQ[I] - CFREQ[J]);
                    IF (TDIFF < DIFF)
                       THEN BEGIN
                          K := I;
                          L := J;

                          DIFF := TDIFF;
                       END;
                 END;

{CALCULATE F [illegible] USING SELECTED PAIR OF FREQUENCIES}

                    D := ABS (SFREQ[K] - CFREQ[L]);

                    IF (D >= SMALL)
                       THEN BEGIN

                          IF (SFREQ [K] > CFREQ[L])
                             THEN PRO := SFREQ[K] / CFREQ[L]
                             ELSE PRO := CFREQ[L] / SFREQ[K];

                          IF (PRO < RMAX)
                             THEN FOMCMP[3] := 1.0-(PRO-1.0) [illegible]

                       END
                       ELSE FOMCMP[3] := 1.0
           END
           ELSE ITERR[3] := 1
   END
     ELSE  [illegible]CK[3] := 1

END; {OF PROCEDURE ACSREF}

END. {OF MODULE ACSREF}
```

```
[illegible] ACSRFI (INPUT, OUTPUT)

PROCEDURE ACSRFI:

(NAME: ACSRFI-(FOM TEST FOR PULSE RATE INTERVAL))
                  ((PRI TEST #2))

(PURPOSE: THIS FOM TEST MEASURE THE LIKELIHOOD BASED ON THE [illegible]
(OF THEIR PULSE REPETITION INTERVALS THAT THE SUBJECT AND CANDIDATE
(EIRS REFER TO THE SAME ENTITY ON THE BATTLEFIELD.  THIS FIGURE OF
(MERIT IS BASED ON THE ABSOLUTE DIFFERENCE BETWEEN THE CANDIDATE PRI
(AND THE SUBJECT PRI (OR A HARMONIC OF THE SUBJECT PRI).)

(RESTRICTIONS: NONE)

(PARAMETER [illegible] := 0E-[illegible])
(INCLUDE [illegible]EDR.COM[illegible])
(INCLUDE [illegible]EIRDR.COM[illegible])
(INCLUDE [illegible]MPRR.COM[illegible])
COMMON [illegible] FOMCMP[illegible], IUCK[illegible], ITERR[illegible]


TYPE
   SHORT      = (ONE,TWO,THREE,FOUR,FIVE,SIX,SEVEN,EIGHT,UNKNOWN)

VAR

[illegible] VARIABLES????)
[illegible]       : REAL:
[illegible]       : REAL,
[illegible]       : REAL:
[illegible]       : REAL;
[illegible]       : REAL:
[illegible]       : REAL;
[illegible]       : REAL;
[illegible]       : REAL;

[illegible]       : SHORT;

(GLOBAL VARIABLES)
[illegible]       : ARRAY [1..5] OF INTEGER:
ITERR       : ARRAY [1..5] OF INTEGER;
FOMCMP      : ARRAY [1..5] OF REAL:
(PERHAPS A FUNCTION PROCEDURE)
VALUEOF: ARRAY [SHORT,SHORT] OF REAL;

(BEGIN FOM TEST FOR PRI #2)

BEGIN

(RETURN TO SUBFUNCTION IF DATA IS INVALID)

IF   [illegible] 0) AND [illegible] 0)
   THEN BEGIN

(RETURN TO SUBFUNCTION IF BOTH ARE CONTINUOUS [illegible])

      IF   [illegible] = [illegible] OR [illegible] = [illegible]
         THEN BEGIN

[illegible]

               [illegible]
               [illegible]
```

```
         IF  (SPRI  < CMPPRI) AND  (CPRI  < CMPPRI)
            THEN BEGIN
               D := ABS(SPRI - CPRI);
               IF ((PIMIN > 0) AND (RPRI > 1.0))
                  THEN IF (D > PIMIN)
                     THEN BEGIN
                        IF (SPRI > CPRI)
                           THEN PRO := SPRI/CPRI
                           ELSE PRO := CPRI/SPRI;
(COMPUTE FOM FOR PULSE RATE INTERVAL)

                        IF (PRO < RPRI)
                           THEN FOMCMP[4] := 1.0*(PRO-1.0)/(RPRI-1.0)
                     END
                  ELSE FOMCMP[4] :=  1.0
               ELSE ITERR[4] := 1

            END

      END
      ELSE FOMCMP[4] := 1.0

   ELSE ITERR[4] := 1
END (OF PROCEDURE)

END (OF MODULE ACSPFI)
```

```
MODULE ACSRFD (INPUT,OUTPUT);

PROCEDURE ACSRFD;

{NAME: ACSRFD-(FOM TEST FOR RADAR PULSE DURATION)}

{PURPOSE: THIS FIGURE OF MERIT TEST MEASURES THE LIKELIHOOD BASED ON}
{THEIR PULSE DURATIONS THAT THE SUBJECT AND CANDIDATE EDRS REFER TO}
{THE SAME ENTITY ON THE BATTLEFIELDS.  THIS FOM IS BASED ON THE RATIO}
{OF THE SUBJECT AND CANDIDATE PULSE DURATIONS.}

{INCLUDE "ACCEDR.COM"}
{INCLUDE "ACCRDR.COM"}
{INCLUDE "XCMPRR.COM"}
{COMMON/CPPLOC/FOMCMP(5),IVCK(5),ITERR(5)}


TYPE

   SHORT    =  (ONE,TWO,THREE,FOUR,FIVE,SIX,SEVEN,EIGHT,UNKNOWN);

VAR {LOCAL VARIABLES ????}

PDMIN    : REAL;
RPDMIN   : REAL;
RPDMAX   : REAL;
SPDUR    : REAL;
PDUR     : REAL;
D        : REAL;
CPDUR    : REAL;
RPD      : REAL;

{GLOBAL VARIABLES}
IVCK    : ARRAY [1..5] OF INTEGER;
ITERR  : ARRAY [1..5] OF INTEGER;
FOMCMP : ARRAY [1..5] OF REAL;

IXSUBJ: SHORT;

{PERHAPS A FUNCTION PROCEDURE}
VALUEOF: ARRAY [SHORT,SHORT] OF REAL;

{EXECUTION TIME}

BEGIN

IXSUBJ := UNKNOWN;
PDMIN  := VALUEOF [IXSUBJ,SIX];
RPDMIN := VALUEOF [IXSUBJ,SEVEN];
RPDMAX := VALUEOF [IXSUBJ,EIGHT];

IF ((SPDUR  = 0.0) OR (CPDUR  = 0.0))
   THEN IVCK[5] := 1
   ELSE IF ((PDMIN <= 0.0) OR (0.0  = 0.0) OR (RPDMAX  = 0.0))
      THEN ITERR[5] := 1
      ELSE IF (RPDMAX  < RPDMIN)
         THEN ITERR[5]:=1
         ELSE BEGIN

{COMPUTE FOM FOR PULSE DURATION}

            D := ABS(SPDUR - CPDUR);

{IF D IS LESS THAN OR EQUAL TO}
[illegible]
```

```
                IF (D > PDMIN)
                   THEN BEGIN

{COMPUTE FOM BETWEEN 0 AND 1}

                         IF (SPDUR > CPDUR)
                            THEN PRO := SPDUR/CPDUR
                            ELSE PRO := CPDUR/SPDUR;

                         IF (PRO <= RPDMIN)
                            THEN FOMCMP[5] := 1.0
                            ELSE IF (PRO < RPDMAX)
                               THEN FOMCMP[5] := 1.0 - (PRO - RPDMIN) /
                                                       (RPDMAX - RPDMIN);

                      END
                      ELSE FOMCMP[5] := 1.0
              END

END; {OF PROCEDURE ACSRFD}

END. {OF MODULE ACSRFD}
```

APPENDIX C

C. DATA BASE ENTRIES

```
1 BE_ACPRPC
  2   BE_ACSRES
   3      BE_ACSCBN
   3      BE_ACSLBQ
  2   BE_ACSPAD
  2   BE_ACSMOD
  2   BE_ACSRDB
   3      BE_ACSRBP
   3      BE_ACSRXQ
   3      BE_ACSRCG
   3      BE_ACSRLB
   3      BE_ACSRCS
  2   BE_A[illegible]RET
   3      BE_ACSRLT
   3      BE_ACSRFT
   3      BE_ACSRFF
   3      BE_ACSRFI
   3      BE_ACSRFD
  2   BE_ACSRAT
   3      BE_ACSLBQ
  2   BE_ACSRIJ
   3      BE_ACSRQX
    4         BE_ACSRQX
TSA158 Loop detected (see level 3) - Structure truncated
    4         BE_ACSSGA
    4         BE_ACSSGD
   3      BE_ACSRGA
   3      BE_ACSRQD
```

Level	Count	Level	Count	Level	Count	Level	Count
1	1	2	7	3	15	4	3

* means UTILIZES

Utilizes Structure

```
 COUNT LEVEL            NAME

   1   1 0     BE_ACPRPC
   2   1 1         BE_ACSRES
   3   1 1 1           BE_ACSCBN
   4   1 1 2           BE_ACSLSQ
   5   1 2         BE_ACSRAD
   6   1 3         BE_ACSMOD
   7   1 4         BE_ACSRDB
   8   1 4 1           BE_ACSRSP
   9   1 4 2           BE_ACSRXG
  10   1 4 3           BE_ACSRCD
  11   1 4 4           BE_ACSRLB
  12   1 4 5           BE_ACSRDS
  13   1 5         BE_ACSRET
  14   1 5 1           BE_ACSRLT
  15   1 5 2           BE_ACSRFT
  16   1 5 3           BE_ACSRFF
  17   1 5 4           BE_ACSRFI
  18   1 5 5           BE_ACSRFD
  19   1 6         BE_ACSPAR
  20   1 6 1           BE_ACSLSQ
  21   1 7         BE_ACSRQU
  22   1 7 1           BE_ACSRQX
  23   1 7 1 1             BE_ACSRQX   *
  24   1 7 1 2             BE_ACSSQA
  25   1 7 1 3             BE_ACSSQD
  26   1 7 2           BE_ACSRQA
  27   1 7 3           BE_ACSRQD
  28   2 0     BE_ACSLSQ
 PSA451 "BE_ACSLSQ" has no SUBPARTS and does not UTILIZE any PROCES
  29   3 0     BE_ACSMOD
 PSA451 "BE_ACSMOD" has no SUBPARTS and does not UTILIZE any PROCES
  30   4 0     BE_ACSPAR
  31   4 1         BE_ACSLSQ
  32   5 0     BE_ACSRAD
 PSA451 "BE_ACSRAD" has no SUBPARTS and does not UTILIZE any PROCES
  33   6 0     BE_ACSRDB
  34   6 1         BE_ACSRSP
  35   6 2         BE_ACSRXG
  36   6 3         BE_ACSRCD
  37   6 4         BE_ACSRLB
  38   6 5         BE_ACSRDS
  39   7 0     BE_ACSRES
  40   7 1         BE_ACSCBN
  41   7 2         BE_ACSLSQ
  42   8 0     BE_ACSRET
  43   8 1         BE_ACSRLT
  44   8 2         BE_ACSRFT
  45   8 3         BE_ACSRFF
  46   8 4         BE_ACSRFI
  47   8 5         BE_ACSRFD
  48   9 0     BE_ACSRQU
```

