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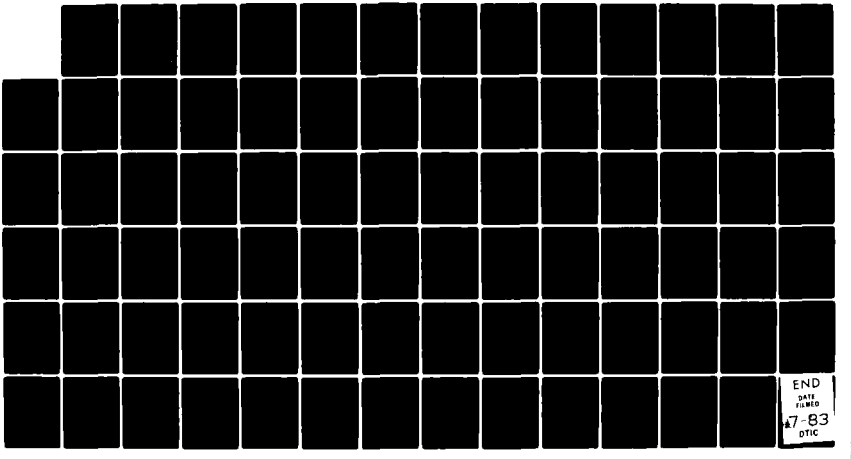

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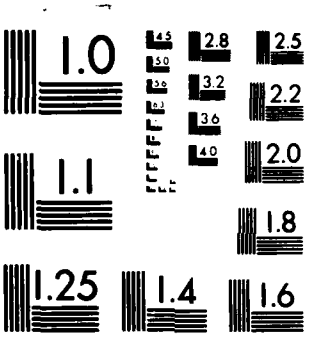

FINAL REPORT

APPLICATIONS OF TIME DELAYS AND MICROPROCESSORS IN CONTROL SYSTEM DESIGN

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PREFACE

This is the final report for the U.S. Army Contract DAAK10-78-C-0325 on the applications of time delays and microprocessors in control system design.

CONTENTS

1. STATE VARIABLE RECONSTRUCTION USING TIME DELAYS

1.1 INTRODUCTION

One of the problems which occur occasionally in designing a satisfactory controller for a deterministic control system is the need of the state variables which are not directly measurable. Luenberger proposed a scheme which approximately reconstructs inaccessible variables, (see ref. [1] and [2]). No methods, however, are currently available for exactly reconstructing the missing variables from observable variables. In this paper a method is presented for exactly reconstructing the inaccessible variables. It uses measurable variables, their delayed values and the control variables on the maximum delay interval. As can be seen in Examples, the method may give satisfactory results in certain cases and can be easily implemented using a microprocessor.

For the sake of simplicity only time invariant linear control systems are considered. The extension to time varying linear systems is straight-forward. The use of delayed state variables was first proposed by Gilchrist in ref. [3], where a similar problem, but from a somewhat different point of view, was investigated. Preliminary results of the method proposed in the paper were reported in ref. [4].

1.2 PROBLEM

Consider the linear time invariant control system

$$\dot{x}(t) = Ax(t) + Bu(t) \tag{1}$$

where x is the n x 1 state vector, u is the r x 1 control vector, A is an n x n constant matrix, and B is an n x r constant matrix. Suppose the observable vector, that is, the variables which can actually be measured, y(t) is given by

$$y(t) = Hx(t) \tag{2}$$

where y is an m x 1 vector, and H is a non-zero m x n constant matrix. Let $0 \leq h_1 < h_2 < \ldots < h_\ell < \infty$ be time delays.

The problem is to reconstruct the state variable vector x(t) from the measurable vectors $y(t - h_1)$, $y(t - h_2)$, ..., $y(t - h_\ell)$ and the measurable control vector u(s), $t - h_\ell \leq s \leq t$.

1.3 RESULTS

The response $x(t)$ of the system (1) is given by

$$x(t) = e^{A(t - (t - h_i))}x(t - h_i) + \int_{t-h_i}^{t} e^{A(t - s)}Bu(s)ds$$

$$= e^{Ah_i}x(t - h_i) + \int_{-h_i}^{o} e^{-As}Bu(t + s)ds, \quad i = 1,2,\ldots,\ell.$$

Multiplying He^{-Ah_i} on both sides,

$$He^{-Ah_i}x(t) = Hx(t - h_i) + He^{-Ah_i}\int_{-h_i}^{o} e^{-As}Bu(t + s)ds$$

$$= y(t - h_i) + He^{-Ah_i}\int_{-h_i}^{o} e^{-As}Bu(t + s)ds. \qquad (3)$$

Since $y(t - h_i)$, $i = 1,2,\ldots,\ell$, and $u(s)$, $t - h_\ell \leq s \leq t$, are measurable, the right hand side of eq. (3) is known for each i, and so eq. (3) is simply linear simultaneous algebraic equations for n unknowns $x(t) = (x_1(t), x_2(t), \ldots, x_n(t))$. Let

$$C \triangleq \begin{bmatrix} He^{-Ah_1} \\ He^{-Ah_2} \\ \vdots \\ He^{-Ah_\ell} \end{bmatrix}$$

and let

$$z(t) \triangleq \begin{bmatrix} y(t - h_1) + He^{-Ah_1} \int_{-h_1}^{o} e^{-As}Bu(t + s)ds \\ y(t - h_2) + He^{-Ah_2} \int_{-h_2}^{o} e^{-As}Bu(t + s)ds \\ \vdots \\ y(t - h_\ell) + He^{-Ah_\ell} \int_{-h_\ell}^{o} e^{-As}Bu(t + s)ds \end{bmatrix} .$$

Eq. (3) can now be rewritten as

$$Cx(t) = z(t) . \tag{4}$$

Obviously C is a known $m\ell$ x n constant matrix and z(t) is a known $m\ell$ x 1 vector for each t. If the rank of the matrix C is n, then x(t) is given by

$$x(t) = [C^TC]^{-1}C^Tz(t) . \tag{5}$$

<u>Result 1</u> If rank (C) = n, then $x(t) = [C^TC]^{-1}C^Tz(t)$.

Note that since C is a constant matrix, if the state x(t) can be reconstructed at some t, then it can be reconstructed for all t. However, the rank of the matrix C is dependent on the delays $h_1, h_2, \cdots, h_\ell$, and so the question is now whether there exists a set of delays $h_1, h_2, \cdots, h_\ell$ such that the corresponding matrix C has rank n for a given system, that is, for the given matrices A, B and H. Let Q be the $m\ell$ x n matrix defined by

$$Q = \begin{bmatrix} H \\ HA \\ \vdots \\ HA^{n-1} \end{bmatrix}$$

The argument used in the proof of the following lemma and also in the proof the next result is similar to that of ref. [5] (pp. 81-82).

Lemma 1 Let I be a non-zero interval. If rank (Q) = n, then the vector space spanned by the n x 1 row vectors of the matrices He^{-Ah} for all h in I is R^n, that is, the row vectors contain n independent vectors.

Proof Suppose the contrary. Then there exists a nonzero 1 x n vector b such that

$$He^{-Ah} b \equiv 0$$

on I. By repeated differentiation with respect to h,

$$He^{-Ah} b = HAe^{-Ah} b = \ldots = HA^{n-1} e^{-Ah} b = 0$$

on I, and so

$$\text{rank } (Q) = \text{rank} \begin{bmatrix} H \\ HA \\ \vdots \\ HA^{n-1} \end{bmatrix} < n.$$

This is a contradiction, and hence the lemma is true.

Result 2 There exists a set of n delays $0 \leq h_1 < h_2 \cdots < h_n \leq a$ for any given $a > 0$ such that the rank of the corresponding matrix C is n if and only if rank (Q) = n.

Proof Consider the necessary condition first. Suppose rank (C) = n, and assume rank (Q) < n. Then there exists a non-zero n x 1 vector b such that

$$Hb = HAb = \ldots HA^{n-1} b = 0.$$

This implies

$$He^{-Ah} b = 0$$

for all h, and hence rank (C) < n. This is a contradiction, and thus rank (Q) = n.

Now consider the sufficiency. If there exist n delays

$$0 \leq h_1 < h_2 \cdots < h_n \leq a$$

such that the matrix

$$C = \begin{bmatrix} He^{-Ah_1} \\ He^{-Ah_2} \\ \vdots \\ He^{-Ah_n} \end{bmatrix}$$

contains n independent row vectors, then the sufficiency is proved.

Let I be the non-zero interval [0, a]. Then by Lemma 1, the matrices He^{-Ah}, $h \in I$, contain n independent n x 1 row vectors. This implies that there exists a set of delays $\{ h_1, h_2, \cdots, h_n \}$ in I such that the matrix C,

$$C = \begin{bmatrix} He^{-Ah_1} \\ He^{-Ah_2} \\ \vdots \\ He^{-Ah_n} \end{bmatrix},$$

contains n independent n x 1 row vectors. Thus, if rank (Q) = n then there exists at least one set of n delays $0 \leq h_1 < h_2 < \cdots < h_n \leq a$, such that rank (C) = n. This completes the proof.

Since a is any given positive constant in the above result, the n delays $\{ h_i \}$ may be chosen arbitrarily small, and, in fact, <u>almost</u> any n different delay values may be used. However, as can be seen in Example 1, this does not necessarily mean that any n different values can be used. In other words, there are delay values which may not be used. Furthermore, because of certain technical reasons, not only the inappropriate delay values but also the delay values near them should be avoided. It is usually convenient to choose $h_1 = 0$.

In almost all of the control systems, certain state variables are usually directly measurable, that is, some components, say, $y_1, y_2, \cdots, y_\ell$, of the observation vector y are the same as the corresponding components

of the state variable x. In this case, by choosing $h_1 = 0$, the first matrix element of the matrix C contains at least ℓ independent row vectors, and hence at most $n - \ell$ additional independent vectors are needed for rank (C) = n. This means that, besides the matrix $He^{-Ah_1} = H$ in the matrix C, at most $n - \ell$ matrices He^{-Ah_i} are required for rank (C) = n. The following lemma summarizes the result.

Lemma 2 If rank (Q) = n, and ℓ components of the observed vector y are identical with the corresponding components of the state variable x, then, in addition to $h_1 = 0$, at most $n - \ell$ additional delays are required for state reconstruction.

1.4 EXAMPLES

Example 1 Consider the linear scalar system

$$v'' + 2v' + 2v = u \ ,$$

and suppose the only quantity measurable is

$$w = v + v'.$$

We wish to reconstruct the original state variables. Let

$$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \ , \qquad x_1 = v \ , \quad x_2 = x_1' = v' \ ,$$

$$y = v + v' = x_1 + x_2.$$

Then

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ -2 & -2 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

$$y = (1, 1)x.$$

The matrix Q is given by

$$Q = \begin{bmatrix} H \\ HA \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

and rank (Q) = 2. Hence the state variable x can be reconstructed. Choose $h_1 = 0$, $h_2 = h$. Then

$$e^{-Ah} = e^{h} \begin{bmatrix} \cos(h) - \sin(h) & -\sin(h) \\ 2\sin(h) & \cos(h) + \sin(h) \end{bmatrix}$$

and

$$C = \begin{bmatrix} H \\ He^{-Ah} \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ e^{h}(\cos(h) + \sin(h)) & e^{h}\cos(h) \end{bmatrix}.$$

Now, rank(C)= 2 if $\sin(h) \neq 0$, that is, $h \neq \ell\pi$, $\ell = 1,2,\cdots$. Note that, although almost any value may be chosen for the delay h to reconstruct the state variable, there are particular numbers, namely, $h = \ell\pi$, which cannot be used. Let $h \neq \ell\pi$, $\ell = 1,2,\cdots$. Then

$$z(t) = \begin{bmatrix} y(t) \\ y(t-h) + He^{-Ah} \int_{-h}^{0} e^{-As} Bu(t+s)ds \end{bmatrix}$$

$$= \begin{bmatrix} y(t) \\ y(t-h) + e^{h}\cos(h) \int_{-h}^{0} e^{s}[\cos(s) + \sin(s)]\, u(t+s)\, ds \\ \qquad - e^{h}[\cos(h) + \sin(h)] \int_{-h}^{0} e^{s}\sin(s)\, u(t+s)\, ds \end{bmatrix},$$

and $x(t) = C^{-1}z(t)$. Thus

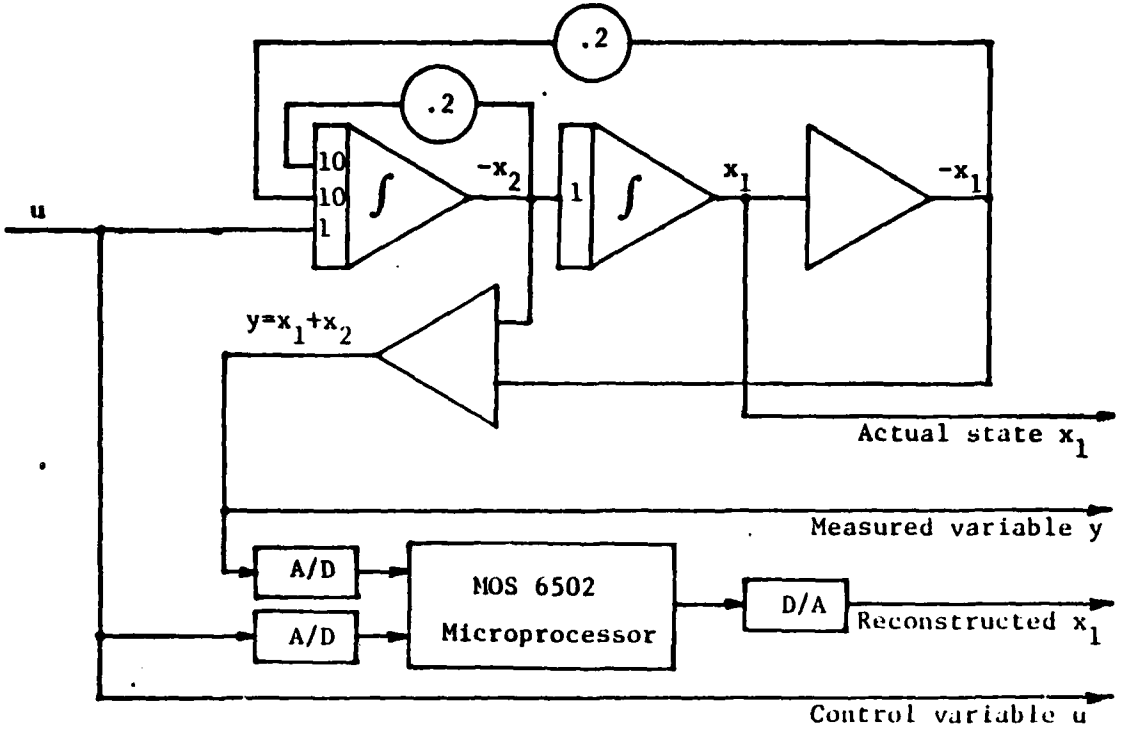
$$x_1(t) = \frac{1}{\sin(h)} \Big[y(t-h) + e^{h}\cos(h) \int_{-h}^{0} e^{s}[\cos(s) + \sin(s)]u(t+s)ds$$

$$- e^{h}[\cos(h) + \sin(h)] \int_{-h}^{0} e^{s}\sin(s)u(t+s)ds - \cos(h)y(t) \Big]$$

$$x_2(t) = \frac{1}{\sin(h)} \Big[[\cos(h) + \sin(h)]y(t) - y(t-h)$$

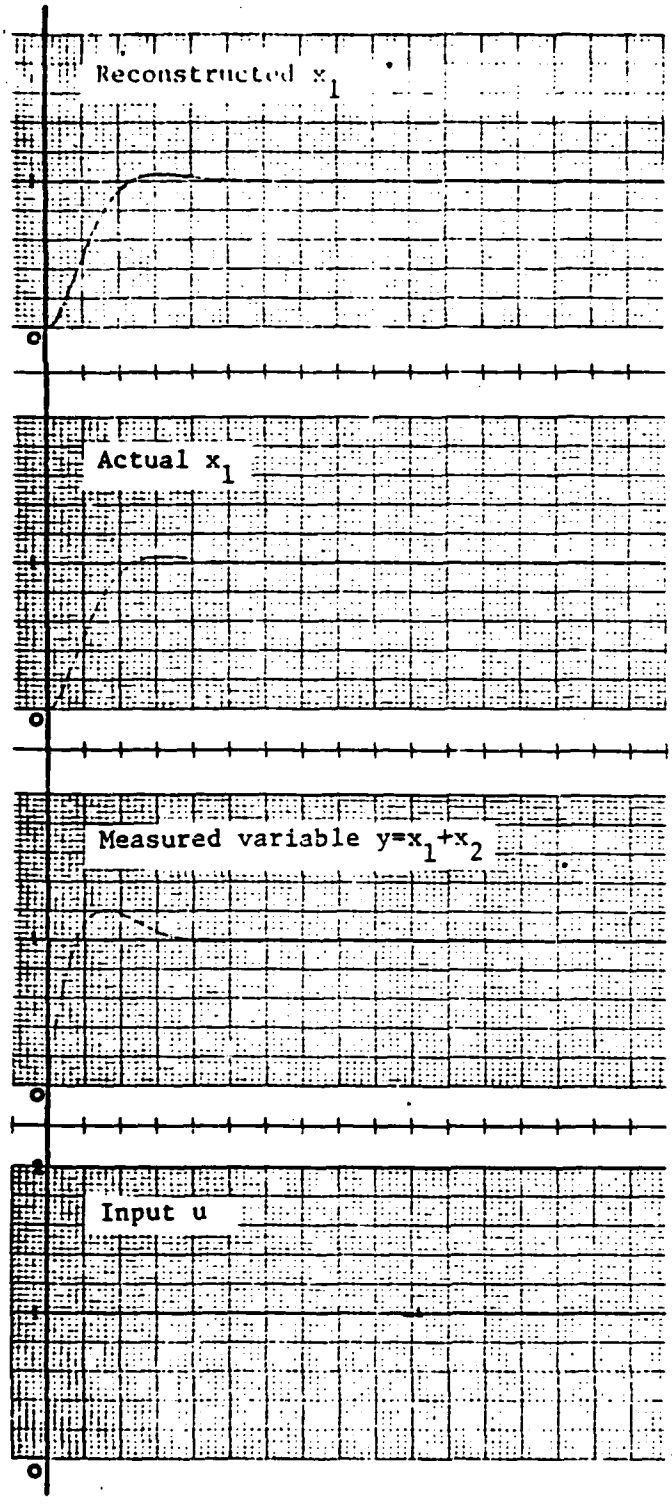
$$- e^{h}\cos(h) \int_{-h}^{0} e^{s}[\cos(s) + \sin(s)]\, u(t+s)\, ds$$

$$+ e^{h}[\cos(h) + \sin(h)] \int_{-h}^{0} e^{s}\sin(s)u(t+s)ds \Big]$$

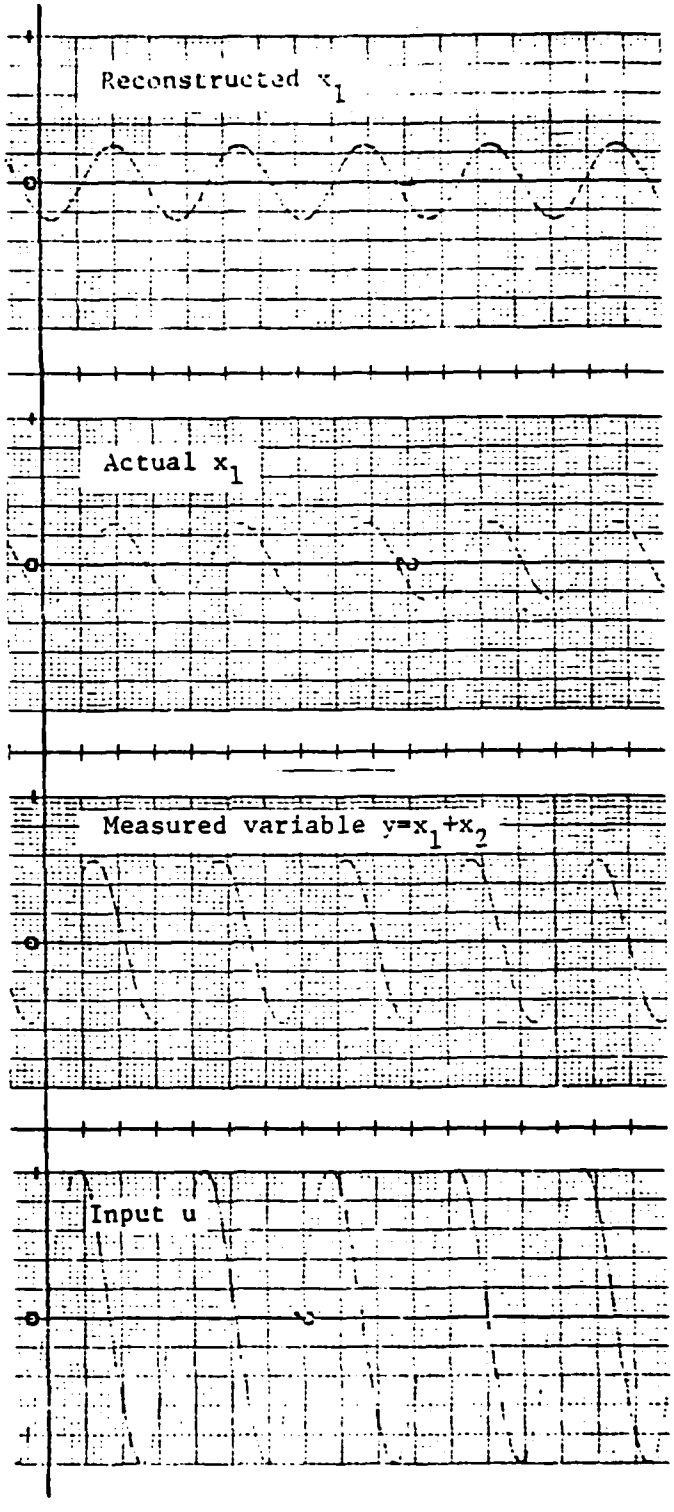


To recorder

Fig. 1 Computer setup for $v''+2v'+2v = u$.



