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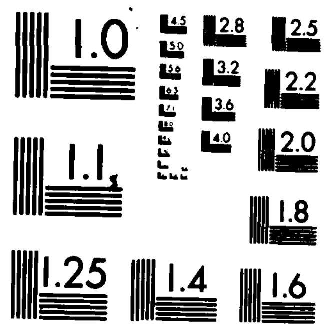
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DEFENSE COMMUNICATIONS ENGINEERING CENTER

AD A128858

TECHNICAL NOTE NO. 7-82

A PROPOSED STUDY PROGRAM FOR THE ENHANCEMENT OF PERFORMANCE OF CLOCKS IN THE DCS TIMING SYSTEM

AUGUST 1982

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Communications System timing survivability can be enhanced by a backup free-running clock capability, but this requires stable, accurate clocks. Stable, accurate cesium clocks are very expensive. Following a calibration period, errors in less expensive quartz or rubidium clocks can most likely be predicted for a free-running period and removed with sufficient accuracy for an adequate length of time to satisfy many DCS requirements. Use of an Autoregressive Integrated Moving Average (ARIMA) model for this clock error prediction is discussed.

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A PROPOSED STUDY PROGRAM FOR THE ENHANCEMENT OF PERFORMANCE OF CLOCKS IN THE DCS TIMING SYSTEM

AUGUST 1982

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A

FOREWORD

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EXECUTIVE SUMMARY

Survivability of uninterrupted slip-free communications in a synchronous switched digital network depends upon the survivability of the timing function. The survivability of the timing function can be greatly enhanced by providing a backup free-running clock mode of timing for each node. The usefulness of a clock in the free-running mode is dependent on its stability and accuracy. This becomes more important at higher data rates such as those for the future DCS. For stability and accuracy the DSCS uses cesium clocks which are controlled to keep them within a few microseconds of the Naval Observatory master clock. Cesium clocks could also be used in the rest of the DCS, but they are very expensive. Other means exist for providing more accuracy and stability during a controlled mode of operation and for short periods of time during free-running modes. These other means are also more survivable than the means presently used to coordinate the DSCS clocks with the Naval Observatory master clock. There is an excellent possibility that high-quality quartz crystal or rubidium clocks could be used as a much lower cost alternative for expensive cesium clocks in many DCS applications provided their errors in a free-running period could be predicted and removed with sufficient accuracy for a long enough period of time. A study by the Naval Observatory under DCA sponsorship shows that prediction techniques can greatly improve the accuracy and stability of clocks during a free-running period following a period of controlled operation. During the period of controlled operation, referred to here as a calibration period, errors in the clock are accurately measured. These measured errors are used to establish initial conditions for a mathematical model which is used to predict clock errors during the free-running period. The results of the study show that there is an extremely high probability of successful application of the technique in the DCS. This offers a high potential capability for both improving survivability and reducing costs compared to other alternatives. However, more study of clock predictability using a larger number of newer clocks is needed. Optimization of the mathematical models should be attempted, and evaluation of possible aliasing due to infrequent measurements (one per hour) should be evaluated. In addition to these evaluations of the predictability of clock performance, the study needs to be extended to include evaluation of the practical application of the technology in the DCS.

This technical note provides a background of the need for accurate, stable timing in a military switched digital communications system such as the DCS. It explains the application of ARIMA models to clock prediction, and it discusses many things that need to be done to answer questions about the practical application of these techniques to the DCS timing system.

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I. INTRODUCTION

This technical note presents a proposed program of test and analysis with a goal of using prediction techniques to enhance the performance of quartz-crystal clocks, rubidium gas cell clocks, and cesium beam clocks for application in the Defense Communications System (DCS). Although this is basically a research project, previous work sponsored by DCA indicates a very high probability of providing results which will make it possible to enhance the survivability of the timing function or reduce costs or both. The prediction models to be employed are members of the general class of Autoregressive Integrated Moving Average (ARIMA) models. Because most readers are probably not familiar with ARIMA models, a discussion of these models as they can be applied to clock prediction is also included.

Studies of timing (synchronization) for the digital Defense Communications System (DCS) have shown the importance of having a backup free-running clock mode of operation to enhance communications system survivability by providing timing when the normal timing capability is not available for any reason. The usefulness of a clock in the free-running mode is dependent on its stability and accuracy. In addition to reducing the need for resynchronization or the occurrence of other communication outages, high stability and accuracy can aid in system monitoring and can be a most important factor in providing rapid recovery of the communications system following an outage.

The Defense Satellite Communications System (DSCS) presently uses expensive cesium clocks to aid the rapid acquisition of its spread spectrum signals. These clocks are coordinated with the Naval Observatory master clock. The techniques used for this coordination do not have a high degree of survivability, but when adequately modified to provide the needed survivability, they would also be effective in other parts of the DCS. However, cesium clocks are very expensive (approximately $40,000 depending on options), and a less expensive alternative is needed. A preliminary study by the Naval Observatory under DCA sponsorship shows a capability to predict errors of high quality quartz-crystal oscillators ($8,000 or less depending on type selection and options) or rubidium clocks ($21,000 or less depending on type and options) with considerable accuracy during a free-running period following a calibration period. Using these predictions, there is a very high probability that errors can be removed with sufficient accuracy for a long enough period of time to permit the use of these clocks as an alternative to much more expensive cesium clocks for many communications applications. An important initial applicat[illegible] might use them as alternate or backup clocks for the principal cesium clocks at some DSCS stations. Later, much more extensive applications would be expected in other parts of the DCS.

An objective of this technical note is to provide sufficient background on the timing needs of the future digital DCS and methods of satisfying them so that the reader will understand: (1) what is needed from this program to enhance the performance of clocks for application in the DCS, (2) why it is important, and (3) how it fits into a complete timing system.

This document describes ARIMA prediction models and how they can be applied to the DCS so that the reader can understand what the models accomplish, how they accomplish it, and the high probability that their application in the DCS could result in very significant cost savings and enhancement of survivability.

An objective of the study program described in this technical note is to further refine clock prediction models for three types of clocks: (1) high quality quartz-crystal clocks, (2) rubidium gas cell clocks, and (3) cesium beam clocks. A further objective is to evaluate the resulting models using clock error data obtained by comparing each of a number of different clocks, from each of the three generic types, with the Naval Observatory master clock. The evaluations are to be made over a large sample of different periods of time in order to obtain statistically meaningful results.

Another aim of the study program is the development of simple recursive microprocessor algorithms for both calibration and prediction (hopefully, ones which can also permit the optimization of parameters) which will permit practical application of the technology to a digital DCS.

A further objective of the study program, which hopefully, will also include a capability for optimizing the parameters in the prediction models, is to include in the programming of the microprocessor the capability to filter timing information and to control the timing system. This includes:

- Elimination of obviously erroneous perturbed timing measurements.
- Filtering the timing information during the controlled mode of operation (most of the time).
- Control of a micro-phase-stepper that controls the output of the oscillator or clock during the controlled mode of operation.
- Calibration of the clock prediction function during the controlled mode of operation.
- Determination of when it is necessary to change from the controlled mode of operation to the free-running mode.
- Making a smooth transition from the controlled mode to the free-running mode.
- Performance of clock correction while in the free-running mode (control of the micro-phase-stepper to remove predicted errors).
- Determination of when to return to the controlled mode.
- Control of a smooth transition back to a controlled mode of operation.
- Accomplishment of any other timing system control functions discussed in the references.