```
COUNT LEVEL             NAME

  49   9.1         BE_ACSRQX
  50   9.1.1           BE_ACSRQX   *
  51   9.1.2           BE_ACSSGA
  52   9.1.3           BE_ACSSGD
  53   9.2         BE_ACSRGA
  54   9.3         BE_ACSRGD
  55   10.0    BE_ACSRQX
  56   10.1        BE_ACSRQX
  57   10.1.1          BE_ACSRQX   *
  58   10.1.2          BE_ACSSGA
  59   10.1.3          BE_ACSSGD
  60   10.2        BE_ACSSGA
  61   10.3        BE_ACSSGD
```

* Loop encountered (a PROCESS is directly or indirectly UTILIZED by itself)

Utilizes Matrix

Explanation of the Utilizes Matrix:

The rows are input PROCESS names, and the columns are PROCESSES UTILIZED by (or a SUBPART of) the rows

(i,j) value	meaning
U	Column j is UTILIZED by Row i
S	Column j is a PART of Row i
B	Column j is both UTILIZED by and a PART of Row i

Utilizes Matrix

1 BE_ACSRES
2 BE_ACSRAD
3 BE_ACSMOD
4 BE_ACSRDB
5 BE_ACSRET
6 BE_ACSPAR
7 BE_ACSRQU
8 BE_ACSCBN
9 BE_ACSLSQ
10 BE_ACSRSP
11 BE_ACSRXQ
12 BE_ACSRCQ
13 BE_ACSRLB
14 BE_ACSRCS
15 BE_ACSRLT
16 BE_ACSRFT
17 BE_ACSRFF
18 BE_ACSRFI
19 BE_ACSRFD
20 BE_ACSRQX
21 BE_ACSRQA
22 BE_ACSRQD
23 BE_ACSSQA
24 BE_ACSRQD

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
1 BE_ACPRPC	U	U	U	U	U	U	U																	
2 BE_ACSRES								U	U															
3 BE_ACSRDB										U	U	U	U	U										
4 BE_ACSRET															U	U	U	U	U					
5 BE_ACSPAR									U															
6 BE_ACSRQU																				U	U	U		
7 BE_ACSRQX																				U			U	U

Count Table for Row Names

Row	Name	Type	SUBPARTS	UTILIZES	Both
1	BE_ACPRPC	PROCESS	0	7	0
2	BE_ACSRES	PROCESS	0	2	0
3	BE_ACSRDB	PROCESS	0	5	[illegible]
4	BE_ACSRIT	PROCESS	0	5	[illegible]
5	BE_ACSFAR	PROCESS	0	1	
6	BE_ACSRGU	PROCESS	0	3	[illegible]
7	BE_ACSFGX	PROCESS	0	3	[illegible]
	Total		0	[illegible]	[illegible]
	Average		0.00	[illegible]	

Count Table for Column Names

Column	Name	Type	PART OF	UTILIZED	Both
1	BE_ACSRES	PROCESS	0	1	0
2	BE_ACSRAD	PROCESS	0	1	0
3	BE_ACSMOD	PROCESS	0	1	0
4	BE_ACSRCD	PROCESS	0	[illegible]	[illegible]
5	BE_ACIRET	PROCESS	0	1	[illegible]
6	BE_ACSRAA	PROCESS	0	[illegible]	[illegible]
7	BE_ACSRIU	PROCESS	0	1	0
8	BE_ACSCBM	PROCESS	0	1	0
9	BE_ACSLTD	PROCESS	0	3	[illegible]
10	BE_ACSREP	PROCESS	0	[illegible]	[illegible]
11	BE_ACSRAQ	PROCESS	0	[illegible]	[illegible]
12	BE_ACSRCD	PROCESS	0	1	[illegible]
13	BE_ACSRLD	PROCESS	0	1	[illegible]
14	BE_ACSRCB	PROCESS	0	[illegible]	[illegible]
15	BE_ACSRLT	PROCESS	0	[illegible]	[illegible]
16	BE_ACSRFT	PROCESS	0	[illegible]	[illegible]
17	BE_ACSRFR	PROCESS	0	[illegible]	[illegible]
18	BE_ACSRRI	PROCESS	0	[illegible]	[illegible]
19	BE_ACSRFD	PROCESS	0	[illegible]	[illegible]
20	BE_ACSRQK	PROCESS	0	2	[illegible]
21	BE_ACSRQA	PROCESS	0	[illegible]	[illegible]
22	BE_ACSRQD	PROCESS	0	[illegible]	[illegible]
23	BE_ACSSVA	PROCESS	0	[illegible]	[illegible]
24	BE_ACSSQD	PROCESS	0	[illegible]	[illegible]
	Total		0	2[illegible]	[illegible]
	Average		0.00	1.[illegible]	

	Tree Level
BE_ACPRPC	ROOT
BE_ACBLBG	leaf
BE_ACSMCD	leaf
BE_ACBPAF	middle
BE_ACBRAD	leaf
BE_ACBRDB	middle
BE_ACBRES	middle
BE_ACCRET	middle
BE_ACBRQU	middle
BE_ACBRQX	middle

Mathematical Field

BE_ACPRPC	Computer Science
BE_ACSLEQ	Statistics
BE_ACSMOD	Computer Science
BE_ACSPAR	"
BE_ACSRAD	"
BE_ACSRDB	Decision Theory
BE_ACSRES	"
BE_ACSRET	Decision Theory, Statistics
BE_ACSRQU	Computer Science
BE_ACSRQX	"

APPENDIX D

D. VECTOR RESEARCH ANALYSIS OF BETA

The following three pages on the BETA system were provided by Vector Research Inc. Section D-1,, "A Location Estimating Algorithm," presents maximum likelihood estimating methods for radar location supported by more than one sensor system. Section D-2, "Algorithm Attributes," considers the BETA system from a Bayesian perspective. Section D-3, "BETA Self-Correlationnn: Evaluation Methodology," discusses some of the more philosophical aspects of algorithm anaysis.

These analyses are referred to within the related sections of the main report, and some of the more significant conclusions incorporated in the final observationss and conclusions (Section 5).

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A LOCATION ESTIMATING ALGORITHM

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1.0 INTRODUCTION

One of the principal functions performed by BETA is the development of estimates of target locations. This chapter examines one algorithm that updates estimates of target locations as successive reports are received and processed. The discussion that follows assumes that the reports are correctly associated, that is, that each provides an estimate of the same location.

2.0 ASSUMPTIONS

Errors in estimating location arise primarily from sensor performance. As such, sources are both random and systematic, examples of the latter being improper calibration or incorrect locations of sensor components. For the discussion which follows we assume that systematic errors are zero or equivalently that calibration has removed bias. We further assume that target location estimates are distributed according to a bivariate normal distribution with the true location given by the mean $\underline{\mu} = \begin{pmatrix} \mu 1 \\ \mu 2 \end{pmatrix}$

Reported locations are denoted by $\underline{X}_i = \begin{pmatrix} X_{1i} \\ X_{2i} \end{pmatrix}$

with covariance matrix $A_i = \begin{pmatrix} \sigma_{1i}{}^2 & \rho_i \sigma_{1i} \sigma_{2i} \\ \rho_i \sigma_{1i} \sigma_{2i} & \sigma_{2i}^2 \end{pmatrix}$

where σ_{1i}^2 and σ_{2i}^2 are the variances of the random variables X_{1i} and X_{2i}, respectively and ρ_i is the correlation coefficient associated with their covariance. Thus, the density function of a reported location x is:

$$\frac{1}{2\pi |A|^{1/2}} \quad \exp\left(- \frac{1}{2} (\underline{x}-\underline{\mu})^T A^{-1} (\underline{x}-\underline{\mu})\right)$$

Reported locations are assumed to be independent, with different covariance matrixes corresponding to different sensors and/or single sensors in different locations relative to the target. We assume that if a report is based on several lines of bearing, or a series of fixes, then the appropriate reduction in variance is reflected in the covariance matrix used in updating the location estimate, i.e., the covariance matrix is always known.

3.0 MAXIMUM LIKELIHOOD ESTIMATE

As an estimate of target location, the maximum likelihood estimate is derived.[1] Consider a collection of reported target locations x_i ; i=i=1,2,...,N with covariance matrixes A_i ; i=1,2,...,N. Then the likelihood the likelihood function is proportional to:

$$\exp\left(-1/2 \sum_{i=1}^{N} (\underline{x}_i-\underline{\mu})^T A_i^{-1}(x_i-\underline{\mu})\right)$$

Taking partial derivatives with respect to μ_1 and μ_2 and equating the results to zero one obtains for the estimates $\hat{\mu}_1$ and $\hat{\mu}_2$:

$$\left(\sum_{i=1}^{N} A_i^{-1}\right)\underline{\hat{\mu}} = \left(\sum_{i=1}^{N} A_i^{-1}\underline{x}_i\right)$$

or

$$\underline{\hat{\mu}} = \left(\sum_{i=1}^{N} A_i^{-1}\right)^{-1} \cdot \sum_{i=1}^{N}\left(A_i^{-1}\underline{x}_i\right).$$

The expected value of $\underline{\hat{\mu}}$ is the true location $\underline{\mu}$ and the covariance matrix is

$$\left(\sum_{i=1}^{N} A_i^{-1}\right)^{-1}$$

Note that in terms of implementing this algorithm for estimating $\underline{\mu}$ one must store two quantities namely:

$$M_N \quad \sum_{i=1}^{N}\left(A_i^{-1}\right)\underline{x}_i$$

and

$$C_N \quad \sum_{i=1}^{N} A_i^{-1}$$

[1] For a more general discussion see: Anderson, T.W., An Introduction to Multivariate Statistics, John Wiley and Sons, Inc., New York, 1958.

Upon receiving the N+1st report, $\underline{X}_{N+1}$, A_{N+1}

$$M_{N+1} = M_N + A_{N+1}^{-1}\underline{X}_{N+1}$$

and $$C_{N+1} = C_N + A_{N+1}^{-1},$$

and $$\hat{\underline{\mu}}_{N+1} = C_{N+1}^{-1}M_{N+1}$$

with covariance C_{N+1}^{-1}.

4.0 ORIENTATION

One of the features of current ELINT sensor systems is that errors in range generally are larger than those in azimuth. Further, because of different sensor system locations error ellipses will have different orientations. Consider a standard Cartesian coordinate system with $\underline{\mu} = \underline{o}$. The the error ellipse is described by $\underline{x}^T A^{-1} \underline{x}$, where

$$A^{-1} = \begin{pmatrix} \frac{1}{(1-\rho^2)\sigma_1{}^2} & \frac{-\rho}{(1-\rho^2)\sigma_1\sigma_2} \\ \frac{-\rho}{(1-\rho^2)\sigma_1\sigma^2} & \frac{1}{(1-\rho_2)\sigma_2{}^2} \end{pmatrix}$$

or

$$\underline{x}^T A^{-1} \underline{x} = \frac{x_1{}^2}{(1-\rho^2)\sigma_1{}^2} - \frac{2x_1x_2}{(1-\rho^2)\sigma_1\sigma_2} + \frac{x_2{}^2}{(1-\rho^2)\sigma_2{}^2}.$$

Consider the rotation

$$y_1 = x_1 \cos\theta - x_2 \sin\theta$$

$$y_2 = x_1 \sin\theta + x_2 \cos\theta$$

where

$$\theta = \frac{1}{2} \tan^{-1} \left(\frac{2\rho\sigma_1\sigma_2}{\sigma_1{}^2 - \sigma_2{}^2} \right)$$

Applying this rotation we obtain a new random vector $\begin{pmatrix} y_1 \\ y_2 \end{pmatrix}$ with covariance matrix:

$$\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \sigma_1^2 & \rho\sigma_1\sigma^2 \\ \rho\sigma_1\sigma_2 & \sigma_2^2 \end{pmatrix} \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$

which has zero correlation, or equivalently θ is the orientation of the error ellipse with the baseline of the coordinate system.

5.0 OBSERVATIONS

