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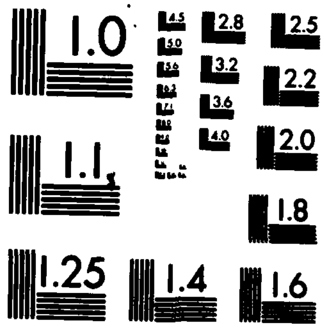

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Office of Naval Research Contract No. N00014-79-C-0555

Abstract

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unclassified

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER SAI-254-83-416-LJ/PRI-60	2. GOVT ACCESSION NO. A129004	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) Electron Orbits in a Free Electron Laser with a Longitudinal Magnetic Wiggler		5. TYPE OF REPORT & PERIOD COVERED technical report 6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) R. E. Aamodt		8. CONTRACT OR GRANT NUMBER(s) N00014-79-C-0555
9. PERFORMING ORGANIZATION NAME AND ADDRESS Science Applications, Inc. 934 Pearl Street Boulder, Colorado 80302		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
11. CONTROLLING OFFICE NAME AND ADDRESS ONR (Code 412) 800 N. Quincy Street Arlington, VA 22217		12. REPORT DATE March 1983 13. NUMBER OF PAGES 16
14. MONITORING AGENCY NAME & ADDRESS(If different from Controlling Office)		15. SECURITY CLASS. (of this report) unclassified 15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

DISTRIBUTION STATEMENT A
Approved for public release;
Distribution Unlimited

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

Approved for public release; distribution unlimited.

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

free electron laser
electron orbits
longitudinal wiggler

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

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S/N 0102-LF-014-6601

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I. INTRODUCTION

In studies of free electron lasers (FEL) systems, in which emission gain is induced through wiggler magnetic field modulation of the electron motion, detailed knowledge of the electron orbits is necessary in order to evaluate the critical features of the amplification scheme. These features include evaluating the frequency range for amplification, gain, rates, saturation mechanisms, and amplifying efficiencies. However, there exist very few magnetic geometries where the electron motion, even in the vacuum fields, can be obtained analytically with sufficient information to assess these critical features. Only recently[1,2] has it been realized that in the "on axis" approximation the helical wiggler orbit problem can be reduced to quadrature, and that particular discovery has already led to significant progress in FEL amplification schemes.[3,4] For FEL systems with strong magnetic guide fields, little or no insightful progress has been made except for this helical case. However, for just this class of magnetic systems, i.e. strong guide fields with a relatively low amplitude magnetic wiggler superimposed, an asymptotic mathematical formalism does exist where the electron equations of motion in the vacuum fields can be greatly simplified, and typically reduced to quadrature in parameter regimes of relevance to applications.

For wiggler fields which are small in amplitude compared to the basic and necessarily geometrically simple guide field, asymptotic methods can be used to simplify the orbits for various ranges of the parameter, $\delta \equiv k_0 v_\parallel / \Omega_c$, where Ω_c is the relativistic electron gyrofrequency in the guide field, and k_0 and $v_\parallel$ are the values of the wiggler wavenumber and electron velocity parallel to this guide field. From the current studies of the exactly soluble "on axis" helical wiggler these same ranges are in fact just the interesting ones.[1,2,3] For $\delta \ll 1$, the wiggler madifications of the quide field orbits can be handled in the well known guiding center approximation,[4] and for $\delta \gg 1$, standard weak perturbation methods easily prove to be adequate. For $|1 - \delta| \ll 1$, the so-called resonant regime, secular perturbation methods have proven successful in simplifying the orbit analysis for particle motion in small amplitude fluctuating rf fields. This analysis can be straightforwardly applied to the wiggler problem as long as only widely separated values of k_0 are present in the system, and as long as these k_0's are very slowly varying in space. Furthermore, this same method can be used when δ is close to the ratio of two discrete integers. However, of particular interest to FEL applications and for the sake of simplicity, we will not here develop a general formalism for an arbitrary wiggler, but will restrict this study to the orbits of particles near the magnetic axis of a longitudinal wiggler field in cylindrical geometry:

$$\vec{B} \simeq B_0\left[1 + \epsilon \cos k_0 z\left(1 + \frac{1 + k_0^2 r^2}{4}\right)\right]\hat{e}_z + \frac{\epsilon}{2} B_0 k_0 r \sin k_0 z \hat{e}_r \quad . \tag{1}$$