(i) Step Input



(ii) Sinusoidal Input

Fig. 2 Responses of $v''+2v'+2v = u$ (Time: 5 mm/sec.).

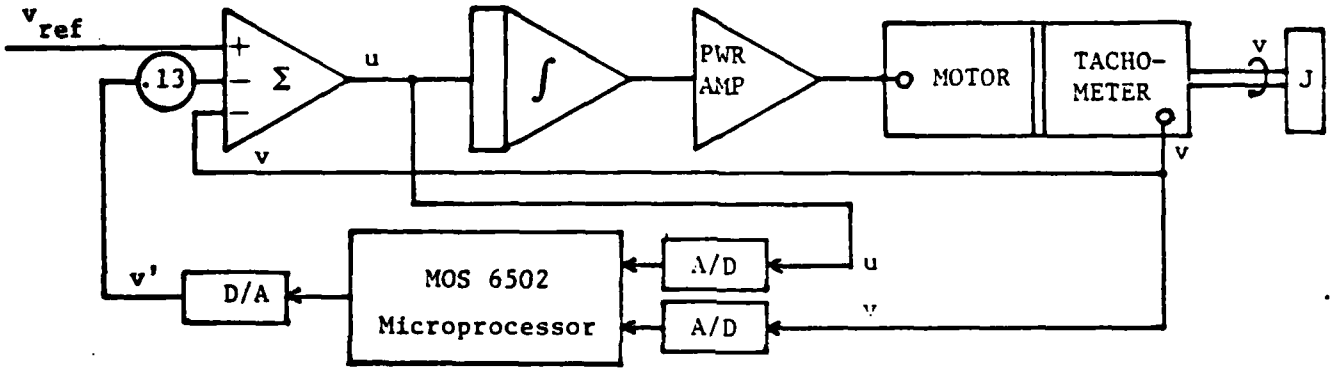
The system was simulated on an analog computer and the reconstruction was carried out on a microprocessor as shown in Fig. 1. The delay value used was h = 0.05 sec., the accuracy of the A/D and D/A converters was 8 binary bits, the sampling interval was 500μ sec., and the microprocessor used was MOS 6502. The result for $x_1(t)$ is given in Fig. 2.

Example 2 To study the effectiveness of the proposed method in a real world environment, a d.c. motor speed regulator was investigated. It was a Motomatic Control Systems Laboratory experiment kit made by Electrocraft, and was consisted of an operational amplifier, a power amplifier and a d.c. motor - tachometer unit. Only the speed of the motor was measurable through the tachometer, and the acceleration variable was inaccessible. On the other hand, the acceleration variable was needed for a satisfactory controller design, and the main problem, therefore, was to reconstruct the acceleration variable from speed variable and the control variable. In addition, we were also interested in the effect of measurement noise, the sensitivity of the proposed method with respect to system parameters and the real-time implementation. No particular effort was made to clean up the noisy measurement of the speed, and the system was modeled as a second order linear system. The real system, however, contained considerable nonlinear friction.

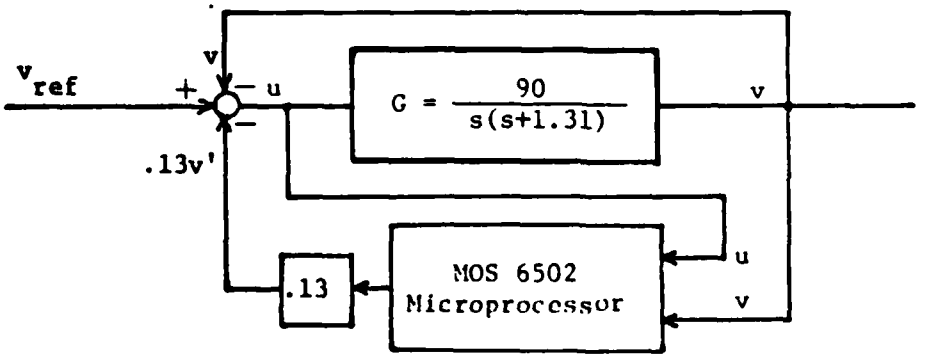
The system diagram is given in Fig. 3. The open-loop transfer function is approximately $G = 90/s(s + 1.31)$. The corresponding differential equation is

$$v'' + 1.31v' = 90u$$

where v is the speed and u is the control function. When only the available speed variable was used in the controller, that is,

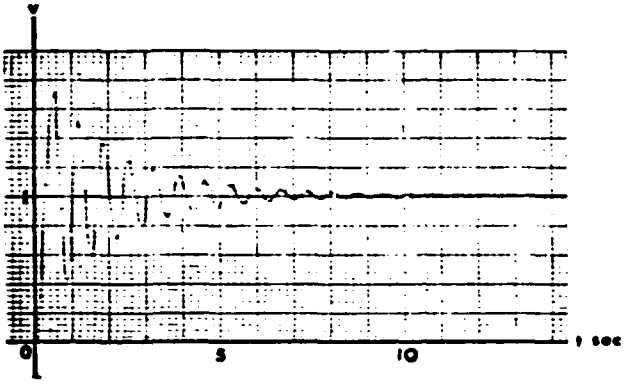


(a)

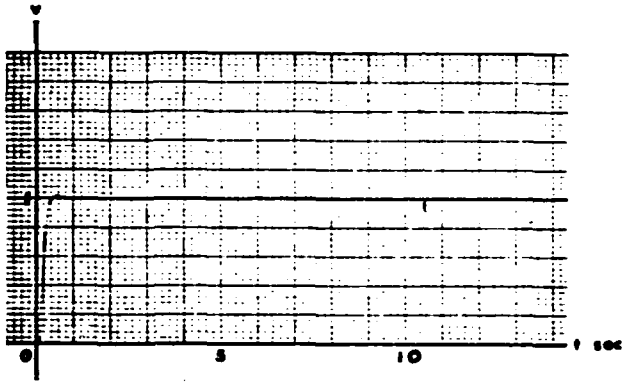


(b)

Fig.3 D.C. Motor Speed Regulator System



(a) Without acceleration feedback



(b) With reconstructed acceleration feedback

Fig. 4 Step Reference Input Response

$$u = v_{ref} - v$$

the system response to a step reference input was as shown in Fig. 4(a). Although the performance could have been improved somewhat using classical compensation network, it was not our objective, and the subject was not pursued any further. Instead, the missing acceleration variable v' was reconstructed, and a new controller

$$= v_{ref} - v - 0.13v'$$

was implemented. The system response to the same step reference input for the new controller is given in Fig. 4(b). As can be seen, the response was quite satisfactory. The small ripples in the response were caused by the tachometer noise.

To reconstruct the acceleration variable, write the system equation in vector form

$$\begin{bmatrix} v' \\ v'' \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & -1.31 \end{bmatrix} \begin{bmatrix} v \\ v' \end{bmatrix} + \begin{bmatrix} 0 \\ 90 \end{bmatrix} u, \qquad A = \begin{bmatrix} 0 & 1 \\ 0 & -1.31 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 90 \end{bmatrix}.$$

$$y = (1, 0) \begin{bmatrix} v \\ v' \end{bmatrix} = v, \qquad H = (1, 0).$$

Let $h_1 = 0$ and $h_2 = h > 0$. Then simple calculation gives

$$Q = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

and

$$C = \begin{bmatrix} 1 & 0 \\ 1 & \frac{1}{1.31}\left(1 - e^{1.31h}\right) \end{bmatrix}$$

Since rank (Q) = 2, the state variables can be reconstructed. Furthermore, rank (C) = 2 for all $h > 0$. Therefore, for any delay time $h > 0$, $v'(t)$ can be reconstructed and is given by

$$v'(t) = \frac{1.31}{(e^{1.31h} - 1)}\left[v(t) - v(t - h)\right] + \frac{90}{(e^{1.31\,h} - 1)}\int_{-h}^{o} (e^{1.31(h+s)} - 1)\, u(t+s)\, ds$$

In this particular example, the delay was $h = 0.04$ sec., the accuracy of A/D and D/A converters was 10 binary bits, and the sampling interval was 200μ sec.

1.5 CONCLUSIONS

A method is developed for exactly reconstructing inaccessible variables in a linear system from the measurable variables, their time delayed values and the control variables on the maximum delay duration. Examples show that the method may give satisfactory results in certain cases.

1.6 REFERENCES

1. D.G. Luenberger, "Observing the State of a Linear System", IEEE Trans. Mil. Electron., Vol. MIL-8, April 1964.
2. D.G. Luenberger, "An Introduction to Observers", IEEE Trans. Automat. Contr., Vol. AC-16, No. 6, December 1971.
3. J.D. Gilchrist, "n-Observability for Linear Systems", IEEE Trans. Automat. Contr., Vol. AC-11, No. 3, July 1966.
4. N.K. Loh and D.H. Chyung, "State Reconstruction from Delayed Observations", Proceedings of the Fifth Annual Pittsburgh Conference on Modeling and Simulation", Vol. 5, April 1974.
5. E.B. Lee and L. Markus, Foundations of Optimal Control Theory, John Wiley, 1967.

2. DELAYED FEEDBACK CONTROLLER FOR A DC MOTOR CONTROL SYSTEM

2.1 INTRODUCTION

In this section, a new controller design, which uses only the observed state variables, is investigaed. It is based on time delayed state variable feedback can be easily implemented by a microprocessor. The method has been applied to several laboratory systems and is found to be almost as effective as an optimal controller.

2.2 PROBLEM STATEMENT

Consider a linear time invariant control system which is given by the vector differential equation

$$\dot{x}(t) = A\,x(t) + B\,u(t)\ ,\ x(0) = x_0$$

$$y(t) = C\,x(t)$$

where x is the n-dimensional state vector, u is the r-dimensional control vector, and y is the m-dimensional observed vector. A, B and C are constant matrices with compatible dimensions. Let J(u) be a cost functional defined by

$$J(u) = \int_0^T f(x(t),u(t))dt\ .$$

The problem is to find a feedback controller u in the form of

$$u = \sum_{j=0}^{N} K_j\, y(t - h_j)$$

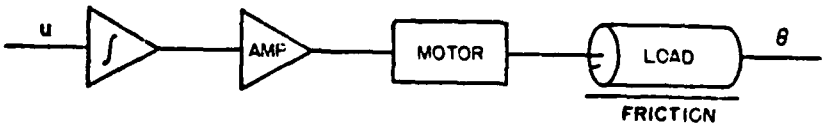
which minimizes the cost functional J(u) over all delayed feedback controllers. Here, $0=h_0 < h_1 < h_2 \ \ldots\ < h_N$ are time delays and K_j, j=0, 1, 2, ...,N, are sxalar constants.

The first question is whether there exists an optimal controller which minimizes the cost functional. If such a control exists, then the next question is how to determine the number of delays N, the delays h_j and the delay coefficients K_j. At present, no answers are available to the above questions. A preliminary study indicates that the minimum number of delays N should be at least equal to n - s, where s is the number of linearly independent variables in y. In the current investigation, the delayed feedback controller is derived in the following way. First, the number of delays is chosen to be equal to n-s. Then the delays h_j are chosen to be large enough so that the absolute value of $y(h_i) - y(h_j)$ is substantially larger than the measurement noise during transient. Then the constants K_j are chosen by numerical iteration. Let K be the vector $(K_0, K_1, \ldots, K_N)$. Once N and h_j are determined, u is uniquely determined by K. Let u = u(K), and K* be the optimal parameter set. Then, assuming the iteration converges, K* may be determined by the iteration

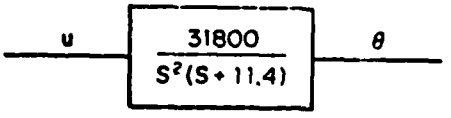
$$K_{i+1} = K_i - \Delta \cdot \text{grad } J(K_i).$$

2.3 EXPERIMENTAL RESULTS

In order to study the feasibility of the proposed method, a third order d.c. motor driven position control system as shown in Fig.1 was investigated.



(a) HARDWARE CONFIGURATION



(b) BASIC MATHEMATICAL MODEL

Fig. 1. Third order d.c. motor position control system.

There are basically two reasons for adding the integrator. The first is to construct a third order system, and the second is to minimize the steady state position error due to the nonlinear Coulomb friction in the system and also to eliminate the staedy state tracking error, that is, the steady state position error when the reference input is a ramp function. The control function u is the input voltage to the integrator, and the output θ is the motor shaft position measured by a potentiometer.

To establish a baseline performance criterion and also for the purpose of comparison, a conventional feedback control using both the position and velocity variables was investigated first. The velocity was measured by a tachometer. The system is shown in Fig.2 below.

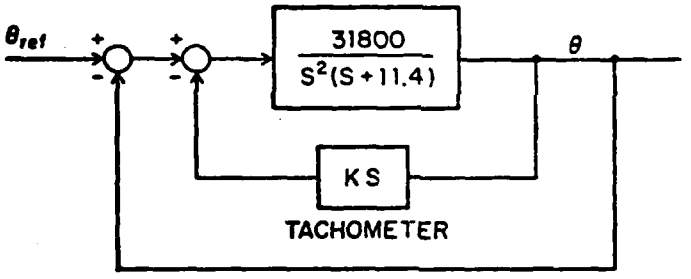


Fig. 2. Conventional feedback control.

The tachometer feedback gain K was determined experimentally so that the system settling time for a step reference input was minimized. It was found that the settling time was minimum when K = 0.77, and the corresponding response was as shown in Fig.3

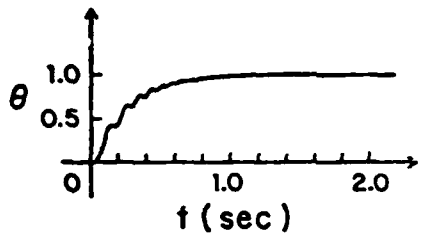


Fig. 3. Step input response of conventional feedback controller (25 mm/sec.).

The settling time was about 1 sec., and there was a sustained small oscillation during the transient. Although the system performance might have been improved by adding compensation network, this possibility was not pursued any further.

The delayed state variable feedback controller was investigated next. It was assumend that only the shaft position was measurable, eliminating the need for measuring the velocity of the shaft of the motor. The mathematical model of the system is now given by

$$\theta''' + 11.4\,\theta'' = 31800\,u$$

$$y = \theta\ .$$

Because the system dimension is 3, and the observed vector y is of dimension 1, two time delays h_1 and h_2 were used in the controller. The values were chosen to be h_1=4.65 msec. and h_2=9.30 msec. The controller is given by

$$u = \theta_{ref}(t) - (K_1\,\theta(t) + K_2\,\theta(t-0.00465) + K_3\,\theta(t-0.0093))$$

Here, θ_{ref} is the reference input. The main objective is to force the output $\theta(t)$ to be the same as the reference input $\theta_{ref}(t)$, that is, $\theta(t) = \theta_{ref}(t)$. A block diagram of the system is shown in Fig.4 below.

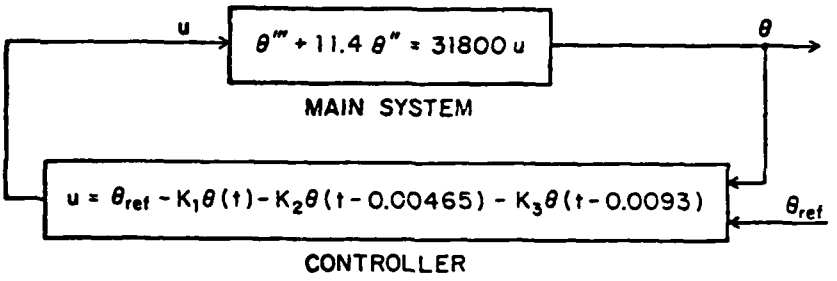


Fig. 4. Delayed variable feedback controller.

The cost functional J(u) is given by

$$J(u) = \int_0^2 \{(\theta_{ref} - \theta(t))^2 + u(t)^2\}\, dt$$

where θ_{ref} is the unit step function and the system is at rest at t=0, that is, $\theta(0) = \theta'(0) = \theta''(0) = 0$. The values K_1, K_2 and K_3 were determined by the iteration method mentioned above, and the values were found to be K_1=113, K_2=-203 and K_3=91. The upper limit of the integral for the cost functional was chosen to be 2 sec. mainly because the conventional feedback control system reached its final value for a step function input in about 1 sec., which is much shorter than the upper limit of 2 sec.

When the optimal delayed state variable feedback controller was implemented, the step reference input response was as shown in Fig.5. The settling time was less than 0.4 sec., and the transient response was very smooth.

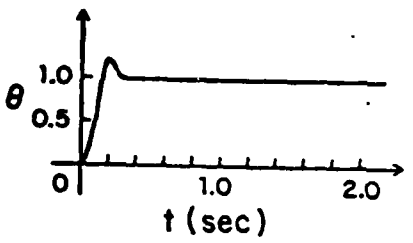


Fig.5 Step input Response of Delyaed Feedback System.

2.4 CONCLUSIONS

It was shown experimentally that the delayed state variable feedback controller is an effective controller. Even though it used only the position variable, the response was at least five times faster than the conventional controller. Furthermore, the transient response of the delayed feedback controller was much smoother than the corresponding response of the conventional controller. The delayed controller, however, was synthesized more or less experimentally, and further studies are needed for developing analytic methods for synthesizing delayed feedback controllers.

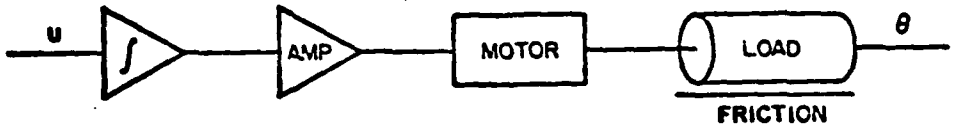
3. MICROPROCESSOR BASED IMPLEMENTATION OF DELAYED FEEDBACK CONTROLLERS

1 INTRODUCTION

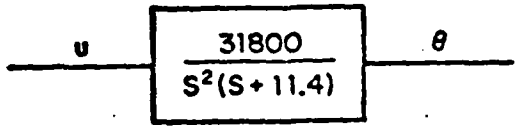
ı this paper, a microprocessor based digital controller for a basically third order d.c. ›tor driven position control system is presented. The controller uses only the output ›sition information, which is measured directly using a digital absolute shaft encoder. ıe result is compared with a conventional feedback controller using both the position and :locity variables. It is shown that the digital controller gives very satisfactory per- ›rmance.

.2 BASIC SYSTEM

ıe system under study is basically a third order d.c. motor driven position control ·stem as shown in Fig. 1. Although the system is modeled as a linear system, there is a ıbstantial nonlinear Coulomb friction. In fact, there is a substantial steady state ›sition error when the loop is closed without the integrator.



(a) HARDWARE CONFIGURATION



(b) BASIC MATHEMATICAL MODEL

Fig. 1. Third order d.c. motor position control system

The function u(t) is the input voltage to the operational amplifier, and the output θ(t) is the shaft position. The main purpose is to design and implement a controller so that the output θ(t) is the same as the input reference function $\theta_{ref}(t)$.

The controller investigated is basically a closed loop, or feedback, digital controller, which is based on a microprocessor as shown in Fig. 2. It should be noted that, although it is basically a third order system, only the position variable θ(t) is used in the design, eliminating the need for a tachometer, an additional system hardware.

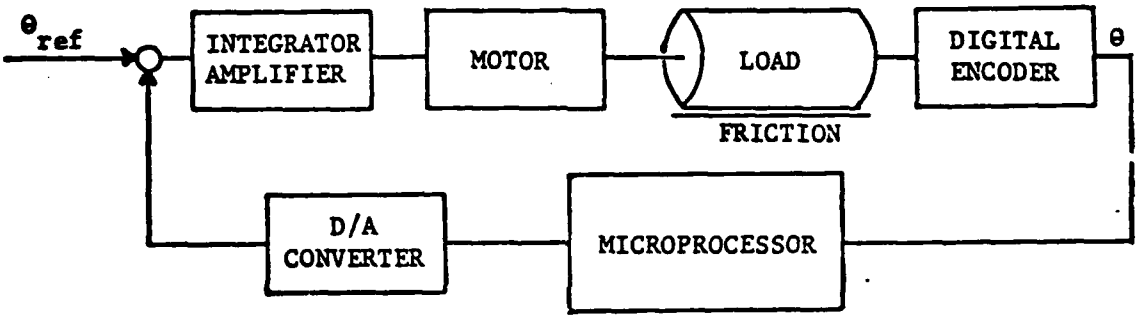


Fig. 2. Digital Controller

3.3 CONVENSIONAL CONTROLLER

To establish a baseline performance criterion and also for the purpose of comparison, a conventional feedback control using both the position and velocity variables is investigated first. The velocity is measured by a tachometer. The system is shown in Fig. 3.