II BACKGROUND

As the DCS becomes more digital--digital transmission, digital switching, digital control--the importance of system timing increases. The timing relationship determines to whom the information belongs and what it means; i.e., particular time slots are assigned for particular purposes. It is very important in a synchronous digital network that any bit originating anywhere in the network be available at the instant when it is needed at any node through which it passes. The loss of satisfactory timing can be catastrophic, causing all received information to be meaningless. Note that this is a phase control problem and not just a frequency control problem, i.e.; syntonization is not adequate and synchronization is required.

Studies by DCA and its contractors over a number of years, using both simulation and analysis, have recommended that all major nodes of the DCS be referenced to Coordinated Universal Time (UTC) when it can be made available. (References [1-11]). When UTC is not available to the network, it is recommended that all major nodes be referenced to the best (highest ranking) clock in the network, and to provide a similar capability for any portion of the network that becomes isolated from the rest of the network. These studies recommended that a time reference for all major nodes be distributed through the network by coordinating the transmission of synchronization codes from every major node with the network reference clock; that minor nodes be slaved to major nodes; and that a free-running clock mode of operation be provided for each node, to be used when other modes of operation are not available. Although reliability was a concern in those studies, the major concern has more recently been survivability during and following an attack upon the communications system prior to or during full scale war. This has led to development of a number of attributes for timing in a digital DCS. A discussion of these attributes [12] indicates their importance to a survivable slip-free digital DCS. They can be provided by methods described in [13]. When these attributes are provided, an accurate timing reference is available at each major node. This timing reference can be used to accurately determine the errors in the local clock.

It was early recognized that the stability and accuracy of a timing system could contribute greatly to the ability of a node to reenter the network quickly after a communications outage during which its clock must free-run. Because of this, stable and expensive cesium clocks which are maintained by the Naval Observatory within acceptable tolerances of UTC (USNO) have been in operation in the Defense Satellite Communications System (DSCS) for a number of years. By accurately distributing time to major nodes throughout the remainder of the DCS, a very stable network timing system will be provided. This will also provide the advantages of accurate clocks at all nodes which can be used for other purposes. With the ability to accurately measure errors in local clocks at major nodes of the DCS, it seemed there would be an excellent opportunity to use this information to predict clock errors which would occur during a free-running period immediately following a period of measurement and calibration. These predicted errors could then be removed during the free-running period, greatly improving the clock performance. This might make it possible to replace some expensive cesium clocks with much less expensive, high quality quartz-crystal clocks. For those applications where errors in quartz-crystal clocks cannot be predicted with sufficient accuracy over a sufficiently long period of time, it is quite likely that errors in much more stable rubidium clocks can be adequately predicted. The cost of rubidium clocks, though higher than quartz crystal clocks, is still much less than the cost of cesium clocks. The Naval Observatory was asked to provide information on the predictability of such clocks. The initial experimental evaluations of such predictability, made by the Naval Observatory under DCA sponsorship, indicate that useful predictions can be made for a significant period of time [14].

Timing stability and accuracy have great importance to the survivability of digital communications. Communications system timing stability can be enhanced by the use of filtering if such filtering employs very long time constants (bandwidths of a few microhertz). Such narrow bandwidths can be provided stably and economically in frequency control loops by employing relatively low cost microprocessors. For survivability of the timing function, every communications link between major nodes of the network should also be capable of serving as a timing reference link. To make the most effective use of this capability, self-organization should be provided. Self-organization will automatically adjust network parameters to compensate for damage to portions of the network. A most practical way to provide this control function is by use of a microprocessor at each major node. Since this function utilizes an insignificant percentage of a microprocessor's capability, the microprocessor can be shared with other timing functions. It can be used for narrowband filtering in a phase control loop, and to provide a double-ended feature which removes from the timing coordination the time that it takes the signal to travel between nodes. This adds greatly to frequency and phase stability of nodal clocks throughout the network. The double-ended feature also contributes greatly to the accuracy of the measurement of timing errors at the nodes. The same microprocessor can be used to make the error measurements independent of error corrections made at other nodes throughout the network. This minimizes the propagation of timing errors through the network. The same microprocessor can be used for combining phase reference (timing) information received over many different paths in a way which will improve measurement accuracy and also simplify the reorganization following damage to the network. Studies [11] have shown that all of these features can be provided using only a portion of a microprocessor's capacity. Therefore, the remaining capacity could be applied to clock error prediction and correction. If not enough remaining capacity were available, the cost of an additional microprocessor would be very low when compared with the cost of a cesium clock. When cesium clocks are used, prediction techniques can also improve their performance; but the major cost saving advantage for the DCS would be the ability to use lower cost clocks in place of cesium clocks.

Although the Naval Observatory has conducted a preliminary study under DCA sponsorship to determine clock predictability, only a limited number of older clocks were used, and there were significant problems with some of the clocks during the measurements. No attempt was made to optimize the parameters in the ARIMA model. Measurements were made once an hour during that study and no attempt was made to determine whether aliasing* was having any effect on the result. It is important to add newer clocks with the latest technology to the clocks which the Observatory already has. It is also important to take data on a larger number of clocks for longer periods of time in order to increase confidence in the statistical studies. Measurements should be made on at least part of the clocks at quarter hour (or more frequent) intervals for use in evaluating possible effects of aliasing. Such an evaluation could be made by using every fourth (or less frequent measurement) and comparing the prediction results with those obtained using every measurement. The effect of averaging several samples prior to application of the ARIMA model should also be investigated. The work needs to be further extended to investigate the programming of a microprocessor to carry out both clock error prediction and the control of a micro-phase-stepper** to correct the output of the free-running clock in a way that could be conveniently applied to the DCS.

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* Aliasing - - The introduction of error in a system employing discrete sampling of continuous data which occurs when that data contains frequencies higher than half the sampling rate. It occurs because the higher frequencies of the original spectrum, as they occur around the sampling rate after sampling, overlap the spectrum of the original continuous data and are indistinguishable from the original frequencies. Aliasing can cause the higher frequencies of the original data to have the same effect on the analysis as if they had been low frequencies in the original data.

**Micro-phase-stepper--a device which can alter the phase/time or frequency of an input reference signal by making precise discrete phase shifts which are digitally controlled. One commercially available unit can make shifts in a 5MHz reference signal of 1 picosecond to 1 microsecond with a resolution of 1 picosecond and frequency changes of $\pm$ 1 part in 10^{-17} to $\pm$ 1 part in 10^{-7} with a resolution of 1 part in 10^{-17}.

III. APPLYING ARIMA MODELS TO CLOCK ERROR PREDICTION

Autoregressive Integrated Moving Average (ARIMA) models to be used in this program are discussed in reference [15], a large text written in the terms of the mathematician and statistician and not easy for most engineers to read. A brief discussion of its application to clock prediction written more for the engineer is provided in this section. This explanation should be adequate to give an engineer an understanding of the application of these models to clock prediction, and it should also put him in a better position to use reference [15]. Some of this discussion results from the author's own observations and is not based directly on the reference.

A model of the form $Z'_t = \phi_1 Z'_{t-1} + \phi_2 Z'_{t-2} + \ldots + \phi_p Z'_{t-p} + a_t$ where $Z'_t = Z_t - \mu$, with Z_t representing the value of the process at time t and μ representing the mean about which the process varies, is called an autoregressive process of order p because it is a regression of the variable Z' on the p previous values of itself. The ϕ's are parameters of the process and a_t is a random shock.