To illustrate the implications of the target location algorithm consider a simplified example in which all reported locations have covariance matrix

$$\begin{pmatrix} \sigma_1^2 & 0 \\ 0 & \sigma_1^2 \end{pmatrix}$$

In this case after N reports, the covariance matrix of the estimate is:

$$\begin{pmatrix} \frac{\sigma_1^2}{N} & 0 \\ 0 & \frac{\sigma_1^2}{N} \end{pmatrix}$$

If artillery accuracy requirements are expressed in terms of a 100 meter CEP we require an equivalent variance of:

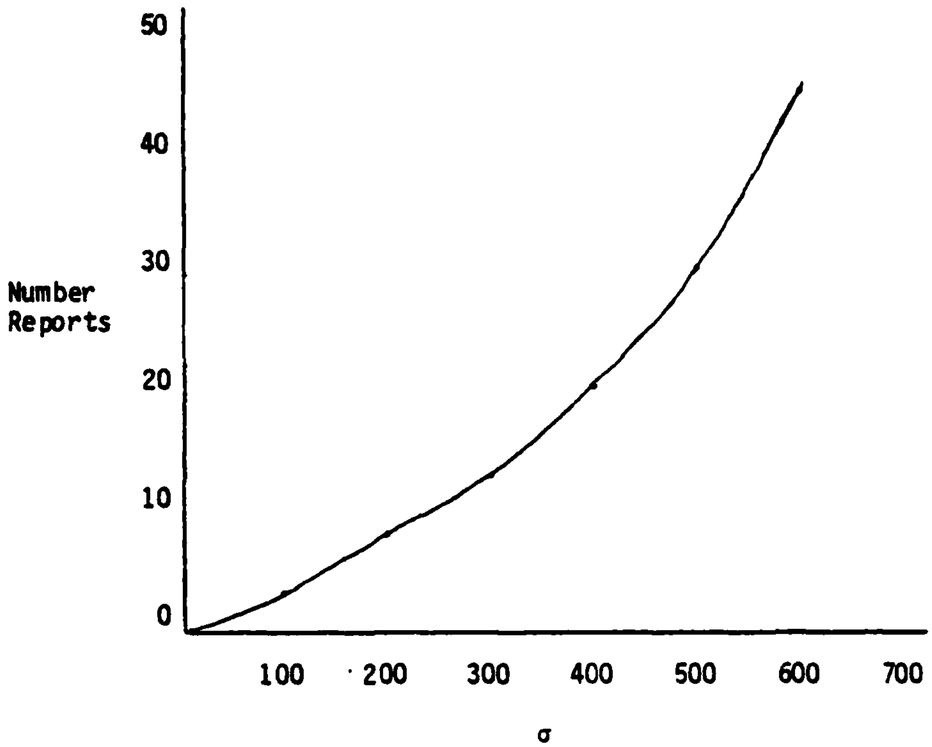
$$\sigma_R = \frac{100}{(2 \ln 2)^{1/2}} = 84.93$$

Since σ_1 is constant across all reports, the number of reports required to achieve targeting accuracy is given by:

$$N = \left[\frac{\sigma}{84.93}\right]^2$$

This function is presented in exhibit 5-1.

EXHIBIT 5-1: NUMBER REPORTS REQUIRED TO ACHIEVE 100 METER CEP



The implications are that achieving adequate accuracy will require a significant number of reports unless sensor performance improves substantially. For example, consider a sensor which has a covariance matrix:

$$\begin{pmatrix} (200)^2 & 0 \\ 0 & (500)^2 \end{pmatrix}$$

representing a system with range errors greater than azimuth errors. Using an approximation to obtain an equivalent CEP it can be shown that 16 reports would be required to achieve an equivalent CEP of 100 meters. In terms of current ELINT systems, and artillery target location accuracy requirements, the implication is that until precision location systems are fielded the number of targets developed and nominated for artillery missions will be limited. This may not preclude use of air assets; both fixed wing and helicopters. Note, moreover, that the introduction of precision location systems contributes to the efficiency of self-correlation, but if errors are sufficiently small, self correlation is not as necessary for target nomination. This raises the issue of emphasis in the system: is it target development, situation development, or both? If target development is emphasized, it is difficult with the current sensor suite. On the other hand, if precision location systems are introduced, location estimation is unnecessary and context may be more important.

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ALGORITHM ATTRIBUTES

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1.0 INTRODUCTION

In order to provide an approach to evaluating and classifying self-correlation algorithms, this paper examines such algorithms in the context of Bayesian decision theory.[1] In particular, comparisons are made of example Bayesian decision criteria with one approach used in BETA, namely:

$$\text{"correlate if} \sum a_i x_i \geq h."$$

In certain instances reports received by BETA do not contain values for one or more of the data elements. In such cases it is our understanding that the procedure implemented involves renormalizing the weights used in the statistic, i.e., if figure of merit x_j cannot be calculated, the weights used are:

$$\frac{a_i}{\sum_{i \neq j} a_i} .$$

The implications of this procedure are also examined.

The purpose of algorithms such as those employed in BETA is to make decisions based on some pre-established criteria. Any particular decision is made by considering data reported by collection systems. As such, the data is subject to a series of errors which cannot be predicted with certainty in advance. For example, suppose an emitter is operating at a frequency f. The frequency f_r reported by a collection system will not in general equal f, but

$$f_r = f + \Delta f_s$$

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[1]For a more general discussion see: DeGroot, Morris H., Optimal Statistical Decisions, McGraw-Hill, Inc., New York, 1970.

where the difference Δf_s is a function of collection system performance, environment, battlefield geometry, etc. The difference cannot be predicted in advance, but can be described probabilistically, i.e., the reported frequency f_r is a random variable with an appropriate distribution function. Similarly, the absolute value fo the difference between two reported frequencies is a random variable, although it need not be a figure of merit in the BETA sense, i.e., take values only between zero and one. However, it is worth noting that if $\underline{X}$ is a random variable with distribution function F(x):

$$\Pr[\underline{X} \leq x] = F(x),$$

then the random variable Y defined by

$$Y = F(\underline{X})$$

takes values only between zero and one and moreover has a *uniform* distribution, i.e.,

$$\Pr[Y \leq y] \begin{cases} = y & 0 \leq y \leq 1 \\ = 0 & y \leq 0 \\ = 1 & y \geq 1. \end{cases}$$

The location difference figure of merit used in BETA, exp(-d/2), has this distribution if the two reported locations are in fact the same.

In general, algorithms such as those in BETA asume that ground truth falls into a set of mutually exclusive categories or states of nature (denoted in this paper by W_i, only one of which is true). The algorithms provide a means of deciding which particular state of nature is true using observed or collected data. The assumption underlying most such algorithms is that the distribution functions describing the random nature of the data depend upon the states of nature. Differences among the distributions are then used to construct appropriate decision criteria and/or analyze algorithm performance. In the remainder of this paper a Bayesian structure is used as a framework to analyze the linear combination of figures of merit used in BETA self-correlation algorithms.

2.0 MODEL

Consider a decision problem involving only two states of nature $\Omega = \{w_1, w_2\}$ and two decisions $D = \{d_1, d_2\}$. Following the observation of a random vector $\underline{x}$, a decision is made with loss matrix:

$$\begin{array}{c} \\ w_1 \\ w_2 \end{array} \begin{array}{c} \begin{matrix} d_1 & d_2 \end{matrix} \\ \begin{pmatrix} 0 & \ell_{12} \\ \ell_{21} & 0 \end{pmatrix} \end{array} .$$

i.e., if the decision d_1 is taken when w_2 is true, a loss of ℓ_{21} is incurred. Prior to the observation of $\underline{x}$, the decision maker believes:

$$\Pr[W = w_1] = p, \text{ and}$$

$$\Pr[W = w_2] = 1-p,$$

with the conditional and unconditional densities of $\underline{x}$ given by $f(\underline{x}|w_i)$ and $f(\underline{x})$, respectively. In this situation it can be shown that the Bayes decision, i.e., that which minimizes the expected loss, is described by:

Decide $d_1 (w = w_1)$ if

$$\Pr[W = w_1|\underline{x}] \geq \frac{\ell_{21}}{\ell_{21} + \ell_{12}} = \ell_0 \text{say}$$

and d_2 $(W = w_2)$ if

$$\Pr[W = w_2|\underline{x}] \geq \frac{\ell_{21}}{\ell_{21} + \ell_{12}}$$

The posterior probability $Pr[W = w_1 | x]$ is given by:

$$\frac{pf(\underline{x}|w_1)}{pf(\underline{x}|w_1) + (1-p)f(\underline{x}|w_2)} .$$

Substituting one obtains the rule: decide d_1 if:

$$\frac{f(\underline{x}|w_1)}{f(\underline{x}|w_2)} > \frac{\ell_0(1-p)}{p(1-\ell_0)} = \gamma \text{ say}.$$

3.0 EXAMPLES

Consider the case when the components of $\underline{X}$ are independent and normally distributed. Suppose:

$$f_i(x_i|w_1) = \frac{1}{\sqrt{2\pi}\sigma_i} \exp\left(\frac{-x_i^2}{2\sigma_i^2}\right)$$

and

$$f_i(x_i|w_2) = \frac{1}{\sqrt{2\pi}\sigma_i} \exp\left(\frac{-(x_i-\mu_i)^2}{2\sigma_i^2}\right) .$$

Then from the previous examples we have:

$$\frac{f(\underline{x}|w_1)}{f(\underline{x}|w_2)} = \exp\left\{-\left\{\sum \frac{x_i^2}{2\sigma_i^2} - \sum \frac{(x_i-\mu_i)^2}{2\sigma_i^2}\right\}\right\}$$

or select d_1 if

$$\exp\left\{-\left(\sum\frac{x_i\mu_i}{\sigma_i^2} - \sum \frac{\mu_i^2}{2\sigma_i^2}\right)\right\} > \gamma .$$

Taking logarithms and rearranging terms:

$$\sum \frac{\mu_i x_i}{\sigma_i^2} < \sum \frac{\mu_i^2}{2\sigma_i^2} - \ln \gamma$$

This expression bears some resemblance to the BETA algorithm and it is worth considering the impact of the BETA procedure for missing data. First, note that if some value, say x_j, is missing, the new decision rule is select d_1 if

$$\sum_{i\neq j} \frac{\mu_i x_i}{\sigma_i^2} < \sum_{i\neq j} \frac{\mu_i^2}{2\sigma_i^2} - \ln \gamma$$

Now consider a normalization of the coefficients. In the first case by normalizing we obtain:

$$\sum\left(\frac{\mu_i}{\sigma_i^2}\right)\left(\sum\frac{\mu_i}{\sigma_i^2}\right)^{-1} x_i \quad < \quad \left(\sum \frac{\mu_i^2}{2\sigma_i^2} - \ln\gamma\right)\left(\sum \frac{\mu_i}{\sigma_i^2}\right)^{-1}$$

If element x_j is missing, renormalization of the coefficients yields for the left hand side

$$\sum_{i\neq j} \left(\left(\frac{\mu_i}{\sigma_i^2}\right)\left(\sum_{i\neq j}\frac{\mu_i}{\sigma_i^2}\right)^{-1} x_i\right)$$

Note, however, that renormalization is only consistent if an appropriate change is made to the right hand side of the inequality both in terms of the sum $\sum \frac{\mu_i^2}{2\sigma_i^2}$ and the normalizing factor.

Now suppose that

$$f_i(x_i|w_1) = \frac{1}{u_i-\ell_i} \qquad \ell_i \leq x_i < u_i$$

$$= 0 \qquad \text{elsewhere.}$$

In this case the x_i are uniformly distributed on the interval $[\ell_i, u_i]$ bearing some similarity to the BETA approach. Then the decision criterion based on $\underline{x}$ is:

select d_1 if

$$\frac{\sum_{i=1}^{n}(u_i-\ell_i)^{-1}}{f(\underline{x}|w_2)} \quad > \quad \frac{\ell_0(1-p)}{(1-\ell_0)p} \quad = \gamma \text{ say}$$

Note that if for any i

$$x_i \leq \ell_i$$