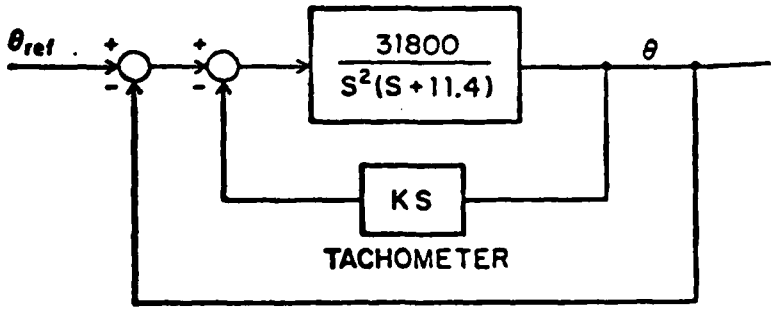


Fig. 3. Conventional feedback control

The tachometer feedback gain K is determined experimentally so that the system settling time for a step reference input is minimized. It is found that the settling time is minimum when K = 0.77, and the corresponding response is as shown in Fig. 4.

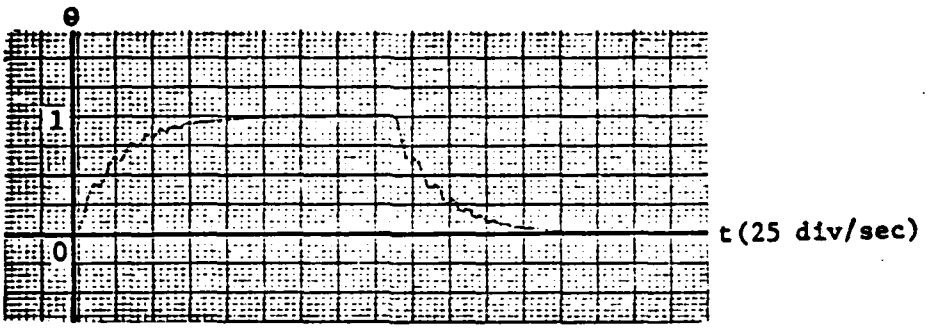


Fig. 4. Step input response of conventional feedback controller

The settling time is about 1 sec., and there is a sustained small oscillation during the transient. Although the system performance could be improved by adding compensation network, this possibility is not pursued any further.

3.4 DIGITAL CONTROLLER

The system given in Fig. 1 can be represented by the differential equation

$$\theta''' + 11.4\theta'' = 31800u \ .$$

Let $y_1 = \theta$, $y_2 = \dot{\theta}$, $y_3 = \ddot{\theta}$ and

$$y = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$$

Then the system equation can now be written as the vector differential equation

$$\dot{y} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -11.4 \end{bmatrix} y + \begin{bmatrix} 0 \\ 0 \\ 31800 \end{bmatrix} u$$

To design a digital controller, it is first necessary to discretize the above equation. Let $x(k) = y(kT)$, and $v(k) = u(t)$, $kT \leq t < (k+1)T$, where T is the sampling interval.

:hen

$$x(k+1) = Ax(k) + Bv(k)$$

:here

$$A = \begin{bmatrix} 1 & T & -\frac{1}{11.4^2} + \frac{T}{11.4} + \frac{1}{11.4} e^{-11.4T} \\ 0 & 1 & \frac{1}{11.4}(1 - e^{-11.4T}) \\ 0 & 0 & e^{-11.4T} \end{bmatrix}$$

ınd

$$B = \begin{bmatrix} 31800 \times \{\frac{T}{11.4^2} - \frac{T^2}{22.8} + \frac{1}{11.4^3}(e^{11.4T} - 1)\} \\ 31800 \times \{\frac{1}{11.4^2} + \frac{T}{11.4} - \frac{1}{11.4^2}(e^{11.4T} - 1)\} \\ 31800 \times \frac{1}{11.4}(e^{11.4T} - 1) \end{bmatrix}$$

:he task is to find an algorithm for v(k) using $x_1(k)$ and θ_{ref} only so that $x_1(k) = \theta_{ref}$ .n steady state. Since it is found that such a feedback controller is not unique, further :estrictions are imposed on the controller v(k). The constraints are that the response :ettling time is minimized and the output θ(t) is smooth during the transient for a step :eference input θ_{ref}.

3y applying a recently developed method for synthesizing an optimal controller based on lelayed feedback [1], the digital controller algorithm

$$v(k) = 0.5\ \theta_{ref} - 80.5\ x(k) + 150\ x(k-4) - 70\ x(k-8)$$

is obtained for the sampling interval of T = 1 msec. The actual implementation of the :ontroller is shown in Fig. 5.

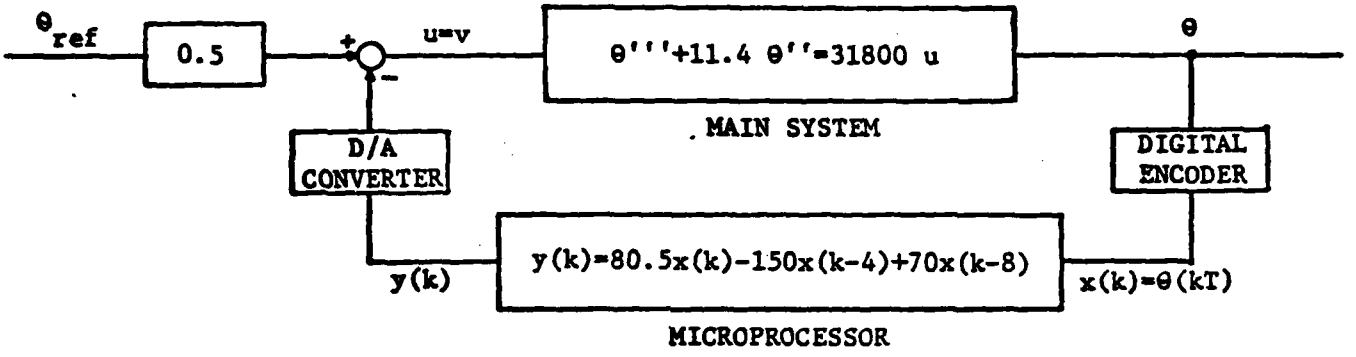


Fig. 5. Microprocessor based digital controller (T = 1 msec.)

Several microprocessors have been used for implementing the digital controller. In all cases, a real time clock of 1 msec is used for initiating the interrupt driven control algorithm. The accuracy of the shaft encoder and the D/A converter is 12-bit. When an LSI-11 microprocessor is used, 132 16-bit words of memory are required. Other microprocessors investigated are MOS6502 (8-bit) and Motorola XM68000 (16-bit).

The step input response of the digital controller is given in Fig. 6. The particular microprocessor used for the experiment is LSI-11.

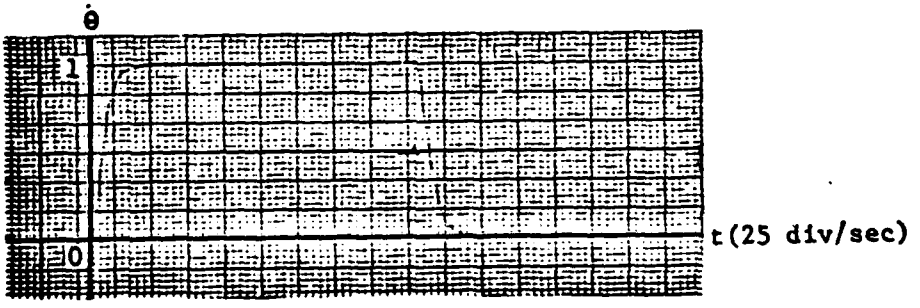


Fig. 6. Step input response of the digital controller

Although only the position variable is used in the digital controller, it gives a much better system response than the conventional feedback controller. The settling time for

a step reference input is now less than 0.36 sec., which is about 3 times faster than the conventional controller. Furthermore, the transient response of the digital controller is very smooth.

3.5 CONCLUSIONS

A microprocessor based digital controller is designed for a third order position control system, and is implemented in an actual system. Although it uses only the position variable it is shown that the system performance is very satisfactory.

3.6 REFERENCE

1. Chyung, D.H., "On a New Closed Loop Controller Design", 1980 Conference on Information Sciences and Systems, Princeton University, March 1980.

4. DELAYED FEEDBACK CONTROLLER FOR AN A.C. MOTOR CONTROL SYSTEM

The system considered is an a.c. motor driven third order position control system. A block diagram is shown in Fig.1 below.

Fig.1 A.C. motor driven position control system.

The output variable is measured by a digital shaft encoder. Since it is the only sensor employed in the system, only the position variable is available for feedback, that is, the feedback compensation may not use velocity and acceleration variables.

The mathematical model of the system, which is determined experimentally, is given by the following transfer function.

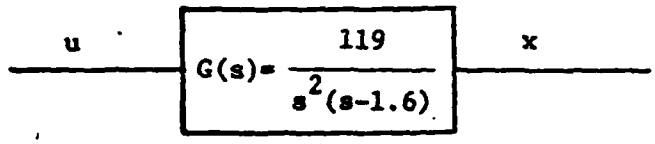


Fig.2 Mathematical model

The problem is to design a feedback compensation H(s), using only the output position variable x, such that the resulting system is stable and follows the reference input x_{ref}. Fig.3 below shows the closed loop control system.

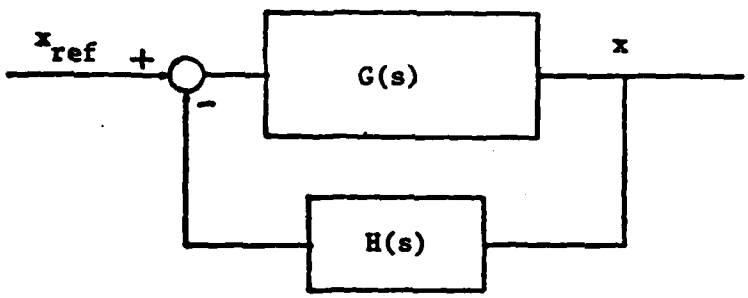


Fig.3 Closed loop system.

The major difficulty in designing a satisfactory compensation H(s) is that the system is a third order system and the acceleration and velocity variables are not available for feedback. That is, H(s) may not contain s and s^2 terms.

To resolve the difficulty, consider the time delayed feedback compensation,

$$H(s) = K_1 + K_2 e^{-T_1 s} + K_2 e^{-T_2 s} .$$

The feedback now contains only the output variable x. However, the feedback loop also contains two time delayed values $x(t-T_1)$ and $x(t-T_2)$. A block diagram of the system is given in Fig. 4.

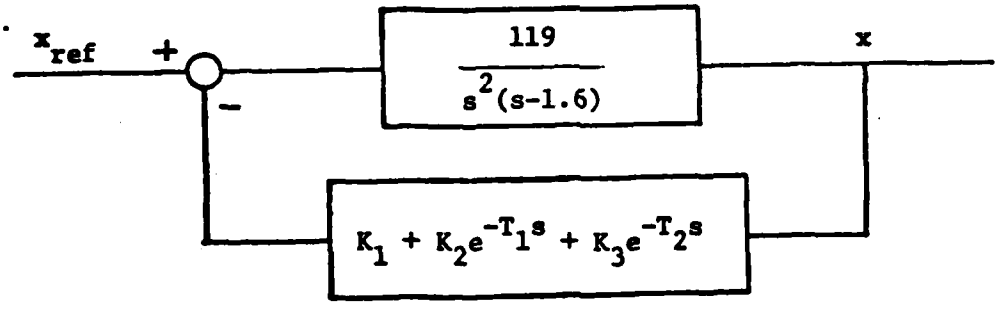


Fig.4 Delayed feedback system.

In the current system, the time delays T_1 and T_2 are chosen to be 0.015 sec. and 0.03 sec., respectively. To satisfy the requirement $x=x_{ref}$ in steady state, the feedback parameters must satisfy the condition

$$K_1 + K_2 + K_3 = 1.$$

The task is now to find the feedback constants K_1, K_2 and K_3. The Bode plot of the forward loop transfer function G(s) is designated by G in Fig.5. For K_1=473, K_2=-916 and K_3=443, the Bode plots of H(s) and GH(s) are shown as H and GH, respectively, in Fig.5. It can be seen from the plots, GH(s) has a gain margin of 20db and phase margin of 50 degrees. hence the system should give satisfactory performance for the particular compensation H(s).

Fig.6(a) shows the step input response ofthe actual system. In order to determine how accurate the model is, the model is simulated on a computer, and the simulation result for the same step input is shown in FIG. 6(b). From the curves, it can be seen that the compensation designed above indeed gives a satisfactory response and the mathematicl model is also satisfactory.

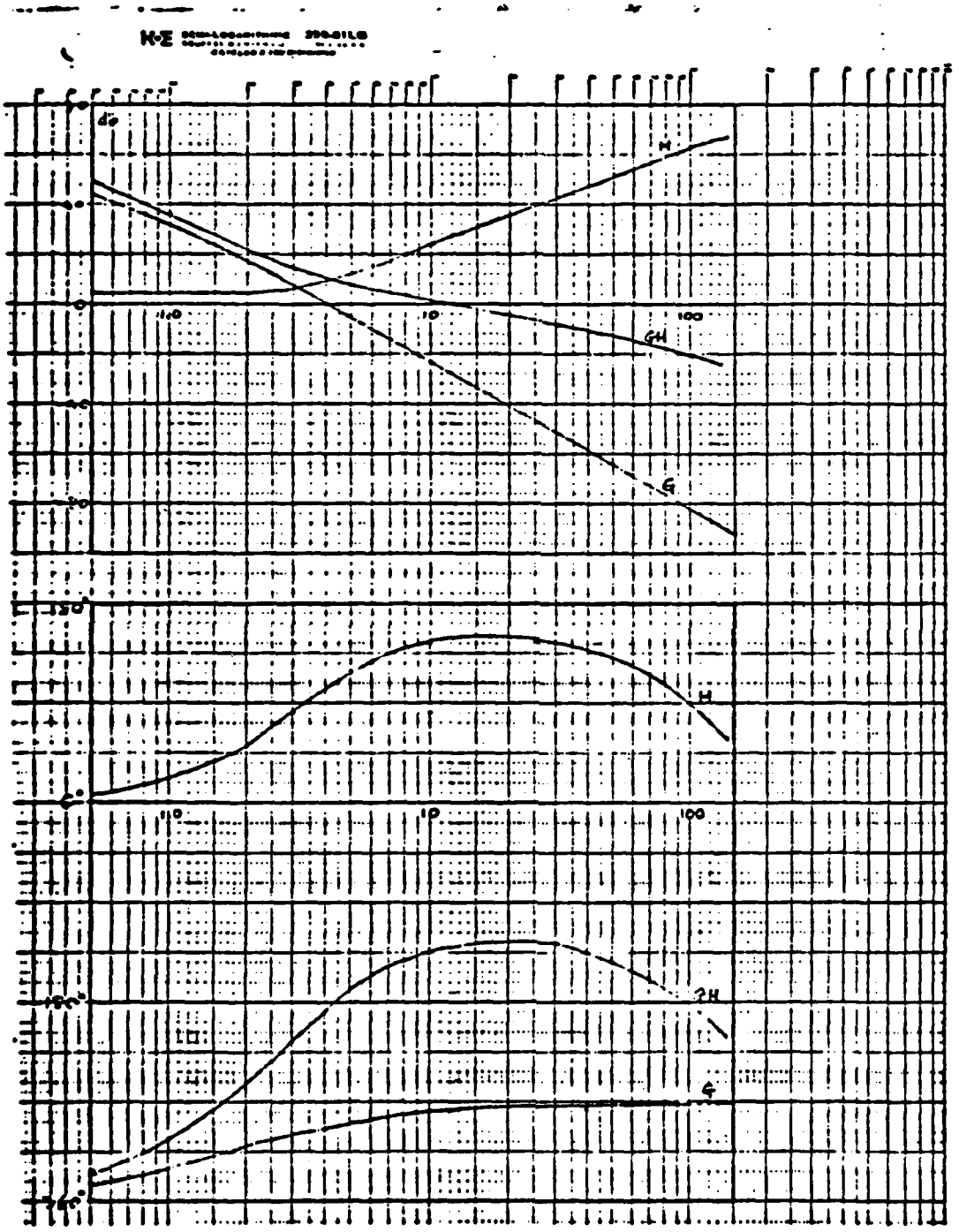
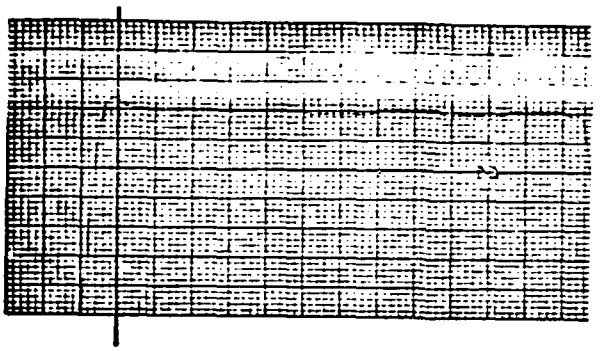
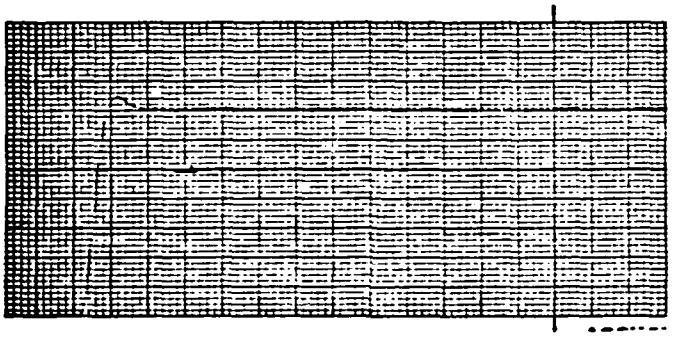


Fig.5 Bode plots of G, H and GH.



(a) Actual system (5 div./sec.)



(b) Computer simulation (5 div./sec.)

Fig. 6 Step input response of delayed feedback system.

5. DISTURBANCE COMPENSATION SCHEME

One of the problems encountered by a position control system on a moving platform is the error due to the disturbance caused by the movement of the platform. In this section, a compensation method for reducing the effects of the disturbance in a d.c. motor driven third order position control system is investigated. Figure 1 below shows the system under study. The

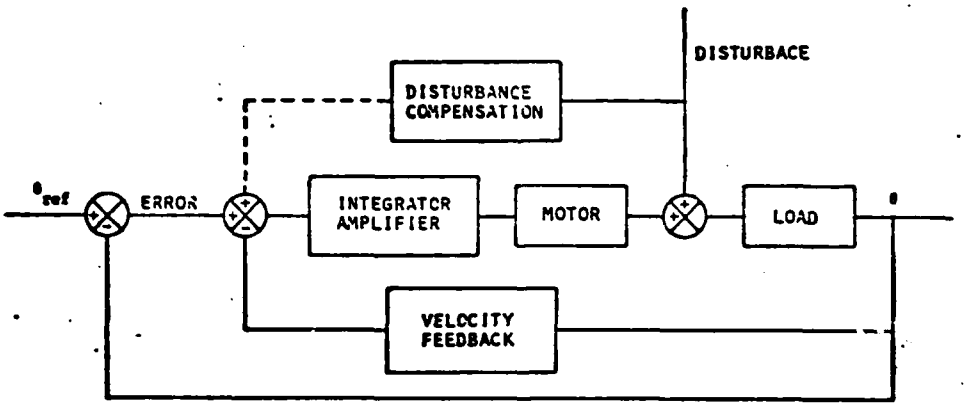


Fig.1 Position Control System

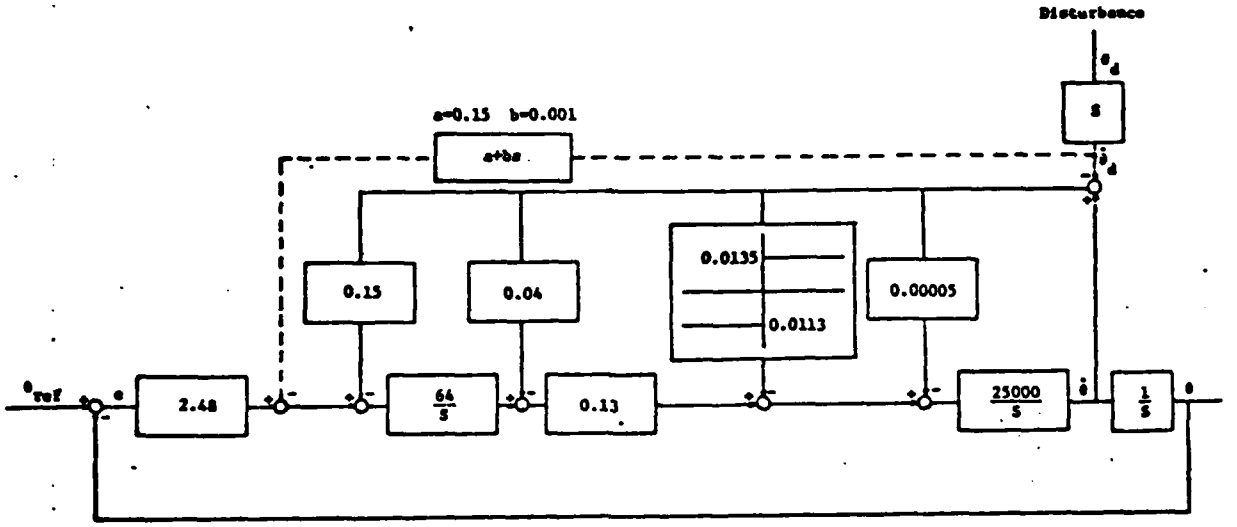


Fig.2 Block Diagram

block diagram model of the system is as shown in Figure 2. As can be seen from the block diagram, there is a substantial nonlinear Coulomb friction between the stator and rotor of the d.c. motor. When a typical second order position control, that is, without the additional integrator in Figure 1, is employed, there is a large steady state error due to the nonlinear friction. The additional integrator is therefore inserted in the system to force the steady state error to be zero. Another reason for the inclusion of the integrator is to investigate the effectiveness of the proposed disturbance compensation scheme in a third order system.