An autoregressive operator of order p may be defined as $\phi(B) = 1 - \phi_1 B - \phi_2 B^2 - \cdots - \phi_p B^p$ in which B is a backward shift operator, i.e., $BZ_t = Z_{t-1}$, and $B^m Z_t = Z_{t-m}$. Then the autoregressive model may be written as $\phi(B)Z'_t = a_t$

Similarly, a model of the form $Z'_t = a_t - \theta_1 a_{t-1} - \theta_2 a_{t-2} - \cdots - \theta_q a_{t-q}$ which is dependent on a finite number, q, of previous a_t's is called a moving average process. Using the operator B, it may be written as $Z'_t = (1-\theta(B))a_t$.

Many time series exhibit a nonstationary behavior and do not vary about a fixed mean. However, the series formed by taking the first, second, or higher difference of these series are frequently stationary. Such behavior can be represented by a generalized operator $\Phi(B)$ in which one or more of the zeros of the polynomial $\Phi(B)$ is unity. Thus, $\Phi(B)$ can be written as $\Phi(B) = \phi(B)(1-B)^d$. Since the differences of a discrete sequence can be related to the derivative of a corresponding continuous function, the inverse of the differences (sum) as sometimes used when working with ARIMA models can be related to an integral of a corresponding continuous function. In solving an equation with d differences, it is common to insert initial values and then sum d times. Therefore, a generalized autoregressive function in which d zeros of the autoregressive operator are unity is said to be integrated with order d. A model containing both a generalized autoregressive part and a moving average part is said to be an Autoregressive Integrated Moving Average Model of order p, d, q, i.e., ARIMA (p, d, q). It has the form $\phi(B)(1-B)^d Z'_t = \theta(B)a_t$ where $\phi(B)$ is an autoregressive operator of order p and $\theta(B)$ is a moving average operator of order q.

Note that when d > 0 the equation contains only differences of Z'_t, i.e., when d = 1 there are first order differences corresponding to differences between adjacent values of Z'_t, and when $d > 1$ there are also higher order differences, such as differences between first order differences, etc. The significance of this is that when $d > 0$, as it always is in clock error measurements, the value of μ in Z'_t drops out and it is only necessary to be concerned with Z_t. However, a value of μ can still be subtracted from Z_t if unreasonably large numbers result otherwise.

In general, a moving average model can be written in terms of an autoregressive model and vice versa, provided certain convergence conditions are met. To illustrate this take the first order autoregressive model $(1-\phi_1 B)Z_t = a_t$ and divide both sides by $1-\phi_1 B$, giving $Z_t = (1-\phi_1 B)^{-1} a_t$. Then $(1-\phi_1 B)^{-1}$ can be expanded into the infinite sequence $1 + \phi_1 B + \phi_1{}^2 B^2 + \phi_1{}^3 B^3 + \cdots$. Therefore, the first order autoregressive model can be written as an infinite order moving average model with $\theta_1 = \phi_1, \theta_2 = \phi_1{}^2, \theta_3 = \phi_1{}^3$, etc. Similarly, the first order moving average model can be written as an infinite order autoregressive model. This begins to show the advantage of using both autoregressive and moving average terms in the model, an ARIMA model. It is usually possible to get a good model without using a large number of either moving average or autoregressive terms.

To put things in a form somewhat more familiar to electronics engineers, the ARIMA model $(1-\phi(B))Z_t = (1-\theta(B))a_t$ can be written in the form $Z_t = ((1-\theta(B))/(1-\phi(B)))\, a_t$ where the expression $(1-\theta(B))/(1-\phi(B))$ is known as the transfer function.

For continuous functions in electronics engineering, the transfer function is usually written as $H(S) = Y(S)/X(S)$ where $X(S)$ is the Laplace transform of the input and $Y(S)$ is the Laplace transform of the output. If the poles of $H(S)$, i.e., the zeros of $X(S)$, have negative real parts, the system is said to be stable. If their real parts are positive, the system is said to be unstable, and the output will increase without bound. With the increased use of digital technology in electronics engineering, more use is being made of discrete transfer functions using the Z transform. These are written in the form $H(Z) = Y(Z)/X(Z)$ and the transfer function of a digital filter has the form $H(Z) = (1 + \sum_{K=1}^{M} a_k Z^{-k})/(1 + \sum_{K=1}^{L} b_k Z^{-k})$. Note that this has the same general form as the transfer function of the ARIMA model if $Z^{-1} = B$, i.e., if the Z operator of the Z transform and the backward shift operator, B, are reciprocals. Indeed, this is the case, because multiplying by Z^{-1} in Z transforms corresponds to a unit step backward in time just the same as the B operator in the ARIMA process. The entire left side of the s-plane used for pole locations in continuous filters (the part where poles of $H(S)$ can be located for a stable system) maps into the inside of the unit circle in the Z-plane. For stability, the poles of $H(Z)$ must fall inside the unit circle. Since B is the reciprocal of Z (the Z transform operator), the zeros of $(1-\phi(B))$ must fall outside the unit circle for stability.

From the preceding discussion, it can be seen that the ARIMA model can be considered to represent the output, Z_t, from a linear filter whose input is white noise, a_t. If d = 0 and all of the zeros of $(1-\phi(B))$ are outside the unit circle, the filter is stable; if the zeros are inside the unit circle, it is unstable. (Alternatively, it could be considered to be a process for transforming Z_t to white noise.) If d = 0 and all of the zeros of $(1-\phi(B))$ are outside the unit circle, the output, Z_t, for a white noise input, a_t, with zero mean will be stationary, i.e., its statistics will be independent of time. If d = 1, i.e., there is one zero on the unit circle, the first difference of the output Z_t will be stationary with zero mean, but the level of Z_t can be nonstationary and wander around. Similarly, if d = 2, both the level and the slope are free to wander around. The applicability of such a model to clock analysis and prediction begins to take shape.

The ARIMA model is a difference equation. Difference equations are relatives of differential equations which are more familiar to many engineers. Like differential equations, difference equations have a general solution that is the sum of two parts: a complementary function, and a particular integral. The complementary function is the solution of the equation when all a_t are zero so that only the autoregressive terms play a part in the complementary function. A distinct real root of $\Phi(B) = 0$ will contribute a negative exponential term to the complementary function. A pair of complex roots contributes a damped sine wave. Multiple equal roots contribute the product of a polynomial and an exponential. If d equal roots are unity, as d unity roots in an integrated autoregressive model, the exponent of the exponential is zero; i.e., the exponential is a constant, 1, and the roots contribute a polynomial of degree d-1. The contributions of various types of roots to the complementary function are very important to the use of the ARIMA model in predicting clock performance. They are important because during the period that is being predicted, the inputs, a_t, will be unknown and it will be necessary to assume that they are zero. Therefore, the prediction will be made using the complementary function and a satisfactory choice of initial conditions. It is not reasonable to expect any clock errors that exist prior to the start of the prediction period to exponentially decrease during the prediction period. Similarly, unless the clock is subjected to some form of cyclic environmental conditions, errors would not be expected to have sine wave terms, either damped or undamped. Therefore, it would be expected that clock errors could best be approximated by some form of polynomial, and it would further be expected that any polynomial that could be estimated with sufficient accuracy to be useful would be relatively simple.

From the foregoing it can reasonably be expected that the ARIMA model will have no autoregressive factors other than the zeros at 1. The ARIMA model should be an ARIMA (0,d,q), i.e., a model with p = 0 and both d and q relatively small integers. If d = 1, the complementary function would be a zero order polynomial which would predict a constant phase error. If d = 2, the complementary function would be a first order polynomial, and it would predict a linear change in phase error added to the initial phase error, i.e., an initial phase error plus a constant frequency error. If d = 3, the complementary function would be a second degree polynomial, and it would predict a linear change in frequency added to the initial frequency error and the initial phase error.