or

$$x_i \geq u_i$$

then d_1 is rejected. If this is not the case an appropriate threshold can be set.

Consider the expression:

$$\Pi f_i(x_i|w_2) < \frac{p(1-\ell_0)}{\ell_0(1-p)} \cdot \Pi(\mu_i-\ell_i)^{-1} = \gamma_i \text{say}$$

Again assume that $f_i(x_i|w_2)$ is a normal density with mean μ_i and variance σ_i^2. Substituting:

$$\frac{1}{\Pi(2\pi\sigma_i^2)^{1/2}} \exp\left\{-\sum \frac{(x_i-\mu_i)^2}{2\sigma_i^2}\right\} < \gamma_i$$

or $$\sum \frac{(x_i-\mu_i)^2}{2\sigma_i^2} > -\ln\gamma_i - \ln(\Pi(2\pi\sigma_i^2)^{1/2}).$$

Provided the figures of merit were in fact the $\frac{(x_i-\mu_i)^2}{2}$, then the coefficients could be considered to be $(\sigma_i^2)^{-1}$. The same normalizing problem as previously discussed exists, namely one must not only normalize but also adjust the decision threshold.

We turn now to the case of dependence between the components of the decision vector $\underline{x}$. For a variety of reasons this is likely to be the case, particularly for the signal parameters. We use the same example, namely decide d_1 if:

$$\frac{f(\underline{x}|w_1)}{f(\underline{x}|w_2)} > \gamma_0$$

As before assume:

$f(\underline{x}|w_1)$ is $N(\underline{0},\Sigma)$

and $f(\underline{x}|w_2)$ is $N(\underline{\mu},\Sigma)$,

with non-zero correlations, i.e.,

$$\Sigma = \begin{pmatrix} \sigma_1^2 & \rho\sigma_1\sigma_2 \\ \rho\sigma_1\sigma_2 & \sigma_2^2 \end{pmatrix}$$

where σ_1^2 and σ_2^2 are the variances of x_1 and x_2 and ρ, the correlation coefficient, is non-zero.

Substituting, the criteria becomes:

$$\frac{\exp\left(-\frac{1}{2}\underline{x}^T\Sigma^{-1}\underline{x}\right)}{\exp\left(-\frac{1}{2}(\underline{x}-\underline{\mu})^T\Sigma^{-1}(\underline{x}-\underline{\mu})\right)} > \gamma_0$$

or $-\frac{1}{2}\underline{x}^T\Sigma^{-1}\underline{x} + \frac{1}{2}(\underline{x}-\underline{\mu})^T\Sigma^{-1}(\underline{x}-\underline{\mu}) > \ln \gamma_0$

or $(\underline{x}-\underline{\mu})^T\Sigma^{-1}(\underline{x}-\underline{\mu}) - \underline{x}^T\Sigma^{-1}\underline{x} > 2\ln \gamma_0$.

This is equivalent to:

$$\underline{x}^T\Sigma^{-1}\underline{\mu} < \frac{1}{2}\underline{\mu}^T\Sigma^{-1}\underline{\mu} - \ln\gamma_0.$$

As in the case of independence, this criteria can be expressed as a linear discriminant. In particular, for n=2, the coefficients are:

$$x_1 : \left\{\frac{\sigma_2^2\mu_1 - \rho_{12}\sigma_1\sigma_2\mu_2}{\sigma_1^2\sigma_2^2(1-\rho_{12}^2)}\right\}$$

$$x_2 : \left\{\frac{-\rho_{12}\sigma_1\sigma_2\mu_1 + \sigma_1^2\sigma_2}{\sigma_1^2\sigma_2^2(1-\rho_{12}^2)}\right\}$$

where ρ_{ij} are correlation coefficients,

and

$$\underline{\mu}^T\Sigma^{-1}\underline{\mu} = \frac{\mu_1^2}{(1-\rho_{12}^2)\sigma_1^2} - \frac{2\rho_{12}\mu_1\mu_2}{\sigma_1\sigma_2(1-\rho_{12}^2)} + \frac{\mu_2^2}{\sigma_2^2(1-\rho_{12}^2)}.$$

For n=3 the coefficient of x_1 becomes:

$$\frac{(1-\rho_{23}^2)\sigma_2^2\sigma_3^2\mu_1 + \sigma_1\sigma_2\sigma_3^2(\rho_{13}\rho_{23}-\rho_{12})\mu_2 + \sigma_1\sigma_2^2\sigma_3(\rho_{12}\rho_{23}-\rho_{13})\mu_3)}{\sigma_1^2\sigma_2^2\sigma_3^2(1-\rho_{12}^2-\rho_{13}^2-\rho_{23}^2 + 2\rho_{12}\rho_{13} + 2\rho_{12}\rho_{23} + 2\rho_{13}\rho_{23})}$$

and for x_2

$$\frac{\sigma_1\sigma_2\sigma_3^2(\rho_{13}\rho_{23}-\rho_{12})\mu_1 + (1-\rho_{13}^2)\sigma_1^2\sigma_3^2\mu_2 + \sigma_1\sigma_2\sigma_3(\rho_{12}\rho_{13}-\rho_{23})\mu_3)}{\sigma_1^2\sigma_2^2\sigma_3^2(1-\rho_{12}^2 -\rho_{13}^2-\rho_{23}^2 + 2\rho_{12}\rho_{13}+2\rho_{12}\rho_{23} + 2\rho_{13}\rho_{23})}.$$

While in the case of independence, provided the threshold was adjusted, normalization made sense, a similar conclusion cannot be made for dependence among the random variables, as can be easily seen by adding the coefficients for x_1 and x_2 in both cases and then dividing the coefficients by the resulting sum and comparing the results to a similar operation when n=2. Thus, the following observation: decision rules exist such that subject to alternation of the decision threshold, normalization to account for missing data can be justified if the various components of the decision variable $\underline{x}$ are independent. If, on the other hand, the components are dependent or correlated, normalization cannot be justified.

4.0 TIME

In the discussion up to this point, no mention has been made of time. Given any two candidates for self-correlation, the time first observed and time last observed are available. Suppose for the two candidates these times are:

Candidate 1: t_{10} t_{20}

Candidate 2: t_{11} t_{21}.

The values of these four variables give rise to three cases:

(1) {overlap, 1st candidate first; overlap, 2nd candidate first;}

(2) {overlap, 1st candidate contains 2nd; overlap, 1st candidate contained in 2nd;} and

(3) {no overlap, 1st candidate first; no overlap, 2nd candidate first} .

The analyst, decision maker, or in this case, algorithm, can and should distinguish among these three general cases. First, all other things considered, the prior probabilities for the first two probably remain the same, as do the conditional distributions or likelihood functions. In fact, in the extreme, e.g., simultaneous observation of an emission by different sensors, differences in reported data should only be due to variations in sensor performance and not to such controllable differences as selection of frequency, etc., whereas, if reports are separated in time, the conditional probabilities $f(x|w_1)$ may be more diffuse. Note that the location estimates are significant in this case: e.g., if the reports should be correlated, the location should be the same. This is not the case if the reported observation intervals do not overlap. For simplicity, consider four cases, distinguished by whether or not location estimates are "close" and whether or not signal parameters are "close." If location estimates are not close and signal parameters are not close, self-correlation is probably inappropriate. If location estimates are close and signal parameters are not close, the possibility of "co-located" emitters must be considered[1] or the possibility that a new emitter has replaced the original. Finally, if location estimates are not close but signal parametrics are consistent, the time and distance relationships must be considered, i.e., the potential displacement must be consistent with teardown, set-up, and travel times. The implications in terms of the simple Bayesian example are described as follows. For cases 1 and 2 above the analyst is likely to retain the same prior and likelihood functions. If locations are close, the signal parametrics will decide; if signal parametrics are close, locations will decide. As time elapses without a report, the probability that the entity has remained in the same location will decrease, as will the probability that its signal parametrics remain constant. For the sake of illustrat n let the time required for a typical displacement be Δt and the distance Δd. Further, suppose that the elapsed time is t, and that $\underline{x}$ measures location difference (x_1) and signal parametric difference (x_2). Then:

$f(x_1, x_2 | w_1)$ may be independent of t

$f(x_1, x_2 | w_2) = 0$ if $\Delta t > t$

$f(x_1, x_2 | w_3)$ may be independent of t

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[1]For example, the LONGTRACK radars at an SA-6 regimental headquarters.

where w_1 = same entity, same location, same parametrics

w_2 = same entity, new location, same parametrics

w_3 = different entity, different location, same or different parametrics.

As an illustration of the characteristics of the problem of time, consider a simplified situation with three decisions:

d_1: correlate: same entity, same location;

d_2: correlate: same entity, new location; and

d_3: do not correlate: different entities, different locations.

Then it can be shown that the same form of decision criteria can be derived, namely select decision d_i if the posterior probability of w_i is greater than a threshold γ_i, or

$$\frac{f(\underline{x}|w_i)p_i}{f(\underline{x}|w_1)p_1 + f(\underline{x}|w_2)p_2 + f(\underline{x}|w_3)p_3} > \gamma_i$$

Two elements are significant here. First, in the absence of reports (and perhaps given the knowledge of dwell time and likelihood of observations) the prior probability p_1 will decrease and the prior probability p_2 will increase. Second, the likelihood $f(\underline{x}|w_2)$ depends upon the time between observations. In particular, let X_1 be a measure of the distance between reported locations at times t_{20} and t_{11}, with $t = t_{11} - t_{20}$. Then

$$f(\underline{x}|w_2) = f(w_2|\underline{x}) \cdot \frac{p_2}{f(x)} .$$

If $t < \Delta_s + \Delta_t$, (tear down plus set up time), then

$$f(w_2|\underline{x}) = 0$$

and

$$f(\underline{x}|w_2) = 0.$$

Similarly, if $\Delta_s + \Delta_t < t < \frac{x_1}{v} + \Delta_s + \Delta_t$

where v is a typical travel speed, then

$$f(w_2|\underline{x}) = 0$$

and

$$f(\underline{x}|w_2) = 0.$$

Finally, if $t > \Delta_s + \Delta_t + \frac{d}{v}$ where d is a typical displacement distance, suppose that $f(x|w_2)$ is non-zero. Now consider the decision d_1. For d_1 we have:

$$\frac{p_1 f(x|w_1)}{p_1 f(x|w_1) + p_2 f(x|w_2) + p_3 f(x|w_3)} > \gamma_1$$

For $t < {}_s + {}_t + \frac{d}{v}$, this expression is