To simulate the disturbance due to the motion of a platorm, the stator, that is, the outer shell, of the motor is rotated, and the effects of the stator motion on the position error of the system is studied. The disturbance motion of the platform, and therefore, the motion of the motor stator, is denoted by θ_d, and the system position, that is, the position of the motor rotor is denoted by θ. The system error is defined by

$$e = \theta_{ref} - \theta$$

where θ_{ref} is the desired position angle. The disturbance stator motion θ_d affects the rotor position θ in three major ways. The first is, of course, the disturbance torque being applied to the rotor shaft through the frictions between the stator and rotor of the motor. The second is the additional velocity feedback due to the disturbances. The last is the electromagnetic interaction between the stator and rotor motions of the motor.

Since the velocity of the disturbance signal, $\dot{\theta}_d$, is practically the most convenient variable to measure, the disturbance compensation is based on the disturbance velocity $\dot{\theta}_d$. The actual structure of the compensation scheme is shown by the dotted line in Figure 2. The values of the constants a and b are determined experimentally and the actual values used in the current experiment are a = 0.15 and b = 0.001.

The step input response of the system without external disturbances is as shown in Figure 3. The settling time is about 0.8 sec. in one direction and 2.6 sec. in the other direction. The difference in settling time in the positive and negative directions is due to unsymmetrical nonlinear frictions.

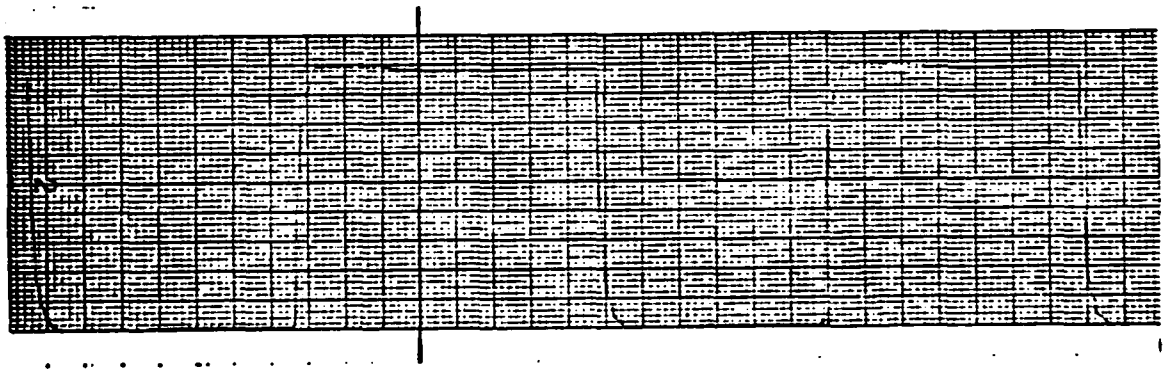


Fig. 3 Step Input Response (0.9°/div., 5 div./sec.)

To study the effects of the disturbances, three different types of disturbance motions are applied to the stator of the motor, and the rotor position error is observed while keeping the reference input θ_{ref} at zero degree. When no disturbance compensation scheme is employed the resulting system position errors are as shown in Figure 4. The curves (a) represent the stator position due to disturbances and the curves (b) represent the actual system position error due to the disturbances. When an impulse of 18° disturbance is applied to the stator, the resulting maximum system error is 19°. For a 35° step disturbance, the maximum error is 25°, and for a 33° peak-to-peak sinusoidal disturbance, the peak-to-peak error is 45°. When the disturbance compensation is employed, the position errors due to the same disturbances are as shown in Figure 5. Now the maximum error due to an 18° impulse disturbance is 6°, the maximum error due to a 35° step disturbance is 9° and the peak-to-peak value of the error due to 35° p-p sinusoidal disturbance is 14°. The results are summerized in Table 1. In general, the disturbance compensation scheme reduces the effects of external disturbances significantly, in the present case, by a factor of 3.

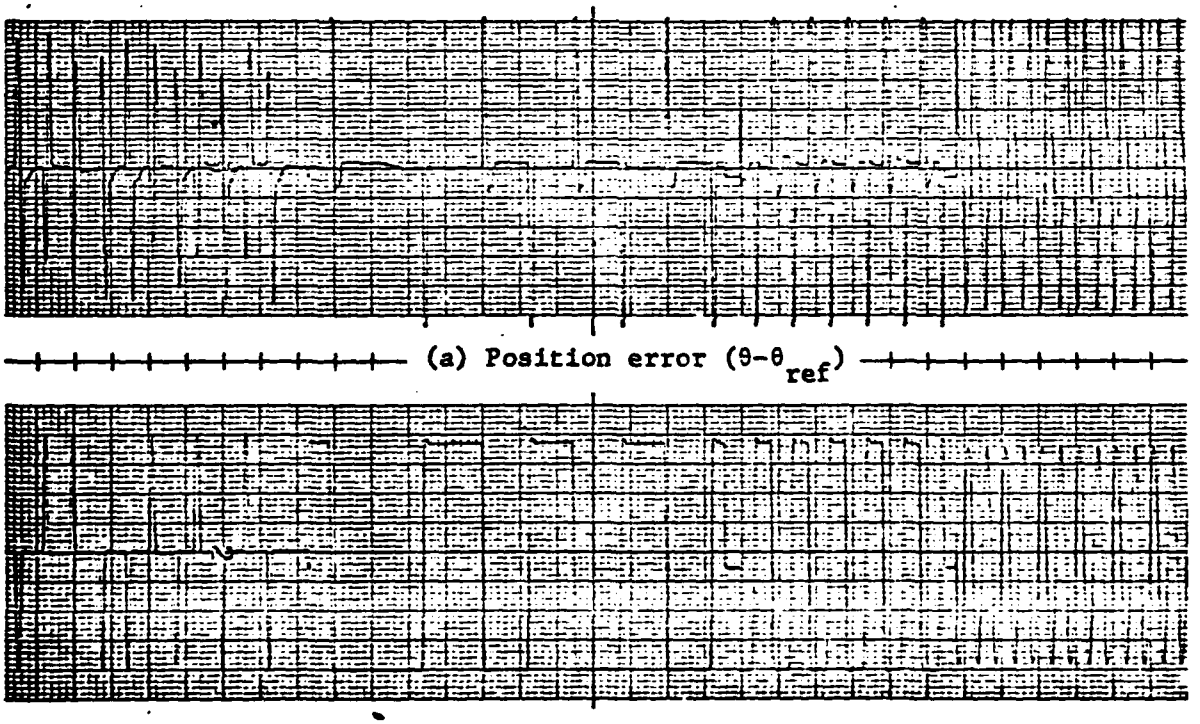
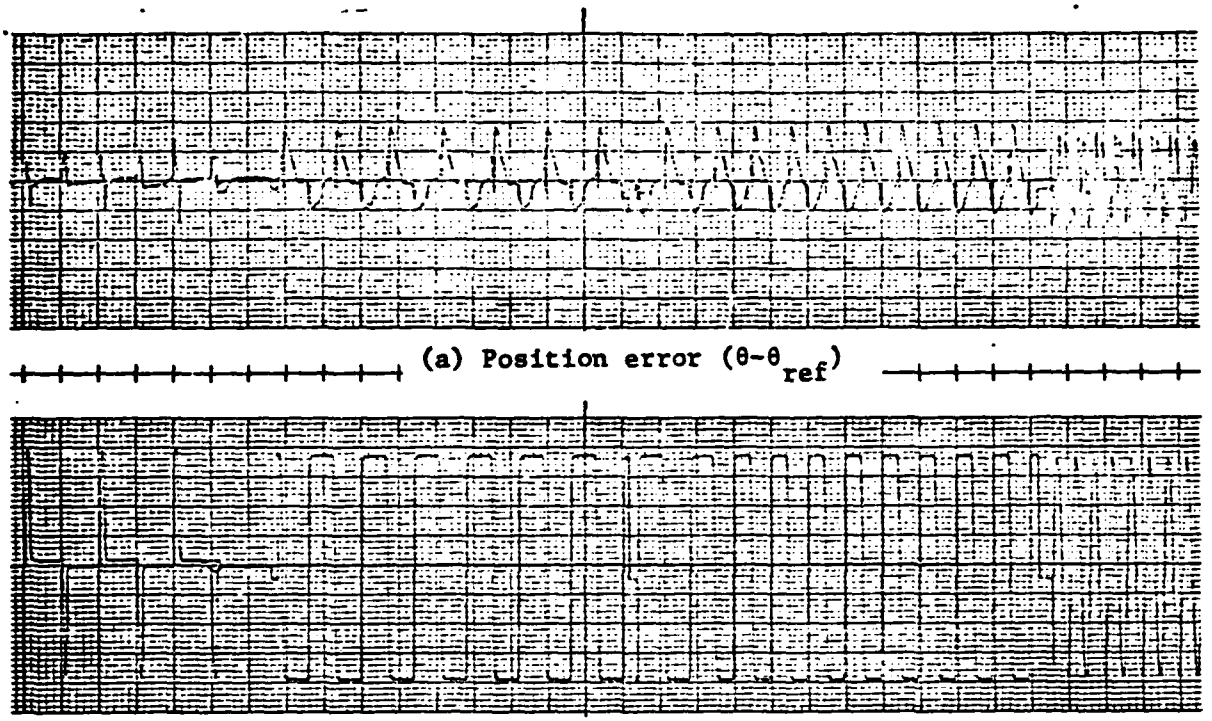


Fig. 4 Position error due to disturbance in the system without compensation (0.9°/div., 5 div./sec.)



(a) Position error ($\theta - \theta_{ref}$)

(b) Disturbance input (θ_d)

Fig. 5 Position error due to disturbance in the system with compensation (0.9°/div., 5 div./sec.)

Disturbance input	Without compensation	With compensation
18° Impulse	19°	6°
35° Step	25°	9°
33° p-p 2Hz.	45° p-p	14° p-p

Table 1. Position error due to external disturbances

In conclusion, it is shown that a disturbance compensation scheme may be employed to reduce the effects of external disturbances in a position control system. Since the major portion of the disturbance is transmitted to the control system through the nonlinear friction, a nonlinear disturbance compensation scheme would be more effective. Further research is currently being carried out to more fully develop the disturbance compensation methods.

6. DISTURBANCE CANCELLING FOR XM-97 TURRET USING TIME DELAYS

In this section, a disturbance cancelling controller is studied for the XM-97 turret control system using time delays. Since the velocity of the disturbance is available for measurement through a hull gyro, the controller uses the rate gyro output. The block diagram of the system with disturbance is given in Fig. 1. Let d(t) be the disturbance hull velocity. Then the disturbance cancelling scheme is basically that of feeding back the cancelling variable z(t) to the input.

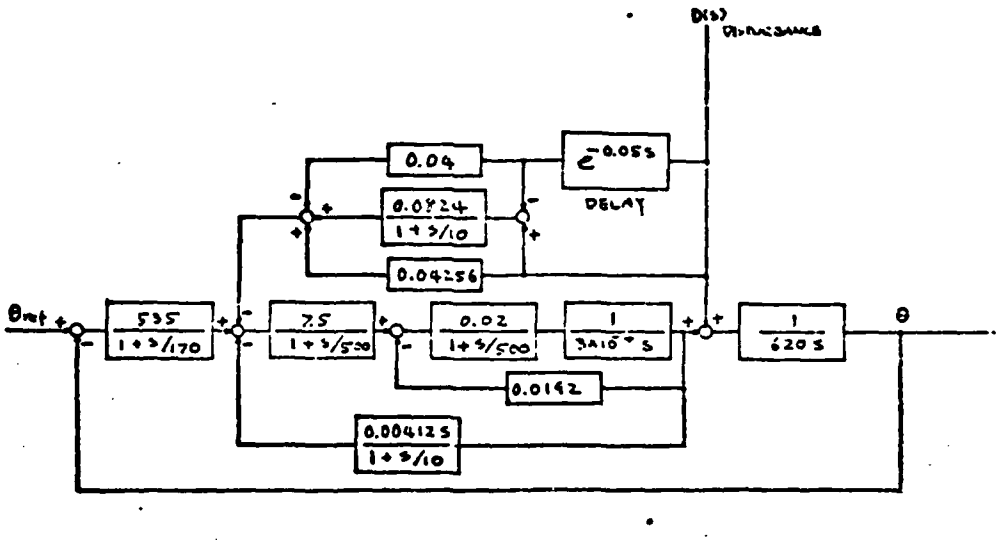


Fig.1 Disturbance Cancelling Control

The variable z(t) is given as

$$z(t) = -\frac{1}{535}\,[0.04256d(t) + 0.04d(t - 0.05) + y(t)]$$

$$\dot{y}(t) + 10y(t) = 0.824(d(t) - d(t - 0.05))$$

Note that this scheme once again uses a time delayed variable. When d(t)= 1000 Cos(10t) and a step reference input of one degree is applied to the system without the disturbance cancelling, the output $\theta(t)$ is as shown in Fig. 2(a). The same response, but with the disturbance cancelling, is given in Fig. 2(a). As can be seen, there is a substantial improvement when disturbance cancelling is employed. The disturbance cancelling scheme is more effective if an optimal control is employed. To show this, the scheme is now applied to the optimal XM-97 turret system as shown in Fig. 3. The response to the same disturbance is given in Fig.4. It is believed that at least a part of the further improvement is due to the elimination of the original tachometer feedback dynamics as well as the application of an optimal control.

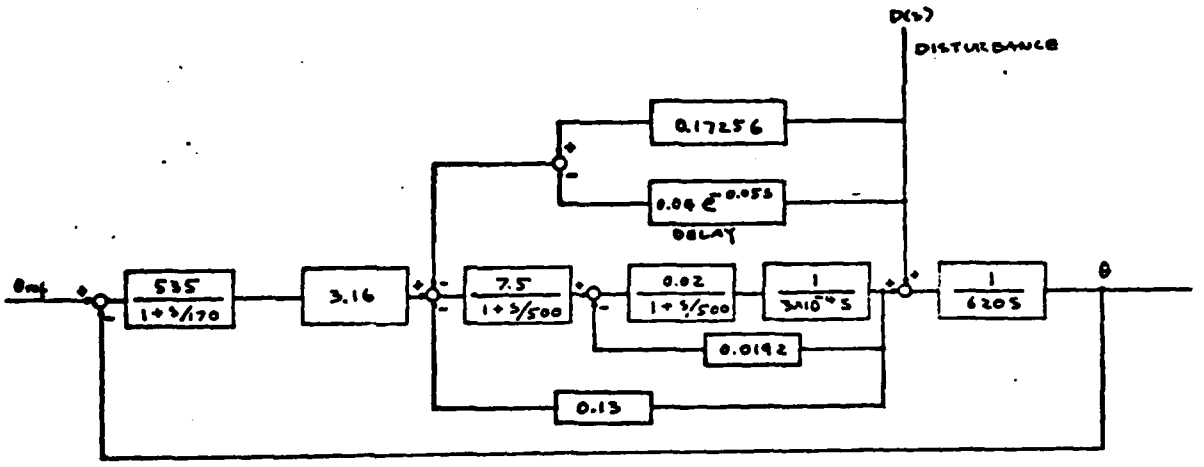


Fig.3 Disturbance Cancelling Control For XM-97 Turret Control System.

MINIMUM X VERSUS TIME MAXIMUM
0.0 1.5402E 00

TIME	X
0.0	0.0
6.0000E-02	4.3581E-01
1.2000E-01	1.1713E 00
1.8000E-01	1.5356E 00
2.4000E-01	1.4420E 00
3.0000E-01	1.1447E 00
3.6000E-01	9.1138E-01
4.2000E-01	8.5065E-01
4.8000E-01	9.1661E-01
5.4000E-01	1.0055E 00
6.0000E-01	1.0454E 00
6.6000E-01	1.0273E 00
7.2000E-01	9.8510E-01
7.8000E-01	9.5743E-01
8.4000E-01	9.6167E-01
9.0000E-01	9.9060E-01
9.6000E-01	1.0243E 00
1.0200E 00	1.0447E 00
1.0800E 00	1.0430E 00
1.1400E 00	1.0253E 00
1.2000E 00	9.9833E-01
1.2600E 00	9.7377E-01
1.3200E 00	9.6066E-01
1.3800E 00	9.6079E-01
1.4400E 00	9.7477E-01
1.5000E 00	9.9603E-01
1.5600E 00	1.0197E 00
1.6200E 00	1.0356E 00
1.6800E 00	1.0372E 00
1.7400E 00	1.0294E 00
1.8000E 00	1.0094E 00
1.8600E 00	9.8609E-01
1.9200E 00	9.6755E-01
1.9800E 00	9.6070E-01
2.0400E 00	9.6687E-01
2.1000E 00	9.8566E-01
2.1600E 00	1.0088E 00
2.2200E 00	1.0290E 00
2.2800E 00	1.0393E 00
2.3400E 00	1.0359E 00
2.4000E 00	1.0200E 00
2.4600E 00	9.9706E-01
2.5200E 00	9.7515E-01
2.5800E 00	9.6192E-01
2.6400E 00	9.6200E-01
2.7000E 00	9.7536E-01
2.7600E 00	9.9732E-01
2.8200E 00	1.0202E 00
2.8800E 00	1.0360E 00
2.9400E 00	1.0393E 00
3.0000E 00	1.0200E 00

(b) With Disturbance Cancelling

MINIMUM X VERSUS TIME MAXIM.
0.0 1.5543E

TIME	X
0.0	0.0
6.0000E-02	4.8540E-01
1.2000E-01	1.2559E 00
1.8000E-01	1.5542E 00
2.4000E-01	1.3540E 00
3.0000E-01	9.9152E-01
3.6000E-01	7.7711E-01
4.2000E-01	8.0621E-01
4.8000E-01	9.0306E-01
5.4000E-01	1.1539E 00
6.0000E-01	1.2124E 00
6.6000E-01	1.1408E 00
7.2000E-01	1.0106E 00
7.8000E-01	8.9419E-01
8.4000E-01	8.2713E-01
9.0000E-01	8.3306E-01
9.6000E-01	8.9827E-01
1.0200E 00	9.9295E-01
1.0800E 00	1.0837E 00
1.1400E 00	1.1424E 00
1.2000E 00	1.1522E 00
1.2600E 00	1.1112E 00
1.3200E 00	1.0331E 00
1.3800E 00	9.4395E-01
1.4400E 00	8.7380E-01
1.5000E 00	8.4693E-01
1.5600E 00	8.7367E-01
1.6200E 00	9.4355E-01
1.6800E 00	1.0340E 00
1.7400E 00	1.1198E 00
1.8000E 00	1.1523E 00
1.8600E 00	1.1306E 00
1.9200E 00	1.0763E 00
1.9800E 00	9.8733E-01
2.0400E 00	9.0275E-01
2.1000E 00	8.5217E-01
2.1600E 00	8.5326E-01
2.2200E 00	9.0562E-01
2.2800E 00	9.9096E-01
2.3400E 00	1.0794E 00
2.4000E 00	1.1402E 00
2.4600E 00	1.1519E 00
2.5200E 00	1.1106E 00
2.5800E 00	1.0307E 00
2.6400E 00	9.4001E-01
2.7000E 00	8.7032E-01
2.7600E 00	8.4592E-01
2.8200E 00	8.7535E-01
2.8800E 00	9.4032E-01
2.9400E 00	1.0393E 00
3.0000E 00	1.1163E 00

(a) Without Disturbance Cancelling

Fig. 4 Response of the System Given in Fig.3

7. MICROPROCESSOR-BASED OPTIMAL DISTURBANCE ACCOMODATING CONTROL FOR XM-97 TURRET CONTROL SYSTEM

7.1 INTRODUCTION

The design of a microcomputer-based optimal disturbance accomodating controller for the XM-97 turret control system is carried out. Simulation studies of the system are made to compare the performance of the system under different control schemes. Responses of the system subject to firing burst torque disturbance as well as sinusoidal torque disturbance are given. Significant improvemnets in the behaviour of the system under disturbances are obtained using the proposed schemes.

The optimal analog control of the system and system disturbance modelling are briefly dealt with in section 7.2. The discrete version of the controller and the discrete disturbance-isolated observer are covered in section 7.3. Simulation results are shown in section 7.4.