The exact description of the particle motion in this multiple mirror geometry can be greatly simplified because of the existence of the two exact constants of the motion--energy and angular momentum--but the orbits cannot be reduced to quadrature except in the limit $\delta \to 0$.[6] Therefore, the success of the asymptotic technique for this case is particularly meaningful. Additionally, a number of FEL studies in this geometry have proceeded neglecting entirely the radial magnetic field in Eq. (1), and therefore have used a B field which is not divergence-free. The consequence of this approximation can now be determined.

II. ASYMPTOTIC TRAJECTORY ANALYSIS

For the specific case of motion in a longitudinal wiggler the relativistic equations of motion in the laboratory frame in standard cartesian components are

$$\frac{dv}{dt} + i\Omega_o v = -\frac{e}{mc}[i\delta B_z v - iv_z(\delta B_x + i\delta B_y)] \tag{2a}$$

$$\frac{dv_t}{dt} = \frac{e}{m_c}[v_x\delta B_y - v_y\delta B_x] \quad , \tag{2b}$$

where $e < 0$ is the electron change, m is the relativistic mass (which is a constant), δB_i are the i^{th} vector components of the wiggler field, $\Omega_o = eB_o/mc$, and $v = v_x + iv_y$. The entire philosophy of the trajectory analysis in all of the previously discussed ranges of δ is that the left-hand sides of Eq. (2) induce only small changes in the orbits described by the right side of Eq. (2) in a cyclotron period $T = 2\pi/\Omega_o$. With such a philosophy and by Fourier analyzing the wiggler fields, a straightforward analysis of the motion can proceed, even in the resonance regime.[5] However, for unification and simplicity here with field given by (1), Eqs. (2) simply become to order ε^2

$$\frac{dv}{dt} + i\Omega v + \frac{i}{2}\frac{d\Omega}{dt}\omega = 0 \tag{3a}$$

and

$$\frac{dv_z}{dt} = -\frac{\varepsilon\Omega_o k_o}{4i}\sin k_o z\,[v\omega^* - v^*\omega] \quad , \tag{3b}$$

with $\omega = x + iy$, and $\Omega = \Omega_o[1 + \varepsilon\cos k_o z(1 + k_o^2r^2/4)]$.

Eqs. (3) are not exactly integrable, so a perturbation theory must be constructed for their solution, however two exact constants of the motion, energy and angular momentum (in a symmetric multiple mirror system) do exist and from (3) these correspond respectively to the conditions

$$\frac{d}{dt}(|v|^2 + v_z^2) = 0 \tag{4a}$$

obtained by multiplying (3a) by v*, and adding this equation to its complex conjugate equation and using (3b)

and

$$\frac{d}{dt}\left[\frac{\dot{\omega}\omega^* - \dot{\omega}^*\omega}{i} + \Omega|\omega|^2\right] = 0 \tag{4b}$$

[obtained by multiplying (3a) by ω^* and subtracting this equation from its complex conjugate].

In the spirit of usual secular perturbation theory, the solutions of (2) proceed by assuming that the $\delta\vec{B}$ corrections induce a slowly varying component of v, and a slow modulation of the phase and amplitude of the fast component of v. So writing

$$v = \bar{v} + \delta v$$

($\bar{v}$, δv are the slow and rapidly varying components of v respectively) and correspondingly

$$\omega = \bar{\omega} + \delta\omega$$

$$v_z = \bar{v}_z + \delta v_z \text{ ,}$$

the slow variation of $\bar{v}$ is determined by

$$\frac{d\bar{v}}{dt} + i\Omega_o\bar{v} = \overline{-\frac{ei}{mc}[\delta B_z v - v_z(\delta B_x + i\delta By)]} \tag{5a}$$