$$\frac{p_1 f(x|w_1)}{p_1 f(x|w_1) + p_3 f(x|w_3)} > \gamma_1$$

For $t > \Delta_s + \Delta_t + \frac{d}{v}$, the expression becomes

$$\frac{p_1 f(x|w_1)}{p_1 f(x|w_1) + p_2 f(x|w_2) + p_3 f(x|w_3)} > \gamma_1 .$$

Note that in the BETA algorithm time is used to derive a figure of merit. While all other figures of merit are random variables, time is not. Effectively it is known with certainty. Even in this very simplified example it is clear that inclusion of time in the decision criteria, treated as a random variable, is incorrect unless collection management and tasking are also considered in which case random variation associated with detection and signature generation must be considered as well.

5.0 TYPE I AND TYPE II ERRORS

In the preceding sections a Bayesian structure was utilized to examine simple algorithms and decision criteria for self-correlation. This structure is attractive because for a number of reasonably general distribution functions it provides computationally simple decision criteria. Given that such distributions were used as approximations to actual distributions (for reasons of clarity and or ease of use) it is likely that the resulting algorithms would be close to optimal. Aspects of the BETA problem are difficult and complex, particularly the nature of the decisions themselves and the prior probabilities. Nevertheless, the structure does offer insights into various algorithm attributes. For example, consider the issue of type I and type II errors. For the simple problem with two alternati. -, the decision rule derived was to select d_1 if:

$$\frac{f(\underline{x}|w_1)}{f(\underline{x}|w_2)} > \frac{\ell_0(1-p)}{p(1-\ell_0)}$$

Suppose the components of $\underline{x}$ are <u>independent</u> and normally distributed and let

$$f(\underline{x}|w_1) = N(0,\Sigma)$$

and

$$f(\underline{x}|w_2) = N(\mu,\Sigma)$$

where

$$\Sigma = \begin{pmatrix} \sigma_1^2 & 0 \\ 0 & \sigma_2^2 \end{pmatrix} .$$

For this example the criterion was:

$$\frac{\mu_1 x_1}{\sigma_1^2} + \frac{\mu_2 x_2}{\sigma_2^2} < \frac{\mu_1^2}{2\sigma_1^2} + \frac{\mu_2^2}{2\sigma_2^2} - \ln\left(\frac{\ell_0(1-p)}{p(1-\ell_0)}\right) .$$

The probability of a type I error in this case is:

$$\Pr\left[\frac{\mu_1 x_1}{\sigma_1^2} + \frac{\mu_2 x_2}{\sigma_2^2} > \frac{\mu_1^2}{2\sigma_1^2} + \frac{\mu_2^2}{2\sigma_2^2} - \ln\left(\frac{(\ell_0(1-p)}{(1-\ell_0)p)}\right) \middle| w_1\right],$$

and the probability of a type II error is:

$$\Pr\left[\frac{\mu_1 x_1}{\sigma_1^2} + \frac{\mu_2 x_2}{\sigma_2^2} < \frac{\mu_1^2}{2\sigma_1^2} + \frac{\mu_2^2}{2\sigma_2^2} - \ln\left(\frac{(\ell_0(1-p)}{p(1-\ell_0)}\right) \middle| \quad w_2\right].$$

In this example, conditional on $W=w_1$:

$$\frac{\mu_1 \underline{X}_1}{\mu_1^2} \quad N\left(0, \frac{\mu_1^2}{\sigma_1^2}\right),$$

and

$$\frac{\mu_2 \underline{X}_2}{\sigma_1^2} \quad N\left(0, \frac{\mu_2^2}{\sigma_2^2}\right).$$

Thus, $\dfrac{\mu_1 \underline{\bar{X}}_1}{\sigma_1^2} + \dfrac{\mu_1 \underline{\bar{X}}_2}{\sigma_2^2} \quad N\left(0, \dfrac{\mu_1^2}{\sigma_1^2} + \dfrac{\mu_2^2}{\sigma_2^2}\right)$

and the probability of a type I error is given by:

$$1 - \Phi\left(\left(\frac{\mu_1^2}{2\sigma_1^2} + \frac{\mu_2^2}{2\sigma_2^2} - \ln\left(\frac{\ell_0(1-p)}{p(1-\ell_0)}\right)\right)\Bigg/\left(\frac{\mu_1^2}{\sigma_1^2} + \frac{\mu_2^2}{\sigma_2^2}\right)^{1/2}\right)$$

where Φ is the cumulative normal distribution function. Similarly, the probability of a type II error is given by:

$$\Phi\left(\left(-\frac{\mu_1^2}{2\sigma_1^2} - \frac{\mu_2^2}{2\sigma_2^2} - \ln\left(\frac{\ell_0(1-p)}{p(1-\ell_0)}\right)\right)\Bigg/\left(\frac{\mu_1^2}{\sigma_1^2} + \frac{\mu_2^2}{\sigma_2^2}\right)^{1/2}\right)$$

A similar procedure can be carried out for any decision rule derived via a Bayesian structure and with modification for any other structure in which the conditional distributions $f(\underline{x}|\underline{w}_1)$ are specified. The critical factor is that this specification for a combat scenario is not simple. Moreover, a range of scenarios probably are required.

6.0 FIGURES OF MERIT ON THE UNIT INTERVAL

To examine the BETA algorithm as currently implemented first note that all the figures of merit lie on the unit interval. To accomplish this, certain transformations have been made. For example, the measure of separation is the complementary cumulative distribution of the random variable $\underline{x}^T \sum^{-1} \underline{\lambda}$ where $\underline{x}$ is the random vector of differences in location. The remainder of the statistics used can be described as follows.

Let S be a measured difference and suppose S has density $f(s|w_1)$. then define a variate $\underline{\bar{X}}$ by:

$$\underline{\bar{X}} = 1 \qquad 0 \le s \le \ell$$

$$\underline{\bar{X}} = 0 \qquad u < s$$

$$\underline{\bar{X}} = \frac{u-s}{u-\ell} \qquad \ell < s \le u.$$

Then:

$$Pr[\underline{X} = 1 | w_1] = \int_0^{\ell} f(s|w_1)ds$$

$$Pr[\underline{X} = 0 | w_1] = \int_u^{\infty} f(s|w_1)ds$$

and

$$Pr[\underline{X} \in x, x+dx] = f(u-(u-\ell)x|w_1)dx \quad \ell \le s < u.$$

Thus, the probability that $\underline{\bar{X}}$ exceeds some threshold h say is:

$$\int_0^{u-h(u-\ell)} f(s|w_1)ds.$$

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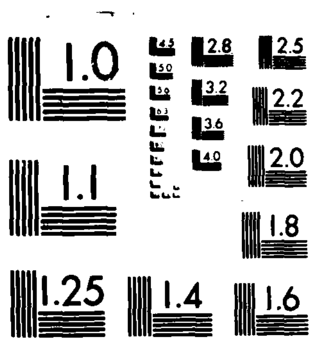


MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS-1963-A

For any linear combination $M=\sum a_i X_i$ of such variables it is possible to determine the probability distribution of M conditional on w_i and thus to determine the type I and type II error probabilities. Consider an example in which

$$f(s\ w_1) = \mu_1 e^{-\mu_1 s}$$

and
$$f(s\ w_2) = \mu_2 e^{-\mu_2 s} .$$

Then

$$\frac{f(s|w_1)}{f(s|w_2)} > \gamma_0$$

is equivalent to

$$\frac{\mu_1}{\mu_2} e^{-s(\mu_1-\mu_2)} > \gamma_0$$

or

$$-s(\mu_1-\mu_2) > \ln\gamma_0 - \ln\left(\frac{\mu_1}{\mu_2}\right)$$

or

$$s < \left(\ln \frac{\mu_1}{\mu_2}\right)-\left(\ln\gamma_0\right)\left(\mu_1-\mu_2\right)^{-1} = \gamma_1$$

The two decision rules are equivalent if $s= {}_1$, $x=h$, or $\frac{\mu-\gamma_1}{\mu-\ell} = h$.

In this case the probability of a type I error is:

$$Pr[s>\gamma_1|w_1] = e^{-\mu_1\gamma_1},$$

and the probability of a type II error is:

$$Pr[s<\gamma_1|w_2] = 1-e^{-\mu_2\gamma_1}$$

If the rules are not equivalent, then for the criterion:

$$x \geq h$$

we have
$$\frac{u-s}{u-\ell} \geq h$$

or
$$s \leq u-h(u-\ell).$$

Hence:

$$Pr[x<h|w_1] = e^{-\mu_1(u-h(u-\ell))}$$

and

$$Pr[x\geq h|w_2] = 1-e^{-\mu_2(\mu-h(u-\ell))}$$

The BETA algorithms make use of a linear combination of figures of merit taking values on the unit interval. As with any algorithm (again provided a scenario is specified) probabilities of type I and type II errors can be determined. As an illustration consider the following example:

Let

$$M = a_1x_1 + a_2a_2$$

and coelate if

$$M \geq h,$$

Where

$$a_1 + a_2 = 1$$

$$0 \leq x_i \leq 1 \qquad i = 1,2.$$

Assume that:

$$f(x_i|w_1) = 1 \qquad 0\leq x_i\leq 1$$

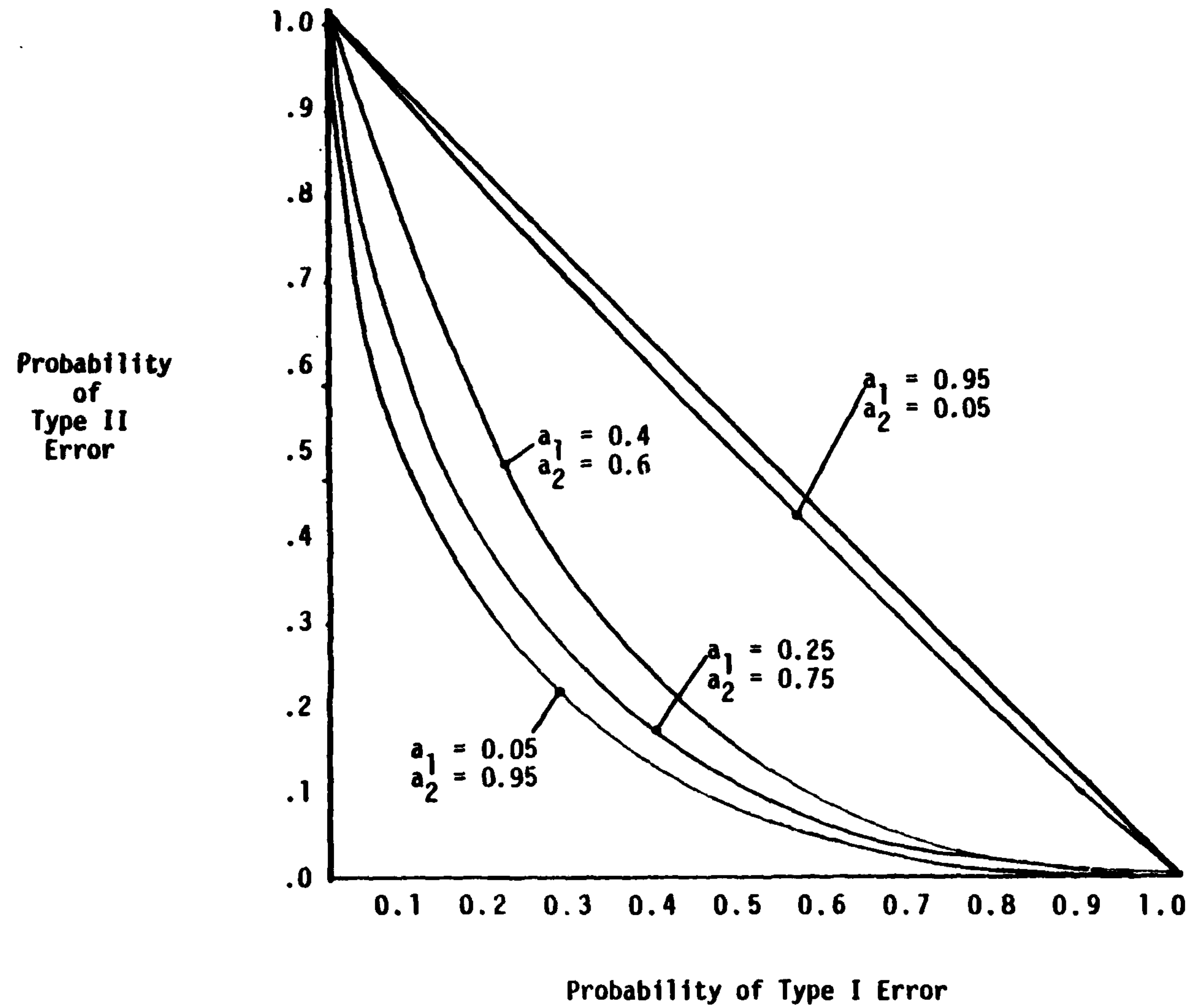
and

$$f(x_1|w_2) = 1 \qquad 0\leq x_1<1$$

$$f(x_2|w_2) = \frac{\mu_1 e^{-\mu_1 x_2}}{1-e^{-\mu_1}} \qquad 0\leq x_2<1 .$$

The assumption of uniform distributions conditioned on $W=w_1$ is not unreasonable because, as noted earlier by choice of transformation, any random variable can be mapped into an equivalent that has such a distribution. In particular, the separation figure of merit exp(-d/2) has a *uniform distribution for the case in which* locations are the same. The distribution assumed for $W=w_2$ has the effect of making values close to zero more probable. Exhibit 6-1 illustrates the resulting type I and

EXHIBIT 6-1: EXAMPLE ERROR PROBABILITIES



type II errors for $\mu = 5.0$. The example shows the contribution of the non-uniformly distributed figure of merit: as more weight is given to x_2, it becomes possible to achieve lower values of the probabilities of both type I and type II errors. This will be true in general, i.e., for any particular set of weights it is possible to determine the curve relating the two probabilities as a function of the decision threshold h. The functions required to carry out the calculations are the conditional distributions of the decision vector. Ideally one wishes to set threshold values for desired error probabilities on the lowest such curve, i.e., the weights defining that curve would specify the best "BETA" algorithm, although not necessarily the best algorithm possible. Note the implications of renormalization. If a data element is missing the preferred procedure would be derive appropriate weights by finding the lowest curve for the remaining elements in the data vector. Provided that the missing element is not superfluous this curve will be above the original. Thus, even if thresholds are changed, it will not be possible to achieve the same performance: one or both probabilities will increase. In a Bayesian context this corresponds to an increase in the expected loss.

The examples in section 3.0 of this paper suggest that if the figures of merit are stochastically dependent the impact of renormalization is to move to an operating point that is above the optimal curve for the reduced data. The examples for both independence and dependence among the data elements suggest that the "best" BETA curve is above the theoretical optimum. The magnitude of the difference should be investigated.