Since quartz-crystal clocks and rubidium gas cells are known to have frequency drift, it would seem that an ARIMA (0,3,q) model might be appropriate to try for these types of clocks. On the other hand, cesium beam clocks have very little frequency drift so that a (0,2,q) model might be more appropriate for them.

If the nature of the prediction function is determined entirely by the complementary function, which in turn is determined entirely by the autoregressive function of order p+d, where does the moving average function play its part? The answer is, for determining the initial values, i.e., how the function determined by the autoregressive terms is fitted to the measured clock error data. For this discussion, let $\hat{Z}_{t+f}$ be the prediction for the error that will exist f measurement periods beyond the measurement time, t, at which the prediction is made. Since there have been no measurements made beyond time t, the values of a_t must be assumed to be zero after time t. In operator form, the ARIMA (0,3,3) model is

$$(1 - B\theta)^3 Z_t = (1 - \theta_1 B + \theta_2 B^2 - \theta_3 B^3) a_t.$$

Written in expanded form, this is

$$Z_t - 3Z_{t-1} + 3Z_{t-2} - Z_{t-3} = a_t - \theta_1 a_{t-1} - \theta_2 a_{t-2} - \theta_3 a_{t-3}$$

and for prediction this can be written as

$$Z_{t+f} - 3Z_{t+f-1} + 3Z_{t+f-2} - Z_{t+f-3} = a_{t+f} - \theta_1 a_{t+f-1} - \theta_2 a_{t+f-2} - \theta_3 a_{t+1-3}$$

Then assuming that for all values of the subscript greater than t, a_s=0, and $Z_s = \hat{Z}_s$ (a predicted value), the following predicted values are obtained.

$$\hat{Z}_{t+1} = 3Z_t - 3Z_{t-1} + Z_{t-2} - \theta_1 a_t - \theta_2 a_{t-1} - \theta_3 a_{t-2}$$

$$\hat{Z}_{t+2} = 3\hat{Z}_{t+1} - 3Z_t + Z_{t-1} - \theta_2 a_t - \theta_3 a_{t-1}$$

$$\hat{Z}_{t+3} = 3\hat{Z}_{t+2} - 3\hat{Z}_{t+1} + Z_t - \theta_3 a_t$$

$$\hat{Z}_{t+4} = 3\hat{Z}_{t+3} - 3\hat{Z}_{t+2} + \hat{Z}_{t+1}$$

$$\hat{Z}_{t+f} = 3\hat{Z}_{t+f-1} - 3\hat{Z}_{t+f-2} + \hat{Z}_{t+f-3} \text{ for } f>3$$

Note that all predictions beyond $\hat{Z}_{t+3}$ are formed completely from previous predictions and no measured data are directly included. Hence, from this point forward the initial conditions have been established and the form determined by the complementary function is followed; the information that p = 0, d = q = 3 along with the values of $\hat{Z}_{t+1}$, $\hat{Z}_{t+2}$, and $\hat{Z}_{t+3}$ completely determine all future predictions. The values of θ_1, θ_2, and θ_3 are important in determining the values of $\hat{Z}_{t+1}$, $\hat{Z}_{t+2}$ and $\hat{Z}_{t+3}$. If θ_3 is zero, the predicted errors for all f > 2 are dependent on the values of Z_t, $\hat{Z}_{t+1}$, and $\hat{Z}_{t+2}$. If both θ_2 and θ_3 are zero, the predicted errors for all f > 1 are dependent on the values of Z_{t-1}, Z_t, and $\hat{Z}_{t+1}$. If θ_1,

θ_2, and θ_3 are all zero, the predicted errors for all $f > 0$ are dependent only on the values of Z_{t-2}, Z_{t-1}, and Z_t. It is clear that if all values of θ are zero, any measurements made prior to Z_{t-2} have no influence on the predictions. Therefore, it appears that the values of θ determine how the measurements made prior to Z_{t-2} influence the predictions. In order to get an idea of how θ can bring the previous measurements into the prediction process, consider the simple moving average model $Z_t = (1-\theta_1 B)a_t$. Remember that in making the measurements only the actual clock errors, Z_t, are observed. There is no direct observation of the random disturbances, a_t. These disturbances must be indirectly inferred from the measurements. Solving the equation, $Z_t = a_t - \theta_1 a_{t-1}$, of the simple first order moving average model to get a_t gives

$$a_t = Z_t + \theta_1 a_{t-1}.$$

Repeated substitution of this expression into itself gives:

$$a_t = Z_t + \theta_1(Z_{t-1} + \theta_1 a_{t-2})$$

$$= Z_t + \theta_1(Z_{t-1} + \theta_1(Z_{t-2} + \theta_1 a_{t-3}))$$

$$= Z_t + \theta_1(Z_{t-1} + \theta_1(Z_{t-2} + \theta_1(Z_{t-3} + \theta_1 a_{t-4})))$$

$$= Z_t + \theta_1 Z_{t-1} + \theta_1{}^2 Z_{t-2} + \theta_1{}^3 Z_{t-3} + \cdots + \theta_1{}^n a_{t-n}$$

In each of these equations, all terms except the last contain a Z, while the last term always has an a instead of Z. If n is large and θ_1 is small, the effect of assuming that $a_{t-n} = 0$ can be very small because it is multiplied by $\theta_1{}^n$. For example, if $\theta_1 = 0.5$ and $n = 10$, the error in a_t from assuming that $a_{t-n} = 0$ is less than 0.1 percent; and if $n = 20$, it is less than 0.0001 percent. However, the value of a_t obtained in this manner contains weighted values of all Z_t *that occur after that value of* a_{t-n} that was assumed to be zero. In effect, θ, determines the rate of decay of influence from older measurements in evaluating the latest a_t.

Taking the expression for the simple moving average model ARIMA (0,0,1) and substituting for each Z_t the corresponding expression for the autoregressive part of an ARIMA (0,3,1) model, the value of a_t can be evaluated for that model as

$$a_t = Z_t + (\theta_1-3)Z_{t-1} + (\theta_1{}^2 - 3\theta_1 + 3)Z_{t-2} + (\theta_1{}^3 - 3\theta_1{}^2 + 3\theta_1 - 1)Z_{t-3} + \theta_1(\theta_1{}^3 - 3\theta_1{}^2 + 3\theta_1 - 1)Z_{t-4} + \cdots + \theta_1{}^{n-3}(\theta_1{}^3 - 3\theta_1{}^2 + 3\theta_1 - 1)\, a_{t-n}.$$

As with the simple moving average model, ARIMA (0,0,1), the ARIMA (0,3,1) model has very little error introduced in the value of a_t from assuming $a_{t-n} = 0$, provided n is large and θ_1 is small.

Reference [15] describes a method of backward estimation whereby measured values are used to estimate what the values would have been prior to the first measurement. By using this approach, it is possible to arrive at more accurate values of a_t more quickly. However, if a microprocessor is employed in making the measurements, there should be no problem in obtaining enough measurements for accurate estimates of a_t without the use of the much more complicated procedure using backward estimates.