7.2 OPTIMAL CONTROLLER AND SYSTEM-DISTURBANCE MODEL

The block diagram for the open-loop XM-97 turret with disturbance input w(t) is given in Fig. 1 and the corresponding equations of motion are given by

$$\dot{\underline{x}}(t) = A\underline{x}(t) + Bu(t) + Fv_r + Dw(t), \tag{1a}$$

$$\underline{y}(t) = C\underline{x}(t), \tag{1b}$$

where $\underline{x}(t) = [x_1(t)\ x_2(t)]^T$, $x_1(t) \triangleq [x_r(t) - x_1'(t)]$, $x_2(t) \triangleq [Nv_r - x_2'(t)]$, $\underline{y}(t) = [y_1(t)\ y_2(t)]^T$ is the observed vector, w(t) is the disturbance torque, and A, B, E, D and C are given by

$$A = \begin{bmatrix} 0 & 1/N \\ 0 & -1.28 \end{bmatrix} \triangleq \begin{bmatrix} 0 & a_{12} \\ 0 & -a_{22} \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ -2.675\times10^5 \end{bmatrix} \triangleq \begin{bmatrix} 0 \\ -b \end{bmatrix},$$

$$E = \begin{bmatrix} 0 \\ 1.28N \end{bmatrix} \triangleq \begin{bmatrix} 0 \\ f \end{bmatrix}, \quad D = \begin{bmatrix} 0 \\ -10^4/3N \end{bmatrix} \triangleq \begin{bmatrix} 0 \\ -g \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

The control objective is to drive $\underline{x}(t)$ to the zero state in the presence of the distrubance torque w(t), and in the same time minimizing a quadratic performance measure. To achieve the control objective, the control u(t) is split into three parts as

$$u(t) = u_{fb}(t) + u_{ff}(t) + u_w(t), \tag{2}$$

where $u_{fb}(t)$ is the feedback component responsible for driving $\underline{x}(t)$ to the zero state, $u_{ff}(t)$ is the feedforward component responsible for accommodating the velocity command v_r, and $u_w(t)$ is the feedforward component responsible for accommodating the distrubance torque w(t). It can easily be shown that $u_{ff}(t)$ and $u_w(t)$ are given by, respectively,

$$u_{ff}(t) = \frac{1.28N}{2.675\text{x}10^5}\ v_r \triangleq k_r v_r\ , \tag{3}$$

$$u_w(t) = -\frac{10^4}{3\text{N}\text{x}2.675\text{x}10^5}\ w(t) \triangleq -k_w\, w(t)\ . \tag{4}$$

Substituting Eqs. (2), (3), (4) into Eq. (1) yields

$$\dot{\underline{x}}(t) = A\underline{x}(t) + Bu_{fb}(t)\ . \tag{5}$$

Consider the performance measure

$$J = \int_0^\infty [q_{11}x_1^2(t) + ru_{fb}^2(t)]dt, \tag{6}$$

where $q_{11} \geq 0$ and $r>0$ are weighting constants.

The optimal control which minimizes J is given by

$$\begin{aligned} u_{fb}(t) &= u_{opt}(t) \\ &= k_1x_1(t) + k_2x_2(t) \\ &= k_1[x_r(t) - x_1'(t)] + k_2[Nv_r - x_2'(t)]. \end{aligned} \tag{7}$$

The numerical values of the optimal gains k_1 and k_2 for different values of q_{11} together with the values of feedforward gains k_r and k_w are given in Table 1.

From Eqs. (2), (3), (4) and (7), the complete control u(t) is given by

$$u(t) = k_1x_1(t) + k_2x_2(t) + k_rv_r(t) - k_ww(t)\ . \tag{8}$$

Since the distrubance w(t) is not known, the control u(t) can be implemented as

$$u(t) = k_1x_1(t) + k_2x_2(t) + k_rv_r - k_w\hat{w}(t)\ , \tag{9}$$

where $\hat{w}(t)$ is an estimate of w(t).

The estimate $\hat{w}(t)$ of w(t) considered in this report will be generated by a discrete Luenberger observer. For simplicity, the distrubance w(t) will be

TABLE 1. OPTIMAL GAINS AND EIGENVALUES OF XM-97 HELICOPTER TURRET CONTROL SYSTEM

(TWO-STATE-VARIABLE MODEL)

Chan..l		Azimuth Channel			Elevation Channel		
q_{11}		5	10	15	5	10	15
Optimal Gains	k_1	2.2361	3.1623	3.8730	2.2361	3.1623	3.8730
	k_2	1.5949×10^{-4}	1.9055×10^{-4}	2.1138×10^{-4}	1.3896×10^{-4}	1.6613×10^{-4}	1.8435×10^{-4}
	k_r	2.9667×10^{-3}	2.9667×10^{-3}	2.9667×10^{-3}	3.8759×10^{-3}	3.8759×10^{-3}	3.8759×10^{-3}
	k_w	2.0098×10^{-5}	2.0098×10^{-5}	2.0098×10^{-5}	1.5380×10^{-5}	1.5380×10^{-5}	1.5380×10^{-5}
Eigenvalues of $A + B[k_1\ k_2]$		-21.972 $\pm$ j21.954	-26.126 $\pm$ j26.111	-28.912 $\pm$ j28.898	-19.226 $\pm$ j19.205	-22.860 $\pm$ j22.842	-25.297 $\pm$ j25.281

approximated by a random step plus ramp function described by

$$\dot{w}(t) = \sigma(t), \quad w(0) = w_o, \tag{10}$$

where $\sigma(t)$ is an unknown sequence of pulses included to take into account of the random change in values of $w(t)$.

Augmenting Eq. (10) to Eq. (1) yields

$$\begin{bmatrix} \dot{\underline{x}}(t) \\ \hline \dot{w}(t) \end{bmatrix} = \left[\begin{array}{c:c} A & D \\ \hline 0 & 0 \end{array}\right] \begin{bmatrix} \underline{x}(t) \\ \hline w(t) \end{bmatrix} + \begin{bmatrix} B \\ \hline 0 \end{bmatrix} u(t) + \begin{bmatrix} E \\ \hline 0 \end{bmatrix} v_r + \begin{bmatrix} 0 \\ \hline 1 \end{bmatrix} \sigma(t)$$

$$\triangleq \tilde{\tilde{A}}\underline{\tilde{x}}(t) + \tilde{B}u(t) + \tilde{E}v_r(t) + \tilde{D}\sigma(t) , \tag{11a}$$

$$= \tilde{\tilde{A}}\underline{\tilde{x}}(t) + \tilde{B}u'(t) + \tilde{D}\sigma(t) \tag{11b}$$

$$\underline{y}(t) = [I_2 \,\vdots\, 0] \begin{bmatrix} \underline{x}(t) \\ \hline w(t) \end{bmatrix} \triangleq \tilde{\tilde{H}}\underline{\tilde{x}}(t) , \tag{11c}$$

where the various vectors and matrices are as defined, and

$$u'(t) \triangleq k_1x_1(t) + k_2x_2(t) - k_ww(t) . \tag{11d}$$

Since the matrix pair $[\tilde{A}, \tilde{H}]$ is completely observable, i.e.,

$$\text{rank } [\tilde{H}^T \,\vdots\, \tilde{A}^T\tilde{H}^T \,\vdots\, \tilde{A}^{2T}\tilde{H}^T]$$

$$= \text{rank} \left[\begin{array}{cc:cc:c} 1 & 0 & 0 & 0 & \\ 0 & 1 & 1/N & -1.28 & \tilde{A}^{2T}\tilde{H}^T \\ 0 & 0 & 0 & -10^4/3N & \end{array}\right] = 3 ,$$

the unknown disturbance $w(t)$ can be estimated by a reduced-order Luenberger observer.

7.3 DISCRETE OPTIMAL CONTROLLER WITH DISTURBANCE-ISOLATED OBSERVER

In this section, a microcomputer based controller with disturbance compensation will be designed. In this scheme, the disturbance w(t) will be estimated as $\hat{w}(k)$ by a discrete disturbance-isolated observer and the optimal control input will be realized as (see Eq. (11d))

$$u'(k) = k_1 x_1(k) + k_2 x_2(k) - k_w \hat{w}(k) \tag{12}$$

~~III-1~~ Design of Discrete Disturbance-Isolated Observer

The augmented system (11b) can be described as

$$\begin{bmatrix} \underline{x}(k+1) \\ \hline w(k+1) \end{bmatrix} = \left[\begin{array}{c|c} A_d & D_d \\ \hline 0 & 1 \end{array}\right] \begin{bmatrix} \underline{x}(k) \\ \hline w(k) \end{bmatrix} + \begin{bmatrix} B_d \\ \hline 0 \end{bmatrix} u(u) + \begin{bmatrix} \bar{D}_d \\ \hline \tau \end{bmatrix} \sigma(t) \tag{13}$$

$$= \tilde{A}_d \tilde{\underline{x}}(k) + \tilde{B}_d u(k) + \tilde{D}_d \sigma(t)$$

where*

$$A_d = e^{A\tau} = \begin{bmatrix} 1 & a_{12}(1-e^{-a_{22}\tau})/a_{22} \\ 0 & e^{-a_{22}\tau} \end{bmatrix}$$

$$D_d = (e^{A\tau}-I)A^{-1}D = \begin{bmatrix} -a_{12}g(e^{-a_{22}\tau}-1 + a_{22}\tau)/a_{22}^2 \\ -g(1-e^{-a_{22}\tau})/a_{22} \end{bmatrix}$$

$$B_d = (e^{A\tau}-I)A^{-1}B = \begin{bmatrix} -a_{12}b(e^{-a_{22}\tau}-1 + a_2\tau)/a_{22}^2 \\ -b(1-e^{-a_{22}\tau})/a_{22} \end{bmatrix}$$

$$\tilde{D}_d = (e^{A\tau}-I-A\tau)A^{-2}D = \begin{bmatrix} a_1 g(e^{-a_{22}\tau}-1 + a_{22}\tau - a_{22}^2\tau^2/2)/a_{22}^3 \\ -g(e^{-a_{22}\tau} - 1 + a_{22}\tau)/a_{22}^2 \end{bmatrix}$$

with u(k) and w(k) assumed to be piece-wise constants i.e., $u(t) = u(k\tau)$ and $w(t) = w(k\tau)$, in the interval $t\in[k\tau,(k+1)\tau]$ and τ being the sampling interval.

*A^{-1} may or may not exist in general. However, the above relations are notationally correct.

For the sampling interval τ of 10 msec, the numerical values of $\tilde{A}_d$, $\tilde{B}_d$ and $\tilde{D}_d$ are given by

$$\tilde{A}_d = \begin{bmatrix} 1 & 1.6026\text{x}10^{-5} & -4.3175\text{x}10^{-7} \\ 0 & 0.98728 & -5.3420\text{x}10^{-2} \\ 0 & 0 & 1 \end{bmatrix}, \quad B_d = \begin{bmatrix} -2.1482\text{x}10^{-2} \\ -2.6580\text{x}10^{3} \\ 0 \end{bmatrix}$$

$$\tilde{D}_d = \begin{bmatrix} -1.4232\text{x}10^{-9} \\ -2.6769\text{x}10^{-4} \\ .01 \end{bmatrix}. \qquad (14)$$

The sampled measurements at $t = k\tau$ are

$$\underline{y}(k) = \underline{x}(k) = [\, I_2 \quad 0 \,] \begin{bmatrix} x(k) \\ w(k) \end{bmatrix} = C_d\, \underline{\tilde{x}}(k). \qquad (15)$$

A discrete disturbance-isolated observer which generates $w(k)$ is given by

$$\dot{z}(k+1) = F\, z(k) + G\, \underline{y}(k) + M\, u(k), \qquad (16a)$$

$$w(k) = z(k) + L\, \underline{y}(k), \qquad (16b)$$

where $z(k)$ is a scalar and

$$F = 1 - L\, D_d,$$

$$G = FL + A_{21} - LA_{11},$$

$$M = B_2 - LB_1.$$

with the choice of L given by

$$L = \tau[\tilde{D}_d^T \tilde{D}_d]^{-1} \tilde{D}_d^T + V(I_2 - [\tilde{D}_d^T \tilde{D}_d]^{-1} \tilde{D}_d^T),$$

where V is chosen such that F is stable.

7.4 Optimal Disturbance Accommodation Controller

With a choice of $\tau = .01$ sec and $V = [7\times10^6 \quad 0]$, and using (14), the observer (16) is given by

$$z(k) = 0.038512\, z(k-1) + [-6.7304\times10^6 \quad -41.430]\underline{y}(k-1) - 4.7839\times10^4\, u'(k-1), \tag{17a}$$

$$\hat{w}(k) = z(k) + [7\times10^6 \quad -74.573]\underline{y}(k), \tag{17b}$$

while the microcomputer control is given by

$$u'(k) = k_1 x_1(k) + k_2 x_2(k) - k_w \hat{w}(k)$$
$$= 2.2361 x_1(k) + 1.5949*10^{-4}\, x_2(k) - 2.0098*10^{-5}\, \hat{w}(k), \tag{18}$$

where the first column entry of Table 1 has been used.

7.5 OPTIMAL DISTURBANCE ACCOMMODATING CONTROLLER WITH DISTURBANCE PREDICTION

It is suggested that an alternative microcomputer scheme to (18) can be realized as

$$\begin{aligned} u'(k) &= k_1 x_1(k) + k_2 x_2(k) - k_w \hat{w}(k) - k_p[(1+\theta)\,\hat{w}(k) - \theta\hat{w}(k)] \\ &= 2.2361\, x_1(k) + 1.5949*10^{-4}\, x_2(k) - 2.0098*10^{-5}\, \hat{w}(k) \\ &\quad -0.5051*10^{-6}\,[(1+\theta)\,\hat{w}(k) - \theta\hat{w}(k)], \end{aligned} \tag{19}$$

where the last term in the control equation is a prediction scheme. In the simulation that follows, it is found that $\theta = 20$ yields improvements in the accommodation of the disturbance.

7.6 SIMULATION RESULTS

Simulation of the performance of the optimal turret control system, subject to different disturbance torque, under 3 types of controls were studied. These controls are

A. Without disturbance compensation (k_w=0)

$$u'(k) = k_1 x_1(k) + k_2 x_2(k)$$

B. With disturbance ocmpensation

$$u'(t) = k_1 x_1(k) + k_2 x_2(k) - k_w \hat{w}(k)$$

C. With disturbance compensation and prediction

$$u'(t) = k_1 x_1(k) + k_2 x_2(k) - k_w \hat{w}(k) - k_p [(1+\theta) \hat{w}(k) - \theta \hat{w}(k-1)]$$

where k_1, k_2, k_w, k_p and θ are as defined in the previous section.

The system was subjected to the following external torque w(t) :

a. torque due to firing bursts, and

b. sinosoidal torque at $\frac{1}{4}$ Hz, 5 Hz and 10 Hz.

Figs. 2(A), 2(B) and 2(C) shows responses of $x_1(t)$ under the controls A, B and C respectively, when the system is subject to external torque w(t) due to firing bursts. The external torque w(t) and its discrete estimates $\hat{w}(k)$ obtained from observer (17) (where control B is used) are shown in Fig. 3. A typical microcomputer control input is shown in Fig. 4 while Fig. 5 shows a typical response of $x_2(t)$.

It is seen that an improvement of about 5 : 1 in the reduction of the maximum amplitude of $x_1(t)$ is obtained when control B is used as compared with control A. A further improvement of about 10 : 1 is obtained when control C is used instead of control A.

Fig. 6(A), 6(B) and 6(C) similarly depict the response of $x_1(t)$ under control A, B and C respectively, when the system is subject to a 1 Hz sinosoidal external disturbance. In Fig. 7 and Fig. 8, the system is subject to a 5 Hz and 10 Hz sinosoidal external disturbance respectively.

In all cases, it is found that control B and C suppress the tranmission of the external disturbance w(t) to the output $x_1(t)$ by an appreciable amount. Control C, which has an element of prediction in its algorithm, exhibits a better disturbance suppression characteristic over the straight disturbance accommodating control B.

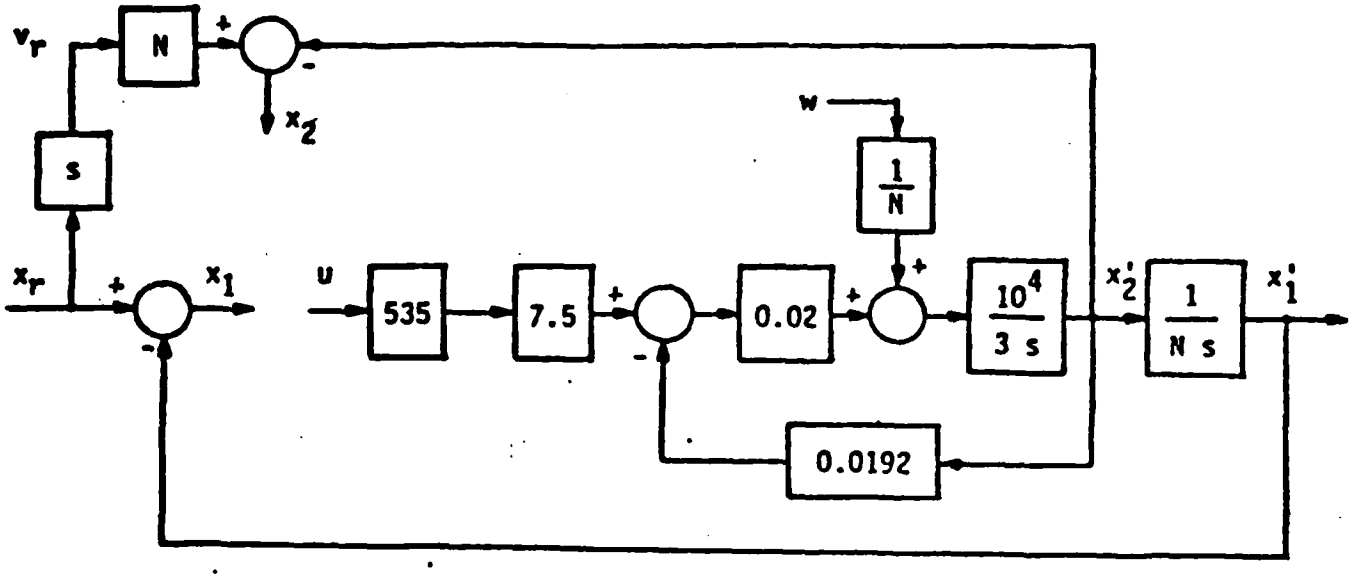


Fig.1 SIMPLIFIED OPEN-LOOP XM-97 HELICOPTER TURRET CONTROL SYSTEM WITH DISTURBANCE INPUT (TWO-STATE-VARIABLE MODEL)

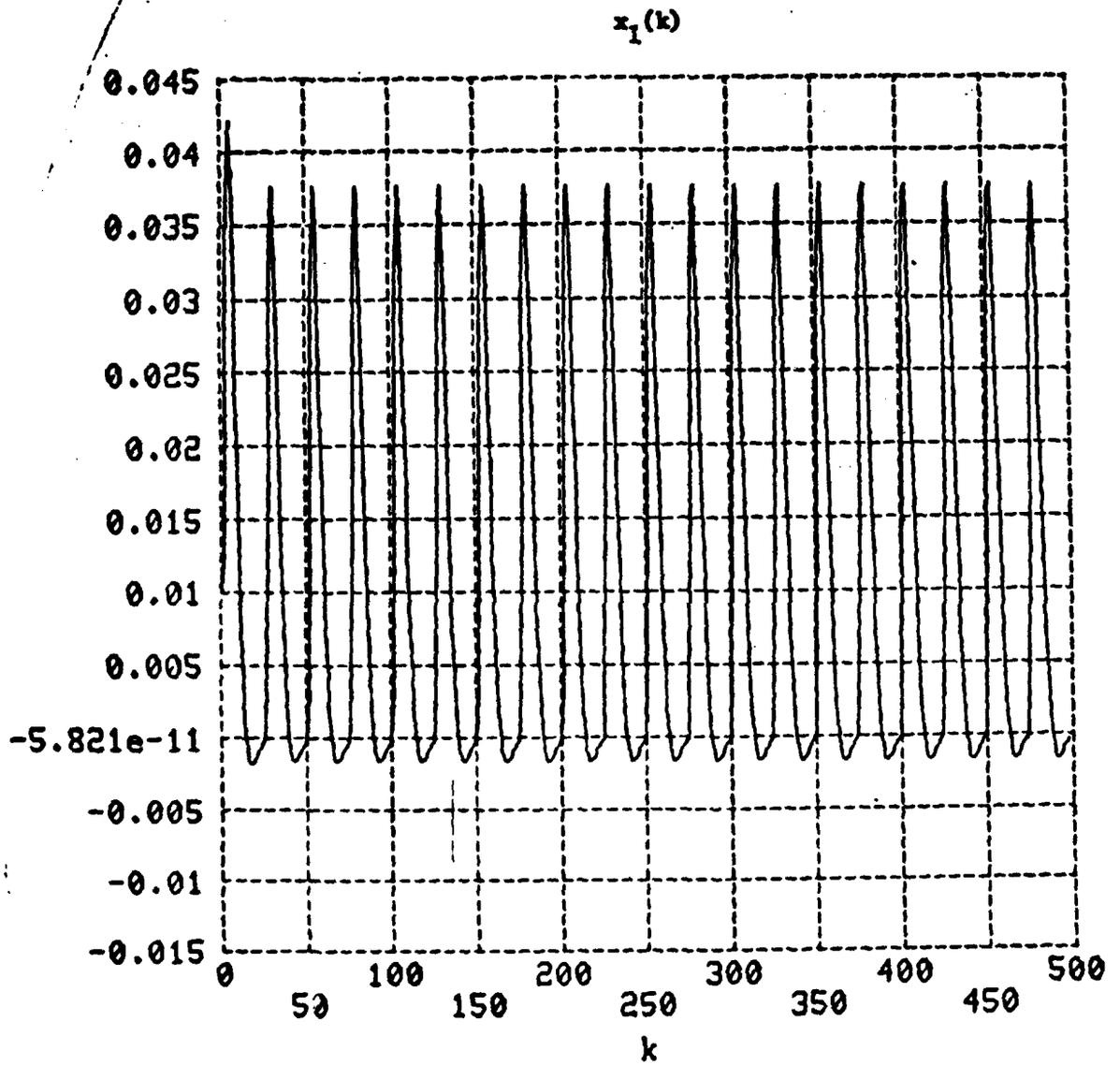


Fig.2(A) System subject to Firing Burst Torque , when Control A is used.