7.0 REMARKS

The scope of the examination of the BETA algorithms was restricted to those used for self-correlation with particular emphasis on ELINT. Two questions were addressed:

(1) How well do the algorithms perform? and

(2) Is the normalization procedure sound?

The question of algorithm performance is closely related to scenario. The Bayesian structure used within this paper represents this relationship by means of losses, prior probabilities, and conditional distributions. Within this structure it is possible to calculate, for any algorithm, the probabilities of type I and type II errors. In the absence of actual numbers and a specific scenario, the procedures were illustrated and an example was provided. The example suggests that the general concept of weights can be explained, i.e., one wishes to give greatest value to the best discriminants. Note, however, that all potential decisions must be considered in such an operation, i.e., both "correlate" and "don't correlate" must be addressed. In the context of linear combinations of figures of merit taking values on the unit interval it may be the case that the probability of a type I error is not critical; rather the algorithms have been designed and weights assigned to minimize the probability of a type II error.

The question of normalization was examined by constructing linear discriminants and examining the impact of missing data. In the case of independent figures of merit, a case can be made for the normalization procedure provided that the threshold is changed appropriately, but it is unlikely the case holds for the figures of merit defined on the unit interval. In the case of dependent figures of merit this is not true and normalization cannot be justified. Since it is likely that the figures of merit are dependent, different procedures should be adopted to account for missing data.

The treatment of temporal data in the current BETA algorithms appears to assume that times are a random variable. Given knowledge of collection management and sensor tasking such an assumption might be appropriate, but it does not appear that this is the case in BETA. A general approach to the issue is provided in section 4.0 in which both priors and conditional distributions are made functions of time between reports.

Currently, results from tests, etc. are the only means of assessing algorithm performance. Results with which we are familiar suggest that the current versions of the algorithms forego self- correlation to avoid incorrect correlation, i.e., type II errors. All other things considered, the BETA alorithms should do reasonably well against stationary ELINT targets. For a vareity of reasons, including the treatment of timing, against targets which displace, this is unlikely to be the case. Nevertheless, given the distances separating most ELINT targets (excluding GUNDISH radars) location differences should be a powerful discriminant. In this respect it may be worthwhile to use an an alternative approach and enhance the distance figure of merit exp(-d/2) by explicitly considering alternate decisions.

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Jeff M. Alden

BETA SELF-CORRELATION:
EVALUATION METHODOLOGY

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1.0 INTRODUCTION

The purpose of this paper is to describe a methodology for evaluating the self-correlation function of BETA correlation. Briefly stated, BETA self-correlation is an aid in deciding if two reports describe the same entity or not. The basic approach of BETA is to calculate statistics or figures-of-merit based on the contents of two reports, determine a weighted sum of the statistics to generate an overall statistic, and then compare the value of the statistic to a preset threshold. If the statistic attains or exceeds the threshold then the reports are correlated, i.e., they are said to describe the same entity; otherwise, they are uncorrelated, i.e., they are said not to describe the same entity.

The remainder of this paper is divided into five chapters. Chapter 2.0 presents a problem description giving the context and concepts required in the discussion of the evaluation methodology. Chapter 3.0 follows with a description of how BETA self-correlation functions. Chapter 4.0 introduces the evaluation criteria as the cost of making correlation errors and presents a cost function that depends on the probabilities of making such errors. Chapter 5.0 gives a mathematical description of the evaluation methodology. Chapter 6.0 identifies some of the ways in which the methodology could be used to improve the performance of BETA. An appendix is included which describes the statistics of BETA in detail.

2.0 PROBLEM DESCRIPTION

This chapter describes the context and explains the concepts required for understanding the evaluation methodology. First, the concept of a "scenario" is explained. Next, an "observable" is defined, the measurement of observables discussed, and the resulting reports are described. Following this, the function of self-correlation is discussed and, lastly, the interpretation of the reports after self-correlation is covered for completeness sake.

2.1 SCENARIO

The evaluation of an aid like BETA requires a knowledge of the various situations in which it may have to operate. The need for this knowledge is twofold. First, the value of the aid is typically sensitive to the importance of the desirable and undesirable events that might occur and their tendencies to occur. For example, if the importance of the possible outcomes of a situation is inconsequential or if all outcomes are equally acceptable except for one undesirable outcome and the tendency for its occurence is negligible, then the aid, regardless of its performance, is of little value. The identification of the desirable and undesirable events of a situation will, to some degree, define the situations in which the aid may have to operate. Second, the need to describe how the aid will operate necessitates a description of its response for any given situation. Then, given the situation, we can characterize its response and measure its performance. The description of the environment or conditions under which the aid is to operate is called a scenario. In summary, a scenario is a description, over time, of all events that play a role in the activities or performance of the aid.

Here, the aid is the BETA self-correlation, which is concerned with the correlation of reports summarizing detections made on electromagnetic emissions such as radar and radio. Some items of a scenario could be the identity of detectable emitters, where they are located, what detectable emissions are made, what sensors are used, what detections are made by the sensors, what are the conditions under which the detections were made, where are the sensors located, and so on, all given as a function of time. Once the scenario has been described, the expected performance of BETA can be specified.

2.2 OBSERVABLES AND MEASUREMENT

As implied in section 2.1, sensors are used to collect information. The physical characteristics of phenomenon detected by a sensor are called observables. The observables of electromagnetic radiation used in BETA are frequency, pulse width, pulse repetition interval, location of the source (possibly derived from lines-of-bearing), and times of observation.

When a sensor detects an emission, it results in a measurement of one or more observables. Normally the estimated value and actual value of the observable differ by some unknown amount. The degree to which they agree is a measure of the performance of the sensor. The probabilistic description of how well a measurement might agree with reality defines the sensor capability.

Given the scenario, the probabilistic nature of the statistics calculated in BETA depends completely upon sensor performance.[1] In general, the more a measurement may deviate from the actual value, the more a dependent statistic will vary in its value. On the other hand, if the sensors are perfect (measured value and true value agree), then the dependent statistics are deterministic. The study of the statistics in BETA will rely primarily on the description of a sensor's capability to measure observables.

2.3 REPORTS

A report is simply a summary of the measurements made by one or more sensors on a single element during the same observation time. Reports are the items correlated during self-correlation. Ideally, a report should describe exactly one entity; otherwise, the concept of "errors" in correlation becomes complicated. For example, if report A describes two unique entities a and b as though they were one entity and report B describes entity b, then is it correct to correlate A and B or to uncorrelate them? Unfortunately, it may be necessary to merge two correlated reports into one report to reduce the number of reports to retain. This may be necessary, for example, when reports are stored in a computer data base with a small memory capacity. A report that represents the merging of two or more reports is called an updated report.

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[1] Assuming that all measurements, once made, remain unaltered.