Returning to the discussion of the ARIMA (0,3,3) prediction model, recall that it was observed that when $\theta_2 = \theta_3 = 0$ an ARIMA (0,3,1) model is obtained. Also, when $\theta_2 = \theta_3 = 0$, all predictions after the first are based on the last two measurements plus the first prediction, with the first prediction based on the last three measurements and a computed value of the random shock, a_t. This indicates the importance of having accurate measurements. The first prediction gives the initial phase error. The first prediction and the last measurement give the initial frequency error. The first prediction and the last two measurements combine to determine the rate of frequency drift. As an example, assume that an ARIMA (0,3,1) prediction model has an error of size Δ in the last measurement, Z_t. In the evaluation of the first prediction $\hat{Z}_{t+1}$, in this model, Z_t is multiplied by 3 introducing a 3Δ error into $\hat{Z}_{t+1}$, but Z_t also appears in a_t which is multiplied by $-\theta_1$ so that the resulting error in the prediction of the initial phase error is $(3-\theta_1)\Delta$. The initial prediction of the frequency error comes from $\hat{Z}_{t+1} - Z_t$. The error in this initial prediction, which results from making an error, Δ, in the last measurement Z_t, is $(3 - \theta_1)\Delta - \Delta = (2 - \theta_1)\Delta$. This error in frequency prediction would cause an error in clock prediction which changes linearly with time. Perhaps of greater importance, the prediction of the frequency drift comes from the second difference of Z_{t-1}, Z_t, and $\hat{Z}_{t+1}$. For a Δ error in the last measurement, the error in drift is $(1-\theta_1)\Delta$. This error in the prediction of the drift would cause a prediction error that would change as the square of time. Hence, it is very important to keep measurement errors small. One obvious method of doing this in a practical clock prediction application using automated measurements is to make measurements once per second and average them for several minutes before using them as a measurement input to the ARIMA model. This could be particularly important for an application such as the DCS where the measurements themselves might sometimes be contaminated by noise. In using this average, care should be taken to eliminate from the average any individual measurements that are obviously displaced from their neighbors enough to indicate probable contamination of that particular measurement.

The ARIMA (0,3,3) model was selected as the basis for discussion and for use in examples partly because it seemed to have characteristics that would make it very useful for clock error prediction. It also has the advantage that by letting $\theta_3 = 0$ in the equations for an ARIMA (0,3,3) model, the ARIMA (0,3,2) model is obtained, and by letting $\theta_2 = \theta_3 = 0$, an ARIMA (0,3,1) model is obtained. The ARIMA (0,0,1) model and the ARIMA (0,3,1) model were chosen for the particular examples of evaluations of a_t because of their simplicity. They do illustrate that the values of the θ's control how earlier clock error measurements contribute to future predictions of clock errors. If the values of the random a_t's were known and the model were absolutely correct, the ARIMA model would be completely deterministic and the prediction would be perfect. If the model were absolutely correct and the values of the a_t's were accurately determined through the last measurement, then the prediction would start with the correct initial conditions. The prediction would be correct except for those random perturbations caused by the a_t's that occur after the beginning of the prediction period, and therefore cannot be predicted. The ARIMA model separates the deterministic predictable performance of the clock from the random unpredictable performance. One object of this study program is to determine the magnitude of the unpredictable part. Since this is dependent on the a_t's, the rms value of the a_t's for a particular clock is of interest. However, it is also of interest to make predictions of clock errors over a long period of time and compare those predicted error values with those that do occur for a large number of samples.

The a_t's can be obtained with a precision that increases during a period of time during which clock error measurements are made and which will be called the calibration period. For the application of clock prediction techniques to the DCS, the calibration period can be the entire period of time during which clock error measurements are made prior to entering a free-running clock mode of operation.

The conditional expectation estimate of $\hat{Z}_{t+f}$, given all Z's up to time t, is the minimum mean square prediction at time t for the value at Z_{t+f}. For a one step ahead estimate the error is the difference between the new measured value and its prediction made just prior to the measurement, and this error is equal to the random impulse a_{t+1}, which is included in the actual measurement but which could not be included in the prediction. One method of evaluating the a_t's is to subtract from each new measurement the value that was predicted for it the previous time period, $a_{t+1} = Z_{t+1} - \hat{Z}_{t+1}$. This process can be started at the initial measurement, $t = 0$, by assuming all unknown a_t information is zero. The accuracy of the estimates of a_t will increase as additional measurements are used. This is illustrated for the ARIMA (0,3,3) model in the following sequence of equations.

$$\hat{Z}_{1+1} = 3Z_1$$

$$a_2 = Z_2 - \hat{Z}_{1+1}$$

$$\hat{Z}_{2+1} = 3Z_2 - 3Z_1 - \theta_1 a_2$$

$$a_3 = Z_3 - \hat{Z}_{2+1}$$

$$\hat{Z}_{3+1} = 3Z_3 - 3Z_2 + Z_1 - \theta_1 a_3 - \theta_2 a_2$$

$$a_4 = Z_4 - \hat{Z}_{3+1}$$

$$\hat{Z}_{4+1} = 3Z_4 - 3Z_3 + Z_2 - \theta_1 a_4 - \theta_2 a_3 - \theta_3 a_2$$

$$a_5 = Z_5 - \hat{Z}_{4+1}$$

$$\hat{Z}_{5+1} = 3Z_5 - 3Z_4 + Z_3 - \theta_1 a_5 - \theta_2 a_4 - \theta_3 a_3$$

$$a_n = Z_n - \hat{Z}_{(n-1)+1}$$

$$\hat{Z}_{n+1} = 3Z_n - 3Z_{n-1} + Z_{n-2} - \theta_1 a_n - \theta_2 a_{n-1} - \theta_3 a_{n-2}.$$

As n grows large the accuracy of the evaluations of a_t increases rapidly, provided the θ's aren't too large.

The discussion to this point should have provided the reader with an understanding of how the ARIMA model might be used to predict clock errors, but has provided no indication of how the model can be optimized for a particular clock or type of clocks. The optimization of the ARIMA model comprises selecting the best set of integers for the parameters p, d, and q, and also selecting the best values for each θ_i and ϕ_j in the model. Reasoning as to what might be expected for clock error performance has indicated that for quartz-crystal or rubidium clocks a good model might have p = 0, d = 3, and q = some small integer. Similar reasoning indicates that a model with p = 0, d = 2, and q = some small integer might be a good model for cesium beam clocks. This needs to be verified by determining how well both of these models fit the measured data. In some cases, it right be desirable to compare results using each of these models with those having small variations in the values of p, d, and q. If the above reasoning about the values of p and d are correct, only q remains to be determined. Taking the third differences of the measurements on the clocks should provide a moving average process. According to reference [15], the autocorrelation function, $_k$, of a moving average process of order q has a finite value for $q \le k$ but is zero for $k > q$. Thus, by taking the autocorrelation function of the third differences (or second differences in the case of cesium clocks), the value of q should be determined.

Once values of p, d, and q have been selected, there still remain some unknown parameters -- the values of the ϕ's, the values of the θ's and σ_a, where σ_a is the standard deviation of the a_t's. Reference [15] provides a general discussion of methods for estimating these parameters using both likelihood and Bayesian approaches. The reference gives the unconditional log-likelihood function as

$$l(\phi,\theta,\sigma_a) = f(\phi,\theta) - n \ln\sigma_a - \frac{S(\phi,\theta)}{2\sigma_a^2}$$

where the ϕ and θ in the equation are

$\phi = \phi_1,\phi_2,\phi_3,\ldots,\phi_m$ and

$\theta = \theta_1,\theta_2,\theta_3,\ldots,\theta_n$.

$$\frac{S(\phi,\theta)}{2\sigma_a} = \sum_{t=-\infty}^{n} [E[a_t \mid \phi,\theta,\nabla^d Z]]$$

where ∇^d is the set of d'th order differences of observed data Z, and $E[A_t \mid \phi,\theta,\nabla^d Z]$ is the expected value of a_t conditioned on ϕ, θ, and $\nabla^d Z$. The $S(\phi,\theta)/2\sigma_a{}^2$ is called the unconditional sum of squares and is also a function of the ϕ's and θ's.