and dv, given by

$$\left(\frac{d}{dt} + i\Omega_o\right)\delta v = -\frac{ei}{mc}\left\{[\delta B_z v - v_z(\delta B_x + i\delta B_y)] - \text{time average}\right\} \tag{5b}$$

and similarly for v_z, and δv_z, where the bar ($\overline{\quad}$) indicates averaging over a cyclotron period, $2\pi/\Omega_o$. Now, the quantity $\delta = k_o\bar{v}_z/\Omega_o$ enters in directly in calculating the time averages. When $\delta \ll 1$ (called the drift approximate or gyrokinetic limit) then δB_i is essentially constant in a cyclotron period, so here we have

$$\frac{\overline{\delta \ll 1}}{} \qquad \frac{d\bar{v}}{dt} + i\Omega_o \bar{v} = -\frac{ei}{mc}[\delta B_z \bar{v} - \bar{v}_z(\delta B_x + i\delta B_y)] \tag{6a}$$

hence

$$\bar{v} = (1 - \frac{d}{i\Omega dt})\frac{e\bar{v}_z}{\Omega mc}(\delta B_x + i\delta B_y) \quad ,$$

$$= \bar{v}_z(\frac{\delta B_x}{B} + \frac{i\delta B_y}{B}) - \bar{v}_z\frac{\varepsilon}{i}(\frac{k_o\bar{v}_z}{\Omega})k_o\bar{\omega}\cos k_o z(1 + \frac{k_o^2\overline{|\bar{\omega} + \delta\omega|^2}}{4}) \quad . \tag{6b}$$

We see from (6b) that in this limit $\bar{v}$ consists of the "x" and "y" components of the velocity parallel to $\vec{B}$, plus the usual curvature drift term, the last term in (6b). The usual grad B drift is neglected in this FEL application because $|v|^2 \ll v_{\parallel}^2$, so it is correspondingly smaller than the curvature drift. Even so, in this drift limit, the curvature drift is of order $\varepsilon\delta k\omega_o$ times v_z, so by the assumed orderings of this limit, this is negligibly small. Note also that the phase of the curvature drift differs from $\bar{\omega}$ by a factor of $\pm i$, hence this drift is in the "θ" direction, i.e. perpendicular to the vector $\vec{r}$.

Further discussions of the drift limit are not necessary because of the vast literature on the subject. However, it is to be noted that an overall amplitude correction is to be added (in the $\delta \ll 1$ limit) to previous orbit analyses[7] which entirely neglected δB_r. This correction, easily derivable from Eq. (2) or (3) corresponds to use of the adiabatic invariant $\mu = e|\delta v|^2/2B$, and corresponds to an amplitude factor multiplying the previously obtained solutions[7] for ω by the factor $\sqrt{\Omega_0/\Omega}$. This geometric factor then properly takes into account that the actual δB must satisfy $\nabla\cdot\delta\vec{B} = 0$, whereas neglect of δB_r does not satisfy this divergence condition.

In the more interesting limit for obtaining large orbit excursions,[1,2] $|\delta| \sim 1$, and the methods of time averaging the exact equations must be carefully reconsidered. In particular the δB_i terms vary as $\sin k_0 z$, the argument of which varies in time roughly as $k_0 v_z t$ + slow modulation. Similarly, from (5b) we see that δv varies as $\exp(-i\Omega_0 t)$* a function with a slow time modulation. Therefore in time averaging (5a) we now obtain, to lowest order in ε,

$$\frac{d\bar{v}}{dt} + i\Omega_0\bar{v} = -\frac{i\Omega_0\varepsilon}{2}\left[\left(1 + \frac{k^2\overline{|\bar{\omega} + \delta\omega|^2}}{4}\right) + \frac{k_0\bar{v}_z}{2\Omega}\right]u\exp(-i\psi) \quad , \tag{7a}$$