2.4 SELF-CORRELATION

The purpose of self-correlation is to declare a belief or disbelief that two reports describe the same entity or emitter. This process aids in resolving the grouping or fusion problem of deciding what set of reports describes the same entity, thereby obtaining a fuller description of the entity and avoiding confusion and erroneous conclusions about the entity due to improper grouping.

The general approach BETA uses in deciding if two reports correlate or not is to calculate a statistic based on the types of information common to both reports and then compare the result to a predetermined threshold. If the threshold is attained or exceeded then the reports are correlated; that is, they are declared to describe the same entity. Otherwise they are uncorrelated; that is, they are declared to describe separate entities.

Envisioning the reports to be stored in a data base of a computer, the effect of self-correlation can be described as changing a relationship between two reports from an "unknown" status to either "correlated" or "uncorrelated." If there are no "unknown" relationships, then the data base has been fully examined and is ready for further interpretation.

2.5 INTERPRETATION

For completeness, the step following the self-correlation function is briefly discussed. Once the reports have been self-correlated, the problem of determining what reports describe the same entity has been at least partially solved. The problem may be not completely solved because a chain of correlated reports may not all correlate with one another (e.g., A correlates with B, B correlates with C, but A does not correlate with C). This problem is avoided when all correlated reports are merged into updated reports, but at the expense of propagating any correlation errors made earlier in the history of an updated report. Assuming such problems are somehow resolved, we are left with reports that have been grouped, hopefully, to provide the highest degree of entity description the reports have to offer.

The next step is to collect groups of correlated reports and examine them to create more complex relationships among them that imply organizational structures, deployment, and missions. The BETA correlation attempts to aid in this interpretation effort, but BETA correlation beyond self-correlation is not considered here.

3.0 SELF-CORRELATION IN BETA CORRELATION

The purpose of this section is to describe, in some detail, how self-correlation is done in BETA.

There are two previously uncorrelated reports selected for correlation: one report is called the "subject" and the other called the "candidate." The perception in BETA has the subject as a new report just received and the candidate as an old, possibly updated, report retrieved from a data base of reports.

The estimates of each observable common to both reports individually compared and a statistic for the type of observable calculated. For example, suppose both reports have an estimate of frequency. Let

u = frequency estimate of subject, and

u_c = frequency estimate of the candidate.

The frequency statistic is defined as follows:

$$V_2 = \begin{cases} 1 & , |U - U_c| < \ell_2 \\ 1 - \dfrac{|U - U_c| - \ell_2}{m_2 - \ell_2} & , \ell_2 \leq |U - U_c| \leq m_2 \\ 0 & , \text{otherwise,} \end{cases}$$

where ℓ_2 and m_2 are arbitrary thresholds.

If U is 1 MHz, U_c is 1.1 MHz, ℓ_2 is .05 MHz, and m_2 is .2 MHz, then the frequency statistic has a value of 2/3. The statistics for the other observables (location, pulse width, pulse repetition interval, and observation times) are given in appendix A. Note that the statistic varies between zero and one and increases as the absolute difference of the measured frequencies decreases. This general behavior is common to all the remaining statistics.

An overall statistic is formed as a weighted sum of the individual statistics calculated. Let H be the overall statistic and V_i the statistic corresponding to observable type i. Then,

$$H = \frac{\sum_{i \in I} d_i V_i}{\sum_{i \in I} d_i} .$$

Here, I is the set of i where observable type i has been estimated in both candidate and subject, and d_i is a weighting factor assigned to observable type i.

The statistic H is then compared to a predetermined threshold h. If H attains or exceeds this threshold, then the two reports are correlated; otherwise, they are uncorrelated. Note that the result cannot be "unknown", which was the status before the reports were correlated.

Although not an important assumption, it is assumed that the threshold h exists independently of the contents of the set I defined above. If h is allowed to depend on I, then there is no real need to renormalize the weights by the division performed above because new thresholds could be defined by multiplying them by the renormalization factor. However, the effect of renormalization is to restrict H to the interval from zero to one which may be a desirable characteristic.

4.0 EVALUATION CRITERIA

This section develops a mathematical expression for evaluating the performance of the BETA self-correlation function. The assumptions leading to the evaluation criteria are first identified, then the expression is given and discussed.

Typically, the way to investigate the performance of an aid like BETA is to study the types of errors it may make and the tendancy to make them under various conditions or scenarios. In BETA there are two types of errors that can be made: type 1 and type 2. A type 1 error occurs when two reports are not correlated and they describe the same entity. A type 2 error occurs when two reports are correlated and they describe two unique entities. To specify the performance of self-correlation for a given scenario and correlation event is to give the probability of making a type 1 and type 2 errors. The concept of a "correlation error" becomes more complicated when one of the reports, i.e., the candidate, of a correlation event is an updated report and conceivably describes more than one entity. If the candidate report describes two or more entities as one entity, then correlating or uncorrelating it with a subject report results in one or more type 2 or type 1 errors, respectively. For example, suppose reports A and B describing entities e_a and e_b, respectively, are merged into a candidate report C. Further, suppose report S describing entity e_b is to be correlated with C. If they are correlated, the effect is to correlate S with A and B, resulting in a type 2 error. If they are uncorrelated, then S is effectively uncorrelated with A and B for one type 1 error. Note that if the candidate report is an updated report, then both types of errors may apply for a given scenario and correlation event. If the candidate is not an updated report, then only one type of error will apply.

In the remainder of this paper, it is assumed the candidate and subject are not updated reports. The primary reason for this assumption is that an evaluation methodology of BETA self-correlation when updated reports are present will depend on how measurements from two reports are combined when creating an updated report. It is not clear how this is done.

The value of an aid like BETA is reflected to the extent undesirable events are avoided and desirable events are encountered through its use. Normally the value of an aid is the extent to which it reduces some kind of average or expected "costs" through its use.

We assume here that a functional relationship exists between cost reduction and performance improvement of BETA. This means we can study the probability of correlation errors and, through a functional relationship, determine the associated costs. If we make the additional assumptions that costs resulting from a correlation error are additive and all pairs of reports are put through the correlation process, then we can write the expected cost of errors in a given scenario, S, as

$$C(S) = \sum_{i \neq j} C_{ij} P_{ij},$$

where

$P_{ij}(S)$ = probability of making a type 1 or type 2 error (only one will apply) when correlating reports i and j under scenario S, and

$C_{ij}(S)$ = the cost associated with making the error indicated by $P_{ij}(S)$.

Although this additive form is not necessary for evaluation, the dependency of expected costs solely on the probability of making type 1 and type 2 errors is desirable in order to simplify the problem to that of determining the probability of making these errors.

To evaluate the performance of BETA over a class or distribution of scenarios with commensurate costs, the overall expected cost is:

$$C = \sum_{S} C(S)\,P(S),$$

where S is summed over all scenarios in the class or distribution, and P(S) is a weighting factor reflecting the relative likelihood or probability that scenario S might occur.

The degree to which the use of BETA reduces C(S) or C is a measure of its value.

5.0 PROBABILITY OF CORRELATION ERRORS

To evaluate BETA self-correlation, we have assumed a cost function dependent only on the probability of making type 1 and type 2 correlation errors. This section developes a mathematical expression for calculating these probabilities.