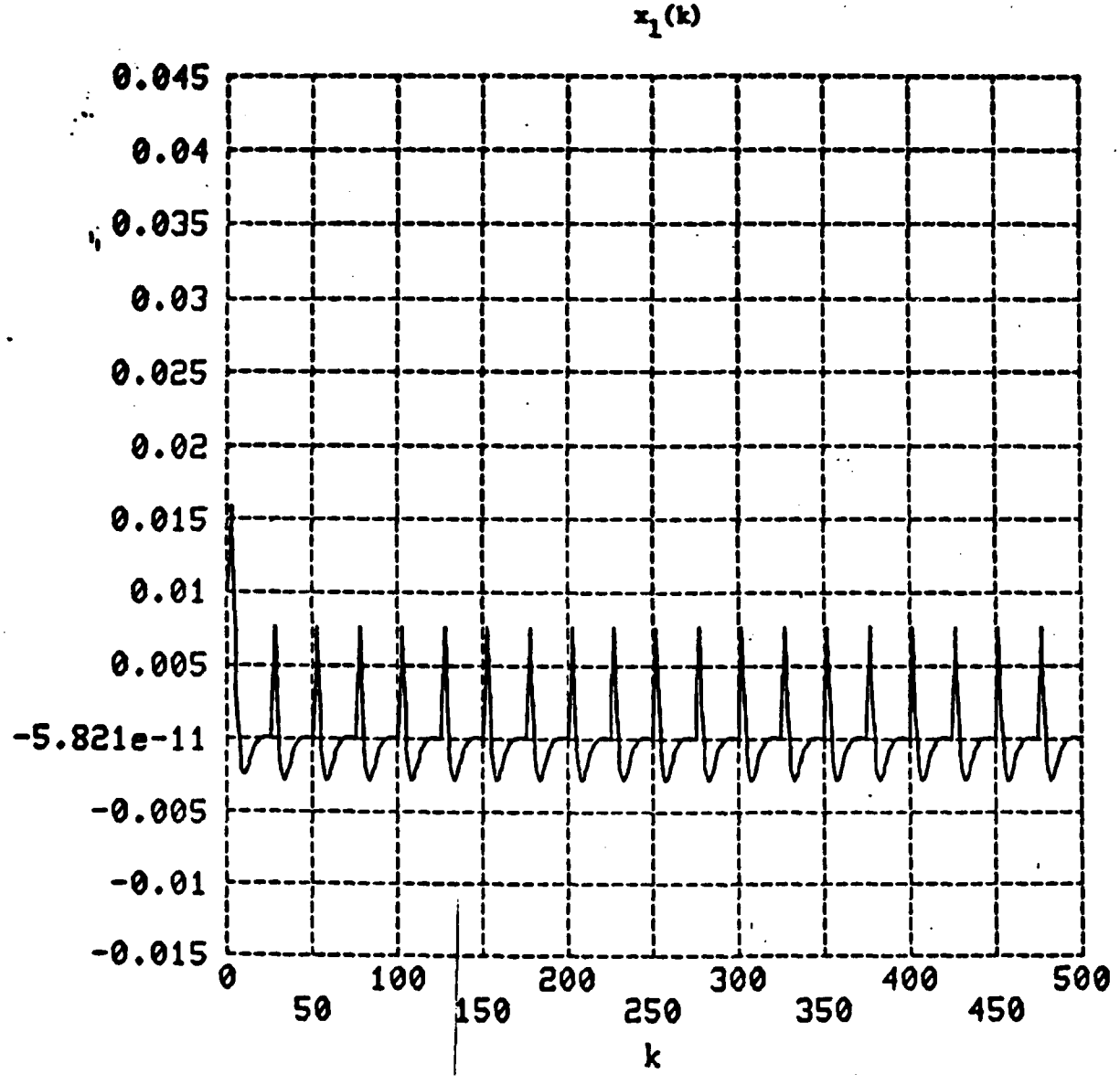


Fig.2(B) System subject to Firing Burst Torque , when control B is used.

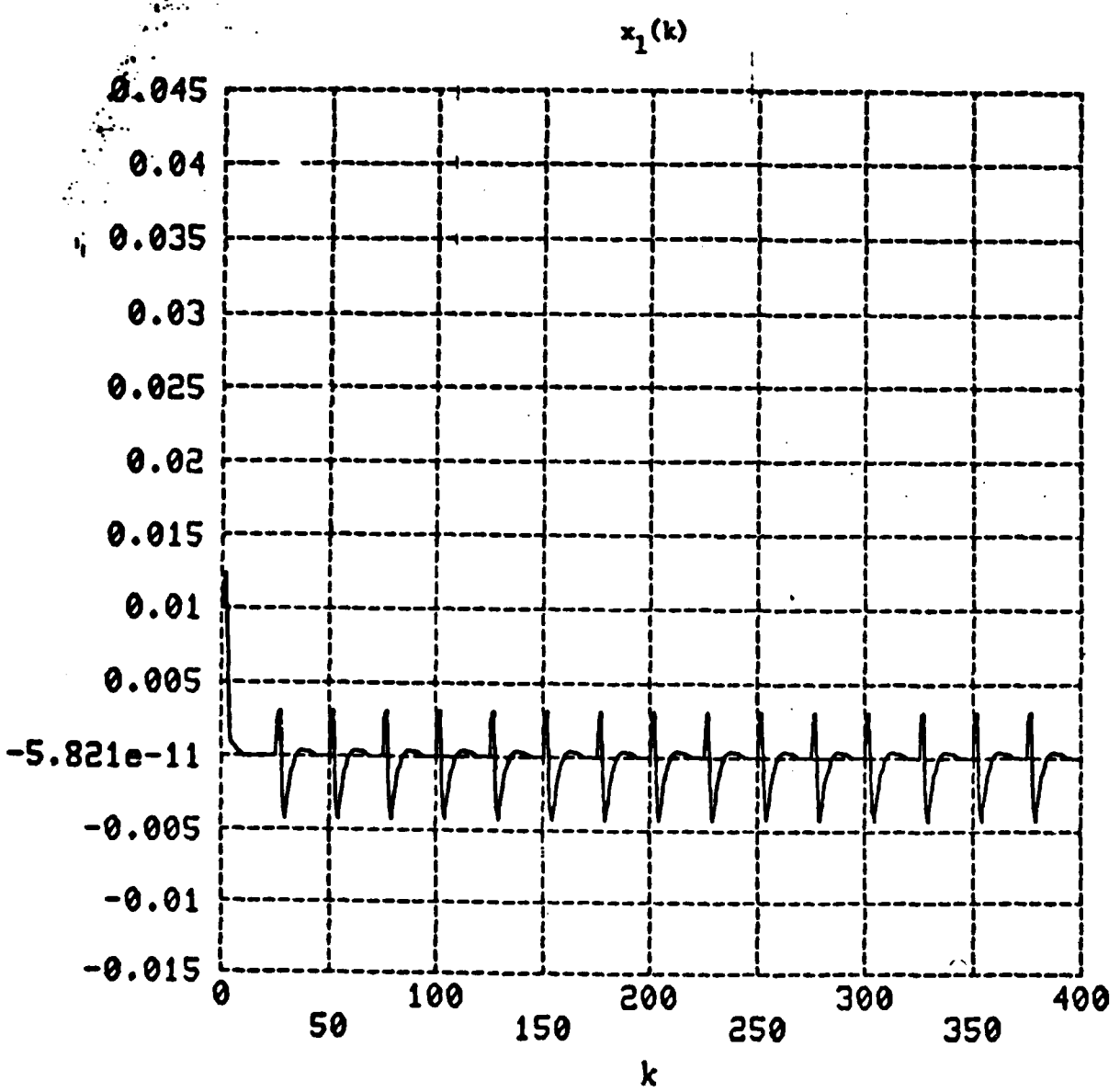


Fig.2(C) System subject to Firing Burst Torque , when Control C is used.

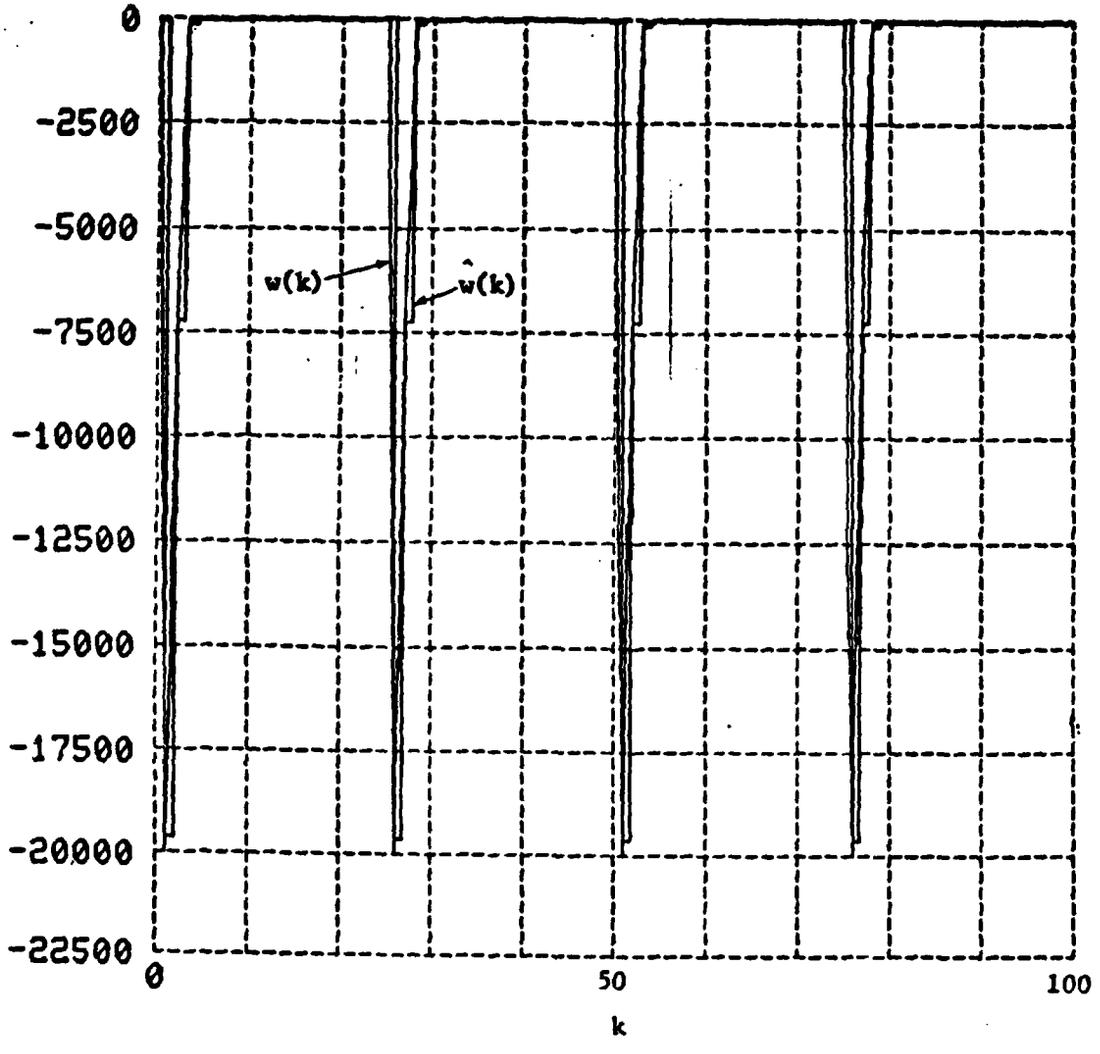


Fig. 3 Firing Burst Torque w(t) and its Discrete Estimates $\hat{w}(k)$.

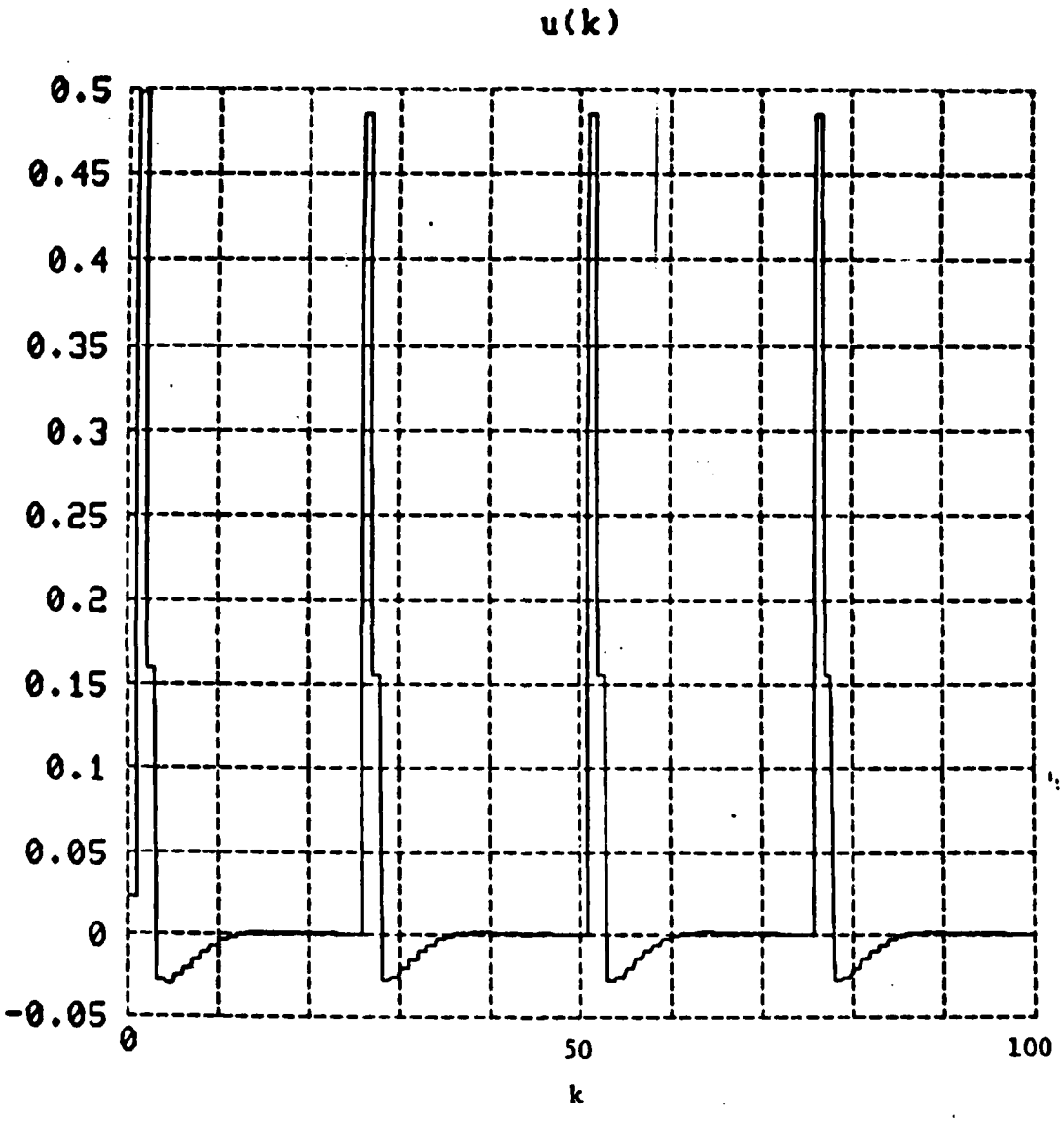


Fig. 4 Typical Microcomputer Control Input (Control B shown).

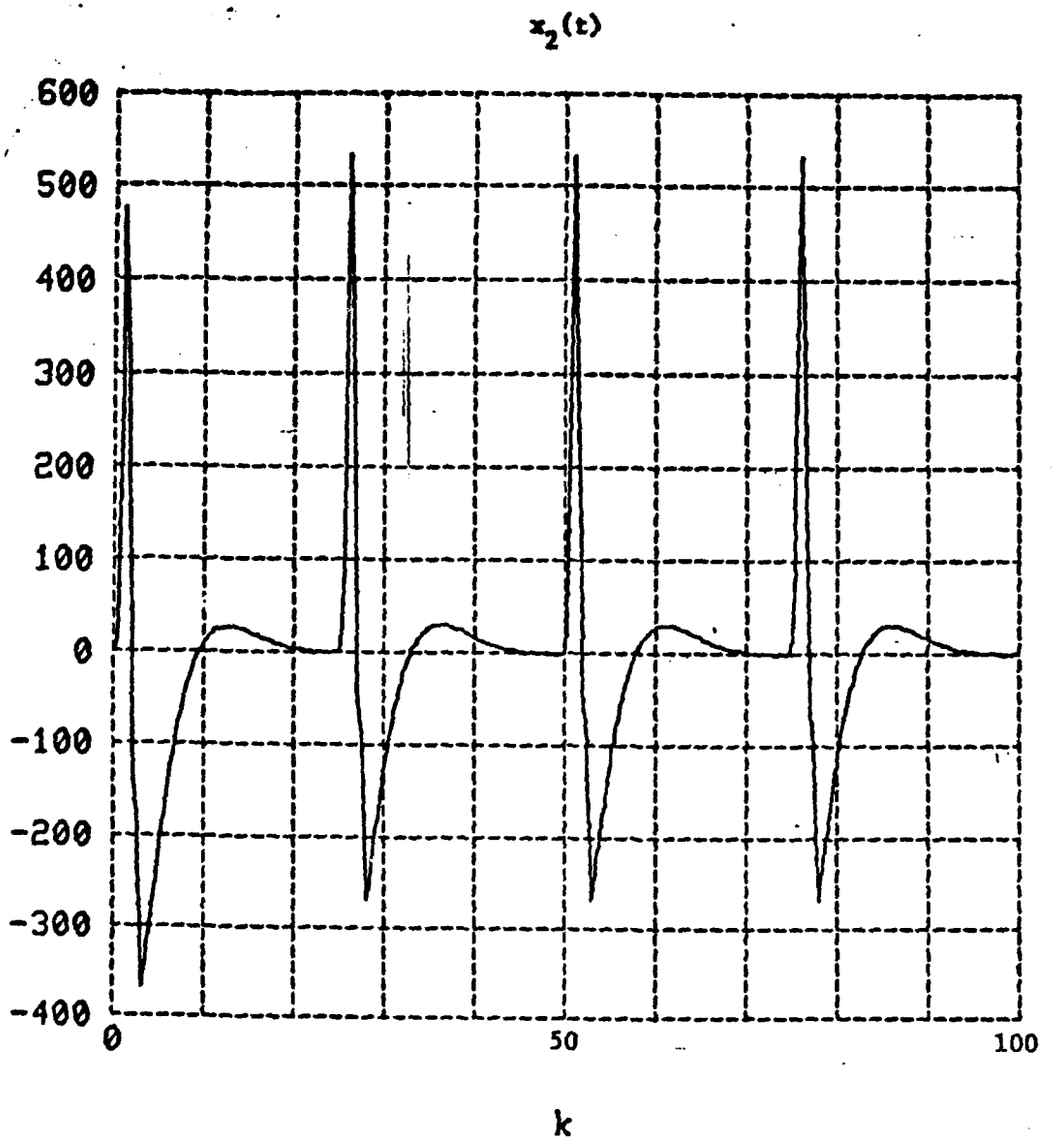


Fig. 5 Typical Response of $x_2(t)$ (with control B)

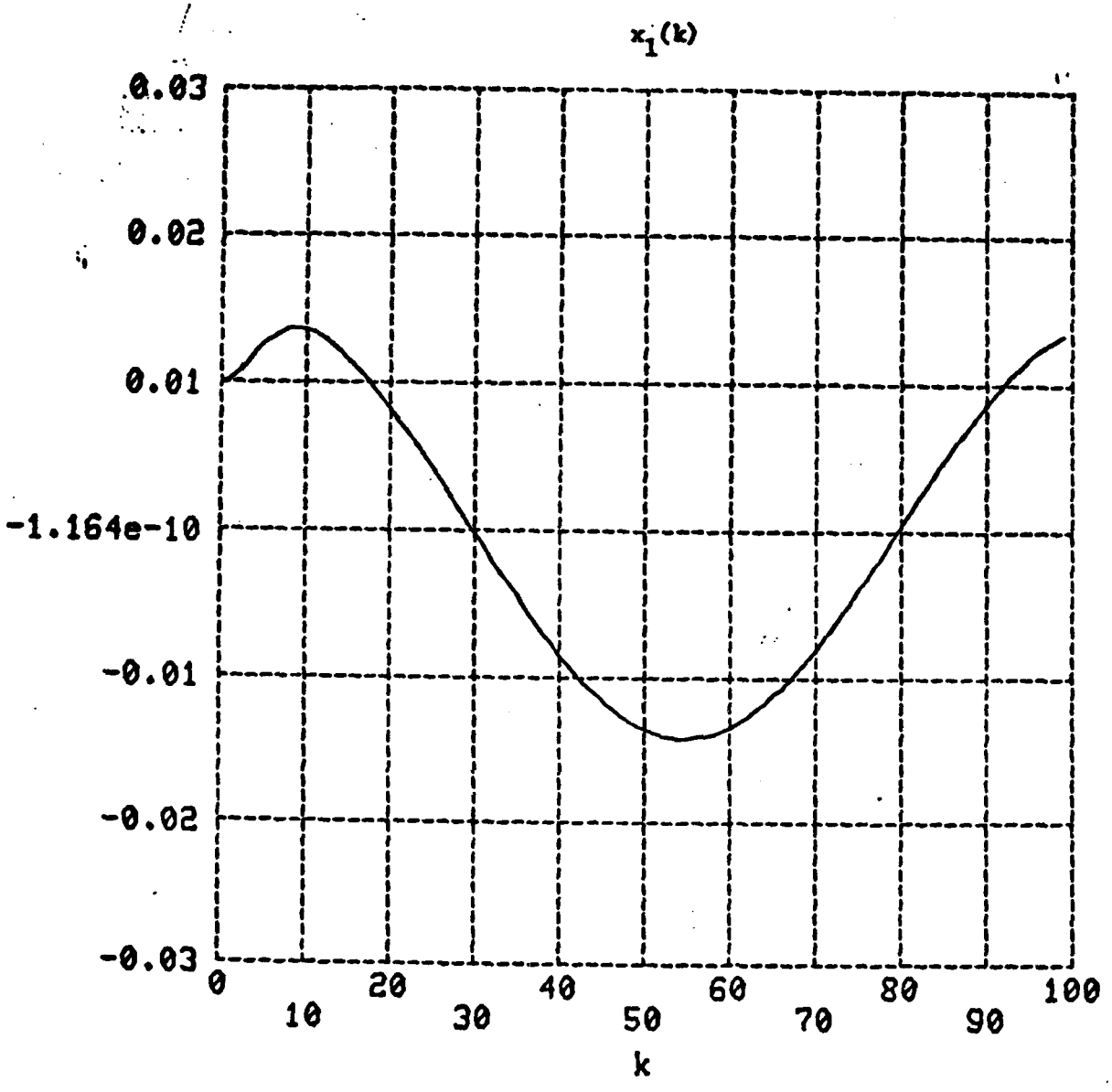


Fig. 6(A) System subject to 1 Hz Sinosoidal Disturbance , when Control A is used.