$$\dot{u} = -\frac{i\varepsilon\Omega_0}{2}\exp(i\psi)\left[\left(1 + \frac{k^2\overline{|\bar{\omega} + \delta\omega|^2}}{4}\right)\bar{v} + \Omega_0\bar{\omega}/2i\right] \quad , \tag{7b}$$

and

$$\frac{d\bar{v}_z}{dt} = -\frac{\varepsilon\Omega_0 k_0}{8}[(\bar{\omega} + i\bar{v}/\Omega_0)\exp(i\psi)u^* + \text{c.c.}] \tag{7c}$$

with the functions $\delta v = u\exp(-i\Omega_0 t)$, $\psi = k_0 z + \Omega_0 t$, and the approximations $\delta\omega \simeq \delta v/(-i\Omega_0)\,[1 + \mathcal{O}\varepsilon]$; $\dot{\psi} \ll \Omega_0$, $\delta v_z \ll |\delta v|$ (by energy conservation $\delta v_z \sim \bar{v}\delta v/\bar{v}_z$), and it was assumed that $k_0\bar{\omega}$ remains small compared to unity. As $\bar{v}$ varies slowly compared to Ω_0, (7a) can be iteratively solved to yield (correct to order ε)

$$\bar{v} = -\frac{\varepsilon}{4}u\exp(-i\psi)\left[1 + \frac{3\dot{\psi}}{\Omega_0} + \frac{k^2\overline{|\bar{\omega} + \delta\omega|^2}}{2}\right] \quad . \tag{8a}$$

Inserting this into (7b) and neglecting terms that are small multiples of ε^2, (7b) becomes

$$\dot{u} - \frac{i\varepsilon^2\Omega_0}{8}u = -\frac{\varepsilon\Omega_0^2}{4}\exp(i\psi)\bar{\omega} \quad . \tag{8b}$$

Defining $p = \exp(-i\varepsilon^2\Omega_o t/8)u$ and $\zeta = \psi - \varepsilon^2\Omega_o/8t$ we find the simple equation set

$$\dot{p} = -\frac{\varepsilon\Omega_o^2}{4}\exp(i\zeta)\bar{\omega} \qquad (9a)$$

$$\bar{v} = \dot{\bar{\omega}} = -\frac{\varepsilon}{4}p\exp(-i\zeta)\left[1 + \frac{3\dot{\psi}}{\Omega_o} + \frac{k^2\overline{|\bar{\omega} + \delta\omega|^2}}{2}\right] \qquad (9b)$$

and

$$\frac{d}{dt}\left[\bar{v}_z - \frac{k_o}{2\Omega_o}|p|^2\right] = 0 \quad . \qquad (9c)$$

Equation (9c) is also a direct result of kinetic energy conservation, Eq. (4a). Using (9a) in the lefthand side of (9b), and neglecting the product of small terms times ε^2 (which then eliminates the $k_o^2r^2/4$ term in B_z, but is valid if k_or remains small)

$$\ddot{p} - i[k_o\bar{v}_z + \Omega_o(1 - \frac{\varepsilon^2}{8})]\dot{p} - \frac{\varepsilon^2\Omega_o^2}{16}p = 0 \quad . \qquad (10)$$

This equation (10) appears to be as complicated as the original starting equations in (3), but in fact now there is only one time scale; the slow time scale compared to $2\pi/\Omega_o$, and with the conservation equation (9c), (10) will be shown to be exactly soluble.