The model is optimized by selecting those values of ϕ and θ which for the particular data, $\nabla^d Z$, maximize the log-likelihood function. In computing the unconditional sum of squares, backward prediction is used to determine the values of $\nabla^d Z$ prior to $t=0$, i.e., $t = -1,-2,\cdots$. These are used in a backward recursion to compute a_{-1}, a_{-2}, $a_{-3},\cdots$. These backward values of a_t and $\nabla^d Z$ can then be used for starting values in a forward recursion. The unconditional sum of squares is obtained by summing the squares of all computed a_t's. Strictly speaking, the unconditional likelihood is needed for parameter estimation. However, the term $f(\phi,\theta)$ in the unconditional log-likelihood function is usually negligible for large values of n. Similarly, if n is large and there are no zeros of $\nabla^d Z$ near the unit circle, a conditional sum of squares can be used which is conditional upon the starting values of $\nabla^d Z$ and a_t. For clock prediction, both of these conditions will usually be satisfied so that the much simpler conditional log-likelihood function can be used. This does not require any backward recursive computations, and the a_t's can be computed by the method already discussed. By summing the squares of a_t's obtained in that way, the conditional sum of squares is obtained. By making evaluations of the sum of squares for various values of θ_1, θ_2, and θ_3, the sum of squares function can be observed. The minimum sum of squares is a close approximation to the maximum likelihood estimate of the parameters θ_1, θ_2, and θ_3. This makes good intuitive sense because the a_t's represent the random unpredictable part of the ARIMA model. Surely, if another set of parameters could be selected which would provide a lower computed value for the sum of the squares of the a_t's, there would be a better set of parameters for the model. Therefore, the optimum set of parameters would seem to be that set of parameters which minimizes the sum of the squares of the computed a_t's.

The ARIMA models offer the advantage for clock prediction that they permit the prediction to be accomplished with relatively simple recursive computations.

As any engineer experienced with digital filtering techniques is well aware, unless the sampling rate (clock error measurement rate for the clock prediction application), is higher by at least a factor of two than the highest frequency component of the sampled data, serious aliasing can result. This aliasing translates frequency components higher than half the sampling rate into lower frequency components where it is very difficult and sometimes impossible to distinguish them from the lower frequency components that legitimately belong there. Aliasing is usually avoided either by filtering away the higher frequencies prior to taking the digital samples or selecting an adequately high sample rate. For the application of clock prediction techniques to the DCS, it is probably much easier to use an adequately high sample rate. Samples of once per second or even more frequent if necessary would cause no particular problem. Many samples could be averaged together to produce a single sample for use in the ARIMA model, since the averaging tends to filter off higher frequencies. Because there has been no evaluation of the part aliasing might play in the ARIMA model prediction of clock errors, the possibility of its existence should be kept in mind until it is shown to not be a factor.

IV. DISCUSSION OF THE INITIAL U.S. NAVAL OBSERVATORY STUDY

Reference [14] describes a preliminary evaluation of "Predictability of Quartz-Crystal Oscillators and Other Devices," which was performed by the Naval Observatory under sponsorship of the DCA. This study resulted directly from a recognition that a communications timing system that has the stability and accuracy to adequately support the survivability requirements of the Defense Communications System could also have the capability to accurately determine the errors in the nodal clocks during those periods of time when the timing system is operating properly. It appeared that it might be possible to use this information to predict the errors that would occur in those clocks when it was necessary for them to free-run. These predicted errors could then be removed to provide much better performance during the free-running period. An important question was whether this increase in performance would be great enough to permit significant cost savings by permitting quartz-crystal or rubidium clocks to be used in place of cesium clocks. Another important question was whether communications survivability would be increased enough by employing such clock error prediction techniques to justify their use even if they did not permit use of lower priced clocks. The preliminary study seems to indicate that either of these reasons would probably justify use of the techniques. However, much additional information is still needed.

In the preliminary study, five different methods were used to make the predictions: (1) a 1st degree polynomial was fitted to the data existing prior to the beginning of the free-running period and an extrapolation of this fitted polynomial was used for the predicted values of error during the free-running period, (2) the same procedure was used for a 2nd degree polynomial, (3) the same procedure was used for a 3rd degree polynomial, (4) the same procedure was used for a 4th degree polynomial, (5) an ARIMA model was used. An ARIMA (0,2,1) model was used and no attempt was made to optimize the model. Although this is probably a good model for cesium clocks, the discussion in the preceding section indicates that an ARIMA (0,3,q) model could be a more appropriate choice for quartz-crystal and rubidium clocks. No attempt was made to determine whether $q = 2$ or $q = 3$ might be better than $q = 1$, and no attempt was made to optimize θ_1 for the various classes of clocks. The value of $\theta_1 = 0.75$ was used in the ARIMA model for all clocks.

For each of the five methods of making the predictions, a specific prediction error was determined by subtracting the actual error, as measured after the prediction time, from the specific prediction obtained using the model. Different sets of data, taken on the same clock at different times, for the same length of prediction, were used for separate calibration periods. Several prediction errors, determined for a given clock with a given calibration period and a given length of prediction, were used to compute a single root-mean-square (rms) error value for each set of parameters.

The choice of an ARIMA (0,3,q) model for either quartz-crystal or rubidium clocks is supported by this preliminary study even though only the ARIMA (0,2,1) model was employed. This support comes from comparing the results of the ARIMA (0,2,1) model with those obtained by fitting 1st, 2nd, 3rd, and 4th degree polynomials to the data taken during calibration periods. In many of the quartz-crystal clock evaluations, the 2nd degree polynomial prediction gave smaller errors than the other polynomials, and in some cases smaller than the ARIMA (0,2,1) model. Since the ARIMA (0,2,1) model results in only a 1st degree polynomial, it would seem that improvement could be made if an ARIMA model resulting in a 2nd degree polynomial were used. With rubidium clocks, the difference in accuracy of prediction between 1st and 2nd degree fitted polynomials was considerably less than the corresponding difference with quartz-crystal oscillators, and the 1st degree fitted polynomial sometimes gave the best prediction. With cesium clocks, for longer prediction lead times, the 1st degree polynomial frequently outperformed the ARIMA (0,2,1) model. This would seem to indicate either the possibility of finding a better ARIMA model than the ARIMA (0,2,1) model or the possibility of making some other improvements such as a better value for θ_1. A good ARIMA model is supposed to produce a prediction with minimum mean square error, so it should not be possible to find a model which gives a lower mean square error in its prediction.

This preliminary study of the predictability of clocks indicated clock errors in high quality quartz-crystal oscillators can be predicted with considerable accuracy during a free-running period following a calibration period. There appears to be a very high probability that by using these predictions to remove errors during the free-running period, enough accuracy can be obtained over a long enough period of time to permit the use of these clocks as an alternative to much more expensive cesium clocks for many communications applications. Since the rubidium clocks are much more predictable than high-quality quartz-crystal clocks, there is an even higher probability that they could be used as an alternative to cesium clocks with correction of the predicted errors.

The digital time error measurements for this preliminary evaluation of the predictability of clocks were made once an hour. This was assumed to be frequent enough to produce satisfactory results, but no evaluations were made of the possibility of significant aliasing occurring of the type discussed in the preceding section. Further work is needed to determine whether there is significant aliasing when measurements are only made once an hour during the calibration period.

The preliminary study used a limited number of older clocks, and there were significant problems with some of the clocks which interrupted measurements. It is important to add newer clocks, using the latest technology, to the clocks which the observatory already has, and to take data on a larger number of clocks for longer periods of time in order to increase confidence in the statistical studies.