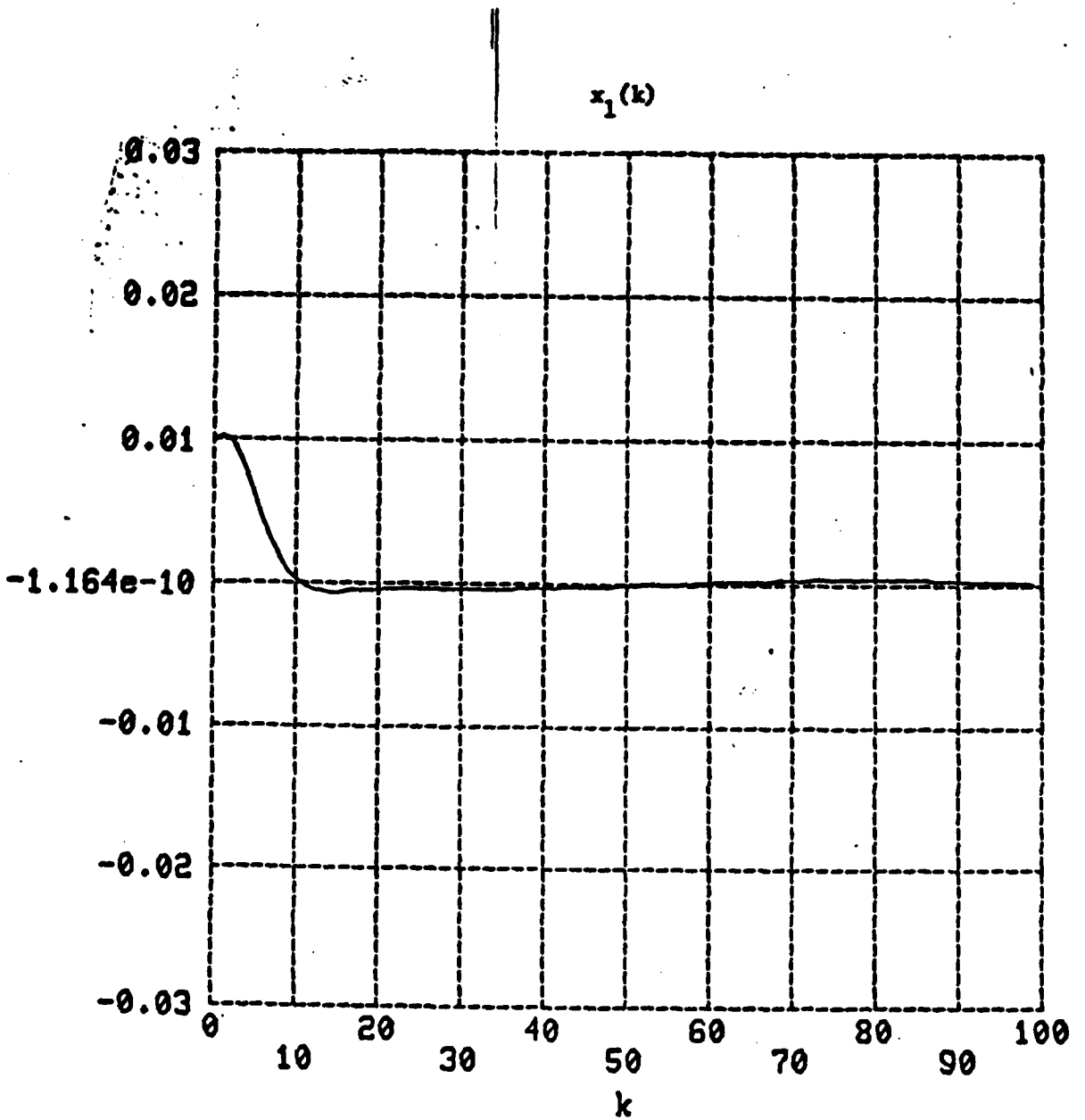


Fig. 6(B) System subject to 1 Hz Sinosoidal Disturbance , when Control B is used.

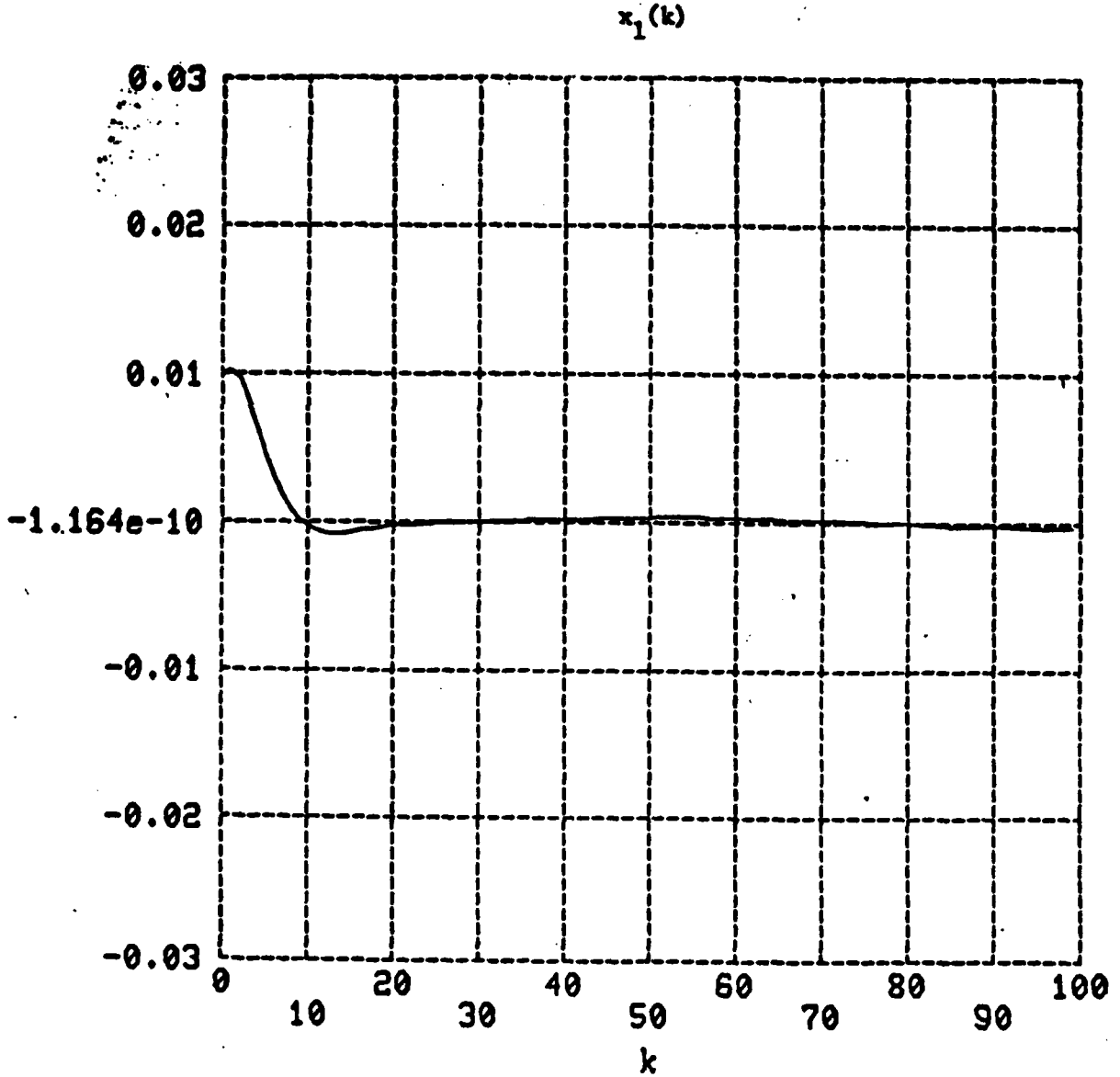


Fig. 6(C) System subject to 1 Hz Sinosoidal Disturbance , when Control C is used.

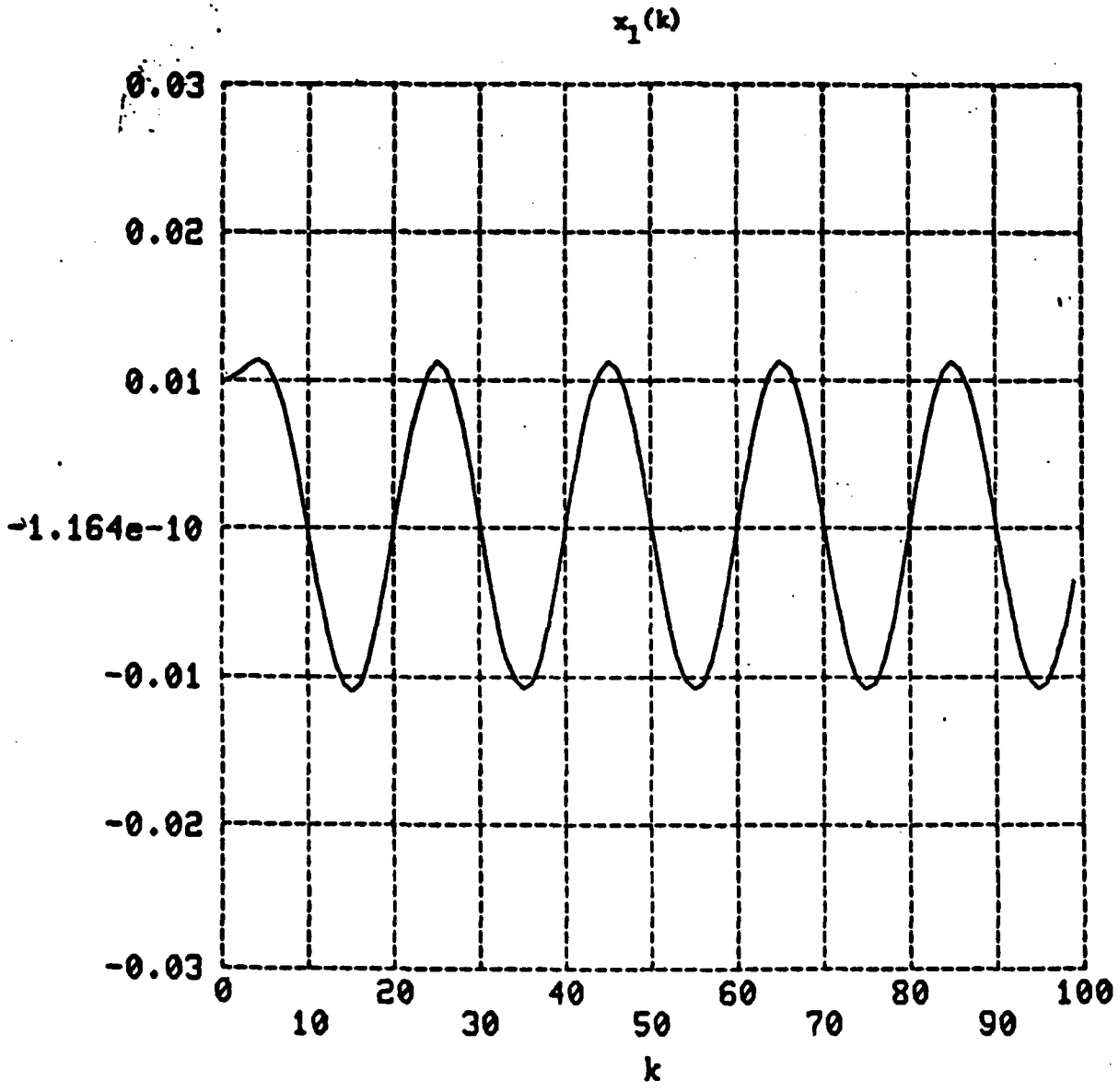


Fig. 7(A) System subject to 5 Hz Sinosoidal Disturbance , when Control A is used.

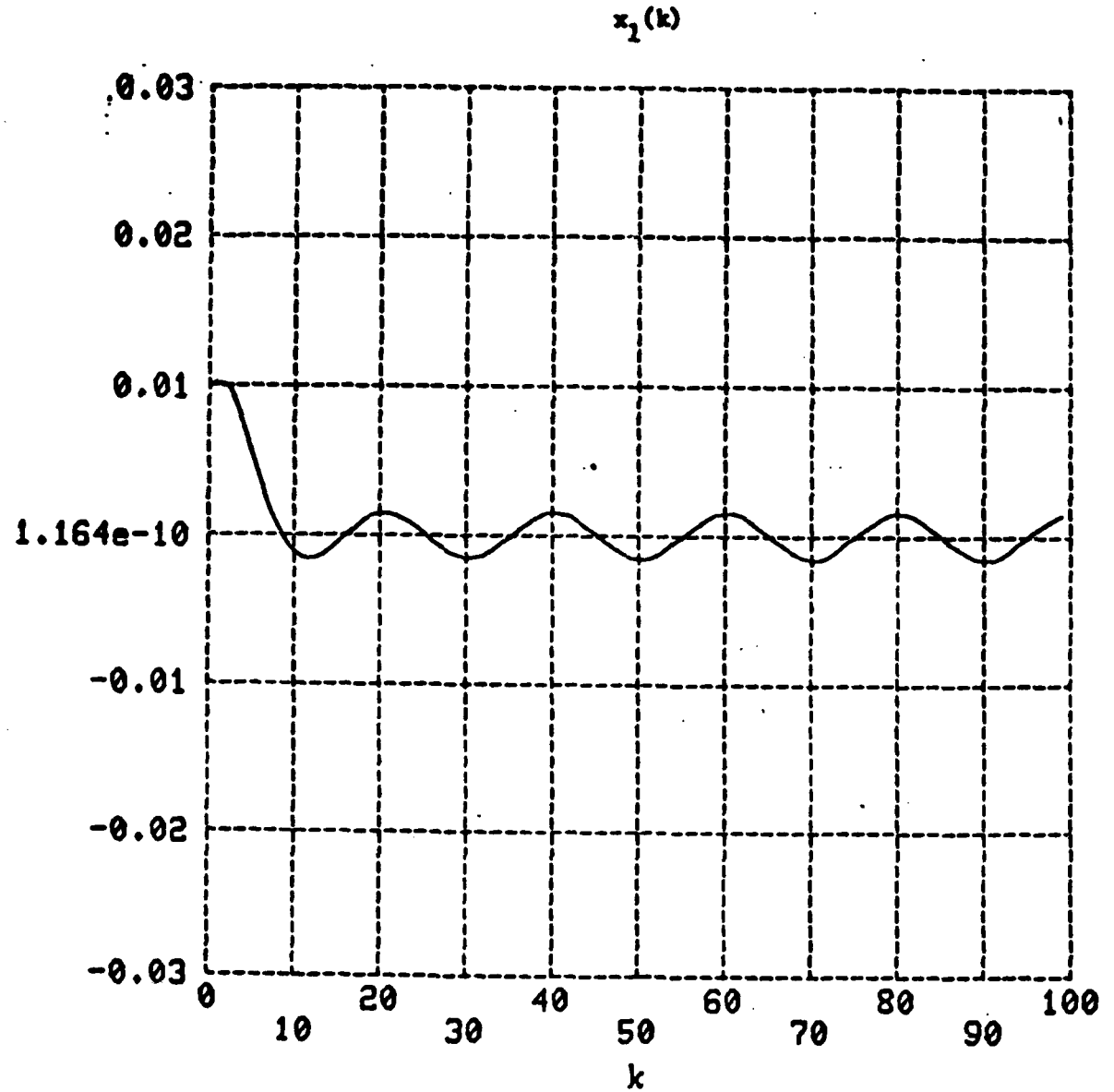


Fig. 7(B) System subject to 5 Hz Sinosoidal Disturbance , when Control B is used.

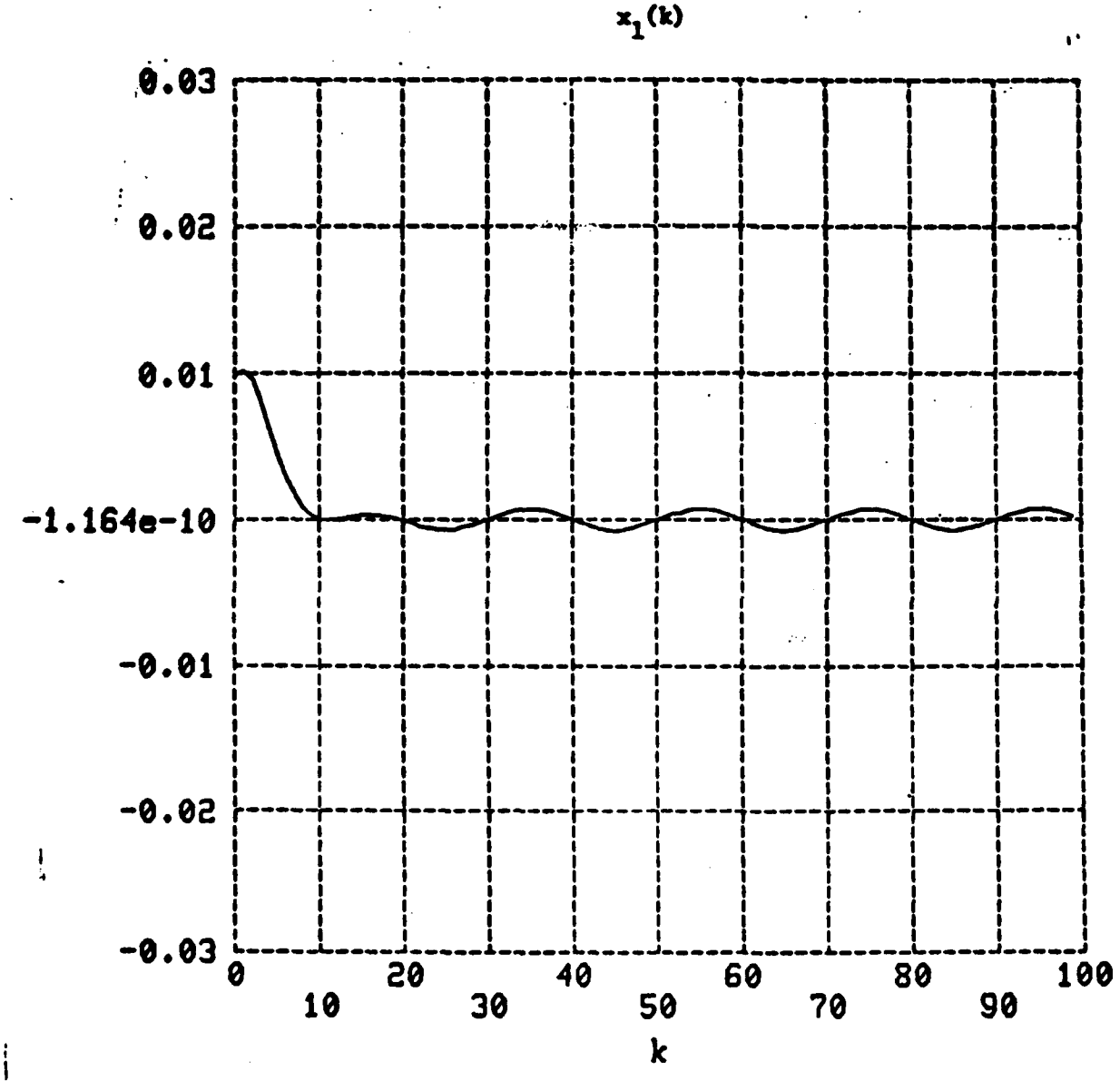


Fig. 7(C) System subject to 5 Hz Sinosoidal Disturbance , when Control C is used.