Introducing the new dependent variables s and θ, $p = \Omega_o s\exp(-i\theta)$, Eqs. (8) reduce to

$$\dot{\theta}s^2 + \frac{q_os^2}{2} + \frac{\Omega_ok_o^2}{8}s^4 = C_2 \quad , \qquad (11a)$$

C_2 a constant, with $q_o = k_oC_1 + (1 - \varepsilon^2/8)\Omega_o$, C_1 the constant defined by Eq. (9c), and

$$\ddot{s} = -\frac{\partial}{\partial s}V(s) \quad , \qquad (11b)$$

with the effective potential V given by

$$V = \frac{C_2^2}{2s^2} + (q_o^2 - \frac{\varepsilon^2\Omega_o^2}{4} - \Omega_ok_o^2C_2)\frac{s^2}{8} + \frac{\Omega_ok_o^2q_os^4}{16} + \frac{\Omega_o^2k_o^4}{128}s^6 \quad . \qquad (11c)$$

Choosing the dimensionless variables $X = k_0 s$, $\eta = q_0/\Omega_0$, and $\zeta = \iota_2 k_0^2/\Omega_0$, we have the final equation, soluble by quadratures

$$\ddot{X} = -\frac{\partial V}{\partial X} \tag{12a}$$

$$V = \frac{\Omega_0^2}{2}\left[\frac{\zeta^2}{X^2} + \frac{1}{4}(\eta^2 - \frac{\epsilon^2}{4} - \zeta)X^2 + \eta\frac{X^4}{8} + \frac{X^6}{64}\right] = \frac{\Omega_0^2}{2}\left[(\frac{\zeta}{X} - \frac{\eta X}{2} - \frac{X^3}{8}) + \eta\zeta - \frac{\epsilon^2}{16}X^2\right]. \tag{12b}$$

By their definition and the assumptions of this analysis

$$X = k_0 s \ll 1 \;, \quad \eta \simeq 1 + \delta \ll 1 \;, \quad \text{and} \quad \zeta \sim s^2 k_0^2 \eta + k^4 s^4 \ll 1 \;,$$

therefore the time scale for the X variation is seen to be much longer than Ω_0, consistent with assumptions. We note that the X^4 and X^6 terms in the effective potential, V, come strictly from the change in $\bar{v}_z$ as $|z|^2$ increases as given by (9c) and is a result of the detuning of the linear resonance, $k_0 v_z \sim -\Omega_0$. Also note that as $\Omega_0 < 0$ for electrons, the conservation law indicates that as $|z| \propto |\delta v|$ increases $\bar{v}_z$ decreases, as expected from constancy of $\gamma = [1 - \vec{v}^2/c^2]^{-1/2}$. The X^6 term in the potential is definitely a stabilizing term, that is, it sets an upper bound on the limits of $|X|$, and these limits are not such as to invalidate the assumption of small X because it clearly dominates the force term when $X \gtrsim \eta$ and as $\eta \ll 1$, the small X approximation must still retain the X^6 term.

By conservation of the effective energy, E, of Eq. (12): $E = \dot{X}^2/2 + V = \text{constant}$, Eq. (12) can be reduced to quadratures and by using the independent variable $y = X^2$, the integrals are all of the elliptic type. However, this result is probably no more useful than the observation that by standardizing the original equation, (3a), and neglecting the $k_0^2 r^2/4$ term in δB_z, the resulting equation is of the Mathieu form. So, some general considerations are in order and will be shown to provide sufficient information for analytically describing the electron orbits to the degree necessary for most FEL applications.

III. RESONANT ORBIT DESCRIPTION

The effective potential described by (12b) yields a very useful method for analytically dealing with the orbits. Recalling the definitions of variables we have that

$$k_0\delta\omega = iX\,\exp\{-i[\Omega_0(1-\varepsilon^2/8)t+\theta]\} \quad , \tag{13a}$$

so that the effective potential diagram describes the modulated Lamor motion of the electron with $X = |k_0\delta\omega|$. Knowing the quantitative features we can derive all of the remaining orbit characteristics including the guiding center position,

$$k_0\bar{\omega} = -\frac{4\,\exp[-i(\theta+\zeta)]}{\varepsilon}\left[\frac{\dot{X}}{\Omega_0} - i\left(\frac{\zeta}{X} - \frac{\eta X}{2} - \frac{X^3}{8}\right)\right] \quad , \tag{13b}$$

or

$$|k_0\bar{\omega}|^2 = \frac{32}{\varepsilon^2\Omega_0^2}\left[E - \frac{\Omega_0^2}{2}\eta\zeta + \frac{\varepsilon^2\Omega_0^2X^2}{32}\right] = \text{constant} + X^2 \quad . \tag{13c}$$