V. THE PROPOSED STUDY

In addition to discussing the knowledge to be gained from this study, this section also discusses an approach to getting the information. It points out some of the things that should be done to acquire the desired information economically in a form that will be useful to both planning and design engineers.

The measurements needed to determine the predictability of high quality quartz-crystal oscillators and other devices should be made relative to a highly accurate reference such as the Naval Observatory master clock, which is based upon an ensemble of a large number of highly accurate cesium clocks. To develop statistical confidence, the measurements should be made on a large number of clocks. At least part of these clocks should employ the most modern technology, and should be of a type that might be employed in the DCS. The measurements must be made over a relatively long period of time because of the statistical character of the information to be determined. The preliminary study by the Naval Observatory used clock error measurements made on a limited number of older clocks. A first major step for the proposed study program is to acquire some additional clocks that use the latest technology. Similarly, any additional test equipment needed to permit the automatic accumulation of measured errors of each clock relative to the Naval Observatory master clock should also be acquired. This equipment should provide for making measurements at frequent intervals.

In order to evaluate possible adverse effects due to aliasing when data is taken at hourly intervals, some data should be taken at intervals of 15 minutes, or more frequently. In order to reduce the effect of errors in making measurements of the clock errors, at least some of the measurements should be made at intervals of one minute or less. These error measurements could be averaged for a half hour or an hour before being applied to the ARIMA model. These more frequent measurements could also be used for a more extensive evaluation of any adverse effects due to aliasing. Another advantage of these more frequent measurements is that they also help to develop a procedure more directly related to practical application in the DCS. In the DCS, frequent measurements are relatively convenient to make, while the problem of making errors in the measurement of clock errors is much more serious than in a laboratory environment. It would be quite reasonable to make clock error measurements once a second and average them for several minutes before application in an ARIMA model. The data needs to be taken over a long period of time in order to acquire enough data from a limited number of clocks to develop confidence in the statistical results.

In addition to increasing the number of clocks to be measured and in addition to acquiring some test equipment to enhance the automatic accumulation of data, the measured data needs to be analyzed, and plans for the practical application of this relatively new technology to the DCS need to be developed.

Analysis of the measured data should concentrate on the application of ARIMA models. A first step in such an analysis should be to develop a near optimum ARIMA model for the data obtained from each individual clock. This step is needed so that some evaluation can be made of how much the optimum model varies from one clock to another. Do they have the same values for p, d, and q? Are the θ parameters nearly the same? Although some comparisons should be made between ARIMA (0,2,q) and ARIMA (0,3,q) models, discussions in a preceding section indicate that the ARIMA (0,3,q) model is expected to be a superior prediction model for both quartz-crystal and rubidium clocks, while the ARIMA (0,2,q) is expected to be superior for cesium clocks. It might also be desirable to make an evaluation of an ARIMA (1,2,q) model where one of the three zeros of the autoregressive part of the model is not on the unit circle. As part of the selection of the best model, the value of q needs to be evaluated. One step that can be taken in determining the value of q is to take the 3rd (or 2nd if an ARIMA (0,2,q) model is used) difference of the measured clock error for a particular clock. For this difference data, compute the autocorrelation function. For some significant offset value of the autocorrelation function, its value, i.e., the value of the autocorrelation function, should approach zero. The largest offset prior to the autocorrelation function approaching zero should correspond to the value of q in the ARIMA model.

Another method of evaluating the value of q for the model is to arbitrarily select a number which is equal to or greater than the expected optimum number, e.g., $q = 3$, as a starting point. Using this number for q, select the values of the θ's to provide an optimum fit to the data. If q is chosen too large, some of the values of the θ's, e.g., θ_2 and θ_3, should be very close to zero. The major problem with this approach is that the amount of effort required to find optimum values of the θ's increases tremendously as the number of different θ's increases. Finding the optimum value of θ when $q = 1$ is quite simple relative to finding the optimum combination of values for three different θ's. Since the a_t's represent the random unpredictable part of the ARIMA model, the θ parameters which produce the smallest root-mean-square (rms) value for the a_t's must be a very good approximation to the optimum values. If several values each of θ_1, θ_2, and θ_3 are used in an ARIMA model with the measured error data for a particular clock, and the rms values of the a_t's are computed for each set of values, this information can be plotted on graphs to obtain a good estimate of the optimum values of the θ's. For greater accuracy, an additional set of values near the first selected optimum can be used and the process repeated. Reference [15] describes an iterative approach for computing optimum values of the θ's, and this approach should be investigated for its possible application to this study.

Once an optimum ARIMA model has been selected for each individual clock, these optimum models should be compared to determine whether a single compromise ARIMA model for all clocks of a given type could be expected to provide near optimum results, i.e., one ARIMA model for all quartz-crystal clocks, another for rubidium clocks and another for all cesium clocks.

The prediction errors for all optimum models selected for individual clocks should be determined by comparing predicted values with actual measured data. To do this, a sequence of measured clock error data is used to perform the calibration process, i.e., evaluate the a_t's for use in determining initial conditions for the predictions. Following the last measured clock error datum used for calibration, prediction is started by assuming all future a_t's to be zero in the recursive process. The datum actually measured for each of the predicted values is subtracted from the predicted value to determine the error in the prediction. For each clock, this should be done for several sets of data taken at different times. It should also be done for a family of different lengths of calibration period. For at least a few clocks, the error in prediction should be plotted as a function of prediction time in order to provide additional insight as to the character of these errors, the unpredictable part of the clock errors. In addition to plotting individual error curves, many different sets of prediction errors for a particular clock should be combined into an rms evaluation of the prediction error for that clock as a function of prediction time.

It is desirable to perform the above evaluations using the optimum ARIMA model for each individual clock, and also using the compromise ARIMA model for each type of clock so that the amount of degradation resulting from the compromise can be evaluated.

When all of the above measurements and computations have been completed, enough information should be available to make estimates of the accuracy with which errors of a particular type of clock can be predicted. Knowing, with some degree of confidence, the accuracy with which clock errors for a particular type of clock can be predicted should be a very valuable tool for design engineers considering the use of such predictions in a practical application. The information is needed to determine whether the predictions can be made with sufficient accuracy over a long enough period of time to satisfy the needs of a particular application, and to evaluate any degradation that might result from the substitution of lower cost clocks for expensive ones.

Since no evaluations have been made to determine the effects of aliasing resulting from measuring clock errors once an hour, this new evaluation of clock error prediction should include determining whether such effects exist and if they do, their nature. One method of doing this is to make several sequences of error predictions using measured error data taken much more frequently than once an hour, then repeating each of these predictions using only a fraction of the measured data, e.g., every 5^{th} or every 10^{th} measurement. By comparing the results of these different sets of predictions, it should be possible to determine whether any degradation in the accuracy of prediction has occurred as a result of using the less frequent measurement data. By using several different rates of clock error measurements, i.e., by allowing different numbers of unused measured data points between those used for making predictions, it should be possible to evaluate any degradation due to aliasing as a function of the frequency with which clock error measurements are made.

Evaluations are also needed to determine the effect of perturbations in the measurement of clock error data. The perturbations might arise from errors produced by the measurement equipment, or as the result of the way the measurement equipment is used. In a communications network, some perturbation can be expected from the noise on the communications link used for clock comparisons. The perturbations due to transmission link noise can be expected to vary with the transmission medium employed. They are expected to be very, very small on fiber optics cable transmission links, but could develop to a significant value on over-the-horizon tropospheric scatter microwave links. Beyond determining how such perturbations affect the accuracy of the prediction results, it is desirable to evaluate methods of minimizing such degradation. As mentioned before, one method is to make very frequent measurements and to average a large number of them together to obtain each measurement data point used in the ARIMA model. In addition to such averaging, steps should be taken to increase the robustness of the prediction process by eliminating from the averages those measurements which are greatly displaced from the mean of the measurements, or by using some other method of robust statistical inference.