Given the scenario, everything is specified except the actual measurements made by the sensors which are probabilistic in nature. This, in turn, implies a probabilistic behavior in making correlation errors. The probabilistic nature of a sensor is given by the general probability function[1] $f(m_1, m_2, \ldots, m_5 \mid a_1, a_2, \ldots, a_5)$ which gives the relative likelihood the measured values of observables 1, 2, ..., 5 is m_1, m_2, ..., m_5 given the actual values are a_1, a_2, ..., a_5, respectively. Although not explicitly stated, this function may well be dependent on other aspects of the scenario (like weather).

The dynamics of an observable i from a particular entity is given by its actual value as a function of time or $a_i(t)$. The probability of a type 1 or type 2 error in correlation is $P(H < h)$ or $P(H \geq h)$, respectively, remembering that only one type of error will apply. The expression for a type 1 error is:

$$P(H < h) = \sum_{\substack{v_1:\ d_1 v_1 \\ < h^*}} \ \sum_{\substack{v_2:\ d_2 v_2 \\ < h^* - d_1 v_1 \cdots}} \cdots \sum_{\substack{v_5:\ d_5 v_5 \\ < h^* - d_1 v_1 - \ldots - d_4 v_4}} g(v_1, v_2, \ldots, v_5),$$

---

[1] A general probability function may be a probability density function if the variable is continuous, a probability mass function if the variable is discrete, or a combination of both.

where $h^* = h\sum d_i$ over $i \in I$, I is set i such that observable i has been measured in both candidate and subject reports, and $g(v_1, v_2, \ldots, v_5)$ is the joint general probability function[1] of the statistics $v_1, v_2, \ldots, v_5$ given the scenario and the two reports involved. Note that depending on the set I, some of the sums above and parameters of g may not appear. An expression for g in terms of the target dynamics, $\bar{a}(t)$ and sensor(s) performance, $f(\bar{m} \mid \bar{a}(t))$, where $\bar{m} = (m_1, m_2, \ldots, m_5)$ and $\bar{a}(t) = (a_1(t), \ldots, a_5(t))$ is:

$$g(\bar{v}) = \sum_{\substack{\bar{m}, \bar{m}_c \\ \text{such that} \\ \bar{V}(\bar{m}, \bar{m}_c) = \bar{v}.}} f(\bar{m} \mid \bar{a}(t)) \; f(\bar{m}_c \mid \bar{a}_c(t_c)),$$

The subscript c represents values associated with the candidate, no subscript refers to the subject, and $\bar{V}(\bar{m}, \bar{m}_c)$ is the vector with components $V_i(m, m_c)$ that represents the functional relationship between statistic V_i for observable type i and the candidate and subject estimates of the observable. Finally, t has a special meaning here: it represents the interval of observation during which the sensor is collecting information to estimate the observables. Note, that in the above, the sensors are assumed to operate independently.

The probability of making a type 2 error is simply $1 - P(H < h)$ under the realization that the two reports now describe separate entities.

---

[1]Thus for the continuous statistics the corresponding sums above is replaced by an integral operator.

To summarize, we will briefly review what has been done mathematically above.

The criteria for correlating two reports depends on the value of a statistic H. Thus, the probability of making a correlation error depends on the probability the statistic has a value that leads to the wrong conclusion. The probability distribution of the statistic depends on two things: the scenario, which includes the system dynamics and all thresholds, and the measurement capabilities of the sensors. These factors define a joint probability distribution of the component statistics which is the function $g(\overline{v})$. To determine the probability of making a correlation error, we simply sum $g(\overline{v})$ over all possible values of the component statistics, which leads to a value of the overall statistic H that produces an erroneous conclusion.

6.0 USES OF EVALUATION METHODOLOGY

There are a number of ways the evaluation methodology could be used to improve the performance of BETA self-correlation under given scenarios. Some of these uses are described here.

There exist numerous thresholds and weights in BETA which are to be determined by some means. Using the evaluation methodology presented here, values for these parameters could be determined for a given cost function and scenario or set of scenarios. The approach is obvious: determine the values of the parameters which minimizes the expected cost function.

The selection of better statistics than those used in BETA could be made using the methodology by selecting those statistics which produce lower expected costs for a given set of scenarios. In fact, the methodology could be used to develop optimal statistics in the sense of minimizing expected costs. The optimal threshold statistic (like the "H" statistic of BETA) and optimal threshold value can, in theory, be determined from the target dynamics, sensor performance and cost function. Of course, the difficult part is characterizing sensor performance and the target dynamics under the scenarios of interest, and developing a reasonable cost function.

Finally, the methodology can be used to answer questions concerning the value of information. Simply put, the expected value of obtaining an additional item of information is the expected reduction in costs that results in obtaining and using the information.

APPENDIX A:

FIGURE-OF-MERIT STATISTICS IN BETA SELF-CORRELATION

The following describes the figure-of-merit (FOM) statistics of the BETA self-correlation algorithm. First, the variables used in the calculation of the FOMs are defined and then the mathematical expression for each FOM is given. There are two reports: the subject (a new report) and the candidate (selected from a data base of old reports).

Variables

X,Y = location estimate;

U = frequency estimate;

W = pulse width;

R = pulse repetition frequency;

a = first time entity observed; and

b = last time entity observed.

Variables with no subscript are estimates of the subject. Variables with a subscript of c, e.g., X_c, are estimates of the candidate. A variable can be "unknown," which means the variable was not estimated in the report. If a variable is unknown, a FOM is not calculated for the variable.

Figure-of-Merit Calculations

The figure of merit statistic for variable i is given by V_i. The mathematical expression for each variable follows.

Location

$$D = \begin{pmatrix} X-X_c \\ Y-Y_c \end{pmatrix}^T \left(\sum + \sum_c\right)^{-1} \begin{pmatrix} X-X_c \\ Y-Y_c \end{pmatrix}$$

$$V_1 = e^{-D/2},$$

where

Σ = Variance - covariance matrix of (X,Y);

$$\begin{pmatrix} \sigma^2_{xx} & \sigma^2_{xy} \\ \sigma^2_{xy} & \sigma^2_{yy} \end{pmatrix}$$

Σ_c = Variance - covariance matrix of (X_c, Y_c).

<u>Frequency</u>

$$V_2 = \begin{cases} 1 & , |U - U_c| < \ell_2 \\ 1 - \dfrac{|U - U_c| - \ell_2}{m_2 - \ell_2} & , \ell_2 \leq |U - U_c| \leq m_2 \\ 0 & , \text{otherwise.} \end{cases}$$

where ℓ_2 and m_2 are given parameters.

<u>Pulse Width</u>

$$V_3 = \begin{cases} 1 & , |W - W_c| < \ell_3 \\ 1 - \dfrac{|W - W_c| - \ell_3}{m_3 - \ell_3} & , \ell_3 \leq |W - W_c| \leq m_3 \\ 0 & , \text{otherwise.} \end{cases}$$

where ℓ_3 and m_3 are given.

Pulse Repetition Frequency

$$V_4 = \begin{cases} 1 - \dfrac{|R/2 - R_c|}{m_4 R/2} & , (1 - m_4) R/2 \leq R_c \leq (1 + m_4) R/2 \\ 1 - \dfrac{|R - R_c|}{m\ 4R} & , (1 - m_4)R \leq R_c \leq (1 + m_4) R \\ 1 - \dfrac{|2R - R_c|}{2m_4 R} & , 2\ (1 - m_4)R \leq R_c \leq 2\ (1 + m_4)R \\ 0 & , \text{otherwise}, \end{cases}$$

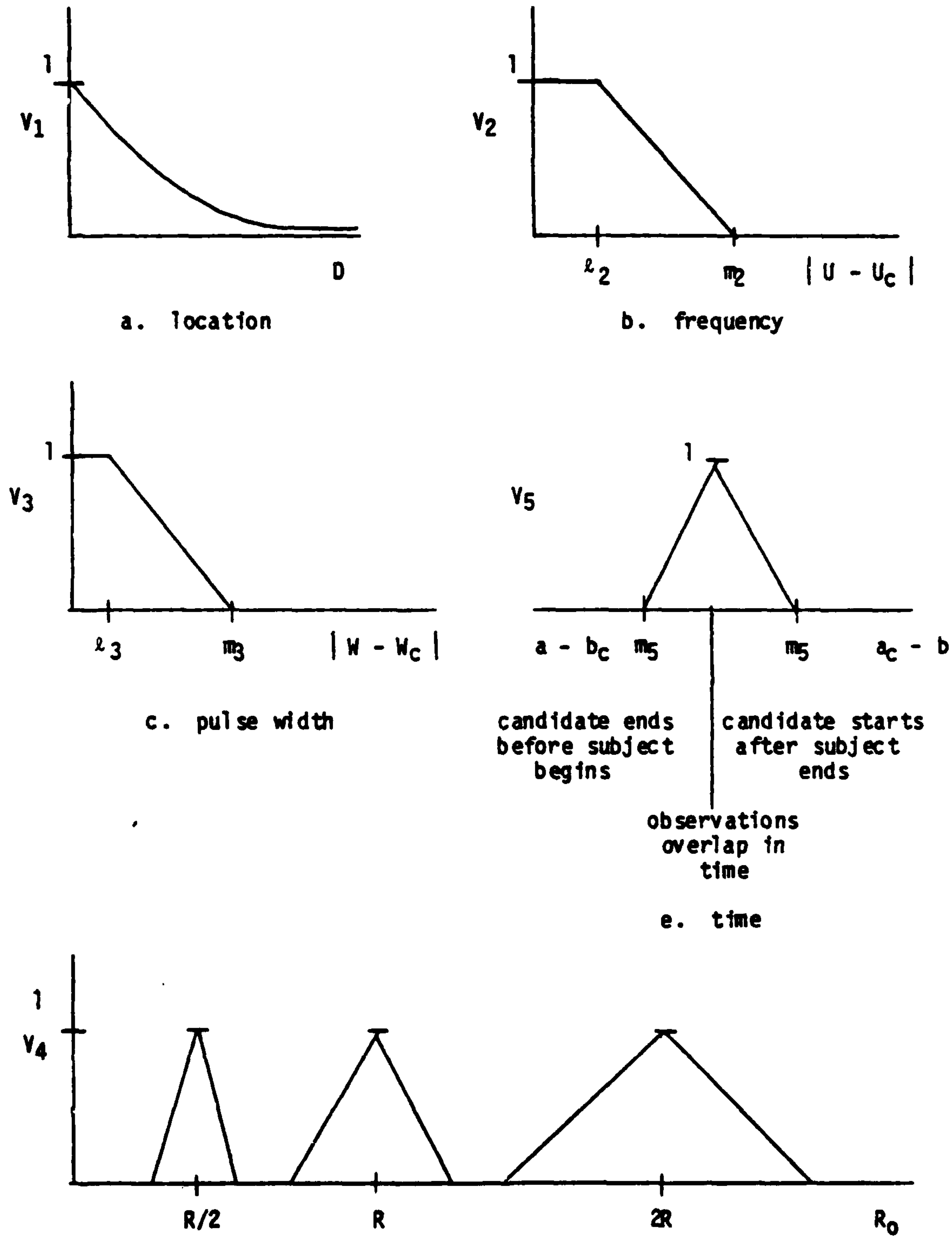
where m_4 is a given parameter.

Time

$$V_5 = \begin{cases} 1 & , a \leq a_c \leq b \text{ or } a \leq b_c \leq b \text{ or} \\ & \ \ a_c \leq a \leq b \leq b_c \\ 1 - (a_c - b)/m_5 & , b < a_c < b + m_5 \\ 1 - (a - b_c)/m_5 & , b - m_5 < b_c < b \\ 0 & , \text{otherwise} \end{cases}$$

Exhibit A-1a through A-1e graphically illustrate the dependence of V_i on the variables defined above.

EXHIBIT A-1: ILLUSTRATION OF FOMS



a. location

b. frequency

c. pulse width

e. time

d. pulse repetition frequency

APPENDIX E

E. SELF-CORRELATION IN THE OTHER SYSTEMS
(bound separately)

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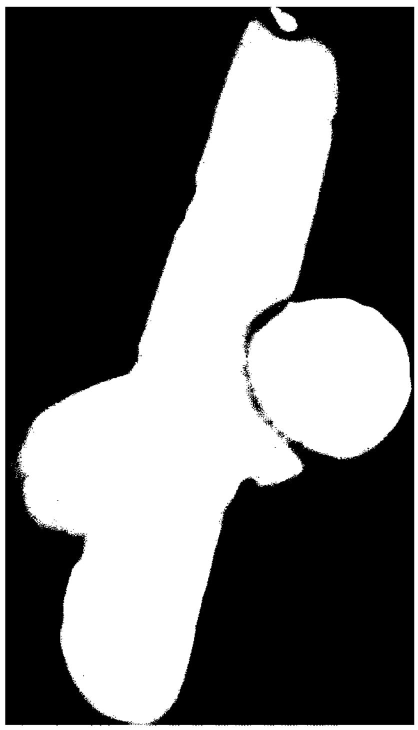

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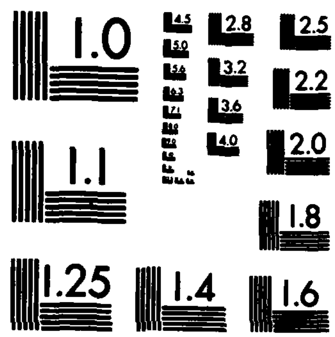

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SUPPLEMENTARY

INFORMATION

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)
This is one in a series of algorithm analysis reports on work performed at the Jet Propulsion Laboratory for the US Army Intelligence Center and School covering selected algorithms in existing Intelligence and Electronic Warfare (IEW) systems. It focuses on self-correlation algorithms in five ELINT systems. These algorithms test a newly reported sighting against information already gathered to determine whether it represents a battlefield entity previously reported. In the systems surveyed (BETA, TCATA

ELINT PROCESSOR, ITEP, QUICKLOOK, AGTELIS) these decision tests were found to be mainly quasi-statistical using location, detection time, and various signal parametrics. The analysis focuses on the underlying assumptions (such as independence of sightings) which must hold for decisions based on these tests to be valid, the environmental factors (such as information available to the test) necessary for the assumptions to hold, and failure modes when some of the assumptions are relaxed.

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