Equation (13c) is also a consequence of angular momentum conservation, equivalent to Eq. (4b). Therefore following the X motion using the effective potential method directly gives a picture of the excursions in Larmor radius, $|k_0\delta\omega(t)| = X$, and the difference in X^2 gives the differences in the square of the guiding center position;

$$|k_0\bar{\omega}(t_1)|^2 - |k_0\bar{\omega}(t_2)|^2 = X^2(t_1) - X^2(t_2) \quad .$$

In general the effective potential has the characteristic shape shown in Fig. 1. The number of minima can either be one or two. These differences are important as a minimum in the potential corresponds to a stable electron orbit with constant radial guiding center and magnitude of the Larmor radius. The details of this potential and the resulting motion can be investigated in depth in various limiting cases of the parameter, ζ, ε, and η.

In the experimentally interesting limit where $|\eta| \gg \varepsilon/2$, the equations of motion can be written in terms of the new variable, $y = X/|\eta|^{1/2}$, and new potential, $\tilde{V}$, as

$$\ddot{y} = -\frac{\partial \tilde{V}}{\partial y} \quad , \quad \tilde{V} = \frac{\Omega_o^2 \eta^2}{2}\left[\frac{\zeta}{\eta^2 y} - yE(\eta) - y^3/8\right]^2 \tag{14}$$

with $E(\eta) = \{ {+1, \; \eta > 0 \atop -1, \; \eta < 0}$. As $\zeta \sim \mathcal{O}(kr)^2\eta$, the constant $\zeta/\eta^2 \sim (kr)^2/\eta$ and is typically of order unity. From (14) we see that the time scale of these defections are of the order $\approx 2\pi/\eta\Omega_o \gg 2\pi/\Omega_o$ by assumption. By simple algebraic calculations the potential $\tilde{V}$ can be shown to have two minima (as in Fig. 1) only if $-2 < \zeta/\eta^2 \le 0$, or $\eta < 0$ and $0 \le \zeta/\eta^2 < 2/3$. Otherwise $\tilde{V}$ has only one minimum for $y \ge 0$.

As a particularly simple example let us determine the quantitative features of a particle with $\zeta = 0$, $|\eta| \gg \varepsilon/2$. For $\eta > 0$, only $y = 0$ is a minimum, and the motion is oscillatory but y remains near $y = 0$. For $\eta < 0$ two minima which are zeros exist in $\tilde{V}$, at $y = 0$, and $y = \sqrt{8}$. A relative maximum of $\tilde{V}$ occurs at $y_m = \sqrt{8/3}$ with a value of $\tilde{V}(y_m| = (16/27)\eta^2\Omega_o^2$. As this value is large compared to the energy, given by (13c) as

$$E = \frac{\varepsilon^2\Omega_o^2}{32}[|\eta\bar{\omega}|^2 - X^2] < \varepsilon^2 ,$$

by assumption then electrons initially near the minimum at $y_o = \sqrt{8}$ will oscillate but remain near to this value of y_o. For small oscillations near y_o the motion is sinusoidal with the exact frequency being given by $\omega_{os} = \sqrt{+\tilde{V}} = \sqrt{2}2^{1/2}\eta\Omega_o$ in this "not too" resonant regime, $|\eta| \gg \varepsilon/2$, one or two equilibrium positions can exist, and motion around these stable singular points is very limited. Additionally, in this not too resonant regime the new non-zero guiding center position is of the order of the size of the Larmor radius, and is η-dependent (eg. $\zeta = 0$ gives the second equilibrium positions as $|k_o\rho| = \sqrt{8|\eta|}$). Hence, small changes in the resonance, $|k_oV_{zo}/\Omega_o| = |\delta|$ can make substantial relative changes in the equilibrium orbit positions.