For applications of clock error prediction techniques in communications systems such as the DCS, it can be expected that it will usually be possible to have an extended calibration period prior to the free-running clock period during which the predictions would be used. This calibration period could usually be expected to last several months or even years. It might be possible to develop a relatively simple microprocessor algorithm to update and more precisely determine the optimum values of the θ's in the ARIMA model for the particular clock on which the predictions would be used. This might make it possible to obtain the most optimum model practicable for use at the specific time when the prediction is needed. This possibility should be investigated as a part of this program.

In addition to the evaluations of the predictability of the clocks as discussed above, investigations need to be made into the practical applications of these techniques in nodes of military communications networks, such as the DCS, to aid in enhancing the survivability of the timing function when the network must withstand massive destruction due to enemy action. Relatively simple algorithms that can be applied to simple reliable low cost microprocessors are needed.

Investigations should also be made into sharing the microprocessor used for applying the ARIMA prediction model to other purposes. Those other purposes might include the ability to assure stable and accurate timing from a node's local clock under all circumstances including a destructive attack upon the network. In order to make good predictions of clock errors during a free-running clock mode of operation, good error measurements must be available during a calibration period preceding the free-running period. This implies that measurement errors due to perturbations in the communications transmission medium should be kept small. To accomplish this, the local clock at each node controls the timing of all transmitted synchronization codes.

The arrival times of all received synchronization codes are measured relative to the local clock. The measurement includes the signal transit time and any difference between the two clocks. The measurement is communicated to the other end of the link. Subtracting the measurement made at one end of the link from that made at the other end and dividing by two gives the measured difference between the two clocks. If all nodes transmit their measured but uncorrected (known) errors to their neighbors, any node wishing to use a particular neighbor for time reference information can use this information from that neighbor, along with the measured difference between its own clock and that of the neighbor, to determine its own measured but uncorrected error. The microprocessor should be programmed to take part in this measurement procedure to assure that good measurement data are available for application in the ARIMA model.

During periods of normal operation when measured errors in the local clock can be used to control the output phase of the local clock, i.e., when it is unnecessary for the local clock to free-run, the clock error measurements need to be filtered before being used for clock error corrections. The microprocessor should be programmed to provide this filtering to assure stability unperturbed by the transmission media or changes in other clocks.

The most stable method of applying clock corrections, i.e., the method which least disturbs the basic stability of the clock, is to provide a phase correction in tandem with the output of the basic clock. Digital control of micro-phase-steppers is a most accurate and effective method of doing this. The local clock signal is normally taken to be the output of this micro-phase-stepper, while the stability of the local clock output is determined by the basic clock or oscillator preceding the micro-phase-stepper. The measurement of the error in the output of the micro-phase-stepper and the known amount of correction provided by it should be used to compute the error in the uncorrected local clock. This evaluation of the uncorrected local clock error is used during the normal periods of operation for calibration of the ARIMA model to be used for making predictions of errors during a free-running period to follow. It is also used for determining clock error corrections to be made by the micro-phase-stepper during normal operations. The microprocessor should be programmed to perform these functions.

During the normal mode of operation, phase error measurement information will be received from several different neighboring nodes. Therefore, the accuracy and stability of the measurement information can be enhanced by optimally combining this information received over several different paths. A method for doing this is discussed in references [4], [7], and [13]. The microprocessor should be programmed to perform the computations which provide this enhancement of measurement precision. When this method of combining the clock error measurement information from different communications paths is used, it also provides possibilities for quantitative timing system self-monitoring which can allow an alarm to be given far in advance of any actual degradation of the communications provided by the system. The microprocessor should also be programmed to provide this function.

In a military communications system subject to massive destruction, provision must be made to automatically reorganize the timing function so that a new network master is selected when necessary, and the transfer of timing information through the network is not degraded more than necessary. Methods for doing this are discussed in references [4], [7], and [13]. The microprocessor should provide this function also.

The microprocessor should also be programmed to minimize the effect of erroneous or perturbed timing error measurements made during normal periods of operation. Also during normal periods of operation, it should filter the measurement data prior to its use for controlling a micro-phase-stepper to correct clock errors. It should control the amount of correction applied by the micro-phase-stepper during the controlled mode of operation. It should also calibrate the clock prediction model during the controlled mode of operation. It should determine when it is necessary to change from the controlled mode of operation to the free-running mode of operation, and provide a smooth transition from the controlled mode to the free-running mode. During the free-running mode, it should predict the clock errors and control the micro-phase-stepper to remove the predicted errors. It should also determine when to return to the controlled mode, and should provide a smooth transition to the controlled mode. It should also provide an alarm capability to attract attention to any timing system faults which occur, and provide automatic diagnosis of the faults for maintenance personnel.

Once a capability to use one or more microprocessors, along with required supporting equipment (such as time interval counters and micro-phase-steppers) to perform all of the desired timing system functions has been demonstrated, a comprehensive report will be needed. This report should present all of the information related to measuring clock errors during normal operation, predicting clock errors and removing them during free-running operation, and otherwise enhancing the performance of the timing system while improving its survivability and lowering its costs. To the extent possible, the report should be written in engineering terms, providing charts and tables with basic parameters and relationships; and it should provide adequate guidance on their use to make it a good tool for both the planning and system design engineers.

VI. CONCLUSIONS AND RECOMMENDATIONS

The survival of the communications function of a military synchronous switched digital network subjected to massive enemy destruction of parts of the network is dependent on the survivability of the timing function. The timing function survivability can be greatly enhanced by providing a backup free-running clock mode of operation at each node of the network. For higher data rates such as those expected between major nodes of the future DCS, the usefulness of a clock in the free-running mode is highly dependent on its stability and accuracy during that free-running period. This clock accuracy also aids the rapid resynchronization of spread spectrum equipment, cryptographic equipment, and other synchronous devices when a node reenters the network after temporarily being isolated from the network. Techniques are available for accurately measuring errors in local clocks during a normal controlled mode of timing system operation. These measurements can be used to calibrate a mathematical model of the error performance of the free-running clock. During any contingency requiring a free-running clock following enemy destruction of parts of the network or following random equipment failures, these predicted errors can be removed, thereby greatly enhancing the timing accuracy.

There is a very high probability that high quality quartz-crystal or rubidium clocks could be used as a much lower cost alternative to expensive cesium beam clocks in many DCS applications when clock error prediction and correction techniques are employed. This could result in lower costs, and enhancement of the short term accuracy. In those cases where quartz-crystal or rubidium clocks cannot be used satisfactorily because their errors cannot be predicted with sufficient accuracy for an adequately long period of time, the same techniques can be used to enhance the accuracy of cesium clocks.

An Autoregressive Integrated Moving Average (ARIMA) model is a good tool to use for these predictions. When properly applied, it should produce a minimum mean square error prediction of the clock errors. It has the advantage of permitting recursive calculation to be used during both the calibration period and the prediction period, and there is some possibility of developing automatic methods for model parameter optimization during lengthy calibration periods.

Although these prediction techniques are extremely promising for a timing system for a digital DCS, considerable information is required before they can be applied most effectively. It is recommended that the general program for acquiring the information described in this document be initiated and carried to a meaningful completion.

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