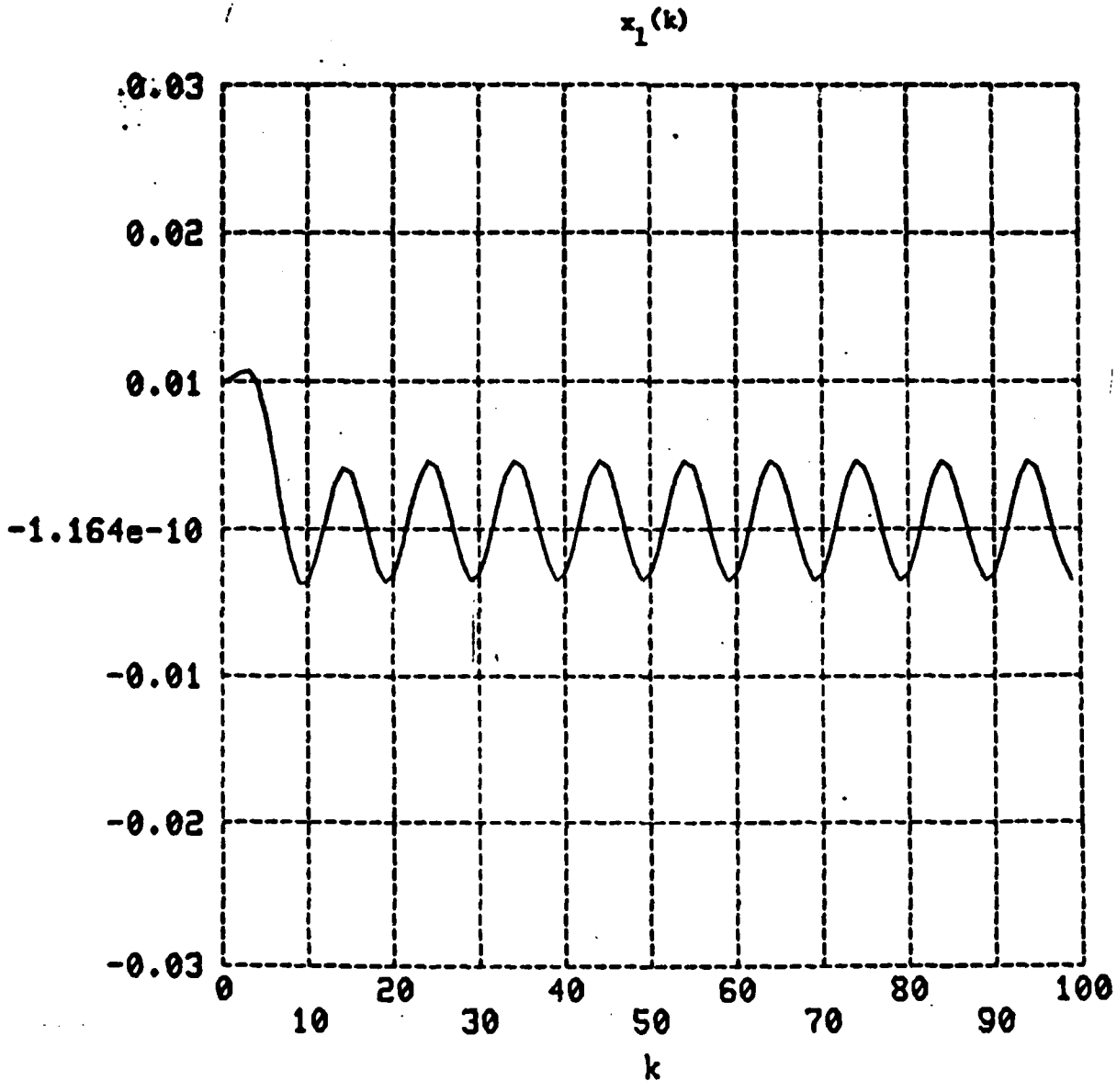


Fig. 8(A) System subject to 10 Hz Sinosoidal Disturbance , when Control A is used.

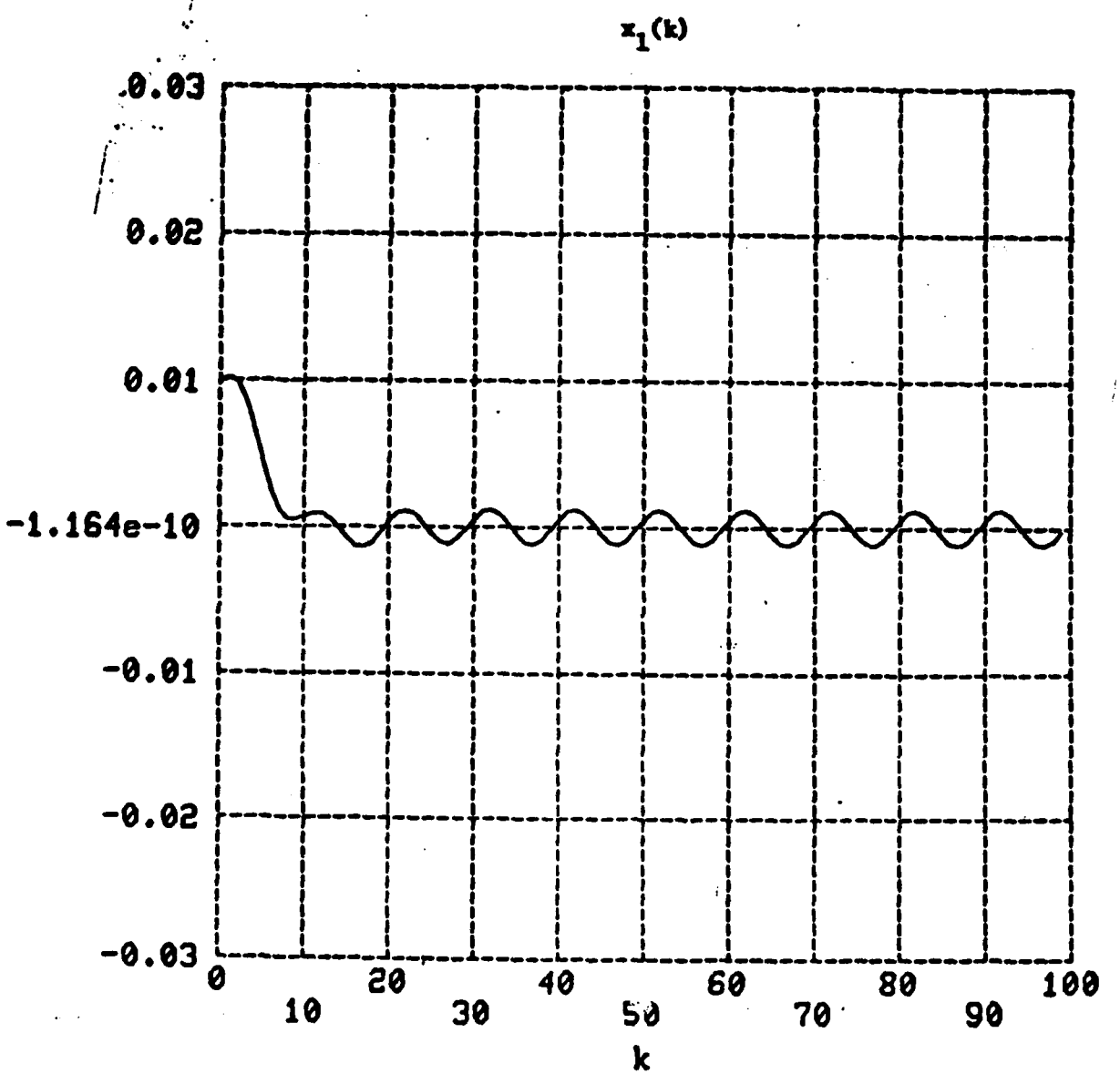


Fig. 8(B) System subject to 10 Hz Sinosoidal Disturbance , when Control B is used.

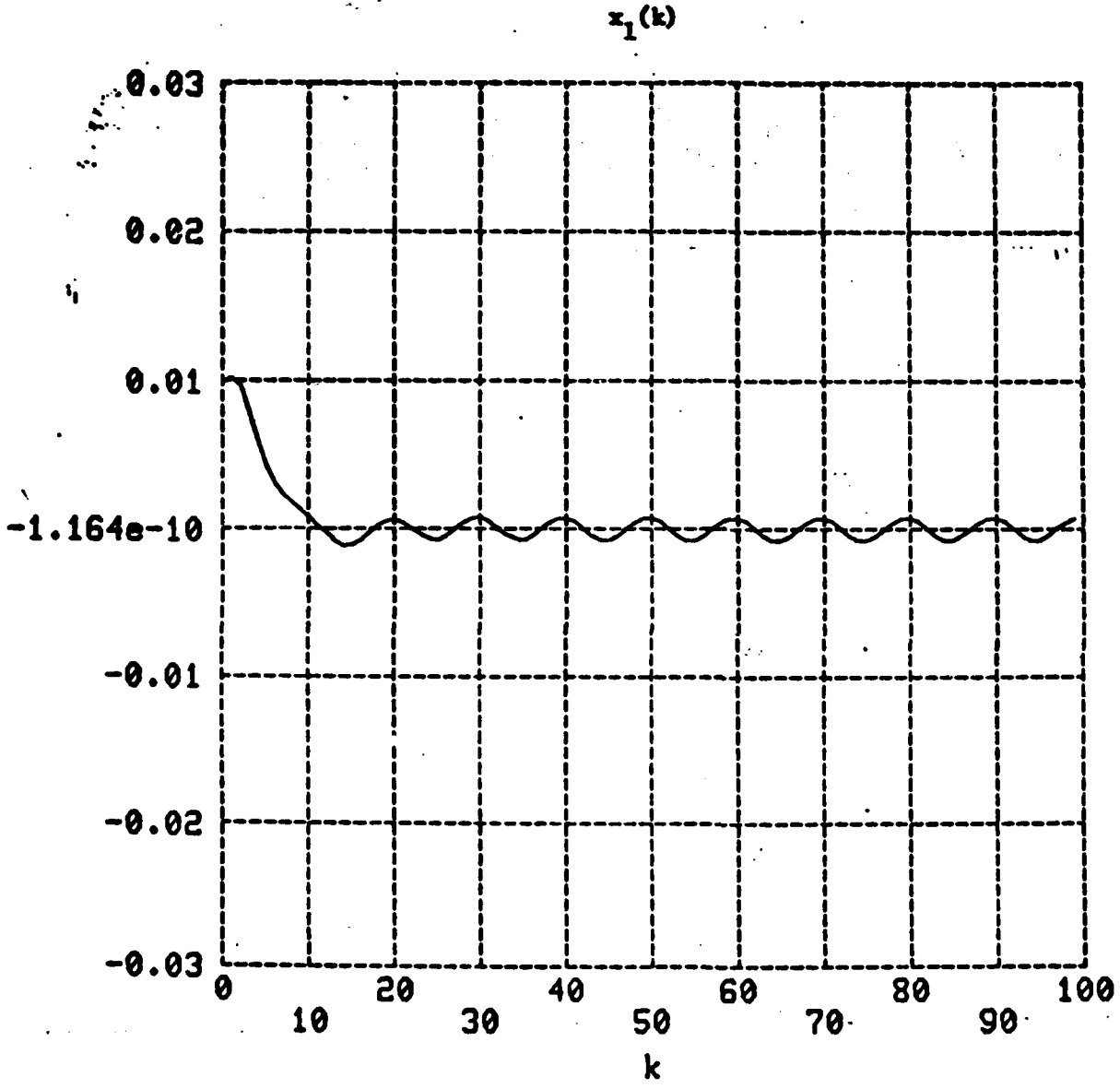


Fig. 8(C) System subject to 10 Hz Sinosoidal Disturbance , when Control C is used.

8. MICROPROCESSOR-BASED COMPENSATION FOR NONLINEAR FRICTION

One of the problems which occur frequently in a mechanical position control system is the non-zero steady state error due to nonlinear frictions such as Coulomb friction and stiction. The steady state error may be reduced by increasing the gain or by gearing down the motor output. It may also be reduced by using the integral of the error as the control input. However, these schemes may also introduce instability, excessive overshoot and long settling time. In this Chapter, we investigate another method which is based on a microprocessor to reduce the steady state error.

The system investigated was a d.c. motor driven position control system shown in Fig. 1.

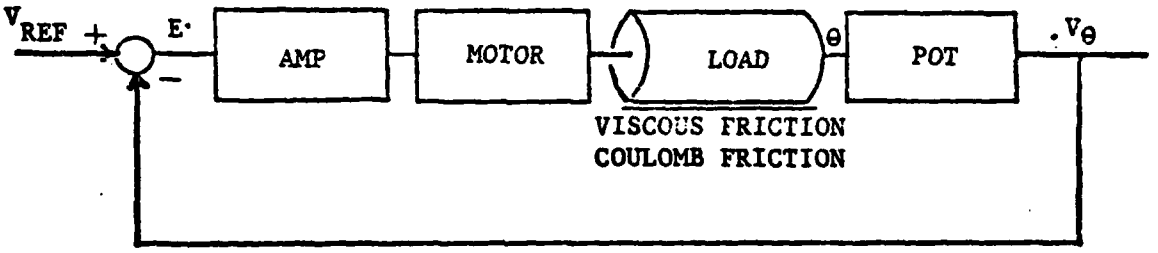


Fig. 1 DC MOTOR POSITION CONTROL SYSTEM

In addition to the Coulomb friction, the system contained some striction as well. However, the compensation scheme was mainly for countering the Coulomb friction. The mathematical model of the system is given in Fig. 2.

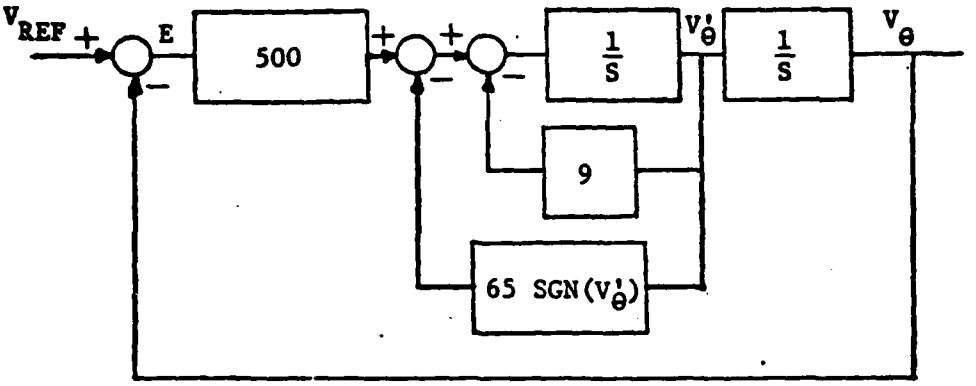


Fig. 2 MATHEMATICAL MODEL

When a step input of 1V was applied, the response was as shown in Fig. 3. As can be seen, the steady state error was approximately 0.2 ∿ 0.3V (20 ∿ 30%). The variation of the error was due to non-uniform frictions.

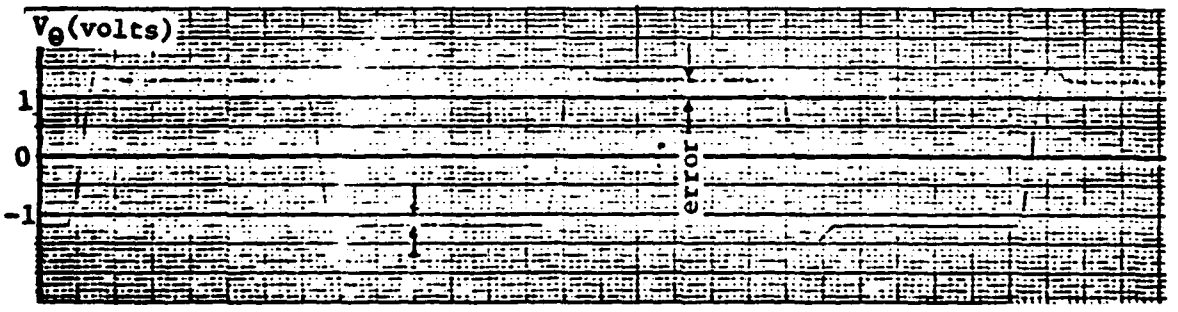


Fig. 3 STEP INPUT RESPONSE (25 mm/sec.)

To reduce the error, a microprocessor based compensation scheme was implemented as shown in Fig. 4.

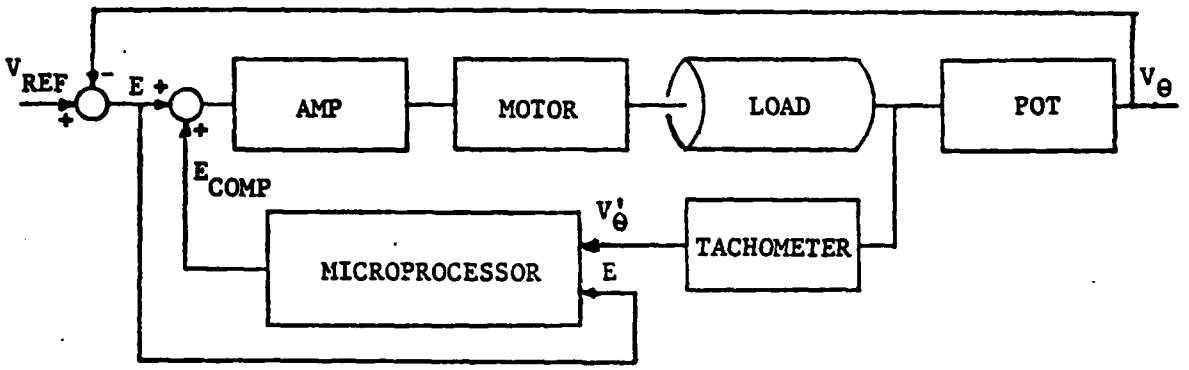


Fig. 4 SYSTEM WITH FRICTION COMPENSATION

The error and the velocity of the motor were sent to the microprocessor through A/D converters, and the compensation command E_{comp} was sent to the system input through a D/A converter. Basically, the compensation signal was determined by the following equation.

$$E_{comp} = \begin{cases} 0 & \text{if} \quad |v'_\theta| \geq 0.1\text{V/sec.} \\ 0 & \text{if} \quad |e| \leq 0.008\text{V} \\ 1.25\text{V} & \text{if} \quad |v'_\theta| \leq 0.1\text{V/sec.}, \; e > 0.008\text{V} \\ -1.25\text{V} & \text{if} \quad |v'_\theta| \leq 0.1\text{V/sec.}, \; e < -0.008\text{V} \end{cases}$$

When the same step input of 1V was applied to the compensated system, the response was as shown in Fig. 5. The actual steady state error was less than 0.001V.

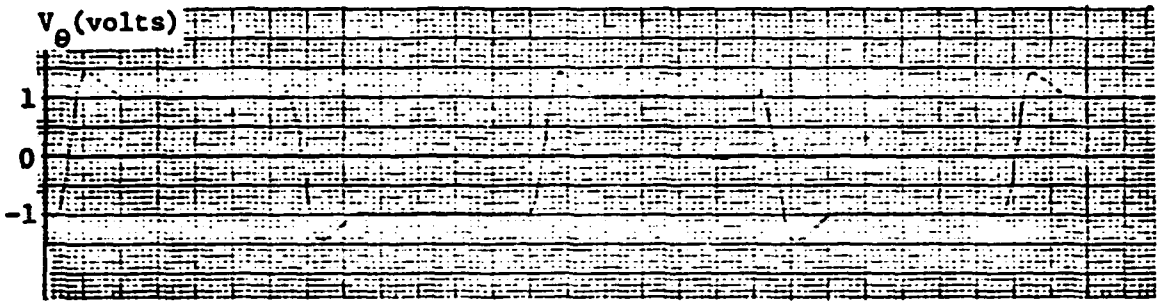
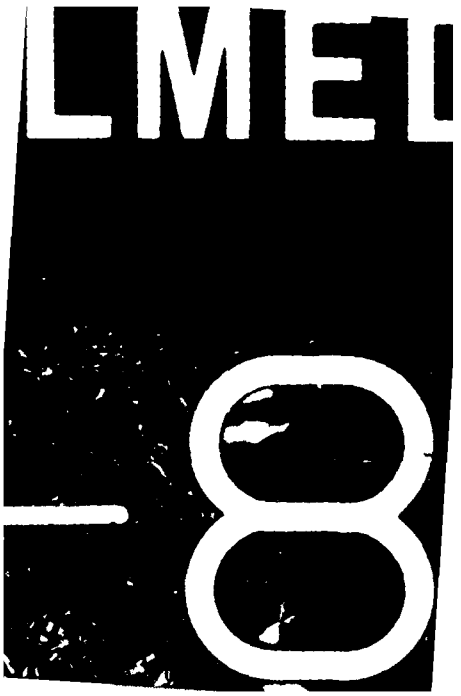


Fig. 5 STEP RESPONSE OF THE COMPENSATED SYSTEM (25 mm/Sec.)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