For smaller values of η, very near resonance such that $\eta^2 - \varepsilon^2/4 - \zeta$ is negative the first minimum in V now is not at $X = 0$ even when $\zeta = 0$. This means that even those electrons that initially have small perpendicular velocities and radial positions near $r = 0$ can still experience significant radial excursions and velocity modulations. For example in the simplest limit $\zeta = 0$, V has an absolute minimum at $X = X_0$, with

$$X_0^2/4 \equiv -5\eta/9 + (4/9)\sqrt{\eta^2 + (9/64)\varepsilon^2} \qquad (15)$$

Therefore an electron will make excursions in X around X_0, turning wherever $E = V(X)$. From (13c), we note that for $\zeta = 0$, the energy $E = (\varepsilon^2\Omega_0^2/32)[X^2 - |k_0\bar{\omega}|^2]$. So for a particle with an initial Larmor radius, (X), equal to its initial guiding center displacement, ($|k_0\bar{\omega}|$), $E = 0$. For these electrons ($\zeta = 0$, $E = 0$, $\eta^2 < \varepsilon^2/4$) the turning points, X_t, can be shown to be $X_t = 0$ and $X_t = 2(\varepsilon/2 - \eta)^{\frac{1}{2}}$. Of course for the particular initial condition of the particle initially being at $r = 0$ with exactly zero perpendicular velocity, i.e., add the special condition $X = 0$ to these initial conditions; the particle just remains at $X \equiv 0$. However, this is an unstable point and very small deviations from the conditions $X = |k_0\bar{\omega}| = 0$ will cause the electron to make significant radial excursions.

In the simplest case of $\zeta = 0$ we should note that the potential has two minima if $\eta^2 > \varepsilon^2/4$ and $\eta < 0$ and these minima are at $X = 0$ and $X = X_0$. If $E = 0$ in this circumstance the particles stability remain at $X = 0$ and $X = X_0$. This demonstrates another parameter range where electrons injected with only a small perpendicular velocity, but on the $r = 0$ beam axis will not remain there. In this case of $E = 0$, the electron would sweep through $r = 0$ every Larmor cycle with its guiding center remaining at the fixed distance $X_0 = |k_0\bar{\omega}|$, from the axis, and slowly rotating in angle.

In both of these cases with $\eta^2 - \varepsilon^2/4$ negative or positive, X_o or the turning point, X_t, defines the amount of excursion possible: $X_{max} \simeq X_t \simeq X_o \simeq |k_o\bar{\omega}|, \approx |k_o \delta v_\perp/\Omega|$, and as such measures both the perpendicular "thermal" spread that a beam can develop; and its typical guiding center displacement. For example when $\eta \ll \varepsilon$, $X_t \simeq \sqrt{2\varepsilon}$, giving a maximum radial displacement of an electron of order $2\sqrt{2\varepsilon}/k_o$; and a perpendicular velocity $\delta v_\perp \simeq \sqrt{2\varepsilon} v_z$. Both of these displacements are significant and must be treated carefully and self-consistently in the modeling of electron beam equilibria and free electron gain factors.

IV. DISCUSSION

The analysis presented here demonstrates that in the vicinity of resonance, $|k_z v_z/\Omega_0| \approx 1$ analytic techniques can be used to obtain the electron orbits even in a longitudinal wiggler including the radial component of the magnetic field. It is also possible to allow for adiabatic variation of the guide field and wiggler wavelength, within this formalism if the quantities vary slowly in a period of this reduced motion, which is of order $2\pi/\Omega_0\varepsilon$, or $2\pi/\eta\Omega_0$ whichever is shortest. This same type of secular analysis has been applied to both the helical and linear wiggler geometries without the assumption used here, of the electrons being near the magnetic axis of the guide field.[8] Additionally, a more general perturbation formalism has been applied to a general wiggler geometry with simple periodicity in Z assuming only a small wiggler field strength to guide field strength. The question of stochasticity of the electron motion has also been addressed.[8]

In all, very important questions of the quality of the <u>in situ</u> electron beam can be answered. Equally significant is that in the vicinity of resonance $|k_\parallel v_\parallel/\Omega_0| \simeq N$, N integer, where large radiation conversion efficiencies can be expected, analytic approximations to the electron orbits can be obtained and these are all important in determining realistic gain factors and saturation mechanisms and radiation strengths for actual free electron lasers.

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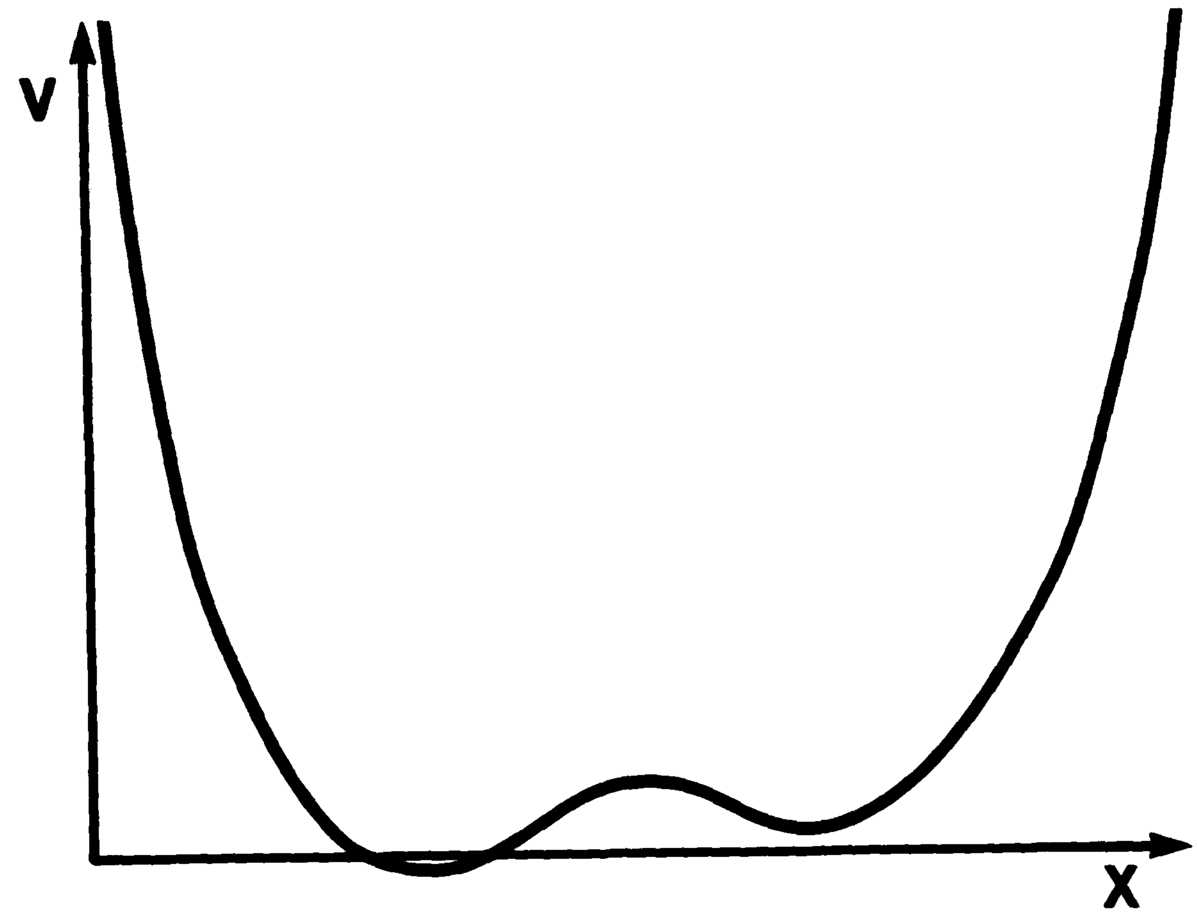

Fig. 1. Effective potential function for the Larmor Displacement, X .